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# Dynamic Asset-Backed Security Design

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**Abstract**

Borrowers obtain liquidity by issuing securities backed by current period payoff and resale price of a long-lived collateral asset. They are privately informed about the payoff distribution. Asset price can be self-fulfilling: higher asset price lowers adverse selection, allows borrowers to raise more funding which makes the asset more valuable, leading to multiple equilibria. Optimal security design eliminates multiple equilibria, improves welfare, and can be implemented as a repo contract. Persistence in adverse selection lowers debt funding, generates volatility in asset price, and exacerbates credit crunch. The theory demonstrates the role of asset-backed securities on stability of market-based financial systems.

**Keywords:** Liquidity; Dynamic Price Feedback; Tradable Assets; Inter-temporal Coordination; Security Design; Multiple Equilibria; Self-fulfilling Prices; Financial Fragility; Haircut; Repo; Repo Runs; Credit Crunch; Asset-Backed Security; Collateral; Limited Commitment; Adverse Selection; Market-Based Financial Intermediation.

**JEL classification:** G10, G01

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# 1 Introduction

In this paper, we propose a theory of security design when the underlying asset is long-lived and traded, and both the current period payoff and the resale price of the asset are used to back the optimally chosen securities. There is asymmetric information about the asset quality: asset owners are better informed about the current period payoff and this information advantage might be short-lived or persistent. There are also gains from trade in the style of Lagos and Wright (2005): the asset-backed securities are used as means of payment for asset owners to fund some outside consumption or investment opportunity. In our model, due to its collateral role, the entire cum-dividend value of the asset creates gains from trade. Consequently, the severity of the adverse selection problem depends on the cum-dividend value of the asset, which activates a price feedback: a high asset price in the future lowers the adverse selection today allowing for more asset-backed security sales today, which in turn justifies the high asset price. In the model, security design and asset price are jointly determined in equilibrium. In this economic setting, we find that when the set of available securities is restricted, there are multiple equilibria with destabilizing self-fulfilling prices. Optimal security design eliminates multiplicity and leads to a unique Pareto-optimal equilibrium where the resale price of the asset is high and the sale of securities backed by the asset raises more funding.

The price feedback mechanism in our paper is empirically relevant because increasingly more markets (exchange or over-the-counter) are created to trade variety of financial assets. Resale prices of these financial assets become collateralizable and are now an important component of the collateral value for borrowing obligations. Financial institutions, consequently, are becoming more dependent on markets to assess the collateral values when intermediating capital flows.[1] For example, short-term asset-backed borrowing facilities including repos or repo-like products are widely adopted as financing instruments for financial institutions. Such securities transform marketable collaterals with varying levels of quality, opacity and information friction to immediate funding, and thereby increase funding liquidity and fuel economic growth. Currently, repo financing remains a crucial source of short-term funding for financial institutions, estimated to have an outstanding notional amount of $12 trillion globally (CGFS, 2017). At the same time, the rise of market-based financial system has sown the seeds of instability: some of the short-term borrowing facilities such as asset-backed commercial papers (ABCPs) experienced runs

[1] According to the Financial Stability Board report on the global shadow banking sector (FSB, 2019) assets under the market-based financial intermediation grew faster than the assets under traditional banks (characterized by the originate-and-hold business models) from 2008 to 2018, and reached 48% of total financial assets at the end of 2018. By the end of 2018, the size of the market-based financial intermediation stood at $184 trillion compared to $148 trillion for banks.

during the Great Recession. Our theory offers a framework to understand the potential fragility in the market-based financial system with wide-spread securitization.

In our model, borrowers value liquidity more than investors, which leads to gains from trade. Borrowers face two commonly observed frictions in raising liquidity. First, they cannot promise to pay back, and thus cannot borrow from the investors unless the promise is made credible. To overcome this lack-of-commitment problem and make the promise credible, borrowers sell securities backed by the value of a long-lived collateral asset which includes the current period payoff and the endogenous resale price. Second, there is asymmetric information about the quality of the collateral asset, which leads to adverse selection that can be short-lived or persistent. This friction limits the collateral asset's effectiveness in raising liquidity. Under these two assumptions on the frictions, we find that a dynamic price feedback emerges in our model since the level of adverse selection depends on the collateral price and the collateral price and security design are mutually dependent. When the set of available financial instruments is restricted, this dynamic feedback loop leads to multiple equilibria in liquidity provision.

To illustrate how this dynamic price feedback leads to multiplicity, we first consider a case where borrowers are restricted to selling asset-backed *equity*.[2] The quality of the collateral asset (captured by the distribution of its payoff) is either high or low and varies period by period with persistence. Borrowers are privately informed about the current period quality at the beginning of each period when they issue equity. Hence, the equity market is subject to adverse selection. The collateral asset is traded in a frictionless asset market at the end of each period. A high resale price of the asset allows borrowers to exchange the asset-backed equity claims for more immediate liquidity and lowers the adverse selection, thereby attracting borrowers with high-quality assets to participate in the equity market.

The dynamic price feedback leads to three possible equilibrium regions in this economy. There is a 'separating' region where adverse selection is severe. In this region, only borrowers with low-quality assets sell their asset-backed equity to obtain funding. Borrowers with high-quality assets choose to not sell any since their equity claims are valued at a large discount due to severe adverse selection. Consequently, the asset-backed equity price today is indeed low, and the asset's resale price is depressed. There is a 'pooling' region where adverse selection is low. In this region, both types raise funding by selling equity claims to realize high gain from trade. Consequently the equity is priced at a high pooling price, and the asset price is booming. There is also a 'multiplicity' region where adverse selection is intermediate, and both separating and pooling equilibria coexist. That is, in this region prices are

[2]This restriction is a natural one since equity instruments are available to all economies including those without developed financial markets.

self-fulfilling.

Next, we introduce security design. For expositional clarity we make the following modeling choices: 1) in each period, the designer chooses a menu of securities backed by the collateral asset's current period dividend and its resale price to maximize ex ante surplus; 2) we assume that securities are traded in dedicated over the counter markets that are informationally segmented; and 3) in each security market investors engage in Bertrand competition. In this environment, we demonstrate that when security design is optimally chosen every period, the dynamic price feedback eliminates the aforementioned dynamic multiplicity and restores the uniqueness in equilibrium.[3] This finding highlights an additional benefit of security design besides the well understood one in the literature – that in a static economy optimal security design improves liquidity.[4] Formally, we show that there is a *unique* equilibrium with security design where the optimal design involves a short-term debt tranche that both borrower types sell in a pooling equilibrium, and a residual equity tranche that only the low type borrower sells in a separating equilibrium. The unique security design equilibrium Pareto-dominates the separating equilibrium and corresponds to the pooling equilibrium in the multiple equilibria region of the equity-only baseline case.

A key economic force in the optimal security design is the dynamic feedback between the asset price and the face value of the debt tranche. As the collateral price increases, the debt-tranche becomes more valuable and the designer is able to raise the face value of the debt further. Conversely, as the face value of debt increases, borrowers raise more liquidity by selling debt and hence realize larger gains from trade, which leads to a higher collateral price. With security design, agents inter-temporally coordinate their beliefs so that the debt tranche is always traded in a pooling equilibrium in each period, leading to higher collateral asset price. Higher asset price, in turn, justifies the debt tranche to be traded in a pooling equilibrium. This dynamic feedback loop removes multiplicity.

In the static setting, it is known since the work of Akerlof (1970) that multiple equilibria may exist in adverse selection models if buyers take prices as given. Wilson (1980) has shown that when buyers are strategic and compete à la Bertrand, there is a unique equilibrium in the static adverse selection environment. However, Wilson's logic does not extend to dynamic settings: it is possible that the expectation of low prices in the future induces adverse selection in the present, leading to a self-

[3]In the online appendix, we show that the assumption that borrowers have the flexibility to adjust the security design at the beginning of each period is important. In practice, security contract terms may not be updated frequently because of administrative costs or simply inattention. When contract terms are rigid in the sense that the face value may not be updated each period, a run equilibrium through a dynamic price feedback might emerge, and the liquidity of the security market may deteriorate.

[4]For example, Leland and Pyle (1977); Myers and Majluf (1984); Nachman and Noe (1994); DeMarzo and Duffie (1995); Biais and Mariotti (2005) and many others reviewed later.

fulfilling low-price equilibrium that survives standard game-theoretic foundations. We show that a new theoretical result, but in a similar spirit, holds in dynamic settings. Expanding the set of available securities between buyers and sellers eliminates the dynamic multiple equilibria and restores uniqueness. The model generates unique implications in the case of persistent private information. When private information is persistent, asset price is state dependent which introduces an additional source of adverse selection. As a result when private information is long-lived, the debt tranche is smaller and supports less funding than when it is short-lived.

Moreover, we demonstrate that these insights on price feedback and security design are robust to alternative modeling choices on the markets for securities and the collateral asset. First, we show that our main result carries over to the case of unsegmented security markets where there is information leakage across securities markets. Second, we demonstrate that issuing long-term securities backed by existing long-term securities is equivalent to issuing securities backed by the current period dividend and the collateral asset's resale price. Third, instead of Bertrand competition among investors, we assume borrowers and investors interact through an intermediary that maximizes total funding from each security. We show that our baseline multiplicity result holds under this alternative microstructure. Fourth, we allow for non-competitive pricing of the collateral asset where buyers and sellers bargain over the price of the asset at the end of each period. We show that non-competitive pricing is equivalent to our main model except that borrowers obtain lower gains from trade.

In the last part of the paper, we focus on one implementation of the optimal security design: short-term repo contracts. In the repo implementation, there is a representation borrower who values funding liquidity more than the investors. The debt tranche in the optimal design has key characteristics of repo and repo-like contracts: short-term, collateralized debt, typically backed by long-term collateral assets.[5] Asset repurchase arises naturally since this borrower has incentive to purchase back the asset in every period to use it for backing securities in the following period. We provide simple and transparent comparative static results that link repo terms with the model primitives for both types of information frictions, hence deriving new testable implications regarding properties of the repo markets.

## 2 Related Literature

In his seminal work on the lemons market, Akerlof (1970) studies the impact of adverse selection on trade volume and efficiency. There is a long lineage of literature security design with asymmetric in-

[5]The residual equity tranche can be thought of as a derivative contract traded on the over-the-counter market.

formation and adverse selection, including Leland and Pyle (1977), Myers and Majluf (1984), Nachman and Noe (1994), DeMarzo and Duffie (1995), DeMarzo and Duffie (1999), Biais and Mariotti (2005).[6] We contribute to this literature by introducing dynamic feedback from asset prices to the security design and illustrating that security design eliminates multiple equilibria. [7]

By studying optimal collateral-backed security design and funding liquidity, our paper is also related to the literature on collateralized lending in monetary economics and macroeconomics starting with the seminal work of Kiyotaki and Moore (1997). Recent works on macroeconomic implications of financial frictions include Gorton and Ordonez (2014), Kuong (2017), Parlatore (2019), and Miao and Wang (2018). Kurlat (2013) and Bigio (2015) study financial frictions that arise endogenously from adverse selection in a dynamic production economy. Our paper demonstrates that security design in a dynamic adverse selection environment eliminates multiple equilibria, pointing to a potentially socially beneficial role of financial intermediaries.

Our paper is aligned with macro-finance literature where multiple equilibria are dynamic in nature (Plantin (2009), Moore (2010; 2013), Chiu and Koeppl (2016), Donaldson and Piacentino (2017), Asriyan, Fuchs, and Green (2017), and Bajaj (2018)). Most closely related to our paper in this literature are Moore (2010; 2013), Chiu and Koeppl (2016), and Asriyan, Fuchs, and Green (2017). They feature dynamic price and liquidity feedback effect under adverse selection. However, the occurrence of multiple equilibria in these papers crucially depends on the persistence of asset quality. In Asriyan, Fuchs, and Green (2017), for example, the gains from trade are in the style of Duffie, Garleanu, and Pedersen (2007): some agents receive a higher utility from the asset dividend, or produce more output using the asset as input, but they have asymmetric information about the quality of the asset. This structure implies that the severity of the adverse selection problem depends only on what creates gain from trade: dividend. If the asset quality is i.i.d., adverse selection is short lived, future high- and low-quality assets look identical and the future resale concern no longer affects today's adverse selection. In our model, the entire cum-dividend value of the asset creates gains from trade, which generates a new feedback. This insight on dynamic multiplicity is unique and does not depend on persistent asset quality. Furthermore,

[6]Relatedly, Guerrieri, Shimer, and Wright (2010) and Chang (2018) study asset markets with asymmetric information and show that, when buyers post a menu of contracts, screening through probabilistic trading (or market tightness) results in a separating equilibrium.

[7]Our result that both borrower types issue debt which is traded in a pooling equilibrium is reminiscent of Gorton and Pennacchi (1990) and Boot and Thakor (1993). Dang, Gorton, and Holmstrom (2013), Farhi and Tirole (2015) and Yang (2020) incorporate endogenous asymmetric information. The fact that information friction affects the moneyness of an asset has also been studied by Lester, Postlewaite, and Wright (2012) and Li, Rocheteau, and Weill (2012). Security design with heterogenous information is studied in Ellis, Piccione, and Zhang (2017).

existing models study dynamic adverse selection in indivisible durable assets whereas our setup admits divisible financial assets (claims to a stream of future dividend payments), providing a natural setting to study security design. The dynamic multiplicity result in our paper is also similar to the multiplicity result in Bajaj (2018). However, Bajaj (2018) does not focus on the price feedback mechanism and does not study security design, whereas our main theoretical result on uniqueness and our application to repo are about security design.[8]

Finally, the repo implementation of our model is related to theoretical works on repo contracts. Among those, Geanakoplos and Zame (2002), Geanakoplos (2003), Fostel and Geanakoplos (2012), and Simsek (2013) model collateralized borrowing in the general equilibrium context. Gottardi, Maurin, and Monnet (2017) model repo contracts and repo chain using the competitive approach of Geanakoplos (1997) with an added feature of the non-pecuniary penalty of default. Dang, Gorton, and Holmstrom (2011) model the haircut as the outcome of repo chain, borrower's quality, lender's liquidity need, and collateral value. Bigio and Shi (2020) study a two-period screening model with adverse selection. To attract the high quality borrowers, lenders offer a repurchase option, a lower rate, but have to reduce loan amounts so that default is not too high. Therefore, in their model, repos resolve adverse selection inducing full participation, but introduce cream-skimming, that can produce worse outcomes than asset sales when adverse selection is mild. Typically in screening models such as Bigio and Shi (2020), the challenge is non-existence rather than multiple equilibria. We depart from the theoretical literature on repo by modeling the joint determination of collateral asset prices and repo terms, highlighting the unique price feedback mechanism and the role of repo in eliminating price multiplicity.

# 3 The Model Setup

The economy is set in discrete time and lasts forever. There is a unit of a long-lived asset which pays out a random perishable payoff in every period. There are many infinitely-lived potential owners, with identical preferences and access to the same information, who can potentially own the asset. We refer to a representative owner as agent $O$. There are also several potential investors who live for a single period and are replaced every period. We refer to them as agent $I$s.

*Gains from Trade.* Agent $O$ values funding provided by agent $I$s which leads to gains from trade in this economy. We denote the value per-unit of funding to agent $O$ by $z$, and assume that it exceeds the

[8]There are also several modeling differences between the two papers. In Bajaj (2018) asset quality is persistent, and the bilateral exchange is modelled as a signaling game.

per-unit cost of providing funding by agent $I$s which is normalized to one.

In a frictionless economy, given that $z > 1$, gains from trade would be potentially unlimited. The key friction that limits gains from trade in our economy is lack of commitment: Agent $O$ cannot promise to pay back, and thus cannot borrow from the investors unless a credible promise is made. The asset provides a way for agent $O$ to partially overcome the commitment friction because it can be used as collateral to back up the securities.

*The Asset Properties and Information Environment.* The asset yields a random payoff at the end of period $t$ which we denote as $s_t \in [\underline{s}, \bar{s}]$, where $0 \leq \underline{s} < \bar{s}$. The payoff state, $s_t$, captures both cash payoff that the asset generates, such as dividend or interest rate payment, and other private benefits that accrue from the asset to agent $O$, such as a convenience yield and rental income. We assume that $s_t$ is distributed according to probability distribution $F_{Q_t}$ where $Q_t \in \{L, H\}$ denotes the quality of the asset. Quality $Q_t$ follows a Markov process where $Q_t = L$ with probability $\lambda_{Q_{t-1}} \in (0,1)$ where $\lambda_L \geq \lambda_H$. The unconditional probability of $Q_t = L$ in the steady state is denoted by $\lambda$, where $\lambda \equiv \lambda_H / (1 - \lambda_L + \lambda_H)$. When quality is i.i.d. across periods, we have $\lambda = \lambda_L = \lambda_H$.

We denote the density of $F_Q$ by $f_Q$ and its survival function, $1 - F_Q(s)$, by $\widetilde{F}_Q(s)$. We assume both distributions have strictly positive density in their domain $[\underline{s}, \bar{s}]$, and $F_H$ stochastically dominates $F_L$ according to the likelihood ratio, i.e., $f_L(s)/f_H(s)$ is decreasing in $s \in [\underline{s}, \bar{s}]$.

We assume that the use of the collateral asset is, however, limited by an additional friction in the economy: asymmetric information. The quality of the collateral asset is privately observed by agent $O$ at the beginning of each period thus introducing an adverse selection problem. The assumption that agent $O$ is better informed of the collateral asset's quality can be motivated or micro-founded in various ways. For example, borrowers hold collateral assets on their balance sheets which may give them an informational advantage on the quality of these assets.[9]

Formally, we assume that at the beginning of each period, the owner of the asset privately learns the asset's quality in that period. The asset's quality and the state are both publicly revealed at the end of each period. We assume that financial markets are segmented across time so that when the period

[9] Ownership of the asset often enables owners to observe the cashflows of the asset, or obtain other cashflow-related information (e.g., on governance). One real-world example of this comes from mortgage loans which are often used as collaterals by loan originators who have better information on their quality. Another example is proprietary investment portfolios of hedge funds which are offered as collaterals to obtain financing and increase leverage. There are also historical incidences where some borrowers, especially when hit by unobservable random negative shocks, debased collateral assets, e.g., by reducing the metallic content of coins below their face value. In recent times, collateral quality has been subject to questioning because of the possibility that borrowers might pledge it multiple times.

is over, newborn agent $I$s in the following period cannot access the past trading or payoff information. Consequently, agent $I$s do not know the quality of the asset in previous periods. When quality is i.i.d., i.e., $\lambda_L = \lambda_H$, this assumption is innocuous because past quality does not provide any information about the future. In an economy with many assets, segmentation across time could arise naturally when the assets look identical to the investors in each period. This assumption allows us to abstract away from the issue of signaling and reputation and focus instead on the dynamic coordination role of security design.[10]

*Asset Price.* We denote the end-of-period ex-payoff price of the asset by $\phi_{Q_t} : \{L, H\} \to \mathbb{R}$. Since $Q_t$ is observed at the end of the period, the asset price depends on the period-$t$ quality realization. In the i.i.d. case we drop the $Q$ subscript and refer to the price simply as $\phi_t$.

