

## **Short Run Bond Risk Premia**

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# Short-Run Bond Risk Premia\*

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## Abstract

In the short-run, bond risk premia exhibit pronounced spikes around major economic and financial crises. In contrast, long-term bond risk premia feature cyclical swings. We empirically examine the predictability of the market variance risk premium—a proxy of economic uncertainty—for bond risk premia and we show the strong predictive power for the one month horizon that almost entirely disappears for horizons above one year. The variance risk premium is largely orthogonal to well-established bond return predictors—forward rates, jumps, yield curve factors, and macro variables. We rationalize our empirical findings in an equilibrium model of uncertainty about consumption and inflation which is coupled with recursive preferences. We show that the model can quantitatively explain the levels of bond and variance risk premia as well as the predictive power of the variance risk premium while jointly matching salient features of other asset prices.

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## Abstract

In the short-run, bond risk premia exhibit pronounced spikes around major economic and financial crises. In contrast, long-term bond risk premia feature cyclical swings. We empirically examine the predictability of the market variance risk premium—a proxy of economic uncertainty—for bond risk premia and we show the strong predictive power for the one month horizon that almost entirely disappears for horizons above one year. The variance risk premium is largely orthogonal to well-established bond return predictors—forward rates, jumps, yield curve factors, and macro variables. We rationalize our empirical findings in an equilibrium model of uncertainty about consumption and inflation which is coupled with recursive preferences. We show that the model can quantitatively explain the levels of bond and variance risk premia as well as the predictive power of the variance risk premium while jointly matching salient features of other asset prices.

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# 1 Introduction

The failure of the expectations hypothesis of the term structure of interest rates, first documented in Fama and Bliss (1987) and Campbell and Shiller (1991), has received unprecedented attention in both the empirical and theoretical academic literature over the past 20 years. In this paper, we first document the large and significant predictive power of the variance risk premium, defined as the difference between the risk-neutral and statistical expectations of realized variance, for bond risk premia at short horizons especially one-month. This short-run forecastability is orthogonal to the well documented long horizon predictability from forward rates (Cochrane and Piazzesi, 2005), macro variables (Ludvigson and Ng, 2009), a hidden factor (Duffee, 2011), and jump risk (Wright and Zhou, 2009).<sup>1</sup> We then posit an economy with time-varying uncertainty risk about real and nominal quantities coupled with agents' preferences for an early resolution of uncertainty and show that these ingredients are enough to quantitatively explain the violation of the expectation hypothesis while matching the moments of the variance risk premium, the equity premium, and risk-free rate.

To capture this short-run uncertainty component of bond risk premia, we rely on the market variance risk premium—or the difference between risk-neutral and objective expectations of the return variation. Following the path of previous work, we proxy the risk neutral expected variance by the popular VIX<sup>2</sup> index, which is termed as “market gauge of fear” (Whaley, 2000). With high frequency intraday data of futures on the S&P 500, we use heterogeneous autoregressive models of realized variance (HAR-RV model, see Corsi, 2009) augmented by lags of implied variances (Drechsler and Yaron, 2011) for estimating the objective expectation of variance risk. Our average variance risk premium is 16.22 (percentage squared monthly basis), well within the typical range of recent empirical estimates. More importantly, our time-series of variance risk premium always remains positive, which makes it a natural candidate measure for economic uncertainty or even stochastic risk aversion.

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<sup>1</sup>Cieslak and Povala (2010) decompose long-term yields into a persistent component and cycles and find that the cyclical component is a strong predictor encompassing several other ones. Huang and Shi (2010) construct a single macro factor using a group lasso method and show that this factor almost doubles the  $R^2$  compared to Ludvigson and Ng (2009).

When using the variance risk premium to forecast bond excess returns, we find that for 2 to 6 month short term Treasury bills with 1 to 5 month holding periods, the estimated coefficients for the variance risk premium are statistically significant. Adding the variance risk premium to a regression with forward rates increases the  $R^2$  by 6% for the 1 month holding period for example. For longer holding periods, the increase in  $R^2$  shrinks to 2%. Moreover, the variance risk premium only has negligible forecasting power for 2 to 5 year Treasury bonds with a 1 year holding period. We show that the short-run forecasting power is robust to the inclusion of other well established bond risk premium predictors such as forward rates, macro variables, a hidden factor, a cyclical component, and jump risk. While these variables have previously been shown to predict bond risk premia for longer maturities, they change little in the significance of the variance risk premium at shorter horizons and in some cases even have zero predictive power.

The intuition for our empirical result becomes more evident when we look at the time series of short term bond risk premia. Bond risk premia at short horizons exhibit pronounced spikes around major economic and financial crises. This pattern is distinctly different from the cyclical swings with a length of up to several years typically observed in long term bond risk premia (see Fama and Bliss, 1987 and Cochrane and Piazzesi, 2005). Interestingly, the variance risk premium exhibits a similar time-series behavior as short-term bonds: It rises sharply before economic or financial crises and then drops again. On the other hand, standard predictors like the CP factor display a strong cyclical behavior (see Koijen, Lustig, Van Nieuwerburgh, 2010). The upshot is that short-term variation in bond risk premia are related to economic uncertainty which are short-lived (see Bloom, 2009) rather than a business cycle component which is more apparent in bond risk premia of longer maturities.

We propose a potential explanation for this short-run predictability in an economy with time-varying economic uncertainty about real and nominal quantities, extending the real uncertainty model of Bollerslev, Tauchen, and Zhou (2009).<sup>2</sup> In an economy with stochastic inflation volatility but with only exogenous shocks, money neutrality holds and there

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<sup>2</sup>Wu (2008), Hasseltoft (2010), and Doh (2010) study the long-run risk models for term structure with both real and nominal uncertainty. However, they also rely on the small persistent growth component and they do not examine predictability of the variance risk premium.

is no inflation risk premium except for the standard Jensen’s inequality term (see Zhou, 2010). In this model with endogenous inflation shocks, we derive a genuine inflation risk premium through two channels. First, we introduce an endogenous stochastic volatility process through the consumption growth channel. Second, we let the stochastic volatility process be correlated with the consumption uncertainty channel. While the equilibrium model developed in this paper is related to the long run risk model of Bansal and Shaliastovich (2010), we explicitly abstain from modeling the small persistent component in consumption growth and inflation as done in their setup. In our model, we allow the volatility of volatility of both inflation and consumption—or the economic uncertainty about these quantities—to speak by themselves on how far the model can go to accommodate the observed level and predictability in bond risk premia.

The key to matching the bond risk premium dynamics is through the calibration of the inflation process, while leaving the choices of preference parameters and real economy dynamics similar to existing studies (see, e.g., Bansal and Yaron, 2004; Bollerslev, Tauchen, and Zhou, 2009). Our calibration exercise shows that an autonomous inflation process, with or without stochastic volatility is not able to replicate the size of the bond risk premium. Combining both a consumption growth channel and a uncertainty channel of non-neutral inflation dynamics, leads to reasonable and rich bond risk premia. Indeed, our calibrated numbers are only several basis points away from their empirical counterparts. We also show that the model produces a reasonable equity premium and risk free rate but overshoots the risk free rate volatility. While the higher order moments (kurtosis and skewness) of the variance risk premium are fitted quite well, the average variance risk premium produced by our model is slightly smaller than its empirical estimate. Finally, the predictive power of the equity variance risk premium for bond risk premia is fitted remarkably well by our preferred inflation uncertainty model.

Previous work has attempted to explain the failure of the expectations hypothesis through the growth channel of consumption, e.g. Wachter (2006) (external habit), Bansal and Shaliastovich (2010) (long-run risk), Gabaix (2009) (rare disasters), Xiong and Yan (2010) (heterogeneous expectations), and Vayanos and Vila (2009) (preferred habitat). We argue that

adding the inflation uncertainty component can go a long way to fit salient features of asset prices and bond risk premia in particular.<sup>3</sup> We also contribute to the growing macroeconomic literature that emphasizes the quantitative importance of time-varying volatility in real and nominal variables to understand the source of aggregate fluctuations, the evolution of the economy, and policy analysis (see, e.g., Fernández-Villaverde and Rubio-Ramírez, 2010). Similarly, Bloom (2009) and Bloom, Floetotto, and Jaimovich (2010) show that higher economic uncertainty, proxied by the VIX, decreases employment and output in near terms. Our empirical finding and modeling approach are broadly consistent with the macroeconomic uncertainty framework driven by real and nominal volatility dynamics.

The rest of the paper is organized as follows. Section 2 describes our data set and the methods used to estimate the variance risk premium and provides empirical finding for the bond return predictability of the variance risk premium. Section 3 presents a structural model of inflation uncertainty with calibration evidence for risk premium dynamics. Section 4 concludes.

## 2 Empirical Analysis

In this section, we first discuss the data we use in our empirical analysis—short term bills and long term bonds for calculating excess returns, macroeconomic financial variables for replicating established return predictors, and high-frequency S&P500 futures and VIX index for measuring realized and implied variances. Our variance risk premium estimate uses a heterogeneous autoregressive forecasting equation of realized variances augmented by multiple lags of implied variances.<sup>4</sup> We then present evidence for the predictive power of the variance risk premium for bond risk premia at the short and the long horizons. Here we first run a set of univariate regressions using the variance risk premium as the sole predictor variable and then control for other well established predictors. We estimate the coefficients

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<sup>3</sup>Papers that study the impact of frictions in bond markets on bond returns include Greenwood and Vayanos (2010) (bond supply) and Fontaine and Garcia (2010) (liquidity premium). Buraschi and Whelan (2010) study the impact of dispersion in forecasts on economic quantities on bond returns and estimate highly significant coefficients.

<sup>4</sup>Additional information on data construction is deferred to a separate Appendix.

using least squares and compute standard errors following Newey and West (1987) and Wei and Wright (2010). The latter approach represents an extension of the well-known reverse regression approach advocated by Hodrick (1992).

## 2.1 Data Description and Variance Risk Premium

Our data runs from January 1990 to September 2010. We use a monthly frequency throughout this paper and thus have 249 observations available.

### Treasury Data:

We use the Fama and Bliss discount bond database from CRSP to compute yields, returns, and forward rates for two to five year bonds. Yields and returns are computed in logs. Yield spreads and excess returns are constructed relative to the one year bond. We denote by  $r_{t+1}^{(\tau)} = p_{t+1}^{(\tau-1)} - p_t^{(\tau)}$ , the return on a  $\tau$  year bond with log price  $p_t^{(\tau)}$ . The excess bond return is defined as:

$$rx_{t+1}^{(\tau)} \equiv r_{t+1}^{(\tau)} - y_t^{(1)},$$

where  $y_t^{(1)}$  is the one year yield. Similarly, we use the Fama Treasury Bill Structures from CRSP to compute yields, returns and forward rates for two to six month T-bills with holding periods one to five month respectively. From the Fama and Bliss discount bond data, we also construct a tent-shaped factor from forward rates, the Cochrane and Piazzesi (2005) factor, CP. Wright and Zhou (2009) document the strong predictive power of the mean jump size for bond risk premia, and accordingly, we measure the 24-month rolling realized jump mean,  $\tilde{J}$ , using five minute frequency data on the 30 year Treasury bond futures, under the assumption that jumps are rare and large. Duffee (2011) estimates a five factor Gaussian model using Treasury yields and extracts a latent factor,  $\tilde{H}$ , that is “hidden” from—or weakly spanned by—the cross-section of yields but has bearing on excess bond returns. Cieslak and Povala (2010) decompose bond yields into a persistent and cyclical components where the persistent component is proxied by a discounted moving average of past core inflation. Similarly, we construct a cyclical term premium factor by first running a contemporaneous regression from yields on the persistent component and then running a predictive regression from bond excess



returns on the fitted residuals from the first regression. The single predictive factor,  $\widehat{cf}$ , is then the fitted value from the second regression.

### **Implied Variance Data:**

As the risk-neutral expectation of return variance for the next 30 days, we use the squared VIX, which is the option-implied variance of the S&P 500 index. We use end-of-month data from the Chicago Board of Options Exchange (CBOE).<sup>5</sup>

### **Stock Index Data:**

To calculate the objective expectation we use intra-day data for S&P 500 futures sampled at the 5 minute interval. Alternatively, we use intra-day cash index data for the S&P 500 index as in Bollerslev, Tauchen, and Zhou (2009). All intra-day data are obtained from Tickdata.

### **Macroeconomic Data:**

We compute the eight static macroeconomic factors  $\widehat{F}_j, j = 1 \dots, 8$ , following Ludvigson and Ng (2009, 2010). We update the time series and we exclude the stock market and interest rate time series in order to have pure macro factors.<sup>6</sup> The macroeconomic data are mainly from Global Insight.<sup>7</sup>

Summary statistics for the bond returns are in Table 1, Panel A, and summary statistics for the macro variables in Panel B. The mean bond returns are increasing with maturity and the numbers are in line with previous studies. While long term bond returns—2 to 5 year bonds for a 1 year holding period—are highly persistent, possibly due to the overlapping return horizon of 11 months, the autocorrelation function for the 1 month holding period returns of the Treasury bills drops much faster with values of first order autocorrelations ranging from 0.23 to 0.55. The CP factor, the mean jump size, the cycle factor, and the hidden factor are all highly persistent with first order autocorrelation coefficients ranging

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<sup>5</sup>The squared VIX is a model-free implied variance (see also Demeterfi, Derman, Kamal, and Zou, 1999; Britten-Jones and Neuberger, 2000). The VIX White Paper (CBOE, 2003) outlines the calculation procedure.

<sup>6</sup>The original data set was previously used in Stock and Watson (2002). The stock market and interest rate time series we exclude are the Ludvigson and Ng (2009) series 82 through 102. In addition, we have to exclude seven variables that are no longer available after 2007. Consequently, we use 104 instead of 132 macroeconomic time series. For a shorter sample period ending in 2007 we use the original factors from Ludvigson and Ng (2010) as a robustness check. Our main results remain unchanged. We defer a more detailed description of the data to a separate Appendix.

<sup>7</sup>In addition, three series are from the BEA and one is from the University of Michigan.

from 0.85 to 0.96. The macro factors,  $\widehat{F}_j$  display much lower autocorrelations, with some components being even negative.

[Insert Table 1 and 2 approximately here.]

Table 2 reports the unconditional correlation among all the predictor variables including variance risk premium. Unsurprisingly, the three factors which are calculated using yields or bond return information, CP,  $\widetilde{H}$  and  $\widehat{cf}$  are quite highly correlated with absolute correlations ranging from 0.32 to 0.72. The variance risk premium is not very highly correlated with the other factors except for the mean jump size,  $\widetilde{J}$  ( $-0.33$ ), and the first macro component,  $\widehat{F}_1$  (0.29), which Ludvigson and Ng (2009) label as the real factor due to its high correlation with measures of real output and employment. This echoes the finding in Bollerslev and Zhou (2007) that the variance risk premium may be intimately related to economic fundamentals in terms of the uncertainty shocks.

