ISSN 0956-8549-842

Information, Market Power and Welfare

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DISCUSSION PAPER NO 842

PAUL WOOLLEY CENTRE WORKING PAPER No 81

September 2021

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September 12, 2021

Abstract

We study a financial market in which agents with interdependent values bid for a risky asset. Some agents are privately informed of their own value for the asset while others seek to infer it from the equilibrium price. Due to adverse selection, uninformed agents are less willing than the informed to provide liquidity, and engage in greater bid shading when prices are more informative. While increased participation by informed agents leads to perfect competition in the limit, the market remains illiquid to some degree even with free entry of uninformed traders. The incentive to produce information is increasing in market size and is maximal in a perfectly competitive economy. Price informativeness, on the other hand, is independent of market size. Curtailing information production by one group can reduce adverse selection, and improve liquidity and welfare for all agents.

Journal of Economic Literature classification numbers: D82, G14.

Keywords: Double auction, interdependent values, market power, adverse selection, information acquisition, welfare.

*This paper has benefited from the feedback of seminar participants at the 2021 North American Summer Meeting of the Econometric Society and the 2021 China Meeting of the Econometric Society. We would also like to thank Péter Kondor and John Moore for their comments.

1 Introduction

We study a financial market in which agents with interdependent values bid for a risky asset. There are several types of bidders distinguished by their value for the asset, which is uncertain ex ante. For a given type, all bidders have the same value but some are privately informed about it while others are not.

The trading protocol is a multi-unit uniform-price double auction. We present our model in terms of bidding for a financial asset, but it is applicable to many divisible-good settings, such as wholesale electricity, spectrum or emission permits. In a financial market, heterogeneity in values can be due to different hedging or liquidity needs, or because of differing investment opportunities.[1] The auction takes the form of a demand submission game wherein each bidder submits a demand schedule, which is a function of his private signal (if he has one) and the price. Agents take account of their price impact and incorporate the informational content of the price into their bids.

The paper builds on Kyle (1989), who analyzes the common values case, with noise traders to prevent prices from being fully revealing. Vives (2011) and Rostek and Weretka (2012, 2015) study the case of interdependent values in a symmetric economy in which all traders submit the same demand function (i.e. with the same weight on their private signal and on the price). Their analysis precludes heterogeneous outcomes such as when some agents choose to collect information while others do not. The model that we present in this paper admits non-symmetric equilibria, and has novel implications for price discovery, liquidity, information acquisition and welfare.

Formally our setup is as follows. There are several types of agents distinguished by their value for the asset. For type i, this value is a random variable θ_i.[2] There are N_i agents of type i who are privately informed; these agents know θ_i. The remaining agents of type i have no private information and rely on the price to infer information about θ_i. There are no noise traders. We assume that the values $\{\theta_i\}$ are jointly normally distributed, and study linear Bayesian Nash equilibria of a demand submission game.

Price informativeness varies for uninformed agents of different types. A lower price is an indicator of lower value, and this effect is stronger if prices are more informative. Thus higher price informativeness is associated with greater adverse selection, which manifests itself in the form of greater bid shading. Demand functions are more inelastic for agents who learn more from the price and are

[1] Vives (2011) discusses heterogeneous values in Treasury and electricity auctions. In Rostek and Weretka (2012) values depend on group affiliations or on the geographic location of traders. In Goldstein et al. (2021) investors with environmental or social concerns have a different value for the asset than traditional investors. Rahi and Zigrand (2018) show how diversity in values can be microfounded by adding hedgers to a model along the lines of Grossman and Stiglitz (1980) or Hellwig (1980). Rahi (2021) provides examples of interdependent values in a production economy with uncertain cost or demand. Glebkin and Kuong (2021) show how differences in trading speed can account for heterogeneous values.

[2] In our model, the asset value for type i is the sum of two independent random variables, θ_i and η_i. Here we assume that $\eta_i = 0$ for ease of exposition.

upward sloping for those who learn the most.

We analyze the effect of market size, as measured by the number of traders, on information aggregation and competitiveness. First, price informativeness does not depend on the size of the economy per se, but on the relative number of informed agents of different types. If we increase the number of agents of all types without changing their relative proportions, price informativeness is unaffected. Second, price-taking behavior is obtained if there is a large number of agents, but this does not require a large number of all types. What is crucial is that there is a large number of informed agents of at least one type. Interestingly, a large number of uninformed agents does not imply price-taking behavior — strategic trading survives even with free entry of uninformed investors. This is due to adverse selection. Uninformed agents who learn the most from prices have upward sloping demands. These agents use up liquidity rather than providing it. Uninformed agents with downward sloping demands do provide liquidity, but as more of these agents enter the market they shade their bids more, limiting the liquidity that they offer.

One aspect of convergence to competitive equilibrium as the number of informed agents goes to infinity is that market depth goes to infinity (or price impact, which is the reciprocal of depth, goes to zero). Depth and welfare go up monotonically if the number of informed agents of all types goes up in the same proportion.[3] But neither depth nor welfare is monotone in the number of informed agents of a given type. It is possible for an increase in the number of informed agents of some type to make all agents worse off.

Next, we study information acquisition. The incentive to acquire information is lower when the market is imperfectly competitive compared to the perfectly competitive case. With imperfect competition, agents not only have price impact but this impact is greater for informed agents, due to adverse selection which impedes liquidity provision by the uninformed. The adverse selection effect can be so strong that informed agents are worse off relative to the uninformed even if information is costless. The incentive to acquire information increases with market size, though price informativeness does not.

We show that there can be complementarities in information acquisition. If more agents of one type become informed, the value of information production can go up for other types, both because price impact is lower for these agents and because prices are less informative for them.

Finally, we study the social optimality of information production. Curtailing information production by one group can reduce adverse selection and enhance liquidity, making all agents better off.

Related literature

The basic model of noisy rational expectations equilibrium in a financial market where informed and uninformed traders compete in demand functions goes back to Kyle (1989). Kyle finds that the information content of prices is lower with

[3] This is true in a subclass of economies in which there is free entry of uninformed agents of one type.

imperfect competition, and is increasing in the number of uninformed agents.

In recent years, a number of papers have sought to generalize Kyle (1989) to allow for interdependent values. In such a setting, prices are partially revealing without the modeling device of noise traders. Vives (2011) studies the constant correlation case, in which the correlation between values is the same for any pair of bidders. Equilibrium is "privately revealing" in the sense that, for every agent, the price together with his own private information is a sufficient statistic for the information of all agents. Price informativeness is increasing in the number of agents. Rostek and Weretka (2012, 2015) allow for a more general correlation of values. They impose an "equicommonality" assumption, namely that the average correlation between the value of the asset for a bidder and those for the remaining bidders is the same for all bidders. Prices are not privately revealing, and as the number of agents goes up, price informativeness can change in an essentially arbitrary way, depending on how the average correlation between values changes with market size.

In our model, correlations between values can be arbitrary. Price informativeness is typically different for agents of different types. A proportionate increase in the number of informed and uninformed agents of all types leaves price informativeness unchanged for all types. This result is in marked contrast to those in the papers cited above. The reason is that when we consider a larger economy we do not alter its characteristics or the relative weight of different types of traders. When Kyle (1989) compares an imperfectly competitive economy to the corresponding competitive economy, the noise trade is assumed to be the same in both economies. In our setting, the noise from the perspective of one group of agents comes from other groups who have a different motive for trade; the degree of competition affects all groups equally. In Vives (2011), adding more agents to the economy injects more information as well since the new agents possess information that the existing ones do not.[4] This is also true in Rostek and Weretka (2012, 2015), with the additional consideration of correlations between values changing as the market grows in size.

In Vives (2011) and Rostek and Weretka (2012, 2015) the analysis is restricted to symmetric equilibria in which all agents submit the same bid function. In our model, strategies differ across agents depending on their information, including the information that they glean from prices (which differs across traders). This allows us to study the impact of increased market participation by a subset of agents. For example, we find that the liquidity provided by uninformed agents is limited by adverse selection to a greater degree than the liquidity provided by informed agents. In Kyle (1989), more uninformed traders make the market more liquid, thereby stimulating informed trade and making prices more informative. This is not the case in our model. While an increase in the number of uninformed traders does increase liquidity if these traders have downward sloping demands, this is exploited by informed traders of all categories, so that there is no impact on price informativeness; there are no noise traders whose trades are unaffected

[4]The same effect is at play in Kawakami (2017), where the price becomes fully revealing as the number of agents goes to infinity.

by the enhanced liquidity.

Thus our analysis shows that the results in Kyle (1989) on market participation and price informativeness depend crucially on the assumption of exogenous noise trade. While Vives (2011) and Rostek and Weretka (2012, 2015) dispense with noise trade, their results are influenced by considerations that go beyond just market size.[5]

Vives (2011) finds that, in an imperfectly competitive economy, adverse selection increases illiquidity as measured by price impact. We expand on this theme by showing that the illiquidity effect is stronger for those agents who learn less from the price. It is most pronounced for informed agents (who do not learn from the price), thereby reducing their incentive to acquire information in the first place. Due to the private revelation property, the price-taking equilibrium in Vives (2011) is ex post first-best efficient. This is not the case in our setting; our welfare analysis involves a comparison of second-best outcomes.

