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Trading and Information Diffusion in Over-the-Counter Markets

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Trading and Information Diffusion in Over-the-Counter Markets*

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Abstract

We propose a model of trade in over-the-counter (OTC) markets in which each dealer with private information can engage in bilateral transactions with other dealers, as determined by her links in a network. Each dealer's strategy is represented as a quantity-price schedule. We analyze the effect of trade decentralization and adverse selection on information diffusion, expected profits, trading costs and welfare. Information diffusion through prices is not affected by dealers' strategic trading motives, and there is an informational externality that constrains the informativeness of prices. Trade decentralization can both increase or decrease welfare. A dealer's trading cost is driven by both her own and her counterparties' centrality. Central dealers tend to learn more, trade more at lower costs and earn higher expected profit.

JEL Classifications: G14, D82, D85

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1 Introduction

A vast proportion of financial assets is traded in over-the-counter (OTC) markets. In these markets, transactions are bilateral, prices are dispersed, trading relationships are persistent, and typically, a few large dealers intermediate a large share of the trading volume. In this paper, we explore a novel approach to modeling OTC markets that reflects these features.

In our model, each dealer with private information can engage in several bilateral transactions with her potential trading partners, as determined by her links in a network. Each dealer's strategy is represented as a quantity-price schedule. Our focus is on how decentralization (characterized by the structure of the dealer network) and adverse selection jointly influence information diffusion, expected profits, trading costs, and welfare. We prove that information diffusion through prices is not affected by strategic considerations in a well-defined sense. We show that each equilibrium price depends on all the information available in the economy, incorporating even the signals of dealers located far from a given transaction. We identify an informational externality that constrains the informativeness of prices. We highlight that decentralization can both increase or decrease welfare and that an important determinant of a dealer's trading cost besides her own centrality is the centrality of her counterparties. Using an example calibrated to securitization markets, we argue that in realistic interdealer networks, more central dealers learn more, trade more at lower costs, and earn higher expected profit. However, we also explain why in some special cases, more-connected dealers can earn a lower expected profit.

In our main specification, there are n risk-neutral dealers organized in a dealer network. Intuitively, a link between i and j indicates that they are potential counterparties in a trade. There is a single risky asset in zero net supply. The final value of the asset is uncertain and interdependent across dealers, with an arbitrary correlation coefficient controlling the relative importance of the common and private components. Each dealer observes a private signal about her value, and all dealers have the same quality of information. Since the values are interdependent, it is valuable to infer each other's signals. Values and signals are drawn from a known multivariate normal distribution. Each dealer simultaneously chooses her trading strategy, understanding her price effect given other dealers' strategies. For any private signal, each dealer's trading strategy is a generalized demand function that specifies the quantity of the asset she is willing to trade with each of her counterparties, depending on the vector of prices in the transactions in which she engages. Each dealer, in addition to trading with other dealers, trades with price-sensitive customers. In equilibrium, prices and quantities must be consistent with the set of generalized demand functions and the market clearing conditions for each link. We refer to this structure as the OTC game. The OTC game is, essentially, a generalization of the Vives (2011) variant of Kyle (1989) to networks. We consider general connected networks.

We show that the equilibrium beliefs in the OTC game are independent of dealers' strategic considerations. In fact, we construct a separate game, in which dealers do not trade, that generates the same posterior beliefs. In this simpler, auxiliary game, dealers are connected in the same network and act in the same informational environment as in the OTC game. However, the dealers' aim is to make a best guess of their own value conditional on their signals and the guesses of the other dealers to which they are connected. We refer to this structure as the conditional guessing game. Because each dealer's equilibrium guess depends on her neighbors' guesses, and through those, on her neighbors' neighbors' guesses, etc., each equilibrium guess partially incorporates the private information of all the dealers in a connected network. However, dealers do not internalize how the informativeness of their guess affects others' decisions, and the equilibrium is typically not informationally efficient. That is, dealers tend to put too much weight on their own signal, thereby making their guess inefficiently informative about the common component.

In the OTC game, we show that each equilibrium price is a weighted sum of the posterior beliefs of the counterparties that participate in the transaction; hence, it inherits the main properties of the beliefs. In addition, each dealer's equilibrium position is proportional to the difference between her expectation and the price. Therefore, a dealer tends to sell at a price higher than her belief to relatively optimistic counterparties and buys at a price lower than her belief from pessimists. This results in dispersed prices and profitable intermediation for dealers with many counterparties, as is characteristic of real-world OTC markets. The proportionality coefficient of a dealer's position is the inverse of her price impact in that transaction. In turn, the dealer's price impact is smaller if her counterparty is less concerned about adverse selection, either because the common value component is less important or because she is more central and learns from several other prices.

To gain further insights into our main topics, we proceed in two distinct ways. First, using a network associated with the securitization market as presented by Hollifield et al. (2016) we show that more-connected dealers learn more, intermediate more, trade a larger gross volume with a lower price impact, and make more profit. We also illustrate how our parameters can be matched to the data and contrast our predictions with the findings from the empirical literature across various markets.

Second, we gain further insights into welfare, expected profits, and illiquidity by analyzing trade in various simple networks. In particular, we isolate the effect of decentralization by comparing the complete OTC network with centralized markets; we illustrate the role of link density by comparing circulant OTC networks in which we successively increase the number of links that each dealer has, and we analyze the effect of asymmetric number of links in the star OTC network. We show that centralized trading might not improve welfare and explain that for certain parameters, more links imply more profits only when the network exhibits assortativity.

Finally, we argue that our one-shot game can be interpreted as a reduced form of the complex dynamic bargaining process that leads to price determination in real-world OTC markets by constructing an explicit, decentralized protocol for the price-discovery process. This exercise also highlights the advantages and limitations of our static approach compared to a full dynamic treatment.

Related literature

Most models of OTC markets are based on search and bargaining (e.g., Duffie et al. (2005); Duffie et al. (2007); Lagos et al. (2008); Vayanos and Weill (2008); Lagos and Rocheteau (2009); Afonso and Lagos (2012); and Atkeson et al. (2012)). By construction, in search models, transactions are between atomistic dealers through non-persistent links. Therefore, our approach is more suitable for capturing the effects of high market concentration implied by the presence of few large dealers intermediating the bulk of the trading volume. At the same time, we collapse trade to a single period, thus missing implications of the dynamic dimension. In this sense, we view these approaches to be complementary. However, models of learning through trade based on search require non-standard structures and are difficult to compare to existing results regarding centralized markets (e.g., Duffie et al. (2009); Golosov et al. (2009)).[1] Our approach is compatible with the standard, jointly normal framework of asymmetric information and learning.

There is a growing literature studying trading in a network (e.g., Kranton and Minehart (2001); Rahi and Zigrand (2006); Gale and Kariv (2007); Gofman (2011); Condorelli and Galeotti (2012); Choi et al. (2013); Malamud and Rostek (2013); Manea (2013); Nava (2013)). These papers typically consider either the sequential trade of a single unit of the asset or a Cournot-type quantity competition.[2] In contrast, we allow agents to form (generalized) demand schedules conditioning the quantities for each of their transactions on the vector equilibrium prices in these transactions. This emphasizes that the terms of the various transactions of a dealer are interconnected in an OTC market. Additionally, to our knowledge, none of the papers within this class addresses the issue of information aggregation which is the focus of our analysis.[3]

A separate literature studies Bayesian (Acemoglu et al. (2011)) and non-Bayesian (Bala and Goyal (1998); DeMarzo et al. (2003); Golub and Jackson (2010)) learning in the context of arbitrary connected social networks. In these papers, agents update their beliefs about a payoff-relevant state after observing the actions of their neighbors in the network. Our model complements these works by considering that (Bayesian) learning occurs through trading.

The paper is organized as follows. The following section introduces the model set-up and the equilibrium concept. In Section 3, we derive the equilibrium and give sufficient conditions for its existence. We characterize the informational content of prices and characteristics of information diffusion in Section 4. In Section 5, we study expected profit, welfare, and illiquidity based on some of the most common networks and calibrate our model to securitization markets. In Section 6, we show how our one-shot game can be interpreted as a reduced form of the complex dynamic bargaining process. Finally, we conclude.

[1]The main focus of these models is the time-dimension of information diffusion across agents. In these models, incentives to share information and to learn are driven by the fact that two agents meet repeatedly or any agent meets with counterparties of their counterparties with zero probability. This is in contrast with our approach, in which dealers understand that the network structure may lead to overlapping information among their counterparties.

[2]As an exception, Malamud and Rostek (2013) also use a multi-unit double-auction set-up to model a decentralized market. However, they do not consider the problem of learning through trade.

[3]Whereas there is another stream of papers (e.g., Ozsoylev and Walden (2011); Colla and Mele (2010); Walden (2013)) that consider that traders have access to the information of their neighbors in a network, in these models, trade takes place in a centralized market.

2 A General Model of Trading in OTC Markets

2.1 The model set-up

We consider an economy with n risk-neutral dealers that trade bilaterally a divisible risky asset.[4] All trades take place at the same time. Dealers, in addition to trading with each other, also serve a price sensitive customer-base. Each dealer is uncertain about the value of the asset. This uncertainty is captured by θ^i, referred to as dealer i's value. We consider that values are interdependent across dealers. In particular, the value of the asset for dealer i can be explained by a component, $\hat{\theta}$, that is common to all dealers and a component, η^i, that is specific to dealer i such that

$$\theta^i = \hat{\theta} + \eta^i,$$

with $\hat{\theta} \sim N(0, \sigma^2_{\hat{\theta}})$, $\eta^i \sim IID\, N(0, \sigma^2_\eta)$, and $\mathcal{V}(\hat{\theta}, \eta^i) = 0$, where $\mathcal{V}(\cdot, \cdot)$ represents the variance-covariance operator. This implies that θ^i is normally distributed with mean 0 and variance $\sigma^2_\theta = \sigma^2_{\hat{\theta}} + \sigma^2_\eta$. The common value component stands for the uncertain cash-flow from the asset. The private value component is a short-cut for unmodeled differences in the utility a dealer derives from this cash-flow, because of differences in background risk, in the usage of the asset as collateral, in technologies to repackage and resell cash flows or in risk-management constraints, for example. The degree of the interdependence between dealers' values is captured by the correlation coefficient

$$\rho = \frac{\sigma^2_{\hat{\theta}}}{\sigma^2_\theta},$$

where $\rho \in [0, 1]$. This representation is useful because we can vary the degree of interdependence, ρ, while keeping the variance σ^2_θ constant.

The asset is in zero net supply. This is without loss of generality, provided supply is constant. We do not assume any constraints on the sizes or signs of dealers' positions.

We assume that each dealer receives a private signal, s^i, such that

$$s^i = \theta^i + \varepsilon^i,$$

where $\varepsilon^i \sim IID\, N(0, \sigma^2_\varepsilon)$ and $\mathcal{V}(\theta^j, \varepsilon^i) = 0$, for all i and j.

[4] While our paper focuses exclusively on over-the-counter markets, in online Appendix C we show how our framework can be generalized to model other partially segmented markets.

Dealers are organized into a trading network, g. A link $ij \in g$ implies that i and j are potential trading partners, or neighbors in the network g. Intuitively, agent i and j know and sufficiently trust each other to trade if they find mutually agreeable terms. Let g^i denote the set of i's neighbors and $m^i \equiv |g^i|$ the number of i's neighbors. If two dealers have a link, let q^i_{ij} denote the quantity that dealer i trades over link ij. The price at which trade takes place is denoted by p_{ij}. Links in the network are undirected, such that if $ij \in g$, then $ji \in g$ also. The notation reflects this property. For instance, $p_{ij} = p_{ji}$ and $q^i_{ij} = q^i_{ji}$.

Whereas our main results hold for any network, throughout the paper, we illustrate the results using two main types of networks as examples.

Example 1 *In an (n, m)* ***circulant network*** *with n odd and $m < n$ even, if dealers are arranged in a ring then each dealer is connected with $m/2$ other dealers on her left and $m/2$ on her right. The $(n, 2)$ circulant network is the circle, whereas the $(n, n-1)$ circulant network is the complete network.*

Example 2 *In a* ***star network****, one dealer is connected with $n-1$ other dealers, and no other links exist.*

We define a one-shot game in which each dealer chooses an optimal trading strategy, provided she takes as given others' strategies but she understands that her trade has a price effect. In particular, the strategy of dealer i is a map from the signal space to the space of *generalized demand functions*. For each dealer i with signal s^i, a generalized demand function is a continuous function $\mathbf{Q}^i : R^{m^i} \to R^{m^i}$ that maps the vector of prices[5], $\mathbf{p}_{g^i} = (p_{ij})_{j \in g^i}$, that prevail in the transactions that dealer i participates in network g into a vector of quantities she wishes to trade with each of her counterparties. The j-th element of this correspondence, $Q^i_{ij}(s^i; \mathbf{p}_{g^i})$, represents her demand function when her counterparty is dealer j, such that

$$\mathbf{Q}^i(s^i; \mathbf{p}_{g^i}) = \left(Q^i_{ij}(s^i; \mathbf{p}_{g^i})\right)_{j \in g^i}.$$

Note that a dealer can buy a given quantity at a given price from one counterparty and sell a different quantity at a different price to another at the same time. When dealer i buys

[5] A vector is always considered to be a column vector unless explicitly stated otherwise.

on the link ij, the quantity $q_{ij}^i = Q_{ij}^i(s^i; \mathbf{p}_{g^i})$ is positive. Conversely, when dealer i sells on the link ij, the quantity q_{ij}^i is negative.

The demand function of dealer i in a transaction with dealer j, $Q_{ij}^i(s^i; \mathbf{p}_{g^i})$, depends on all the prices $\mathbf{p}_{g^i}$. For example, if k is linked to i who is linked to j, a high demand from dealer k might raise the bilateral price p_{ki}. This might make dealer i revise her estimate of her value upwards and adjust her quantity supplied to both k and j accordingly. However, $Q_{ij}^i(s^i; \mathbf{p}_{g^i})$ depends only on $\mathbf{p}_{g^i}$, not on the full price vector. This emphasizes a critical feature of OTC markets, namely, that the price and quantity traded in a bilateral transaction are known only by the two counterparties involved in the trade and not immediately revealed to all market participants. Whereas OTC trading protocols do not typically involve the submission of full demand schedules, we think of generalized demand functions as a reduced-form price determination mechanism that captures the repeated exchange of limit and market orders (i.e., the offer and acceptance of quotes) across fixed counterparties that have persistent links, within a short time-interval. To illustrate this mapping, we explicitly model the price-discovery process in Section 6. This also shows why our specification need not rely on the implicit assumption of a Walrasian auctioneer.

Apart from trading with each other, each dealer also serves a price-sensitive customer base. Customers have quadratic preferences for holding a quantity q of the asset. We assume that a dealer i uses each link ij to satisfy an exogenously given fraction of her customer base. In particular, we consider that dealer i trades with the customers she associates to the link ij at the same price she trades with dealer j, p_{ij}, adjusted by an exogenous markup. This implies that for each transaction between i and j, the customer base generates a downward-sloping demand

$$D_{ij}(p_{ij}) = \beta_{ij} p_{ij}, \tag{1}$$

where the constant $\beta_{ij} < 0$ is a summary statistic for dealer i and j's customers' preferences and the markup that the dealers charge.[6] Just as the dealers do, customers in our model take the network structure as given and do not search across dealers for better prices. This specification captures in reduced form the fact that clients in OTC markets typically have very

[6]For example, suppose that the marginal utility of each customer buying quantity q is $\frac{1}{\beta} q$. If a customer is associated to a link ij, she will pay $p_{ij}(1 + \mu_{ij})$ per unit where μ_{ij} is the markup. Then, her inverse demand function is given by $\frac{1}{\beta} q = p_{ij}(1 + \mu_{ij})$, that is $\beta_{ij} = \beta(1 + \mu_{ij})$.

few and long-lasting dealer relationships. For instance, Hendershott et al. (2016) document that a large group of clients in the corporate bond market trade with a single dealer annually.

The expected payoff for dealer i corresponding to the strategy profile $\left\{\mathbf{Q}^i\left(s^i;\mathbf{p}_{g^i}\right)\right\}_{i\in\{1,\ldots,n\}}$ is

$$E\left[\sum_{j\in g^i} Q^i_{ij}(s^i;\mathbf{p}_{g^i})\left(\theta^i - p_{ij}\right)|s^i,\mathbf{p}_{g^i}\right], \tag{2}$$

where p_{ij} are the elements of the bilateral clearing price vector $\mathbf{p}$ defined by the smallest element of the set

$$\widetilde{\mathbf{P}}\left(\left\{\mathbf{Q}^i\left(s^i;\mathbf{p}_{g^i}\right)\right\}_i,\mathbf{s}\right)\equiv\left\{\mathbf{p}\;\middle|\;Q^i_{ij}\left(s^i;\mathbf{p}_{g^i}\right)+Q^j_{ij}\left(s^j;\mathbf{p}_{g^j}\right)+\beta_{ij}p_{ij}=0,\,\forall\; ij\in g\right\} \tag{3}$$

by lexicographical ordering[7], if $\widetilde{\mathbf{P}}$ is non-empty. While in equilibrium q^i_{ij} and q^j_{ij} tend to have the opposite sign, $q^i_{ij}\neq -q^j_{ij}$, because the customers also trade a quantity $\beta_{ij}p_{ij}$. If $\widetilde{\mathbf{P}}$ is empty, we choose $\mathbf{p}$ to be the infinity vector and say that the market breaks down and define all dealers' payoff to be zero. We refer to the collection of rules that define a unique vector $\mathbf{p}$ for any given realization of signals and strategy profile as $\mathbf{P}\left(\left\{\mathbf{Q}^i\left(s^i;\mathbf{p}_{g^i}\right)\right\}_i,\mathbf{s}\right)$. Introducing the set (3) ensures that we can evaluate dealers' payoffs for any demand functions that dealers may choose. This will allow us to search for a Bayesian Nash equilibrium, as explained in the following section.

2.2 Equilibrium concept

The environment described above represents a Bayesian game, henceforth referred to as the OTC game. The risk-neutrality of dealers and the normal information structure allows us to search for a linear equilibrium of this game, which is defined as follows.

Definition 1 *A Linear Bayesian Nash equilibrium of the OTC game is a vector of linear generalized demand functions* $\left\{\mathbf{Q}^1(s^1;\mathbf{p}_{g^1}),\mathbf{Q}^2(s^2;\mathbf{p}_{g^2}),\ldots,\mathbf{Q}^n(s^n;\mathbf{p}_{g^n})\right\}$ *such that* $\mathbf{Q}^i(s^i;\mathbf{p}_{g^i})$

[7] The specific algorithm we choose to select a unique price vector is immaterial. To ensure that our game is well defined, we need to specify dealers' payoffs as a function of strategy profiles both on and off the equilibrium path.

solves the problem

$$\max_{(Q^i_{ij})_{j\in g^i}} E\left\{\left[\sum_{j\in g^i} Q^i_{ij}(s^i;\mathbf{p}_{g^i})\left(\theta^i - p_{ij}\right)\right]\Big| s^i,\mathbf{p}_{g^i}\right\}, \tag{4}$$

for each dealer i*, where* $\mathbf{p} = \mathbf{P}(\cdot,\mathbf{s})$.

A dealer i chooses a demand function, $Q^i_{ij}(\cdot)$, for each transaction ij, to maximize her expected profits, given her information, s^i, and given the demand functions chosen by the other dealers. Implicit in the definition of the equilibrium is that each dealer understands that she has a price impact when trading with the counterparties given by the network g. Solving problem (4) is equivalent to finding a fixed point in demand functions.

3 The Equilibrium

In this section, we derive the equilibrium in the OTC game. First, we derive the equilibrium strategies as a function of posterior beliefs. Second, we construct posterior beliefs. Third, we provide sufficient conditions for the existence of the equilibrium in the OTC game for any network.

3.1 Derivation of demand functions

Our derivation follows Kyle (1989) and Vives (2011). We conjecture an equilibrium in linear demand functions, such that the demand function of any given dealer i in the transaction with a counterparty j is

$$Q^i_{ij}(s^i;\mathbf{p}_{g^i}) = t^i_{ij}(y^i_{ij}s^i + \sum_{k\in g^i} z^i_{ij,ik}p_{ik} - p_{ij}). \tag{5}$$

We refer to t^i_{ij} as the *trading intensity* of dealer i on the link ij, whereas y^i_{ij} and $z^i_{ij,ik}$ capture the effects specific to the dealer's private signal and the price p_{ik} on the quantity that dealer i demands on the link ij. As will become clear below, dealer i's best response is (5) when all other agents' demand functions are given by (5).

As is standard in similar models, we simplify the optimization problem (4), which is defined over a function space, to finding the functions $Q^i_{ij}(s^i;\mathbf{p}_{g^i})$ point-by-point. For this, we fix a realization of the vector of signals, $\mathbf{s}$. Then, we solve for the optimal quantity q^i_{ij} that each dealer i demands when trading with a counterparty j as she takes the demand functions of the other dealers as given. Thus, we obtain dealer's i best response quantity q_{ij}^i in the transaction with dealer j for each realization of the signals. This essentially gives us a map from prices to quantities, or her demand function. We describe the procedure in detail below.