### 2.1.1 Forecasting Realized Variance and the Variance Risk Premium

To estimate realized variance, we use tick data from S&P 500 futures, which is one of the most heavily traded assets on the Chicago Mercantile Exchange (CME). Let  $RV_{t,\tau}$  be the realized variance from day  $t - \tau$  to day  $t$ , with  $\tau$  being typically a month or equivalently 22 days. To estimate the objective expectation of return variation of the next period  $\mathbb{E}_t^{\mathbb{P}}(RV_{t+\tau,\tau})$ , we first consider the realized variance  $RV_t$  at day  $t$ , which is defined as:

$$RV_t = \sum_{i=1}^M r_{t,i}^2, \quad (1)$$

where  $r_{t,i} = \log P(t - 1 + \frac{i}{M}) - \log P(t - 1 + \frac{i-1}{M})$  is the intra-daily log return in the  $i^{th}$  sub-interval of day  $t$  and  $P(t - 1 + i/M)$  is the asset price at time  $t - 1 + i/M$ . For each day, we take  $r_{t,i}$  between 9:30 and 16:00 at every five minute interval to calculate  $RV_t$ . The normalized monthly realized variation  $RV_{t,\text{mon}}$  is defined by the average of the 22 daily measures,  $RV_{t,\text{mon}} = \frac{1}{22} \sum_{j=0}^{21} RV_{t-j}$ . The normalized weekly realized variation  $RV_{t,\text{week}}$  is correspondingly defined by the average of the five daily measures,  $RV_{t,\text{week}} = \frac{1}{5} \sum_{j=0}^4 RV_{t-j}$ .

To better capture the long memory behavior of volatility, we use the daily, weekly and monthly realized variance estimates to estimate the heterogeneous autoregressive model of realized volatility (HAR-RV) proposed by Corsi (2009). HAR-RV estimators have become increasingly popular in the financial econometrics literature in the past years (see Corsi, Pirino, and Renò, 2010, Bollerslev and Todorov, 2011 and Patton and Sheppard, 2011). HAR-RV is a parsimonious version of high-order auto-regressions. We augment the HAR monthly forecasting model with additional lags of implied variance:

$$RV_{t+22,\text{mon}} = \alpha + \beta_D RV_t + \beta_W RV_{t,\text{week}} + \beta_M RV_{t,\text{mon}} + \sum_{i=1}^k \beta_{V,i} VIX_{t-i}^2 + \epsilon_{t+22,\text{mon}}, \quad (2)$$

where  $VIX_t^2$  is the square of the daily VIX index divided by  $12 \times 10^4 \times 30$  to be comparable to  $RV_{t,\text{mon}}$ . Equation 2 is motivated by the large literature in derivatives pricing showing that implied variance is a more efficient forecast for future realized variance than its own lag (Jiang and Tian, 2005) and extends the forecasting model of Drechsler and Yaron (2011) that uses one lag realized variance and one lag implied variance.

Estimated coefficients of equation (2) are presented in Table 3. As we can see, adding  $VIX^2$  to the regression, increases the adjusted  $R^2$  by 2% (columns 1 and 2). When we include more lags of the  $VIX^2$ , there is little gain in terms of  $R^2$ . To test more formally for the optimal number of lags in equation (2), we apply the standard information criteria. When restricting the maximum lag to 8, the optimal numbers of lags chosen by the AIC and BIC are 4 and 3, respectively (see Figure 1). Using the specification with 4 lags, we calculate the predicted values which serve as our estimate for our realized variance,  $\mathbb{E}_t^{\mathbb{P}}(RV_{t+22,\text{mon}}) = 22 \times \widehat{RV}_{t+22,\text{mon}}$ ; while the realized variance  $RV_{t,\text{mon}}$  always refers to the aggregated monthly measure.<sup>8</sup>

[Insert Table 3 and Figure 1 approximately here.]

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<sup>8</sup>The unconditional correlation between all the different forecasting specifications is very high, ranging from 0.94 to 0.99.

The variance risk premium is formally defined as the difference between the expected future variation under the risk-neutral and actual probability measures between day  $t$  and  $T$ :

$$VRP_{t,\tau} \equiv \mathbb{E}_t^{\mathbb{Q}}(RV_{t+\tau,\tau}) - \mathbb{E}_t^{\mathbb{P}}(RV_{t+\tau,\tau}), \quad (3)$$

where  $\tau = T - t$  denotes the time horizon which typically is a month or 22 trading days.<sup>9</sup>

To proxy for the risk-neutral variance, we take the VIX squared of the S&P 500 index with a one month (22 trading days) horizon, using a model-free approach. Under some regularity assumptions and even if the underlying asset follows a general jump diffusion (see Jiang and Tian, 2005 and Carr and Wu, 2009), this risk-neutral expected variance can be computed by as a portfolio of European calls on the underlying. We plot the  $VIX^2$ , the expected realized variance, together with the variance risk premium in Figure 2 and summary statistics are reported in Table 1, Panel C.

**[Insert Figure 2 approximately here.]**

The figure reveals that most of the peaks in the variance risk premium occur at financial events such as the LTCM default in August 1998, the burst of the dot com bubble in 2000, and the most recent financial crisis in the late 2008.

Looking at the summary statistics in Table 1, Panel C, there are several interesting points to note. First, the average variance risk premium (last column,  $VRP_{\text{HAR}}$ ) is around 16% which is comparable to the numbers found previously in the literature (see, e.g., Drechsler and Yaron, 2011; Bekaert and Engstrom, 2009; and Bekaert, Hoerova, and Lo Duca, 2010). Second, contrary to the previous literature, our measure never turns negative, which is partly driven by our HAR-RV forecasting model which is augmented by 4 lags of implied variance. This is important not only from an empirical point of view but also given the theoretical underpinnings—the variance risk premium is usually interpreted as an insurance premium for investors who pay for an asset whose payoff is high when return variation is large. Third, the first and second order autocorrelation coefficients of our variance risk premium are 0.85 and 0.70, which alleviates econometric concerns of regressing on highly persistent variables.

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<sup>9</sup>For notational simplicity, we subsequently drop the subscript  $\tau$  as we always consider the one month horizon variance risk premium  $VRP_t$ .

## 2.2 Short and Long Horizon Bond Return Predictability

Predictive regressions face two challenges: (i) predictors are highly persistent and (ii) the regressand is an over-lapping sum of short-term returns. Ang and Bekaert (2007) show that for longer overlapping horizons with highly persistent regressors, the Newey and West (1987) statistics tend to over-reject the null of no predictability. Bollerslev, Marrone, Xu, and Zhou (2011) find that, for shorter overlapping horizons with less persistent regressors, Hodrick (1992) standard errors have less power in detecting the alternative of predictability. Since our bond return predictability exercise involves both short and long overlapping horizons and includes both persistent and non-persistent predictors, we report both the more robust Hodrick t-statistics—the reverse regression approach extended to the multivariate case by Wei and Wright (2010)—and the more powerful Newey-West t-statistics. These two test statistics are valid under both the null of no predictability and the alternative of predictability.

### 2.2.1 Bond Risk Premia Predictability from the Variance Risk Premium

We run the following regressions for U.S. Treasury bills:

$$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)}(h) + \beta_1^{(\tau)}(h)VRP_t + \epsilon_{t+h}^{(\tau)},$$

where  $rx^{(\tau)}$  are either the excess returns on Treasury bills,  $h = 1, 2, 3, 4, 5$  and  $\tau = 2, 3, 4, 5, 6$  months or the excess returns on Treasury bonds,  $h = 12$  and  $\tau = 24, 36, 48, 60$  months.  $VRP_t$  is the market variance risk premium described earlier. The regression results are presented in Table 4, Panel A and B. Both the Newey and West (1987) and Wei and Wright (2010) standard errors deliver the very similar results.<sup>10</sup>

[Insert Table 4 approximately here.]

For Treasury bills with a 1 month holding period in Panel A, the variance risk premium is statistically significant, with t-values ranging from 1.97 to 2.37 for the Wei and Wright (WW)

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<sup>10</sup>We thank Min Wei and Jonathan Wright for sharing code with us.

standard errors and between 1.78 and 2.08 for the Newey and West (NW) standard errors. As we increase the holding period, we see a slight decrease in the t-statistics, however, the market variance risk premium still remains significant for all holding periods and all different maturities. The average  $R^2$  is around 2% in all regressions with slightly higher numbers for the 1 month holding period. When we compare the  $R^2$  with the  $R^2$  from a regression using the CP factor only, we find that when we add the variance risk premium to the regression, the average increase is around 6% for the 1 month maturity and drops to around 2% for longer holding periods. In summary, the variance risk premium does seem to possess non-trivial forecasting power for short-term Treasury bill excess returns.

In contrast, for Treasury bonds with a 1 year holding period in Panel B, the variance risk premium has little predictive power with t-statistics ranging from 0.83 to 1.88 and  $R^2$ s near zero. Importantly, however, we find that both for short and long term bonds, the estimated slope coefficients of the variance risk premium are always positive, which is in line with the intuition that higher risk is compensated with a higher return on the underlying asset.

In essence, we find that the variance risk premium predicts well bond risk premia with short maturities (at short horizons) but has little predictive power for long maturity bonds (at long horizons). To further test this preliminary empirical regularity, we construct one month holding period returns for longer maturity bonds using the dataset of Gürkaynak, Sack, and Wright (2006, GSW dataset), which has maturities up to 30 years. We also construct annual returns using the same database. Using these risk premia, we then run the same univariate regressions from the 1 and 12 month bond excess returns on the variance risk premium. The estimated slope coefficients are summarized in Figure 3. We compute standardized coefficients, meaning, all variables have mean 0 and a standard deviation of 1 to make coefficients comparable in terms of the economic significance as well as the statistical significance summarized by the 95 percent confidence band.

**[Insert Figure 3 approximately here.]**

The upper left panel of Figure 3 summarizes the results from Table 4 Panel A: The variance risk premium is a good predictor of short maturity bonds, in fact, the economic

significance is comparable to the CP factor and is even stronger for the longer maturities for which the CP factor has marginal predictive power. The upper right panel exhibits estimated coefficients for longer maturity bonds up to 15 years in the GSW dataset, but with a 1 month holding period returns. Interestingly, the variance risk premium becomes more and more significant and remains positive in predicting the bond risk premia—the longer the maturity, the higher the economic significance. The CP factor on the other hand, although significant, turns negative in forecasting 1 month bond returns, which continues the trend observed in the top-left panel. We repeat the same exercises for 12 month holding periods with bond maturities up to 15 years in the lower two panels. These plots reinforce the findings in Table 4 Panel B: The variance risk premium does not predict bond risk premia with holding periods of 12 month except for the 2 and 3 year maturity bond. However, the CP factor becomes highly significant and remains positive for these long term bond risk premia.

These findings are important because the market variance risk premium may capture a unique component of bond risk premia that is relevant for the short horizon and is driven by economic uncertainty shocks but that is at the same time orthogonal to the long horizon component captured by established predictors. To further justify our empirical conjecture, we next turn to a host of robustness checks involving well established bond return predictors.

### **2.3 Controlling for Other Bond Return Predictors**

A large literature has been devoted to studying different factors that predict bond risk premia at long horizons like 1 year, and it is natural to ask the question whether the predictive power of the variance risk premium is captured by those predictor variables. Cochrane and Piazzesi (2005) find that a linear combination of forward rates is the most powerful predictor for long-term bond returns. Wright and Zhou (2009) show that the mean jump size explains a significant fraction of the variation in long term bond excess returns and doubles the adjusted  $R^2$  when combined with the CP factor. Duffee (2011) estimates a latent factor from a five factor Gaussian model which has predictive power for bond excess returns but is not (or only weakly) spanned by the cross-section of yields. Ludvigson and Ng (2009) include macro factors extracted from a large set of macro variables using principal components analysis to

explain a highly significant fraction of the time variation in bond excess returns. Cieslak and Povala (2010) find high  $R^2$ s when running predictive regressions from bond excess returns on cycles which represent deviations from the long-run relationship between yields and the slow-moving component of inflation and savings. Our extended regression is of the following form:

$$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)}(h) + \beta_1^{(\tau)}(h)\text{VRP}_t + \beta_2^{(\tau)}(h)\text{CP}_t + \beta_3^{(\tau)}(h)\tilde{J}_t + \beta_4^{(\tau)}(h)\tilde{H}_t + \beta_5^{(\tau)}(h)\widehat{cf}_t + \sum_{j=1}^8 \beta_{5+j}^{(\tau)}(h)\widehat{F}_{j,t} + \epsilon_{t+h}^{(\tau)},$$

which is the univariate regression augmented by the Cochrane-Piazzesi factor, CP, the mean jump size,  $\tilde{J}$ , the Duffee hidden factor,  $\tilde{H}$ , the cycle factor,  $\widehat{cf}$ , and the Ludvigson and Ng macro factors,  $\widehat{F}_j$ .

**[Insert Tables 5, 6, 7, and 8 approximately here.]**

Tables 5 to 8 present the estimated coefficients for the 2 to 6 month Treasury bills for a 1, 2, and 3 month holding period.<sup>11</sup> Again, standard errors are calculated using both Newey and West (1987) and Wei and Wright (2010). Most importantly, we note that the variance risk premium is statistically significant across all different specifications for the 1 month holding period. The coefficients remain remarkably stable independent of the variables added to the regression. The statistical power becomes lower at the 2 month holding period and nearly disappears at the 3 month holding period. For the 1 month holding period, adding other predictors does not change the significance of the variance risk premium very much, but the hidden factor and the macro variables seem to drive out the effect of variance risk premium for longer horizons of 2 and 3 months. The striking short-run predictability of the variance risk premium for bond returns is similar to the one reported by Zhou (2010) and mirrors the findings for stock returns by Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011).

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<sup>11</sup>For longer holding periods (4 and 5 months), the variance risk premium has almost zero predictive power in the presence of other control variables. To save space, we defer these results to a separate Appendix.



As for the other predictors, the estimated coefficients for the CP factor are highly significant for all holding periods except when adding the hidden factor and the macro variables to the regression. The mean jump size is in general not significant at all for predicting short term Treasury bill returns. The estimated coefficient for the cycle factor seems to lose power when adding the macro factors to the regression but is highly significant for the shortest holding periods. Overall, the hidden factor is statistically significant in all regressions and the estimated coefficients remain remarkably stable across all different holding periods and different specifications. It is interesting to note, however, that the hidden factor seems to crowd out both the CP and  $\widehat{cf}$  factor at short horizons, which may not be surprising since  $\tilde{H}$ , CP, and  $\widehat{cf}$  are all estimated from bond yields. Out of the eight macro factors, only  $\widehat{F}_3$  is statistically highly significant and stable at short holding periods. As we increase the holding period however, the macro factors become more important.