Vives (2011) analyzes information acquisition (in the online appendix), but due to symmetry restrictions, his model only admits the case where all agents acquire information of the same precision. Kyle (1989) studies an equilibrium in which informed and uninformed agents coexist, but he does not examine its welfare properties.

Glebkin and Kuong (2021) and Manzano and Vives (2021) study variants of the Vives (2011) model that feature some heterogeneity in bid functions, but in settings that are different from ours. In Glebkin and Kuong (2021) there are two types, one of which consists of price-taking agents. Manzano and Vives (2021) have two types that differ in the precision of agents' private signals; the equilibrium is privately revealing as in Vives (2011).

Rostek and Yoon (2020) provide a general overview of the literature on uniform-price double auctions in a linear-normal setting. A number of papers study competitive equilibria with interdependent values, sidestepping the difficulties that arise when agents have market power and act strategically: Vives (2014) is a perfectly competitive version of Vives (2011); Rahi and Zigrand (2018) and Rahi (2021) analyze learning externalities in information production.[6]

The rest of the paper is organized as follows. We introduce the model in the next section and provide an equilibrium existence result in Section 3. In Section 4 we discuss how private information affects liquidity. In Section 5 we present our results on convergence to competitive equilibrium, followed by an analysis of incentives to acquire information in Section 6. In Section 7 we introduce an economy with free entry and show how market participation affects depth and

[5]Later we discuss the precise sense in which the notion of market size in these papers differs from ours (see Proposition 5.5).

[6]Other papers in which agents have interdependent values and equilibrium prices convey information include Bergemann et al. (2021) and Heumann (2021), who introduce multidimensional signals into the Vives (2011) model, Babus and Kondor (2018), in which dealers engage in bilateral trading on a network, Bernhardt and Taub (2015) on learning about common and private values in a duopoly, and Du and Zhu (2017) on the optimal frequency of trading. Kyle et al. (2018) discuss the similarities between a model with interdependent values and one with overconfident traders who agree to disagree.

welfare. We endogenize information acquisition in Section 8. Concluding remarks follow in Section 9. All proofs are in the appendix.

2 The Economy

There is a single risky asset in zero net supply. There are several types of agents distinguished by their value for the asset. Let $L := \{1, \ldots, L\}$ denote both the set and the number of types. The asset value for an agent of type $i \in L$ is $v_i = \theta_i + \eta_i$. Agents of type i may be informed or uninformed; an informed agent privately observes θ_i. The random variables $\{\theta_i, \eta_i\}_{i \in L}$ are jointly normal with mean zero. For each i, η_i is independent of $\theta := (\theta_i)_{i \in L}$, and $\mathrm{Var}(\theta_i)$ is the same for all i. Let R denote the correlation matrix of θ, with ij'th element $\rho_{ij} := \mathrm{corr}(\theta_i, \theta_j)$; in the two-type case we drop the subscripts and write ρ_{12} simply as ρ. We assume that R is positive definite.

The payoff of an agent of type i is $W_i := (v_i - p)q - (k/2)q^2$, where p is the asset price, and q is the number of units of the asset bought by the agent. The scalar k is positive and can be interpreted as an inventory cost parameter or proxy for risk aversion.

The price is determined in a trading game as follows. Each agent submits a demand function that is linear in his private signal (if he has one) and in the price,[7] whereupon the "auctioneer" finds a price at which excess demand is zero, and allocates to each agent the quantity demanded by him at that price. If there are multiple market-clearing prices, the price with the lowest absolute value is chosen (the positive value in case of ties). If there is no market-clearing price, no trade takes place.

For type $i \in L$, the number of informed traders is N_i and the number of informed traders is M_i, with $N_i + M_i \geq 1$ for all i. Let $L_I := \{i \in L | N_i \geq 1\}$ and $L_U := \{i \in L | M_i \geq 1\}$. Thus L_I is the set of types that have at least one informed agent, and L_U is the set of types that have at least one uninformed agent. We will use the notation L_I and L_U to also denote the cardinality of the sets L_I and L_U, respectively. We assume that $2 \leq L_I \leq L$. This rules out equilibria in which the price is fully revealing for some type. We put no restriction on L_U; thus $0 \leq L_U \leq L$. Let $N := \sum_{i \in L} N_i$ and $M := \sum_{i \in L} M_i$. It will sometimes be convenient to use the shorthand notation γ for the vector $(N_i)_{i \in L}$ and ν for the vector $(M_i)_{i \in L}$. All vectors are column vectors by default. We assume that $R_i^\top \gamma \geq 0$ for all i. As we shall see, this is equivalent to $\mathrm{Cov}(\theta_i, p) \geq 0$ for all i.

We denote the demand functions of informed and uninformed agents of type i by $q_i^I(p, \theta_i)$ and $q_i^U(p)$, respectively. Given our linearity assumption, these functions take the form

$$
\begin{aligned}
q_i^I(p, \theta_i) &= \mu_i \theta_i - \alpha_i p, \quad && i \in L_I, \\
q_i^U(p) &= -\beta_i p, && i \in L_U,
\end{aligned}
\tag{1}
$$

[7] As in Kyle (1989), we can consider strategies that are more general but lead to the same linear equilibrium.

for some scalars μ_i, α_i and β_i. Hence, aggregate demand $D(p,\theta)$ is given by

$$\begin{aligned} D(p,\theta) &= \sum_{i\in L_I} N_i q_i^I(p,\theta_i) + \sum_{i\in L_U} M_i q_i^U(p) \\ &= \sum_{i\in L_I} N_i\mu_i\theta_i - \left[\sum_{i\in L_I} N_i\alpha_i + \sum_{i\in L_U} M_i\beta_i\right] p. \end{aligned}$$

Letting

$$\Phi := \sum_{i\in L_I} N_i\alpha_i + \sum_{i\in L_U} M_i\beta_i, \tag{2}$$

we can write

$$D(p,\theta) = \sum_{i\in L_I} N_i\mu_i\theta_i - \Phi p. \tag{3}$$

We assume that $\Phi > 0$; we will verify shortly that this assumption is always satisfied. Thus Φ is the absolute value of the slope of the aggregate demand function (for any given θ). The market-clearing condition is $D(p,\theta) = 0$. If there is an exogenous market order z, the market-clearing condition becomes $\sum_{i\in L_I} N_i\mu_i\theta_i - \Phi p + z = 0$, so that $\Phi = (\partial p/\partial z)^{-1}$. Thus we can interpret Φ as the overall depth of the market, which we will call *market depth*.

Agents behave strategically, taking into account the impact of their bids on the equilibrium price. Given the market-clearing condition, an informed agent of type i understands that if he buys q units of the asset, the equilibrium price is determined by the equation $q + D(p,\theta) - (\mu_i\theta_i - \alpha_i p) = 0$. Hence, from (3), the inverse demand function p_i^I that this agent faces is

$$p_i^I(q) = \phi_i^{-1} q + \phi_i^{-1}\left[\sum_{j\in L_I} N_j\mu_j\theta_j - \mu_i\theta_i\right], \tag{4}$$

where

$$\phi_i = \Phi - \alpha_i. \tag{5}$$

Similarly, the inverse demand function for an uninformed agent of type i is obtained from the equation $q + D(p,\theta) + \beta_i p = 0$. It is given by

$$p_i^U(q) = \delta_i^{-1} q + \delta_i^{-1}\sum_{i\in L_I} N_i\mu_j\theta_i, \tag{6}$$

where

$$\delta_i = \Phi - \beta_i. \tag{7}$$

Thus ϕ_i^{-1} and δ_i^{-1} are the price impact parameters, and ϕ_i and δ_i the corresponding depth parameters, for informed and uninformed agents of type i, respectively. Depth differs across agents depending on their type and on whether they are informed or not. This is because the residual supply function that an agent faces depends on his own contribution to net aggregate demand. We will restrict attention to equilibria at which the depth parameters of informed and uninformed agents of every type are strictly positive.[8]

[8] In fact, depths must be strictly positive at any interior equilibrium (at which the trades of all agents are finite).

Definition 2.1 (Equilibrium) *A profile of demand schedules* $\{\{q_i^I\}_{i\in L_I}, \{q_i^U\}_{i\in L_U}\}$ *is a Bayesian Nash equilibrium of the trading game if* $q_i^I(p,\theta_i)$ *maximizes*

$$\mathbb{E}(W_i|\theta_i, p) = \big[\theta_i - p_i^I(q)\big]q - \frac{k}{2}q^2, \tag{8}$$

and $q_i^U(p)$ *maximizes*

$$\mathbb{E}(W_i|p) = \big[\mathbb{E}(\theta_i|p) - p_i^U(q)\big]q - \frac{k}{2}q^2. \tag{9}$$

Note that market clearing is implicit in this definition and the equilibrium price satisfies $p = p_i^I(q_i^I(p,\theta_i))$ for all $i \in L_I$, and also $p = p_i^U(q_i^U(p))$ for all $i \in L_U$. In the next section we show that there exists a unique equilibrium.