Given the conjecture (5) and market clearing

$$Q_{ij}^i(s^i;\mathbf{p}_{g^i}) + Q_{ij}^j(s^j;\mathbf{p}_{g^j}) + \beta_{ij}p_{ij} = 0, \tag{6}$$

the residual inverse demand function of dealer i in a transaction with dealer j is

$$p_{ij} = -\frac{t_{ij}^j(y_{ij}^j s^j + \sum_{k\in g^j, k\neq i} z_{ij,jk}^j p_{jk}) + q_{ij}^i}{\beta_{ij} + t_{ij}^j\left(z_{ij,ij}^j - 1\right)}. \tag{7}$$

Denote

$$I_{ij}^j \equiv -\frac{t_{ij}^j(y_{ij}^j s^j + \sum_{k\in g^j, k\neq i} z_{ij,jk}^j p_{jk})}{\beta_{ij} + t_{ij}^i\left(z_{ij,ij}^j - 1\right)} \tag{8}$$

and rewrite (7) as

$$p_{ij} = I_{ij}^j - \frac{1}{\beta_{ij} + t_{ij}^j\left(z_{ij,ij}^j - 1\right)} q_{ij}^i. \tag{9}$$

The uncertainty that dealer i faces about the signals of others is reflected in the random intercept of the residual inverse demand, I_{ij}^j, whereas her capacity to affect the price is reflected in the slope $-1/\left(\beta_{ij} + t_{ij}^j\left(z_{ij,ij}^j - 1\right)\right)$. Thus, the price p_{ij} is informationally equivalent to the intercept I_{ij}^j. This implies that finding the vector of quantities $\mathbf{q}^i = \mathbf{Q}^i(s^i;\mathbf{p}_{g^i})$ for one particular realization of the signals, $\mathbf{s}$, is equivalent to solving

$$\max_{(q_{ij}^i)_{j\in g^i}} \sum_{j\in g^i} q_{ij}^i \left(E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) + \frac{1}{\beta_{ij} + t_{ij}^j\left(z_{ij,ij}^j - 1\right)} q_{ij}^i - I_{ij}^j \right).$$

From the first-order conditions, we derive the quantities q_{ij}^i for each link of i and for each realization of $\mathbf{s}$ as

$$2\frac{1}{\beta_{ij} + t_{ij}^j\left(z_{ij,ij}^j - 1\right)} q_{ij}^i = I_{ij}^j - E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right).$$

Then, using (9), we can find the optimal demand function for each dealer i when trading with dealer j:

$$Q^i_{ij}(s^i;\mathbf{p}_{g^i}) = -\left(\beta_{ij} + t^j_{ij}\left(z^j_{ij,ij} - 1\right)\right)\left(E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.) - p_{ij}\right). \tag{10}$$

Furthermore, given our conjecture (5), equating coefficients in equation (10) implies that

$$E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.) = y^i_{ij} s^i + \sum_{k \in g^i} z^i_{ij,ik} p_{ik}.$$

However, the projection theorem implies that the belief of each dealer i can be described as a unique linear combination of her signal and the prices she observes. Thus, it must be that $y^i_{ij} = y^i$ and $z^i_{ij,ik} = z^i_{ik}$ for all i, j, and k. In other words, the posterior belief of a dealer i is given by

$$E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) = y^i s^i + \mathbf{z}_{g^i}\mathbf{p}_{g^i}, \tag{11}$$

where $\mathbf{z}_{g^i} = \left(z^i_{ij}\right)_{j \in g^i}$ is a *row* vector of size m^i. Then, we obtain that the trading intensity of dealer i is the inverse of her price impact in the transaction with dealer j, or

$$t^i_{ij} = t^j_{ij}\left(1 - z^j_{ij}\right) - \beta_{ij}. \tag{12}$$

Substituting (11) back into our conjecture (5), we obtain that the demand of dealer i in a transaction with dealer j is given by

$$Q^i_{ij}(s^i;\mathbf{p}_{g^i}) = t^i_{ij}\left(E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) - p_{ij}\right). \tag{13}$$

That is, the quantity that dealer i trades with j is the perceived gain per unit of the asset, $\left(E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) - p_{ij}\right)$, multiplied by the endogenous trading intensity parameter, t^i_{ij}. Moreover, by substituting the optimal demand function (13) into the bilateral market clearing condition (6), we obtain the equilibrium price between any pair of dealers i and j as a linear combination of the posterior beliefs of i and j:

$$p_{ij} = \frac{t^i_{ij} E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.) + t^j_{ij} E(\theta^j \left| s^i, \mathbf{p}_{g^j}\right.)}{t^i_{ij} + t^j_{ij} - \beta_{ij}}, \tag{14}$$

At this point, we depart from the standard derivation. The standard approach is to determine the coefficients of the demand function (5) using a fixed-point argument. In particular, given our conjecture (5), the bilateral clearing conditions represent a system of linear equations from which prices can be derived as an affine combination of signals. Then, the projection theorem implies that for each dealer i, the coefficients y^i and $\mathbf{z}_{g^i}$ must satisfy the following fixed-point condition:

$$\begin{bmatrix} y^i \\ \mathbf{z}_{g^i}^\top \end{bmatrix} = \mathcal{V}\left(\theta^i, \begin{bmatrix} s^i \\ \mathbf{p}_{g^i} \end{bmatrix}\right) \times \left(\mathcal{V}\begin{bmatrix} s^i \\ \mathbf{p}_{g^i} \end{bmatrix}\right)^{-1}. \tag{15}$$

Note that if (15) has a solution for each dealer i, equation (10) implies that our conjecture (5) is verified.

In general networks, this procedure yields a high dimensional problem. First, the system of bilateral clearing conditions (6) has as many equations as the number of links in the network. Second, for each dealer, we need to solve a fixed-point problem that is itself a function of her position in the network.

Our main methodological innovation is that we derive the equilibrium of the OTC game in two steps. First, we construct the equilibrium posterior beliefs without solving for the demand curve or the implied quantities and prices. For this, in Section 3.2, we introduce an auxiliary game called the *conditional guessing game*.

Second, based on the equilibrium beliefs in the conditional guessing game, we construct the equilibrium demand functions of the OTC game in Section 3.3. We provide conditions for the existence of an equilibrium. In Section 4, we also formally state and qualify the one-to-one mapping of the posterior beliefs in the two games.

3.2 Deriving posterior beliefs: The conditional guessing game

We define the conditional guessing game as follows. Consider a set of n agents that are connected in the same network g as in the corresponding OTC game. The information structure is also the same as in the OTC game. Before the uncertainty is resolved, each agent i makes a guess, e^i, about her value of the asset, θ^i. Her guess is the outcome of a function that has as arguments the guesses of other dealers she is connected to in the network g. In particular, given her signal, dealer i chooses a guess function, $\mathcal{E}^i\left(s^i; \mathbf{e}_{g^i}\right)$, that maps the vector of guesses of her neighbors, $\mathbf{e}_{g^i}$, into a guess e^i. When the uncertainty is resolved, agent i receives a payoff

$-\left(\theta^i - e^i\right)^2$, where e^i is an element of the guess vector $\mathbf{e}$ defined by the smallest element of the set

$$\Xi\left(\left\{\mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right)\right\}^i, \mathbf{s}\right) \equiv \left\{\mathbf{e} \,\middle|\, e^i = \mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right), \forall\, i\right\}, \tag{16}$$

by lexicographical ordering. We assume that if a fixed-point in (16) did not exist, then dealers would not make any guesses and their payoffs would be set to minus infinity. Essentially, the set of conditions (16) is the counterpart in the conditional guessing game of the market clearing conditions in the OTC game.

Definition 2 *An equilibrium of the conditional guessing game is given by a strategy profile* $\left(\mathcal{E}^1, \mathcal{E}^2, ..., \mathcal{E}^n\right)$ *such that each agent* i *chooses strategy* $\mathcal{E}^i : R \times R^{m^i} \to R$ *to maximize her expected payoff*

$$\max_{\mathcal{E}^i}\left\{-E\left(\left(\theta^i - \mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right)\right)^2 \left| s^i, \mathbf{e}_{g^i}\right.\right)\right\},$$

where $\mathbf{e} = \Xi\left(\cdot, \mathbf{s}\right)$.

As in the OTC game, we simplify this optimization problem and find the guess functions $\mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right)$ point-by-point. That is, for each realization of the signals, $\mathbf{s}$, an agent i chooses a guess that maximizes her expected profits, given her information, s^i, and the guess functions chosen by the other agents. Her optimal guess function is then given by

$$\mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right) = E\left(\theta^i | s^i, \mathbf{e}_{g^i}\right). \tag{17}$$

In the next proposition, we state that the guessing game has an equilibrium in any network.

Proposition 1 *In the conditional guessing game, for any network* g*, there exists an equilibrium in linear guess functions such that*

$$\mathcal{E}^i\left(s^i;\mathbf{e}_{g^i}\right) = \bar{y}^i s^i + \bar{\mathbf{z}}_{g^i}\mathbf{e}_{g^i}$$

for any i*, where* $\bar{y}^i$ *is a scalar and* $\bar{\mathbf{z}}_{g^i} = \left(\bar{z}^i_{ij}\right)_{j \in g^i}$ *is a row vector of length* m^i*.*

We derive the equilibrium in the conditional guessing game as a fixed-point problem in the space of $n \times n$ matrices. In particular, consider an arbitrary $n \times n$ matrix $V' = \left[\mathbf{v}^{i'}\right]_{i=1,..n}$ and let the guess of each agent i be

$$\overset{'}{e}^{i} = \overset{'}{\mathbf{v}}^{i}\mathbf{s}, \tag{18}$$

given a realization of the signals $\mathbf{s}$. It follows that when dealer j takes as given the choices of her neighbors, $\overset{'}{\mathbf{e}}_{g^j}$, her best response guess is

$$\overset{''}{e}^{j} = E\left(\theta^j | s^j, \overset{'}{\mathbf{e}}_{g^j}\right). \tag{19}$$

Since each element of $\overset{'}{\mathbf{e}}_{g^j}$ is a linear function of the signals and the conditional expectation is a linear operator for jointly normally distributed variables, equation (19) implies that there is a unique vector $\overset{''}{\mathbf{v}}^{j}$, such that

$$\overset{''}{e}^{j} = \overset{''}{\mathbf{v}}^{j}\mathbf{s}. \tag{20}$$

In other words, the conditional expectation operator defines a mapping from the $n \times n$ matrix $\overset{'}{V} = \left[\overset{'}{\mathbf{v}}^{i}\right]_{i=1,..n}$ to a new matrix of the same size $\overset{''}{V} = \left[\overset{''}{\mathbf{v}}^{i}\right]_{i=1,..n}$. An equilibrium of the conditional guessing game exists if this mapping has a fixed point. Proposition 1 shows the existence of a fixed point and describes the equilibrium as given by the coefficients of s^i and $\mathbf{e}_{g^i}$ in $E\left(\theta^i | s^i, \mathbf{e}_{g^i}\right)$ at this fixed point.

Next, we use the conditional guessing game to establish conditions for the existence of an equilibrium in the OTC game, and we show how to solve for the equilibrium coefficients. In the following section, we also prove that posterior beliefs of the OTC game coincide with the equilibrium beliefs in the conditional guessing game.

3.3 Solving for equilibrium coefficients and existence

In this subsection, we prove the main results of this section. In particular, we provide sufficient conditions under which we can construct an equilibrium of the OTC game building on an equilibrium of the conditional guessing game.

Proposition 2 *Let $\bar{y}^i$ and $\bar{\mathbf{z}}_{g^i} = \left(\bar{z}^i_{ij}\right)_{j \in g^i}$ be the coefficients that support an equilibrium in the conditional guessing game and let $e^i = E(\theta^i \left| s^i, \mathbf{e}_{g^i}\right)$ be the corresponding equilibrium expectation of agent i. Then, there exists a Linear Bayesian Nash equilibrium in the OTC game* *whenever $\rho < 1$ and the following system has a solution $\left\{y^i, z^i_{ij}\right\}_{i=1,..n, j\in g^i}$ such that $z^i_{ij} \in (0,2)$:*

$$\frac{y^i}{\left(1 - \sum_{k\in g^i} z^i_{ik} \frac{2-z^k_{ki}}{4-z^i_{ik}z^k_{ki}}\right)} = \bar{y}^i \tag{21}$$

$$z^i_{ij} \frac{\frac{2-z^i_{ij}}{4-z^i_{ij}z^j_{ji}}}{\left(1 - \sum_{k\in g^i} z^i_{ik} \frac{2-z^k_{ki}}{4-z^i_{ik}z^k_{ki}}\right)} = \bar{z}^i_{ij}, \forall j \in g^i.$$

All z^i_{ij} are determined by ρ and the ratio $\sigma \equiv \frac{\sigma^2_\varepsilon}{\sigma^2_\theta}$ and independent of β_{ij}. The equilibrium demand functions are given by (5) with

$$t^i_{ij} = -\beta_{ij} \frac{2 - z^j_{ij}}{z^i_{ij} + z^j_{ji} - z^i_{ij} z^j_{ji}}. \tag{22}$$

The equilibrium beliefs are $E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) = y^i s^i + \sum_{j\in g^i} z^i_{ij} p_{ij}$, whereas the equilibrium prices and quantities are

$$p_{ij} = \frac{t^i_{ij} e^i + t^j_{ij} e^j}{t^i_{ij} + t^j_{ij} - \beta_{ij}} \tag{23}$$

$$q^i_{ij} = t^i_{ij}\left(e^i - p_{ij}\right). \tag{24}$$

The conceptual advantage of our method of constructing the equilibrium compared with the standard approach is that our method is based on a simpler fixed-point problem. Indeed, in the conditional guessing game we solve for a fixed point in beliefs. This simplifies the fixed-point problem because there are only n guessing functions as opposed to $\left(\Sigma_i m^i\right)$ demand functions. Then, the system of equations (21) ensures we can map n expectations, e^i, from the conditional guessing game into $M \geq n$ prices in the OTC game in a consistent manner.

Note also that Proposition 2 also describes a simple numerical algorithm to find the equilibrium of the OTC game for any network. In particular, the conditional guessing game gives parameters $\bar{y}^i$ and $\bar{z}_{ij}$, and conditions (21) imply parameters y^i and z_{ij}. Making use of (22), we then obtain the demand functions that imply prices and quantities by (23)-(24).

The next proposition strengthens the existence result for our specific examples.

Proposition 3

1. *In any network in the circulant family, the equilibrium of the OTC game exists.*

2. *In a star network, the equilibrium of the OTC game exists.*

For the star network and the complete network, closed-form solutions are derived in Appendix B.

We showed in Proposition 2 that an equilibrium exists when the solution, z^i_{ij}, of the system (21) is in the interval $(0, 2)$. As Section 5 illustrates, apart from the networks characterized in Proposition 3, we found that the equilibrium exists for a large range of parameters for empirically relevant networks. [8]

We conclude this section with the observation that customers' demand plays a limited role in our analysis. Whereas there is no equilibrium for $\beta_{ij} = 0$, for any choice of $\beta_{ij} < 0$, prices, beliefs and scaled quantities $\frac{q^i_{ij}}{\beta_{ij}}$ are not affected. We summarize this in the following Corollary.

Corollary 1 *Prices, beliefs and scaled quantities,* $\frac{q^i_{ij}}{\beta_{ij}}$*, are independent of the slope of customers' demand,* β_{ij}*. Furthermore, if* $\beta_{ij} = \hat{\beta} \cdot \hat{\beta}_{ij}$*, where each* $\hat{\beta}_{ij}$ *is an arbitrary negative scalar and* $\hat{\beta}$ *is a positive constant, then prices, beliefs and scaled quantities remain constant and well-defined as* $\hat{\beta} \to 0$*. When* $\hat{\beta} = 0$*, the equilibrium in the OTC game does not exist.*

This result follows immediately from (22) and (24). Clearly, beliefs must be independent of customers' demand as they can be derived from the conditional guessing game where there are no customers. Quantities, q^i_{ij}, are proportional to β_{ij}, because trading intensities, t^i_{ij}, are. This follows immediately from the fact that β_{ij} is a parallel shift in expression (12), which drives the equilibrium trading intensities.

Intuitively, we need a non-zero β_{ij} because $\frac{1}{\beta_{ij}}$ serves as a finite upper bound for the price impact of an additional unit supplied in a transaction between i and j. This is apparent from (9). To see why this is essential, it is useful to think about equation (26) as a best response function for trading intensities. If β_{ij} were 0, then the counterparties' best responses would

[8] There are irregular networks for which the conditions of Proposition 2 are not satisfied for some parameters. In these cases, there is at least one agent who puts negative weight on at least one of her neighbors' expectations, that is, $\bar{z}^i_{ij} < 0$ for some i and ij. This is possible because the correlation between θ^i and e^j, conditional on all the other expectations of i's neighbors and s^i might be negative. Whereas this is still a valid equilibrium of the conditional guessing game, it results in a negative z^i_{ij} in the OTC game, which violates the second-order conditions. More details are available on request.

converge to zero as $|(1 - z_{ij})| < 1$ by the conditions required in Proposition 2. That is, trade would collapse. This is a well-known property of similar games (e.g., Kyle (1989) for the case of two agents). Based on Corollary 1, we argue that the exogenous demand from customers solves this technical problem with minimal impact on the results.

4 Information Diffusion

In this section, we discuss the informational properties of prices in the OTC market. First, we characterize the role of the market structure in the diffusion of information through prices. Second, we introduce a measure of informational efficiency and highlight inefficiencies in how agents learn from prices.

4.1 Prices and Information Diffusion

We study how the market structure affects the diffusion of information through the network or trades. For this purpose, we analyze two dimensions. First, we are interested in finding out to what extent the ability of agents to behave strategically and impact prices influences how much information gets revealed. Second, we investigate how the network structure interacts with the role of prices as information aggregators.

To evaluate the role of agents' strategic motives when trading, the conditional guessing game is a useful benchmark. This is because any considerations related to price manipulation are not present in the conditional guessing game. We establish the following result.

Proposition 4 *In any Linear Bayesian Nash equilibrium of the OTC game the vector with elements e^i defined as*

$$e^i = E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.)$$

is an equilibrium expectation vector in the conditional guessing game.

The idea behind this proposition is as follows. We have already shown that in a linear equilibrium, each bilateral price p_{ij} is a linear combination of the posteriors of i and j, $E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.)$ and $E(\theta^j \left| s^j, \mathbf{p}_{g^j} \right.)$, as described in (14). Therefore, in each transaction, given that a dealer knows her own belief, the price reveals the belief of her counterparty. Thus, when a dealer chooses her generalized demand function, she essentially conditions her expectation about the asset value on the expectations of the other dealers she is trading with. Consequently, the set of posteriors implied in the OTC game works also as an equilibrium in the conditional guessing game.

The equivalence of beliefs on the two games implies that any feature of the beliefs in the OTC game must be unrelated in any way to price manipulation, market power or other profit-related motives.

Next, we analyze the role of the network structure in how prices aggregate information. We obtain the following result for general connected networks.

Proposition 5 *Suppose that there exists an equilibrium in the OTC game. Then in any connected network g, each bilateral price is a linear combination of all signals in the economy, with strictly positive weight on each signal.*

This result suggests that a decentralized trading structure can be surprisingly effective in transmitting information. Indeed, although we consider only a single round of transactions, each price partially incorporates all the private signals in the economy. A simple way to see this is to consider the residual demand curve and its intercept, I^i_{ij}, defined in (8)-(9). This intercept is stochastic and informationally equivalent with the price p_{ij}. The chain structure embedded in the definition of I^i_{ij} is critical. The price p_{ij} gives information on I^j_i, which gives some information about the prices at which agent j trades in equilibrium. For example, if agent j trades with agent k, then p_{jk} affects p_{ij}. By the same logic, p_{jk} in turn is affected by the prices agent k trades at with her counterparties, etc. Therefore, p_{ij} aggregates the private information of signals of every agent, dealer i is indirectly connected to, even if this connection is through several intermediaries.

Typically, however, dealers in the OTC market do not learn from prices all the relevant information in the economy. This is because in a network g, a dealer i can use only m^i linear combinations of the vector of signals, $\mathbf{s}$, to infer the informational content of the other $(n-1)$ signals. In contrast, as Vives (2011) shows, in a centralized market in which each agent chooses one demand function and the market clears at a single price, a dealer i learns all the relevant information in the economy, and her posterior belief is given by $E\left(\theta^i|\mathbf{s}\right)$.

There are two special cases in which the prices are privately fully revealing if agents trade over the counter. In our context, the equilibrium prices are *privately fully revealing* if for each dealer i, $(s^i, \mathbf{p}_{g^i})$ is a sufficient statistic of the vector of signals $\mathbf{s}$, in the estimation of θ^i. The following result describes these cases.

Proposition 6

1. *In the complete network, prices are privately fully revealing.*

2. *In any connected network, g, when an equilibrium in the OTC game exists, prices are arbitrary close to privately fully revealing as ρ approaches 1. That is,*

$$\lim_{\rho \to 1} E\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) = E\left(\theta^i | \mathbf{s}\right)$$
$$\lim_{\rho \to 1} \mathcal{V}\left(\theta^i | s^i, \mathbf{p}_{g^i}\right) = \mathcal{V}\left(\theta^i | \mathbf{s}\right).$$

The first case follows immediately. In a complete network, each agent has $m^i = n - 1$ neighbors; thus, she observes $n - 1$ prices. Given that she know her own signal, she can in equilibrium invert the prices to obtain the signals of the other dealers.

The second case in Proposition 6 shows that in the common value limit, decentralization per se does not impose any friction on the information transmission process in any network. To shed more light on the intuition behind the latter result, we build intuition based on the learning process in the conditional guessing game and appeal to the equivalence of beliefs with the OTC game.

Consider the case in which $\rho = 1$. As we show in the Appendix, this implies a unique equilibrium in the conditional guessing game where each agent guess is the best guess they could obtain by observing all the signals:

$$e^i = E\left(\theta^j | \mathbf{s}\right) = E\left(\theta^i | \mathbf{s}\right) = e^j.$$

The key idea is that at $\rho = 1$, there is no private value component; hence, each agent wants to make the best guess about the common value component only. Once i can learn $E\left(\theta^i | \mathbf{s}\right)$ from its neighbor j, i can and will make the same guess. That is, this is a fixed point of the system (18)-(20) and hence an equilibrium in the conditional guessing game. Because the conditional guessing game is continuous in ρ, any equilibrium in the conditional guessing game is close to this one when ρ is close to 1. That is, it is close to be privately fully revealing in the sense of the statement. By Proposition 2, the equilibrium in the OTC game for ρ close to 1 inherits this property.

Note that we use a limit argument because when $\rho = 1$, an equilibrium in the OTC game does not exist. The intuition is essentially the Grossman-Stiglitz paradox. If prices reveal the common value, dealers do not have incentives to put weight on their private signal. However, in this case, market clearing cannot channel the private information into the prices. In contrast, we formally define the equilibrium of the conditional guessing game as a fixed point of guesses. As a consequence, the equilibrium of the conditional guessing game is well defined, even when $\rho = 1$.

4.2 Informational Efficiency

In this section, we discuss the informational efficiency of prices. We defer the discussion of allocative inefficiency to Section 5.1.

As we have seen above, information is generally not fully revealed in the equilibrium of the OTC trading game, apart from the two cases discussed in Proposition 6. Moreover, no single price fully reveals all of the information, except in the common value limit. Thus, we propose a measure of informational efficiency based on dealers' beliefs, taking into account that their learning is constrained by the network structure. More precisely, we exploit the equivalence of beliefs in Proposition 4 and define a measure of constrained informational efficiency as the negative sum of squared deviations from the true value,

$$U\left(\left\{\bar{y}^i, \bar{\mathbf{z}}_{g^i}\right\}_{i\in\{1,\ldots n\}}\right) \equiv -E\left[\sum_i \left(\theta^i - \mathcal{E}^i\left(s^i; \mathbf{e}_{g^i}\right)\right)^2 \middle| \mathbf{s}\right], \tag{25}$$

where $\mathcal{E}^i(\cdot)$ is the guess function of a dealer i in the conditional guessing game. Then, we can find conditional guessing functions $\left\{\mathcal{E}^i\left(s^i; \mathbf{e}_{g^i}\right)\right\}_{i=1\ldots n}$ that maximize our measure of constrained informational efficiency (25) subject to $\mathbf{e} = \Xi(\cdot, \mathbf{s})$ and (16). This is the planner's solution in the conditional guessing game. Alternatively, we can also look for marginal deviations in dealers' equilibrium strategies in the conditional guessing game (which, by Proposition 2, would correspond to marginal deviations from equilibrium strategies in the OTC game), which would improve constrained informational efficiency.

In general, we find that beliefs are not constrained informationally efficient. We illustrate the underlying informational externality on the circle and star networks in this section, and show that this observation is robust to a large set of random networks in Section 5.2.

Since in a circle, all dealers are symmetric, and each can learn only from two prices, this is the simplest example that can be used to recover the learning externality that leads to informational inefficiencies. To see the intuition, we use expressions (18)-(20) as an iterated algorithm of best responses. That is, in the first round, each agent i receives an initial vector of guesses, $\mathbf{e}'_{g^i}$, from her neighbors. Given this, each agent i chooses her best guess, e''^{i}, as in (19). The vector of guesses $\mathbf{e}''_{g^i}$, with elements given by (20), is the starting point for i in the following round. By definition, if the algorithm converges to a fixed point, then this is an equilibrium of the conditional guessing game.