**[Insert Table 9 approximately here.]**

These results naturally lead us to look more carefully at long term bond excess returns and the associated predictive power of the variance risk premium over yearly horizons. As shown in Table 9, the variance risk premium is significant at a one percent level for 2, 3 and 4 year bonds, but becomes less significant for longer maturity bonds. While the significance does not seem to be affected by adding the CP factor to the regression, we see a decline in the significance when adding the mean jump size and the hidden factor. We also note that adding the mean jump size almost doubles the adjusted  $R^2$  from on average 20% to nearly 40%. The macro and the hidden factors seem to crowd out the significance of both the variance risk premium and CP factor, leaving only the jump size and cycle factor as powerful predictors for long term bond excess returns. In other words, the variance risk premium does not seem to have predictive power for long term bond returns, once controlling for other predictors which are known to predict long maturity bond returns.

**[Insert Figure 4 approximately here.]**

The contrast between short and long term bond risk premia is best seen from Figure 4. It is clear that short term bond risk premia (top panel) have large spikes around major financial

crises and economic recessions, but these shocks are generally short lived—uncertainty comes and goes. On the other hand, the long term bond risk premia (bottom panel) seem to have gradual persistent swings at least several years apart and sometimes even as long as the business cycle frequency—like the 2001–2008 cycle. Put together, we have a whole picture of bond risk premia responding both slowly to long term cyclical risk and quickly to short term uncertainty shocks.

In summary, we find that the stock market variance risk premium is a robust predictor of bond returns at the short end, but the predictive power becomes weaker at longer maturities and vanishes to zero for very long maturity bonds, as also previously reported in Baele, Bekaert, and Inghelbrecht (2010). The short term predictive power of the variance risk premium is also robust to the inclusion of other standard predictors such as the CP factor, the hidden factor, the jump mean, the cyclical factor, and macro variables.

### **3 Economic Uncertainty and Inflation Dynamics**

To understand why the variance risk premium has significant predictive power for short-run bond risk premia, we present a stylized structural model where the real bond risk premium is present because of agents' preference for an early resolution of uncertainty and the nominal bond risk premia is non-redundant because the inflation process co-varies with both cash flow and uncertainty shocks. The nominal bond risk premium in our economy works only through the conditional volatility channel and does not rely on the conditional mean channel (as in Pennacchi, 1991; Sun, 1992). As such, our model may be viewed as an extension of the consumption uncertainty model by Bollerslev, Tauchen, and Zhou (2009).

Our calibration result suggests that the proposed inflation uncertainty model not only has the capability to replicate the predictability pattern of the variance risk premium for bond risk premia documented in recent research, but also matches the level of bond risk premia typically hard to pin down in structural economic models. The volatility or uncertainty channel to resolve the Expectations Hypothesis puzzles is in contrast with those relying

on the consumption growth channel—e.g., habit formation (Wachter, 2006), long-run risk (Bansal and Shaliastovich, 2009), or rare disasters (Gabaix, 2009).

### 3.1 Economic Uncertainty and Variance Risk Premia

The representative agent in the economy has Epstein-Zin-Weil recursive preference and has the value function  $V_t$  of her life-time utility given as:

$$V_t = \left[ (1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t [V_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \quad (4)$$

where  $C_t$  is consumption at time  $t$ ,  $\delta$  denotes the subjective discount factor,  $\gamma$  refers to the coefficient of risk aversion,  $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ , and  $\psi$  equals the intertemporal elasticity of substitution (IES). The key assumption that  $\psi > 1$  hence  $\theta < 0$  implies that agents prefer an earlier resolution of economic uncertainty, such that the uncertainty or volatility risk in asset markets carries a *positive* risk premium.

The log consumption growth and its volatility follow the joint dynamics:

$$g_{t+1} = \mu_g + \sigma_{g,t} z_{g,t+1}, \quad (5)$$

$$\sigma_{g,t+1}^2 = a_\sigma + \rho_\sigma \sigma_{g,t}^2 + \sqrt{q_t} z_{\sigma,t+1}, \quad (6)$$

$$q_{t+1} = a_q + \rho_q q_t + \varphi_q \sqrt{q_t} z_{q,t+1}, \quad (7)$$

where  $\mu_g > 0$  denotes the constant mean growth rate.<sup>12</sup> The time-variation in  $\sigma_{g,t+1}^2$  is one of the two components that drives the equity risk premium, or the “consumption risk”; while the time-variation in  $q_t$  is not only responsible for the “uncertainty risk” component in equity risk premium, but also constitutes the main driver of variance and bond risk premia as explained below.

Let  $w_t$  denote the logarithm of the wealth-consumption ratio, of the asset that pays the consumption endowment,  $\{C_{t+i}\}_{i=1}^\infty$ ; and conjecture a solution for  $w_t$  as an affine function of

<sup>12</sup>The parameters satisfy  $a_\sigma > 0, a_q > 0, |\rho_\sigma| < 1, |\rho_q| < 1, \varphi_q > 0$ ; and  $\{z_{g,t}\}, \{z_{\sigma,t}\}$  and  $\{z_{q,t}\}$  are iid  $\mathcal{N}(0, 1)$  processes jointly independent with each other.

the state variables,  $\sigma_{g,t}^2$  and  $q_t$ ,  $w_t = A_0 + A_\sigma \sigma_{g,t}^2 + A_q q_t$ . One can solve for the coefficients  $A_0$ ,  $A_\sigma$  and  $A_q$  using the standard Campbell and Shiller (1988) approximation  $r_{t+1} = \kappa_0 + \kappa_1 w_{t+1} - w_t + g_{t+1}$ . The restriction that  $\psi > 1$  hence  $\theta < 0$  implies that the impact coefficient associated with both consumption and volatility state variables are negative; i.e.,  $A_\sigma < 0$  and  $A_q < 0$ . So if consumption and uncertainty risks are high, the price-dividend ratio is low, hence risk premia are high. In response to high economic uncertainty risks, agents sell risky assets, and consequently the wealth-consumption ratio falls; so that risk premia rise.

The conditional variance of the time  $t$  to  $t+1$  return,  $\sigma_{r,t}^2 \equiv \text{Var}_t(r_{t+1})$ , is given by:  $\sigma_{r,t}^2 = \sigma_{g,t}^2 + \kappa_1^2 (A_\sigma^2 + A_q^2 \varphi_q^2) q_t$ . The variance risk premium can then be defined as the difference between risk-neutral and objective expectations of the return variance,<sup>13</sup>

$$\begin{aligned} \text{VRP}_t &\equiv \mathbb{E}_t^Q (\sigma_{r,t+1}^2) - \mathbb{E}_t^P (\sigma_{r,t+1}^2), \\ &\approx (\theta - 1) \kappa_1 [A_\sigma + A_q \kappa_1^2 (A_\sigma^2 + A_q^2 \varphi_q^2) \varphi_q^2] q_t > 0. \end{aligned} \quad (8)$$

One key observation here is that any temporal variation in the endogenously generated variance risk premium is due solely to the volatility-of-volatility or economic uncertainty risk,  $q_t$ , but not the consumption growth risk,  $\sigma_{g,t+1}^2$ . Moreover, provided that  $\theta < 0$ ,  $A_\sigma < 0$ , and  $A_q < 0$ , as would be implied by the agents' preference of an earlier resolution of economic uncertainty, this difference between the risk-neutral and objective expectations of return variances is guaranteed to be positive. If consumption volatility is not stochastic or there is no recursive preference, the variance risk premium is zero by construction.

### 3.2 Inflation Dynamics and Bond Return Predictability

In order for the real economy model outlined above to have realistic implications for nominal bond risk premia, one needs to impose rich inflation dynamics, which are capable to incorpo-

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<sup>13</sup>The approximation comes from the fact that the model-implied risk-neutral conditional expectation cannot be computed in closed form, and a log-linear approximation is applied.

rate stochastic volatility, money non-neutrality, and perhaps both cash flow and uncertainty shocks. Our preferred specification for expected inflation  $\pi_t$  is:

$$\pi_{t+1} = a_\pi + \rho_\pi \pi_t + \varphi_\pi z_{\pi,t+1} + \varphi_{\pi g} \sigma_{g,t} z_{g,t+1} + \varphi_{\pi \sigma} \sqrt{q_t} z_{\sigma,t+1}, \quad (9)$$

where  $\rho_\pi$  is the persistence and  $\frac{a_\pi}{1-\rho_\pi}$  is the long-run level of the inflation process. The innovations in the inflation dynamics consist of three parts: (1) a constant volatility part  $\varphi_\pi$  with exogenous shock  $z_{\pi,t+1}$  that is uncorrelated with all shocks in the real model, (2) a stochastic volatility part  $\varphi_{\pi g} \sigma_{g,t}$  that works through the consumption growth channel  $z_{g,t+1}$ , and (3) another stochastic volatility part  $\varphi_{\pi \sigma} \sqrt{q_t}$  that works through the volatility channel  $z_{\sigma,t+1}$ . Note that  $\varphi_{\pi g}$  and  $\varphi_{\pi \sigma}$  “leverage up” the inflation exposure to the growth and uncertainty risks, hence money-neutrality is implicitly violated.

We can examine each component of inflation shocks separately to assess which channel affects more the bond risk premia and to what degree:

$$\begin{aligned} \text{Model I:} \quad & \pi_{t+1} = a_\pi + \rho_\pi \pi_t + \varphi_\pi z_{\pi,t+1} \\ \text{Model II:} \quad & \pi_{t+1} = a_\pi + \rho_\pi \pi_t + \varphi_\pi \sigma_{g,t} z_{\pi,t+1} \\ \text{Model III:} \quad & \pi_{t+1} = a_\pi + \rho_\pi \pi_t + \varphi_\pi z_{\pi,t+1} + \varphi_{\pi g} \sigma_{g,t} z_{g,t+1} \\ \text{Model IV:} \quad & \pi_{t+1} = a_\pi + \rho_\pi \pi_t + \varphi_\pi z_{\pi,t+1} + \varphi_{\pi \sigma} \sqrt{q_t} z_{\sigma,t+1} \\ \text{Model V:} \quad & \pi_{t+1} = a_\pi + \rho_\pi \pi_t + \varphi_\pi z_{\pi,t+1} + \varphi_{\pi g} \sigma_{g,t} z_{g,t+1} + \varphi_{\pi \sigma} \sqrt{q_t} z_{\sigma,t+1} \end{aligned} \quad (10)$$

Model I includes only the autonomous inflation and constant volatility. Even with stochastic volatility, Model II still has no genuine inflation risk premium, since the inflation innovation is exogenous.<sup>14</sup> When there is stochastic volatility either through the growth channel (Model III) or uncertainty channel (Model IV), a genuine inflation risk premium exists and money neutrality is broken implicitly. Our preferred inflation specification (9) or Model V incorporates all three channels.<sup>15</sup>

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<sup>14</sup>The ability or inability of Model II in explaining both the level of bond risk premia and the predictability of the variance risk premium is also examined by Zhou (2010).

<sup>15</sup>There is a growing literature that examines the stochastic volatility or uncertainty effect in real macroeconomic variables (see, e.g., Bloom, 2009; Bloom, Floetotto, and Jaimovich, 2010; Benigno, Ricci, and Surico, 2010; Fernández-Villaverde and Rubio-Ramírez, 2010).

For each of the five model specifications, one can solve for the bond yield, the bond risk premium, and the predictability slope coefficient and  $R^2$  when regressing the bond risk premium on the variance risk premium. We present the general result of Model V here, as others are either special cases or very easy to derive.<sup>16</sup> The nominal bond yield can be expressed as an affine function of the state variables:

$$y_t^n = -\frac{1}{n} \begin{bmatrix} A(n) & B(n) & C(n) & D(n) \end{bmatrix} \begin{bmatrix} 1 & \sigma_{g,t}^2 & q_t & \pi_t \end{bmatrix}' \quad (11)$$

where the coefficients  $A(n)$ ,  $B(n)$ ,  $C(n)$ , and  $D(n)$  are solutions to ordinary difference equations.

Let  $rx_{t+1}^{n-1}$  be the bond excess return from  $t$  to  $t+1$  for an  $n$ -period bond holding one period, then its expected value  $\text{brp}_t^n$  or bond risk premium is given by:

$$\begin{aligned} \text{brp}_t^n &= D(n-1)\varphi_{\pi g} \left( -\frac{\theta}{\psi} + \theta - 1 - \varphi_{\pi g} \right) \sigma_{g,t}^2 \\ &\quad \{ [B(n-1) + D(n-1)\varphi_{\pi\sigma}] [(\theta-1)\kappa_1 A_\sigma - \varphi_{\pi\sigma}] + C(n-1)(\theta-1)\kappa_1 A_q \varphi_q^2 \} q_t \\ &\quad - D(n-1)\varphi_\pi^2. \end{aligned} \quad (12)$$

The first two items reflect consumption and uncertainty risk premia that are amplified by the endogenous inflation shock parameters  $\varphi_{\pi g}$  and  $\varphi_{\pi\sigma}$ , while the third item captures the autonomous inflation shock rough  $\varphi_\pi$ .

Our modeling framework also has implications for the predictability pattern of the bond risk premium by the variance risk premium. In a regression  $\text{brp}_t^n = a + b\text{VRP}_t$ , the model-implied slope coefficient and  $R^2$  are given by:

$$\begin{aligned} b &= \frac{\text{Cov}(\text{brp}_t^n, \text{VRP}_t)}{\text{Var}(\text{VRP}_t)} = \frac{\{\cdot\}}{(\theta-1)\kappa_1 [A_\sigma + A_q \kappa_1^2 (A_\sigma^2 + A_q^2 \varphi_q^2) \varphi_q^2]}, \\ R^2 &= \frac{b^2 \text{Var}(\text{VRP}_t)}{\text{Var}(\text{brp}_t^n)} = \frac{\{\cdot\}^2 \text{Var}(q_t)}{\{\cdot\}^2 \text{Var}(q_t) + D(n-1)^2 \varphi_{\pi g}^2 \left( -\frac{\theta}{\psi} + \theta - 1 - \varphi_{\pi g} \right)^2 \text{Var}(\sigma_{g,t}^2)}, \end{aligned}$$

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<sup>16</sup>The analytical solutions for bond prices, bond risk premia, and the predictability  $R^2$  and slope coefficients for Models I-V and the real economy are provided in a technical note (Zhou, 2011).

where  $\{\cdot\} \equiv [B(n-1) + D(n-1)\varphi_{\pi\sigma}][(\theta-1)\kappa_1 A_\sigma - \varphi_{\pi\sigma}] + C(n-1)(\theta-1)\kappa_1 A_q \varphi_q^2$ . Using these two metrics, we can evaluate whether the proposed inflation dynamics can reproduce the empirical pattern of bond return predictability from the variance risk premium as presented in Section 2.