3 Equilibrium

Calculating agents' optimal portfolios, we find that the depth parameter for informed agents is the same for all types. We denote this common depth parameter by ϕ. We begin by characterizing demand functions, equilibrium prices, and market depth in terms of ϕ and $\{\delta_i\}_{i\in L_U}$. For random variables x and y, we denote the covariance of x and y by σ_{xy}, and the variance of x by σ_x^2. Given our assumption that the variance of θ_i is the same for all i, we write $\mathrm{Var}(\theta_i)$ as σ_θ^2.

Proposition 3.1 (Demand functions, price function) *For an economy with depth parameters* ϕ *and* $\{\delta_i\}_{i\in L_U}$*, agents' demand functions are given by*

$$q_i^I = \alpha(\theta_i - p), \qquad i \in L_I, \tag{10}$$

$$q_i^U = \frac{\delta_i}{k\delta_i + 1}\big[\mathbb{E}(\theta_i|p) - p\big] = -\beta_i p, \quad i \in L_U, \tag{11}$$

where

$$\alpha = \Phi - \phi = \frac{\phi}{k\phi + 1}, \tag{12}$$

$$\beta_i = \Phi - \delta_i \tag{13}$$

$$= \frac{\delta_i}{k\delta_i + 1}\left[1 - \frac{\sigma_{\theta_i p}}{\sigma_p^2}\right] \tag{14}$$

$$= \frac{\delta_i}{k\delta_i + 1}\left[1 - \frac{R_i^\top \gamma}{\gamma^\top R\gamma}(k\phi + 2)\right]. \tag{15}$$

The price function is given by

$$p = (k\phi + 2)^{-1}\gamma^\top\theta, \tag{16}$$

and market depth is

$$\Phi = \phi\frac{k\phi + 2}{k\phi + 1}. \tag{17}$$

Comparing (10) with (1), we see that $\mu_i = \alpha_i = \alpha$ for all $i \in L_I$. Demand functions are completely described by the slope parameters α and $\{\beta_i\}_{i\in L_U}$. Note that β_i and δ_i are defined only for $i \in L_U$.

The price function takes a very simple form. It depends only on the number of informed agents across types, $\{N_i\}_{i\in L}$, and on ϕ, the depth parameter of informed agents. We will see shortly that ϕ depends on the correlation matrix R, on $N := \sum_{i\in L} N_i$, and on $\{M_i\}_{i\in L}$. The depth parameters for uninformed agents, $\{\delta_i\}_{i\in L_U}$, do not explicitly appear in the price function. Market depth Φ is positive since ϕ is positive, and is increasing in ϕ. The slope parameter α is positive as well. We will discuss the signs of the slope parameters $\{\beta_i\}_{i\in L_U}$ later.

Proposition 3.1 gives us prices and demand functions in terms of depths. In order to complete our equilibrium characterization, we need to calculate the depths. Substituting for α and Φ in (2) gives us

$$\sum_{i\in L_U} M_i\beta_i = \frac{\phi}{k\phi+1}\big[(k\phi+2)-N\big]. \tag{18}$$

From (13), (15), (17) and (18), we obtain the following system of equations:

$$\sum_{i\in L_U} M_i\left[\phi\frac{k\phi+2}{k\phi+1}-\delta_i\right] = \frac{\phi}{k\phi+1}\big[(k\phi+2)-N\big], \tag{19}$$

$$\phi\frac{k\phi+2}{k\phi+1} = \delta_i\left[\frac{1-\frac{R_i^\top\gamma}{\gamma^\top R\gamma}(k\phi+2)}{k\delta_i+1}+1\right], \quad i=1,\dots,L_U. \tag{20}$$

An equilibrium can then be described in reduced form as a vector of depths $(\phi,\delta_1,\dots,\delta_{L_U}) \in \mathbb{R}_{++}^{L_U+1}$ that solves (19) and (20). This equation system has a simple solution if there are no uninformed traders (the set L_U is empty) and $N \geq 3$. Then we have $k\phi+2 = N$ and hence $p = N^{-1}\gamma^\top\theta$, from (16). The same equilibrium arises if there are uninformed agents but their optimal bids are zero ($\beta_i = 0$ for all $i \in L_U$); we shall see later that this is the case if price informativeness is the same for all types (Proposition 4.3 (iv)). In general, however, there is no closed-form solution.

Proposition 3.2 (Existence) *For any given distribution of agents $\{N_i, M_i\}_{i\in L}$ satisfying $N \geq 3$ and $R_i^\top\gamma/\gamma^\top R\gamma \leq 1/2$ for all $i \in L_U$, there exists a unique equilibrium. It is completely characterized by ϕ; for $i \in L_U$, $\delta_i = g_i(\phi)$, where g_i is a strictly increasing function.*

Given the characteristics of the economy, described by the correlation matrix R and the distribution of agents across types $\{N_i, M_i\}_{i\in L}$, there exists a positive solution ϕ to the equation system (19)–(20). The value of ϕ in turn pins down $\delta_i = g_i(\phi)$ for all $i \in L_U$, and also the market depth Φ, from (17). Uniqueness of equilibrium follows from the specification of the trading game. If there are multiple solutions for ϕ, the trading game stipulates that the highest solution be chosen, since this corresponds to the price with the lowest absolute value (due to (16)). This also corresponds to the highest level of δ_i, for each $i \in L_U$, and of Φ.

Proposition 3.2 requires that $R_i^\top\gamma/\gamma^\top R\gamma \leq 1/2$ for all $i \in L_U$. Sufficient conditions for this to hold can be deduced from the following lemma.[9]

Lemma 3.3 *Suppose one of the following conditions is satisfied: (i) $N_i \geq 2$; (ii) $N_i \geq 1$ and $R \geq 0$; or (iii) $\rho_{ij} = \rho$ for all $i \neq j$. Then $R_i^\top\gamma/\gamma^\top R\gamma \leq 1/2$.*

4 Adverse Selection, Liquidity and Bid Shading

In this section we study the connection between learning from prices and adverse selection. Adverse selection in turn impacts liquidity and bid shading. We use the terms liquidity and depth interchangeably. It will be clear from the context if we are referring to depth for a specific agent type (e.g. ϕ for informed agents or δ_i for uninformed agents of type i) or for the market as a whole (as measured by Φ). We also speak more informally of the elasticity of an agent's demand function as a measure of his willingness to provide liquidity.[10] In our linear setting, bid shading by an agent means that his demand function is less elastic than in a perfectly liquid market with no informational frictions.

Uninformed agents make inferences from the price about their own value. We use the following measure of *price informativeness* for type i:

$$\mathcal{V}_i := \frac{\mathrm{Var}(\theta_i) - \mathrm{Var}(\theta_i|p)}{\mathrm{Var}(\theta_i)}.$$

Given our assumption that $N_i \geq 1$ for at least two types, it follows from the price function (16) that $\mathcal{V}_i \in [0, 1)$; prices are partially revealing for each type. We say that $A \propto B$ if A and B have the same sign ($A = cB$, for some $c > 0$). In the next proposition we collect some results about price informativeness from Rahi and Zigrand (2018).

Proposition 4.1 (Price informativeness) *Given $\gamma := (N_i)_{i\in L}$, price informativeness for type i is*

$$\mathcal{V}_i = \frac{(R_i^\top\gamma)^2}{\gamma^\top R\gamma}. \tag{21}$$

Furthermore,

$$\frac{\partial\mathcal{V}_i}{\partial N_i} \propto R_i^\top\gamma. \tag{22}$$

Given our assumption that $R_i^\top\gamma \geq 0$ for all i, price informativeness for each type is increasing in the number of informed agents of that type. Price informativeness does not depend on the number of uninformed agents of any type. Moreover, $\mathcal{V}_i$ is homogeneous of degree zero in $\gamma := (N_i)_{i\in L}$; if we scale the number of informed

[9] Proposition 3.2 does not require any upper bound on the average correlation as in the existence results in Rostek and Weretka (2012, 2015). This is because we do not require individual demand functions to be downward sloping. Indeed, in our model, some demand functions will typically be upward sloping (see Proposition 4.3).

[10] This is the same notion of liquidity provision as in Glebkin and Kuong (2021).

agents up or down, keeping fixed their relative proportions across types, price informativeness is unaffected.

Since depths and slopes are two sides of the same coin, due to the relation $\beta_i = \Phi - \delta_i$ (equation (13)), we state our results on both and then discuss them together.