We chose an example with eleven dealers to have a sufficient number of iterations. We illustrate the iteration rounds in Figure 1 from the point of view of dealer 6. We plot the weights with which signals are incorporated in the guess of dealer 5, 6 and 7, i.e. $\mathbf{v}^5, \mathbf{v}^6, \mathbf{v}^7$. In each figure, the dashed lines show the posteriors of dealer 5 and 7 at the beginning of each round, and the solid line shows the posterior of agent 6 at the end of each round after she observes her neighbors' guesses. We start the algorithm by assuming that the posteriors of dealer 5 and 7 are the posteriors in the common value limit, $\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}\mathbf{1}^\top s$, as illustrated by the straight dashed lines that overlap in panel A. The best response guess of dealer 6 at the end of round 1 is shown by the solid line peaking at s^6 in Panel A. The reason dealer 6 puts more weight on her signal, s^6, is that it is more informative about her value, θ^6, than the rest of the signals. Clearly, this is not a fixed point because all other agents choose their guesses in the same way. Thus, in round 2, agent 6 observes posteriors that are represented by the dashed lines shown in Panel B; these are the mirror images of the round-1 guess of dealer 6. Note that the posteriors that dealers 5 and 7 hold at the beginning of round 2 are less informative for dealer 6 than the equal-weighted sum of signals $\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}\mathbf{1}^\top s$. The reason is that, for dealer 6, her signal together with the equal-weighted sum of signals is a sufficient statistic for all the information in the economy. Thus, whereas in round 1, she learned everything she wanted to learn, in round 2, she cannot do so. The weight that dealer 5 and 7 place on their own private signals "jams" the information content of the guesses that dealer 6 observes. Nevertheless, the round-2 guesses are informative, and dealer 6 updates her posterior by placing a larger weight on her own signal, as the solid line on Panel B indicates. Since all other agents update their posterior in a similar way, the guesses that dealer 6 observes in round 3 are a mirror image of her own guess, as indicated by the dashed lines in Panel C. The solid line in Panel C represents dealer 6's guess in round 4. In Panel D, we depict the guess of dealer 6 in each round until round 5, where we reach the fixed point.

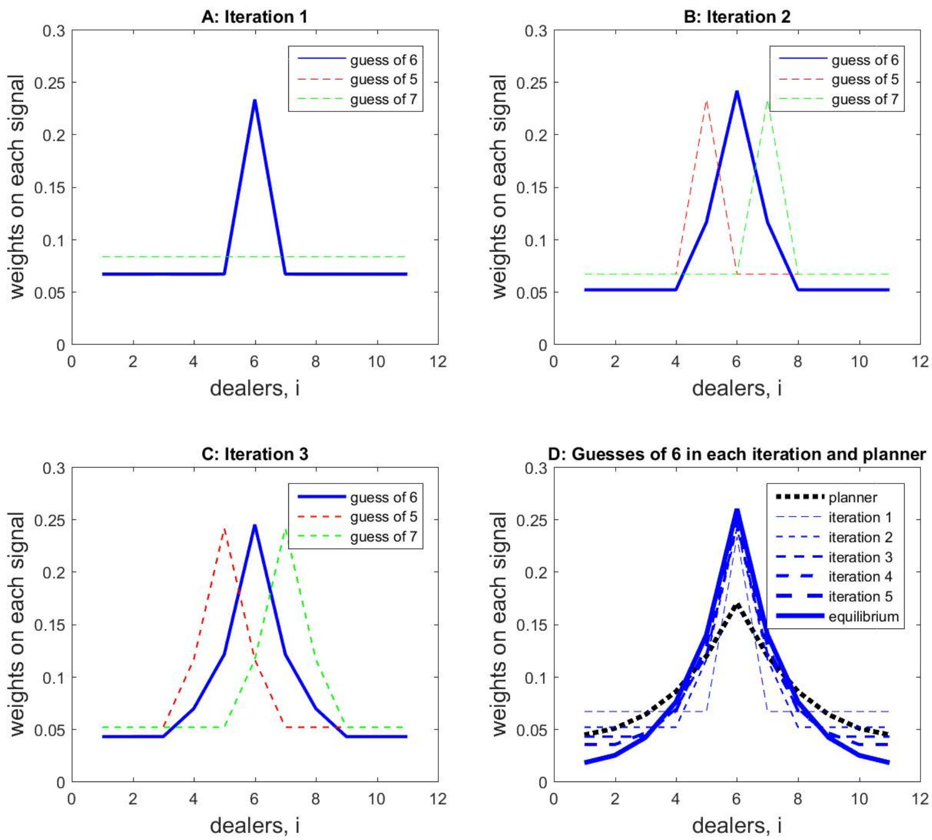

Figure 1: Best responses in the conditional guessing game in a ring network. Panel A shows of player 6's best response weight on each signal when her neighbors' guess weighs each signal uniformly at $\frac{\sigma_\theta^2}{n\sigma_\theta^2+\sigma_\varepsilon^2}\mathbf{1}^\top$. Panel B-D shows further iterations of best responses. Panel D also shows the planner's solution. The parameters are $n = 11, \rho = 0.8, \sigma_\theta^2 = \sigma_\varepsilon^2 = 1, \beta_{ij} = -1$.

The thick dashed curve in the last panel of Figure 1 shows the optimal weights on each signal in the belief of dealer 6 in the planner's solution. As is apparent, the dealer places more weight on her own signal in equilibrium than what is informationally efficient. The reason is that each agent's conditional guess function affects how much her neighbors can learn from her guess. This, in turn, affects the learning of her neighbors' neighbors, etc. Although dealers optimally choose guesses that are tilted towards their own signals, they do not internalize that they distort the informational content of these guesses for others.

In the following proposition, we show that this observation is not unique to the example. Indeed, in any star network, the sum of payoffs would increase if, starting from the decentralized equilibrium, both central and periphery dealers would put less weight on their respective signal and more weight on their neighbors' guesses.

Proposition 7 *Let* $U\left(\left\{\bar{y}^i, \bar{\mathbf{z}}_{g^i}\right\}_{i\in\{1,\ldots n\}}\right)$ *be the sum of payoffs in an star network for any given strategy profile* $\left\{\bar{y}^i, \bar{\mathbf{z}}_{g^i}\right\}_{i\in\{1,\ldots n\}}$. *Then, if* $\left\{\bar{y}^{i*}, \bar{\mathbf{z}}^*_{g^i}\right\}_{i\in\{1,\ldots n\}}$ *is the decentralized equilibrium, then*

$$\lim_{\delta\to 0}\frac{\partial U\left(\left\{\bar{y}^{i*}-\delta, \bar{\mathbf{z}}^*_{g^i}+\delta\mathbf{1}\right\}_{i\in\{1,\ldots n\}}\right)}{\partial\delta} > 0.$$

That is, starting from the equilibrium solution, marginally decreasing weights on a dealer's signal or marginally increasing weights on other dealers' guesses increase the sum of payoffs.

The intuition that we provide about why dealers overweight their signal in a circle network is informative as well about why the central dealers overweight their signal in a star network. The planner would prefer the central dealer to put less weight on her own signal because this would make her guess more informative on the common value component, that is, more useful for the periphery agents. In turn, once the guess of the central agent is more informative, the periphery agents should put more weight on that and less weight on their own signal. This explains why periphery agents overweight their signal in the decentralized solution.

Note that this informational inefficiency does not arise as a result of imperfect competition or strategic trading motives that agents have. Indeed, the equivalence between dealers' beliefs in the conditional guessing game and in the OTC game implies that this is not the case. Instead, it is a consequence of the learning externality arising from the interaction between the interdependent value environment and the network structure.

An interesting question is whether the informational inefficiency can be corrected to some degree. It is a reasonable conjecture that when signals are costly to acquire, dealers may put less weight on their signal relative to the information they learn from prices than when the signals are costless. However, how dealers would best respond to each others' choices of information precision, how the properties of the remaining equilibrium would change with the network structure, and how it would compare to the planner's solution are non-trivial questions which we leave for future research.

5 Simple networks and real-world OTC markets

In this section, we further explore the implications of our model. We proceed in two distinct ways.

First, we gain further insights into welfare, expected profits, and illiquidity by analyzing trade in simple networks. In particular, we isolate the effect of decentralization by comparing the complete OTC network with centralized markets, we illustrate the role of link density by comparing different circulant networks, and we analyze the effect of asymmetric number of links in the star OTC network.

Second, using a filtered network associated to the securitization market as presented by Hollifield et al. (2016), we argue that we should expect more connected dealers to learn more, intermediate more, trade a larger gross volume with a lower price impact, and make more profit. We illustrate how our parameters can be matched to the data and contrast our predictions with findings from the empirical literature across various markets.

5.1 Profit, welfare, and illiquidity

In this section, we start with some general observations about how the OTC market structure and adverse selection affect dealers expected profit, welfare, and illiquidity. Then, we proceed to give further insights by analyzing two simple OTC networks: the complete network and the star network.

To keep the market structures comparable, we assume that dealers have an identically sized customer pool. To simplify the welfare analysis, we assume that dealers charge zero mark-up. As before, a dealer i in the OTC market uses each link ij to satisfy an exogenous fraction of her customer base. This implies that in the centralized market, the absolute slope of the customers demand is $-\beta_V = nB$, whereas in any OTC markets with K total links the customers' demand in any transaction between dealer i and j is $-\beta_{ij} = \frac{nB}{K}$, where $B > 0$ is an exogenous constant.

5.1.1 General observations

Before the formal analysis, it is instructive to explain the intuition about what might determine traders' profit and total welfare in our economy. First of all, recall that each dealer is risk-neutral and their valuation has a private component. This implies that if all dealers would take unboundedly large negative or positive positions, that could lead to unboundedly large expected profit and welfare. As an illustration, consider the following (non-equilibrium) allocation. Let the posterior expectations e^i be determined in the equilibrium of the conditional guessing game and let prices and traded quantities be fixed at

$$p_{ij} = \frac{e^i + e^j}{2},\ q^i_{ij} = t\left(e^i - p_{ij}\right),$$

where the trading intensity, t, is the same arbitrary positive constant for each agent. It is easy to check that as each dealer trades in the direction of her posterior, increasing t without bound would increase expected profit and total welfare without bound.

In equilibrium, dealers do not take infinite positions because they are concerned about adverse selection. Whereas expressions (23) and (24) for prices and quantities are similar to the thought experiment above, the trading intensity of each dealer, t^i_{ij}, is determined as in equilibrium from the best response function given by expressions (11) and

$$t^i_{ij} = t^j_{ij}\left(1 - z^j_{ij}\right) - \beta_{ij}. \tag{26}$$

The coefficients of prices in posteriors, z^j_{ij}, depend on the network structure, and so do the trading intensities. By solving for the trading intensities while keeping z^j_{ij} and z^i_{ij} constant, we obtain the equilibrium expression (22). Note that this expression implies $\frac{\partial t^i_{ij}}{\partial z^j_{ij}} < 0$. That is, the trading intensity of dealer i is smaller if her counterparty puts a larger weight on the price p_{ij} when forming her expectation. We should expect z^j_{ij} to be higher when the price p_{ij} is a more important source of information for j because either i observes more prices, j observes fewer prices, or the correlation across values is small. Therefore, z^j_{ij} is a natural measure of how much dealer j is concerned about adverse selection when trading with dealer i.

From (9) and (26), $\frac{1}{t^i_{ij}}$ is the price impact of a unit of trade of i at link ij. That is, the more j is concerned about adverse selection, the less liquid the trade is for dealer i. Hence, she trades with a lower trading intensity. Averaging $\frac{1}{t^i_{ij}}$ over the links of i provides a natural, dealer-level illiquidity measure similar to the one used in Li and Schürhoff (2014) and Hollifield et al. (2016) for instance. We use this measure to compare illiquidity across market structures from i's perspective. We use illiquidity, cost of trading, and price impact interchangeably.

We naturally expect the average profit of dealer i

$$E\left(\sum_{ij\in g^i} q^i_{ij}\left(\theta^i - p_{ij}\right)\right) = \sum_{ij\in g^i} t^i_{ij} E\left(\left(e^i - p_{ij}\right)^2\right), \tag{27}$$

to increase with the number of links because this implies both more opportunities to trade and intermediate, in addition to higher trading intensities. Although the expected profit also depends on the gains per unit of trade, $E\left(\left(e^i - p_{ij}\right)^2\right)$ at each link, we find in all our examples that variation in trading intensities and opportunities for intermediation are the driving forces.

As a fraction of assets are allocated to customers in equilibrium, we also need their expected utility for a full welfare analysis. Customers' expected utility at link ij is proportional to the variance of the price p_{ij} since

$$E\left(\frac{\left(-\left(q^j_{ij} + q^i_{ij}\right)\right)^2}{2\beta_{ij}} + \left(q^i_{ij} + q^j_{ij}\right) p_{ij}\right) = \frac{\beta_{ij}}{2} E\left(p^2_{ij}\right) - \beta_{ij} E\left(p^2_{ij}\right) = -\frac{\beta_{ij}}{2} E\left(p^2_{ij}\right), \tag{28}$$

as follows from market clearing.

The total welfare is then the sum of profits and customers' utility summed over each link of the network.

$$\sum_{ij\in g}\left(-\frac{\beta_{ij}}{2} E\left(p^2_{ij}\right) + E\left(q^i_{ij}\left(\theta^i - p_{ij}\right)\right) + E\left(q^j_{ij}\left(\theta^j - p_{ij}\right)\right)\right). \tag{29}$$

Sometimes, it will be easier to work with the equivalent formula

$$\sum_{ij\in g}\left(\frac{\beta_{ij}}{2} E\left(p^2_{ij}\right) + E\left(\theta^i q^i_{ij}\right) + E\left(\theta^j q^j_{ij}\right)\right). \tag{30}$$

where we net out the transfers across agents obtaining the sum of expected value of allocations for dealers and customers. Provided $\beta_{ij} = \beta$ for all links, welfare is linear in β.

Finally, note that from (23), it immediately follows that price dispersion arises naturally in this model. A dealer with multiple trading partners is trading the same asset at various prices because she is facing different demand curves along each link. Just as a monopolist does in a standard price-discrimination setting, this dealer sets a higher price in markets in which the demand is higher. In fact, from (23), we can foresee that the price dispersion in our framework must be closely related to the dispersion in posterior beliefs.

5.1.2 The effect of decentralized trading: the centralized and the complete network OTC market

Comparing the equilibrium in a centralized market as described in Vives (2011) with the equilibrium in the OTC complete network isolates the effect of trade decentralization. In both cases, each trader can trade with all of the others and from Proposition 6 we have that the posterior expectations are the same (and efficiently incorporate all the information in the market). Nevertheless the prices, allocations, and welfare differ.

The main observation in this subsection is that the effect of trade decentralization on welfare and illiquidity depends on the correlation across dealer's values. Close to the common value limit, the OTC market is more liquid and provides higher total welfare than the centralized market, whereas for lower correlations across values, the opposite is typically true.

In Appendix B.1, we report closed-form solutions for the price, p_V, quantity, q_V, and the price coefficient in expectations, z_V, for centralized markets (i.e., Vives (2011)). The trading intensity of a dealer in a centralized market is given by $t_V = \frac{-\beta_V}{n(z_V-1)+2-z_V}$, which is the fixed point of expression

$$t^i = (n-1)\, t^{-i} \left(1 - z_V^{-i}\right) - \beta_V. \tag{31}$$

Equation (31) shows how the trading intensity, t^i, of dealer i responds to the trading intensity of all other agents, t^{-i}, and to their adverse selection concern, z_V^{-i}. This is the centralized counterpart of (26). The expected profit and welfare in a centralized market are calculated by trivial modifications of (27) and (29).

Importantly, Vives (2011) shows that there is linear equilibrium in centralized markets, if and only if $1 - \frac{1}{n-1} < z_V$. As we argued above, adverse selection concerns determine the slope of demand curves when dealers are risk-neutral. In a centralized market, this concern must be sufficiently strong, or the equilibrium cannot be sustained. The same condition is also required for an equilibrium to exist in an OTC market. However, with bilateral trades, it reduces to $0 < z_{ij}^j$.

In a complete network, the trading intensity is $t_{ij}^j = t_{ij}^i = t_{CN} = -\beta_{CN}\frac{1}{z_{CN}}$, and a closed-form for the adverse selection parameter z_{CN} is given in Appendix B.3. Additionally, as we explained above, we keep the total mass of customers constant across the two market structures, implying $-\beta_{CN} = \frac{2B}{n-1}$ and $-\beta_V = Bn$ for some $B > 0$.

Panels A-D in Figure 2 illustrate how dealers' profit, customers' utility, illiquidity and total welfare compare across the two markets for different values of ρ, fixing all other parameters.

In the next proposition, we state our analytical results corresponding to these figures.

Proposition 8 *Comparing a centralized market with a complete-network OTC market*

1. *When ρ or $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ is sufficiently low, such that z_V converges to $1 - \frac{1}{1-n}$ from above, the total welfare and dealers' profits are larger and illiquidity is smaller in the centralized market.*

2. *When ρ is sufficiently close to 1, then*

   (a) total welfare and customers' utility are higher and illiquidity is lower in the OTC market, whereas

   (b) dealers' profits are higher in the centralized market.

The intuition is as follows. Note first that as $z_V \to 1 - \frac{1}{n-1}$ from above, trading intensity grows without bound, $t_V \to \infty$, and illiquidity falls to zero. Because the information content of the price increases with ρ and $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ in a centralized market, so does z_V. This implies that for sufficiently low ρ and $\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$, welfare and dealers' profit are increasing without bound in a centralized market. This holds because as adverse selection becomes weaker, dealers are ready to take on very large bets. Because the private value component implies gains from trade, these large trades translate into high expected profit and high welfare. Given that quantities and profits are finite in the OTC market as long as ρ is not close to 0, it immediately follows that at least when z_V is close to $1 - \frac{1}{n-1}$, profit and welfare are larger and illiquidity is lower in centralized markets.

Perhaps more surprising is that in the common value limit, when ρ is close to 1, total welfare is higher and illiquidity is lower in the OTC market than in the centralized market.

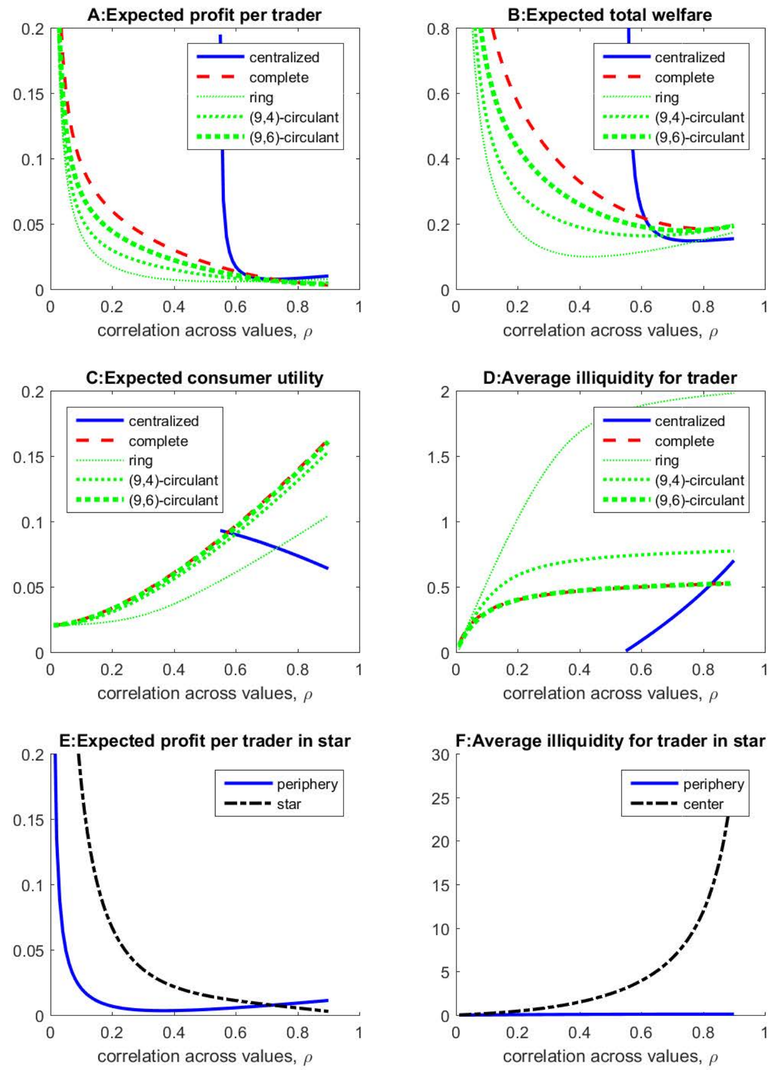

Figure 2: Expected profit, expected welfare, expected customer utility, average trading cost (illiquidity) per trader in various networks. Parameters: $n = 9, B = 1$, $\sigma_\theta^2 = 0.1, \sigma_\varepsilon^2 = 1$.

We start with the result on illiquidity. There are two forces that drive this result. First, even if price aggregated the same amount of information under the two market structures, that is, if z_V and z_{CN} were equal, mechanical differences in best responses in (31) and (26) would lead to a different outcome. Namely, the absolute values of both the slope and and the intercept of best responses are higher in the centralized market. The slope is higher because the aggregate response of $(n-1)$ counterparties is higher than that of a single counterparty, whereas the intercept is higher because all customers are present in the centralized market:

$-\beta_V = Bn > \frac{2B}{n-1} = -\beta_{CN}$. Whereas the slope and intercept have opposite effects, simple algebra shows that the sum of these forces would result in higher illiquidity in the OTC market as $\frac{\frac{1}{t_V}}{\frac{1}{t_{CN}}}|_{z_v=z_{CN}=z} < 1$.

Second, however, the single price in the centralized market aggregates more information than each of the individual prices separately in the OTC market. Indeed, it is easy to check that $z_{CN} < z_V$ for any parameter values. This tends to make illiquidity higher in the OTC market. Note that increasing the ratio $\frac{z_V}{z_{CN}}$ increases illiquidity in the centralized market relative to the OTC market as

$$\frac{\partial \frac{\frac{1}{t_V}}{\frac{1}{t_{CN}}}|_{\frac{z_V}{z_{CN}}=x}}{\partial x} = \frac{\partial \frac{-\frac{2B}{n-1}\frac{1}{z}}{\frac{-nB}{n(xz-1)+2-zx}}}{\partial x} > 0.$$

Because $\frac{z_V}{z_{CN}}$ is monotonically increasing in ρ, this force is strongest at the common value limit. As we prove in the proposition, this effect is sufficient to make illiquidity higher for in OTC market in the common value limit.

To understand the result on welfare, we start by comparing customers utility. Note first that the ratio of customers' utility in the complete network OTC market and the centralized market is the ratio of the price variance in each market: $\frac{\frac{B}{n-1}\frac{n(n-1)}{2}E\left(p_{CN}^2\right)}{\frac{nB}{2}E\left(p_V^2\right)} = \frac{E\left(p_{CN}^2\right)}{E\left(p_V^2\right)}$. Also, in the common value limit, the price variance is larger under the OTC structure as

$$\lim_{\rho\to 1}\frac{E\left(p_{CN}^2\right)}{E\left(p_V^2\right)} = \lim_{\rho\to 1}\frac{\left(\frac{1}{2+z_{CN}}\right)^2 4\left(\mathcal{V}\left(e^i\right)+\mathcal{V}\left(e^i,e^j\right)\right)}{\left(\frac{1}{2+(n-1)z_V}\right)^2 2n\left(\mathcal{V}\left(e^i\right)+(n-1)\mathcal{V}\left(e^i,e^j\right)\right)} = \left(\frac{2n-3}{n-1}\right)^2 > 1.$$

As is apparent from the second expression above, there are two forces. On the one hand, in a centralized market the variance of the price is connected to the variance of the sum of all expectations, whereas in an OTC market, it is connected to the variance of the sum of the two expectations at each link. The first one is higher, which makes customers' expected utility higher on centralized markets. On the other hand, as $z_V > z_{CN}$, the multiplier coming from trading intensities tends to push customers' utility higher in the OTC market. The ratio $\frac{z_V}{z_{CN}}$ is maximal in the common value limit, and the second force turns out to dominate the first. So in this limit, the utility is higher under the OTC structure. As Panels A-D in Figure 2 demonstrate, when ρ is smaller, the first force might dominate, thus implying that utility tends to be larger under the centralized structure.