*Securities.* Agent $O$ raises funding from investors through the sale of asset backed securities. A security $y : [\underline{s}, \bar{s}] \times \{\phi_L, \phi_H\} \to \mathbb{R}_+$ is a payoff and price contingent contract. Security payment is fulfilled at the end of a period when the state and the price become public information. We assume that securities are monotone in the total payoff generated by the asset, so:

$$y(s, \phi_Q) \geq y(s', \phi_{Q'}) \text{ if } s + \phi_Q \geq s' + \phi_{Q'} \tag{1}$$

for all $s, s' \in [\underline{s}, \bar{s}]$ and $Q, Q' \in \{L, H\}$.

A *security design* is a finite set of securities, $\mathcal{J} = \left\{y^1, \cdots, y^J\right\}$, backed by the asset, that is,

$$\sum_{y^j \in \mathcal{J}} y^j(s, \phi) \leq s + \phi \tag{2}$$

for all $s \in [\underline{s}, \bar{s}]$ and $\phi \in \{\phi_L, \phi_H\}$.

*Security Markets.* The securities are sold in dedicated over-the-counter markets after agent $O$ obtains private information about the asset's quality and before the quality becomes public information. In each security market, several investors make bids à la Bertrand and the seller – the asset owner – decides how much to sell at the highest bid.[11] We denote the price of security $j$ by $q_t^j$, and the quantity of security $j$ exchanged when the underlying asset quality is $Q$ by $a_{t,Q}^j$.[12] For expositional clarity, we further assume that an investor has access only to one security market, so that trading information is segmented across security markets.[13]

[10]See Chari, Shourideh, and Zetlin-Jones (2014) on the reputation effect under adverse selection.

[11]If several investors are tied for the highest bid, agent $O$ equally splits the amount she would like to sell between them.

[12]The price of the security does not depend on the underlying asset quality because investors are not able to distinguish between low and high quality when they make offers for the security, but the quantity exchanged depends on the quality because the owner is privately informed.

[13]In section 7 we show that the segmentation assumption is not necessary for the main results in the paper.

*Determination of the Asset Price.* The asset price at the end of period $t$, $\phi_{t,Q}$ for $Q \in \{L, H\}$, is equal to its discounted value given time $t$ quality and time $t+1$ security design $\mathcal{J}_{t+1}$:

$$\begin{aligned}\phi_{t,Q} = \beta \Bigg\{ \lambda_Q \left[ \int_{\underline{s}}^{\bar{s}} \left( \sum_{j \in \mathcal{J}_{t+1}} a^j_{t+1,L} \left( z q^j_{t+1} - y^j_{t+1}(s, \phi_{t+1,L}) \right) + (s + \phi_{t+1,L}) \right) dF_L(s) \right] \\ + (1 - \lambda_Q) \left[ \int_{\underline{s}}^{\bar{s}} \left( \sum_{j \in \mathcal{J}_{t+1}} a^j_{t+1,H} \left( z q^j_{t+1} - y^j_{t+1}(s, \phi_{t+1,H}) \right) + (s + \phi_{t+1,H}) \right) dF_H(s) \right] \Bigg\},\end{aligned} \tag{3}$$

where $\beta$ is the discount factor, $0 < \beta < 1/z$.

The above asset price is equal to the continuation value to the owner of retaining the asset at the end of period $t$. We assume that the asset price is set in a frictionless *competitive centralized* market. In our economy, frictions exist exclusively in the securities market so that we focus purely on the role of security design under dynamic adverse selection.[14]

*Security Design Problem.* The goal of security design is to choose at the beginning of each period the set of securities that are available for trading in that period to maximize ex-ante surplus. The security design is flexible since the set of securities is updated at the beginning of each period. The design takes prices and quantities of the securities as given. Hence, formally, in the beginning of each period $t$, the security design $\mathcal{J}_t$ takes the prices, $q^j_t$, and quantities, $a^j_{t,Q}$, as given to maximize:

$$\begin{aligned}V_t = &\lambda \left[ \int_{\underline{s}}^{\bar{s}} \left( \sum_{j \in \mathcal{J}_t} a^j_{t,L} \left( z q^j_t - y^j_t(s, \phi_{t,L}) \right) + (s + \phi_{t,L}) \right) dF_L(s) \right] \\ &+ (1 - \lambda) \left[ \int_{\underline{s}}^{\bar{s}} \left( \sum_{j \in \mathcal{J}_t} a^j_{t,H} \left( z q^j_t - y^j_t(s, \phi_{t,H}) \right) + (s + \phi_{t,H}) \right) dF_H(s) \right].\end{aligned} \tag{4}$$

Security design takes place at the beginning of each period before the arrival of any private information and is independent of trading history. We have in mind an environment where there are several competing short-lived intermediaries who offer security design services to the asset owners. Competing intermediaries end up maximizing the overall expected surplus in the economy.

*Timing.* In each period, the security design takes place first. Then, agent O receives private information and trading in the security markets occurs. Once trading in the security markets is completed, both $Q_t$ and $s_t$ are revealed and asset price is determined. Finally, agent $O$ pays the investors who hold the securities, and consumption takes place.[15] Figure 1 graphs this timeline.

We now define the equilibrium concept in our economy.

[14] In section 7, we allow for a non-competitive mechanism where the asset price is set via Nash bargaining.

[15] The dynamic framework is borrowed from Lagos and Wright (2005).

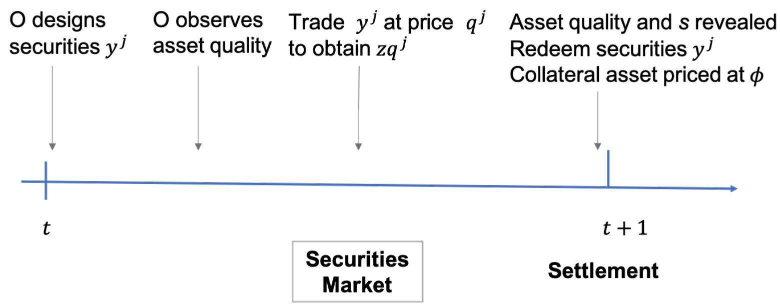


Figure 1: Timeline

**Definition 1.** An equilibrium with security design consists of asset prices $\{\phi_{t,L}, \phi_{t,H}\}$, a security design $\mathcal{J}_t = \{y_t^1, \cdots, y_t^J\}$, security prices $q_t^j$ and quantities $\left\{a_{t,L}^j, a_{t,H}^j\right\}$ for all securities such that:

1. The price of security $j$, $q_t^j$, is determined through Bertrand competition in each security market, and thus $q_t^j$ is equal to the expected value of a unit of the security given $a_{t,Q}^j$:

$$q_t^j = \lambda a_{t,L}^j E_L y_t^j (s, \phi_{t,L}) + (1-\lambda) a_{t,H}^j E_H y_t^j (s, \phi_{t,H}). \tag{5}$$

2. Quantities sold by each type must be optimal given the price, i.e., for each $Q \in \{L, H\}$,

$$a_{t,Q}^j \in \arg\max_{a\in[0,1]} a \left( zq_t^j - E_Q y_t^j (s, \phi_{t,Q}) \right). \tag{6}$$

3. Asset prices $\{\phi_{t,L}, \phi_{t,H}\}$ satisfy (3).

4. Security design $\mathcal{J}_t$ satisfies constraints (1) and (2) and maximizes (4) among all security designs satisfying (1) and (2) where security prices and quantities are given by (5) and (6).

# 4 Equilibrium in Security Markets

The value of securities affects the optimal security design. We begin the analysis by describing the equilibrium in the market for an arbitrary security $y$.[16] We assume that the expected payoff of the security when issued by the high type is weakly more than that issued by the low type, i.e., $E_L y(s, \phi_L) \leq E_H y(s, \phi_H)$.[17] We define the degree of information insensitivity as the ratio of the expected value of the security under the low versus the high distribution, i.e., $E_L y(s, \phi_L)/E_H y(s, \phi_H)$. As this ratio increases,

[16]In this section we drop the time subscript $t$ and the security index $j$ from all the variables to ease the notation.
[17]This assumption is automatically satisfied for monotone securities.

the expected values of the security under the low versus high distribution become closer, and the adverse selection problem becomes less severe.

Since our focus is not on multiple equilibria in the static setting, following Wilson (1980), we assume that buyers are strategic and compete à la Bertrand which ensures that equilibrium in each security market is generically unique. That is, in the market for security $y$, investors simultaneously make price offers taking into account which types of borrower would sell the security at that price. Agent $O$ observes these offers, and decides how much of the security to allocate to each investor.[18] Due to Bertrand competition, investors make zero surplus in expectation, and the equilibrium price of the security, $q$, is given by (5). The quantities sold by each type of agent $O$, $a_Q$, is optimal for that type and satisfies (6). The next proposition characterizes the equilibrium in the security market.

**Proposition 1.** *If* $E_L y(s,\phi_L)/E_H y(s,\phi_H) > \zeta \equiv 1-(z-1)/(\lambda z)$, *in the market for security* $y$ *the price of the security is* $q = \lambda E_L y(s,\phi_L) + (1-\lambda) E_H y(s,\phi_H)$ *and* $a_L = a_H = 1$. *If* $E_L y(s,\phi_L)/E_H y(s,\phi_H) < \zeta$ *the price of the security is* $q = E_L y(s,\phi_L)$ *and* $a_L = 1$ *and* $a_H = 0$.[19]

Proposition 1 shows that when $E_L y(s,\phi_L)/E_H y(s,\phi_H)$ is above the threshold $\zeta$, the adverse selection problem is not too severe and both types sell the security. In this case, the security price is the pooling price $q = \lambda E_L y(s,\phi_L) + (1-\lambda) E_H y(s,\phi_H)$. When $E_L y(s,\phi_L)/E_H y(s,\phi_H)$ is below the threshold, the adverse selection problem is severe and only the low type sells the security. In this case, the security price is the separating price $q = E_L y(s,\phi_L)$. A security traded in a pooling equilibrium commands a higher price and generates more liquidity for the borrower than the one traded at a lower separating equilibrium price. The above proposition also indicates that in addition to the parameters that characterize quality heterogeneity, the gains from trade parameter, $z$, is also an important determinant of adverse selection: a lower $z$ leads to a higher $\zeta$. In particular, even if there is very little asymmetric information about the quality of the security i.e., when $E_L y(s,\phi_L)/E_H y(s,\phi_H)$ is slightly below 1, as $z$ approaches 1 (so that $\zeta$ is close 1), the security will be sold in a separating equilibrium. In other words, when gains from trade is low, even a slight amount of asymmetric information results in a severe adverse selection problem.

[18] In this formulation agent $O$ has all the bargaining power, but this is not crucial for any of our results.

[19] When $E_L y/E_H y = \zeta$, there are multiple equilibria. In particular, both pooling and separating (and even semi-separating) equilibria are possible. To simplify exposition in this knife edge case, we select the pooling equilibrium.

# 5 The Baseline: Multiple Equilibria of the Dynamic Lemons Market

In this section, we consider a baseline case where the borrower is restricted to issuing only the equity claim, or a passthrough security, to the collateral asset in the security market. We demonstrate that this economy is fragile and exhibits dynamic multiplicity in prices. Specifically, we show that there might be multiple equilibria in the security market justified by different asset prices. The multiple asset prices are themselves justified by the different equilibria in the security market.

For this baseline case, we use the notion of equilibrium in Definition 1 with the restriction that the equity claim to the asset is the only available security. Choice of security design to maximize (4) becomes trivial since there is only a single feasible security. By Proposition 1 the price of the equity claim to the asset in the security market is given by $q_t^P = \lambda \left(E_L s + \phi_{t,L}\right) + (1-\lambda)\left(E_H s + \phi_{t,H}\right)$ if $(E_L s + \phi_{t,L})/(E_H s + \phi_{t,H}) \geq \zeta$ and $q_t^S = E_L s + \phi_{t,L}$ otherwise. Using (3), we obtain the price of the collateral asset in the asset market as

$$\phi_{t,Q} = \begin{cases} \beta z q_{t+1}^P, & \text{if } \frac{E_L s + \phi_{t+1,L}}{E_H s + \phi_{t+1,H}} \geq \zeta, \\ \beta \left[ z \lambda_Q q_{t+1}^S + (1-\lambda_Q)\left(E_H s + \phi_{t+1,H}\right)\right], & \text{if } \frac{E_L s + \phi_{t+1,L}}{E_H s + \phi_{t+1,H}} < \zeta. \end{cases} \tag{7}$$

For the rest of the paper, we study *stationary equilibria* and hence drop the time subscripts. In particular, in the baseline case, in a stationary equilibrium, the collateral asset is either always traded in a pooling equilibrium, or it is always traded in a separating equilibrium.

## 5.1 Pooling Equilibrium

Plugging $q^P$ into (7) we observe that a pooling equilibrium, in which both types of agent $O$ sell the equity claim in the security market for the intermediate goods, exists if and only if

$$\frac{E_L s + \phi_L^P}{E_H s + \phi_H^P} \geq \zeta, \tag{8}$$

where the asset prices in the pooling equilibrium are given by

$$\phi_Q^P = \beta z \left(\lambda \left(E_L s + \phi_L^P\right) + (1-\lambda)\left(E_H s + \phi_H^P\right)\right).$$

Solving for the pooling prices we obtain

$$\phi_L^P = \phi_H^P = \frac{\beta z \left(\lambda E_L s + (1-\lambda) E_H s\right)}{1-\beta z}. \tag{9}$$

By plugging (9) into (8) we see that a pooling equilibrium exists if and only if $E_L s/E_H s \geq \kappa_P$, where

$$\kappa_P = \frac{\zeta - \beta z\left(1-\left(1-\zeta\right)\lambda\right)}{1-\beta z\left(1-\left(1-\zeta\right)\lambda\right)}. \tag{10}$$

## 5.2 Separating Equilibrium

A separating equilibrium, in which only the low type of agent $O$ sells the equity claim in the security market, exists if and only if

$$\frac{E_L s + \phi_L^S}{E_H s + \phi_H^S} < \zeta, \tag{11}$$

where the asset prices in the separating equilibrium are given by,

$$\begin{aligned}
\phi_L^S &= \beta\left(z\lambda_L E_L s + \left(1-\lambda_L\right)E_H s + z\lambda_L \phi_L^S + \left(1-\lambda_L\right)\phi_H^S\right), \\
\phi_H^S &= \beta\left(z\lambda_H E_L s + \left(1-\lambda_H\right)E_H s + z\lambda_H \phi_L^S + \left(1-\lambda_H\right)\phi_H^S\right).
\end{aligned}$$

Solving for the separating prices we obtain

$$\phi_L^S = \beta\frac{z\left(\lambda_L - \beta\left(\lambda_L-\lambda_H\right)\right)E_L s + \left(1-\lambda_L\right)E_H s}{1-\beta-\beta\left(z\lambda_L-\lambda_H\right)+\beta^2 z\left(\lambda_L-\lambda_H\right)}, \tag{12}$$

$$\phi_H^S = \beta\frac{z\lambda_H E_L s + \left(1-\lambda_H-\beta z\left(\lambda_L-\lambda_H\right)\right)E_H s}{1-\beta-\beta\left(z\lambda_L-\lambda_H\right)+\beta^2 z\left(\lambda_L-\lambda_H\right)}. \tag{13}$$

By plugging (12) and (13) into (11) we see that a separating equilibrium exists if and only if $E_L s/E_H s < \kappa_S$, where

$$\kappa_S = \frac{\zeta - \beta\left(1-\left(1-\zeta z\right)\lambda_L\right)}{1-\beta\left(1-\left(1-\zeta z\right)\lambda_H\right)}.$$

## 5.3 Properties of Equilibria and Multiplicity

As the ratio $E_L s/E_H s$ increases, the expected payoff with respect to the two distributions becomes closer, and adverse selection is ameliorated. The following proposition shows that there is always a range with an intermediate degree of information insensitivity such that multiple equilibria exist.

**Proposition 2.** *(i) If $E_L s/E_H s \geq \kappa_S$, then there is a unique equilibrium in which the collateral asset is sold in a pooling equilibrium in the security market and the pooling price is given by (9).*

*(ii) If $\kappa_P > E_L s/E_H s$, then there is a unique equilibrium in which the collateral asset is sold in a separating equilibrium in the security market and the separating prices are given by (12) and (13).*

*(iii) If*

$$\kappa_S > \frac{E_L s}{E_H s} \geq \kappa_P, \tag{14}$$

*then both the pooling equilibrium described in (i) and the separating equilibrium described in (ii) exist.*

Next, we demonstrate the liquidity service of the asset by comparing the asset price in our economy with the discounted present value of future payoffs from the asset in state $Q$, which we denote by $\phi_{0,Q}$:

$$\phi_{0,Q} = \beta \left( \lambda_Q \left( E_L s + \phi_{0,L} \right) + \left( 1 - \lambda_Q \right) \left( E_H s + \phi_{0,H} \right) \right).$$

Solving for $\phi_{0,L}$ and $\phi_{0,H}$ we obtain

$$\phi_{0,L} = \beta \frac{\left( \lambda_L - \beta \left( \lambda_L - \lambda_H \right) \right) E_L s + \left( 1 - \lambda_L \right) E_H s}{\left( 1 - \beta \right) \left( 1 - \beta \left( \lambda_L - \lambda_H \right) \right)}, \tag{15}$$

$$\phi_{0,H} = \beta \frac{\lambda_H E_L s + \left( 1 - \lambda_H - \beta \left( \lambda_L - \lambda_H \right) \right) E_H s}{\left( 1 - \beta \right) \left( 1 - \beta \left( \lambda_L - \lambda_H \right) \right)}. \tag{16}$$

We show in the next proposition that in both states $L$ and $H$, the price of the asset in a separating (pooling) equilibrium is strictly higher than the discounted value of future payoffs whenever a separating (pooling) equilibrium exists. The results are immediate by comparing the expressions of prices.

**Proposition 3.** *If $E_L s / E_H s < \kappa_S$ then $\phi_Q^S > \phi_{0,Q}$, and if $E_L s / E_H s > \kappa_P$ then $\phi_Q^P > \phi_{0,Q}$ for $Q \in \{L, H\}$.*

In our model equilibrium prices exceed the present value of all future payoffs from the asset because of the liquidity service provided by the asset. The asset backs the owner's borrowing from the investors which she values more than the payoffs that she gives up and this liquidity service is reflected in the asset price.