### 3.3 Calibrating Bond Risk Premia and Return Predictability

The key to match the bond risk premium dynamics is through calibrating the inflation process, while leaving the choices of preference parameters and real economy dynamics similar to the existing studies (see, e.g., Bansal and Yaron, 2004; Bollerslev, Tauchen, and Zhou, 2009). Across all five models, as seen in Panel A of Table 10, we choose the same inflation level and persistence such that the annualized inflation rate is 2.4 percent. The choices of the volatility parameters are such that the annualized inflation volatility is 4.5 percent. When there are two or three innovations in inflation shocks as in Models III-V, the parameters are set such that each component contributes equally to the total inflation volatility. Note that the inflation dynamics—level, persistence, volatility—are almost the same as the exogenous process in Gallmeyer, Hollifield, Palomino, and Zin (2009), which imposes certain disciplines on our calibration exercise. The choices of preference structure and real economy parameters, as seen in Panel B of Table 10, are largely similar to those in Zhou (2010).

[Insert Table 10 approximately here.]

Our main calibration result on short term bond risk premia level is reported in Panel A of Table 11. The observed bond risk premia of 2 to 6 month Treasury bills for a 1 month holding period range from 33 to 75 basis points (bps). It is instructive to use the real bond as a benchmark—174 to 338 bps, which is far exceeding the observed levels. Therefore, it does not come as a surprise that exogenous inflation, either with stochastic volatility (Model II) or without (Model I), will overshoot bond risk premia even more since the exogenous inflation shock only adds on to the bond risk premium, which is purely driven by the Jensen’s inequality term but not by a genuine risk premium effect.

[Insert Table 11 approximately here.]

In essence, we are facing a challenge of simultaneously matching the levels of bond risk premia and the moments of variance risk premium. As shown in Panel B of of Table 11, our sample variance risk premium has a mean of 16.22 and a standard deviation of 15.94, well within the typical range of around 10 to 22 percent found in recent empirical studies (Bollerslev, Tauchen, and Zhou, 2009 and Drechsler and Yaron, 2011). Our real economy model can match the observed variance risk premium quite well with a mean of 10.84 and a standard deviation of 10.34. Our model also does a decent job matching the skewness (2.30) and kurtosis (9.51), producing model implied values of 1.87 (skewness) and 8.04 (kurtosis).

Of course, our calibration strategy for the real model, and consequently for the exogenous inflation Models I and II, is to match the variance risk premium as best as we can but sacrifice by overfitting the bond risk premia about 6 to 7 times larger. A similar trade-off is also reported in Zhou (2010), where the real economy model or autonomous inflation model II can match well the bond risk premia (44-86 bps in data and 73-94 bps in model), but severely undershoots the variance risk premia (17.45 in the data compared to 4.62 in model). Therefore, without dropping the money neutrality assumption implicit in the autonomous inflation dynamics, there is little hope one can simultaneously match bond and variance risk premia.

It is interesting to note that when money neutrality is violated as in Model III, the model-implied bond risk premia can be dramatically lowered to around 86-113 bps, compared to the exogenous inflation Model II (around 185-385 bps). This improvement is primarily driven by the negative comovement between inflation and consumption innovations ( $\varphi_{\pi g} = -0.157 < 0$ , Panel A of Table 10). The negative correlation between inflation and consumption shocks is consistent with more recent empirical evidence when both growth and inflation are in a moderate range (see, e.g., Piazzesi and Schneider, 2006; Campbell, Sunderam, and Viceira, 2009). Similarly, when the inflation shock is positively correlated with the uncertainty shock as in Model IV ( $\varphi_{\pi\sigma} = 0.1897 > 0$ , Panel A of Table 10), bond risk premia also moderate to around 126-243 bps from the exogenous inflation Model II (around 185-385 bps). Intuitively this could happen as volatility shocks—although uncorrelated with consumption shocks—are



negatively correlated with market risk premia (see, e.g. Bansal and Yaron, 2004), therefore any inflation shock which works through the uncertainty channel reduces bond risk premia through a discount rate effect.

Finally, Model V combining both cash flow and uncertainty channels of inflation effects seems to produce reasonable bond risk premia—62 to 84 bps—the closest to observed range of 33 to 75 bps. This is indeed a combined effect from lower risk premium of inflation’s growth channel and lower risk premium of inflation’s uncertainty channel, and as such Model V may prove to be a more flexible way of modeling inflation risk in matching bond risk premium dynamics. Our result on the nominal risk-free rate and 5-year yield are also reasonable—4.38 and 2.93 percent, while the other four models produce a similar size of the risk free rate but with a 5-year yield ranging from  $-2.05$  to negative infinity. Again this result reflects the challenge of simultaneously matching bond risk premia and variance risk premium.

Long-term bond risk premia cannot be matched well by our model with only three underlying shocks. Model V implies bond risk premia of 3-10 bps versus the observed levels of 95-278 bps, while other models imply negative bond risk premia and some are near negative infinity. In terms of the real economy model, Panel B of Table 11, it produces a reasonable equity premium of 5.61 percent and an equity volatility of 21.91 percent. The model also matches quite well the real risk-free rate—1.12, but the risk-free rate volatility of 14.61 percent is much higher than historical average of around 3.37 percent. The overshooting of the risk-free rate volatility and underfitting of long term bond risk premia are closely related outcomes of limiting the setup to only three risk factors.

**[Insert Figure 5 approximately here.]**

The model-implied predictability regression slope coefficients and  $R^2$ s are plotted in Figure 5, along with the empirical estimated ones. As shown in the top panel, the predictability slope coefficients of 1 month excess bond return regressions clearly show a gradual upward trend for 2-6 month Treasury bills. Model I and Model II overfit the predictability slopes by quite a large margin. Model III and IV improve significantly and fall within the 95 percent confidence bands for 5-6 month t-bills. Our preferred Model V seems to fit reasonable

well the slope coefficients and is the closest to the 95 percent confidence bands—in fact it matches the 5-6 month t-bills almost exactly. In the bottom panel, Models I and VI produce predictability  $R^2$ s of 100 percent by construction, since both bond risk premium and variance risk premium are driven by the same uncertainty factor  $q_t$  alone (Zhou, 2010). Model II seems to improve significantly as the bond risk premium also loads on the consumption growth risk  $\sigma_{g,t}^2$ . The  $R^2$ 's implied by Models III and V actually get very close to the observed ones.

In summary, our preferred Model V, which incorporates inflation's exposure to both growth and uncertainty risks, seems to have the potential to match both the observed bond risk premia levels and the predictability pattern from the variance risk premium.

## 4 Conclusion

This paper documents the predictive power of the equity market variance risk premium, defined as the difference between the risk-neutral and objective expectations of return variations, for bond excess returns. The predictive power is shown to be particularly strong in the short-run for a one month horizon and orthogonal to other known predictors of bond risk premia—forward rates, jump risk, yield curve factors and macro variables. The previously documented predictors for bond risk premia are particularly powerful for longer horizons of one year and more and hence also for longer maturity bonds. Short term bond risk premia exhibit pronounced spikes around major economic and financial crises, which is in contrast to the cyclical swings typically observed in long term bond risk premia.

We then propose a model that features time-varying uncertainty about real and nominal quantities together with investors' preferences for early resolution for uncertainty to produce the level and predictability of bond risk premia which have been found difficult to pin down in standard asset pricing models. While the real side of the economy follows earlier literature, the inflation process consists of two key ingredients—one stochastic volatility process that covaries with the consumption growth and the other that covaries with the consumption uncertainty, which gives rise to a genuine inflation risk premium.

In our calibration exercise, the model implied bond risk premia are only several basis points away from their empirical counterparts. The model also produces a reasonable equity premium, risk free rate and equity volatility but overshoots the risk free rate volatility. The average variance risk premium produced by our model is only slightly lower than the estimated empirical variance risk premium. In addition, the higher order moments (variance, skewness and kurtosis) of the variance risk premium are also fitted quite nicely. Finally, the model is able to replicate the predictive power of the equity variance risk premium for bond risk premia remarkably well.

## References

- Ang, Andrew and Geert Bekaert (2007), “Stock Return Predictability: Is It There?” *Review of Financial Studies*, vol. 20, 651–707.
- Baele, Lieven, Geert Bekaert, and Koen Inghelbrecht (2010), “The Determinants of Stock and Bond Return Comovements,” *Review of Financial Studies*, vol. 23, 2374–2428.
- Bansal, Ravi and Ivan Shaliastovich (2009), “Learning and Asset-Price Jumps,” Working Paper.
- Bansal, Ravi and Ivan Shaliastovich (2010), “A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets,” Working Paper.
- Bansal, Ravi and Amir Yaron (2004), “Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles,” *Journal of Finance*, vol. 59, 1481–1509.
- Bekaert, Geert and Eric Engstrom (2009), “Asset Return Dynamics under Bad Environment-Good Environment Fundamentals,” Working Paper.
- Bekaert, Geert, Marie Hoerova, and Marco Lo Duca (2010), “Risk, Uncertainty and Monetary Policy,” Working Paper NBER 16397.
- Benigno, Pierpaolo, Luca Antonio Ricci, and Paolo Surico (2010), “Unemployment and Productivity in the Long Run: The Role of Macroeconomic Volatility,” Working Paper.
- Bloom, Nicholas (2009), “The Impact of Uncertainty Shocks,” *Econometrica*, vol. 77, 623–685.
- Bloom, Nicholas, Max Floetotto, and Nir Jaimovich (2010), “Really Uncertain Business Cycles,” Working Paper.
- Bollerslev, Tim, James Marrone, Lai Xu, and Hao Zhou (2011), “Stock Return Predictability and Variance Risk Premia: Statistical Inference and International Evidence,” Working Paper.
- Bollerslev, Tim, George Tauchen, and Hao Zhou (2009), “Expected Stock Returns and Variance Risk Premia,” *Review of Financial Studies*, vol. 22, 4463–4492.
- Bollerslev, Tim and Viktor Todorov (2011), “Tails, Fears, and Risk Premia,” *forthcoming, Journal of Finance*.

- Bollerslev, Tim and Hao Zhou (2007), “Expected Stock Returns and Variance Risk Premia,” Finance and Economics Discussion Series 2007-11, Federal Reserve Board.
- Britten-Jones, Mark and Anthony Neuberger (2000), “Option Prices, Implied Price Processes, and Stochastic Volatility,” *Journal of Finance*, vol. 55, 839–866.
- Buraschi, Andrea and Paul Whelan (2010), “Macroeconomic Uncertainty, Difference in Beliefs, and Bond Risk Premia,” Working Paper.
- Cambpell, John Y. and Robert J. Shiller (1991), “Yield Spreads and Interest Rate Movements: A Bird’s Eye View,” *Review of Economic Studies*, vol. 58, 495–514.
- Campbell, John Y. and Robert J. Shiller (1988), “The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors,” *Review of Financial Studies*, vol. 1, 195–228.
- Campbell, John Y., Adi Sunderam, and Luis M. Viceira (2009), “Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds,” Working Paper, Harvard University.
- Carr, Peter and Liuren Wu (2009), “Variance Risk Premiums,” *Review of Financial Studies*, vol. 22, 1311–1341.
- CBOE (2003), “VIX: CBOE Volatility Index,” CBOE White Paper.
- Cieslak, Anna and Pavol Povala (2010), “Understanding Bond Risk Premia,” Working Paper.
- Cochrane, John H. and Monika Piazzesi (2005), “Bond Risk Premia,” *American Economic Review*, vol. 95, 138–160.
- Corsi, Fulvio (2009), “A Simple Approximate Long-Memory Model of Realized Volatility,” *Journal of Financial Econometrics*, vol. 7, 174–196.
- Corsi, Fulvio, Davide Pirino, and Roberto Renò (2010), “Threshold Bipower Variation and the Impact of Jumps on Volatility Forecasting,” *forthcoming, Journal of Econometrics*.
- Demeterfi, Kresimir, Emanuel Derman, Michael Kamal, and Joseph Zou (1999), “A Guide to Volatility and Variance Swaps,” *Journal of Derivatives*, vol. 6, 9–32.
- Doh, Taeyoung (2010), “Long Run Risks in the Term Structure of Interest Rates: Estimation,” Working Paper, Kansas City Fed RWP 08-11.
- Drechsler, Itamar and Amir Yaron (2011), “What’s Vol Got to Do With It,” *Review of Financial Studies*, vol. 24, 1–45.

- Duffee, Gregory (2011), “Information in (and not in) the Term Structure,” *forthcoming, Review of Financial Studies*.
- Fama, Eugene F. and Robert R. Bliss (1987), “The Information in Long-Maturity Forward Rates,” *American Economic Review*, vol. 77, 680–692.
- Fernández-Villaverde, Jesús and Juan Rubio-Ramírez (2010), “Macroeconomics and Volatility: Data, Models, and Estimation,” NBER Working Paper 16618.
- Fontaine, Jean-Sebastian and Rene Garcia (2010), “Bond Liquidity Premia,” Working Paper.
- Gabaix, Xavier (2009), “Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance,” Working Paper, NYU Stern.
- Gallmeyer, Michael, Burton Hollifield, Francisco Palomino, and Stanley Zin (2009), “Term Premium Dynamics and the Taylor Rule,” Working Paper.
- Greenwood, Robin and Dimitri Vayanos (2010), “Bond Supply and Excess Bond Returns,” Working Paper.
- Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright (2006), “The U.S. Treasury Yield Curve: 1961 to the Present,” Working Paper, Federal Reserve Board.
- Hasseltoft, Henrik (2010), “Stocks, Bonds, and Long-Run Consumption Risks,” Working Paper.
- Hodrick, Robert J. (1992), “Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement,” *Review of Financial Studies*, vol. 5, 357–386.
- Huang, Jay and Zhan Shi (2010), “Determinants of Bond Risk Premia,” Working Paper.
- Jiang, George J. and Yisong S. Tian (2005), “Model-Free Implied Volatility and Its Information Content,” *Review of Financial Studies*, vol. 18, 1305–1342.
- Koijen, Ralph S. J., Hanno N. Lustig, and Stijn Van Nieuwerburgh (2010), “The Cross-Section and Time-Series of Stock and Bond Returns,” Working Paper, University of Chicago.
- Ludvigson, Sydney C. and Serena Ng (2009), “Macro Factors in Bond Risk Premia,” *Review of Financial Studies*, vol. 22, 5027–5067.
- Ludvigson, Sydney C. and Serena Ng (2010), “A Factor Analysis of Bond Risk Premia,” *forthcoming, Handbook of Applied Econometrics*.