Finally, we explain why welfare is higher but the expected profit of dealers is lower in the OTC market in the common value limit. For this, we substitute in the closed-form expressions to (30), the sum of the value of allocations to dealers and customers. Taking the limit, it is easy to show that the sum of terms corresponding to dealers is actually greater in the OTC market than in the centralized market in the common value limit $\rho \to 1$. This is due to the larger trading intensity in OTC markets in this limit. The difference between formulas (29) and (30) represents, essentially, a transfer from dealers to customers. Since this transfer is larger under the OTC market structure, this explains why welfare and profit move in opposite directions. As is apparent from the middle expression in (28), the total transfer is $\sum_{ij}\left(-\beta_{ij}E\left(p_{ij}^2\right)\right)$, twice the utility of customers, which, as we argued above, is indeed higher in the OTC complete network than in the centralized market in the common value limit.

5.1.3 The effect of more links: circulant networks with varying density

Panels A-D in Figure 2 also illustrate how welfare, customers' utility, dealers' profit and illiquidity compares in various (n,k)-circulants. With fewer links, welfare and customers' utility tends to decrease and illiquidity tends to increase, whereas dealers' profit might go either way.

As there are no explicit solutions for the conditional guessing game for circulant networks, we do not have analytical results for the circulant OTC networks either. Nonetheless, because of the symmetry, the intuition behind the numerical results is relatively simple. Decreasing the number of links in symmetric fashion has two main effects: each dealer learns less and each dealer has fewer opportunities to trade and intermediate. Learning less implies more concern about adverse selection, lower trading intensities on average, higher illiquidity and smaller variance of prices at each link (as fewer links implies lower variation in expectations as weights on the common prior increase and weights on signals decrease). Fewer opportunities to trade and smaller trading intensities imply a smaller trading volume which is the dominating force in reduced welfare. The lower price variance implies a reduced customers' utility and, by the logic explained above, a smaller total transfer from dealers to customers. Profits can go either way because the net effect of less trade and smaller transfers is ambiguous. As we see in the figure, close to the common value limit, less dense networks might be more profitable for dealers.

5.1.4 The effect of asymmetry: periphery and the central dealer in a star network

The star network is an ideal case to study the effect of asymmetry on allocations and welfare. The main result in this subsection is that central agents do not always earn higher expected profit than periphery agents. In fact, expected profit is higher for periphery agents in the common value limit.

Simple, closed-form solutions that characterize the equilibrium in a star network are spelled out in Appendix B. The next proposition and Panels E-F in Figure 2 show analytical and numerical results, respectively, concerning illiquidity, profit, and welfare.

Proposition 9 *In a star network, the following statements hold*

1. *The adverse selection concern and the trading intensity of periphery traders are higher,* $z_P > z_C$, $t_P > t_C$, *or, equivalently, the central dealer faces a more illiquid market than the periphery dealers for any* ρ.

2. *In the common value limit,* $\rho \to 1$, *central dealer's profit converge to zero while the periphery dealer"s profit is bounded away from zero as* $t_P \to -\beta$, $t_C \to 0$.

We start by comparing the trading intensities, t_P and t_C. As we noted before, for the central agent prices are privately fully revealing. That is, her posterior belief is the same as the belief of dealers in a complete network or in a centralized market. In contrast, the learning of periphery dealers is limited by the fact that they observe a single price only. As a result, the weight that a periphery dealer puts on the price is larger than the weight the central dealer puts on the same price, so $z_P > z_C$ always holds. Intuitively, the periphery dealer is more concerned about adverse selection than the central dealer, as the central dealer knows more. Therefore, from (22), the trading intensity of periphery traders is always larger, as

$$\frac{t_P}{t_C} = \frac{2 - z_C}{2 - z_P} > 1.$$

Hence, at each link the central dealer trades with a smaller intensity, or equivalently, the market is less liquid for the central dealer than for the periphery dealer.

The lesson from the above intuition is that a dealer trading with less-connected counterparties should face a higher price impact. In the special case of a star, the dealer with the least-connected counterparties is the central dealer. Thus, in the star, there is a positive correlation between price impact and number of own links. However, this is an artifact of the special structure of the star. In more general core-periphery networks the average number of links of more-connected dealers' counterparties is often greater. Indeed, in our calibrated example in Section 5.2, there is a negative relationship both between price impact and number of counterparty's links (just as in the star) and between price impact and number of own links (unlike in a star).

Now, we turn to profits and allocations in the star network. Although the central dealer trades with less intensity, she also trades and intermediates across more links, and by (23), the distance between her expectation and the price is higher than for the periphery dealer. As illustrated in Panels E-F in Figure 2, for a large set of parameter values, the effect of the smaller trading intensity is dominated, and trading as a central agent is more profitable in expectation than as a periphery agent. However, this is not always the case. As is apparent in the figure, this statement is reversed as we approach the common value limit. In fact, in the limit the expected profit of the central dealer is zero, whereas it is strictly positive for periphery dealers, as we state in Proposition 9. Again, this is related to the strong negative assortativity in a star.

To see the intuition, it is illustrative to specify how profits are determined close to the common value limit. In the limit, all dealers put diminishing weight on their own signal as they form expectations. Instead, in the conditional guessing game as $\rho \to 1$, periphery dealers put a weight of $\bar{z}_P \to 1$ on the expectation of the central dealer, while the central dealer puts equal weight on each of the periphery agents' expectations, implying $\bar{z}_C \to \frac{1}{n-1}$. Thus, by Proposition 9, as we approach the common value limit in the OTC game, this implies trading intensities of $t_P \to -\beta$, $t_C \to 0$. That is, central dealers do not trade in this limit at all, and periphery dealers trade only with customers. In the common value limit, the central agent has better information about the common value of the asset than periphery agents. Thus, as a manifestation of the no-trade theorem, there cannot be an equilibrium where these agents trade with each other. Therefore, the only remaining question is who trades with the customers. As periphery agents are more concerned about adverse selection, the price impact of the central dealer is larger. This implies that there is a price-quantity pair at which the central dealer stops trading, but at which the periphery dealer is still willing to trade. This results in positive trade between periphery and customers only.

5.2 Real-world OTC markets: a calibrated example

An attractive feature of our model is that it generates a rich set of empirical predictions. As we emphasize in this section, for any given information structure and dealer network, our model generates the full list of demand curves and the joint distribution of bilateral prices and quantities, which we can use to calculate the expectations of price dispersion, intermediation, trading volume and other financial variables. Therefore, in principle, our results could be compared to stylized facts from the growing empirical literature using transaction-level OTC data.

In this subsection, we give guidelines for future empirical work by an illustrative example. Namely, we fit our parameters to the securitization market, i.e., the secondary market of ABSs, CDOs, CMBs and non-agency CMOs, as presented by Hollifield et al. (2016). In particular, conditioning on the reported dealer network, we explore the sensitivity of various variables of interest to the parameters of our model, we find the parameters that match few selected moments reported by Hollifield et al. (2016), and we analyze the model with the fitted parameters.

5.2.1 The dealer network and sensitivity of moments

Hollifield et al. (2016) analyze the effect of the interdealer network on bid-ask spreads in various segments of the securitization market using a transaction level data set from FINRA during eight months in 2011-2012. Conveniently, in the working paper version, Hollifield et al. (2013) report the filtered, 78-dealer representation of the trading network where only links with sufficient frequency and size of transactions are reported. We reproduce this network on Figure 3. Using this network, we calculate the equilibrium of our model as described by Propositions 1and 2.

To give a sense of which empirical moments one can choose to fit our model to the data, on Figure 4 we plot a number of objects for a range of parameters. In particular, we show the mean price impact, the ratio of expected profit per dealer and expected net positions per dealer, absolute and relative price dispersion, expected total gross volume, and mean expected profit

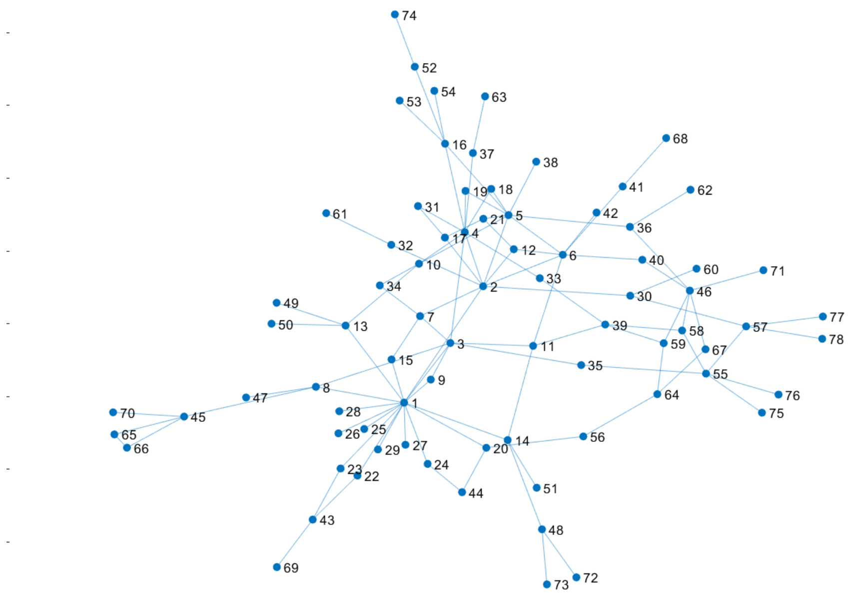

Figure 3: The inter-dealer trading network on the securitization market reproduced from Hollifield et al. (2013, Figure 5). Links are defined by trading relationships with at least 50 trade reports and at least \$10 million of original balance transacted in the sample from May 16, 2011 to February 29, 2012. Dealers are ordered and numbered by their eigenvalue centrality. (The five links unconnected to the main network are omitted).

per dealer as a function of $\sigma \equiv \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ and ρ, keeping $\sigma_\theta^2 = -\beta = 1$ fixed. Formal definitions of the variables are provided in Table 1. Note that we do not have to simulate the random variables θ^i and ε^i. For any network, given the equilibrium coefficients, z_{ij}^i, defined in Proposition 2, we can analytically calculate these objects. There is also no need to check the sensitivity to β and σ_θ^2 because the relative price dispersion and mean profit over the net position depend only on $\sigma \equiv \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ and ρ, mean price impact linearly scales with $(-\beta)$, absolute price dispersion linearly scales with σ_θ, gross volume linearly scales with $\sigma_\theta(-\beta)$ and expected profit linearly scales with $\sigma_\theta^2(-\beta)$ by Corollary 1 and Proposition 2. To make the sensitivities comparable, we scale and normalize all the quantities to be proportional to their calibrated value calculated in the next section. Note that the smallest ρ, σ combination which is plotted in each surface corresponds to the calibrated value of ρ, σ; therefore, each surface takes the value of 0 at that point.

moments	formula
mean price impact	$\frac{1}{m}\Sigma_i \frac{1}{\lvert g^i \rvert}\Sigma_{j\in g^i}\frac{1}{t^i_{ij}}$
ratio of expected profit to expected net positions per dealer	$\frac{E\left(\frac{1}{n}\Sigma_i \sum_{ij\in g^i} q^i_{ij}\left(\theta^i - p_{ij}\right)\right)}{E\left(\frac{1}{n}\Sigma_i \lvert \sum_{ij\in g^i} q^i_{ij}\rvert\right)}$
absolute price dispersion	$\sqrt{E\left(\frac{1}{m}\Sigma_{ij}\left(p_{ij}-\bar{p}\right)^2\right)}$
relative price dispersion	$\frac{\sqrt{E\left(\frac{1}{m}\Sigma_{ij}\left(p_{ij}-\bar{p}\right)^2\right)}}{E(\lvert\bar{p}\rvert)}$
expected total gross volume	$E\left(\Sigma_i \Sigma_{j\in g^i}\lvert q^i_{ij}\rvert\right)$
mean expected profit per dealer	$E\left(\frac{1}{n}\Sigma_i \sum_{ij\in g^i} q^i_{ij}\left(\theta^i - p_{ij}\right)\right)$

Table 1: Moments and definitions.

5.2.2 Matching parameters

Ideally, we would estimate our parameters and test the fit by GMM in an over-identified system of a large number of moments including the ones in Figure 4. However, because we do not have access to the data set, we limit ourselves to the following exercise. We use three moments that are reported in Hollifield et al. (2016) that have a natural counterpart in our model and respond differently to our parameters: the average customer bid-ask spread, relative price dispersion, and total volume.[9] As we show in this subsection, these three moments exactly identify the three free parameters of the model, $\sigma \equiv \frac{\sigma^2_\varepsilon}{\sigma^2_\theta}$, $(-\beta)\sigma^2_\theta$ and ρ. Then, using the fitted parameter values we calculate various indicators to assess the strength and weaknesses of our framework. We provide more details of this exercise in Appendix C.

Hollifield et al. (2016) constructs customer bid-ask spreads as follows. They identify trades when a customer sells a given quantity to a dealer, which the dealer (potentially through other dealers) passed on to another customer. The bid-ask spread is the difference between the two transaction prices with the customers as a percentage of the value of the transaction. A single dealer involved in 83% of these chains.

The variable that corresponds in our model to the customer bid-ask spread is price impact. Indeed, from (13), the theoretical price difference for customers associated to a link ij of selling and buying Δ units from dealer j, $p_{ij}(-\Delta) - p_{ij}(\Delta)$, as a percentage of the marginal valuation

[9]For simplicity, we focus on one of the two main explored segments: the "Rule 144a" securities. We choose this segment because the average trade size is larger; therefore, we expect that the filtering of our base network is less limiting.

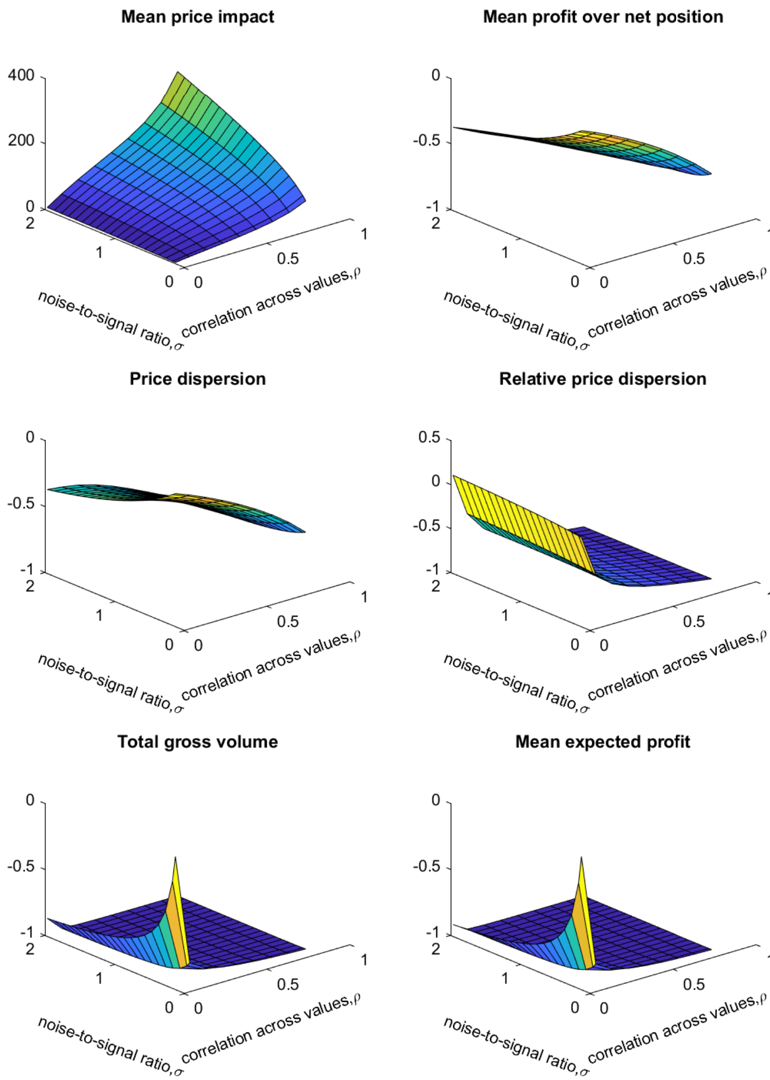

Figure 4: Moments as a function of correlation across values, ρ, and noise-to-signal ratio, $\sigma \equiv \frac{\sigma_\epsilon^2}{\sigma_\theta^2}$. From left to right and from top to bottom, we show the mean price impact, the ratio of expected profit per dealer to the expected net positions per dealer, the absolute and relative price dispersion, the expected total gross volume and the mean expected profit per dealer, keeping $\sigma_\theta^2 = (-\beta) = 1$ fixed. Each plot is scaled and normalized to show deviations proportional to implied values at calibrated parameters. The minimal $\rho = 0.014, \sigma = 0.1584$ combination in each plot is the pair of calibrated parameters.

of that customer is

$$\frac{p_{ij}(-\Delta) - p_{ij}(\Delta)}{\frac{1}{\beta}\Delta} = \frac{\left(e^i - \frac{-\Delta}{t_{ij}^i}\right) - \left(e^i - \frac{\Delta}{t_{ij}^i}\right)}{\frac{1}{\beta}} = \frac{2\beta}{t_{ij}^i}. \tag{32}$$

By (22), for any given dealer and link, this is a constant pinned down by $\sigma \equiv \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$ and ρ only. Therefore, we match the average value of (32) implied by our model to the average customer bid-ask spread in Hollifield et al. (2016, Table 3).

Hollifield et al. (2016, Table 10) report the relative dispersion of customer bid-ask spread separately for the top 5% of the highest eigenvalue centrality dealers (core) and for the rest of the dealers (periphery). Because we expect that our filtered network contains all the core dealers, we choose the relative dispersion measure corresponding to core dealers as our second moment. We match this with the model implied ratio of expected price dispersion to the absolute mean of prices in those transactions where one of the counterparties is a core dealer.

We match the total volume in identified customer to customer chains in the sample with the expected total volume in our model as reported in Hollifield et al. (2016, Table 2).

This procedure gives the parameter values of $\rho = 0.014$, $\sigma = 0.1584$, $(-\beta)\,\sigma_\theta^2 = 7.3835$. With the estimated parameter values we verify that our model implies that the average spread for core dealers is less than for periphery dealers and the average spread for large trades is smaller than that for small trades. These results are qualitativly consistent with Hollifield et al. (2016, Table 3, Table 10). Figure 4 gives a good idea of how the moments identify the parameters. Relative price dispersion essentially pins down ρ as the corresponding surface is very sensitive to ρ but almost flat in $\sigma \equiv \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$. This is intuitive as a principal source of price dispersion in our model is the private component in dealers' values. Given ρ, the average price impact pins down $\sigma \equiv \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$. Given these two parameters, the total volume pins down $(-\beta)\sigma_\theta^2$ because it scales linearly in that variable.

5.2.3 Stylized facts and other OTC markets

In this section, we further explore the qualitative implications of our model and constrast them with empirical facts from other OTC markets. For this exercise, we use the same network described above and the parameters we have calibrated.

We start by illustrating how dealer's centrality is related to a set of standard financial indicators using our calibrated parameters in Figure 5. Panels A-D show dealer level measures. In particular, we plot each dealer's expected profit, expected gross volume, intermediation, calculated as the expected gross volume over expected net volume and posterior precision (as percentage of the corresponding precision under fully revealing prices), against the number of trading partners of the given dealer. Panels E-F show link level measures. In particular, we plot the price impact a dealer faces on a given link against the number of trading partners, and against the sum of the trading partners of the two counterparties connected by the link, respectively.

The relationship between the degree of the dealer and her profit, gross volume, gross-to-net volume ratio or precision is strong and positive. The shape of the scatter plots suggests that, given the calibrated parameters, the dealer's degree centrality summarizes almost all the relevant information of her network position to determine her profit, volume, information precision and gross-to-net volume ratio. While there is only a weak negative relationship between the degree of the dealer and her price impact, there is a strong relationship between the sum of the degrees of both counterparties and price impact. This is consistent with our discussion in Section 5. The price impact a dealer faces is smaller when her counterparty puts less weight on the given price, for example, because she trades with a large number of counterparties. Therefore, we should observe the smallest price impact when both parties have a large number of trading partners.[10]

These observations are qualitatively consistent with the empirical literature considering various markets. Similarly to Hollifield et al. (2016), Di Maggio et al. (2017) find in the context of the corporate bond market that central dealers offer lower spreads compared to periphery dealers. Consistently with our analysis, they also find that the centrality of both counterparties matter for price impact of the trade. They find that the price impact is lowest between two high-centrality dealers, highest between two low-centrality dealers, and in-between when only one of the counterparty has high centrality. This is as predicted by Panel F on Figure 5.[11]

Li and Schürhoff (2014) also show that central agents in the municipal bond market trade more, their trades are more profitable, and they seem to be better informed than others. The positive relationship between centrality and trading volume is also confirmed by Roukny et al. (2014) for a data-set of European CDSs.

[10]As parameters vary the relationships shown in Figure 5 are always strong and have the same sign. However, especially as correlation, ρ, increases, degree centrality does not suppress all the other network characteristics to the same extant. We illustrates this observation in Appendix C.

[11]Interestingly, in contrast to Hollifield et al. (2016) and Di Maggio et al. (2017), Li and Schürhoff (2014) finds that central traders offer higher spreads than periphery traders in the context of the municipal bond market. Hollifield et al. (2016) suggests that the difference might be due to the different level of sophistication in these markets. Our model suggests that in the municipal bond market adverse selection might not be the first order determinant of variation in spreads.