Next, we discuss the intuition for the existence of multiple equilibria in the baseline setting which arises due to a dynamic price feedback effect. When the asset price is high, the degree of information insensitivity of the equity, $\left( E_L s + \phi_L^P \right) / \left( E_H s + \phi_H^P \right)$, is above the threshold $\zeta$. Hence, the adverse selection problem is mild and the high-type agent $O$ is willing to pool with the low type and issue equity in the security market. In turn, if agents anticipate the equity claims of the asset to be traded in a pooling equilibrium in future periods, the liquidity service of the asset is large hence the asset price today is high. Conversely, when the asset price is low, the degree of information insensitivity of the equity, $\left( E_L s + \phi_L^S \right) / \left( E_H s + \phi_H^S \right)$, is below the threshold $\zeta$. Therefore, the adverse selection problem is severe and the high type agent $O$ retains the asset and chooses not to issue equity in the security market. In turn, if agents anticipate the equity claim of the asset traded in a separating equilibrium in future periods, the liquidity service of the asset is limited thus the asset price today is low. As a result, the asset prices are self-fulfilling in this economy.

Comparative static analyses reveal how the size of the multiple equilibrium region varies with the underlying parameters. For example, persistence in quality increases when $\lambda_L - \lambda_H$ increases holding

$\lambda \in (0,1)$ constant.[20] We find that the separating equilibrium threshold ($\kappa_S$) is increasing and the pooling equilibrium threshold ($\kappa_P$) remains the same as persistence in quality increases. That is, the region of multiplicity expands as persistence goes up. Interestingly, the multiplicity region does not disappear when the asset quality is i.i.d. which we state in the following corollary.

**Corollary 1.** *When quality is i.i.d. across periods, i.e., $\lambda = \lambda_L = \lambda_H$, multiple equilibria in Proposition 2 exist whenever*

$$\frac{\zeta - \beta\left(1-(1-\zeta z)\lambda\right)}{1-\beta\left(1-(1-\zeta z)\lambda\right)} > \frac{E_L s}{E_H s} \geq \frac{\zeta - \beta z\left(1-(1-\zeta)\lambda\right)}{1-\beta z\left(1-(1-\zeta)\lambda\right)}. \tag{17}$$

*The asset price in the separating equilibrium is lower than that in the pooling equilibria, $\phi^S < \phi^P$, where*

$$\phi^S = \frac{\beta\left(z\lambda E_L s + (1-\lambda) E_H s\right)}{1-\beta}, \tag{18}$$

$$\phi^P = \frac{\beta z\left(\lambda E_L s + (1-\lambda) E_H s\right)}{1-\beta z}. \tag{19}$$

The existence of multiple equilibria when the asset quality is i.i.d. indicates that the sources of multiple equilibria in our setting are distinct from those in the existing literature. In the static setting, multiple equilibria exist under perfect competition as in Akerlof (1970). If prices are low, only low-quality assets are sold, which justifies low prices; if prices are high, higher-quality assets are also sold which in turn justifies higher prices. For some parameter values both equilibria exist. Wilson (1980) has shown that when buyers are strategic and compete à la Bertrand, there is a unique equilibrium in the static adverse selection environment. The reason is that uninformed buyers do not take prices as given, they recognize the link between price and quality, and only the highest zero-profit price survives as a Nash equilibrium. However, this logic does not extend to dynamic settings. The expectation of low prices in the future could induce adverse selection in the present thus lead to a self-fulfilling low-price equilibria that survives standard game-theoretic foundations.

In the next section, we show that increasing the flexibility of security design by removing the restriction on the set of available securities, restores the unique equilibrium in the economy.

# 6 The Main Model: Optimal Security Design

In this section, we solve the equilibrium described in Definition 1 with security design, and show that the equilibrium is unique. Hence, optimal security design eliminates multiple equilibria that arise when

[20] When either high- or low-quality is the absorbing state but not both, that is, when $\lambda_L = 1$ and $\lambda_H \neq 0$ or when $\lambda_L \neq 1$ and $\lambda_H = 0$, asymmetric information disappears and there is a unique stationary equilibrium. When both high- and low-qualities are absorbing states, that is, $\lambda_L = 1$ and $\lambda_H = 0$, multiple equilibria arise again.

agents are restricted to trading only the equity claim of the underlying asset.[21]

### 6.1 Unique Equilibrium with Optimal Security Design

The next proposition characterizes the optimal security design, and shows that it involves at most two securities: One security, $y_D(s,\phi_Q)$, which is traded in a pooling equilibrium is a debt contract; the other security, $y_E(s,\phi_Q)$, which is traded in a separating equilibrium is the residual equity tranche. That is, both high and low quality borrowers sell one unit of the debt contract, only low quality borrowers sell one unit of the equity contract and high quality borrowers retain the equity contract.

**Proposition 4.** *Suppose that either $f_L$ or $f_H$ is log-concave. The optimal security traded in a pooling equilibrium is a debt contract given by:*

$$y_D(s,\phi_Q) = \min(s+\phi_Q, D), \tag{20}$$

*for some $D \in (\underline{s}+\phi_L, \bar{s}+\phi_H]$. The residual tranche is an equity contract traded in a separating equilibrium and is given by $y_E(s,\phi_Q) = \max(0, s+\phi_Q - D)$. Moreover, $D$ is unique for given $\phi_L$ and $\phi_H$.*

The amount $D$ is the face value of the debt contract, and it is pinned down by the high quality asset owner's participation constraint. It always exceeds $\underline{s}+\phi_L$ since this amount is free from adverse selection. As the persistence in asset quality vanishes and $\lambda_L$ and $\lambda_H$ get closer, and the resale prices $\phi_L$ and $\phi_H$ approach each other. In the limit, $\phi_L = \phi_H = \phi$, and the face value of debt always exceeds $\underline{s}+\phi$, incorporating all of the resale price.

Using Proposition 4 and letting $d \equiv D - \phi_L$ and $\Delta\phi \equiv \phi_H - \phi_L$, we simplify the statement of equilibrium given in Definition 1.[22] With this notation, we write the prices of the debt and equity tranches, $q_D$ and $q_E$, as:

$$q_D = \lambda\left(\phi_L + E_L s - \int_d^{\bar{s}} \widetilde{F}_L(s)ds\right) + (1-\lambda)\left(\phi_H + E_H s - \int_{d-\Delta\phi}^{\bar{s}} \widetilde{F}_H(s)ds\right), \tag{21}$$

$$q_E = \int_d^{\bar{s}} \widetilde{F}_L(s)ds. \tag{22}$$

[21] In an Online Appendix, we show that multiple equilibria might re-emerge when the security design is rigid, that is, when the contract terms are not always updated at the beginning of each period.

[22] When the asset quality is low, $d$ is the threshold up to which the current period's payoff is incorporated into the debt tranche. When the asset quality is high and $d-\Delta\phi < 0$, the debt tranche incorporates a fraction of the asset price $\phi_H$, namely $D < \phi_H$, and none of the current payoff. When the asset quality is high and $d - \Delta\phi > 0$, the debt tranche incorporates the asset price $\phi_H$ and the current payoff up to the threshold $d - \Delta\phi$.

Both types of borrowers sell the debt tranche but only the low type sells the equity tranche. As a result, the expected amount of funding raised by selling the securities depends on the state $Q$ and equals $q_D + \lambda_Q q_E$. From (3), we write the asset price as:

$$\phi_Q = \beta\left[zq_D + z\lambda_Q q_E + (1-\lambda_Q)\left(\int_{d-\Delta\phi}^{\bar{s}} \widetilde{F}_H(s)ds\right)\right], \, Q \in \{L, H\}. \tag{23}$$

Solving for equilibrium then comprises solving the designer's optimization problem in (4) to find the optimal threshold $d \in [\underline{s}, \bar{s} + \Delta\phi]$ given the prices of debt and equity tranches $q_D$ and $q_E$, and the asset prices $\phi_L$ and $\phi_H$.

For our main result we strengthen the standard hazard rate dominance condition, and assume that $F_H$ and $F_L$ satisfy,

$$\left(f_H(s)/\widetilde{F}_H(s)\right)/\left(f_L(s)/\widetilde{F}_L(s)\right) \le \left[1-\beta(\lambda_L-\lambda_H)\right]^2. \tag{24}$$

In the i.i.d. case, where $\lambda_L = \lambda_H$, (24) is the standard hazard rate dominance condition, which automatically follows from the likelihood ratio dominance assumption.[23]

Now, we state the main theorem of the paper.

**Theorem 1.** *Suppose that either $f_L$ or $f_H$ is log-concave, and (24) holds, then there is a unique equilibrium with security design. If*

$$E_L s/E_H s < \kappa_P, \tag{25}$$

*then $d \in (\underline{s}, \bar{s} + \Delta\phi)$, otherwise, that is, if $E_L s/E_H s \ge \kappa_P$,* then *$d = \bar{s} + \Delta\phi$. In the former case, the equilibrium with security design strictly Pareto-dominates the (unique) separating equilibrium in the baseline case. In the latter case, security design uniquely selects the pooling equilibrium. It thus strictly Pareto-dominates the separating equilibrium in the baseline case when there is one and it replicates the pooling equilibrium otherwise.*

The region given by (25) is the region identified in Proposition 2 where a unique separating equilibrium exists when only equity is allowed to be traded. Hence, security design improves liquidity when there is a unique separating equilibrium in the baseline case. The formal proof of the theorem is in the Appendix.

We note that for the self-fulfilling multiple equilibria result in Proposition 2 and the uniqueness under optimal security design result in Theorem 1 to hold, we only need the following two assumptions: lack

[23]This is because when $F_H$ stochastically dominates $F_L$ according to the likelihood ratio, i.e., $f_L(s)/f_H(s)$ is decreasing in $s$, then $F_H$ stochastically dominates $F_L$ according to hazard rate, that is,

$$\left(f_H(s)/\widetilde{F}_H(s)\right)/\left(f_L(s)/\widetilde{F}_L(s)\right) \le 1.$$

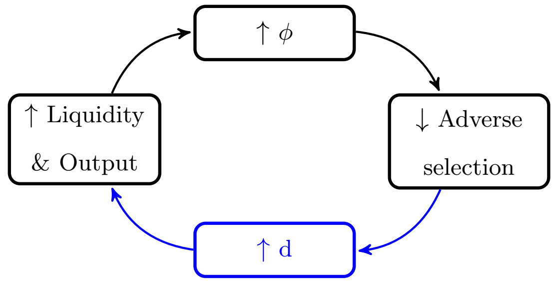


Figure 2: Asset Price $\phi$ and Debt Face Value $\phi + d$

of commitment from borrowers and asymmetric information about the quality of the (only pledgeable) collateral asset. We demonstrate later in the paper that the main results are robust to the alternative securities and asset market microstructures. The modeling choices we have made in the main model: such as segmented securities market, competitive asset markets, Bertrand competition in securities market, security design objective function, as well as maturity structures are mainly for expositional clarity.

Figure 2 illustrates the feedback loop between the asset price, which depends on the future value of the collateral, and the current face value of the debt contract, which is the underlying mechanism in Theorem 1. As the face value of the debt tranche, $D = \phi + d$, increases, agent $O$ obtains more liquidity, and gains from trade increase because the marginal value of liquidity for agent $O$ exceeds the marginal cost of providing liquidity by the investors. The feedback loop involves inter-temporal coordination since the increase in gains from trade in future periods leads to an increase in $\phi$. Higher asset prices are incorporated into the face value of debt, alleviating the adverse selection problem and pushing the face value even higher. We provide an intuitive discussion of the result in Theorem 1 and the economic mechanism behind it in the next two subsections.

### 6.2 IID Asset Quality

In this subsection, for expositional clarity, we focus on the case where the asset quality is i.i.d. across periods to demonstrate the feedback between price and face value of the debt contract and the role of dynamic coordination. This allows us to provide an intuitive derivation of the result in Proposition 1 and to highlight the underlying mechanism of dynamic coordination.

When asset quality is i.i.d. across periods, both high- and low-quality assets are sold at the same price at the end of the period which we denote by $\phi$, and the following corollary of Proposition 1 holds as long as $F_H$ dominates $F_L$ in likelihood ratio, i.e., $f_L(s)/f_H(s)$ is decreasing. We state the formal result in the following corollary.[24]

**Proposition 5.** *In the case of i.i.d asset quality, if $E_L s/E_H s < \kappa_P$, then there is a unique equilibrium where the debt threshold $d \in (\underline{s}, \bar{s})$ and the asset price $\phi$ are solutions to the participation constraint and the Euler equation that are given by*

$$\phi = \frac{z}{z-1}\lambda \int_{\underline{s}}^{d} \left[\widetilde{F}_H(s) - \widetilde{F}_L(s)\right] ds - \int_{\underline{s}}^{d} \widetilde{F}_H(s) ds - \underline{s}, \tag{26}$$

$$\phi = \frac{\beta}{1-\beta z}\left\{z\left[\lambda E_L s + (1-\lambda)E_H s\right] - (1-\lambda)(z-1)\int_{d}^{\bar{s}} \widetilde{F}_H(s) ds\right\}. \tag{27}$$

*If $E_L s/E_H s \geq \kappa_P$, then there is a unique equilibrium where $d = \bar{s}$ and $\phi = \frac{\beta z}{1-\beta z}\left[\lambda E_L s + (1-\lambda)E_H s\right]$.*

This proposition links optimal security design equilibrium to the equilibria of the baseline case in Section 5 as stated in Corollary 1. There are two scenarios. First, with security design there is uniquely determined and welfare-improving tranching in the separating equilibrium region of the baseline case, i.e., when $E_L s/E_H s < \kappa_P$. Second, the pooling equilibrium is selected as the unique equilibrium in the multiple equilibria region of the baseline case, i.e., when $E_L s/E_H s \geq \kappa_P$.

We now graphically construct the optimal security design equilibrium. For any $d$, let $\phi(d)$ be the asset price in the asset market satisfying (27). Similarly, for any $\phi$, let $d(\phi)$ be the debt threshold satisfying 26. We graph the former with a solid line and latter with a dash-dotted line in Figures 3 and 4. An intersection of these two lines is a solution to 26 and (27) and constitutes an equilibrium.

Observe from the Euler equation (27) that if agents coordinate on a higher debt threshold tomorrow, the asset price today will be higher, since $\phi$ is increasing in $d$.[25] The function $\phi(d)$ has a few noteworthy aspects. Let $\underline{\phi} = \phi(\underline{s})$ and $\phi^P = \phi(\bar{s})$. From Corollary 1 in Section 5 we observe that $\phi^S$ is the asset price when only the low type sells the asset and the high type retains both the resale price and the current period payoff. In contrast, the asset price calculation in (27) takes into account that both types of borrowers sell the debt claim backed by the future resale price as part of the collateral. As a result, $\underline{\phi} > \phi^S$. On the other hand, $\phi(\bar{s})$ is the same as the pooling price $\phi^P$. To see this, we observe that the $\phi^P$ calculation takes into account that both types use the resale price and the entire current period payoff of the asset as collateral, which is equivalent to setting the face value of the debt contract to $D = \phi^P + \bar{s}$.

Next, consider the designer's choice of debt threshold, $d(\phi)$, which is depicted by the dash-dotted

[24]In the i.i.d case we drop (24) since it becomes the standard hazard rate dominance condition. We also do not need log-concavity of either distribution.

[25]Note that $\phi$ is strictly increasing for $d \in [\underline{s}, \bar{s})$, $\partial\phi/\partial d$ is decreasing and is zero at $d = \bar{s}$.

line in Figure 3 for the case where $E_L s/E_H s < \kappa_P$.[26] Optimal security design chooses $d$ to be as large as possible making sure that the debt tranche is traded in a pooling equilibrium. We discussed in the previous paragraph that as $d$ increases, $\phi$ increases, which is depicted by the solid line in Figure 3. This relaxes the high type's participation constraint. However, as the debt tranche incorporates more of the high payoff states, eventually the high type's participation constraint begins to tighten because, by the likelihood ratio dominance, the likelihood of the high payoff states according to the high type relative to the low type keeps increasing, and the adverse selection problem worsens. If $d$ is too high, the high type, who values those states much more than the low type, might prefer to retain the debt tranche rather than pool with the low type. Optimal security design pushes $d$ to the unique point where the high type is indifferent between selling or retaining the debt. Crucially, optimal security design solves the coordination problem that we observed in the baseline case where lenders face strategic uncertainty about the high type's participation in the security market. Optimal security design eliminates this uncertainty by ensuring that both types participate in trading the debt tranche.

Figure 3 illustrates that regardless of how low the asset price is, as long as tranching is feasible, optimal security design involves a debt tranche that incorporates some of the current period payoff. That is, $d\left(\underline{\phi}\right) > \underline{s}$. In the region depicted in Figure 3, adverse selection is severe, and even when the asset price is as high as possible, the high type prefers to retain the equity tranche. That is, $d\left(\phi^P\right) < \bar{s}$.

Using these two curves, $\phi\left(d\right)$ and $d\left(\phi\right)$, we can find the equilibrium values $\left(d^*, \phi^*\right)$. The equilibrium is where the two curves intersect, i.e., when $\phi^* = \phi\left(d^*\right)$ and $d^* = d\left(\phi^*\right)$. As Figure 3 shows, when $E_L s/E_H s < \kappa_P$, the unique equilibrium debt threshold is $d^* \in \left(\underline{s}, \bar{s}\right)$. This explains the optimal security design equilibrium and its difference relative to the baseline case in the first scenario.

Second, we consider the scenario when $E_L s/E_H s > \kappa_P$ in Figure 4. In this case, adverse selection is less severe and the $d(\phi)$ function is shifted to the right as the same asset price can sustain a higher face value where the debt tranche is traded in a pooling equilibrium. When the asset price is above a threshold denoted by $\widehat{\phi}$, optimal security design incorporates all payoff states $\bar{s}$ to the face value of debt, which is captured by the vertical part of the $d(\phi)$ function. This vertical portion of $(d(\phi))$ is a special feature of debt contract: the debt threshold cannot exceed the maximum payoff which the collateral asset can yield. The two curves intersect only at the upper right corner, $\left(\bar{s}, \overline{\phi}\right)$. As a result, there is a unique equilibrium for the security design problem and it involves setting the debt threshold $d^* = \bar{s}$. That is, the optimal security is a pass-through security, which means that the optimal security's payoff

[26] This is the left boundary of multiple equilibria region in 14. In this region, without security design, adverse selection leads to a unique separating equilibrium.