Proposition 4.2 (Depths) *The depth parameters ϕ and $\{\delta_i\}_{i \in L_U}$ satisfy the following properties:*

i. $\delta_i \geq \phi$ *for all* $i \in L_U$, *and* $\delta_i = \phi$ *if and only if* $\mathcal{V}_i = 0$.

ii. $\delta_i = \delta_j$ *if and only if* $\mathcal{V}_i = \mathcal{V}_j$, *and* $\delta_i > \delta_j$ *if and only if* $\mathcal{V}_i > \mathcal{V}_j$.

iii. If $M = 0$, *then* $k\phi + 2 = N$. *If* $M \geq 1$, *then* $k\phi + 2 \leq N + M$, *with equality if and only if* $\mathcal{V}_i = 0$ *for all* $i \in L_U$.

Proposition 4.3 (Slopes) *The slope parameters α and $\{\beta_i\}_{i \in L_U}$ satisfy the following properties:*

i. $\alpha > 0$.

ii. $\beta_i \leq \alpha$ *for all* $i \in L_U$, *and* $\beta_i = \alpha$ *if and only if* $\mathcal{V}_i = 0$.

iii. $\beta_i = \beta_j$ *if and only if* $\mathcal{V}_i = \mathcal{V}_j$, *and* $\beta_i < \beta_j$ *if and only if* $\mathcal{V}_i > \mathcal{V}_j$.

iv. Suppose $L_U = L$. *Then* $\beta_i = 0$ *for all* i *if and only if* $\mathcal{V}_i = \mathcal{V}_j$ *for all* i, j.

v. Suppose $L_I = L_U = L$, *and* $\mathcal{V}_i \neq \mathcal{V}_j$ *for some* i, j. *Then,* $\min_{i \in L} \beta_i < 0 < \max_{i \in L} \beta_i$.

In order to interpret these results, it is useful to compare the economy to one in which agents are "naive" in the sense that they ignore the information contained in prices. If an uninformed agent of type i is naive, he behaves as though $\mathcal{V}_i = 0$. The following observation is immediate from Propositions 4.2 and 4.3, and equation (12).

Lemma 4.4 (Naive economy) *If all uninformed agents are naive,* $k\phi + 2 = N + M$. *Furthermore,* $\delta_i = \phi$ *and* $\beta_i = \alpha = \phi/(k\phi + 1)$, *for all* $i \in L_U$.

In an economy with naive agents the slope and depth parameters are the same for all types and the same for informed and uninformed agents. Note that $\alpha < \lim_{\phi \to \infty} \alpha = k^{-1}$, i.e. the common slope parameter is lower than what would arise in a perfectly liquid naive economy (we will discuss competitive equilibrium in detail later; see Proposition 5.1). Thus there is some bid shading in a naive economy due to imperfect competition, but none due to adverse selection.

Now we ask what happens when we introduce adverse selection through learning from prices. All informed agents have the same slope parameter α and the corresponding depth parameter ϕ. But these parameters are lower than in the economy with naive agents; from Proposition 4.2 (iii), $k\phi + 2 < N + M$ if prices are informative for at least one type. The additional bid shading by informed

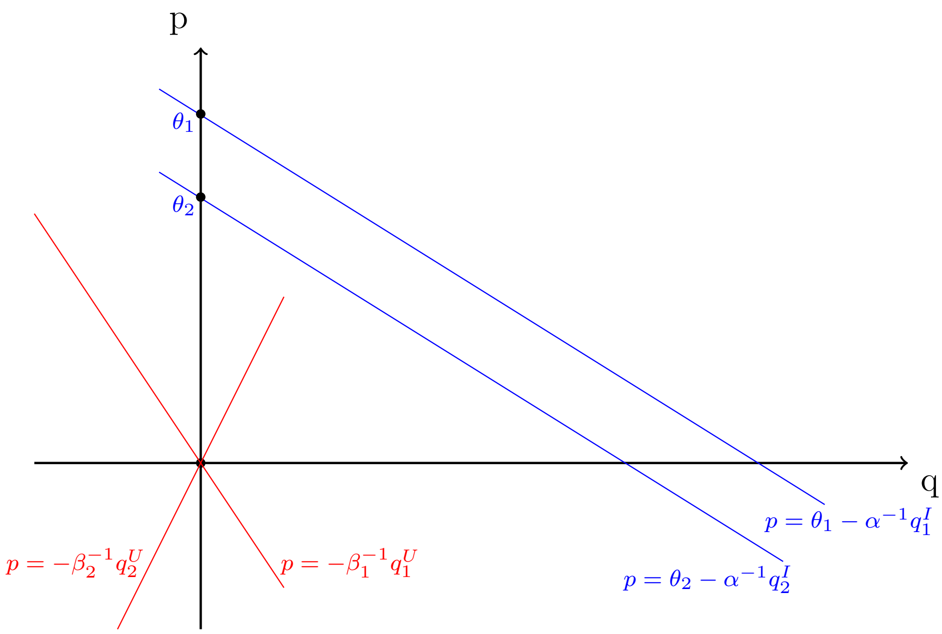

Figure 1: Inverse demand functions

agents, beyond that in the naive economy, is due to lower liquidity provision by uninformed agents. For an uninformed agent, a lower price is bad news about his value for the asset (since $\mathrm{Cov}(\theta_i, p) \propto R_i^\top \gamma \geq 0$ for all i). Learning from prices induces him to reduce his quantity response to a lower price. Indeed, this learning effect can be so strong that an uninformed agent buys less when the price falls. Thus adverse selection induces uninformed agents to shade their bids, the more so the more they learn from prices. This in turn implies that they provide less liquidity to informed agents, so the latter have greater price impact (ϕ is lower).

In Figure 1, we show inverse demand functions for the case of two types, with a nonzero number of informed and uninformed agents of both types (see (10) and (11)). In a naive economy, the blue curves are flatter and the red curves (that pass through the origin) are parallel to the blue curves. When uninformed agents learn from prices, demand becomes more inelastic for all agents but more so for the uninformed. If price informativeness is the same for both types, we have $\beta_1 = \beta_2 = 0$, and demands are perfectly inelastic (at zero quantity) for all uninformed agents. If price informativeness differs for the two types, the demand curve of the less informed type is downward sloping while that of the more informed type is upward sloping.

More generally, suppose that there are informed and uninformed agents of all types. Then the following statements are equivalent: (a) $\mathcal{V}_i > \mathcal{V}_j$, (b) $\delta_i > \delta_j$, and (c) $\beta_i < \beta_j$. Among uninformed agents, those who learn the least from prices have the most elastic demand and contribute the most to liquidity provision. The ones who learn the most have an upward sloping demand curve; these agents use up liquidity instead of providing it. Agents who have the most elastic demand are also those whose counterparties have less elastic, or even upward sloping, demands. As a result, any deviation by the former from their equilibrium demand at any given price requires a greater price adjustment in order for the market to absorb it. Thus uninformed agents for whom price informativeness is the lowest, by virtue of having the most elastic demands also have the greatest price impact, or lowest depth. Conversely, agents who learn the most from prices have upward sloping demands and the least price impact, or highest depth.

From the expression for β_i given by (14), we see that the parameter that measures adverse selection for uninformed agents of type i is the regression coefficient of θ_i on p, given by

$$\psi_i := \frac{\sigma_{\theta_i p}}{\sigma_p^2} = \frac{R_i^\top \gamma}{\gamma^\top R \gamma}(k\phi + 2). \tag{23}$$

We refer to ψ_i as the *price sensitivity* for type i. At a given equilibrium, $\psi_i > \psi_j$ if and only if $\mathcal{V}_i > \mathcal{V}_j$. This allows us to rank depths and slopes by price informativeness.[11] It is worth emphasizing that bid shading and price impact for an agent of type i depends on the sensitivity of θ_i to p, not on how much he knows about θ_i. If $\mathcal{V}_i = 0$, $\beta_i = \alpha$ and $\delta_i = \phi$; an uninformed agent who learns nothing shades his bid no more than an informed agent, and has the same price impact. On the other hand, if $\mathcal{V}_i$ is close to one, an uninformed agent shades his bid more, and has a lower price impact, than an informed agent, even though the two agents have almost the same information in equilibrium.

We have used an economy with naive agents as a benchmark for our economy. Another instructive benchmark is the full-information economy in which all agents of type i observe θ_i, for all i. Let H be the total number of agents, informed or uninformed. If all agents are informed ($N = H$), then $k\phi + 2 = H$, by Proposition 4.2 (iii). If some of the H agents are uninformed but naive, we again have $k\phi + 2 = H$, by Lemma 4.4. On the other hand, if there are uninformed agents who extract information from prices, we have $k\phi + 2 < H$ (using Proposition 4.2 (iii) once again). These observations imply that ϕ, and therefore market depth Φ, is the same in the full-information economy and the naive economy, but is lower if there are some agents who are rational and uninformed. Learning from prices leads to more bid shading, and hence lower market depth, than in an economy with no informational frictions. It is in this sense that adverse selection impacts liquidity in our setting.