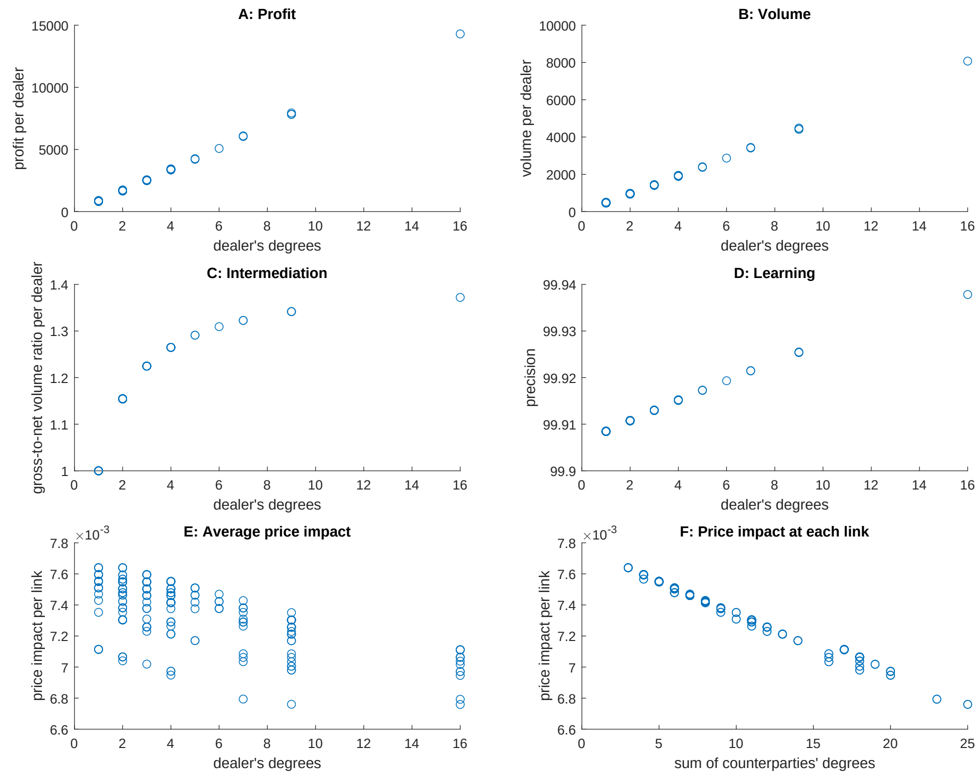

Figure 5: Panels A-D show each dealer's expected profit, gross volume, intermediation, and posterior information precision (as percentage of precision under fully revealing prices) against the number of the dealer's trading partners. Panels E and F show the price impact a dealer faces at a given link against the number of her trading partners, and against the sum of the trading partners of the two counterparties at the given link, respectively. Parameter values are $\rho = 0.014, \sigma = 0.1584, \beta = -1, \sigma_\theta^2 = 7.3835$

Our model can also be used as a basis for counter-factual analysis. As an illustration, in Appendix C, we analyze the effect of market distress in our calibrated example. In particular, we remove the most connected dealer from the network. We find that network-wide price dispersion and average price impact goes up, while overall trading volume goes down. These observations are consistent with findings in the empirical literature that study the the effect of a stress event on market indicators, such as Friewald et al. (2012) for the corporate bond market, Afonso and Lagos (2012) for the Fed Funds market, and Agarwal et al. (2012) for the MBS market.

Importantly, thinking about the underlying trading network structure might be useful even when the econometrician has only limited information about dealers' characteristics. Indeed, given our results, we should expect that larger transaction size is associated with lower trading costs, higher profits, profitable and more informative trades. The reason is that these are properties of transactions of more connected dealers. From this set of predictions, the pattern that percentage cost is decreasing in the size of the transaction is a robust observation in many different contexts (see Green et al. (2007) and Li and Schürhoff (2014) on municipal bonds and Edwards et al. (2007) and Randall (2015) on US corporate bonds).

Finally, consistent with our observations Atkeson et al. (2012), Li and Schürhoff (2014) and Hollifield et al. (2016) all report the CDS and the securitized loan markets are highly concentrated. While the same is true when US corporate bonds are evaluated in the aggregate, Schultz (2001) reports that trading in specific bonds seems to be spread across multiple dealers. Our model is silent on these differences.

6 The Price-Discovery Game

In this section, we guide the reader to better connect our OTC game to the real world features of trading in OTC markets. In the OTC game dealers' trading strategies are represented by generalized demand curves, and all trades take place simultaneously. This is a very tractable and rich theoretical structure, and we consider the one-shot OTC game as a reduced-form representation of how equilibrium prices and quantities are determined in reality.

In real world OTC markets, dealers do not post full demand curves. Instead, dealers engage in bilateral negotiations with their counterparties by quoting prices which are valid for a certain quantity. To capture this feature, we introduce a variant of the OTC game where dealers find the equilibrium prices and quantities through a sequence of bilateral exchange of quotes.

In particular, consider that each node in a given network is a trading desk. Each desk i consists of a desk-head, whom we continue to refer as dealer i, and one or more traders. Dealer i designs a bidding strategy and the traders have to implement this strategy. The bidding strategy describes how the traders should respond to bids they receive from counterparties. Bidding takes place sequentially, in rounds. In each bidding round τ, each trading desk i makes an offer $\pi^i_{ij,\tau} = \left\{p^i_{ij,\tau}, q^i_{ij,\tau}\right\}$ to each trading desk j with whom she has a link, indicating that she is willing to trade quantity $q^i_{ij,\tau}$ for price $p^i_{ij,\tau}$. Thus, in any bidding round all dealers make and receive offers to and from their counterparties in the network. If the price offered by i to j is arbitrarily close to the price offered by j to i, and the two quantities differ only to the extent of the order of the customers' demand at the given price, then the offer is accepted. Otherwise, a trading desk j that receives the bid in round τ, responds with a counter-offer $\pi^j_{ij,\tau+1} = \left\{p^j_{ij,\tau+1}, q^j_{ij,\tau+1}\right\}$ in round $\tau + 1$. This process can continue for any number of rounds, until all trading desks accept the offers. At that point, trades are executed both across dealers and with customers. We define this game formally in the appendix and call it a price-discovery game.

The following proposition proves our claim that dealers can find the equilibrium prices and quantities in the OTC game by playing the price-discovery game.

Proposition 10 *Suppose that there exists an equilibrium in the OTC game, with $\bar{y}^i \geq 0$ and $\bar{z}^i_{ij} \geq 0$ whenever $j \in g^i$.*

1. *There exists a set of bidding strategies that are an equilibrium in the price-discovery game.*
2. *The resulting prices and quantities are the same as the equilibrium prices and quantities in the OTC game.*

Although the construction of the price-discovery game is arguably artificial, it illustrates some important features of our OTC game. On the one hand, finding the equilibrium prices and quantities in the OTC game needs not rely on any kind of auctioneer. The price-discovery game shows that equilibrium prices and quantities can be found via an iterative, decentralized process. Moreover, the coefficients of the generalized demand curves (and, consequently the coefficients of the bidding strategies) can be derived without observing the realization of signals. Indeed, dealers can find the equilibrium coefficients just by understanding the structure of the game they are facing.

On the other hand, the price discovery game emphasizes an important limitation of the OTC game. Namely, the equivalence between the price discovery game and the OTC game relies on the fact that the bidding strategies are static. That is, they do not depend on the bidding round, and they are conditional only on the outcome of the last bidding round, as opposed to all previous rounds. This restriction on the strategy space of dealers that dealers can use is necessary as the OTC game is nevertheless a static game. Thus, in our framework, we are unable to capture any dynamic learning considerations inherent in real-world OTC markets.

7 Conclusions

In this paper, we proposed a model of trading and information diffusion in OTC markets. Dealers trade on a fixed network, and each dealer's strategy is represented as a quantity-price schedule. We showed that information diffusion through prices is unaffected by dealers' strategic trading motives and that each price partially incorporate the private information of all dealers, and we identified an informational externality that constrains the informativeness of prices. We also highlighted that trade decentralization can both increase or decrease welfare and that the main determinant of a dealer's trading cost is not her centrality but rather the centrality of her counterparties. We used a calibrated example and various robustness checks to illustrate that in realistic interdealer networks, more-central dealers learn more, intermediate more, trade more at lower costs and earn higher expected profit.

Importantly, trading protocols in OTC markets have become increasingly diverse. There are a number of protocols (e.g., dealer runs, broker-assisted work-up protocols), which our paper does not address explicitly. Given this increasing diversity, it is important to develop frameworks that put limited emphasis on any one particular trading protocol and can still capture robust features of OTC markets. Our approach emphasizes that links are persistent, that the market structure is concentrated, and that dealers intermediate trade between otherwise-disconnected counterparties. Our model yields price-quantity pairs that are consistent with each dealers information, potential trading partners and objectives. We implicitly suggest that if such pairs exists, it is likely that the market will converge to these points, independently of the trading protocol.

Demand and supply curves have been a powerful tool to model equilibrium in centralized good markets since the beginning of economic thinking. Using our approach of generalized demand curves on networks, we have also found a method to obtain insights for decentralized markets.

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A Appendix: Proofs

In some proofs we use the convention that a network, g, can be represented by an *adjacency* matrix $A(g)$ with elements $A_{ij} = 1$ if $ij \in g$, and $A_{ij} = 0$ if $ij \notin g$. Also, sometimes we use matrix notation as follows. $V, \bar{Z}, \bar{Y}$ are $n \times n$ matrices of row vectors $\mathbf{v}^i, \bar{\mathbf{y}}^i, \bar{\mathbf{z}}^i,$, $i = 1, ..., n$ respectively. The individual elements of V are v_{ij}. The individual element of $\bar{Z}$ is $\bar{z}^i_{ij}$ iff $A_{ij} = 1$ and 0 otherwise, while $\bar{Y}$ is a diagonal matrix with $\bar{y}^i$ at its i-th diagonal element.

Proof of Proposition1

We prove the statement in a more general form than stated. It proves existence for a general Gaussian information structure. We need only that if ω^i is a column vector of the covariances between θ^i and each of the signals (in our case it is $\omega_{ii} = \sigma^2_\theta$ and $\omega_{ij} = \rho\sigma^2_\theta$ for $i \neq j$), then $\omega^i > 0$ for all i. Note that we can rewrite the problem as follows. We are looking for a V^* matrix of which each row $\mathbf{v}^i$ is the solution of the problem of

$$\begin{aligned} &\max_{\bar{\mathbf{y}}^i, \bar{\mathbf{z}}^i} \left(2\mathbf{v}^i\omega^i - \mathbf{v}^i\Sigma(\mathbf{v}^i)^\top\right) \\ &\text{s.t. } \mathbf{v}^i = \left[\bar{\mathbf{y}}^i + \bar{\mathbf{z}}^i V^*\right] \\ &z^i_{ij} = 0 \Longleftrightarrow A_{ij} = 0 \end{aligned} \tag{A.1}$$

where Σ is the covariance matrix of signals, $\mathbf{s}$, and $\bar{\mathbf{y}}^i$ is a row vector with a $\bar{y}^i$ at the $i-th$ place and 0 otherwise, while $\bar{\mathbf{z}}^i$ is a row vector of size n with elements of some $(\bar{z}^i_{ij})_{j=1,..n}$ (subject to the last constraint).

To see that this V^* exists, let us first define the matrix mapping $F : R^{n\times n} \to R^{n\times n}$, which maps any $n \times n$ matrix V^0 to another one with rows $(\mathbf{v}^i)_{i=1,...,n}$ defined by

$$\begin{aligned} \mathbf{v}^i \equiv &\arg\max_{\bar{\mathbf{y}}^i, \bar{\mathbf{z}}^i} \left(2\mathbf{v}^i\omega^i - \mathbf{v}^i\Sigma(\mathbf{v}^i)^\top\right) \\ &\text{s.t. } \mathbf{v}^i = \left[\bar{\mathbf{y}}^i + \bar{\mathbf{z}}^i V^0\right] \\ &z^i_{ij} = 0 \Longleftrightarrow A_{ij} = 0 \end{aligned} \tag{A.2}$$

Further, let

$$\mathbb{V}^i \equiv \{\mathbf{v}^i : \mathbf{v}^i\Sigma(\mathbf{v}^i)^\top - 2\mathbf{v}^i\omega^i + \frac{\omega^2_{ii}}{\Sigma_{ii}} \leq 0\}.$$

and $\mathbb{V}^{n\times n} \equiv \mathbb{V}^1 \times \mathbb{V}^2.. \times \mathbb{V}^n$ be the set of matrices with rows $\mathbf{v}^i \in \mathbb{V}^i$.

We need to show that F is a continuous self map with respect to the set of matrices $\mathbb{V}^{n\times n}$ and that $\mathbb{V}^{n\times n}$ is a convex compact set. Hence, the Brouwer fixed-point theorem applies. We proceed in steps.

1. We show that F defined by (A.2) is a self-map.

   For this, note that increasing the number of 0-s in the $i-th$ row of A (decreasing the number of links to i in the network) adds more constraints to the problem (A.2). So we consider the extreme problem where the $i-th$ row and column of A has only zeros, that is, each $\bar{z}^i_{ij} \equiv 0$. It is easy to show that in this case the problem reduces to

$$\frac{\omega_{ii}^2}{\Sigma_{ii}} = \max_{v_{ii}} \left[2v_{ii}\omega_{ii} - v_{ii}\Sigma_{ii}v_{ii}\right]$$

with a solution of $y^i = v_{ii} = \frac{\omega_{ii}}{\Sigma_{ii}}$ and $v_{ij} = 0$ for all $i \neq j$. Thus, for any A with non-zero elements in the $i-th$ row and column, $\frac{\omega_{ii}^2}{\Sigma_{ii}}$ is a lower bound on the value agent i can achieve, that is, the solution $\mathbf{v}^i$ will satisfy $\frac{\omega_{ii}^2}{\Sigma_{ii}} \leq 2\mathbf{v}^i\omega^i - \mathbf{v}^i\Sigma(\mathbf{v}^i)^\top$. Implying that for any V_0 and A, F projects to $\mathbb{V}^{nxn}$.

2. F is continuous in V^0 by the Maximum Theorem.

3. Given that the Cartesian product of convex and compact sets is also convex and compact, we only have to show that each $\mathbb{V}^i$ is convex, closed and bounded

   (a) $\mathbb{V}^i$ is convex. Under the assumption that Σ is positive definite, $\mathbf{v}^i\Sigma(\mathbf{v}^i)^\top - 2\mathbf{v}^i\omega^i$ is a convex function (the sum of a convex and a linear function). From the fact that the sub-level sets of a convex functions are convex, it follows that the set $\mathbb{V}^i$ is convex.

   (b) $\mathbb{V}^i$ is closed. Let $g(\mathbf{v}^i) \equiv \mathbf{v}^i\Sigma(\mathbf{v}^i)^\top - 2\mathbf{v}^i\omega^i + \frac{\omega_{ii}^2}{\Sigma_{ii}}$ be a function from R^n to R. Clearly, g is continuous and $\mathbb{V}^i \equiv \{\mathbf{v}^i : g(\mathbf{v}^i) \leq 0\}$. Let $\mathbf{v}^i_n$, $n = 1...\infty$ be a convergent series of vectors in $\mathbb{V}^i$ with $\mathbf{v}^i_\infty$ being the limit point of this series. Since g is continuous, we have $g(\mathbf{v}^i_\infty) = \lim_{n\to\infty} g(\mathbf{v}^i_\infty) \leq 0$. Hence $\mathbf{v}^i_\infty \in \mathbb{V}^i$.

   (c) $\mathbb{V}^i$ is bounded. Note that the function $g(\mathbf{v}^i)$ is strictly convex, continuous, and twice-differentiable. Hence, there exists a minimum $\mathbf{v}^i_{\text{min}}$ that $g(\mathbf{v}^i_{\text{min}}) \leq g(\mathbf{v}^i)$ for all $\mathbf{v}^i \in \mathbb{V}^i$ Also, from the definition $\mathbb{V}^i$, $g(\mathbf{v}^i) \leq 0$ for all $\mathbf{v}^i \in \mathbb{V}^i$. Note also that $g(\cdot)$ is strongly convex on $\mathbb{V}^i$ as there exists $m > 0$ such that $\nabla^2 g(\mathbf{v}) - m\mathbf{I}$ $=2\Sigma - m\mathbf{I}$ is positive definite (for example, one can pick $m = \sigma_\theta^2 + \sigma_\varepsilon^2$). Also, from strong convexity

   $$g(\mathbf{v}^{''}) \geq g(\mathbf{v}^{'}) + \nabla g(\mathbf{v}^{'})(\mathbf{v}^{''} - \mathbf{v}^{'})^\top + \frac{m}{2}||\mathbf{v}^{''} - \mathbf{v}^{'}||_2^2.$$

   for any $\mathbf{v}^{'}, \mathbf{v}^{''} \in R^n$. In particular, for $\mathbf{v}^{'} = \mathbf{v}^i_{\text{min}}$, we have $\nabla g(\mathbf{v}^i_{\text{min}}) = 0$ implying that

   $$g(\mathbf{v}^{''}) - g(\mathbf{v}^i_{\text{min}}) \geq \frac{m}{2}||\mathbf{v}^{''} - \mathbf{v}^i_{\text{min}}||_2^2.$$

   Let us pick $\mathbf{v}^{''} = \mathbf{v}^i$ an arbitrary element of $\mathbb{V}^i$. Then $g(\mathbf{v}^{''}) \leq 0$ implying

   $$-\frac{2}{m} g(\mathbf{v}^i_{\text{min}}) \geq ||\mathbf{v}^i - \mathbf{v}^i_{\text{min}}||_2^2$$

   proving the claim.

Proof of Proposition 2 and Corollary 1

Consider an equilibrium of the conditional guessing game in which

$$E(\theta^i \left| s^i, \mathbf{e}_{g^i} \right.) = \bar{y}^i s^i + \sum_{k\in g^i} \bar{z}^i_{ik} E(\theta^k \left| s^k, \mathbf{e}_{g^k} \right.)$$

for every i. If the system (21) has a solution, then

$$E(\theta^i \left| s^i, \mathbf{e}_{g^i} \right.) = \frac{y^i}{\left(1 - \sum_{l \in g^i} z_{il}^i \frac{2 - z_{li}^l}{4 - z_{il}^i z_{li}^l}\right)} s^i + \sum_{k \in g^i} z_{ik}^i \frac{\frac{2 - z_{ik}^i}{4 - z_{ik}^i z_{ki}^k}}{\left(1 - \sum_{l \in g^i} z_{il}^i \frac{2 - z_{li}^l}{4 - z_{il}^i z_{li}^l}\right)} E(\theta^k \left| s^k, \mathbf{e}_{g^k} \right.) \quad \text{(A.3)}$$

holds for every realization of the signals, and for each i. Now we show that choosing the prices and demand functions (23) and (24) is an equilibrium of the OTC game.

First note that (26) for i and j at a given link implies (22). Also, the choice (24) implies

$$E(\theta^i \left| s^i, \mathbf{e}_{g^i} \right.) = y^i s^i + \sum_{k \in g^i} z_{ik}^i p_{ij} = E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.). \quad \text{(A.4)}$$

The second equality comes from the fact that the first equality holds for any realization of signals and the projection theorem determines a unique linear combination with this property for a given set of jointly normally distributed variables. Thus, (24) for each ij link is equivalent with the corresponding first order condition (10). Finally, (A.4) also implies that the bilateral clearing condition between a dealer i and dealer j that have a link in network g

$$t_{ij}^i \left(E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.) - p_{ij}\right) + t_{ij}^j \left(E(\theta^j \left| s^j, \mathbf{p}_{g^j} \right.) - p_{ij}\right) + \beta_{ij} p_{ij} = 0$$

is equivalent to (23). That concludes the statement.

Corollary 1 follows from the direct observation of (23) and (24) and (22).

Proof of Proposition 3

Case 1: Circulant networks

We provide here a draft of the proof. Further details of each step is available on request.

Step 1: We show that for circulant networks, each $\bar{z}_{ij}^i > 0$ for any $\rho \in (0, 1)$.

1. First, we show that $\bar{z}_{ij}^i > 0$ in the limit $\rho \to 0$. Clearly when $\rho = 0$, the equilibrium V is a diagonal matrix as the signals of others are uninformative in this pure private value case. The starting point is to show that for diminishingly small ρ, the off-diagonal elements of V which are corresponding to first neighbors are diminishing at a slower rate than the rest of the off-diagonal elements. In particular, we conjecture and verify that there are constants a_0 and a_1 that
$$\lim_{\rho \to 0} \left(\frac{V - a_0 I}{\rho}\right) = a_1 A$$
where I and A are the identity matrix and the adjacency matrix respectively. For this, we calculate the matrices $\bar{Y}$ and $\bar{Z}$ which correspond to a starting matrix $V^0 = a_0 I + a_1 A$ for a given a_0 and a_1 in problem (A.2), obtain the resulting new matrix $V^1 = \left(\bar{Y} + \bar{Z} V^0\right)$, observe that each non-zero element in $\bar{Z}$, $\bar{z}_{ij}^i > 0$ are positive, and verify there are indeed a_0 and a_1 values for which $\lim_{\rho \to 0} \left(\frac{V^1 - a_0 I}{\rho}\right) = a_1 A$.

2. Given that all $\bar{z}_{ij}^i$ are positive in this limit, let us counterfactualy assume that there is

$\rho \in (0,1)$ for which at least one $\bar{z}^i_{ij} < 0$. By continuity, then must be a ρ_0 for which all $\bar{z}^i_{ij} \geq 0$ but at least one of them is zero. But this implies that for these parameters dealer i finds the expectation of one of her neighbors uninformative. Let $\{i_k\}_{k=1,...,m^i}$ be the set of i's neighbors and, without loss of generality, suppose that the index of this neighbor is m^i. The only way this holds is that there is a linear combination of s^i and $\left\{e^{i_k}\right\}_{k=1,...,(m^i-1)}$ which replicates $e^{i_{m^i}}$, that is, that there is an arbitrary vector $[\lambda_0, \lambda_1..\lambda_{m^i-1}]$, that

$$\lambda_0 s^i + \lambda_1 e^{i_1} + ...\lambda_{m^i-1} e^{i_{(m^i-1)}} = e^{i_{m^i}} \tag{A.5}$$

(a) Note that if the network is circulant, there must be an equilibrium where V is also circulant. To see this, note that problem (A.2) maps circulant networks into circulant networks. Also, given that we prove the properties of $\mathbb{V}^{n\times n}$ vector-by-vector in the proof of 1, repeating those steps proves the existence of a circulant V fixed point. Furthermore, in this equilibrium the rows corresponding to the expectation of agent i and j has to have the structure of $v_{i(i+l)} = v_{i(i-l)} = v_{j(j+l)} = v_{j(j-l)}$ for every $l \geq 0$ as long as $n \geq i-l, i+l, j-l, j+l \geq 1$. This is, the weight of each signal in the equilibrium expectation of a given dealer can depend only on whether that signal belongs to a first neighbor, or a second neighbor etc. of the given dealer. This is coming from the symmetry across dealers in circulant networks and the symmetric informational content of their expectations in this equilibrium.

(b) However, given this symmetric structure of the equilibrium V matrix, there are no v_{ij} and $[\lambda_0, \lambda_1..\lambda_{m^i-1}]$ values which can solve the equations (A.5) unless all v_{ij} are the same. For instance, let us spell this out the implied equation system for the first agent in a $(7,4)$ circulant network with $\bar{k}$ being the second neighbor. If the row of V corresponding to the expectation of the first neighbor of 1 has the structure of $\begin{matrix} v_1 & v_0 & v_1 & v_2 & v_3 & v_3 & v_2 \end{matrix}$ then his second neighbor must have the structure of $\begin{matrix} v_2 & v_1 & v_0 & v_1 & v_2 & v_3 & v_3 \end{matrix}$. Thus, we need

$$\lambda_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + (\lambda_1 + \lambda_2) \begin{pmatrix} v_1 \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_3 \\ v_2 \end{pmatrix} = (1 - \lambda_3) \begin{pmatrix} v_2 \\ v_1 \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_3 \end{pmatrix}$$

to hold for some scalars. It is easy to check that this implies that all $v-s$ are identical. However, it is also easy to check that a V with identical elements cannot be a fixed point.