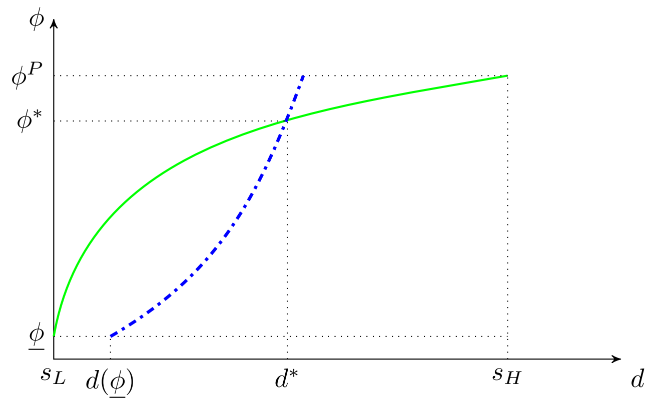


Figure 3: $\phi(d)$ and $d(\phi)$ when $E_L s/E_H s < \kappa_P$

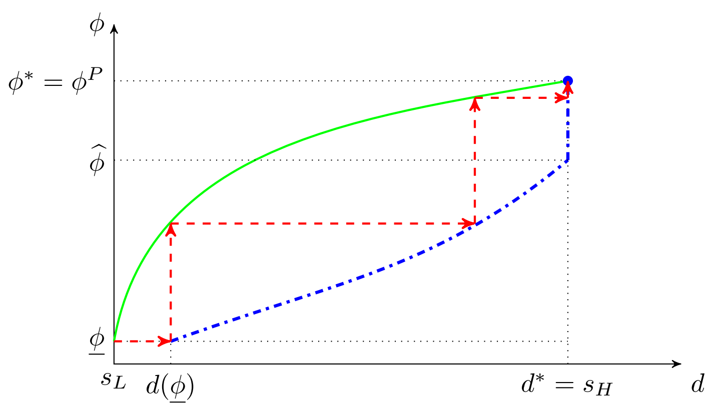


Figure 4: $\phi(d)$ and $d(\phi)$ when $E_L s/E_H s > \kappa_P$

is mapped one-to-one from the asset's cashflow at the realization date, equivalent to an equity contract.

The scenario depicted in Figure 4 may seem surprising since, as we illustrated in Section 5, without the possibility of security design there is a coordination problem leading to multiple equilibria in part of this region. Security design solves this coordination problem, and we obtain a unique equilibrium in which agent $O$ sells the entire "pass-through" debt tranche in a pooling equilibrium. Intuitively, without security design the high type decides among only two options: whether to use the resale price and the current period payoff of the asset as collateral versus retaining both parts. The outcome depends on the asset price. In the good equilibrium $\phi = \phi^P$and the high type sells the asset. In the bad equilibrium, $\phi = \phi^S$ and the high type retains the asset. The bad equilibrium cannot survive with security design because even if the asset price was $\phi^S$, the optimal security design would be able to improve this separating equilibrium by creating a debt tranche with the face value $\phi^S$, which in turn would increase the asset price above $\phi^S$. Both graphs in Figures 3 and 4 in fact show that the equilibrium asset price in the optimal security design equilibrium is no less than $\underline{\phi} = \phi(\underline{s}) > \phi^S$ (since the face value of the debt tranche is never below $\underline{\phi} + \underline{s}$). Given the increase in the asset price to $\underline{\phi}$ from $\phi^S$, the high type's participation constraint is relaxed, which leads to the optimal security design to incorporate more of the current period payoff into the debt tranche (that is, $d > \underline{s}$). A higher $d$ will increase the asset price $\phi$ and so on, triggering the dynamic price feedback loop. This unravelling process is illustrated in Figure 4 with the dashed arrows. As the graph in Figure 4 shows when the asset price is $\underline{\phi}$, the face value of the debt rises to $\underline{\phi} + d(\underline{\phi})$. When the face value of the debt increases to $\underline{\phi} + d(\underline{\phi})$, the asset price further increases, and so on. The process ends when the price rises to $\phi^P$.

### 6.3 Persistent Asset Quality

In this section, we discuss how introducing persistence in asset quality affects the feedback loop and adverse selection problem in our model. When the asset quality is persistent, the adverse selection problem is more severe because there is an additional source of information asymmetry. In addition to the current period payoff, lenders and borrowers are also asymmetrically informed about the future asset price. The next proposition shows that an increasing persistence in asset quality leads to a lower debt threshold.

**Proposition 6.** *Fix $\lambda > 0$. When asset quality becomes more persistent, i.e., as $\lambda_L - \lambda_H$ increases, debt threshold $d$ decreases.*[27]

The gap between high- and low-quality asset prices, $\Delta\phi$, reflects the degree of adverse selection introduced by persistence in asset quality. Since prices are forward looking this second source of adverse selection is dynamic. To see how this additional dynamic effect works, suppose high- and low-quality assets sell at different prices, i.e., $\Delta\phi = \phi_H - \phi_L > 0$, and the designer initially sets the debt face value at $D = \underline{s} + \phi_L$, which is completely safe regardless of asset quality. At $D = \underline{s} + \phi_L$, the high type strictly prefers to sell the debt tranche and the participation constraint does not bind. Increasing the debt threshold initially increases both $\phi_L$ and $\phi_H$ due to the feedback loop in this economy. In the i.i.d. case, when $d$ is large enough, the debt tranche incorporates more of the high payoff states, tightening the high type's participation constraint. When the asset quality is persistent, the additional dynamic effect

[27] If $d \in (\underline{s}, \bar{s})$ then as $\lambda_L - \lambda_H$ increases, $d$ decreases strictly.

may relax the high type's participation constraint as the debt threshold goes up. The reason is that, when $d$ is higher, the debt tranche incorporates more of the high payoff states that are relatively more likely under the high distribution and therefore benefits the low-quality asset owner relatively more. As a result, the low quality asset price $\phi_L$ increases faster than the high-quality asset price $\phi_H$, narrowing the price gap $\Delta\phi$ and lowering the adverse selection in asset prices. This effect may potentially relax the participation constraint, and necessitates the condition (24) to guarantee a unique security design equilibrium.[28] The following proposition shows that as persistence increases, the price difference $\Delta\phi$ - a proxy for price volatility in our model - increases and the dynamic effect gets stronger.

**Proposition 7.** *Fix $\lambda > 0$. When asset quality becomes more persistent, i.e., as $\lambda_L - \lambda_H$ increases, $\Delta\phi$ increases.*

Finally we observe that when persistence increases, consistent with the above proposition, condition (24) becomes more stringent.[29]

## 7 Robustness: Alternative Modeling Choices

We now demonstrate the robustness of our results derived from the main model by allowing for alternative modeling choices. For expositional clarity, we only provide summary and intuitions in this section. The formal results are presented in Appendix B.

*Unsegmented Security Markets.* We first change the segmentation assumption on security markets by assuming security markets are unsegmented, that is, we allow lenders to make inferences about the type of the borrower from their trades across markets. We show in Appendix B.1. that our main result – there is a unique equilibrium with security design – is robust to this modification.

With unsegmented markets, the security designer chooses at most two securities. When the high and the low type borrowers trade different securities, the design is separating. When both types trade the same security, the design is pooling. In the separating case, the designer chooses a debt tranche and the pass-through equity. The high type trades only the debt tranche (and retains the residual equity) and the low type trades the pass-through equity. Because security markets are unsegmented, in the separating case, the lenders learn the borrower's type. We show that in this case the only constraint

[28]The required condition, (24), is stronger than the hazard rate dominance condition, and is not implied by likelihood dominance.

[29]Similarly, when agents become more patient, i.e., as $\beta$ increases, price gap increases and condition (24) becomes more stringent as well.

that binds is the low type's incentive compatibility (IC) constraint which makes sure that the low type does not mimic the high type by selling debt instead of pass-through equity. In the separating case, this constraint pins down the debt threshold. In the pooling case, the designer chooses a single debt tranche. Both types trade the debt and retain the residual equity. We show that in this case the only constraint that binds is the high type borrower's participation constraint which makes sure that the high type has the incentive to sell debt instead of retain it.[30] In the pooling case, this constraint pins down the debt threshold. We show that overall there is still a unique equilibrium in which either the design is separating and the equilibrium asset price is low, or the design is pooling and the asset price is high. An immediate corollary of the analysis is that the designer obtains a higher payoff with segmented compared with unsegmented markets since the low type's IC constraint is not needed in the former case.

*Long Term Securities.* Next, we introduce long term securities that specify payments from the borrower to the investor in every period and state. We assume that an investor who buys a long term security becomes a borrower in the next period and raises funding by selling another long term security which is backed by the existing long term security that she owns.[31]

We compare the setting with long term securities with the one we consider in our main model where in each period the borrower sells a security backed by the current period dividend and the resale price of the long-lived asset. To make the comparison as stark as possible we restrict attention to the case where there is symmetric information about the quality of the long lived asset and the quality is i.i.d. In Appendix B.2, we show that these two environments are equivalent in the sense that the asset price and the amount of funding raised in the securities market are the same in the two settings. This result is intuitively similar to the principle of optimality in dynamic programming: the asset price captures all future gains from trade and is akin to the value function which captures the value of the dynamic program under future optimal behavior.

*Securities Market Microstructure.* In the main model we stay close to the standard lemons market à la Akerlof which is the simplest model of a lemons market and provide closed-form solutions. To show that our results are robust to a different security market microstructure, we solve the model where the borrower and the investors trade securities through an intermediary in Appendix B.3. We assume that the intermediary's goal is to maximize the expected funding while making sure that the lenders break even. An alternative interpretation of this setting is that borrowers signal their types through the quantities that they trade (or equivalently probability of trade) and we select the most

[30]This constraint is the same as the high type's participation constraint in our main model.

[31]Alternatively, this environment can be viewed as one where the long lived asset is transferred from a borrower to a lender who becomes a borrower in the next period.

efficient equilibrium. The latter interpretation is closely related to the undefeated equilibrium concept of Mailath, Okuno-Fujiwara, and Postlewaite (1993) invoked in Bajaj (2018). We show that securities are still traded either in a pooling or a separating equilibrium. However, unlike in the main model, the high type is able to sell a fraction of the security in the separating case. Despite this difference, there is still a discontinuous drop in the amount of funding the borrower can raise when equilibrium switches from pooling to separating. We show that, as in our main model, when the borrower is restricted to issuing only the equity claim to the collateral asset, the economy exhibits dynamic multiplicity in prices.

*Nash Bargaining for Asset Resale Price.* In the main model, we assume that at the end of each period there is a competitive market where the borrower buys back the asset from the investor. Suppose instead, that the two parties bargain over the resale price of the asset via Nash bargaining where $\theta \in (0, 1]$ is the bargaining power of the borrower. We show in Appendix B.4 that with this alternative model is equivalent to the main model where the gains from trade parameter $z$ in the asset price is replaced with $\hat{z} = 1 - \theta + \theta z$. This result is intuitive since only proportion $\theta$ of gains from trade is captured by the borrower and hence, reflected in the asset price.

# 8 Implementation as Repo Contract

Optimal securities derived in the paper describe contract terms on cashflows between borrowers and lenders upon realization of the state. In practice, the optimal security can be implemented in several ways. In this section, we demonstrate one prominent implementation which is a one-period repo contract traded in a pooling equilibrium, and a residual equity-like contract traded in a separating equilibrium. In this implementation, there is a representative borrower who values the funding liquidity (at $z > 1$) more than investors, and hence has incentive to purchase back the asset in every period to be able to use it for backing securities in the next period. We first map terms of repo contracts in the context of our model. These contract terms are endogenously determined given the underlying information and preference parameters. Next we provide analytical solutions using a two-point distribution in order to link the primitives of the model to the repo contract terms.

## 8.1 Terms of the Repo Contract

When two parties enter a repo contract, one party sells an asset to another party at one price (which in our model corresponds to the loan value or the price of the debt tranche $q_D$) and commits to repurchase the same or another part of the same asset from the second party at a different price at a future date (which in our model corresponds to the face value of the debt tranche $D$). If the seller defaults during the life of the repo, the buyer (as the new owner) can sell the asset to a third party to offset the loss.[32] The most straightforward mapping of the optimal contract in the model to reality is as follows. During the term of the repo the lender receives $s$, the cash flow or the convenience yield/service flow from the asset (in this sense, lender is the legal owner) in an escrow account.[33] When the repo term is finished, there are two possibilities: (i) if the face value $D$ is more than $s+\phi$, the borrower obtains the asset back from escrow by paying its price $\phi$; (ii) if the face value $D$ is less than $s+\phi$, then the borrower pays the lender remaining $D-s$, so that the lender obtains the promised face value $D$ and the borrower takes the asset back from escrow.[34]

Our model complements the existing repo literature by offering an alternative explanation for why in a repo contract, an asset is sold and agreed to be repurchased.[35] This feature naturally arises in our model since borrowers always buy the collateral back at the end of borrowing -- hence repurchase leg endogenously arises in equilibrium.

We now describe the two terms of the repo contract: repo rate, $r$, and haircut, $h$. The definition of repo rate is straightforward:

$$r \equiv \frac{\text{face value}}{\text{loan value}} - 1 = \frac{D - q_D}{q_D}. \tag{28}$$

From the definition of repo rate $r$, we observe that the relationship between asset quality and interest rate is not straightforward because asset quality has two opposing effects on the repo rate. When asset quality worsens (improves), loan value is lower (higher), leading to a high (low) repo rate. At the same time, the face value of the debt might be adjusted down (up), resulting in a lower (higher) repo rate.

[32] We take the definition of a fixed term repo contract from the International Capital Market Association (ICMA).

[33] Escrow guarantees that the lender returns the asset. This is consistent with our model which focuses on limited commitment on the borrower side.

[34] According to this implementation, the pass through security follows the most commonly conceived form of repo: the borrower sells the security to obtain funding, the lender owns it via a custodian and consume the benefit of being an owner which is $s$, the cash flow/service flow of the asset, and the borrower repurchases the security back at the end of the repo term (at price $\phi$).

[35] The feature of asset repurchase is modeled differently in the repo literature. In Gottardi, Maurin, and Monnet (2017), asset repurchase arises from the need of lender and borrower to share risk since the collateral asset price is volatile. In Duffie (1996) and Parlatore (2019), the reason for asset repurchases comes from the illiquidity in the secondary market – if the secondary market is illiquid, it will be difficult for the borrowers to find the collateral asset to buy and hence they would like to repurchase the collateral asset back directly from the lender. In Bigio and Shi (2020), the asset repurchase option is introduced to meet the high quality borrowers' incentive compatibility constraint, since they can exercise the option to obtain their high quality asset back. In ours, borrowers want to repurchase the asset back so that they can use it to obtain short-term liquidity in the future.

To define the haircut, we first need to define the collateral value from the lender's perspective. In the context of our model, the lender expects to generate on average $E\phi/\beta$ from the sale of the collateral in case of default, where $E\phi \equiv \lambda\phi_L + (1-\lambda)\phi_H$. Below, we refer to this amount as the collateral value.[36]

The definition of repo haircut in our model is

$$h \equiv 1 - \frac{\text{loan value}}{\text{collateral value}} = 1 - \frac{q_D}{E\phi/\beta}. \tag{29}$$

From (23) we write the collateral value as:

$$E\phi/\beta = zq_D + \lambda z q_E + (1-\lambda)e_H, \tag{30}$$

where $e_H = \int_{d-\Delta\phi}^{\bar{s}} \widetilde{F}_H(s)ds$ is agent $O$'s expected value of a high-quality equity tranche. Substituting (30) into (29), we obtain the following expression for haircut:

$$h = \underbrace{(z-1)\frac{q_D + \lambda q_E}{E\phi/\beta}}_{\text{gain from trade/collateral value}} + \underbrace{\frac{\lambda q_E + (1-\lambda)e_H}{E\phi/\beta}}_{\text{equity/collateral value}}. \tag{31}$$

We observe from (31) that the repo haircut has two components. The first component arises because the borrowers, who price the collateral asset, value the liquidity service of the asset to realize gain from trade, while lenders, who price the loan, do not value it. The term $z-1$ is the net marginal value of the liquidity service; it reflects heterogenous valuation over the collateral asset between the lenders and the borrowers in our model.[37] The second component is the value of the equity tranche relative to the collateral value and arises mechanically because equity tranche by definition is excluded from the repo debt.

## 8.2 Repo Contract with a Two-point Distribution

In this section, we illustrate the properties of optimal repo contract in closed form when quality follows a two-point payoff distribution. We first consider the i.i.d. case and return briefly to the persistent case at the end of the section. The purpose of this exercise is to provide simpler expressions for haircut and repo rate with respect to the primitives of the model. Using these expressions we are able to find clean comparative statics of repo contract terms that generate empirically testable hypothesis, especially in the i.i.d. case which captures the characteristics of the vast treasury repo market in the US.

[36] $E\phi$ is the end-of-period expected value of the collateral asset. Because the repo contract is an intra-period short term contract, the collateral value in the definition of haircut refers to the beginning-of-period value, which equals $E\phi/\beta$.

[37] In the case of debt tranche as a passthrough security, the equity tranche disappears, and the haircut is $(z-1)/z$, solely driven by the marginal value of liquidity service.

Suppose, the high (low) quality asset pays one unit of payoff with probability $\pi_H$ ($\pi_L$) and pays zero otherwise where $0 \leq \pi_L < \pi_H \leq 1$ and $\lambda = \lambda_L = \lambda_H$. The debt contract takes a simple form. Regardless of the realization of the payoff, it pays the resale price $\phi$. In addition, it pays $d$ units if the current payoff is one.[38]

Let the expected value of the payoff (based on public information) be given by $Es \equiv (1-\lambda)\pi_H + \lambda\pi_L$. Expressions for repo rate and haircut given in (28) and (31) become much simpler. Repo rate is

$$r = \frac{1 - Es}{\nu}, \tag{32}$$

and haircut is

$$h = 1 - \frac{\beta}{1 - \frac{Es}{\nu}}, \tag{33}$$

where $\nu \equiv \lambda(\pi_H - \pi_L)/(z-1)$ captures the degree of adverse selection. Severity of adverse selection increases in the probability that the asset is low quality ($\lambda$) and the difference in the probability of obtaining a positive payoff under high versus low quality ($\pi_H - \pi_L$) and decreases in gains from trade $(z-1)$. We observe from equation (33) that haircut is increasing in adverse selection $\nu$ holding $Es$ fixed, and the sensitivity of haircut to adverse selection is increasing in $\beta$. The latter observation is another manifestation of the dynamic feedback between collateral price and contract terms: when agents become more forward looking, the role of resale price in backing the loan becomes more important. Hence, higher adverse selection lowers price and leads to a higher haircut. Comparative statics on haircut with respect to information friction that ignore the dynamic feedback and take resale price as exogenously given would then be inaccurate. The following proposition describes the comparative statics of repo rate and haircut.