5 Convergence to Competitive Equilibrium

In this section we show that the economy converges to a (perfectly) competitive limit as the number of agents grows without bound. Our competitive benchmark is the economy described in Section 2 but with a continuum of informed and uninformed agents of each type, reinterpreting N_i and M_i as the mass (rather than the number) of informed and uninformed agents of type i. Thus an individual agent has zero price impact, or equivalently all depths are infinite. We begin

[11] From (14), β_i is the product of two terms: $(1 - \psi_i)$ which captures the direct learning effect, and $\delta_i/(k\delta_i + 1)$ which reflects the effect of learning on depth. If $\mathcal{V}_i > \mathcal{V}_j$, then the direct learning term is lower for i while the depth term is higher. The former dominates, so that $\beta_i < \beta_j$.

by characterizing the equilibrium of a competitive economy, denoting the price function by $\hat{p}$ and the slope parameters by $\hat{\alpha}$ and $\{\hat{\beta}_i\}_{i\in L_U}$ in order to distinguish them from the price function and slope parameters of the economy in Section 2.

Proposition 5.1 (Competitive equilibrium) *In a competitive economy with the mass of agents given by $\{N_i, M_i\}_{i\in L}$, the price function is*

$$\hat{p} = \lambda^{-1}\gamma^\top \theta, \tag{24}$$

where

$$\lambda := \frac{N + M}{1 + \sum_{i\in L_U} M_i \frac{R_i^\top \gamma}{\gamma^\top R \gamma}}, \tag{25}$$

and the slope parameters are

$$\hat{\alpha} = k^{-1}, \tag{26}$$

$$\hat{\beta}_i = k^{-1}\left[1 - \frac{\sigma_{\theta_i \hat{p}}}{\sigma^2_{\hat{p}}}\right], \qquad i \in L_U. \tag{27}$$

The slope parameters satisfy all the properties in Proposition 4.3.

Comparing (26) with (27), we see that uninformed agents shade their bids when they learn from prices. This is due to adverse selection just as in the imperfectly competitive case. We provide a fuller discussion of this point after Proposition 7.2.

Recall that $\gamma := (N_i)_{i\in L}$ and $\nu := (M_i)_{i\in L}$. It will be convenient to refer to the equilibrium of the imperfectly competitive economy and the equilibrium of the corresponding competitive economy by $\mathcal{E}(\gamma, \nu)$ and $\hat{\mathcal{E}}(\gamma, \nu)$, respectively, where

$$\mathcal{E}(\gamma, \nu) := \Big(p, (\mathcal{V}_i)_{i\in L}, \alpha, \phi, (\beta_i, \delta_i)_{i\in L_U}\Big),$$
$$\hat{\mathcal{E}}(\gamma, \nu) := \Big(\hat{p}, (\mathcal{V}_i)_{i\in L}, \hat{\alpha}, \hat{\phi}, (\hat{\beta}_i, \hat{\delta}_i)_{i\in L_U}\Big).$$

For notational convenience, we suppress the dependence of the parameters describing an equilibrium on (γ, ν). The parameters $\hat{p}, \hat{\alpha}$, $\{\hat{\beta}_i\}_{i\in L_U}$ are given by Proposition 5.1, while $\hat{\phi} = \infty$ and $\hat{\delta}_i = \infty$ for all $i \in L_U$. Price informativeness for each type is the same for both economies; even though p and $\hat{p}$ are not equal, they are both proportional to $\gamma^\top\theta$.[12]

If $M \geq 1$, we define $\delta := M^{-1}\sum_{i\in L_U} M_i\delta_i$; thus δ is the (weighted) average depth parameter for uninformed agents. We parametrize the economy by ξ, $\xi \geq 1$.

Proposition 5.2 (Convergence I) *The equilibrium $\mathcal{E}$ converges to $\hat{\mathcal{E}}$ as the number of agents increases in fixed proportion: $\lim_{\xi\to\infty} \mathcal{E}(\xi\gamma, \xi\nu) = \hat{\mathcal{E}}(\gamma, \nu)$. Furthermore, ϕ and δ are strictly increasing in ξ.*

[12] In Rostek and Weretka (2012, 2015), the equilibrium price function is the same for the imperfectly competitive and the perfectly competitive economy. Since we allow for a non-symmetric equilibrium, this is not the case in our model.

We interpret the original economy as one for which $\xi = 1$. As we increase ξ, the number of informed and uninformed agents of each type goes up, but their relative proportions remain the same. The proposition says that the limiting equilibrium is competitive.[13]

Next, we show that the market becomes perfectly liquid, or infinitely deep, even if we fix the number of uninformed agents of each type, increasing (in fixed proportion across types) only the number of informed agents. Somewhat more surprisingly, the market becomes perfectly liquid even if we only let the number of informed agents of a single type go to infinity, keeping fixed not only the number of uninformed agents of each type, but also the number of informed agents of all other types. The limiting equilibrium in these cases is not the competitive equilibrium described in Proposition 5.1. Rather, it coincides with the corresponding limit of the competitive equilibrium.

Proposition 5.3 (Convergence II) *We have the following convergence results:*

i. $\lim_{\xi\to\infty} \mathcal{E}(\xi\gamma, \nu) = \lim_{\xi\to\infty} \hat{\mathcal{E}}(\xi\gamma, \nu)$. *Furthermore, ϕ and δ are strictly increasing in ξ.*

ii. Suppose $R_\ell \geq 0$. Then, $\lim_{N_\ell\to\infty} \mathcal{E}(\gamma, \nu) = \lim_{N_\ell\to\infty} \hat{\mathcal{E}}(\gamma, \nu)$.

While the market becomes infinitely deep when the number of informed agents (of any type) goes to infinity, this is not the case when the number of uninformed agents becomes large. When we consider the effect of a change in M_i on ϕ and β_i, we write $\phi(M_i)$ and $\beta_i(\phi(M_i))$ to make this dependence explicit. Note that β_i depends on ϕ but not directly on M_i.

Proposition 5.4 *Suppose $N_i \geq 2$ and $R_i^\top \gamma > 0$ for all i. Then:*

i. There exist strictly positive scalars $\underline{\kappa}$ and $\bar{\kappa}$ such that $\{\phi, \delta_1, \ldots, \delta_{L_U}\} \subset [\underline{\kappa}, \bar{\kappa}]$ for all $(M_1, \ldots, M_L) \in \mathbb{R}_+^L$.

ii. $\phi(M_i) - \phi(M_i') \propto \beta_i(\phi(M_i'))$, *for all* $M_i > M_i' \geq 1$.

iii. $\lim_{M_i\to\infty} \beta_i = 0$ *and* $\lim_{M_i\to\infty} M_i\beta_i < \infty$.

Thus the depth parameter ϕ is a bounded function of M_i for all i, and it is also bounded away from zero. These properties are inherited by $\{\delta_i\}_{i\in L_U}$, as well as by market depth Φ, as these are pinned down by ϕ. Any change in ϕ is accompanied by a change in $\{\delta_i\}_{i\in L_U}$ and Φ in the same direction. If β_i is (initially) positive, entry of uninformed agents of type i improves market liquidity, but since β_i converges to zero, the market remains illiquid to some degree (all the depth parameters are bounded) even when entry of these agents is unrestricted. If β_i is (initially) negative, greater market participation by uninformed agents of type i lowers market liquidity; these agents absorb liquidity rather than providing it. In Figure 1, as M_i increases, the inverse demand functions of uninformed

[13] By $p \to \hat{p}$ we mean that p converges to $\hat{p}$ almost surely (we simply check that the coefficients of p converge to the corresponding coefficients of $\hat{p}$).

agents of type i eventually become steeper, converging to the vertical axis as M_i goes to infinity. The relative positions of the inverse demand functions for uninformed agents of different types are the same for all M_i. This is because the slope parameters $\{\beta_j\}_{j \in L_U}$ are ranked by price informativeness (Proposition 4.3 (iii)), which does not depend on M_i.

The last observation about price informativeness being invariant with respect to M_i highlights the point that we made earlier about adverse selection being measured by price sensitivity, given by (23). An increase in M_i leaves price informativeness $\mathcal{V}_i$ unchanged, but it does affect price sensitivity ψ_i through its effect on ϕ. If $\beta_i > 0$, ϕ goes up, increasing ψ_i and driving β_i to zero as M_i grows without bound. While liquidity improves, it is limited by adverse selection. If $\beta_i < 0$, an increase in M_i reduces liquidity due to the upward sloping demands of these agents, even though this is offset to some extent by a reduction in ψ_i.

In the foregoing analysis we have investigated the consequences of increasing market size, where market size is interpreted as $(\gamma, \nu) := (N_i, M_i)_{i \in L}$. Price informativeness does not depend on ν, and it is invariant to any scaling of γ. We can also ask what happens to price informativeness if we think of market size as the number of types L. This is essentially the approach taken by Rostek and Weretka (2012). In their model, $N_i = 1$ and $M_i = 0$ for all i, and θ satisfies the "equicommonality" assumption, which means that the average correlation of θ_i with $\{\theta_j\}_{j \neq i}$ is the same for all i, i.e. $(L-1)^{-1} \sum_{j \neq i} \rho_{ij} = \bar{\rho}$, for all i. They postulate a function $\bar{\rho}(L)$, which describes how the "commonality" parameter $\bar{\rho}$ varies with L. The shape of this commonality function depends on how heterogeneity in values arises (e.g. through differences in geographical location or from group affiliations). The following proposition is the analog of their price informativeness result in our setting. The symbol Δ denotes a change in a variable when the number of types goes up from L to $L+1$.