- Patton, Andrew and Kevin Sheppard (2011), “Good Volatility, Bad Volatility: Signed Jumps and the Persistence of Volatility,” Working Paper.
- Pennacchi, George (1991), “Identifying the Dynamics of Real Interest Rates and Inflation: Evidence Using Survey Data,” *Review of Financial Studies*, vol. 4, 53–86.
- Piazzesi, Monika and Martin Schneider (2006), “Equilibrium Yield Curve,” *NBER Macroeconomics Annual*, forthcoming.
- Stock, James H. and Mark W. Watson (2002), “Macroeconomic Forecasting using Diffusion Indexes,” *Journal of Business and Economic Statistics*, vol. 20, 147–162.
- Sun, Tong-Sheng (1992), “Real and Nominal Interest Rate: A discrete-Time Model and Its Continuous-Time Limit,” *Review of Financial Studies*, vol. 5, 581–611.
- Vayanos, Dimitri and Jean-Luc Vila (2009), “A Preferred-Habitat Model of the Term-Structure of Interest Rates,” Working Paper.
- Wachter, Jessica A. (2006), “A Consumption-Based Model of the Term Structure of Interest Rates,” *Journal of Financial Economics*, vol. 79, 365–399.
- Wei, Min and Jonathan H. Wright (2010), “Reverse Regressions and Long-Horizon Forecasting,” Working Paper.
- West, Whitney K. and Kenneth D. West (1987), “A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix,” *Econometrica*, vol. 55, 703–708.
- Whaley, Robert E. (2000), “The Investor Fear Gauge,” *Journal of Portfolio Management*, vol. 26, 12–17.
- Wright, Jonathan H. and Hao Zhou (2009), “Bond Risk Premia and Realized Jump Risk,” *Journal of Banking and Finance*, vol. 33, 2333–2345.
- Wu, Shu (2008), “Long-Run Consumption Risk and the Real Yield Curve,” Working Paper, Department of Economics, University of Kansas.
- Xiong, Wei and Hongjun Yan (2010), “Heterogeneous Expectations and Bond Markets,” *Review of Financial Studies*, vol. 23, 1433–1466.
- Zhou, Hao (2010), “Variance Risk Premia, Asset Predictability Puzzles, and Macroeconomic Uncertainty,” Working Paper, Federal Reserve Board.

Zhou, Hao (2011), "Term Structure of Interest Rates with Inflation Uncertainty," Technical Report, Federal Reserve Board.



**Table 1**  
**Summary Statistics**

This table presents summary statistics for all monthly data from January 1990 to September 2010. In Panel A we report the 1 month holding period excess returns on the Treasury bills and the annual excess returns on the Fama and Bliss bonds. Panel B reports the Cochrane and Piazzesi (2005) factor, CP, the mean jump size,  $\tilde{J}$ , the cycle factor,  $\widehat{cf}$ , the Duffee (2011) hidden factor,  $\tilde{H}$ , and the eight Ludvigson and Ng (2009) macro factors,  $\hat{F}_j$ . To construct the statistics for the mean jump and the hidden factor, we scale the series by  $10^2$ . Panel C reports the VIX<sup>2</sup>, the realized variance calculated using 5 min returns on the index and the futures, respectively, the projected value from equation (2) and the corresponding variance risk premia.

PANEL A: SUMMARY STATISTICS BOND VARIABLES

	2m	3m	4m	5m	6m	2y	3y	4y	5y
Mean	0.32	0.45	0.44	0.66	0.74	0.96	1.78	2.46	2.83
Max	2.50	3.31	4.54	5.81	7.65	3.64	7.31	10.30	12.54
Min	-0.57	-0.87	-1.19	-1.43	-6.07	-2.37	-5.24	-6.88	-8.37
StDev	0.43	0.52	0.72	0.91	1.22	1.40	2.65	3.69	4.52
Skewness	1.74	1.58	1.73	1.67	0.79	-0.14	-0.25	-0.28	-0.34
Kurtosis	7.69	7.33	8.77	7.93	11.16	2.06	2.31	2.40	2.55
AC(1)	0.55	0.31	0.23	0.28	0.26	0.95	0.94	0.93	0.92
AC(2)	0.39	0.25	0.20	0.20	0.21	0.88	0.86	0.83	0.81

PANEL B: SUMMARY STATISTICS MACRO VARIABLES

	CP	$\tilde{J}$	$\tilde{H}$	$\widehat{cf}$	$\hat{F}_1$	$\hat{F}_2$	$\hat{F}_3$	$\hat{F}_4$	$\hat{F}_5$	$\hat{F}_6$	$\hat{F}_7$	$\hat{F}_8$
Mean	1.51	0.02	-0.05	1.21	-0.06	0.01	0.08	0.03	0.03	0.00	0.01	-0.02
Max	4.39	0.16	0.05	4.55	21.73	11.85	7.22	15.75	6.69	5.02	5.85	5.72
Min	-4.12	-0.21	-0.24	-1.32	-7.41	-14.97	-10.14	-10.39	-8.68	-5.19	-6.32	-5.63
StDev	1.03	0.08	0.05	1.37	4.65	3.22	2.64	2.02	1.90	1.86	1.73	1.68
Skewness	-0.46	-0.69	-0.31	0.23	1.71	-0.68	-0.54	1.53	0.07	0.28	-0.15	0.21
Kurtosis	7.00	3.24	2.54	2.31	7.25	6.90	4.26	19.96	5.29	2.80	3.69	3.84
AC(1)	0.87	0.96	0.85	0.95	0.86	-0.12	0.35	0.11	0.02	0.48	0.20	-0.18
AC(2)	0.78	0.93	0.71	0.91	0.83	-0.27	0.42	0.25	0.10	0.50	0.26	0.14

PANEL C: SUMMARY STATISTICS VARIANCE RISK PREMIA

	VIX <sup>2</sup>	$RV_{Idx}^{5min}$	$RV_{Fut}^{5min}$	HAR	$VRP_{Idx}$	$VRP_{Fut}$	$VRP_{HAR}$
Mean	39.10	16.91	27.79	22.88	22.20	11.32	16.22
Max	298.90	374.75	607.47	223.96	119.63	86.42	91.86
Min	9.05	1.48	2.86	6.60	-75.85	-308.57	0.90
StDev	36.79	31.09	48.64	23.59	19.02	26.52	15.94
Skewness	3.40	7.73	8.25	4.56	1.34	-7.29	2.30
Kurtosis	19.25	80.89	91.01	31.35	11.04	91.42	9.51
AC(1)	0.82	0.66	0.60	0.68	0.50	0.23	0.85
AC(2)	0.61	0.39	0.31	0.40	0.40	0.05	0.70

**Table 2**  
**Cross Correlations of Predictor Variables**

This table presents the cross correlation for the Cochrane and Piazzesi (2005) factor, CP, the mean jump size,  $\tilde{J}$ , the cycle factor,  $\widehat{cf}$ , the Duffee (2011) hidden factor,  $\tilde{H}$ , the eight Ludvigson and Ng (2009) macro factors,  $\widehat{F}_j$ , and the variance risk premium, VRP. We use monthly data from January 1990 to September 2010.

	CP	$\tilde{J}$	$\tilde{H}$	$\widehat{cf}$	$\widehat{F}_1$	$\widehat{F}_2$	$\widehat{F}_3$	$\widehat{F}_4$	$\widehat{F}_5$	$\widehat{F}_6$	$\widehat{F}_7$	$\widehat{F}_8$	VRP
CP	1.00												
$\tilde{J}$	-0.01	1.00											
$\tilde{H}$	-0.69	0.08	1.00										
$\widehat{cf}$	0.72	-0.15	-0.32	1.00									
$\widehat{F}_1$	0.42	-0.24	-0.24	0.20	1.00								
$\widehat{F}_2$	-0.11	-0.01	0.09	-0.04	0.00	1.00							
$\widehat{F}_3$	-0.41	-0.22	0.29	-0.32	0.25	-0.05	1.00						
$\widehat{F}_4$	-0.07	-0.28	0.08	-0.01	0.16	-0.04	0.01	1.00					
$\widehat{F}_5$	0.03	-0.20	-0.04	0.01	0.05	0.07	-0.18	0.07	1.00				
$\widehat{F}_6$	-0.16	-0.32	0.30	0.12	0.06	0.05	0.13	0.06	0.01	1.00			
$\widehat{F}_7$	0.30	-0.04	-0.14	0.19	0.18	-0.07	0.04	-0.06	0.01	-0.07	1.00		
$\widehat{F}_8$	-0.10	-0.09	0.14	0.10	-0.13	0.06	0.09	-0.09	0.12	0.07	0.03	1.00	
VRP	0.02	-0.33	-0.03	-0.10	0.29	-0.03	0.11	0.22	0.11	0.17	0.14	-0.10	1.00

**Table 3**  
**Forecasting Realized Variance**

This table presents estimated coefficients of the following regression:

$$RV_{t+22,\text{mon}} = \alpha + \beta_D RV_t + \beta_W RV_{t,\text{week}} + \beta_M RV_{t,\text{mon}} + \sum_{i=1}^k \beta_{V,i} VIX_{t-i}^2 + \epsilon_{t+22,\text{mon}},$$

where coefficients are estimated with ordinary least squares and t-statistics are calculated using Newey and West (1987). We use daily data from January 1990 to February 2011.

Constant	0.0000 (1.13)	0.0000 (6.37)	0.0000 (1.20)	0.0000 (1.65)	0.0000 (1.59)
$RV_t$	0.0519 (3.43)	0.0809 (3.41)	0.0502 (2.24)	0.0118 (1.07)	0.0120 (1.03)
$RV_{t,\text{week}}$	0.3587 (3.75)	0.3915 (3.59)	0.3677 (6.39)	0.4304 (5.24)	0.4317 (5.13)
$RV_{t,\text{mon}}$	0.0230 (0.18)	0.2137 (2.00)			
$VIX_t^2$	0.3668 (2.30)		0.3846 (4.12)	0.7571 (3.84)	0.7567 (3.86)
$VIX_{t-1}^2$				-0.1071 (-2.43)	-0.1253 (-2.32)
$VIX_{t-2}^2$				-0.0253 (-0.69)	-0.0216 (-0.54)
$VIX_{t-3}^2$				-0.1798 (-2.97)	-0.2758 (-1.48)
$VIX_{t-4}^2$				-0.1113 (-0.61)	
Adj. $R^2$	0.55	0.53	0.55	0.55	0.55

**Table 4**  
**Short and Long Run Predictability Regressions**

Each month we run the following regression:  $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)}(h) + \beta_1^{(\tau)}(h)VRP_t + \epsilon_{t+h}^{(\tau)}$ , where  $rx_{t+h}^{(\tau)}$  are either the excess returns on Treasury bills,  $h = 1, 2, 3, 4, 5$  and  $\tau = 2, 3, 4, 5, 6$  months (Panel A) or the excess returns on Treasury bonds,  $h = 12$  and  $\tau = 24, 36, 48, 60$  months (Panel B).  $VRP_t$  is the market variance risk premium. Coefficients are estimated with ordinary-least squares.  $t$ -statistics are in brackets and are calculated using Newey and West (1987) and Wei and Wright (2010) standard errors, respectively. The estimates for the constant are omitted from the table. The sample spans the period from January 1990 to September 2010.

PANEL A: FAMA TREASURY BILLS					
	2m	3m	4m	5m	6m
VRP	0.009	0.010	0.010	0.012	0.017
nw	(1.78)	(1.97)	(1.91)	(1.74)	(2.08)
ww	(2.37)	(2.03)	(2.12)	(1.97)	(2.35)
Adj. $R^2$	0.06	0.04	0.02	0.02	0.02
VRP		0.006	0.008	0.010	0.016
nw		(2.14)	(2.27)	(1.96)	(2.29)
ww		(2.01)	(2.08)	(1.73)	(2.35)
Adj. $R^2$		0.02	0.02	0.02	0.02
VRP			0.005	0.007	0.013
nw			(2.15)	(1.92)	(1.86)
ww			(2.13)	(1.71)	(1.89)
Adj. $R^2$			0.02	0.02	0.02
VRP				0.008	0.016
nw				(1.93)	(2.87)
ww				(2.02)	(2.51)
Adj. $R^2$				0.02	0.02
VRP					0.009
nw					(1.78)
ww					(2.37)
Adj. $R^2$					0.05

PANEL B: FAMA BLISS TREASURY BONDS				
	2y	3y	4y	5y
VRP	0.019	0.033	0.036	0.032
nw	(2.02)	(1.89)	(1.45)	(1.05)
ww	(1.88)	(1.77)	(1.64)	(0.83)
Adj. $R^2$	0.02	0.02	0.01	0.01

**Table 5**  
**Predictability Regressions of Treasury Bills: 1m Holding Period**

Each month we run the following regression:  $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)}(h) + \beta_1^{(\tau)}(h)VRP_t + \sum_{j=2}^{11} \beta_j^{(\tau)}(h)M_{t,j} + \epsilon_{t+h}^{(\tau)}$ , where  $h = 1$  months and  $\tau = 2, 3, 4, 5, 6$  months.  $rx_{t+h}^{(\tau)}$  are the holding period returns of Treasury bills in excess of the yield on a 1-month zero coupon bond,  $VRP_t$  is the variance risk premium and  $M_{t,j}$  collectively are the CP factor, the mean jump size, the cycle factor, the Duffee hidden factor, and the macro factors.  $t$ -statistics are in brackets and are calculated using Newey and West (1987) and Wei and Wright (2010) standard errors, respectively. The estimates for the constant are omitted from the table. The sample spans the period from January 1990 to September 2010.