This is a contradiction which concludes step 1.
Step 2: We show that $\bar{z}^i_{ij} < 1$for any $\rho \in (0,1)$.
For this, note that by using forward induction on the fixed point equation $V = \bar{Y} + \bar{Z}V$, we

obtain that the equilibrium matrix V must satisfy

$$V = \bar{Y} \lim_{u\to\infty} \sum_{0}^{u} \left(\bar{Z}\right)^{u} + \lim_{u\to\infty} \left(\bar{Z}\right)^{u+1} V.$$

As $\rho \in (0,1)$ the diagonal of $\bar{Y}$ must be strictly positive, as s^i must contain residual information on the private value element of θ^i relative to the guesses of others. We know from Proposition 1 that V exists. From the fact that all elements of $\bar{Z}$ are non-negative and from the fact that the Neumann series $\lim_{u\to\infty} \sum_0^u \left(\bar{Z}\right)^u$ converges if and only if $\lim_{u\to\infty} \left(\bar{Z}\right)^{u+1} = 0$ (see Meyer (2000) page 618), we must have that indeed $\lim_{u\to\infty} \left(\bar{Z}\right)^{u+1} = 0$. As $\bar{Z}$ must be symmetric for a circulant network, and all elements are non-negative, if any elements were larger than 1, then there were some elements of $\lim_{u\to\infty} \left(\bar{Z}\right)^{u+1}$ which would not diminish (as the elements $\left(\bar{z}^i_{ij}\right)^{u+1}$ will be a component in some elements of the the matrix $\left(\bar{Z}\right)^{u+1}$ for any i and j).

Step 3: Now, we search for equilibria such that beliefs are symmetric, that is

$$z^i_{ij} = z^j_{ji}$$

for any pair ij that has a link in network g.

The system (21) becomes

$$\frac{y^i}{\left(1 - \sum_{k\in g^i} z^i_{ik} \frac{2 - z^i_{ik}}{4 - \left(z^i_{ik}\right)^2}\right)} = \bar{y}^i$$

$$z^i_{ij} \frac{\frac{2 - z_{ij}}{4 - z^2_{ij}}}{\left(1 - \sum_{k\in g^i} z^i_{ik} \frac{2 - z^i_{ik}}{4 - \left(z^i_{ik}\right)^2}\right)} = \bar{z}^i_{ij}$$

for any $i \in \{1, 2, ..., n\}$. Working out the equation for z^i_{ij}, we obtain

$$\frac{z^i_{ij}}{2 + z^i_{ij}} = \bar{z}^i_{ij} \left(1 - \sum_{k\in g^i} \frac{z^i_{ik}}{2 + z^i_{ik}}\right)$$

and summing up for all $j \in g^i$

$$\sum_{j\in g^i} \frac{z^i_{ij}}{2 + z^i_{ij}} = \sum_{j\in g^i} \bar{z}^i_{ij} \left(1 - \sum_{k\in g^i} \frac{z^i_{ik}}{2 + z^i_{ik}}\right).$$

Denote

$$S^i \equiv \sum_{k\in g^i} \frac{z^i_{ik}}{2 + z^i_{ik}}.$$

Substituting above and summing again for $j \in g^i$

$$S^i \left(1 + \sum_{j \in g^i} \bar{z}^i_{ij}\right) = \sum_{j \in g^i} \bar{z}^i_{ij}$$

or

$$S^i = \frac{\sum\limits_{j \in g^i} \bar{z}^i_{ij}}{\left(1 + \sum\limits_{j \in g^i} \bar{z}^i_{ij}\right)}.$$

We can now obtain

$$z^i_{ij} = \frac{2\bar{z}^i_{ij}\left(1 - S^i\right)}{1 - \bar{z}^i_{ij}\left(1 - S^i\right)} \tag{A.6}$$

and

$$y^i = \bar{y}^i \left(1 - S^i\right).$$

Finally, the following logic show that $z^i_{ij} \leq 2$. As $\bar{z}^i_{ij} < 1$, $2\bar{z}^i_{ij} < \left(1 + \sum\limits_{j \in g^i} \bar{z}^i_{ij}\right)$ implying that $2\bar{z}^i_{ij}\left(1 - S^i\right) < 1$ or $2\bar{z}^i_{ij}\left(1 - S^i\right) < 2\left(1 - \bar{z}^i_{ij}\left(1 - S^i\right)\right)$, which gives the result by A.6.

Case 2: Star networks

We give the closed-form solutions for the star network in Appendix B.2. One can check by straightforward algebra that the resulting z^i_{ij} are indeed in the $[0, 2]$ interval.

Proof of Proposition 4

In an equilibrium of the OTC game, prices and quantities satisfy the first order conditions (10) and must be such that all bilateral trades clear.

Since market clearing conditions (6) are linear in prices and signals, we know that each price (if an equilibrium price vector exists) must be a certain linear combination of signals. Thus, each price is normally distributed.

From the first order conditions we have that

$$q^i_{ij}(s^i, \mathbf{p}_{g^i}) = t^i_{ij}\left(E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.) - p_{ij}\right).$$

The bilateral clearing condition between a trader i and trader j that have a link in network g implies that

$$t^i_{ij}\left(E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.) - p_{ij}\right) + t^i_{ij}\left(E(\theta^j \left| s^j, \mathbf{p}_{g^j}\right.) - p_{ij}\right) + \beta_{ij} p_{ij} = 0$$

and solving for the price p_{ij} we have that

$$p_{ij} = \frac{t^i_{ij} E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.) + t^i_{ij} E(\theta^j \left| s^j, \mathbf{p}_{g^j}\right.)}{t^i_{ij} - \beta_{ij}}$$

Since agent i knows $E(\theta^i \left| s^i, \mathbf{p}_{g^i}\right.)$, by definition, the vector of prices $\mathbf{p}_{g^i}$ is informationally equivalent for her with the vector of posteriors of her neighbors $\mathbf{E}_{g^i} = \left\{E(\theta^j \left| s^j, \mathbf{p}_{g^j} \right.)\right\}_{j \in g^i}$. This implies that

$$E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.) = E(\theta^i \left| s^i, \mathbf{E}_{g^i} \right.).$$

Note also that as each price is a linear combination of signals and $E\left(\theta^j | \cdot\right)$ is a linear operator on jointly normal variables, there must be a vector $\mathbf{w}^i$ that $E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.) = E(\theta^i \left| s^i, \mathbf{E}_{g^i} \right.) = \mathbf{w}^i\mathbf{s}$. That is, the collection of $\left\{\mathbf{w}^i\right\}_{i=1,...n}$ has to satisfy the system of n equations given by

$$\mathbf{w}^i\mathbf{s} = E(\theta^i \left| s^i, \left\{\mathbf{w}^j\mathbf{s}\right\}_{j \in g^i} \right.)$$

for every i. However, the collection $\left\{\mathbf{w}^i\right\}_{i=1,...n}$ that is a solution of this system, is also an equilibrium of the conditional guessing game by construction.

Proof of Proposition 5

Equation (23), the fact that $t^i_{ij} > 0$ for all i and j, and $E(\theta^i \left| s^i, \mathbf{p}_{g^i} \right.) = E(\theta^i \left| s^i, \mathbf{E}_{g^i} \right.) = \mathbf{v}^i\mathbf{s}$ from Proposition 4 implies that we only have to show that all elements of the equilibrium V matrix defined in the proof of Proposition 1 are strictly positive.

We use the notation of Proposition 1. $\mathbb{V}^i$ is a convex set and must contain some strictly positive vectors as the one minimizing $g(\cdot)$, defined above, $\left(\mathbf{v}^i\right)^{\min} \equiv \Sigma^{-1}\omega^i$, is strictly positive by assumption. Now we show that there it contains only a single point, the one for $v_{ii} = \frac{\omega_{ii}}{\Sigma_{ii}}$ and $v_{ij} = 0$ for all $i \neq j$, which is in any of the axes of R^n. This is sufficient to prove that vectors in $\mathbb{V}^i$ cannot have negative elements as for a convex set to cross any of the axes, it should have at least two points on that given axis. We show this by contradiction. Assume that $\mathbf{v}^i \in \mathbb{V}^i$ has an other elements on any of the axes, e.g. a $\bar{v}^i$ such that $v_{i1} = v_{i2} = ... = v_{in-1} = 0$ and $v_{in} = x$. Then $g(\cdot)$ simplifies to

$$g(\mathbf{v}^i) = x^2\Sigma_{nn} - 2x\omega_{in} + \frac{\omega_{ii}^2}{\Sigma_{ii}},$$

The function attains a minimum at $x^* = \frac{\omega_{in}}{\Sigma_{nn}}$. The minimum value of the function is $-\frac{\omega_{in}}{\Sigma_{nn}}$. Therefore $\mathbf{v}^i$ is not an element of $\mathbb{V}^i$ if

$$\frac{\omega_{in}^2}{\Sigma_{nn}} < \frac{\omega_{ii}^2}{\Sigma_{ii}}.$$

But this always holds in our parameterization (as $\frac{\omega_{in}^2}{\Sigma_{nn}} = \frac{\left(\rho\sigma_\theta^2\right)^2}{\sigma_\theta^2+\sigma_\varepsilon^2}$ and $\frac{\omega_{ii}^2}{\Sigma_{ii}} = \frac{\left(\sigma_\theta^2\right)^2}{\sigma_\theta^2+\sigma_\varepsilon^2}$).

As we showed in Proposition 1, for any network and any parameters V must be in the Cartesian product of $\mathbb{V}^{n\times n}$. However, the previous argument shows that there is a single matrix which has not strictly positive elements, the diagonal matrix with $v_{ii} = \frac{\omega_{ii}}{\Sigma_{ii}}$ for all i. But it is simple to check that this cannot be a fixed point of our system for any connected network and any parameters as long as $\rho \neq 0$.

Proof of Proposition 6

1. From Proposition 4 we know that in any equilibrium of the OTC game

$$E(\theta^i|s^i, \mathbf{p}_{g_i}) = E(\theta^i|s^i, \mathbf{e}_{g_i}).$$

Also, Lemma 5 shows that each equilibrium expectation in the conditional guessing game is a linear combination of all signals in the economy

$$E(\theta^i|s^i, \mathbf{e}_{g_i}) = \mathbf{v}^i\mathbf{s}$$

where $\mathbf{v}_i > 0$ for all i. That is, when the dealer network is complete, through prices, each dealer observes $(n-1)$ linear combinations of $\mathbf{s}$ apart from her own private signal. Clearly, as long as these linear combinations are independent, the statement holds. Instead, let us assume that there exists a β, that $E(\theta^i|s^i, \mathbf{e}_{g_i}) = \mathbf{v}^i\mathbf{s} = \beta\mathbf{v}^j\mathbf{s} = E(\theta^j|s^j, \mathbf{e}_{g_j})$ for some i and j. These two agents are connected (as everyone else). However, for any $\rho < 1$, this is impossible, because at least one of the agents would find that putting a different weight on its own signal s^i improves over its estimate of θ^i over the best estimate of agent j on θ^j. Implying that either $\mathbf{v}^i\mathbf{s}$ $\mathbf{v}^j\mathbf{s}$ is not an equilibrium of the conditional guessing game, implying that it cannot be an equilibrium guess in the OTC game either (by Proposition 2).

2. Given joint normality, we only have to show that $\lim_{\rho\to1} E\left(\theta^i|s^i, \mathbf{p}_{g^i}\right) = E\left(\theta^i|\mathbf{s}\right)$ and $\lim_{\rho\to1} var\left(\theta^i|s^i, \mathbf{p}_{g^i}\right) = var\left(\theta^i|\mathbf{s}\right)$. We proceed in steps.

   (a) Note that when $\rho = 1$, there exists an equilibrium of the conditional guessing game, $\left(\mathbf{e}_{g^i}\right)_{i=1,\dots n}$ where $E\left(\theta^i|s^i, \mathbf{e}_{g^i}\right) = E\left(\theta^i|\mathbf{s}\right)$ and $var\left(\theta^i|s^i, \mathbf{e}_{g^i}\right) = var\left(\theta^i|\mathbf{s}\right)$ for all i.This is so, because when $\rho = 1$, the best guess, $E\left(\theta^i|\mathbf{s}\right)$, for each agent conditionally of observing each signal is the same across agents. Therefore, if any agent were to pick this guess, all of its neighbors would and could pick the same guess (by putting a weight of 1 on this guess). That is, it is a fixed point as defined by the equilibrium in the conditional guessing game.

   (b) We argue this equilibrium is unique. For this, note that with $\rho = 1$, the equilibrium guess of agent i, $\mathbf{v}^i\mathbf{s}$ has to be identical of that of agent $\mathbf{v}^j\mathbf{s}$, for any pair of agents. The reason is that otherwise, there exists two linked agents, i, k with $\mathbf{v}^j\mathbf{s} \neq \mathbf{v}^k\mathbf{s}$, which implies either (1) differing values,e.g., $\left(2\mathbf{v}^i\omega^i - \mathbf{v}^i\Sigma(\mathbf{v}^i)\right)^\top > \left(2\mathbf{v}^k\omega^k - \mathbf{v}^k\Sigma\left(\mathbf{v}^k\right)^\top\right)$, where we use the notation in the proof of Proposition 1. However, in this case, because $\rho = 1$, agent k would be motivated to put a weight of 1 to the guess of agent i, to obtain the same value as agent i does. Which is a contradiction of the claim that $\mathbf{v}^k\mathbf{s}$ is an equilibrium guess. Or (2) if $\left(2\mathbf{v}^i\omega^i - \mathbf{v}^i\Sigma\left(\mathbf{v}^i\right)^\top\right) = \left(2\mathbf{v}^k\omega^k - \mathbf{v}^k\Sigma\left(\mathbf{v}^k\right)^\top\right)$ but $\mathbf{v}^j\mathbf{s} \neq \mathbf{v}^k\mathbf{s}$, then a convex combination of $\mathbf{v}^j\mathbf{s}$ and $\mathbf{v}^k\mathbf{s}$ is a feasible guess for any of the agents which improves their value. This is also a contradiction of the claim that $\mathbf{v}^j\mathbf{s}, \mathbf{v}^k\mathbf{s}$ were equilibrium guesses. However, if all agents have the same guesses, then this guess has to be the best guess $E\left(\theta^i|\mathbf{s}\right)$. Otherwise, any agent would put a non-zero weigh on his own signal to improve on this guess, leading to different equilibrium guesses across agents; a contradiction.

(c) By the continuity of the conditional guessing game in ρ, whenever ρ is sufficiently close to one any equilibrium must be arbitrarily close to the one at $\rho = 1$, implying

$$\lim_{\rho \to 1} E\left(\theta^i | s^i, \mathbf{e}_{g^i}\right) = E\left(\theta^i | \mathbf{s}\right)$$
$$\lim_{\rho \to 1} \mathcal{V}\left(\theta^i | s^i, \mathbf{e}_{g^i}\right) = \mathcal{V}\left(\theta^i | \mathbf{s}\right).$$

(d) Following the argument in Proposition 2, if the equilibrium of the OTC game exists for a ρ sufficiently close to 1, it is based on an equilibrium of the conditional guessing game. Therefore for any agent the equilibrium price vector $\mathbf{p}_{g_i}$ is informationally equivalent to the guess vector $\mathbf{e}_{g^i}$ in the conditional guessing game, implying the result.

Proof of Proposition 7

Observe that by symmetry across periphery dealers, $V = \left(I - \bar{Z}\right)^{-1} \bar{Y}$, for star network has the elements of

$$v_{11} = \bar{y}_C \frac{1}{1 - (n-1)\bar{z}_C \bar{z}_P}$$
$$v_{i1} = \bar{y}_C \frac{\bar{z}_P}{1 - (n-1)\bar{z}_C \bar{z}_P}$$
$$v_{ii} = \bar{y}_P \frac{1 - (n-2)\bar{z}_C \bar{z}_P}{1 - (n-1)\bar{z}_C \bar{z}_P}$$
$$v_{1i} = \bar{y}_P \frac{\bar{z}_C}{1 - (n-1)\bar{z}_C \bar{z}_P}$$
$$v_{ij} = \bar{y}_P \frac{\bar{z}_C \bar{z}_P}{1 - (n-1)\bar{z}_C \bar{z}_P}$$

where $\bar{y}_C, \bar{y}_P$ are the weights on the private signal and $\bar{z}_C, \bar{z}_P$ are the weights on the others' guesses in the central and periphery agents' guessing function respectively. As maximizing $E\left(-\left(\theta - e^i\right)^2\right)$ is equivalent with maximizing

$$2tr\left(V\Sigma_{\theta\mathbf{s}}\right) - tr\left(V\Sigma V^\top\right)$$

where $\Sigma_{ii} = 1 + \sigma^2, \Sigma_{ij} = \rho, [\Sigma_{\theta s}]_{ii} = 1, [\Sigma_{\theta s}]_{ij} = \rho$, we calculate the expressions for the components of this objective function.

$$\left[V\Sigma V^\top\right]_{11} = \left(1+\sigma^2\right) v_{11}^2 + \left(1+\sigma^2\right)(n-1) v_{1i}^2 + \rho 2 (n-1) v_{1i} v_{11} + \rho (n-1)(n-2) v_{1i}^2$$
$$= \frac{\left(\left(1+\sigma^2\right)\bar{y}_C^2 + \left(\left(1+\sigma^2\right) + \rho(n-2)\right)(n-1)\bar{y}_P^2 \bar{z}_C^2 + \rho 2 (n-1) \bar{y}_C \bar{y}_P \bar{z}_C\right)}{\left(1-(n-1)\bar{z}_C \bar{z}_P\right)^2}$$

and

$$\left[V\Sigma V^{\top}\right]_{ii} = \left(\left(n-2\right)\left(n-3\right)v_{ij}^2 + \left(n-2\right)2\left(v_{i,1}+v_{i,i}\right)v_{ij} + 2v_{i,1}v_{i,i}\right)\rho + \left(\sigma^2+1\right)\left(v_{ii}^2 + \left(n-2\right)v_{ij}^2 + v_{i,1}^2\right)$$
$$= \frac{\left(\bar{y}_C + \bar{z}_C\bar{y}_P(n-2)\left(1-\frac{(n-1)}{2}\bar{z}_C\bar{z}_P\right)\right)2\bar{z}_P\bar{y}_P\rho + \left(\sigma^2+1\right)\left((1-(n-2)\bar{z}_C\bar{z}_P)^2\bar{y}_P^2 + (n-2)\bar{y}_P^2\bar{z}_P^2\bar{z}_C^2 + \bar{y}_C^2\bar{z}_P^2\right)}{(1-(n-1)\bar{z}_C\bar{z}_P)^2}$$

and

$$tr\left(V\Sigma V^{\top}\right) = \left[V\Sigma V^{\top}\right]_{11} + (n-1)\left[V\Sigma V^{\top}\right]_{ii}.$$

Also,

$$tr\left(V\Sigma_{\theta s}\right) = v_{11} + \left(n-1\right)v_{ii} + \rho\left(n-1\right)\left(v_{1i}+v_{i1}\right) + \rho\left(n-1\right)\left(n-2\right)v_{ij} =$$
$$= \frac{\bar{y}_C + \rho\left(n-1\right)\bar{y}_P\bar{z}_C}{\left(1-\left(n-1\right)\bar{z}_C\bar{z}_P\right)} + \left(n-1\right)\frac{\bar{y}_P\left(1-\left(n-2\right)\bar{z}_C\bar{z}_P\left(1-\rho\right)\right) + \rho\bar{y}_C\bar{z}_P}{\left(1-\left(n-1\right)\bar{z}_C\bar{z}_P\right)}$$

This implies that

$$\lim_{\delta\to 0}\frac{\partial U\left(\bar{z}_C+\delta, \bar{z}_P+\delta, \bar{y}_C-\delta, \bar{y}_P-\delta\right)}{\partial\delta} = -\frac{f\left(\bar{z}_P, \bar{z}_C, \bar{y}_C, \bar{y}_P; n, \rho, \sigma\right)}{\left(-1+(n-1)\bar{z}_C\bar{z}_P\right)^3},$$

where $f\left(\cdot\right)$ is a polynomial. Then we substitute in the analytical expressions for the decentralized optimum $\bar{z}_C^*, \bar{z}_P^*, \bar{y}_C^*, \bar{y}_P^*$ given in closed form in Appendix B.2 and rewrite $\lim_{\delta\to 0}\frac{\partial U\left(\bar{z}_C^*+\delta, \bar{z}_P^*+\delta, \bar{y}_C^*-\delta, \bar{y}_P^*-\delta\right)}{\partial\delta}$ as a fraction. Both the numerator and the denominator are polynomials of σ^2.of order 9. A careful inspection reveals that each of the coefficients are positive for any $\rho\in(0,1)$ and $n\geq 3$.(Details on the resulting expressions in these calculations are available from the authors on request.)

Proof of Proposition 8

The first part comes by the observation that as $z_V \to 1-\frac{1}{n-1}$, $t_V\to\infty$, while t_{CN} is finite for these parameters. The second part comes from taking the limit $\rho\to 1$ of the ratio of the corresponding closed-form expressions we report in Appendices B.1 and B.3. In particular,

$$\lim_{\rho\to 1}\frac{t_{CN}}{t_V} = \frac{2n-3}{n-1} > 1$$
$$\lim_{\rho\to 1}\frac{-\frac{\beta_{CN}}{2}E\left(p_{ij}^2\right)}{-\frac{\beta_V}{2}E\left(p_V^2\right)} = \left(\frac{2n-3}{n-1}\right)^2 > 1$$
$$\lim_{\rho\to 1}\frac{\frac{n(n-1)}{2}E\left(q_{CN}\left(\theta^i - p_{CN}\right)\right)}{E\left(q_V\left(\theta^i - p_V\right)\right)} = \frac{2n-3}{(n-1)^2} < 1$$
$$\lim_{\rho\to 1}\frac{\frac{n(n-1)}{2}E\left(q_{CN}\theta^i\right) + \frac{\beta_{CN}}{2}E\left(p_{ij}^2\right)}{E\left(q_V\theta^i\right) + \frac{\beta_V}{2}E\left(p_V^2\right)} = \frac{3-8n+4n^2}{(3(n-1)^2} > 1.$$

Proof of Proposition 9

The statements come with simple algebra from the closed-form expressions we report in Appendix B.2.

The first part comes by the observation that as $z_V \to 1 - \frac{1}{n-1}$, $t_V \to \infty$, while t_C and t_P are finite for these parameters. The second part comes from taking the limit $\rho \to 1$ of the ratio of the corresponding closed-form expressions we report in Appendices B.3 and B.2. In particular,

$$\lim_{\rho\to 1} \frac{t_V}{t_C} = (n-1) \frac{z_C + z_P - z_C z_P}{(2 - z_P)\left((n-1) z_V - (n-2)\right)} = \infty$$
$$\lim_{\rho\to 1} \frac{t_V}{t_P} = (n-1) \frac{z_C + z_P - z_C z_P}{(2 - z_C)\left((n-1) z_V - (n-2)\right)} = \frac{n-1}{n} < 1$$

Proof of Proposition 10

Formally, we define the price-discovery game as follows. In round 0, each dealer i chooses a bidding strategy $B^i_{ij}(s^i; \left\{\pi^j_{ij,\tau}\right\}_{j\in g^i})$ that describes the counter-offers that traders at desk i should make in round $\tau + 1$, conditional on the bids they received in round $\tau \geq 0$, such that

$$B^i_{ij}(s^i; \left\{\pi^j_{ij,\tau}\right\}_{j\in g^i}) = \pi^i_{ij,\tau+1}, \tag{A.7}$$

for each $j \in g^i$. If there exists a price and quantity vector $\left\{\bar{p}^i_{ij}, \bar{q}^j_{ij}\right\}_{ij\in g}$ with

$$\bar{p}^i_{ij} = \bar{p}^j_{ij},$$
$$\bar{q}^i_{ij} + \bar{q}^j_{ij} + \beta_{ij}\bar{p}^i_{ij} = 0,$$

and

$$\lim_{\tau\to\infty} \pi^i_{ij,\tau} = \left(\bar{p}^i_{ij}, \bar{q}^i_{ij}\right),$$

for every $ij \in g$ and for any random starting vector $\left\{\pi^i_{ij,0}\right\}_{ij\in g}$, then trade takes place.