Proposition 5.5 (Equicommonal auctions) *Suppose θ satisfies the equicommonality assumption, and $N_i = \bar{N} \geq 1$ for all i. Then $\mathcal{V}_i = \bar{\mathcal{V}} := \left[1 + (L-1)\bar{\rho}\right] L^{-1}$, for all i. Given a commonality function $\bar{\rho}(L)$, $\Delta \bar{\mathcal{V}} > 0$ if and only if $\Delta \bar{\rho} > (1-\bar{\rho})/L^2$.*

Under the assumption of equicommonality, and with an equal number of informed agents of each type, price informativeness is the same for all types. It goes up with L provided there is a sufficiently large increase in the commonality parameter $\bar{\rho}$. Unlike the result in Rostek and Weretka (2012), price informativeness falls if $\bar{\rho}$ is constant. This includes what they call the "fundamental value" case ($\rho_{ij} = \rho > 0$, for all $i \neq j$) and the "independent values" case ($\rho_{ij} = 0$, for all $i \neq j$). We obtain a different result because in our setting, there are multiple bidders who share the same value. The price is informative about θ_i because it reflects the bids of informed agents of type i. The bids of types $j \neq i$ cloud this information. Adding another type clouds it even further.

6 Incentives for Information Production

In this section we lay the groundwork for our welfare analysis and analyze the incentives of agents to acquire information. We can calculate ex ante utilities by plugging in the demand function of each agent into his objective function (given by (8) or (9)). In order to interpret the resulting expressions some definitions will be useful. As in Rahi (2021), we define the *gains from trade* for type i by

$$G_i := \frac{\sigma^2_{\theta_i - p}}{\sigma^2_\theta}. \tag{28}$$

Agents of type i have more profitable trading opportunities the greater the distance between their own value θ_i and the market value p. Indeed, if $p = \theta_i$, there are no gains from trade for these agents and their optimal trade is zero. We define the function $F : (0, \infty) \to (0, \infty)$ by

$$F(x) := \frac{x(kx + 2)}{(kx + 1)^2}. \tag{29}$$

It is easy to check that F is strictly increasing. We denote the ex ante utilities of informed and uninformed agents of type i by $\mathcal{U}_i^I$ and $\mathcal{U}_i^U$, respectively.

Lemma 6.1 (Utilities) *Ex ante utilities are given by*

$$\mathcal{U}_i^I = \frac{\sigma^2_\theta}{2} F(\phi) G_i, \qquad i \in L_I, \tag{30}$$

$$\mathcal{U}_i^U = \frac{\sigma^2_\theta}{2} F(\delta_i)\big[G_i - (1 - \mathcal{V}_i)\big], \qquad i \in L_U. \tag{31}$$

Note that $\delta_i \geq \phi$, and hence $F(\delta_i) \geq F(\phi)$, with equality if and only if $\mathcal{V}_i = 0$ (see Proposition 4.2 (i)). Comparing the utilities of informed and uninformed agents of the same type, we see that privileged information is a double-edged sword. If there is no information leakage ($\mathcal{V}_i = 0$), informed agents are unambiguously better off. If prices reveal some information, however, adverse selection kicks in and liquidity (as measured by depth) is lower for the informed. As we shall see below, the adverse impact on liquidity can outweigh the informational advantage of informed agents so that they are worse off relative to the uninformed, even if information is costless. We denote the *utility differential* between the informed and uninformed of type i by $\Delta\mathcal{U}_i := \mathcal{U}_i^I - \mathcal{U}_i^U$, which we can think of as a measure of the incentive to acquire information for agents of type i. Henceforth, when we refer to $\mathcal{U}_i^I$, $\mathcal{U}_j^U$ or $\Delta\mathcal{U}_\ell$, it is understood that $i \in L_I$, $j \in L_U$ and $\ell \in L_I \cap L_U$, respectively. We denote the corresponding competitive equilibrium variables with a "hat".

Proposition 6.2 (Incentives for information production) *For given $\{N_i, M_i\}_{i \in L}$, utility differentials satisfy the following properties:*

i. $\Delta\mathcal{U}_i < \Delta\mathcal{U}_j$ *if and only if* $\mathcal{V}_i > \mathcal{V}_j$.

ii. There exists a threshold level of price informativeness $\mathcal{V}^*$ such that $\Delta\mathcal{U}_i > 0$ if and only if $\mathcal{V}_i < \mathcal{V}^*$.

iii. $\Delta\mathcal{U}_i < \Delta\hat{\mathcal{U}}_i = \sigma_\theta^2(2k)^{-1}(1-\mathcal{V}_i)$.

Parts (i) and (ii) of Proposition 6.2 provide a comparison of utility differentials for different types at a given equilibrium. The incentive to become informed is lower for types with higher price informativeness. In fact, informed agents are worse off relative to uninformed agents of the same type if price informativeness exceeds a certain threshold level. Part (iii) says that the incentive to acquire information is weaker with imperfect competition than with perfect competition. This is because agents not only have price impact when the market is imperfectly competitive, but this impact is greater for informed agents. In a competitive economy, agents can trade in an infinitely deep market with no price impact, and information always has positive value (though this value declines with price informativeness).

We now provide an example that shows that informed agents may be worse off compared to the uninformed due to adverse selection, as suggested by our discussion of Lemma 6.1 and by Proposition 6.2 (ii).[14]

Example 6.1 Suppose there are three types, with $N_1 \geq 2$, $N_2 = N_3 \geq 2$, $M_i \geq 1$ for all i, and

$$\theta_1 = \tilde{\theta}_1, \quad \theta_2 = a\tilde{\theta}_2 + \epsilon_1, \quad \theta_3 = -a\tilde{\theta}_2 + \epsilon_2,$$

where $\{\tilde{\theta}_1, \tilde{\theta}_2, \epsilon_1, \epsilon_2\}$ are mutually independent, $0 < a < 1$, and

$$\sigma^2_{\tilde{\theta}_1} = \sigma^2_{\tilde{\theta}_2} = 1, \quad \sigma^2_{\epsilon_1} = \sigma^2_{\epsilon_2} = \sigma^2_\epsilon, \quad a^2 + \sigma^2_\epsilon = 1.$$

Hence $\rho_{12} = \rho_{13} = 0$ and $\rho_{23} = -a^2$, so that $R_1^\top\gamma = N_1$ and $R_2^\top\gamma = R_3^\top\gamma = N_2(1-a^2)$. By Proposition 3.2, there exists a unique equilibrium (the assumption that $N_i \geq 2$ ensures that $R_i^\top\gamma/\gamma^\top R\gamma \leq 1/2$, by Lemma 3.3).

We consider limits as σ^2_ϵ goes to zero, and hence a goes to 1. From (21), we see that

$$\mathcal{V}_1 = \frac{(R_1^\top\gamma)^2}{\gamma^\top R\gamma} = \frac{N_1^2}{N_1^2 + 2N_2^2(1-a^2)},$$

which converges to 1 as σ^2_ϵ goes to 0. Thus, in the limit, prices become perfectly informative for type 1. We claim that gains from trade for type 1 do not vanish, however, i.e. $\lim_{\sigma^2_\epsilon\to 0} G_1 > 0$. From (16) and (28),

$$G_i = 1 + \frac{\sigma_p^2}{\sigma_\theta^2} - 2\frac{\sigma_{\theta_i p}}{\sigma_\theta^2} = 1 + \frac{\gamma^\top R\gamma}{(k\phi+2)^2} - 2\frac{R_i^\top\gamma}{k\phi+2}. \tag{32}$$

It is straightforward to check that if $\lim_{\sigma^2_\epsilon\to 0} G_1 = 0$, then $\lim_{\sigma^2_\epsilon\to 0}(k\phi+2) = N_1$, implying from (15) that $\lim_{\sigma^2_\epsilon\to 0}\beta_i \geq 0$ for all i. Hence, from (18), $\lim_{\sigma^2_\epsilon\to 0}(k\phi + 2) \geq N > N_1$, a contradiction.

[14]This result should be distinguished from the Hirshleifer effect (Hirshleifer (1971)), which refers to a welfare loss due to more information. Here we are comparing the welfare of informed and uninformed agents rather than providing a comparative static with respect to information. Our result is due to an adverse depth effect; it does not arise in a competitive economy.