	2m					3m					4m				
VRP	0.009	0.012	0.009	0.010	0.012	0.009	0.011	0.008	0.009	0.010	0.010	0.011	0.007	0.009	0.009
nw	(1.84)	(2.67)	(2.21)	(3.18)	(4.11)	(2.09)	(2.66)	(2.08)	(2.91)	(3.61)	(2.04)	(2.10)	(1.50)	(2.18)	(2.82)
ww	(2.36)	(3.00)	(2.47)	(2.78)	(3.14)	(1.99)	(2.30)	(1.67)	(1.94)	(2.07)	(2.13)	(1.50)	(1.86)	(1.91)	
CP	0.167	0.167	0.305	0.126	0.008	0.157	0.157	0.326	0.142	-0.083	0.194	0.194	0.377	0.115	-0.226
nw	(4.45)	(4.88)	(5.92)	(1.64)	(0.10)	(4.00)	(4.21)	(6.61)	(1.90)	(-0.82)	(3.90)	(3.97)	(6.10)	(1.26)	(-1.69)
ww	(4.09)	(4.18)	(4.63)	(1.19)	(0.08)	(2.65)	(2.67)	(3.75)	(1.15)	(-0.71)	(2.85)	(2.85)	(3.94)	(0.80)	(-1.78)
$\tilde{J}$		1.180	0.766	1.082	0.645		0.808	0.304	0.629	0.396		0.404	-0.142	0.321	0.254
nw		(2.08)	(1.81)	(2.39)	(1.67)		(1.37)	(0.67)	(1.37)	(0.84)		(0.59)	(-0.23)	(0.50)	(0.39)
ww		(1.96)	(1.27)	(1.70)	(1.03)		(1.02)	(0.38)	(0.77)	(0.46)		(0.49)	(-0.17)	(0.38)	(0.30)
$\widehat{cf}$			-0.121	-0.072	-0.051			-0.147	-0.097	-0.082			-0.159	-0.088	-0.084
nw			(-3.94)	(-2.24)	(-1.52)			(-4.15)	(-2.55)	(-2.17)			(-3.49)	(-1.90)	(-1.89)
ww			(-2.60)	(-1.39)	(-0.93)			(-2.49)	(-1.51)	(-1.25)			(-2.57)	(-1.28)	(-1.20)
$\tilde{H}$				-0.311	-0.281				-0.319	-0.357				-0.454	-0.564
nw				(-2.19)	(-2.22)				(-2.72)	(-2.94)				(-3.17)	(-3.59)
ww				(-2.01)	(-2.02)				(-1.91)	(-2.18)				(-2.23)	(-2.82)
$\widehat{F}_1$					0.011					0.041					0.072
nw					(0.76)					(2.26)					(3.01)
ww					(0.52)					(1.51)					(2.33)
$\widehat{F}_2$					-0.010					-0.017					-0.030
nw					(-1.39)					(-1.29)					(-1.57)
ww					(-1.01)					(-1.28)					(-1.74)
$\widehat{F}_3$					-0.036					-0.069					-0.102
nw					(-2.32)					(-3.66)					(-3.96)
ww					(-1.66)					(-2.39)					(-3.11)
$\widehat{F}_4$					-0.067					-0.060					-0.055
nw					(-3.52)					(-2.39)					(-1.57)
ww					(-2.19)					(-1.46)					(-1.27)
$\widehat{F}_5$					-0.022					-0.026					-0.056
nw					(-1.88)					(-1.68)					(-2.86)
ww					(-1.39)					(-1.16)					(-2.30)
$\widehat{F}_6$					0.027					0.028					0.036
nw					(2.59)					(1.88)					(1.79)
ww					(1.92)					(1.38)					(1.59)
$\widehat{F}_7$					0.011					0.004					0.000
nw					(0.73)					(0.19)					(0.01)
ww					(0.52)					(0.14)					(0.00)
$\widehat{F}_8$					-0.004					-0.012					-0.034
nw					(-0.25)					(-0.59)					(-1.27)
ww					(-0.18)					(-0.42)					(-1.07)
Adj. $R^2$	0.17	0.21	0.28	0.33	0.37	0.11	0.12	0.18	0.22	0.26	0.08	0.08	0.12	0.17	0.22

**Table 6**  
**Predictability Regressions of Treasury Bills: 1m Holding Period**

Each month we run the following regression:  $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)}(h) + \beta_1^{(\tau)}(h)VRP_t + \sum_{j=2}^{11} \beta_j^{(\tau)}(h)M_{t,j} + \epsilon_{t+h}^{(\tau)}$ , where  $h = 1$  months and  $\tau = 2, 3, 4, 5, 6$  months.  $rx_{t+h}^{(\tau)}$  are the holding period returns of Treasury bills in excess of the yield on a 1-month zero coupon bond,  $VRP_t$  is the variance risk premium and  $M_{t,j}$  collectively are the CP factor, the mean jump size, the cycle factor, the Duffee hidden factor, and the macro factors.  $t$ -statistics are in brackets and are calculated using Newey and West (1987) and Wei and Wright (2010) standard errors, respectively. The estimates for the constant are omitted from the table. The sample spans the period from January 1990 to September 2010.

	5m				6m					
VRP	0.011	0.011	0.006	0.008	0.009	0.016	0.014	0.007	0.010	0.011
nw	(1.95)	(1.88)	(1.62)	(1.84)	(2.22)	(2.25)	(1.95)	(1.12)	(1.75)	(2.05)
ww	(1.95)	(1.75)	(1.99)	(1.63)	(2.40)	(2.37)	(1.88)	(0.96)	(1.69)	(1.80)
CP	0.256	0.256	0.509	0.151	-0.261	0.263	0.262	0.647	0.152	-0.472
nw	(3.98)	(3.97)	(6.95)	(1.42)	(-1.53)	(3.12)	(3.08)	(6.36)	(1.18)	(-2.10)
ww	(2.80)	(2.80)	(3.97)	(0.82)	(-1.50)	(2.28)	(2.27)	(3.94)	(0.71)	(-2.18)
$\tilde{J}$		-0.162	-0.919	-0.284	-0.392		-0.919	-2.073	-1.195	-1.068
nw		(-0.21)	(-1.48)	(-0.42)	(-0.57)		(-0.99)	(-2.84)	(-1.44)	(-1.25)
ww		(-0.15)	(-0.82)	(-0.26)	(-0.34)		(-0.69)	(-1.59)	(-0.92)	(-0.78)
$\widehat{cf}$			-0.221	-0.123	-0.113			-0.337	-0.201	-0.185
nw			(-3.93)	(-2.16)	(-2.13)			(-4.08)	(-2.61)	(-2.54)
ww			(-2.62)	(-1.36)	(-1.24)			(-3.16)	(-1.81)	(-1.70)
$\tilde{H}$				-0.622	-0.738				-0.861	-1.056
nw				(-4.09)	(-4.28)				(-5.66)	(-5.48)
ww				(-2.43)	(-2.87)				(-2.94)	(-3.54)
$\widehat{F}_1$					0.089					0.148
nw					(2.94)					(3.50)
ww					(2.21)					(2.93)
$\widehat{F}_2$					-0.038					-0.060
nw					(-1.52)					(-1.79)
ww					(-1.68)					(-2.24)
$\widehat{F}_3$					-0.113					-0.147
nw					(-3.33)					(-3.59)
ww					(-2.54)					(-2.76)
$\widehat{F}_4$					-0.082					-0.122
nw					(-1.73)					(-1.84)
ww					(-1.40)					(-1.67)
$\widehat{F}_5$					-0.062					-0.079
nw					(-2.51)					(-2.53)
ww					(-1.91)					(-1.96)
$\widehat{F}_6$					0.040					0.037
nw					(1.55)					(1.06)
ww					(1.33)					(1.01)
$\widehat{F}_7$					0.006					0.000
nw					(0.19)					(-0.01)
ww					(0.12)					(-0.01)
$\widehat{F}_8$					-0.036					-0.074
nw					(-1.14)					(-1.45)
ww					(-0.84)					(-1.21)
Adj. $R^2$	0.08	0.08	0.13	0.19	0.24	0.06	0.06	0.12	0.18	0.24

**Table 7**  
**Predictability Regressions of Treasury Bills: 2m Holding Period**

The table reports the regression results for bond excess returns. Each month we run the following regression:  $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)}(h) + \beta_1^{(\tau)}(h)VRP_t + \sum_{j=2}^{11} \beta_j^{(\tau)}(h)M_{t,j} + \epsilon_{t+h}^{(\tau)}$ , where  $h = 2$  months and  $\tau = 3, 4, 5, 6$  months.  $rx_{t+h}^{(\tau)}$  are the holding period returns of Treasury bills in excess of the yield on a 2-month zero coupon bond,  $VRP_t$  is the variance risk premium and  $M_{t,j}$  collectively are the CP factor, the mean jump size, the Duffee hidden factor, and the Ludvigson and Ng macro factors. Coefficients are estimated with ordinary-least squares.  $t$ -statistics are in brackets and are calculated using Newey and West (1987) and Wei and Wright (2010) standard errors, respectively. The estimates for the constant are omitted from the table. The sample spans the period from January 1990 to September 2010.

	3m				4m				5m				6m							
VRP	0.006	0.008	0.005	0.006	0.008	0.008	0.008	0.004	0.005	0.006	0.009	0.007	0.001	0.003	0.005	0.015	0.010	0.001	0.005	0.008
nw	(2.26)	(2.67)	(1.83)	(1.77)	(2.56)	(2.36)	(2.31)	(1.21)	(1.25)	(1.59)	(2.05)	(1.54)	(0.24)	(0.61)	(0.95)	(2.40)	(1.62)	(0.17)	(0.60)	(0.92)
ww	(1.64)	(1.92)	(1.77)	(1.75)	(2.38)	(2.08)	(2.27)	(1.05)	(1.40)	(1.26)	(1.74)	(1.49)	(0.25)	(0.84)	(0.96)	(2.40)	(0.92)	(0.10)	(0.39)	(0.60)
CP	0.158	0.158	0.309	0.177	-0.021	0.188	0.188	0.389	0.151	-0.232	0.291	0.291	0.587	0.219	-0.319	0.352	0.352	0.838	0.224	-0.639
nw	(4.88)	(4.91)	(6.79)	(2.11)	(-0.23)	(3.65)	(3.65)	(5.61)	(1.35)	(-1.89)	(3.91)	(3.90)	(5.14)	(1.32)	(-1.74)	(3.50)	(3.50)	(5.21)	(1.02)	(-2.48)
ww	(2.80)	(2.83)	(3.76)	(1.52)	(-0.16)	(2.18)	(2.18)	(3.48)	(1.04)	(-1.45)	(2.44)	(2.42)	(3.89)	(1.12)	(-1.41)	(2.13)	(2.12)	(4.00)	(0.87)	(-2.01)
$\hat{J}$		0.757	0.304	0.537	0.248		0.049	-0.555	-0.134	-0.075		-1.023	-1.913	-1.260	-1.132		-2.260	-3.715	-2.628	-2.343
nw		(1.91)	(0.79)	(1.10)	(0.50)		(0.09)	(-0.92)	(-0.17)	(-0.09)		(-1.34)	(-1.86)	(-1.22)	(-1.07)		(-2.18)	(-2.80)	(-1.92)	(-1.65)
ww		(0.94)	(0.36)	(0.64)	(0.30)		(0.05)	(-0.49)	(-0.12)	(-0.07)		(-0.73)	(-1.30)	(-0.89)	(-0.79)		(-1.20)	(-1.91)	(-1.42)	(-1.23)
$\hat{c}\hat{f}$			-0.132	-0.096	-0.071			-0.176	-0.111	-0.109			-0.260	-0.159	-0.154			-0.425	-0.257	-0.242
nw			(-4.49)	(-2.48)	(-1.88)			(-3.90)	(-1.94)	(-2.24)			(-3.39)	(-1.91)	(-2.16)			(-3.79)	(-2.24)	(-2.49)
ww			(-2.24)	(-1.53)	(-1.10)			(-2.24)	(-1.40)	(-1.33)			(-2.47)	(-1.51)	(-1.42)			(-2.92)	(-1.80)	(-1.67)
$\hat{H}$				-0.229	-0.240			-0.413	-0.540				-0.640	-0.815					-1.066	-1.355
nw				(-2.45)	(-2.58)			(-3.42)	(-4.18)				(-3.46)	(-4.15)					(-4.51)	(-5.25)
ww				(-1.57)	(-1.64)			(-2.19)	(-2.71)				(-2.46)	(-2.97)					(-3.14)	(-3.74)
$\hat{F}_1$					0.033					0.095					0.138					0.209
nw					(1.98)					(3.70)					(3.78)					(4.14)
ww					(1.17)					(2.47)					(2.53)					(2.77)
$\hat{F}_2$					0.005					-0.001					-0.003					-0.028
nw					(0.63)					(-0.05)					(-0.19)					(-1.21)
ww					(0.38)					(-0.03)					(-0.11)					(-0.76)
$\hat{F}_3$					-0.050					-0.095					-0.120					-0.199
nw					(-2.97)					(-3.80)					(-3.27)					(-4.11)
ww					(-1.59)					(-2.17)					(-1.93)					(-2.30)
$\hat{F}_4$					-0.088					-0.080					-0.122					-0.180
nw					(-4.17)					(-2.13)					(-2.26)					(-2.30)
ww					(-2.48)					(-1.73)					(-1.91)					(-2.01)
$\hat{F}_5$					-0.021					-0.051					-0.067					-0.089
nw					(-1.53)					(-2.19)					(-2.18)					(-2.37)
ww					(-0.97)					(-1.80)					(-1.79)					(-1.69)
$\hat{F}_6$					0.026					0.046					0.063					0.070
nw					(1.91)					(1.90)					(1.82)					(1.52)
ww					(1.36)					(1.75)					(1.72)					(1.36)
$\hat{F}_7$					-0.018					-0.018					-0.020					-0.010
nw					(-1.00)					(-0.62)					(-0.52)					(-0.17)
ww					(-0.52)					(-0.48)					(-0.39)					(-0.14)
$\hat{F}_8$					-0.055					-0.080					-0.122					-0.194
nw					(-3.50)					(-2.94)					(-3.14)					(-3.38)
ww					(-1.96)					(-2.26)					(-2.66)					(-2.90)
Adj. $R^2$	0.11	0.12	0.19	0.21	0.28	0.07	0.07	0.12	0.16	0.26	0.08	0.08	0.14	0.19	0.29	0.06	0.07	0.15	0.22	0.33

**Table 8**  
**Predictability Regressions of Treasury Bills: 3m Holding Period**

The table reports the regression results for bond excess returns. Each month we run the following regression:  $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)}(h) + \beta_1^{(\tau)}(h)VRP_t + \sum_{j=2}^{11} \beta_j^{(\tau)}(h)M_{t,j} + \epsilon_{t+h}^{(\tau)}$ , where  $h = 3$  months and  $\tau = 4, 5, 6$  months.  $rx_{t+h}^{(\tau)}$  are the holding period returns of Treasury bills in excess of the yield on a 3-month zero coupon bond,  $VRP_t$  is the variance risk premium and  $M_{t,j}$  collectively are the CP factor, the mean jump size, the cycle factor, the Duffee hidden factor, and the macro factors.  $t$ -statistics are in brackets and are calculated using Newey and West (1987) and Wei and Wright (2010) standard errors, respectively. The sample spans the period from January 1990 to September 2010.