**Proposition 8.** *Both the repo rate and the haircut are decreasing in the expected value of the payoff based on public information, $Es$, holding $\nu$ fixed. Repo rate is decreasing and repo haircut is increasing in the degree of adverse selection, $\nu$, holding $Es$ fixed.*

This proposition maps out how the degree of adverse selection and the expected value of the payoff, which are functions of the primitives of the model, affect repo rate and haircut. The only part of

[38] The expressions for the terms of this repo contract are as follows:

$$\begin{aligned} d &= D - \phi = \frac{\frac{\beta}{1-\beta z}\left[z\lambda\pi_L + (1-\lambda)\pi_H\right]}{\frac{z}{z-1}\lambda\left(\pi_H - \pi_L\right) - \frac{1-\beta[1+\lambda(z-1)]}{1-\beta z}\pi_H} < 1, \\ \phi &= \frac{\beta}{1-\beta z}\left[z\lambda\pi_L + (1-\lambda)\pi_H + (1-\lambda)(z-1)\pi_H d\right]. \end{aligned}$$

Proposition 8 that may seem counter-intuitive is the statement that repo rate is decreasing in adverse selection. In fact, this result is in the same spirit as the standard result in the credit rationing models (Stiglitz and Weiss (1981)). When adverse selection increases, haircut goes up, which means that the face value of repo loan is lower, making the repo loan safer and leading to a lower repo rate.

The results in the above proposition point out that the impact of adverse selection on repo terms is intricate. When testing how adverse selection affects repo rates and haircuts, empiricists need to control for changes in the expected value of the asset's payoff. The degree of adverse selection and the expected value of the payoff can be inferred from secondary information (such as prices, dividends, credit ratings, convenience yields, etc.). With these implementable metrics, the simple analytical solution provides new testable implications for cross-sectional repo contracts.

In segments of the repo market that uses low quality collaterals, private information advantage can be long-lived. Motivated by this observation, we next demonstrate the effect of persistent private information for the two-point distribution case. The main message is that higher persistence in private information leads to higher price volatility, lower collateral values and funding, larger haircuts and repo rates. To allow for persistence we let $\lambda_L > \lambda_H$ where $\Delta\lambda = \lambda_L - \lambda_H$. To simplify the analysis and obtain closed-form solutions for the repo terms, we further assume that $\pi_H = 1$ and $\pi_L = 0$, i.e., the high quality asset always pays one unit of payoff and the low-quality asset always pays zero. The following proposition describes the comparative statics for outcomes of economic interest as persistence in quality $\Delta\lambda$ increases holding the steady state quality distribution $\lambda$ constant.

**Proposition 9.** *Keeping $\lambda$ constant, as $\Delta\lambda$ increases (i) the debt threshold, $d$ decreases; (ii) price volatility, $\Delta\phi$ , increases; (iii) collateral value $E\phi$ decreases; (iv) loan value, $q_D$, decreases; (v) haircut, $h$, increases; (vi) repo rate, $r$, increases.*

# 9 Conclusion

Our paper studies optimal security design in a dynamic lemons market. We show that the implementation of our optimal security design involves short-term collateralized debt. Because optimal security design helps coordinate investors' inter-temporal decisions, the dynamic lemons market under optimal security design is robust to multiple equilibria induced by inter-temporal miscoordination. We also explore economic implications of an implementation of optimal security, short-term repos, and derive dynamic equilibrium properties of repo rates, haircuts, and asset price premium. Our setup in general can be applied to any collateralized borrowing where the collaterals are traded in the capital market.[39] The underlying economic mechanism of our theory is the price liquidity feedback effect derived from the fact that collateral assets can be resold and resale prices can back security payments. Optimal security design eliminates multiplicity, generates greater amounts of liquidity, and restores the economy to a unique Pareto-optimal equilibrium.

According to the current understanding, the shadow banking system of overnight repurchase agreements, asset-backed securities, broker-dealers, and investments contributed to the Great Depression and the runs on the shadow banking system were classic bank runs à la Diamond and Dybvig (1983). However, this popular explanation ignores the fact that most of the securitized products and the short-term funding instruments of these shadow banks are backed by the resale prices of the assets on their balance sheet (in addition to dividend/interest payments). Our model implies that in a dynamic economy, when financial intermediaries can flexibly tranche their assets, self-fulfilling price dynamics can be removed and the amount of funding liquidity as well as the real output in the economy will be greatly improved. Securitization in fact eliminates multiple equilibria and excessive volatility in asset prices and liquidity. Nevertheless, our theory identifies a new source of financial fragility that potentially emerges via the price-liquidity feedback loop. For example, we find, in a repo implementation of our model, that more persistence in private information results in more adverse selection, volatile asset prices, a lower amount of repo debt financing, exacerbating credit crunch. [40] We conclude, therefore, by pointing out that as the current global financial system moves from bank-based towards market-governed, it is crucial to understand the dynamic feedback mechanism between asset prices and funding liquidity identified in this paper. It may generate more funding and promote greater economic growth, but at the same time it also might ignite destabilizing self-fulfilling crises. Economic policymakers and financial regulators need to closely monitor this new source of financial instability.

[39] It has been observed that more firms raise funding and manage their working capital directly from investors by issuing securities backed by marketable collateral assets on their balance sheets, sidestepping banks or other traditional financial intermediaries. For instance, Apple Inc. reported $5.2 billion of repo borrowing in its 2020 10-K filing to support its working capital need during the Covid-19 pandemic. An implication of this practice is that firms now have incentives to acquire marketable assets (such as high grade sovereign and corporate bonds) to access funding liquidity directly.

[40] Rigidity in security design may lead to failure of inter-temporal coordination and result in runs. We discuss the implications of rigidity in follow up work which is available upon request.

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# A Appendix

## A.1 Proof of Proposition 1

Let $\overline{q} \equiv \lambda E_L y + (1-\lambda) E_H y$. Note that $z\overline{q} - E_H y \gtreqless 0$ iff $E_L y / E_H y \gtreqless \zeta$.

Consider the case $E_L y / E_H y > \zeta$. Suppose that the equilibrium price $q$ is strictly less than $\overline{q}$. In this case an investor can deviate and bid $\overline{q} - \epsilon$ where $\epsilon > 0$. For $\epsilon$ small enough, $z\left(\overline{q} - \epsilon\right) - E_H y > 0$. Hence at this price both types sell the security and the deviation generates strictly positive surplus. This means that the equilibrium price must be at least $\overline{q}$. At price $\overline{q}$ or above both types sell the security, hence the only price that is consistent with zero profit condition is $q = \overline{q}$.

Now consider the case $E_L y / E_H y < \zeta$. In this case high type will sell the security only if $q$ is sufficiently larger than $\overline{q}$. However, at prices above $\overline{q}$, investors make negative profit. Hence equilibrium price must be below $\overline{q}$. If $q$ is below $\left(E_L y\right)/z$ then neither type sells the security. In this case, one of the investors can deviate and bid $E_L y - \epsilon$ where $\epsilon > 0$. For $\epsilon$ small enough, $z\left(E_L y - \epsilon\right) - E_L y > 0$ so the low type sells the security and the deviating agent makes strictly positive surplus. If $q$ is at least $\left(E_L y\right)/z$ but less than $E_L y$ then the low type sells the security to the investors who bid that price. In this case, one of the investors who bids $E_L y$ or less can deviate and bid slightly above $q$. This agent then buys the security alone and increases her surplus. At prices greater than equal to $E_L y$ (and below $\overline{q}$), the low type alone sells the security. Hence the only price that is consistent with zero profit condition is $q = E_L y$.

## A.2 Proof of Proposition 2

By the discussion in the text we know that a pooling equilibrium exists if and only if $E_L s / E_H s \geq \kappa_P$, and a separating equilibrium exists if and only if $E_L s / E_H s < \kappa_S$. To complete the proof of the proposition we need to show $\kappa_P < \kappa_S$. To see this note that,

$$[\zeta - \beta z (1 - (1-\zeta)\lambda)] - [\zeta - \beta (1 - (1-\zeta z)\lambda_L)] = -\beta\lambda_L (z-1)\left(\frac{1-\lambda}{\lambda}\right)$$

and similarly,

$$[1 - \beta z (1 - (1-\zeta)\lambda)] - [1 - \beta (1 - (1-\zeta z)\lambda_H)] = -\beta\lambda_H (z-1)\left(\frac{1-\lambda}{\lambda}\right).$$

Using the equalities above and the fact that $\lambda_L > \lambda_H$ we obtain:

$$\begin{aligned}\kappa_P = \frac{\zeta - \beta z (1-(1-\zeta)\lambda)}{1 - \beta z (1-(1-\zeta)\lambda)} &< \frac{\zeta - \beta z (1-(1-\zeta)\lambda) + \beta\lambda_L (z-1)\left(\frac{1-\lambda}{\lambda}\right)}{1 - \beta z (1-(1-\zeta)\lambda) + \beta\lambda_H (z-1)\left(\frac{1-\lambda}{\lambda}\right)} \\ &= \frac{\zeta - \beta (1-(1-\zeta z)\lambda_L)}{1 - \beta (1-(1-\zeta z)\lambda_H)} = \kappa_S.\end{aligned}$$

## A.3 Statement and Proof of Lemma 1

**Lemma 1.** *If two securities, $y$ and $y'$, are both traded in a pooling (separating) equilibrium, then $y + y'$ is also traded in a pooling (separating) equilibrium. Moreover, if a feasible security design contains $y$ and $y'$, replacing the two securities by $y + y'$ is also a feasible security design and the value of the designer's objective remains the same in these two cases. Hence, w.l.o.g. we can restrict attention to security designs that contain at most two securities, one traded in a pooling equilibrium and the other traded in a separating equilibrium.*

*Proof.* If two securities, $y$ and $y'$, are both traded in a pooling equilibrium, $E_L y \geq \zeta E_H y$ and $E_L y' \geq \zeta E_H y'$. Then combining these two securities results in a security traded in a pooling equilibrium. Similarly, combining two securities traded in a separating equilibrium results in a security traded in a separating equilibrium. To see the second statement in the lemma, first note that replacing the two securities with their combination is clearly feasible. In addition, when $y$, $y'$ and $y + y'$ all trade in a pooling (separating) equilibrium, $q''$, the price of $y + y'$, is the sum of $q$ and $q'$, the prices of $y$ and $y'$. Now consider the pooling case. Ignoring the irrelevant terms, agent $O$'s payoff when the two securities are separate is:

$$\lambda \int \{a[zq - y(s)] + a[zq' - y'(s)]\}\, dF_L(s) + (1-\lambda)\int \{a[zq - y(s)] + a[zq' - y'(s)]\}\, dF_H(s)$$

and when they are combined is:

$$\lambda \int \{a[zq'' - (y(s) + y'(s))]\}\, dF_L(s) + (1-\lambda)\int \{a[zq'' - (y(s) + y'(s))]\}\, dF_H(s).$$

Since $q'' = q + q'$, when the securities are combined agent $O$'s payoff is unchanged.

Next consider the separating case. Once again ignoring the irrelevant terms, agent $O$'s payoff when the two securities are separate is:

$$\lambda \int \{a[zq - y(s)] + a[zq' - y'(s)]\} dF_L(s) + (1-\lambda) \int \{ay(s) + ay'(s)\} dF_H(s)$$

and when they are combined is:

$$\lambda \int \{a[zq'' - (y(s) + y'(s))]\} dF_L(s) + (1-\lambda) \int \{a(y(s) + y'(s))\} dF_H(s).$$

Once again, when the securities are combined agent $O$'s payoff is unchanged. $\square$

## A.4 Proof of Proposition 4

Using Lemma 1 we restate the optimal security design problem as choosing the pooling tranche of the asset, $y_D(s, \phi_Q)$, to maximize the value of a high-quality debt tranche:

$$E_H y_D(s, \phi_H) \tag{A.1}$$

subject to

$$s + \phi_Q - y_D(s, \phi_Q) \geq 0, \forall s \in [\underline{s}, \bar{s}] \text{ and } Q \in \{L, H\}, \tag{A.2}$$

$$E_L y_D(s, \phi_L) - \zeta E_H y_D(s, \phi_H) \geq 0, \tag{A.3}$$

and

$$y_D(s, \phi_Q) \geq y_D(s', \phi_{Q'}) \text{ if } s + \phi_Q \geq s' + \phi_{Q'}, \forall s \in [\underline{s}, \bar{s}] \text{ and } Q, Q' \in \{L, H\}. \tag{A.4}$$

We obtain the objective function by plugging the security prices and the quantities given in Proposition 1 into the designer's objective (4). The first constraint above is the simplified feasibility constraint (2) and requires $y_D(s, \phi_Q)$ to be backed by the underlying asset. The second is the requirement in Proposition 1 that the security is sold in a pooling equilibrium. The third constraint restates (1) that requires the pooling security to be monotone in the total payoff generated by the asset.[41][42]

We introduce some notations that we use in the rest of the proof. Let $\widetilde{Y}$ be the set of non-decreasing functions from $[\underline{s} + \phi_L, \bar{s} + \phi_H]$ to $\mathbb{R}_+$. Define

$$G_L(x) = \begin{cases} F_L(x - \phi_L) & \text{if } x \in [\underline{s} + \phi_L, \bar{s} + \phi_L] \\ 1 & \text{if } x \in (\bar{s} + \phi_L, \bar{s} + \phi_H] \end{cases}$$

[41] Equation (1) needs to hold for the residual equity tranche as well but this constraint is not binding.

[42] The uniqueness of equilibrium does not depend on the restriction of issuing monotone securities, and also holds when the borrower issues Arrow securities against the dividend payment and the resale value of the asset. This result is available upon request.

and

$$G_H(x) = \begin{cases} 0 & \text{if } x \in [\underline{s}+\phi_L, \underline{s}+\phi_H) \\ F_H(x-\phi_H) & \text{if } x \in [\underline{s}+\phi_H, \bar{s}+\phi_H] \end{cases}.$$

The corresponding density functions are:

$$g_L(x) = \begin{cases} f_L(x-\phi_L) & \text{if } x \in [\underline{s}+\phi_L, \bar{s}+\phi_L] \\ 0 & \text{if } x \in (\bar{s}+\phi_L, \bar{s}+\phi_H] \end{cases}$$

and

$$g_H(x) = \begin{cases} 0 & \text{if } x \in [\underline{s}+\phi_L, \underline{s}+\phi_H) \\ f_H(x-\phi_H) & \text{if } x \in [\underline{s}+\phi_H, \bar{s}+\phi_H] \end{cases}.$$

Let $E_{G_Q}$ denote the expectation with respect to $G_Q$. Consider the following problem:

$$\max_{\widetilde{y}\in\widetilde{Y}} E_{G_H}\widetilde{y} \tag{A.5}$$

subject to:

$$x-\widetilde{y}(x) \geq 0, \forall x \in [\underline{s}+\phi_L, \bar{s}+\phi_H], \tag{A.6}$$

$$E_{G_L}\widetilde{y} - \zeta E_{G_H}\widetilde{y} \geq 0. \tag{A.7}$$

Next, we provide three lemmas which help us prove the result.

**Lemma 2.** *For any $y(x,\phi_Q)$ where $Q \in \{L, H\}$ that satisfies (A.2)-(A.4), there is $\widetilde{y} \in \widetilde{Y}$ that satisfies (A.6) and (A.7) and*

$$E_H y(s,\phi_H) = E_{G_H}\widetilde{y}. \tag{A.8}$$

*Proof.* Fix an arbitrary $y(s,\phi_Q)$ and let $\widetilde{y}(x) = y(x,\phi_Q)$ for $x \in [\underline{s}+\phi_Q, \bar{s}+\phi_Q]$ and $Q \in \{L, H\}$. If $\underline{s}+\phi_H < \bar{s}+\phi_L$, then by (A.4), $y(x,\phi_L) = y(x,\phi_H)$ for $x \in [\underline{s}+\phi_H, \bar{s}+\phi_L]$. Hence, $\widetilde{y}$ is well-defined in this range. If $\bar{s}+\phi_L < \underline{s}+\phi_H$, then let $\widetilde{y}(x) = y(\bar{s},\phi_L)$ for $x \in [\bar{s}+\phi_L, \underline{s}+\phi_H]$.[43] It is easy to see that $\widetilde{y}$ is increasing by (A.4) and satisfies (A.6) and (A.7) by (A.2) and (A.3). Moreover, by construction of $G_H$, (A.8) is satisfied. □

**Lemma 3.** *Suppose $\widetilde{y}^* \in \widetilde{Y}$ maximizes (A.5) subject to (A.6) and (A.7). Let $y_D(s,\phi_Q) = \widetilde{y}^*(s+\phi_Q)$ for $s \in [\underline{s}, \bar{s}]$ where $Q \in \{L, H\}$. Then $y_D(s,\phi_Q)$ maximizes (A.1) subject to (A.2)-(A.4).*

[43] In this range, $\widetilde{y}$ can be chosen in any way as long as it is non-decreasing.

*Proof.* By construction $y_D(s, \phi_Q)$ satisfies (A.2)-(A.4) and $E_H y_D(s, \phi_H) = E_{G_H} \widetilde{y}^*$. Lemma 2 then implies that $y_D(s, \phi_Q)$ maximizes (A.1) subject to (A.2)-(A.4). $\square$

**Lemma 4.** *If $f_L(x)/f_H(x)$ is decreasing for $x \in [\underline{s}, \bar{s}]$ and either $f_L$ or $f_H$ is log-concave then $f_L(x-\phi_L)/f_H(x-\phi_H)$ is decreasing in $x$ for $x \in [\underline{s}+\phi_H, \bar{s}+\phi_L]$.*

*Proof.* Note that the derivative of $f_L(x-\phi_L)/f_H(x-\phi_H)$ is negative if:

$$\frac{f_L'(x-\phi_L)}{f_L(x-\phi_L)} < \frac{f_H'(x-\phi_H)}{f_H(x-\phi_H)}. \tag{A.9}$$

Since $f_L(x)/f_H(x)$ is decreasing we have:

$$\frac{f_L'(x-\phi_Q)}{f_L(x-\phi_Q)} < \frac{f_H'(x-\phi_Q)}{f_H(x-\phi_Q)} \tag{A.10}$$

where $Q \in \{L, H\}$. Moreover, $f_Q$ is log-concave if and only if $f_Q'(x)/f_Q(x)$ decreasing in $x$. Hence if $f_H$ is log-concave then

$$\frac{f_H'(x-\phi_L)}{f_H(x-\phi_L)} < \frac{f_H'(x-\phi_H)}{f_H(x-\phi_H)} \tag{A.11}$$

and if $f_L$ is log-concave then

$$\frac{f_L'(x-\phi_L)}{f_L(x-\phi_L)} < \frac{f_L'(x-\phi_H)}{f_L(x-\phi_H)} \tag{A.12}$$

Inequality (A.10) together with either (A.11) or (A.12) implies (A.9). $\square$

**Proposition 10.** *Assume that $f_L(x)/f_H(x)$ is decreasing for $x \in [\underline{s}, \bar{s}]$, and either $f_L$ or $f_H$ is log-concave. Then there is a unique solution to (A.5) where the optimal solution $\widetilde{y}^*$ is such that*

$$\widetilde{y}^*(x) = \min(x, D),$$

*for some $D \in (\underline{s}+\phi_L, \bar{s}+\phi_H]$.*

*Proof.* We can write any right-continuous monotone security $\widetilde{y}(s)$ as:

$$\widetilde{y}(s) = \phi_L + \underline{s} + \int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H} \chi(j)dj,$$

where $\chi(j) \geq 0$ for all $j \in [\underline{s}+\phi_L, \bar{s}+\phi_H]$.