The pay-off for a dealer i is the expected profit $E\left[\sum_{j\in g^i} \bar{q}^i_{ij}\left(\theta^i - \bar{p}^i_{ij}\right)\right]$, provided $\left\{\bar{p}^i_{ij}, \bar{q}^j_{ij}\right\}_{ij\in g}$ exist, and minus infinity otherwise. Thus, taking each other dealers' bidding strategy as given, dealer i solves

$$\max_{\left\{B^i_{ij}(s^i;\left\{\pi^j_{ij,\tau}\right\}_{j\in g^i})\right\}_{j\in g^i}} E\left[\sum_{j\in g^i} \bar{q}^i_{ij}\left(\theta^i - \bar{p}^i_{ij}\right)|s^i\right].$$

Starting from an equilibrium in the OTC game, we construct a bidding strategy for dealer i as follows. When a trader at desk i receives a bid $\pi^j_{ij,\tau} = \{p^j_{ij,\tau}, q^j_{ij,\tau}\}$ from each of his counterparties $j \in g^i$, she transforms $p^j_{ij,\tau}$ to

$$e^j_\tau = \frac{p^j_{ij,\tau}\left(t^i_{ij} + t^j_{ij} - \beta_{ij}\right) - t^i_{ij} e^i_{\tau-1}}{t^j_{ij}}$$

for each $j \in g^i$. Then, she updates her expectation about the asset value to be

$$e^i_{\tau+1} = \bar{y}^i s^i + \bar{\mathbf{z}}_{g^i}\mathbf{e}_{g^i,\tau}. \tag{A.8}$$

Finally, she constructs the counter-offer $\pi^i_{ij,\tau+1}$ with elements

$$p^i_{ij,\tau+1} = \frac{t^i_{ij}e^i_{\tau+1} + t^j_{ij}e^j_\tau}{t^i_{ij} + t^j_{ij} - \beta_{ij}}$$
$$q^i_{ij,\tau+1} = t^i_{ij}\left(e^i_{\tau+1} - p^i_{ij,\tau+1}\right).$$

First, we show that if bidding functions are defined as above, the OTC price-discovery process converges to the equilibrium prices and quantities in the OTC game. To see this, we write (A.8) in matrix form as

$$\mathbf{e}_{\tau+1} = \bar{Y}\mathbf{s} + \bar{Z}\mathbf{e}_\tau.$$

where $\mathbf{e}_{\tau+1} = \left(e^i_{\tau+1}\right)_{i=1..n}$ and $\bar{Y}, \bar{Z}$ are constructed from $\bar{y}^i$ and $\bar{\mathbf{z}}_{g^i}$ respectively. Note that starting from any random vector $\mathbf{e}_0$ we'll have

$$\mathbf{e}_{\tau+1} = \left(I + \bar{Z} + ... + \left(\bar{Z}\right)^\tau\right)\bar{Y}\mathbf{s} + \left(\bar{Z}\right)^{\tau+1}\mathbf{e}_0.$$

In step 2 of the proof of Proposition 3, we show that the fact that all elements of $\bar{Z}$ are positive together with the existence of equilibrium in the conditional guessing game imply that $\lim_{u\to\infty}\left(\bar{Z}\right)^{u+1} = 0$, which in turn implies that $\left(I - \bar{Z}\right)$ is nonsingular (see Meyer (2000) page 618), $\left(I - \bar{Z}\right)^{-1} \geq 0$ and

$$\left(I - \bar{Z}\right)^{-1} = \sum_{\tau=1}^{\infty}\left(\bar{Z}\right)^\tau.$$

(see Meyer (2000) pp. 620 & 618.)

Thus, we have that

$$\lim_{\tau\to\infty}\mathbf{e}_{\tau+1} = \left(I - \bar{Z}\right)^{-1}\bar{Y}\mathbf{s},$$

or the equilibrium expectations in the OTC game. But then, by definition, $\left\{\bar{p}^i_{ij}, \bar{q}^i_{ij}\right\}_{ij\in g}$ exist and coincide with the equilibrium of the OTC game.

The last step is to show that dealer i would not want to change her bidding strategy unilaterally. Note that for any such deviation to be meaningful, it has to imply alternative limit price and quantity vectors. If there is no convergence, dealer i receives a payoff of minus infinity. However, by construction, if a modified bidding strategy converges to different price and quantity vectors, then these vectors are also fixed points of generalized demand curves in the OTC game. However, the other dealers' bidding strategies are constructed based on their equilibrium demand functions in the OTC game. This implies that if dealer i wants to deviate from the equilibrium bidding strategies in the price-discovery game, he wants to deviate from his generalized demand curve in the original OTC game as well. But this is a contradiction.

B Appendix: Closed forms in special cases

Throughout, we use the notation $\sigma \equiv \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$.

B.1 Centralized market

Following Vives (2011), we have

$$q_V = t_V \left(e^i - p_V\right)$$
$$p_V = \frac{t_V}{(\beta_V + nt_V)} \sum_i e^i,$$

where $t_V = \frac{-\beta_V}{n(z_V-1)+2-z_V}$ and $z_V = \frac{2\sigma\rho}{(1-\rho)(1+\rho(n-1))+\sigma}$ implying $\frac{\partial z_V}{\partial \rho}, \frac{\partial z_V}{\partial \sigma} > 0$ and expectations are privately fully revealing

$$e^i = E(\theta^i | s^i, \mathbf{e}_{g^i}) = E(\theta^i|\mathbf{s}) = \frac{1-\rho}{1+\sigma-\rho}\left(s^i + \frac{\rho\sigma}{(1-\rho)(1-\rho+n\rho+\sigma)}\sum_{i=1}^n s^i\right).$$

Substituting in these expressions into expressions (27)-(29) gives closed-form solutions for expected profit, expected utility of customers and welfare.

B.2 Star network

Without loss of generality, we characterize a star network with dealer 1 at the centre. There exist at least one equilibrium of the conditional guessing game such that for dealer 1

$$\bar{z}_{1i}^1 = \bar{z}_C \tag{B.1}$$

for any i. Similarly, for any dealer i in the periphery

$$\bar{z}_{i1}^i = \bar{z}_P.$$

We start with dealer 1, who chooses her demand function conditional on the beliefs of the other $(n-1)$ dealers. Given that she knows s_1, she can invert the signals of all the other dealers. Hence, her belief is given by

$$E(\theta^1|s^1, \mathbf{e}_{g^1}) = E(\theta^1|\mathbf{s}) = \frac{1-\rho}{1+\sigma^2-\rho}\left(s_1 + \frac{\rho\sigma^2}{(1-\rho)(1+\sigma^2-\rho+n\rho)}\sum_{i=1}^n s^i\right).$$

Or

$$E(\theta^1|s^1, \mathbf{e}_{g^1}) = v_{11}s^1 + \sum_{j=2}^n v_{1j}s^j,$$

where

$$v_{11} = \frac{1-\rho}{1+\sigma^2-\rho}\left(1+\frac{\rho\sigma^2}{(1-\rho)\left(1+\sigma^2-\rho+n\rho\right)}\right) \tag{B.2}$$

$$v_{1j} = \frac{1-\rho}{1+\sigma^2-\rho}\frac{\rho\sigma^2}{(1-\rho)\left(1+\sigma^2-\rho+n\rho\right)} \tag{B.3}$$

for all $j \neq 1$.

Further, the belief of a periphery dealer i is given by

$$E(\theta^i|s^i, e^1) = \begin{pmatrix} 1 \\ \tilde{\mathcal{V}}\left(\theta^i, e^1\right) \end{pmatrix}^\top \begin{pmatrix} 1+\sigma^2 & \tilde{\mathcal{V}}\left(s^i, e^1\right) \\ \tilde{\mathcal{V}}\left(s^i, e^1\right) & \mathcal{V}\left(e^1\right) \end{pmatrix}^{-1} \begin{pmatrix} s^i \\ e^1 \end{pmatrix},$$

where $\tilde{\mathcal{V}}(\cdot,\cdot) \equiv \frac{\mathcal{V}(\cdot,\cdot)}{\sigma_\theta^2}$ is the scaled covariance operator and

$$\tilde{\mathcal{V}}\left(e^1\right) = \frac{(1-\rho)\left(1+(n-1)\rho\right)+\sigma^2\left(1+(n-1)\rho^2\right)}{\left(1+\sigma^2-\rho\right)\left(1+\sigma^2+(n-1)\rho\right)}$$

$$\tilde{\mathcal{V}}\left(s^i, e^1\right) = \rho$$

$$\tilde{\mathcal{V}}\left(\theta^i, e^1\right) = \rho\frac{(1-\rho)\left(1+(n-1)\rho\right)+\sigma^2\left(2+(n-2)\rho\right)}{\left(1+\sigma^2-\rho\right)\left(1+\sigma^2+(n-1)\rho\right)}.$$

Since

$$\begin{aligned} E(\theta^i|s^i, e^1) &= \frac{\tilde{\mathcal{V}}\left(e_1\right)-\tilde{\mathcal{V}}\left(\theta^i, e_1\right)\rho}{\tilde{\mathcal{V}}\left(e_1\right)\left(1+\sigma^2\right)-\rho^2}s^i + \frac{\tilde{\mathcal{V}}\left(\theta^i, e_1\right)\left(1+\sigma^2\right)-\rho}{\tilde{\mathcal{V}}\left(e_1\right)\left(1+\sigma^2\right)-\rho^2}e^1 \\ &= v_{ii}s^i + v_{i1}s^i + \sum_{\substack{j=2 \\ j\neq i}}^{n} v_{1j}s^j \end{aligned}$$

for any $i \neq 1$, it follows that

$$v_{i1} = \frac{\tilde{\mathcal{V}}\left(\theta^i, e^1\right)\left(1+\sigma^2\right)-\rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right)-\rho^2}v_{11} \tag{B.4}$$

$$v_{ii} = \frac{\tilde{\mathcal{V}}\left(e^1\right)-\tilde{\mathcal{V}}\left(\theta^i, e^1\right)\rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right)-\rho^2} + \frac{\tilde{\mathcal{V}}\left(\theta^i, e^1\right)\left(1+\sigma^2\right)-\rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right)-\rho^2}v_{1j} \tag{B.5}$$

$$v_{ij} = \frac{\tilde{\mathcal{V}}\left(\theta^i, e^1\right)\left(1+\sigma^2\right)-\rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right)-\rho^2}v_{1j} \tag{B.6}$$

and

$$\bar{y}_P = \frac{\tilde{\mathcal{V}}\left(e^1\right) - \tilde{\mathcal{V}}\left(\theta^i, e^1\right)\rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right) - \rho^2}$$

$$\bar{z}_P = \frac{\tilde{\mathcal{V}}\left(\theta^i, e^1\right)\left(1+\sigma^2\right) - \rho}{\tilde{\mathcal{V}}\left(e^1\right)\left(1+\sigma^2\right) - \rho^2}.$$

Moreover, since

$$e^1 = E(\theta^1 | s^1, \mathbf{e}_{g^1}) = \bar{y}_C s^1 + \sum_{j=2}^{n} \bar{z}_C e^j = \bar{y}_C s^1 + \sum_{j=2}^{n} \bar{z}_C \left(\bar{y}_P s^i + \bar{z}_P e^1\right),$$

then

$$E(\theta^1 | s^1, \mathbf{e}_{g^1}) = \frac{\bar{y}_C}{1-(n-1)\,\bar{z}_C \bar{z}_P} s^1 + \sum_{j=2}^{n} \frac{\bar{z}_C \bar{y}_P}{1-(n-1)\,\bar{z}_C \bar{z}_P} s^i.$$

This implies that

$$\bar{z}_C = \frac{v_{1j}}{\bar{y}_P + (n-1)\,\bar{z}_P v_{1j}}$$

and

$$\bar{y}_C = \frac{v_{11}\bar{y}_P}{\bar{y}_P + (n-1)\,\bar{z}_P v_{1j}}.$$

We now solve the system (21) with substituting the expression for $\bar{z}_C, \bar{y}_C, \bar{z}_p, \bar{y}_P$ above giving the solution

$$z_P = 2\bar{z}_P$$

and

$$z_C = \bar{z}_C\left(n + 2\bar{z}_P - n\bar{z}_P - 1\right) + 1 - \sqrt{\left(\left(\bar{z}_C\left(n\left(1-\bar{z}_P\right) + 2\bar{z}_P - 1\right) + 1\right)\right)^2 - 4\bar{z}_C}$$

and

$$y_C = \bar{y}_C\left(1 - n z_C \frac{2 - z_P}{4 - z_C z_P}\right)$$

$$y_P = \bar{y}_P\left(1 - z_P \frac{2 - z_C}{4 - z_C z_P}\right).$$

B.3 Complete network

In the complete network, each dealer i chooses her demand function conditional on the beliefs of the other $(n-1)$ dealers. Given that she knows s^i, she can invert the signals of all the other dealers. Hence, her belief is given by

$$E(\theta^i | s^i, \mathbf{e}_{g^i}) = E(\theta^i | \mathbf{s}) = \frac{1-\rho}{1+\sigma-\rho}\left(s^i + \frac{\rho\sigma}{(1-\rho)\left(1-\rho+n\rho+\sigma\right)} \sum_{i=1}^{n} s^i\right).$$

Then, following the same procedure as above (for a star), and taking into account that in a complete network trading strategies are symmetric, we obtain that

$$E(\theta^i|s^i,\mathbf{e}_{g^i}) = \bar{y}s^i + \bar{z}\sum_{\substack{j=1\\j\neq i}}^{n} e^j$$

where

$$e^j = E(\theta^j|s^j,\mathbf{e}_{g^j})$$

and

$$\bar{y} = \frac{(1-\rho)(1+(n-1)\rho)}{1-\rho+\rho(1-\rho)(n-1)+\sigma(1+(n-2)\rho)}$$
$$\bar{z} = \frac{\rho\sigma_\varepsilon^2}{\sigma_\theta^2+\sigma_\varepsilon^2+\rho^2\sigma_\theta^2-2\rho\sigma_\theta^2-2\rho\sigma_\varepsilon^2-n\rho^2\sigma_\theta^2+n\rho\sigma_\theta^2+n\rho\sigma_\varepsilon^2}.$$

Solving the system (21), we obtain

$$y^i = \frac{\sigma_\theta^2(1-\rho)(1+(n-1)\rho)}{\left(\sigma_\theta^2(1-\rho)(1+(n-1)\rho)+\sigma_\varepsilon^2(1+2(n-3)\rho)\right)+3\rho\sigma_\varepsilon^2}, \forall i$$
$$z_{ij}^i = \frac{2\rho\sigma_\varepsilon^2}{\left(\sigma_\theta^2(1-\rho)(1+(n-1)\rho)+\sigma_\varepsilon^2(1+2(n-3)\rho)\right)+2\rho\sigma_\varepsilon^2}, \forall ij.$$

Substituting in the expressions for t_{ij}^i in Proposition (2) we obtain

$$t_{ij}^i = -\beta_{ij}\frac{\left(\sigma_\theta^2(1-\rho)(1+(n-1)\rho)+\sigma_\varepsilon^2(1+2(n-3)\rho)\right)+2\rho\sigma_\varepsilon^2}{2\rho\sigma_\varepsilon^2}.$$

C Appendix: Details on calibrated example

C.1 Baseline

We have calibrated our model by finding the equilbrium and calculating the matched moments for a grid of parameter values. We have targeted values are in Table C.1. We kept refining the grid to the point where the match was sufficiently close. The code and a detailed explanation is available on Peter Kondor's website.

moments	model at $\left(\rho = 0.014, \sigma = 0.1584, (-\beta)\sigma_\theta^2 = 7.3835\right)$	data
average spread (%)	0.742	0.742
relative price dispersion, core (%)	371.3	371.3
total volume ($M)	277676	277676

Table C.1: Matched and implied moments in the model and in the data. The data moments are from Hollifield et al. (2016, Table 2,3 and 10). We match the 71% of total volume as this is the fraction of fully identified chains. For model the average customer spread for a given dealer is $\frac{1}{|g^i|}\Sigma_{j\in g^i}\frac{2\beta}{t^i_{ij}}$ which we average over the whole sample. Relative price dispersion for core dealers is ratio of expected price dispersion to the absolute mean of prices in those transactions where one of the counterparties is a core dealer.

C.2 Market distress

To model the effect of market distress, we drop the most connected node from our network. On Figure 3, this is dealer 1. As dealers 22-23, 25-29, 43 and 69 are connected to the rest of the market only through dealer 1, we drop those dealers too. We have emphasized in the main text, under our calibrated parameters the first order determinant of price impact, intermediation volume and expected profit is dealer centrality. Consistently with this observation, the main channel our treatment affects the new equilibrium is that all the dealers who were connected to dealer 1 now have less trading partners. Therefore, these dealers face larger price impact, trade less and earn smaller expected profit. Averaging over dealers implies larger average price impact and less volume. Price dispersion also increases as the disruption of information flows leads to less convergence in posterior expectations which is the main determinant of price dispersion by expression (14).

C.3 Robustness

We searched over the parameter range used in Figure 4 and observed that the connection between degree centrality and expected profit, gross volume, intermediation, information precision and average price impact are qualitatively the same as illustrated in Figure 5. However, especially for larger correlation across dealers' values, ρ, degree centrality does not suppress all the other network characteristics to the same extent. Figure C.1, a variant of Figure 5 with $\rho = 0.8$ illustrates this observation.

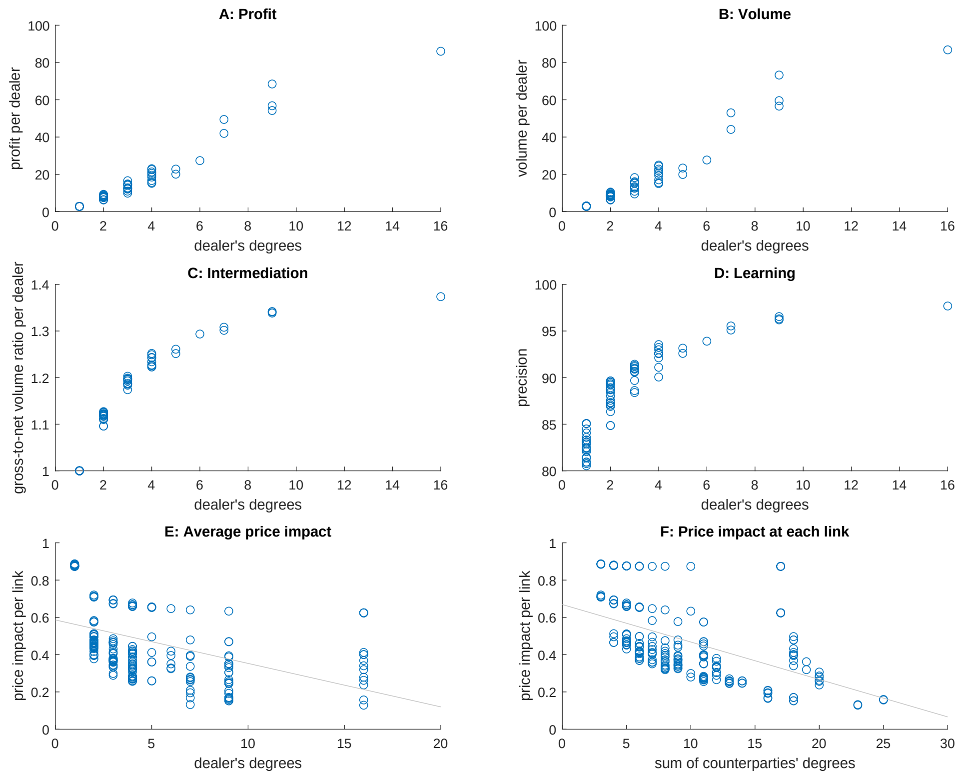

Figure C.1: Panels A-D show each dealer's expected profit, gross volume, intermediation, and posterior information precision (as percentage of precision under fully revealing prices) against the number of the dealer's trading partners. Panels E and F show the price impact a dealer faces at a given link against the number of her trading partners, and against the sum of the trading partners of the two counterparties at the given link, respectively. Parameter values are $\rho = 0.8, \sigma = 0.1584, \beta = -1, \sigma_\theta^2 = 7.3835$. We added a least-squares line to Panels E and F.

D Appendix: Trading in Segmented Markets

D.1 General set-up

Our framework can provide insights about trade in segmented markets as well. Markets are segmented when investors, such as hedge funds and asset management firms, trade in some markets but not in others. Although segmented, markets can be connected, in the sense agents are able to trade in multiple venues at the same time. To study the implications in segmented markets, we extend our model in the following way.

We consider an economy in which there are N trading posts connected in a network g. At each trading post, I, there exist n^I risk-neutral dealers. The entire set of dealers is denoted $\mathcal{N} = \bigcup_{I=N}^{N} I$. Each dealer $i \in I$ can trade with other dealers in his own trading post and with dealers at any trading post J that is connected with the trading post I by a link IJ. Essentially, the link IJ represents a market in which dealers at trading posts I and J can trade with each other. However, they have access to trade in other markets at the same time. Let g^I denote the set of trading posts that are linked with I in the network g, and $m^I \equiv \left|g^I\right|$ represent the number of I's links.

As before, dealers trade a risky asset in zero net supply, and all trades take place at the same time. Each dealer is uncertain about the value of the asset. In particular, a dealer's value for the asset is given by θ^i, which is a random variable normally distributed with mean 0 and variance σ_θ^2. Moreover, we consider that values are interdependent across all dealers. In particular, $\mathcal{V}(\theta^i, \theta^j) = \rho\sigma_\theta^2$ for any two agents $i, j \in \mathcal{N}$. Each dealer receives a private signal, $s^i = \theta^i + \varepsilon^i$, where $\varepsilon^i \sim IID\, N(0, \sigma_\varepsilon^2)$ and $\mathcal{V}(\theta^j, \varepsilon^i) = 0$, for all i and j.

A dealer $i \in I$ seeks to maximize her final wealth

$$\sum_{J \in g^I} q_{IJ}^i \left(\theta^i - p_{IJ}\right),$$

where q_{IJ}^i is the quantity traded by dealer i in market IJ, at a price p_{IJ}. Similarly to the OTC model, the trading strategy of the dealer i with signal s^i is a generalized demand function $\mathbf{Q}^i : R^{m^i} \to R^{m^i}$ which maps the vector of prices, $\mathbf{p}_{g^I} = (p_{IJ})_{J \in g^I}$, that prevail in the markets in which dealer i participates in network g into a vector of quantities she wishes to trade

$$\mathbf{Q}^i(s^i; \mathbf{p}_{g^I}) = \left(Q_{IJ}^i(s^i; \mathbf{p}_{g^I})\right)_{J \in g^I},$$

where $Q_{IJ}^i(s^i; \mathbf{p}_{g^I})$ represents her demand function in market IJ.

Apart from trading with each other, dealers also serve a price-sensitive customer base. In particular, we assume that for each market IJ, the customer base generates a downward sloping demand

$$D_{IJ}(p_{IJ}) = \beta_{IJ} p_{IJ}, \tag{D.1}$$

with an arbitrary constant $\beta_{IJ} < 0$. The exogenous demand (D.1) ensures the existence of the equilibrium when agents are risk neutral, and facilitates comparisons with the OTC model.