	4m					5m					6m				
VRP	0.004	0.004	0.005	0.004	0.004	0.008	0.007	0.008	0.002	0.002	0.017	0.013	0.015	0.001	0.002
nw	(2.16)	(2.20)	(2.58)	(0.85)	(0.91)	(2.63)	(2.15)	(2.49)	(0.28)	(0.29)	(3.67)	(2.83)	(3.07)	(0.12)	(0.22)
ww	(1.77)	(1.72)	(1.95)	(0.74)	(0.74)	(2.03)	(1.86)	(1.68)	(0.24)	(0.24)	(2.25)	(1.98)	(1.94)	(0.10)	(0.18)
CP	0.170	0.172	0.250	0.100	-0.163	0.214	0.207	0.316	0.109	-0.435	0.273	0.254	0.442	0.152	-0.755
nw	(5.07)	(5.06)	(5.80)	(1.27)	(-1.63)	(3.79)	(3.59)	(4.22)	(0.84)	(-2.35)	(3.31)	(2.99)	(3.94)	(0.81)	(-2.67)
ww	(3.03)	(2.98)	(3.31)	(0.90)	(-1.15)	(2.16)	(1.99)	(2.59)	(0.63)	(-1.72)	(1.82)	(1.61)	(2.33)	(0.64)	(-2.12)
$\bar{J}$		0.273	0.091	0.376	0.075		-1.039	-1.296	-1.009	-1.067		-2.775	-3.218	-2.988	-2.927
nw		(0.65)	(0.23)	(0.60)	(0.12)		(-1.53)	(-1.93)	(-1.02)	(-1.03)		(-2.65)	(-3.11)	(-2.04)	(-1.95)
ww		(0.37)	(0.13)	(0.47)	(0.09)		(-0.84)	(-1.04)	(-0.76)	(-0.77)		(-1.55)	(-1.79)	(-1.65)	(-1.55)
$c\bar{J}$			-0.090	-0.067	-0.062			-0.126	-0.117	-0.121			-0.218	-0.232	-0.225
nw			(-3.20)	(-1.74)	(-1.82)			(-2.47)	(-1.63)	(-1.90)			(-2.62)	(-2.10)	(-2.37)
ww			(-1.10)	(-0.98)	(-0.98)			(-1.36)	(-1.17)	(-1.18)			(-1.57)	(-1.64)	(-1.58)
$\bar{H}$				-0.419	-0.452				-0.757	-0.907				-1.243	-1.506
nw				(-4.53)	(-4.44)				(-5.23)	(-5.17)				(-6.06)	(-5.94)
ww				(-2.99)	(-3.05)				(-3.18)	(-3.50)				(-3.79)	(-4.15)
$\hat{F}_1$					0.057					0.139					0.226
nw					(2.82)					(3.71)					(3.91)
ww					(1.85)					(2.38)					(2.73)
$\hat{F}_2$					-0.008					-0.023					-0.047
nw					(-0.88)					(-1.77)					(-1.89)
ww					(-0.59)					(-0.92)					(-1.32)
$\hat{F}_3$					-0.077					-0.138					-0.219
nw					(-3.56)					(-3.40)					(-3.62)
ww					(-2.33)					(-2.18)					(-2.43)
$\hat{F}_4$					-0.073					-0.111					-0.188
nw					(-2.83)					(-2.25)					(-2.35)
ww					(-2.05)					(-1.88)					(-2.20)
$\hat{F}_5$					-0.034					-0.078					-0.110
nw					(-2.01)					(-2.79)					(-2.70)
ww					(-1.50)					(-2.02)					(-1.90)
$\hat{F}_6$					0.036					0.057					0.076
nw					(2.39)					(2.06)					(1.77)
ww					(2.10)					(1.93)					(1.70)
$\hat{F}_7$					0.048					0.033					0.043
nw					(1.78)					(1.12)					(0.84)
ww					(1.58)					(0.66)					(0.65)
$\hat{F}_8$					-0.013					-0.048					-0.116
nw					(-0.57)					(-1.56)					(-2.20)
ww					(-0.40)					(-0.98)					(-1.52)
Adj. $R^2$	0.08	0.08	0.11	0.23	0.30	0.05	0.05	0.07	0.24	0.36	0.04	0.05	0.07	0.26	0.38



Table 9

Predictability Regressions of Treasury Bonds Robustness

The table reports the regression results for bond excess returns. Each month we run the following regression:  $rx_{t+12}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}VRP_t + \sum_{j=2}^{11}\beta_j^{(\tau)}M_t + \epsilon_{t+12}^{(\tau)}$ , where  $rx_{t+12}^{(\tau)}$  are the annual holding period returns of Treasury bonds in excess of the 1 year yield,  $VRP_t$  is the variance risk premium and  $M_{t,j}$  collectively are the CP factor, the mean jump size, the Duffee hidden factor, and the Ludvigson and Ng macro factors. Coefficients are estimated with ordinary-least squares.  $t$ -statistics are in brackets and are calculated using Newey and West (1987) and Wei and Wright (2010) standard errors, respectively. The sample spans the period from January 1990 to September 2010 and data is monthly.

	2y					3y					4y					5y				
VRP	0.022	0.014	0.016	-0.007	-0.009	0.040	0.023	0.026	-0.013	-0.018	0.044	0.022	0.024	-0.022	-0.028	0.047	0.020	0.019	-0.031	-0.036
nw	(3.24)	(1.94)	(2.05)	(-0.57)	(-0.70)	(3.07)	(1.76)	(1.82)	(-0.59)	(-0.76)	(2.43)	(1.22)	(1.23)	(-0.69)	(-0.85)	(2.03)	(0.88)	(0.79)	(-0.77)	(-0.88)
ww	(2.68)	(1.44)	(1.61)	(-0.48)	(-0.59)	(2.55)	(1.29)	(1.42)	(-0.50)	(-0.63)	(2.06)	(0.90)	(0.97)	(-0.59)	(-0.71)	(1.84)	(0.69)	(0.66)	(-0.70)	(-0.77)
CP	0.630	0.582	0.743	0.774	0.202	1.089	0.991	1.194	1.042	0.230	1.545	1.413	1.524	1.203	0.242	1.647	1.485	1.364	0.737	-0.214
nw	(6.21)	(5.38)	(5.01)	(3.81)	(0.82)	(5.70)	(5.01)	(4.44)	(2.69)	(0.48)	(5.88)	(5.26)	(4.24)	(2.20)	(0.35)	(5.06)	(4.51)	(3.05)	(1.07)	(-0.24)
ww	(4.04)	(3.50)	(3.73)	(2.77)	(0.57)	(3.67)	(3.18)	(3.33)	(1.97)	(0.34)	(3.77)	(3.30)	(3.20)	(1.62)	(0.26)	(3.41)	(2.97)	(2.45)	(0.82)	(-0.19)
$\hat{J}$		-6.861	-7.240	-8.091	-7.882		-13.793	-14.269	-15.426	-14.491		-18.554	-18.814	-19.774	-18.209		-22.839	-22.555	-23.009	-21.385
nw		(-5.90)	(-6.25)	(-6.15)	(-5.72)		(-6.20)	(-6.28)	(-5.78)	(-5.21)		(-5.82)	(-5.81)	(-5.07)	(-4.50)		(-5.47)	(-5.47)	(-4.59)	(-4.08)
ww		(-4.24)	(-4.44)	(-4.65)	(-4.26)		(-4.63)	(-4.69)	(-4.73)	(-4.24)		(-4.66)	(-4.61)	(-4.41)	(-3.91)		(-4.80)	(-4.67)	(-4.26)	(-3.83)
$c\hat{f}$			-0.187	-0.329	-0.366			-0.235	-0.436	-0.535			-0.129	-0.356	-0.505		0.140	-0.055	-0.214	
nw			(-1.85)	(-3.33)	(-3.86)			(-1.20)	(-2.18)	(-2.77)			(-0.47)	(-1.22)	(-1.78)		(0.41)	(-0.15)	(-0.60)	
ww			(-1.40)	(-2.16)	(-2.39)			(-0.93)	(-1.50)	(-1.82)			(-0.37)	(-0.87)	(-1.22)		(0.33)	(-0.11)	(-0.43)	
$\hat{H}$				-0.857	-1.069				-1.889	-2.315				-2.638	-3.239			-3.535	-4.169	
nw				(-3.14)	(-3.46)				(-3.39)	(-3.79)				(-3.27)	(-3.70)			(-3.44)	(-3.76)	
ww				(-2.38)	(-2.76)				(-2.68)	(-3.10)				(-2.63)	(-3.07)			(-2.86)	(-3.25)	
$\hat{F}_1$				0.175				0.277						0.336					0.332	
nw				(4.33)				(3.55)						(2.98)					(2.34)	
ww				(2.47)				(2.07)						(1.78)					(1.48)	
$\hat{F}_2$				-0.016				-0.031						-0.055					-0.069	
nw				(-0.78)				(-0.76)						(-0.92)					(-0.94)	
ww				(-0.48)				(-0.50)						(-0.63)					(-0.66)	
$\hat{F}_3$				-0.141				-0.188						-0.220					-0.212	
nw				(-3.10)				(-2.00)						(-1.60)					(-1.19)	
ww				(-2.01)				(-1.38)						(-1.13)					(-0.91)	
$\hat{F}_4$				-0.076				-0.026						0.037					0.054	
nw				(-1.13)				(-0.19)						(0.18)					(0.20)	
ww				(-0.90)				(-0.16)						(0.16)					(0.19)	
$\hat{F}_5$				-0.099				-0.174						-0.234					-0.274	
nw				(-2.86)				(-2.47)						(-2.29)					(-2.10)	
ww				(-2.12)				(-1.92)						(-1.83)					(-1.76)	
$\hat{F}_6$				0.091				0.148						0.177					0.203	
nw				(2.79)				(2.25)						(1.88)					(1.70)	
ww				(1.95)				(1.67)						(1.44)					(1.37)	
$\hat{F}_7$				0.028				0.024						0.021					-0.002	
nw				(0.62)				(0.27)						(0.16)					(-0.01)	
ww				(0.44)				(0.20)						(0.12)					(-0.01)	
$\hat{F}_8$				-0.021				-0.056						-0.100					-0.131	
nw				(-0.51)				(-0.72)						(-0.89)					(-0.92)	
ww				(-0.35)				(-0.50)						(-0.64)					(-0.70)	
Adj. $R^2$	0.19	0.33	0.35	0.51	0.56	0.16	0.32	0.32	0.48	0.52	0.16	0.31	0.30	0.45	0.47	0.12	0.27	0.26	0.41	0.42

**Table 10**  
**Model Calibration Parameter Setting**

This table reports the calibration parameter values for the real economy model similar as in Bollerslev, Tauchen, and Zhou (2009) and Zhou (2010). The Campbell-Shiller linearization constants are  $\kappa_1 = 0.9$  and hence  $\kappa_0 = 0.3251$ . The inflation dynamics parameters are adapted from Gallmeyer, Hollifield, Palomino, and Zin (2009) for our sample period of January 1990 to September 2010.

**PANEL A: INFLATION DYNAMICS**

	Model I	Model II	Model III	Model IV	Model V
Constant	$a_\pi = 8 \times 10^{-4}$	$a_\pi = 8 \times 10^{-4}$	$a_\pi = 8 \times 10^{-4}$	$a_\pi = 8 \times 10^{-4}$	$a_\pi = 8 \times 10^{-4}$
Persistence	$\rho_\pi = 0.60$	$\rho_\pi = 0.60$	$\rho_\pi = 0.60$	$\rho_\pi = 0.60$	$\rho_\pi = 0.60$
Autonomous	$\varphi_\pi = 0.0104$	$\varphi_\pi = 0.2221$	$\varphi_\pi = 0.0073$	$\varphi_\pi = 0.0073$	$\varphi_\pi = 0.006$
Consumption			$\varphi_{\pi g} = -0.1570$		$\varphi_{\pi g} = -0.1282$
Uncertainty				$\varphi_{\pi\sigma} = 0.2324$	$\varphi_{\pi\sigma} = 0.1897$

**PANEL B: REAL ECONOMY**

	$\delta = 0.997$
Preference	$\gamma = 2$
	$\psi = 1.5$
	$\mu_g = 0.0015$
Endowment	$a_\sigma = 0.0011$
	$\rho_\sigma = 0.5$
	$a_q = 2 \times 10^{-5}$
Uncertainty	$\rho_q = 0.98$
	$\varphi_q = 0.006$

**Table 11**  
**Calibrated Model-Implied Financial Market Risk Premia**

This table reports the calibration output values for bond risk premia and variance risk premia from the stochastic uncertainty model of consumption and inflation dynamics used in this paper. The observed bond risk premia and variance risk premia are from the empirical exercise of this paper for the sample period of 1990-2010.

**PANEL A: BOND RISK PREMIA**

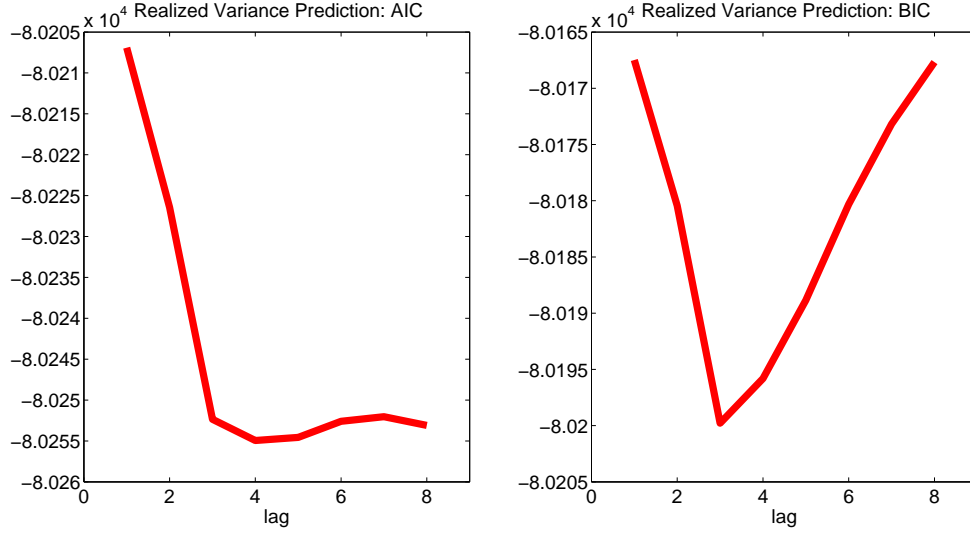
	Data	Real Model	Model I	Model II	Model III	Model IV	Model V
2 Month Bill	0.33	1.74	1.82	1.85	0.93	1.26	0.69
3 Month Bill	0.46	2.61	2.74	2.84	1.13	1.89	0.84
4 Month Bill	0.45	3.05	3.20	3.38	1.08	2.19	0.80
5 Month Bill	0.67	3.27	3.44	3.68	0.97	2.34	0.71
6 Month Bill	0.75	3.38	3.56	3.85	0.86	2.42	0.62

**PANEL B: VARIANCE RISK PREMIUM, EQUITY PREMIUM AND RISK-FREE RATE**

Variance Risk Premium	Data	Model
Mean	16.22	10.84
Std Dev	15.94	10.34
Skewness	2.30	1.87
Kurtosis	9.51	8.04

	Data	Model
Equity Premium	3.58	5.61
Equity Volatility	14.60	21.91
Risk-Free Rate	1.13	1.12
Risk-Free Rate Volatility	3.37	14.61