Let $\widetilde{G}_Q(x) = 1 - G_Q(x)$ for $Q \in \{L, H\}$ and $x \in [\underline{s}+\phi_L, \bar{s}+\phi_H]$. Then,

$$E_{G_Q}\widetilde{y} = \phi_L + \underline{s} + \int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H} \widetilde{G}_Q(j)\chi(j)dj.$$

The optimization problem (A.5) is equivalent to the following problem:

$$\arg\max_{\chi\geq 0}\int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H}\widetilde{G}_H(x)\chi(x)dx, \tag{A.13}$$

$$s.t.\underline{s}+\phi_L+\int_{\underline{s}+\phi_L}^{x}\chi(j)dj\leq x,\forall x\in\left[\underline{s}+\phi_L,\bar{s}+\phi_H\right], \tag{A.14}$$

$$\int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H}\left[\widetilde{G}_L(x)-\zeta\widetilde{G}_H(x)\right]\chi(x)dx+(1-\zeta)\left(\underline{s}+\phi_L\right)\geq 0, \tag{A.15}$$

$$\chi(x)\geq 0,\forall x\in\left[\underline{s}+\phi_L,\bar{s}+\phi_H\right] \tag{A.16}$$

Note that the feasible set is compact, convex and nonempty so the optimization problem must have a solution. Moreover, since the objective function is bounded above, the solution must be finite. The Lagrangian of the optimization problem is

$$\begin{aligned}\mathcal{L}\left(x;\gamma,\mu,\mu_\chi\right)=&\int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H}\widetilde{G}_H(x)\chi(x)dx+\int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H}\gamma(x)\left[x-\left(\underline{s}+\phi_L\right)-\int_{\underline{s}+\phi_L}^{x}\chi(j)dj\right]dx\\&+\mu\left\{\int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H}\left[\widetilde{G}_L(x)-\zeta\widetilde{G}_H(x)\right]\chi(x)dx+(1-\zeta)\left(\underline{s}+\phi_L\right)\right\}+\int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H}\mu_\chi(x)\chi(x)dx\\=&\int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H}\left\{\widetilde{G}_H(x)+\mu\left[\widetilde{G}_L(x)-\zeta\widetilde{G}_H(x)\right]-\eta\left(x\right)+\mu_\chi(x)\right\}\chi(x)dx\\&+\mu(1-\zeta)\left(\underline{s}+\phi_L\right)+\int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H}\eta\left(x\right)dx,\end{aligned}$$

where the second equality is obtained by using integration by parts on the second term of the Lagrangian, and then setting $\eta\left(x\right)=\int_x^{\bar{s}+\phi_H}\gamma(j)dj$. Let $\mathcal{L}^*=\min_{\gamma\geq 0,\mu\geq 0,\mu_\chi\geq 0}\left[\max_{\chi\geq 0}\mathcal{L}\left(x;\gamma,\mu,\mu_\chi\right)\right]$. Note that $\mathcal{L}^*$ is the value of the original optimization problem. The quantity inside the curly brackets must be zero or otherwise the value of the optimization problem would be infinite. Consider the following problem,

$$\begin{aligned}\min_{\mu\geq 0}\quad\min\nolimits_{\eta\geq 0,\mu_x\geq 0}\quad&\mu(1-\zeta)\left(\underline{s}+\phi_L\right)+\int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H}\eta\left(x\right)dx\\s.t.\qquad&\widetilde{G}_H(x)+\mu\left[\widetilde{G}_L(x)-\zeta\widetilde{G}_H(x)\right]-\eta\left(x\right)+\mu_\chi(x)=0.\end{aligned}$$

The value of this problem is $\mathcal{L}^*$. Let $H_\mu\left(x\right)=\widetilde{G}_H(x)+\mu\left[\widetilde{G}_L(x)-\zeta\widetilde{G}_H(x)\right]$, and rewrite one more time as:

$$\begin{aligned}\min_{\mu\geq 0}\quad\min\nolimits_{\eta\geq 0}\quad&\mu(1-\zeta)\left(\underline{s}+\phi_L\right)+\int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H}\eta\left(x\right)dx\\s.t.\qquad&\eta\left(x\right)\geq H_\mu\left(x\right),\end{aligned}$$

and the constraint that $\eta(x)$ is a decreasing function in $x$. Note, $h_\mu\left(x\right)\equiv\frac{\partial H_\mu(x)}{\partial x}=-\left[g_H\left(x\right)+\mu\left(g_L\left(x\right)-\zeta g_H\left(x\right)\right)\right]$, $H_\mu\left(\underline{s}+\phi_L\right)=1+\mu\left[1-\zeta\right]>0$ and $H_\mu\left(\bar{s}+\phi_H\right)=0$.

If $\mu = 0$ then $h_\mu(x) < 0$. Suppose $\mu > 0$.

- Case 1) Suppose $\bar{s} + \phi_L < \underline{s} + \phi_H$:
  - If $x \in [\underline{s} + \phi_L, \bar{s} + \phi_L]$ then $h_\mu(x) = -\mu(f_L(x - \phi_L)) < 0$.
  - If $x \in (\bar{s} + \phi_L, \underline{s} + \phi_H)$ then $h_\mu(x) = 0$.
  - If $x \in [\underline{s} + \phi_H, \bar{s} + \phi_H]$ then $h_\mu(x) = -f_H(x - \phi_H)[1 - \mu\zeta]$.

    If $1 - \mu\zeta \geq 0$ then $h_\mu(x) \leq 0$ for all $x \in [\underline{s} + \phi_L, \bar{s} + \phi_H]$. If $1 - \mu\zeta < 0$ then $h_\mu(x) \leq 0$ for $x \in [\underline{s} + \phi_L, \underline{s} + \phi_H]$ and $h_\mu(x) > 0$ for $x \in [\underline{s} + \phi_H, \bar{s} + \phi_H]$. Hence, $h_\mu$ changes its sign at most once from negative to positive.

- Case 2) Suppose $\underline{s} + \phi_H < \bar{s} + \phi_L$:
  - If $x \in [\underline{s} + \phi_L, \underline{s} + \phi_H)$ then $h_\mu(x) = -\mu(f_L(x - \phi_L)) < 0$.
  - If $x \in [\underline{s} + \phi_H, \bar{s} + \phi_L]$ then $h_\mu(x) = -f_H(x - \phi_H)\left[1 + \mu\left(\frac{f_L(x-\phi_L)}{f_H(x-\phi_H)} - \zeta\right)\right]$.
  - If $x \in (\bar{s} + \phi_L, \bar{s} + \phi_H]$ then $h_\mu(x) = -f_H(x - \phi_H)[1 - \mu\zeta]$.

    By Lemma 4, $h_\mu(x)$ can change sign from negative to positive only once over $[\underline{s} + \phi_H, \bar{s} + \phi_L]$. Moreover, at $x = \bar{s} + \phi_L$, $h_\mu(x)$ may only jump up and its sign remains unchanged on $(\bar{s} + \phi_L, \bar{s} + \phi_H]$. Hence, $h_\mu$ changes its sign at most once from negative to positive.

Since $h_\mu$ changes its sign at most once from negative to positive, and since $H_\mu(\underline{s} + \phi_L) > 0$ and $H_\mu(\bar{s} + \phi_H) = 0$, either there exists a unique $x^*_\mu \in (\underline{s} + \phi_L, \bar{s} + \phi_H)$ such that $H_\mu(x^*_\mu) = 0$, or $H_\mu(x) > 0$ for all $x \in (\underline{s} + \phi_L, \bar{s} + \phi_H)$. In the latter case, we let $x^*_\mu = \bar{s} + \phi_H$.

Note that for given $\mu \geq 0$ optimal $\eta_\mu$ is given by:

$$\eta_\mu(x) = \begin{cases} H_\mu(x) & \text{if} \quad x \leq x^*_\mu \\ 0 & \text{if} \quad x > x^*_\mu \end{cases}.$$

Plugging this into the minimization problem we get:

$$\min_{\mu \geq 0} \mu(1-\zeta)\phi + \int_{\underline{s}+\phi_L}^{x^*_\mu} \left(\widetilde{G}_H(x) + \mu\left[\widetilde{G}_L(x) - \zeta\widetilde{G}_H(x)\right]\right) dx.$$

The first order condition for this problem is:

$$(1-\zeta)\phi + \int_{\underline{s}+\phi_L}^{x^*_\mu} \left[\widetilde{G}_L(x) - \zeta\widetilde{G}_H(x)\right] dx + \frac{\partial x^*_\mu}{\partial \mu} H_\mu(x^*_\mu) \geq 0$$

Because $H_\mu(x^*_\mu) = 0$,

$$(1-\zeta)\phi + \int_{\underline{s}+\phi_L}^{x^*_\mu} \left[\widetilde{G}_L(x) - \zeta\widetilde{G}_H(x)\right] dx \geq 0$$

with complementary slackness.

Let $x^* \in (\underline{s}+\phi_L, \bar{s}+\phi_H]$ be the unique $s$ for which

$$(1-\zeta)\phi + \int_{\underline{s}+\phi_L}^{x^*} \left[\widetilde{G}_L(x) - \zeta \widetilde{G}_H(x)\right] dx = 0$$

if it exists. If

$$(1-\zeta)\phi + \int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H} \left[\widetilde{G}_L(x) - \zeta \widetilde{G}_H(x)\right] dx > 0$$

for all $x \in [\underline{s}+\phi_L, \bar{s}+\phi_H]$, then $x^* = \bar{s}+\phi_H$.

If $x^* < \bar{s}+\phi_H$ then $\mu > 0$, $x^*_\mu = x^*$, and

$$\mathcal{L}^* = \mu(1-\zeta)\phi + \int_{\underline{s}+\phi_L}^{x^*} \left(\widetilde{G}_H(x) + \mu\left[\widetilde{G}_L(x) - \zeta\widetilde{G}_H(x)\right]\right) dx = \int_{\underline{s}+\phi_L}^{x^*} \widetilde{G}_H(s) ds.$$

If $x^* = \bar{s}+\phi_H$ then $\mu = 0$, $x^*_\mu = \bar{s}+\phi_H$, and

$$\mathcal{L}^* = \int_{\underline{s}+\phi_L}^{\bar{s}+\phi_H} \widetilde{G}_H(s) ds.$$

To complete the proof, let $D = x^*$ and note that $\chi(x) = 1$ for $x \in [\underline{s}+\phi_L, D)$ and $\chi(x) = 0$ for $x \in [D, \bar{s}+\phi_H]$ achieves the value $\mathcal{L}^*$ and it is feasible, and must be optimal for the original problem. $\square$

To finish the proof we observe from Lemma 3 that solution to (A.1) is $y_D(s, \phi_Q) = \min(s+\phi_Q, D)$ for $s \in [\underline{s}, \bar{s}]$ where $Q \in \{L, H\}$ and $D \in (\underline{s}+\phi_L, \bar{s}+\phi_H]$.

## A.5 Proof of Theorem 1

Using Proposition 4, we write the designer's problem as:

$$\max_{d \in [\underline{s}, \bar{s}+\Delta\phi]} \int_{\underline{s}}^{d-\Delta\phi} \widetilde{F}_H(s) ds \tag{A.17}$$

subject to

$$\underline{s} + \phi_L + \int_{\underline{s}}^{d} \widetilde{F}_L(s) ds - \zeta\left(\underline{s} + \phi_H + \int_{\underline{s}}^{d-\Delta\phi} \widetilde{F}_H(s) ds\right) \geq 0, \tag{A.18}$$

where $d \equiv D - \phi_L$ and $\Delta\phi \equiv \phi_H - \phi_L$. To obtain (A.17) and (A.18), we substitute (20) into the designer's objective given in (A.1) and into (A.3) which guarantees that the debt tranche is sold in a pooling equilibrium.[44]

[44] Condition (A.4) holds automatically for the debt contract given in Equation (20).

Define Agent $O$'s value over the equity tranche, $e_H$ and $e_L$, as:

$$e_L(d) \equiv \int_d^{\bar{s}} \widetilde{F}_L(s)ds, \tag{A.19}$$

$$e_H(d) \equiv \int_{d-\Delta\phi}^{\bar{s}} \widetilde{F}_H(s)ds. \tag{A.20}$$

Note that the flow payments in the debt tranche are

$$\int_{\underline{s}}^{d} \widetilde{F}_L(s)ds = E_L s - e_L(d), \tag{A.21}$$

$$\int_{\underline{s}}^{d-\Delta\phi} \widetilde{F}_H(s)ds = E_H s - e_H(d). \tag{A.22}$$

The participation constraint (A.18) can now be written as:

$$\Gamma(d) = \phi_L + E_L s - e_L(d) - \zeta(\phi_H + E_H s - e_H(d)) \geq 0. \tag{A.23}$$

To prove that there is a unique equilibrium with security design, we show that either there is a unique $d \in (\underline{s}, \bar{s} + \Delta\phi)$ at which the participation constraint is binding, or the constraint does not bind and $d = \bar{s} + \Delta\phi$.

As a first step, we solve $\phi_Q$ as functions of $e_H$ and $e_L$. Then, we evaluate the derivative of $e_H$ and $e_L$ with respect to $d$. Substituting $q_D$ and $q_E$ which are given by (21) and (22) into $\phi_Q$ which is given by (23), and using (A.21) and (A.22), we obtain:

$$\phi_Q = \beta\left[z\left(\lambda\left(\phi_L + E_L s - e_L(d)\right) + (1-\lambda)\left(\phi_H + E_H s - e_H(d)\right)\right) + z\lambda_Q e_L(d) + (1-\lambda_Q)e_H(d)\right]$$

and

$$\Delta\phi = \phi_H - \phi_L = \beta(\lambda_L - \lambda_H)(e_H(d) - z e_L(d)). \tag{A.24}$$

Note that the prices are functions of all the underlying parameters of the model even though we do not explicitly express their dependence on them. Next, we solve for the prices $\phi_L$ and $\phi_H$.

$$\phi_L = c_{0L} + c_{HL}e_H + c_{LL}e_L \tag{A.25}$$

where

$$c_{0L} = \frac{\beta z\left(\lambda E_L s + (1-\lambda)E_H s\right)}{1-\beta z},$$

$$c_{HL} = \frac{\beta\left[1 - \lambda_L - z(1-\lambda) + z(1-\lambda)\beta(\lambda_L - \lambda_H)\right]}{1-\beta z},$$

$$c_{LL} = \frac{\beta z\left[-\lambda + \lambda_L - (1-\lambda)\beta(\lambda_L - \lambda_H)z\right]}{1-\beta z},$$

and

$$\phi_H = c_{0H} + c_{HH} e_H + c_{LH} e_L \tag{A.26}$$

where

$$\begin{aligned}
c_{0H} &= \frac{\beta z \left(\lambda E_L s + (1-\lambda) E_H s\right)}{1-\beta z}, \\
c_{HH} &= \frac{\beta \left[1 - \lambda_H - z(1-\lambda) - z\lambda\beta(\lambda_L - \lambda_H)\right]}{1-\beta z}, \\
c_{LH} &= \frac{\beta z \left[-\lambda + \lambda_H + \lambda\beta(\lambda_L - \lambda_H) z\right]}{1-\beta z}.
\end{aligned}$$

Using (A.19), (A.20), and (A.24), we obtain derivatives of $e_L$ and $e_H$ with respect to $d$ as:

$$e_L'(d) = -\widetilde{F}_L(d), \tag{A.27}$$

$$e_H'(d) = -\frac{G_L(d)}{G_H(d)} \widetilde{F}_H(d - \Delta\phi), \tag{A.28}$$

where

$$G_L(d) = 1 - \beta(\lambda_L - \lambda_H) z \widetilde{F}_L(d),$$

and

$$G_H(d) = 1 - \beta(\lambda_L - \lambda_H) \widetilde{F}_H(d - \Delta\phi).$$

We can now write the derivative of $\Gamma$ with respect to $d$ as:

$$\Gamma'(d) = \kappa_L e_L'(d) + \kappa_H e_H'(d), \tag{A.29}$$

where

$$\begin{aligned}
\kappa_L =& c_{LL} - \zeta c_{LH} - 1 \\
=& \frac{\beta z}{1-\beta z} \left[\lambda_L - \zeta\lambda_H - (1-\zeta)\lambda - \beta(\lambda_L - \lambda_H)\right] - 1, \\
\kappa_H =& c_{HL} - \zeta c_{HH} + \zeta \\
=& \frac{\beta}{1-\beta z} \left[1 - \lambda_L - \zeta(1-\lambda_H) - (1-\zeta) z (1-\lambda) + \beta(\lambda_L - \lambda_H)\right] + \zeta.
\end{aligned}$$

Since $e_L(\underline{s}) = E_L s$ and $e_H(\underline{s}) = E_H s + \Delta\phi$,

$$\Gamma(\underline{s}) = \phi_L - \zeta\left(\phi_H - \Delta\phi\right) = (1-\zeta)\,\phi_L > 0,$$

and

$$\Gamma(\bar{s} + \Delta\phi) < 0 \text{ if and only if } E_L s / E_H s < 1 - (z-1) / \left(z\lambda\left(1-\beta\right)\right).$$

Substituting (A.27) and (A.28) into (A.29), we obtain:

$$\frac{\Gamma'(d)}{\widetilde{F}_L(d)} = -\kappa_L - \kappa_H \frac{e'_H(d)}{e'_L(d)}.$$

If $\kappa_H > 0$ and $\frac{d\left(e'_H(d)/e'_L(d)\right)}{dd} > 0$, then $\Gamma(d) = 0$ has at most one solution for $d \in (\underline{s}, \bar{s} + \Delta\phi)$. To see this, note that $\Gamma'$ can change sign at most once from positive to negative. Since $\Gamma(\underline{s}) > 0$, if $\Gamma(\bar{s} + \Delta\phi) < 0$ then there exists a unique solution to $\Gamma(d) = 0$ where $d \in (\underline{s}, \bar{s} + \Delta\phi)$, and if $\Gamma(\bar{s} + \Delta\phi) > 0$ then $\Gamma(d) > 0$ for all $d \in (\underline{s}, \bar{s} + \Delta\phi)$. If $\kappa_H < 0$ and $\frac{d\left(e'_H(d)/e'_L(d)\right)}{dd} > 0$, then $\Gamma'(d) > 0$. Hence, $\Gamma(d) > 0$ for all $d \in (\underline{s}, \bar{s} + \Delta\phi)$.[45]