Recall that $\delta_i \geq \phi$, with equality if and only if $\mathcal{V}_i = 0$ (Proposition 4.2 (i)). Since $\lim_{\sigma_\epsilon^2 \to 0} \mathcal{V}_1 = 1$, we must have $\lim_{\sigma_\epsilon^2 \to 0} \delta_1 > \lim_{\sigma_\epsilon^2 \to 0} \phi$. Therefore, from Lemma 6.1, $\Delta\mathcal{U}_1 < 0$ for sufficiently small σ_ϵ^2. The informed have lower ex ante utility than the uninformed because the informed have greater price impact even as their informational advantage vanishes. ||

7 Market Size, Depth and Welfare

For the remainder of the paper we focus on a limiting case of our economy in which there is free entry of uninformed agents of one type. This is reminiscent of the analysis in Kyle (1989) of free entry of uninformed speculators, but with interdependent rather than common values, and no noise traders. As in Kyle (1989), we use the limiting economy to study information acquisition. But we also go beyond Kyle (1989) in providing a number of welfare results. In this section we investigate the impact of market size on depth and welfare. In the next section we endogenize information acquisition and address the question of whether incentives to collect information are aligned with social objectives.

Definition 7.1 ($\mathcal{F}_\ell$-economy) *Suppose $N_\ell \geq 2$ for some $\ell \in L_I$, and $R_i^\top \gamma > 0$ for all $i \in L_I$. Then we refer to the limiting economy as $M_\ell \to \infty$ as an $\mathcal{F}_\ell$-economy.*

The symbol $\mathcal{F}$ serves as a mnemonic for "free entry", and the subscript ℓ indicates that there is free entry of uninformed agents of type ℓ.[15]

Lemma 7.1 *An $\mathcal{F}_\ell$-economy has a unique equilibrium with $\phi > 0$ and $\beta_\ell = 0$. The equilibrium price is given by*

$$p = \mathbb{E}(\theta_\ell | p) = \frac{R_\ell^\top \gamma}{\gamma^\top R \gamma} \gamma^\top \theta, \tag{33}$$

and ϕ solves

$$\psi_\ell = \frac{R_\ell^\top \gamma}{\gamma^\top R \gamma}(k\phi + 2) = 1. \tag{34}$$

Free entry of uninformed agents of type ℓ wipes out their trading profits. In the limiting economy, $\beta_\ell = 0$. Thus each uninformed agent of type ℓ trades a zero amount and his equilibrium utility $\mathcal{U}_\ell^U$ is zero. From (11), the equilibrium price is given by $p = \mathbb{E}(\theta_\ell | p)$, and hence the regression coefficient of θ_ℓ on p, which is equal to the price sensitivity ψ_ℓ defined in (23), is equal to 1. Equation (34) can be interpreted as a "zero-profit" condition.

Uninformed agents of type ℓ enter the market as long as there are profits to be made. If $\psi_\ell < 1$, then $\beta_\ell > 0$ (see equation (15)), i.e. their demand functions are downward sloping. As more of them enter, they provide higher liquidity to other traders. On the hand, if $\psi_\ell > 1$, then $\beta_\ell < 0$. In this case, entry of uninformed

[15]Note the joint restrictions on R and $\{N_i\}_{i \in L_I}$ implied by the assumption that $R_i^\top \gamma > 0$ for all $i \in L_I$. For example, in the two-type case, we require that $\rho > -\min\{N_2/N_1, N_1/N_2\}$.

agents of type ℓ drains liquidity from the market. An equilibrium with free entry is established when ϕ satisfies (34). An $\mathcal{F}_\ell$-economy is tractable since we can obtain a closed-form solution for ϕ, and moreover because ϕ does not depend on the number of uninformed agents of any type (apart from the condition that $M_\ell \to \infty$).

For an $\mathcal{F}_\ell$-economy, we can strengthen Proposition 5.2 on convergence to competitive equilibrium to monotone convergence (Proposition 5.2 shows monotone convergence for ϕ and $\delta := M^{-1}\sum_{i\in L_U} M_i\delta_i$, but not for the individual depths $\{\delta_i\}_{i\in L_U}$ or for agents' utilities). As in the general case, we distinguish variables for a competitive $\mathcal{F}_\ell$-economy with a "hat". The competitive depth parameters are equal to infinity.

Proposition 7.2 (Monotone convergence) *Consider an $\mathcal{F}_\ell$-economy parametrized by $\xi\gamma$, $\xi \geq 1$. The price function $p(\xi\gamma)$ and the vector*

$$\Xi(\xi\gamma) := \Big(\alpha, \phi, (|\beta_i|, \delta_i)_{i\in L_U}, (\mathcal{U}_i^I)_{i\in L_I}, (\mathcal{U}_i^U)_{i\in L_U}\Big)$$

do not depend on M_i, $i \neq \ell$. We have $p(\xi\gamma) = p(\gamma) = \hat{p}(\gamma)$, $\Xi(\xi\gamma)$ is increasing in ξ, and $\lim_{\xi\to\infty}\Xi(\xi\gamma) = \hat{\Xi}(\gamma)$.[16] *Furthermore, if $N_\ell \geq 4$, then $\Delta\mathcal{U}_i(\xi\gamma)$ is strictly increasing in ξ, for all $i \in L_I \cap L_U$.*

As we mentioned earlier, ϕ does not depend on M_i, $i \neq \ell$, which is a consequence of the zero-profit condition (34). Hence the same property holds for p and Ξ. If a change in M_i, $i \neq \ell$, disturbs (34), more uninformed agents of type ℓ enter until (34) is restored.

As the number of informed agents of all types goes up in the same proportion, the price function remains unchanged, the depth parameters increase monotonically to infinity, and demand functions become more responsive to the price. Thus increased competition leads to lower price impact but has no effect on prices. All agents are better off as a result: utilities increase monotonically, converging to their competitive equilibrium values. The utility of the informed increases at a faster rate than that of the uninformed, so that the incentive to acquire information increases with market size. These welfare effects are driven entirely by depth; since p does not depend on ξ, gains from trade and price informativeness are not affected by market size.

An $\mathcal{F}_\ell$-economy provides a useful way to understand the connection between bid shading, adverse selection and liquidity. Consider first the case of perfect competition. Then agents can trade with no price impact and bid shading is purely a consequence of adverse selection. Comparing (26) and (27), uninformed agents shade their bids if they learn from prices and agents who learn the most have upward sloping demands. The magnitude of bid shading for type i is measured by $\sigma_{\theta_i\hat{p}}/\sigma^2_{\hat{p}}$. In the case of imperfect competition, we can compare (12) and (14). Since $p = \hat{p}$, we see that the adverse selection component of bid shading, which is measured by $\sigma_{\theta_i p}/\sigma^2_p$ for type i, is the same as in the corresponding competitive economy. But there is an additional effect due to price impact, for both

[16] If $\beta_j = 0$ (this must be the case for $j = \ell$), then $\beta_j = \hat{\beta}_j = 0$ and $\mathcal{U}_j^U = \hat{\mathcal{U}}_j^U = 0$ for all ξ. Monotonicity in ξ is strict for all other elements of Ξ.

informed and uninformed agents. This depth effect reduces α and $|\beta_i|$ below their competitive levels.

While depth and welfare increase monotonically if the number of informed agents goes up in the same proportion for all types, they are not in general monotone in the number of informed agents of a given type.[17]

Proposition 7.3 (Depth) *In an $\mathcal{F}_\ell$-economy,*

i. *ϕ is strictly convex in N_i for all $i \in L_I$.*

ii. *$\partial\phi/\partial N_\ell > 0$ if and only if $\mathcal{V}_\ell > 1/2$.*

iii. *Consider $i \in L_I$, $i \neq \ell$. Suppose $\rho_{mj} = \rho$ for all $m \neq j$, and $N_\ell + N_i \geq N_j$ for all $j \neq \ell, i$. Then $\partial\phi/\partial N_i > 0$.*[18]

The relationship between ϕ and N_ℓ is U-shaped. For given $\{N_j\}_{j\in L}$, ϕ is pinned down by the zero-profit condition (34). A simple calculation shows that, keeping ϕ fixed, price sensitivity ψ_ℓ is increasing in N_ℓ if and only if $\mathcal{V}_\ell < 1/2$. Hence, for low values of N_ℓ, an increase in N_ℓ leads to a higher ψ_ℓ, inducing uninformed agents of type ℓ to tilt their demand functions so that they are upward sloping ($\beta_\ell < 0$; see (15)). More uninformed agents enter, reducing ϕ (due to their upward sloping demands; see Proposition 5.4 (ii)), and thereby restoring the zero-profit condition. For large values of N_ℓ, ψ_ℓ is decreasing in N_ℓ (for fixed ϕ). In this case, β_ℓ becomes positive, and entry of uniformed agents raises ϕ. Thus the U-shaped relationship between ϕ and N_ℓ is attributable to the hump-shaped relationship between adverse selection, as measured by ψ_ℓ, and N_ℓ.