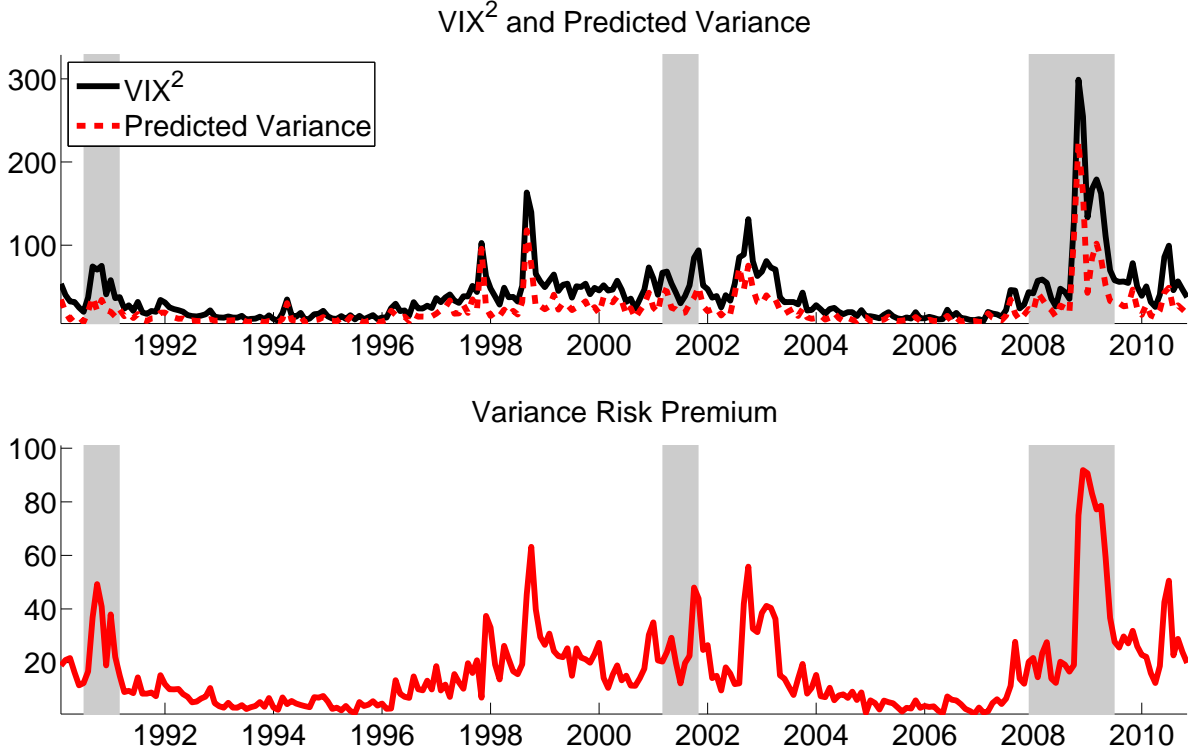


**Figure 1. AIC and BIC**

This Figure plots the AIC (left) and BIC (right) using different lag length  $k$  in equation:

$$RV_{t+22,\text{mon}} = \alpha + \beta_D RV_t + \beta_W RV_{t,\text{week}} + \beta_M RV_{t,\text{mon}} + \sum_{i=1}^k \beta_{V,i} \text{VIX}_{t-i}^2 + \epsilon_{t+22,\text{mon}},$$

where  $RV_{t,\text{week}} = 1/5 \sum_{j=0}^4 RV_{t-j}$ ,  $RV_{t,\text{mon}} = 1/22 \sum_{j=0}^{21} RV_{t-j}$  and  $\text{VIX}_t^2$  is the square of the daily VIX index divided by  $12 \times 10^4 \times 30$ . We use daily data from January 1990 to February 2011.

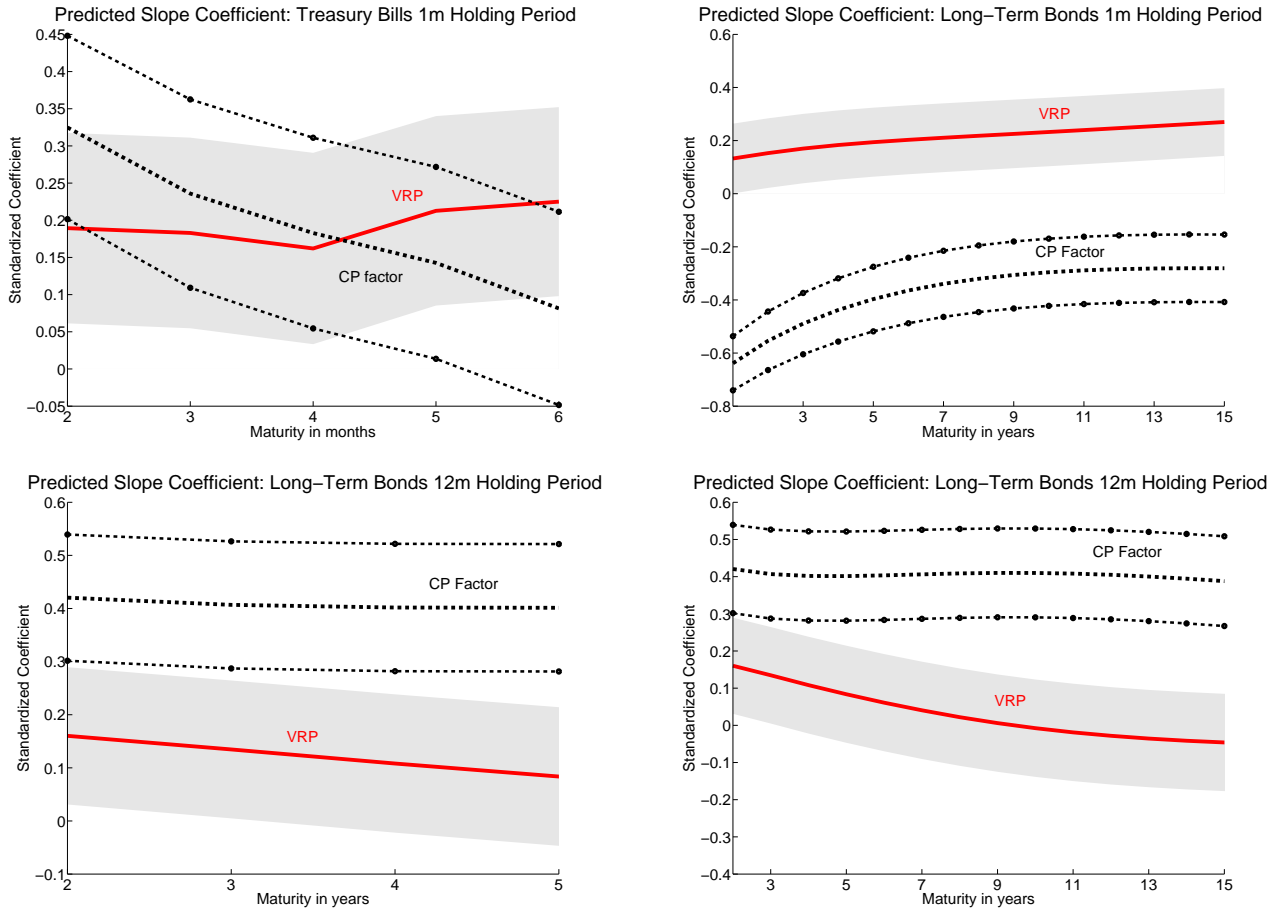


**Figure 2. Market Variance Risk Premium**

The upper panel plots the  $VIX^2$  together with the forecasted realized variance which we calculate from a projection:

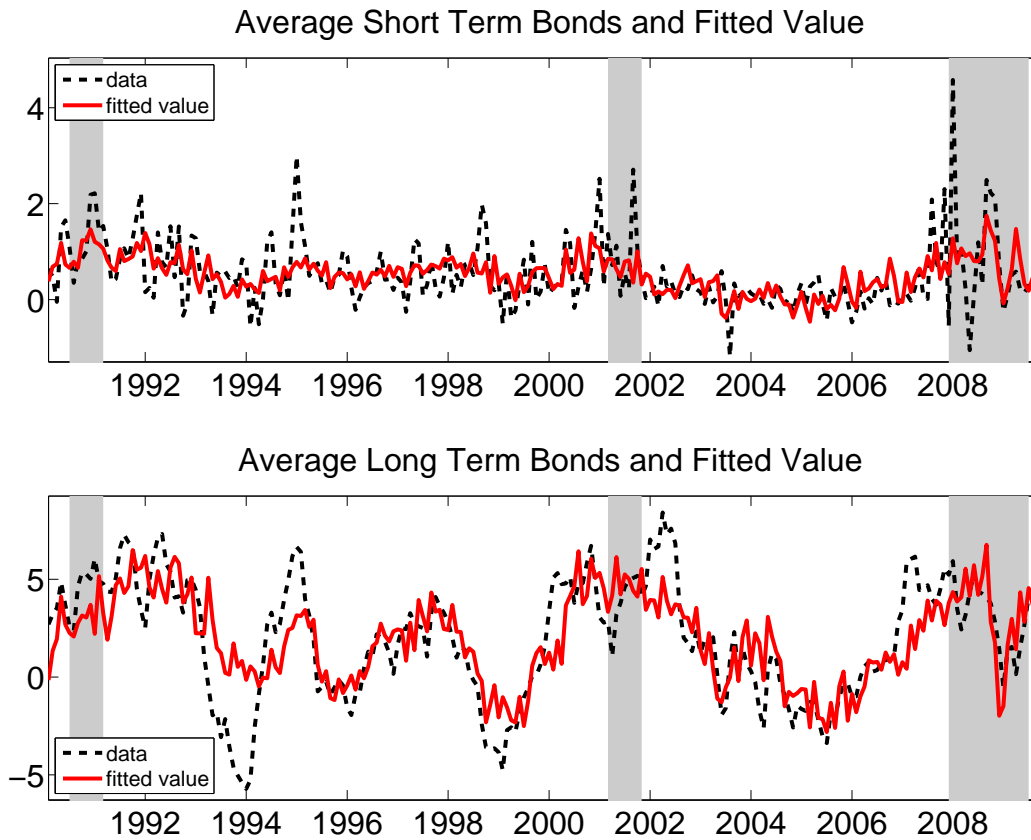
$$RV_{t+22,\text{mon}} = \alpha + \beta_D RV_t + \beta_W RV_{t,\text{week}} + \beta_M RV_{t,\text{mon}} + \sum_{i=1}^k \beta_{V,i} VIX_{t-i}^2 + \epsilon_{t+22,\text{mon}},$$

where  $RV_{t,\text{week}} = 1/5 \sum_{j=0}^4 RV_{t-j}$ ,  $RV_{t,\text{mon}} = 1/22 \sum_{j=0}^{21} RV_{t-j}$  and  $VIX_t^2$  is the square of the daily VIX index divided by  $12 \times 10^4 \times 30$  to convert numbers into a monthly quantity that is comparable to  $RV_{t,\text{mon}}$ .  $RV_t$  represents the daily realized variance calculated using 5 min squared returns on the S&P 500 futures.



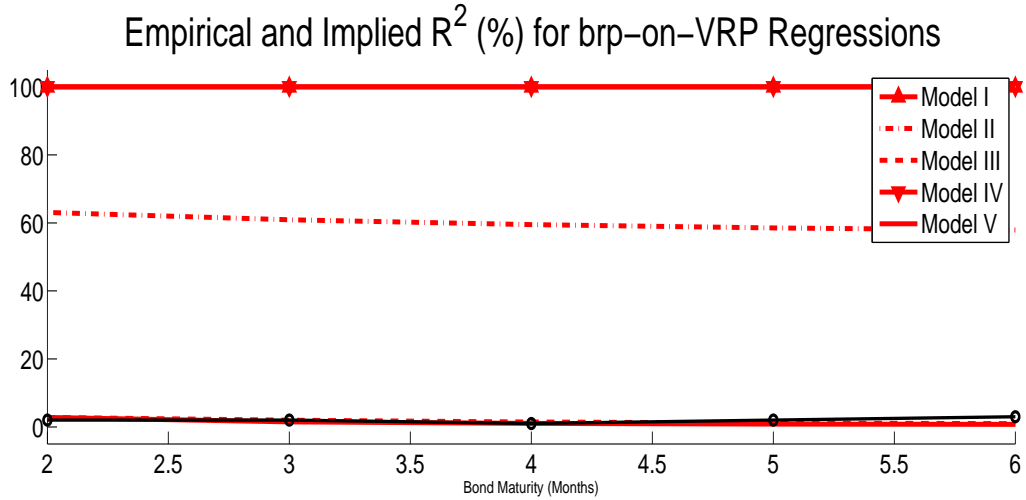
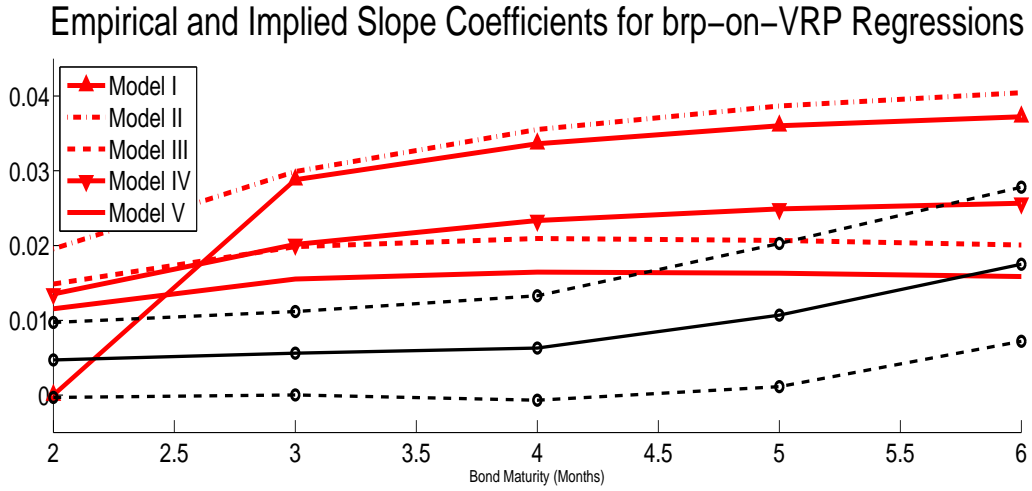
**Figure 3. Estimated Slope Coefficients from Univariate Regressions**

The upper left (lower left) figure plots the estimated slope coefficient from regressing risk premia of Treasury bills (bonds) with a 1 (12) month holding period on the variance risk premium (bold line). The upper right (lower right) figure plots the estimated slope coefficient from regressing Treasury bond risk premia with a 1 (12) month holding period on the variance risk premium (bold line). The dashed line represents the estimated slope coefficient for the Cochrane and Piazzesi (2005) factor, CP. The dash-dot lines are the 95 percent confidence bands for the CP factor, and the shaded areas represent 95 percent confidence bounds for the variance risk premium. The right two panels are using Treasury bonds up to 15 year maturity from the GSW dataset. The slope coefficients are standardized meaning the variables have zero mean and a standard deviation of one. Coefficients are estimated using monthly data from January 1990 to September 2010.



**Figure 4. Ex Post versus Ex Ante Bond Risk Premia in Short and Long Horizons**

The upper panel plots the average 1 month bond risk premium for Treasury bills with maturities 2 to 6 months (dashed line) together with the fitted value from a regression (bold line). The lower panel plots the average 1 year bond risk premium for Treasury bonds with maturities 2 to 5 years (dashed line) together with the fitted value from a regression (bold line).



**Figure 5. Model-Implied and Estimated Slopes and  $R^2$ s for 2-6 Month T-bills**

The figure shows the calibrated model-implied slope coefficients and  $R^2$ s (thick lines) for regressing the 2-6 months Treasury bill 1 month excess returns on the variance risk premium, along with their estimated empirical counterparts (thin lines with circles) and 95 percent confidence bands of the slope coefficients (thin dashed lines with circles).