To finish the proof we observe that,

$$\frac{d\Delta\phi}{dd} = -\frac{G_L(d) - G_H(d)}{(G_L(d))^2 G_H(d)}. \tag{A.30}$$

So,

$$\frac{d}{dd} \ln\left[e'_H(d)/e'_L(d)\right] = \frac{1}{\chi_L}\left[\frac{f_L(d)}{\widetilde{F}_L(d)} - \frac{f_H(d-\Delta\phi)}{\widetilde{F}_H(d-\Delta\phi)}\left\{\frac{(G_L(d))^2 G_H(d) + G_L(d) - G_H(d)}{G_L(d)(G_H(d))^2}\right\}\right].$$

Hence, $\frac{d\left(e'_H(d)/e'_L(d)\right)}{dd} > 0$ if

$$\frac{f_L(d)}{\widetilde{F}_L(d)} - \frac{f_H(d-\Delta\phi)}{\widetilde{F}_H(d-\Delta\phi)}\left\{\frac{(G_L(d))^2 G_H(d) + G_L(d) - G_H(d)}{G_L(d)(G_H(d))^2}\right\} > 0. \tag{A.31}$$

Note that

$$\frac{(G_L(d))^2 G_H(d) + G_L(d) - G_H(d)}{G_L(d)(G_H(d))^2} < \frac{1}{(G_H(d))^2}.$$

Moreover, if $f_H$ is log-concave:

$$\frac{f_H(d-\Delta\phi)}{\widetilde{F}_H(d-\Delta\phi)} \Big/ \frac{f_L(d)}{\widetilde{F}_L(d)} < \frac{f_H(d)}{\widetilde{F}_H(d)} \Big/ \frac{f_L(d)}{\widetilde{F}_L(d)}$$

and if $f_L$ is log-concave

$$\frac{f_H(d-\Delta\phi)}{\widetilde{F}_H(d-\Delta\phi)} \Big/ \frac{f_L(d)}{\widetilde{F}_L(d)} < \frac{f_H(d-\Delta\phi)}{\widetilde{F}_H(d-\Delta\phi)} \Big/ \frac{f_L(d-\Delta\phi)}{\widetilde{F}_L(d-\Delta\phi)}.$$

Combining we observe that (A.31) holds if

$$\frac{f_H(d)}{\widetilde{F}_H(d)} \Big/ \frac{f_L(d)}{\widetilde{F}_L(d)} < \frac{1}{(G_H(d))^2}$$

which is (24). It is immediate that when $d \in (\underline{s}, \bar{s} + \Delta\phi)$, the security design equilibrium Pareto dominates the the (unique) separating equilibrium in the baseline case. Moreover, security design uniquely selects the pooling equilibrium when $d = \bar{s} + \Delta\phi$, and hence it strictly Pareto dominates the separating equilibrium if there are multiple equilibria in the baseline case which completes the proof.

[45] This case is consistent with Condition (25) because when $\kappa_H < 0$, $E_L s/E_H s > 0 \geq 1 - (z-1)/(z\lambda(1-\beta))$, and optimal debt threshold is $d = \bar{s} + \Delta\phi$.

## A.6 Proof of Proposition 6

Fix $\lambda \in (0,1)$. Since $\lambda_H = \frac{\lambda}{1-\lambda}(1-\lambda_L)$, if $\lambda_L$ increases by $d > 0$ then $\lambda_H$ decreases by $\frac{\lambda}{1-\lambda}d > 0$ so that $\lambda$ remains constant. Fix some small $d > 0$. Let $\Gamma_d$ denote the participation constraint (A.23) as a function of $d$. We show next that $\Gamma_d(d) < \Gamma_0(d)$ for all $d \in [\underline{s}, \bar{s}]$ which establishes that the unique debt threshold must be decreasing in $d$. Note that we can write $\Gamma_d$ as:

$$\Gamma_d(d) = \kappa_H e_H(d) + \kappa_L e_L(d)$$

plus other terms that do not depend on $d$. For any $d \in [\underline{s}, \bar{s}]$,

$$\Gamma_d(d) - \Gamma_0(d) = d\beta/(1-\lambda)(1-\beta z)[-(1-\beta) + \lambda(1-\zeta)](e_H(d) - ze_L(d)).$$

Since,

$$-(1-\beta) + \lambda(1-\zeta) \leq 0 \Leftrightarrow 1 - \frac{z-1}{z(1-\beta)} = \kappa_P \geq 0$$

and $e_H(d) - ze_L(d) = \Delta\phi/(\beta(\lambda_L - \lambda_H)) > 0$, we have $\Gamma_d(d) < \Gamma_0(d)$.

## A.7 Proof of Proposition 7

Let $\Delta\lambda = \lambda_L - \lambda_H$. From (A.24), we obtain

$$\begin{aligned}
\frac{\partial \Delta\phi}{\partial d} &= -\frac{\beta\Delta\lambda\left[\widetilde{F}_H(d-\Delta\phi) - z\widetilde{F}_L(d)\right]}{1-\beta\Delta\lambda\widetilde{F}_H(d-\Delta\phi)}, \\
\frac{\partial \Delta\phi}{\partial \Delta\lambda} &= \frac{\beta\left[\int_{d-\Delta\phi}^{\bar{s}}\widetilde{F}_H(s)ds - z\int_d^{\bar{s}}\widetilde{F}_L(s)ds\right]}{1-\beta\Delta\lambda\widetilde{F}_H(d-\Delta\phi)} \\
&= \frac{\beta(e_H - ze_L)}{1-\beta\Delta\lambda\widetilde{F}_H(d-\Delta\phi)}.
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{d\Delta\phi}{d\Delta\lambda} &= \frac{\partial\Delta\phi}{\partial d}\frac{dd}{d\Delta\lambda} + \frac{\partial\Delta\phi}{\partial\Delta\lambda} \\
&= \frac{\beta}{1-\beta\Delta\lambda\widetilde{F}_H(d-\Delta\phi)}\left\{-\Delta\lambda\left[\widetilde{F}_H(d-\Delta\phi) - z\widetilde{F}_L(d)\right]\frac{dd}{d\Delta\lambda} + e_H - ze_L\right\} \qquad \text{(A.32)}
\end{aligned}$$

Because the designer takes asset prices, $\phi_H$ and $\phi_L$, as given when choosing the debt threshold $d$, the optimal debt threshold must be such that increasing $d$ tightens the IC when $d \in (\underline{s}, \bar{s})$. If optimal $d = \bar{s}$ then $\partial\Gamma/\partial d = 0$. So, the partial effect of increasing $d$ on $\Gamma$ at the optimal $d$ must be negative,

$$\begin{aligned}
\frac{\partial\Gamma}{\partial d} &= -e_L'(d) + \zeta e_H'(d) \\
&= \widetilde{F}_L(d) - \zeta\widetilde{F}_H(d-\Delta\phi) \leq 0,
\end{aligned}$$

which is equivalent to $\widetilde{F}_H(d-\Delta\phi)-\frac{1}{\zeta}\widetilde{F}_L(d)\geq 0$. Since $1/\zeta = z\left[1/\left(1-((z-1)(1-\lambda)/\lambda)\right)\right] > z$,

$$\widetilde{F}_H(d-\Delta\phi)-z\widetilde{F}_L(d)>0.$$

Because high-quality equity seller is not willing to sell the equity, $e_H\,(d)-ze_L\,(d)>0$.[46] From Proposition 6, $dd/d\Delta\lambda\leq 0$. Therefore, by (A.32), $\frac{d\Delta\phi}{d\Delta\lambda}>0$.

## A.8 Proof of Proposition 8

Specializing the model to this case, we can write the expressions for the debt threshold, $d$, and the asset price, $\phi=\phi_L=\phi_H$:

$$d = \frac{\frac{\beta}{1-\beta z}\left[z\lambda\pi_L+(1-\lambda)\pi_H\right]}{\frac{z}{z-1}\lambda\left(\pi_H-\pi_L\right)-\frac{1-\beta z+\beta(1-\lambda)(z-1)}{1-\beta z}\pi_H}, \tag{A.33}$$

$$\phi = \frac{\beta\left[z\lambda\pi_L+(1-\lambda)\pi_H\right]}{1-\beta z-\beta\frac{(1-\lambda)(z-1)\pi_H}{\frac{z}{z-1}\lambda(\pi_H-\pi_L)-\pi_H}}. \tag{A.34}$$

Plugging the expressions for $d$ and $\phi$, Equations (A.33) and (A.34), into (28) and (29) we obtain (32) and (33). The result follows immediately from these expressions.

## A.9 Proof of Proposition 9

From Equations (A.25) and (A.26) in the proof for Theorem 1,

$$\phi_L = c_{0L}+c_{HL}e_H,$$

where

$$c_{0L} = \frac{\beta z(1-\lambda)}{1-\beta z},$$
$$c_{HL} = \frac{\beta\left[1-\lambda_L-z(1-\lambda)+z(1-\lambda)\beta(\lambda_L-\lambda_H)\right]}{1-\beta z},$$

and

$$\phi_H = c_{0H}+c_{HH}e_H,$$

where

$$c_{0H} = \frac{\beta z(1-\lambda)}{1-\beta z},$$
$$c_{HH} = \frac{\beta\left[1-\lambda_H-z(1-\lambda)-z\lambda\beta(\lambda_L-\lambda_H)\right]}{1-\beta z}.$$

[46]Alternatively, $e_H\,(d)-ze_L\,(d)=\Delta\phi/\left(\beta(\lambda_L-\lambda_H)\right)>0$.

Hence,

$$\begin{aligned}\Delta\phi &= (c_{HH} - c_{HL})e_H \\ &= \frac{\beta(\lambda_L - \lambda_H)\,(1-d)}{1-\beta(\lambda_L - \lambda_H)}\end{aligned} \tag{A.35}$$

and

$$E\phi = \frac{\beta z(1-\lambda)}{1-\beta z} - \frac{\beta(z-1)(1-\lambda)}{1-\beta z}\,(1-d+\Delta\phi)\,.$$

From the proof of Proposition 1, the participation constraint for the owner of a high-quality debt tranche is

$$\begin{aligned}\Gamma(d) &= \phi_L + E_L s - e_L\,(d) - \zeta\,(\phi_H + E_H s - e_H\,(d)) \\ &= \phi_L - \zeta\,(\phi_L + d) \geq 0.\end{aligned}$$

When the IC constraint is binding,

$$d = \frac{1-\zeta}{\zeta}\phi_L = \frac{\beta(z-1)\frac{1-\lambda_L}{1-\beta(\lambda_L-\lambda_H)}}{1-z+(1-\beta)z\lambda+\beta(z-1)\frac{1-\lambda_L}{1-\beta(\lambda_L-\lambda_H)}}.$$

As $\Delta\lambda$ increases keeping $\lambda$ constant, $\lambda_L$ decreases. Then, from this expression we observe that $d$ is decreasing in $\Delta\lambda$ keeping $\lambda$ constant.

From equation A.35, $\Delta\phi$ is increasing in $\Delta\lambda$ keeping $\lambda$ constant. Because

$$E\phi = \frac{\beta z(1-\lambda)}{1-\beta z} + \frac{\beta(z-1)(1-\lambda)}{1-\beta z}\,(-1+d-\Delta\phi)\,,$$

the expected asset price decreases in $\Delta\lambda$ because $d$ is decreasing and $\Delta\phi$ is increasing in $\Delta\lambda$ keeping $\lambda$ constant.

Loan volume equals $q_D$

$$\begin{aligned}q_D &= \lambda\phi_L + (1-\lambda)\phi_H + \lambda(Es_L - e_L) + (1-\lambda)(Es_H - e_H) \\ &= \frac{1-\lambda}{1-\beta z}\,[\beta + (1-\beta)(d-\Delta\phi)]\end{aligned}$$

Since $d - \Delta\phi$ decreases in $\Delta\lambda$ keeping $\lambda$ constant, $q_D$ is decreasing in $\Delta\lambda$ keeping $\lambda$ constant.

We compute the haircut as,

$$\begin{aligned}h &= \beta((z-1)q_D + \lambda z e_L + (1-\lambda)e_H)/E\phi \\ &= \frac{z-1+(2-z-\beta)\,(1-d+\Delta\phi)}{z-(z-1)\,(1-d+\Delta\phi)}.\end{aligned}$$

Notice that

$$\frac{\partial h}{\partial\left(1-d+\Delta\phi\right)} \propto \left(2-z-\beta\right)\left[z-\left(z-1\right)\left(1-d+\Delta\phi\right)\right]$$
$$+\left(z-1\right)\left[z-1+\left(2-z-\beta\right)\left(1-d+\Delta\phi\right)\right]$$
$$=1-\beta z>0.$$

So,

$$\frac{dh}{d\Delta\lambda}=-\frac{\partial h}{\partial(1-d+\Delta\phi)}\frac{d(d-\Delta\phi)}{d\Delta\lambda}>0.$$

Haircut increases in $\Delta\lambda$ keeping $\lambda$ constant.

The repo rate is given by

$$r=\frac{d+\phi_L}{q_D}-1=\frac{1}{1-\zeta}\frac{1-\beta z}{\lambda}\frac{\left(1-\beta\Delta\lambda\right)d}{\beta-\beta\Delta\lambda+\left(1-\beta\right)d}-1.$$

Note that:

$$\frac{\partial}{\partial\Delta\lambda}\left\{\frac{\left(1-\beta\Delta\lambda\right)d}{\beta-\beta\Delta\lambda+\left(1-\beta\right)d}\right\}=\frac{\beta d\left(1-\beta\right)\left(1-d\right)-\frac{\partial d}{\partial\Delta\lambda}\left\{(1-\beta)d+\beta^2\Delta\lambda\left(1-\Delta\lambda\right)\right\}}{\left[\beta-\beta\Delta\lambda+\left(1-\beta\right)d\right]^2}>0.$$

So, finally, $r$ is increasing $\Delta\lambda$ keeping $\lambda$ constant.

# B Robustness and Extensions

## B.1 Unsegmented Security Markets

For simplicity we restrict attention to the iid case. As in the main model a *security design* is a menu of securities backed by the asset and each type chooses a subset of the securities to trade. With unsegmented markets, we can combine all the securities that are traded by a given type. Hence w.l.o.g we can restrict attention to a menu of at most two securities $\left\{y^L,y^H\right\}$ where each security is backed by the asset, that is,

$$y^Q(s,\phi)\leq s+\phi \tag{B.1}$$

for all $s\in[\underline{s},\bar{s}]$. After agent $O$ obtains private information about the asset's quality, he sells only one of the securities from the menu in the security market. Note that the two securities may be identical, in which case the security is traded in a pooling equilibrium. Otherwise, since markets are unsegmented, lenders learn the type of the borrower and securities are traded in a separating equilibrium. Security market is otherwise as in the main model.

The asset price at the end of period $t$, $\phi_t$ is equal to :

$$\phi_t = \beta \left\{ \lambda \left[ \int_{\underline{s}}^{\bar{s}} \left( \left( zq^L_{t+1} - y^L_{t+1}(s, \phi_{t+1}) \right) + (s + \phi_{t+1}) \right) dF_L(s) \right] \right. \\ \left. + (1-\lambda) \left[ \int_{\underline{s}}^{\bar{s}} \left( \left( zq^H_{t+1} - y^H_{t+1}(s, \phi_{t+1}) \right) + (s + \phi_{t+1}) \right) dF_H(s) \right] \right\}, \tag{B.2}$$

where $\beta$ is the discount factor, $0 < \beta < 1/z$.

In the beginning of each period $t$, designer takes the prices, $q^Q_t$ and asset price $\phi_t$ as given to maximize:

$$V_t = \lambda \left[ \int_{\underline{s}}^{\bar{s}} \left( \left( zq^L_{t+1} - y^L_{t+1}(s, \phi_{t+1}) \right) + (s + \phi_{t+1}) \right) dF_L(s) \right] \\ + (1-\lambda) \left[ \int_{\underline{s}}^{\bar{s}} \left( \left( zq^H_{t+1} - y^H_{t+1}(s, \phi_{t+1}) \right) + (s + \phi_{t+1}) \right) dF_H(s) \right] \tag{B.3}$$

subject to the low type and the high type's incentive compatibility constraints:

$$zq^L_{t+1} - E_L y^L_{t+1}(s, \phi_{t+1}) \geq zq^H_{t+1} - E_L y^H_{t+1}(s, \phi_{t+1}) \tag{B.4}$$

$$zq^H_{t+1} - E_H y^H_{t+1}(s, \phi_{t+1}) \geq zq^L_{t+1} - E_H y^L_{t+1}(s, \phi_{t+1}) \tag{B.5}$$

and the high type's participation constraint[47]

$$zq^H_{t+1} - E_H y^H_{t+1}(s, \phi_{t+1}) \geq 0 \tag{B.6}$$

We now define the equilibrium concept in our economy.

**Definition 2.** An equilibrium with security design consists of the asset price $\phi_t$, a security design $\{y^L_t, y^H_t\}$, security prices $q^Q_t$ such that:

1. The price, $q^Q_t$, of security $y^Q_t$ for $Q \in \{L, H\}$ is determined through Bertrand competition in each security market:

$$q^Q_t = \begin{cases} \lambda E_L y^Q_t(s,\phi) + (1-\lambda) E_H y^Q_t(s,\phi) & \text{if } y^L_t = y^H_t \\ E_Q y^Q_t(s,\phi) & \text{if } y^L_t \neq y^H_t \end{cases} \tag{B.7}$$

2. Asset price $\phi$ satisfies (B.2).

3. Security design satisfies (B.1) and maximizes (B.3) among all security designs satisfying the incentive compatibility constraints (B.4) and (B.5) and the participation constraint (B.6).

[47]The low type's IC is automatically satisfied by combining (B.4) and (B.5).