The expected payoff for dealer $i \in I$ corresponding to the strategy profile $\left\{\mathbf{Q}^i\left(s^i; \mathbf{p}_{g^I}\right)\right\}_{i \in \mathcal{N}}$ is

$$E\left[\sum_{J \in g^I} Q_{IJ}^i(s^i; \mathbf{p}_{g^I}) \left(\theta^i - p_{IJ}\right) | s^i\right]$$

where p_{IJ} are the prices for which all markets clear. That is, prices satisfy

$$\sum_{i \in I} Q_{IJ}^i\left(s^i; \mathbf{p}_{g^I}\right) + \sum_{j \in J} Q_{IJ}^j\left(s^j; \mathbf{p}_{g^J}\right) + \beta_{IJ} p_{IJ} = 0, \ \forall\ IJ \in g. \tag{D.2}$$

D.2 Equilibrium concept

As in the OTC game, we use the concept of Bayesian Nash equilibrium. For completeness, we reproduce it below.

Definition 3 *A Linear Bayesian Nash equilibrium of the segmented market game is a vector of* *linear generalized demand functions* $\left\{\mathbf{Q}^i\left(s^i;\mathbf{p}_{g^I}\right)\right\}_{i\in\mathcal{N}}$ *such that* $\mathbf{Q}^i(s^i;\mathbf{p}_{g^I})$ *solves the problem*

$$\max_{(Q^i_{IJ})_{J\in g^I}} E\left\{\left[\sum_{J\in g^I} Q^i_{IJ}(s^i;\mathbf{p}_{g^I})\left(\theta^i - p_{IJ}\right)\right]|s^i\right\}, \tag{D.3}$$

for each dealer i*, where the prices* p_{IJ} *satisfy (D.2) .*

A dealer i chooses a demand function in each market IJ, in order to maximize her expected profits, given her information, s^i, and given the demand functions chosen by the other dealers.

D.3 The Equilibrium

In this section, we outline the steps for deriving the equilibrium in the segmented market game for any network structure. First, we derive the equilibrium strategies as a function of posterior beliefs. Second, we construct posterior beliefs that are consistent with dealers' optimal choices. In the OTC game we used the conditional guessing game as an intermediate step in constructing beliefs. Here, we employ the same line of reasoning, although we do not explicitly introduce the conditional guessing game structure that would correspond to the segmented market game.

D.3.1 Derivation of demand functions

We conjecture an equilibrium in demand functions, where the demand function of dealer i in market IJ is given by

$$Q^i_{IJ}(s^i;\mathbf{p}_{g^I}) = t^I_{IJ}(y^I_{IJ}s^i + \sum_{K\in g^I} z^I_{IJ,IK}p_{IK} - p_{IJ}) \tag{D.4}$$

for any $i \in I$ and J. As evident in the notation, we consider that all dealers at trading post I are symmetric in their trading strategy, and weigh in same way the signal they receive and the prices they trade at. Nevertheless, they end up trading different quantities, as they have different realizations of the signal.

We solve the optimization problem (D.3) pointwise. That is, for each realization of the vector of signals, $\mathbf{s}$, we solve for the optimal quantity q^i_{IJ} that each dealer $i \in I$ demands in market IJ. Given the conjecture (D.4) and the market clearing conditions (D.2), the residual inverse demand function of dealer i in market IJ is

$$p_{IJ} = -\frac{t^I_{IJ}y^I_{IJ}\sum\limits_{k\in I,k\neq i} s^k + t^J_{IJ}y^J_{IJ}\sum\limits_{k\in J} s_k + (N_I-1)\sum\limits_{L\in g^I,L\neq J} t^I_{IJ}z^I_{IJ,IL}p_{IL} + N_J\sum\limits_{L\in g^J,L\neq I} t^J_{IJ}z^J_{IJ,JL}p_{JL} + q^i_{IJ}}{(N_I-1)\,t^I_{IJ}\left(z^I_{IJ,IJ}-1\right) + N_J t^J_{IJ}\left(z^J_{IJ,IJ}-1\right) + \beta_{IJ}}. \tag{D.5}$$

Denote

$$I^J_i \equiv -\frac{t^I_{IJ}y^I_{IJ}\sum\limits_{k\in I,k\neq i} s^k + t^J_{IJ}y^J_{IJ}\sum\limits_{k\in J} s_k + (N_I-1)\sum\limits_{L\in g^I,L\neq J} t^I_{IJ}z^I_{IJ,IL}p_{IL} + N_J\sum\limits_{L\in g^J,L\neq I} t^J_{IJ}z^J_{IJ,JL}p_{JL}}{(N_I-1)\,t^I_{IJ}\left(z^I_{IJ,IJ}-1\right) + N_J t^J_{IJ}\left(z^J_{IJ,IJ}-1\right) + \beta_{IJ}} \tag{D.6}$$

and rewrite (D.5) as

$$p_{IJ} = I_i^J - \frac{1}{(N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1 \right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1 \right) + \beta_{IJ}} q_{IJ}^i. \tag{D.7}$$

The uncertainty that dealer i faces about the signals of others is reflected in the random intercept of the residual inverse demand, I_i^J, while her capacity to affect the price is reflected in the slope $-1/\left((N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1 \right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1 \right) + \beta_{IJ} \right)$. In the segmented markets game, however, the random intercept I_i^J reflects not only the signals of the dealers at the trading post J, but also the signals of the other dealers at the trading post I.

Then, solving the optimization problem (D.3) is equivalent to finding the vector of quantities $\mathbf{q}^i = \mathbf{Q}^i(s^i; \mathbf{p}_{g^I})$ that solve

$$\max_{(q_{IJ}^i)_{j \in g^I}} \sum_{J \in g^I} q_{IJ}^i \left(E\left(\theta^i | s^i, \mathbf{p}_{g^I} \right) - \left(I_i^J - \frac{q_{IJ}^i}{(N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1 \right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1 \right) + \beta_{IJ}} \right) \right)$$

From the first order conditions we derive the quantities q_{IJ}^i that dealer $i \in I$ trades in each market IJ, for each realization of $\mathbf{s}$, as

$$2 \frac{1}{(N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1 \right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1 \right) + \beta_{IJ}} q_{IJ}^i = I_i^J - E\left(\theta^i | s^i, \mathbf{p}_{g^I} \right),$$

This implies that the optimal demand function

$$Q_{IJ}^i(s^i; \mathbf{p}_{g^i}) = -\left((N_I - 1)\, t_{IJ}^I \left(z_{IJ,IJ}^I - 1 \right) + N_J t_{IJ}^J \left(z_{IJ,IJ}^J - 1 \right) + \beta_{IJ} \right) \left(E(\theta^i \left| s^i, \mathbf{p}_{g^I} \right.) - p_{IJ} \right) \tag{D.8}$$

for each dealer i in market IJ.

Further, given our conjecture (D.4), equating coefficients in equation (D.8) implies that

$$E(\theta^i \left| s^i, \mathbf{p}_{g^I} \right.) = y_{IJ}^I s^i + \sum_{K \in g^I} z_{IJ,IK}^I p_{IK}.$$

However, the projection theorem implies that the belief of each dealer i can be described as a unique linear combination of her signal and the prices she observes. Thus, it must be that $y_{IJ}^I = y^I$, and $z_{IJ,JK}^I = z_{IK}^I$ for all I, J, and K. In other words, the posterior belief of a dealer i is given by

$$E(\theta^i \left| s^i, \mathbf{p}_{g^I} \right.) = y^I s^i + \mathbf{z}_{g^I} \mathbf{p}_{g^I}, \tag{D.9}$$

where $\mathbf{z}_{g^I} = \left(z_{IJ}^I \right)_{J \in g^I}$ is a *row* vector of size m^i. Then, we obtain that the trading intensity of dealer at trading post I satisfies

$$t_{IJ}^I = (N_I - 1)\, t_{IJ}^I \left(1 - z_{IJ}^I \right) + N_J t_{IJ}^J \left(1 - z_{IJ}^J \right) - \beta_{IJ}. \tag{D.10}$$

If we further substitute this into the market clearing conditions (D.2) we obtain the price in market IJ as follows

$$p_{IJ} = \frac{t_{IJ}^{I}\left(\sum_{i\in I} E\left(\theta^{i}|s^{i},\mathbf{p}_{g^{I}}\right)\right) + t_{IJ}^{J}\left(\sum_{j\in J} E\left(\theta^{j}|s^{j},\mathbf{p}_{g^{J}}\right)\right)}{N_{I}t_{IJ}^{I} + N_{J}t_{IJ}^{J} - \beta_{IJ}}. \tag{D.11}$$

From (D.10) and the analogous equation for t_{IJ}^{J}, it is straightforward to derive the trading intensity that dealers at trading post I and J have. This implies that we can obtain the price in each market IJ as

$$p_{IJ} = w_{IJ}^{I}\left(\sum_{i\in I} E\left(\theta^{i}|s^{i},\mathbf{p}_{g^{I}}\right)\right) + w_{IJ}^{J}\left(\sum_{j\in J} E\left(\theta^{j}|s^{j},\mathbf{p}_{g^{J}}\right)\right), \tag{D.12}$$

where

$$w_{IJ}^{I} \equiv \frac{z_{IJ}^{J} - 2}{\left(N_{J} + N_{I} - 1\right) z_{IJ}^{I} z_{IJ}^{J} - 2\left(N_{I} - 1\right) z_{IJ}^{I} - 2\left(N_{J} - 1\right) z_{IJ}^{J} - 4}.$$

This expression is useful to relate the belief of a dealer $i \in I$ to the beliefs of other dealers at the same trading post, and at trading posts that are connected to I.

D.3.2 Derivation of beliefs

We follow the same solution method that we developed in Section 3.1. As before, the key idea is to reduce the dimensionality of the problem and use our conjecture about demand functions to derive a fixed point in beliefs, instead of the fixed point (D.8).

In the OTC game we constructed each dealer's equilibrium belief as a linear combination of the beliefs of her neighbors in the network. For this, we introduced the conditional guessing game. The conditional guessing game was a useful intermediate step in making the derivations more transparent, as well as an informative benchmark about the role of market power for the diffusion of information.

In the segmented market game it is less straightforward to formulate the corresponding conditional guessing game. Since there are multiple dealers at each trading post, it is not immediate how each dealer forms her guess. In particular, we would need to make additional assumptions about the linear combination of the guesses of dealers in the same trading post and dealers of the neighboring trading post, that each agent can condition her guess on.

Thus, in the segmented market game we construct beliefs directly as linear combinations of signals. We conjecture that for each dealer $i \in I$, her belief is an affine combination of the signals of all dealers in the economy

$$E\left(\theta^{i}|s^{i},\mathbf{p}_{g^{I}}\right) = \bar{v}_{II}^{I} s^{i} + \sum_{K=1}^{N} v_{IK}^{I} S^{K}, \tag{D.13}$$

where $S^{K} = \sum_{k\in K} s^{k}$, $\forall K$. This further implies that

$$\sum_{i\in I} E\left(\theta^{i}|s^{i},\mathbf{p}_{g^{I}}\right) = \bar{v}_{II}^{I} S^{I} + N_{I} \sum_{K=1}^{N} v_{IK}^{I} S^{K}.$$

Before we derive the fixed point equation for beliefs, it is useful to write (D.12) in matrix form, for each trading post I. For this we introduce some more notation. Unless specified otherwise, in the notation below we keep I fixed and vary $J \in \{1, ..., N\}$. Let $\mathbf{p}^I$ be a $N-$column vector with elements p_{IJ} if $IJ \in g$, and 0 otherwise. Let $\mathbf{z}^I$ be a $N-$column vector with elements z^I_{IJ} if $IJ \in g$, and 0 otherwise. Similarly, let $\mathbf{w}^I$ be the $N-$column vector with elements w^I_{IJ} if $IJ \in g$, and 0 otherwise, while W_I be a matrix with elements w^J_{IJ} on diagonal if IJ have a link, and 0 otherwise (all elements off-diagonal are 0, as well). Further, let $\mathbf{v}^I$ be the $N-$row vector with elements v^I_{IJ}, and $\bar{\mathbf{v}}^I$ be the $N-$row vector with elements $\bar{v}^I_{II}$ at position I and 0 otherwise. Let V be the square matrix with rows $\mathbf{v}^I$, and $\bar{V}$ be the matrix with rows $\bar{\mathbf{v}}^I$. Let $\mathbf{S}$ be the $N-$column vector with elements S^I. Let N be a square matrix with elements n^I on diagonal and 0 otherwise. Let $\mathbf{1}$ be the $N-$column vector of ones.

Substituting our conjecture for beliefs (D.13) in the equation for the price (D.12), we obtain the vector of prices which dealers at each trading post I are trading as

$$\mathbf{p}^I = \mathbf{w}^I \left(\bar{\mathbf{v}}^I + n^I \mathbf{v}^I\right) \mathbf{S} + W^I \left(\bar{V} + \mathsf{N}V\right) \mathbf{S}.$$

We are now ready to formalize the result.

Proposition D.1 *There exists an equilibrium in the segmented markets game if the following system of equations*

$$\mathbf{v}^I = \left(\mathbf{z}^I\right)^\top \left(\mathbf{w}^I \left(\bar{\mathbf{v}}^I + n^I \mathbf{v}^I\right) + W^I \left(\bar{V} + \mathsf{N}V\right)\right) \mathbf{1}, \forall I \tag{D.14}$$

and

$$\bar{v}^I_{II} = y^I, \forall I$$

admits a solution in $\mathbf{v}^I$*, for each* I*.*

Proof. As for the OTC game, the proof is constructive. Note that showing that equation (D.14) admits a solution is equivalent to showing that there exists a fixed point in $\mathbf{v}^I$. This is because, the projection theorem implies that $\mathbf{z}^I$, and inherently, $\mathbf{w}^I$ are a function of $\mathbf{v}^I$.

Let $\mathbf{v}^I$ be a fixed point of (D.14) and $\bar{v}^I_{II} = y^I$, for each I. We construct an equilibrium for the segmented-market game with beliefs given by (D.13), as follows. We choose conveniently $\mathbf{z}^I$ and $\mathbf{w}^I$ such that

$$E\left(\theta^i | s^i, \mathbf{p}_{g^I}\right) = y^I s^i + \left(\mathbf{z}^I\right)^\top \left(\mathbf{w}^I \left(\bar{\mathbf{v}}^I + n^I \mathbf{v}^I\right) + W^I \left(\bar{V} + \mathsf{N}V\right)\right) \mathbf{S}$$

is satisfied. Then, it follows that the prices given by (D.11) and demand functions given by (D.8) is an equilibrium of the OTC game. ■

The derivation we have outlined above also highlights the main technical difficulty of the segmented market game relative to the OTC game. That is, the signals of dealers in the same trading post obscure the (sum of) beliefs of the dealers in neighboring trading posts, such that a dealer can no longer invert the prices she observes and infer what his neighbors posteriors.

D.4 Learning and illiquidity in a star network

In this section, we illustrate the effects of market integration on learning from prices and market liquidity in an example.In particular, we restrict ourselves to considering a star network, in which there are n_P dealers at each periphery trading post, and n_C dealers at the central trading post. In particular, we conduct the following numerical exercise. We consider an economy with nine agents. Keeping their information set fixed, we compare the following four market structures:

1. 8 trading posts connected in a star network, with one agent in each trading post ($N = 8$, $n_P = 1$, $n_C = 1$), that is, 8 trading venues. This is our baseline model with a star network.

2. 4 trading posts connected in a star network, with two agents in each periphery node and one agent in the central node ($N = 4$, $n_P = 2$, $n_C = 1$), that is, 4 trading venues.

3. 2 trading posts connected in a star network, with four agents in each periphery node and one agent in the central node ($N = 2$, $n_P = 4$, $n_C = 1$), that is, 2 trading venues.

4. A centralized market ($N = 1$, $n_P = 9$, $n_C = 0$), that is, a single trading venue.

We consider two directions. First, we investigate what drives the illiquidity central and periphery agents face for changing degrees of market segmentation. We concentrate on (il)liquidity as this is a more commonly reported variable in the empirical literature, and we leave the analysis of welfare and expected profits to Appendix D. Second, to complement the analysis in Section 4 , we also analyze how much dealers can learn from prices under these market structures.

The left and center panels in Figure D.2 show the average illiquidity that a periphery, $\frac{1}{t_P}$, and a central dealer, $\frac{1}{t_C}$, face in each of the scenarios described above. We also plot the average illiquidity that any agent in a centralized market, $\frac{1}{t_V}$, faces. For easy comparison, all the parameters are the same as in Section 5.1.

To highlight the intuition, we start with the extreme cases of market segmentation comparing illiquidity under a star network and in a centralized market.

D.4.1 Extreme cases of market segmentation with a star network

In this part, we compare illiquidity of dealers in a centralized market and that of a periphery or central dealer in a star network.

The solid curve in Panels D and the curves in panel F in Figure 2 illustrate that compared to any agent in a centralized market, the central agent in the star faces higher trading price impact in general, but the periphery agents tend to face smaller price impact when the correlation across values is sufficiently high. We partially prove this result. The following proposition states that if ρ is sufficiently large, illiquidity for the central agent is larger, while illiquidity for the periphery agents is lower than that for an agent in a centralized market and, when ρ is sufficiently small, illiquidity for any agent in a star network is larger than the illiquidity for any agent in a centralized market.

Proposition D.2

1. *When ρ is sufficiently small, such that z_V is sufficiently close to $1 - \frac{1}{n-1}$, then illiquidity for any agent in a star network is larger than for any agent in a centralized market*

2. *In the common value limit, when* $\rho \to 1$,

   (a) *illiquidity for a central agent is higher in a star network than for any agent in a centralized market, and*

   (b) *illiquidity for a periphery agent is lower in a star network than for any agent in a centralized market.*

Proof. The first part comes by the observation that as $z_V \to 1 - \frac{1}{n-1}$, $t_V \to \infty$, while t_C and t_P are finite for these parameters. The second part comes from taking the limit $\rho \to 1$ of the ratio of the corresponding closed-form expressions we report in Appendices B.3 and B.2. In particular,

$$\lim_{\rho \to 1} \frac{t_V}{t_C} = (n-1) \frac{z_C + z_P - z_C z_P}{(2 - z_P)((n-1) z_V - (n-2))} = \infty$$
$$\lim_{\rho \to 1} \frac{t_V}{t_P} = (n-1) \frac{z_C + z_P - z_C z_P}{(2 - z_C)((n-1) z_V - (n-2))} = \frac{n-1}{n} < 1$$

■

Similarly to the comparison between the complete OTC network and the centralized market in Section 5.1.2, there are two main forces that drive the illiquidity ratios $\frac{t_V}{t_C}$ and $\frac{t_V}{t_P}$. First, the best response function (31) of a dealer in a centralized market is steeper and has a larger intercept than the best response function (26) of central and periphery dealers in the star OTC network. Simple algebra shows that if, counterfactually, the adverse selection parameters were equal, $z_P = z_C = z_V$ then $\frac{t_V}{t_C}|_{z_V = z_C = z_P} = \frac{t_V}{t_P}|_{z_V = z_C = z_P} > 1$, that is, illiquidity for any agents in the OTC market would be higher than for any agent in the centralized market. This is the effect which dominates when ρ is small.

Second, parameters z_C, z_V and z_P differ. As we stated in Proposition 9 central agents face less liquid markets than periphery agents, $\frac{1}{t_P} < \frac{1}{t_C}$ because periphery agents are more concerned about adverse selection ($z_C < z_P$). This implies that $\frac{t_V}{t_C} > \frac{t_V}{t_P}$ and difference is increasing for higher ρ. In fact, in the common value the central agent faces an infinitely illiquid market in the sense that $t_C \to 0$, but consumers provide a relatively liquid trading environment for periphery agents. For periphery agents this is sufficiently strong to reduce their price impact below the centralized market level as stated in the second part of the proposition.

D.4.2 Intermediate cases of market segmentation with a star network

Interestingly, while the illiquidity a central agent faces is monotonic in segmentation, the illiquidity a periphery agents face is not. We see in left panel of Figure D.2 is how the relative strength of the two forces identified in Section D.4.1 plays out in the four scenarios we consider. First, related to the effect of decentralization on best response functions, illiquidity for any agent decreases as the market structure approaches a centralized market. Second,the effect coming from the differing weights of z_C and z_P is weaker in more centralized markets. The reason is that as central dealers observe less prices in more centralized markets, they put a larger weight, z_C in each price, implying a smaller difference between z_P and z_C. This is the reason why the illiquidity a periphery agent faces under the 2 trading venues structure increases with ρ almost as fast as in centralized markets. With 4 venues the effect of ρ is weaker.

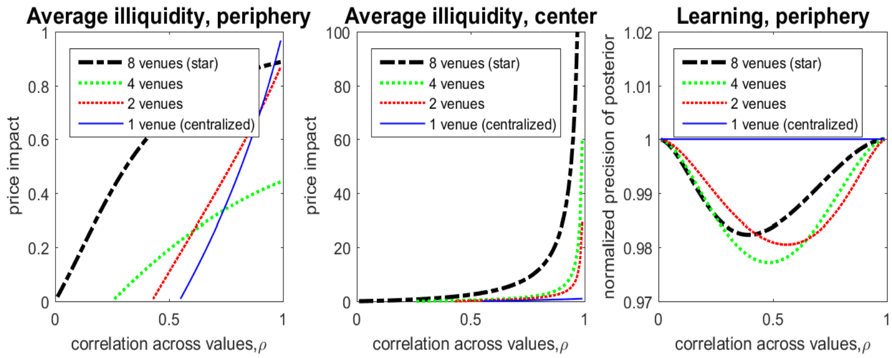

Figure D.2: Illiquidity on segmented markets. We show our measure of illiquidity for central agents , $\frac{1}{t_C}$, (left panel) and for periphery agents, $\frac{1}{t_P}$, (right panel) when there are 8 trading venues (dotted), 4 trading venues (dashed), 2 trading venues (dash-dotted), and in the centralized market (solid) as a function of the correlation across values, ρ. Other parameter values are $\sigma_\theta^2 = 1$, $\sigma_\varepsilon^2 = 0.1$, $B = 1$.

Turning to the effect of segmentation on learning, note that for the central dealer prices are fully revealing under any of the segmented market structures in this exercise. This is because each price she observes is a weighted sum of her own signal and the sum of signals of the periphery dealers trading in each venue. Hence, the prices the central dealer observes represent a sufficient statistic for all the useful information in the economy. This would not be the case if there were more than one dealer at the central trading post.

In contrast, as it is shown in the right panel of Figure D.2 a periphery agent in a segmented market always learns less than the central agent, or any agent in a centralized market. Interestingly, for small correlation across values, ρ, a periphery agent in a more segmented market learns more, while for a sufficiently large correlation across values the opposite is true. The intuition relies on the relative strength of opposing forces. The price a periphery agent learns from is a weighted average of the sum of posteriors of periphery agents in the same trading post and the posterior of the central agent. The posterior of the central agent is more informative than any of the posteriors that periphery agent at the same trading post have. The more segmented the market is, the easier is for a dealer at a periphery trading post to isolate the posterior of the central dealer (for example, in the baseline star network, any price reveals the posterior of the central dealer perfectly). At the same time, the sum of the posteriors of periphery dealers at a periphery trading post is more informative in a less segmented market, as the noise in the signal, as well as the private value components tend to cancel out. This latter effect helps learning more when the private value component is more important, that is, when ρ is small. This explains the pattern in the right panel of Figure D.2.

D.5 Welfare and expected profit in the star network

Finally, we illustrate with the following figure how expected profit and welfare changes with market segmentation. We leave the detailed analysis for future research and highlight only two interesting observations. First, as trading intensities were not monotonic for the periphery in the degree of segmentation, expected profit is not monotonic either. Also, total welfare is also not monotonic in segmentation.