For the rest of this section we restrict attention to threshold securities where

$$y_t^Q(s,\phi) = \begin{cases} s+\phi & \text{if } s \le \delta^Q \\ \delta^Q + \phi & \text{if } s > \delta^Q \end{cases}$$

Next we look at two cases:

**Pooling case:**

In a pooling equilibrium $y_t^L = y_t^H$. Denote the pooling debt threshold by $\delta^P$. Note that incentive compatibility constraint are automatically satisfied. High type's participation constraint is satisfied iff:

$$(z-1)\left(\underline{s} + \phi + \int_{\underline{s}}^{\delta^P} \widetilde{F}_H(s)ds\right) \ge z\lambda\left(\int_{\underline{s}}^{\delta^P} \widetilde{F}_H(s)ds - \int_{\underline{s}}^{\delta^P} \widetilde{F}_L(s)ds\right).$$

In a pooling equilibrium designer's value is:

$$z\left(\underline{s} + \phi + \lambda\int_{\underline{s}}^{\delta^P} \widetilde{F}_L(s)ds + (1-\lambda)\int_{\underline{s}}^{\delta^P} \widetilde{F}_H(s)ds\right) + \lambda\int_{\delta^P(\phi)}^{\overline{s}} \widetilde{F}_L(s)ds + (1-\lambda)\int_{\delta^P(\phi)}^{\overline{s}} \widetilde{F}_H(s)ds$$

**Separating case:**

Clearly, for the low type's incentive compatibility constraint to hold we must have $\delta^L > \delta^H$. The low type's incentive compatibility constraint (B.4) can be written as:

$$z\int_{\underline{s}}^{\delta^L} \widetilde{F}_L(s)ds - \int_{\delta^H}^{\delta^L} \widetilde{F}_L(s)ds \ge z\int_{\underline{s}}^{\delta^H} \widetilde{F}_H(s)ds$$

The high type's incentive compatibility constraint (B.5) can be written as:

$$z\int_{\underline{s}}^{\delta^H} \widetilde{F}_H(s)ds \ge z\int_{\underline{s}}^{\delta^L} \widetilde{F}_L(s)ds - \int_{\delta^H}^{\delta^L} \widetilde{F}_H(s)ds$$

Combining we obtain:

$$z\int_{\underline{s}}^{\delta^L} \widetilde{F}_L(s)ds - \int_{\delta^H}^{\delta^L} \widetilde{F}_L(s)ds \ge z\int_{\underline{s}}^{\delta^H} \widetilde{F}_H(s)ds \ge z\int_{\underline{s}}^{\delta^L} \widetilde{F}_L(s)ds - \int_{\delta^H}^{\delta^L} \widetilde{F}_H(s)ds.$$

Note that designer's payoff is increasing in $\delta^H$. Suppose the first inequality is a strict inequality and the second inequality holds with equality. Then increasing $\delta^H$ slightly, relaxes the second inequality without violating the first one. (This is because derivative of the middle expression, $z\widetilde{F}_H(\delta^H)$, is strictly larger than the derivative of the third expression, $\widetilde{F}_H(\delta^H)$.) Hence the high type's incentive compatibility constraint cannot be binding.

In addition, increasing $\delta^L$ relaxes low type's incentive compatibility constraint. Since designer's payoff increasing in $\delta^L$, we must have have $\delta^L = \bar{s}$. Hence in the separating case we only need to check low type's incentive compatibility constraint:

$$z \int_{\underline{s}}^{\overline{s}} \widetilde{F}_L(s)ds - \int_{\delta^H}^{\overline{s}} \widetilde{F}_L(s)ds \geq z \int_{\underline{s}}^{\delta^H} \widetilde{F}_H(s)ds$$

From the above constraint, we can see that $\delta^H$ does not depend on $\phi$. In a separating equilibrium designer's value is

$$z \left( \phi + \lambda E_L s + (1-\lambda) \left( \underline{s} + \int_{\underline{s}}^{\delta^H} \widetilde{F}_H(s)ds \right) \right) + (1-\lambda) \int_{\delta^H}^{\overline{s}} \widetilde{F}_H(s)ds$$

To summarize: there are two possible types of stationary equilibria. In a pooling equilibrium, both types sell debt and retain equity. In this case, only high type's participation constraint may be binding. In a separating equilibrium, high type sells debt and low type sells the entire equity. In this case, only low type's incentive compatibility constraint may be binding. (It is also possible that none of the constraints are binding and both types sell the entire equity.)

Next we look at which type of design is selected by the security designer. The answer to this question depends on the asset price which itself depends on the chosen design. But before solving for the full equilibrium, we first take asset price as exogenous and see how the chosen design depends on the asset price. Clearly, if none of the constraints are binding at $\delta^H = \delta^P = \bar{s}$, then the designer chooses the pass through security and there is no distinction between pooling and separating cases. So suppose the relevant constraints are binding.

Comparing the designer's payoff in pooling versus separating, we see that there is some threshold $\hat{\delta}$ satisfying $\overline{s} > \hat{\delta} > \delta^H$,such that pooling design generates higher payoff iff $\delta^P \geq \hat{\delta}$. Moreover neither $\hat{\delta}$ nor $\delta^H$ depend on $\phi$. Since $\delta^P$ is increasing in $\phi$, there is some $\hat{\phi}$ such that for $\phi \geq \hat{\phi}$ we have $\delta^P \geq \hat{\delta}$. Hence, pooling design generates a higher payoff iff $\phi \geq \hat{\phi}$.

Now we turn to solving for equilibrium. We consider the two cases separately.

**Separating case:**

Consider the following mapping:

$$\phi' = \beta z \left( \phi + \lambda E_L s + (1-\lambda) \left( \underline{s} + \int_{\underline{s}}^{\delta^H} \widetilde{F}_H(s)ds \right) \right) + \beta (1-\lambda) \int_{\delta^H}^{\overline{s}} \widetilde{F}_H(s)ds. \tag{B.8}$$

Note:

$$\frac{\partial \phi'}{\partial \phi} = \beta z + \beta (1-\lambda) (z-1) \frac{\partial \delta^H}{\partial \phi} \widetilde{F}_H(\delta^H) = \beta z < 1.$$

Hence, there can only be one stationary separating equilibrium where the separating asset price is:

$$\phi^S = \frac{1}{1-\beta z}\left(\beta z\left(\lambda E_L s + (1-\lambda)\left(\underline{s} + \int_{\underline{s}}^{\delta^H} \widetilde{F}_H(s)ds\right)\right) + \beta(1-\lambda)\int_{\delta^H}^{\overline{s}} \widetilde{F}_H(s)ds\right).$$

**Pooling case:**

Consider the following mapping:

$$\phi' = \beta z\left(\underline{s} + \phi + \lambda \int_{\underline{s}}^{\delta^P(\phi)} \widetilde{F}_L(s)ds + (1-\lambda)\int_{\underline{s}}^{\delta^P(\phi)} \widetilde{F}_H(s)ds\right) + \beta\left(\lambda \int_{\delta^P(\phi)}^{\overline{s}} \widetilde{F}_L(s)ds + (1-\lambda)\int_{\delta^P(\phi)}^{\overline{s}} \widetilde{F}_H(s)ds\right) \tag{B.9}$$

where $\delta^P(\phi)$ is the solution to

$$(z-1)\left(\underline{s} + \phi + \int_{\underline{s}}^{\delta^P} \widetilde{F}_H(s)ds\right) = z\lambda\left(\int_{\underline{s}}^{\delta^P} \widetilde{F}_H(s)ds - \int_{\underline{s}}^{\delta^P} \widetilde{F}_L(s)ds\right).$$

Note:

$$\frac{\partial \delta^P}{\partial \phi} = \frac{(z-1)}{\left((z\lambda - z + 1)\widetilde{F}_H(\delta^P) - z\lambda\widetilde{F}_L(\delta^P)\right)}$$

$$\frac{\partial \phi'}{\partial \phi} = \beta z + \beta\frac{(z-1)^2\left(\lambda\widetilde{F}_L(\delta^P) + (1-\lambda)\widetilde{F}_H(\delta^P)\right)}{\left((z\lambda - z + 1)\widetilde{F}_H(\delta^P) - z\lambda\widetilde{F}_L(\delta^P)\right)}$$

$$\frac{\partial^2 \phi'}{\partial \phi^2} = -\beta\lambda(z-1)^2 \frac{f_L(\delta^P)\widetilde{F}_H(\delta^P) - f_H(\delta^P)\widetilde{F}_L(\delta^P)}{\left((z\lambda - z + 1)\widetilde{F}_H(\delta^P) - z\lambda\widetilde{F}_L(\delta^P)\right)^2} < 0.$$

Hence, there can only be one stationary pooling equilibrium where the pooling asset price is:

$$\phi^P(1-\beta z) = \beta z\left(\underline{s}\lambda \int_{\underline{s}}^{\delta^P(\phi^P)} \widetilde{F}_L(s)ds + (1-\lambda)\int_{\underline{s}}^{\delta^P(\phi^P)} \widetilde{F}_H(s)ds\right) + \beta\left(\lambda \int_{\delta^P(\phi^P)}^{\overline{s}} \widetilde{F}_L(s)ds + (1-\lambda)\int_{\delta^P(\phi^P)}^{\overline{s}} \widetilde{F}_H(s)ds\right).$$

Finally, we show that there is in fact a unique equilibrium. If the equilibrium is separating then the asset price is less than $\hat{\phi}$. If the equilibrium is pooling then the asset price is more than $\hat{\phi}$. To see this fact, graph the mappings (B.8) and (B.9) from $\phi$ to $\phi'$. The mapping (B.9) begins below (B.8) and the two intersect at $\phi = \hat{\phi}$. It is clear from the graph that 45-degree line intersects both of these mapping either to the left of $\hat{\phi}$ or to the right of $\hat{\phi}$. If the intersections are to the left of $\hat{\phi}$, by our earlier argument, designer chooses a separating design every period. So, the unique stationary equilibrium is the one where the design is separating every period. If the intersections are to the right of $\hat{\phi}$, designer chooses a pooling design every period. So, the unique stationary equilibrium is the one where the design is pooling every period.

## B.2 Long Term Securities

A long term security in period $t$ is a mapping $y_t : \{t, t+1, \ldots\} \times [\underline{s}, \bar{s}] \to \mathbb{R}_+$ that specifies payments from the borrower to the investor in every period and state. We assume that time $t$ long term security is backed by a time $t-1$ long term security. That is $y_t(\tau, s) \leq y_{t-1}(\tau, s)$ for $\tau \in \{t, t+1, \ldots\}$ and $s \in [\underline{s}, \bar{s}]$. In addition, time 0 security is backed by the long term asset, $y_0(\tau, s) \leq s$ for $\tau \in \{1, 2, \ldots\}$.

Denote the price of security at time $t$ by $q_t^L$. q_{t}^{L} is the amount that the lender pays the borrower for long term security in period $t$. Let's assume that lenders compete for the security (or equivalently, the borrower has all bargaining power.) In that case price of the security at time $t$ is the lender's reservation value of the security which is $q_t^L = Ey_t(t, s) + \beta z q_{t+1}^L$ where $y_t(t, s) \leq y_{t-1}(t, s)$. In the pricing equation, the first term is the expected value of the payment of security $t$ at period $t$, and the second term is the value of the security at the end of period t for a period $t+1$ borrower. Since the borrower would like to raise as much funding as possible $q_t^L = Ey_{t-1}(t, s) + \beta z q_{t+1}^L$. Since in period 0 the borrower would set $y_0(t, s) = s$ we obtain: $q_t^L = Es + \beta z q_{t+1}^L$. In steady state we obtain $q^L = \frac{Es}{1-\beta z}$.

In our model with symmetric information the security price is $q = Ey + \phi = Es + \phi$. Asset price is $\phi = \frac{\beta z}{1-\beta z} Es$. Combining we find $q = \frac{Es}{1-\beta z}$.

Thus, the amount of funding raised is the same with long term securities backed by long term securities versus short term securities backed by the long lived asset's resale price.

## B.3 An Alternative Security Market Microstructure

In this section we introduce security trading through an intermediary that maximizes expected funding in the security market. Fix an arbitrary security $y$. Assume $E_H y > E_L y$. The intermediary posts $(a_Q, q_Q)$ for $Q \in \{L, H\}$. Borrower of type $Q$ sells $a_Q$ units of the security and the investor receives this amount and pays $q_Q$ to the borrower through the intermediary. We assume that the investor breaks even. incentive compatibility constraints require:

$$zq_H - a_H E_H y \geq zq_L - a_L E_H y \tag{B.10}$$

and

$$zq_L - a_L E_L y \geq zq_H - a_H E_L y \tag{B.11}$$

where $a_Q \in [0, 1]$ and $q_Q \leq E_Q y$. High type's incentive to participate requires:

$$zq_H - a_H E_H y \geq 0. \tag{B.12}$$

(Type $L's$ incentive to participate is automatically satisfied.)

Hence the intermediary's problem can be written as:

$$\max_{q_L \in [0, E_L y], q_H \in [0, E_H y]} \lambda q_L + (1-\lambda) q_H$$

subject to (B.13) and (B.12) and lenders break even.

We consider two cases:

Pooling: In this case $q_L = q_H = q$ and $a_L = a_H = a$. incentive compatibility constraints (B.10) and (B.11) are automatically satisfied. Break even constraint implies $q = a(\lambda E_L y + (1-\lambda) E_H y)$. Hence $a = 1$. From (B.12) we obtain:

$$\frac{E_L y}{E_H y} \geq 1 - \frac{z-1}{z\lambda} = \zeta.$$

Separating: In this case $q_L \neq q_H$. Since lenders know the type of the borrower, break even implies $q_Q = a_Q E_Q y$. Let $\sigma(y) = \frac{E_L y}{E_H y}$. Plugging into (B.10) and (B.11) we obtain:

$$(\sigma(y) - z) q_L + (z-1) q_H \geq 0 \tag{B.13}$$

$$-(z-1) q_L + (z - \sigma(y)) q_H \leq 0 \tag{B.14}$$

Solution to the problem is $q_L = E_L y$, $a_L = 1$, $q_H = \frac{\sigma(y)(z-1)}{z-\sigma(y)} E_H y$ and $a_H = \frac{\sigma(y)(z-1)}{z-\sigma(y)} < 1$. Funding raised is $\lambda E_L y + (1-\lambda) \frac{\sigma(y)(z-1)}{z-\sigma(y)} E_H y$ which is strictly less that in the pooling case.

Note that unlike the situation in the main model, in the separating case the high type sells a fraction of the security and retains the rest. Yet, there is still a "discontinuity" at the threshold $\zeta$. When $\sigma(y)$ goes above $\zeta$, there is a discrete drop in the amount of funding the borrower can raise.

### B.3.1 Multiple Equilibria with Equity Claims

Suppose the borrower is restricted to issuing only the equity claim, or a passthrough security, to the collateral asset in the security market. We show that the multiple equilibria in the security market justified by different asset prices can happen under the alternative formulation.

As in the baseline case, we can solve for the asset price in a pooling equilibrium:

$$\begin{aligned} \phi^P &= \beta z(\phi^P + \lambda E_L s + (1-\lambda) E_H s), \\ \phi^P &= \frac{\beta z(\lambda E_L s + (1-\lambda) E_H s)}{1 - \beta z}. \end{aligned} \tag{B.15}$$

The asset price in the separating equilibrium is given by:

$$\phi^S = \beta z \left[ \lambda \left( E_L s + \phi^S \right) + (1-\lambda) \frac{(z-1)\left(E_H s + \phi^S\right)}{z - \frac{E_L s + \phi^S}{E_H s + \phi^S}} \right] + \beta(1-\lambda) \frac{z(E_H s - E_L s)}{z - \frac{E_L s + \phi^S}{E_H s + \phi^S}}.$$

As usual multiple equilibria exist when:

$$\frac{E_L s+\phi^S}{E_H s+\phi^S}<\zeta \leq \left(\frac{E_L s+\phi^P}{E_H s+\phi^P}\right).$$

Although a simple closed for condition is not easy to get, we can show numerically that multiplicity is possible. Suppose $\beta = 0.9$, $z = 1.05$, $\lambda = 0.5$, $E_L s = 1$, $E_H s = 10$. Then $\zeta = 0.9$, $\phi^S = 70.58$ and $\phi^P = 94.5$. Hence,

$$\frac{E_L s+\phi^S}{E_H s+\phi^S} = 0.88 \text{ and } \frac{E_L s+\phi^P}{E_H s+\phi^P} = 0.91$$

and both prices are consistent with equilibria.

## B.4 Nash Bargaining over the Asset Price

Let $\theta$ denote the bargaining power of Agent $O$. Agent $O$ designs the security at the beginning of a period, anticipating bargaining over the ownership of the asset at the end of the period. Denote the resale value of the asset from renegotiation $\phi$.

For simplicity, we focus on a stationary equilibrium. Given the security design $\{y_i(s)\}$. The reservation value of the asset for agent $O$ is

$$V_O = \beta\left[\lambda a_{iL}\left(zq_i - E_L y_i\right) + (1-\lambda)a_{iH}(zq_i - E_H y_i) + Es + \phi\right]$$

Taken $\phi$ as given, the optimal security design is the same as in the main text. The reservation value can then be expressed as

$$V_O = \beta\left\{(z-1)\left[\int_{s_L}^{s_H} (\min(d,s)+\phi)\, d\left(\lambda F_L(s) + (1-\lambda)F_H(s)\right) + \lambda\int_d^{s_H}(s-d)dF_L(s)\right] + Es + \phi\right\}$$

The reservation value of the asset for suppliers is

$$V_S = \beta\left(Es+\phi\right).$$

In Nash bargaining the resale value $\phi$ maximizes:

$$(V_O-\phi)^\theta(\phi - V_S)^{1-\theta}$$

Hence,

$$\begin{aligned}\phi &= \theta V_O + (1-\theta)V_S \\ &= \beta\left\{\theta(z-1)\left[\int_{s_L}^{s_H} (\min(d,s)+\phi)\, d\left(\lambda F_L(s) + (1-\lambda)F_H(s)\right) + \lambda\int_d^{s_H}(s-d)dF_L(s)\right] + Es + \phi\right\}\end{aligned}$$

The asset price is equivalent to that in our main model with a different private return from investment for agent $O$, $\hat{z} = 1-\theta+\theta z$.

## B.5 Online Appendix: Continuous time extension of Corollary 1

Interestingly, the next corollary shows that in the continuous time limit of the i.i.d. asset quality case in Corollary 1, there still exists a region of multiple equilibria, indicating the importance of adverse selection problem at the short horizon. When we take the model to the continuous time limit, we change notations in the following way: denote the duration of a period in the discrete time model $dt$, the dividend payment of the asset ($s$) $sdt$, the gain from trade ($z$) $1 + zdt$, the time discount factor ($\beta$) $1 - \rho dt$, where $\rho > z > 0$. We can derive the condition of multiple equilibria in the continuous time limit by taking equation (17) to the limit.

**Corollary 2.** *In the continuous time limit of $dt \to 0$, multiple equilibria exist whenever*

$$\frac{\rho - z}{\rho + \frac{1-\lambda}{\lambda} z} > \frac{E_L s}{E_H s} \geq \frac{\rho - \frac{1}{\lambda} z}{\rho}. \tag{B.16}$$

Notice that the upper bound in (B.16) is always above the lower bound since $0 \geq -(1-\lambda)z/\lambda^2$. So, there always exists multiple equilibria in the continuous time limit. Intuitively, in the limit, both the gains of trade and the heterogeneity in asset value become smaller, $1 + zdt \to 1$, $(E_L sdt + \phi)/(E_H sdt + \phi) \to 1$. But the heterogeneity in value *relative* to the gains from trade may still be large, so that adverse selection may still be present in the limit.