From Lemma 6.1, welfare depends on depth and gains from trade. On the latter, we have the following result:

Lemma 7.4 (Gains from trade) *Consider an $\mathcal{F}_\ell$-economy. Suppose $\rho_{ij} = \rho$ for all $i \neq j$. Then*

$$G_i = (1 - \mathcal{V}_i) + \frac{(1-\rho)^2(N_i - N_\ell)^2}{\gamma^\top R\gamma}. \tag{35}$$

In particular,

$$G_\ell = 1 - \mathcal{V}_\ell. \tag{36}$$

From (28), G_i is a measure of the distance between θ_i and p. The first term of (35) indicates that this distance is inversely related to price informativeness. Indeed, it is intuitive to think of price informativeness for type i as being high when p is close to θ_i and hence G_i is low. This intuition is correct for type ℓ. For other types, however, it is incomplete, and we need to take account of the second term in (35). This term captures the distance between θ_i and p in terms of the distance

[17] Rostek and Weretka (2015) show that market participation can have an arbitrary effect on depth and welfare depending on how the average correlation between values changes with market size. This is a different comparative statics exercise from ours; see the discussion of Proposition 5.5.

[18] In the two-type case, $\partial\phi/\partial N_i > 0$ for $i \neq \ell$, since the condition on N_i is trivially satisfied.

between N_i and N_ℓ, the number N_ℓ being key in determining the equilibrium price given by (33).

We now bring together our results on depth and gains from trade to show that an increase in N_ℓ can make all agents worse off. The following proposition is for the case of two types; under stronger assumptions, it can be generalized to arbitrarily many types.

Proposition 7.5 (Welfare) *Consider an $\mathcal{F}_2$-economy with two types. Suppose $\rho \leq 1/2$ and $N_2 \leq N_1/3$. Then $\mathcal{U}_2^U = 0$ for all N_2, and the utility of all other agents is strictly decreasing in N_2.*

Under the conditions of the proposition, an increase in N_2 leads to lower welfare for all agents (other than the uninformed of type 2 whose utility is zero for any level of N_2 due to free entry). This is a consequence of lower depth as well as lower gains from trade. The depth effect comes from the downward sloping part of the U-shaped relationship between ϕ and N_ℓ discussed earlier. Gains from trade for type 2 are lower because $G_2 = 1 - \mathcal{V}_2$ and $\mathcal{V}_2$ is increasing in N_2. For type 1, price informativeness falls as N_2 increases but, for low values of N_2 relative to N_1, gains from trade are nevertheless lower due to the second term in (35).

8 Information Acquisition

In order to study information acquisition we restrict ourselves to an $\mathcal{F}_\ell$-economy, i.e. an economy with free entry of uninformed agents of type ℓ, and we ignore integer constraints. An agent of type i can choose to become informed by paying a fixed positive cost c_i. We first investigate the value of information for agents of type ℓ, keeping fixed the number of informed and uninformed agents of all other types. Since $\mathcal{U}_\ell^U = 0$, the utility differential $\Delta\mathcal{U}_\ell$ is equal to $\mathcal{U}_\ell^I$.

Lemma 8.1 *In an $\mathcal{F}_\ell$-economy,*

$$\Delta\mathcal{U}_\ell = \mathcal{U}_\ell^I = \frac{\sigma_\theta^2}{2} F(\phi)(1 - \mathcal{V}_\ell). \tag{37}$$

If $N_\ell \geq 3$, we have $\partial(\Delta\mathcal{U}_\ell)/\partial N_\ell < 0$, and $\lim_{N_\ell \to \infty} \Delta\mathcal{U}_\ell = 0$.

Equation (37) is immediate from (30) and (36). Recalling that F is an increasing function, we see that informed agents of type ℓ are better off if they have lower price impact (higher ϕ) and if less of their private information leaks through prices (lower $\mathcal{V}_\ell$). An increase in N_ℓ leads to a higher $\mathcal{V}_\ell$ (see (22)), while the effect on depth is U-shaped (Proposition 7.3 (ii)). However, the first effect dominates if $N_\ell \geq 3$, so that $\Delta\mathcal{U}_\ell$ declines as more agents become informed.

Let N_ℓ^e be the equilibrium number of informed agents of type ℓ, and $\hat{N}_\ell^e$ the corresponding number when the market is perfectly competitive.[19]

[19] Strictly speaking, $\hat{N}_\ell^e$ is the mass of informed agents of type ℓ, but we can think of it as a number in order to compare it to N_ℓ^e, with the proviso that these agents act as though they have no price impact.

Proposition 8.2 (Information acquisition) *In an $\mathcal{F}_\ell$-economy, there exists a scalar $\bar{c} > 0$ such that for any $\{N_i\}_{i \in L \setminus \{\ell\}}$ and $c_\ell < \bar{c}$, there is a unique $N_\ell^e \geq 3$. Furthermore, $N_\ell^e < \hat{N}_\ell^e$, $\partial N_\ell^e / \partial c_\ell < 0$ and $\lim_{c_\ell \to 0} N_\ell^e = \infty$.*

The proposition says that for a given cost c_ℓ, where c_ℓ is low enough to allow at least three agents to profitably collect information, there is a unique N_ℓ^e that is decreasing in c_ℓ. Moreover, in comparison to the corresponding perfectly competitive economy, the number of informed agents is lower because the incentive to acquire information is weaker (Proposition 6.2 (iii)).

Our next task is to investigate how N_ℓ^e is affected by the number of informed agents of other types (the number of uninformed agents has no impact).

Proposition 8.3 (Complementarities) *Consider an $\mathcal{F}_\ell$-economy. Suppose $c_\ell < \bar{c}$. Then we have the following results for $i \in L_I$, $i \neq \ell$:*

i. Suppose $R_i \geq 0$. Then $\lim_{N_i \to \infty} N_\ell^e = \infty$.

ii. Suppose $\rho_{mj} = \rho$ for all $m \neq j$, and $N_\ell + N_i \geq N_j$ for all $j \neq \ell, i$. Then $\partial \phi / \partial N_i > 0$, $\partial \mathcal{V}_\ell / \partial N_i < 0$, and $\partial N_\ell^e / \partial N_i > 0$.

The result says that there is a complementarity in information acquisition. A large number of agents of type ℓ collect information if there are many informed agents of some other type. In the constant correlation case, we get a stronger result, provided $N_\ell + N_i$ exceeds the number of agents of any other type (this condition is vacuously satisfied if $L_I = 2$): a higher N_i induces more agents of type ℓ to acquire information. The reason is two-fold. A higher N_i implies a higher depth ϕ. It also implies that price informativeness $\mathcal{V}_\ell$ is lower, and hence there is less leakage of private information through prices for type ℓ agents.

We now consider an $\mathcal{F}_2$-economy with two types, and study the welfare properties of equilibrium with endogenous information acquisition by both types. Let H_1 be the number of agents of type 1. Thus $H_1 = N_1 + M_1$ and $N_1 \leq H_1$. There is no upper bound on N_2 since there is free entry of uninformed agents of type 2 and any number of these agents can choose to become informed. Let (N_1^e, N_2^e) be an equilibrium allocation of information. In order to evaluate the welfare properties of this equilibrium, we consider a hypothetical planner who can choose an alternative allocation of information (N_1^*, N_2^*), with informed agents of type i paying c_i. Any such allocation chosen by the planner gives rise to a unique equilibrium of the demand submission game, by Lemma 7.1. When we say that an equilibrium is Pareto inefficient, we mean that the planner can find an allocation of information which leads to a Pareto improvement.

Proposition 8.4 (Inefficient information acquisition) *Consider an $\mathcal{F}_2$-economy with two types and an equilibrium allocation of information (N_1^e, N_2^e). Suppose $\rho \leq 1/2$ and $N_2^e \leq H_1/3$. Then the equilibrium is Pareto inefficient. If $N_1^e < H_1$, scaling up (N_1, N_2) is Pareto improving. If $N_1^e = H_1$, a Pareto improvement can be achieved by lowering N_2.*

If $N_1^e < H_1$, increasing the number of informed agents of both types is welfare improving by Proposition 7.2. For the case of $N_1^e = H_1$, the result follows from Proposition 7.5, and the observation that $\mathcal{U}_2^I - c_2 = \mathcal{U}_2^U = 0$, which implies that the agents of type 2 who switch from informed to uninformed as a consequence of lowering N_2 are no worse off. Given the monotonic relationship between c_2 and N_2 (Proposition 8.2), the condition $N_2^e \leq H_1/3$ is equivalent to c_2 being higher than some cutoff value.

9 Conclusion

In this paper we analyze an imperfectly competitive asset market with interdependent values. Some agents are privately informed of their own value for the asset while others extract information about their value from the equilibrium price. Learning from prices is the conduit through which adverse selection affects trading behavior. Agents who learn more engage in greater bid shading, thereby limiting the liquidity that they provide to others. While a large number of informed traders leads to perfect competition, markets remain illiquid to some degree even with free entry of uninformed traders. An increase in the number of informed traders in one sector of the economy can be Pareto worsening. While the incentive to acquire information is lower with imperfect competition, there may nevertheless be excessive information production in equilibrium.

Appendix: Proofs

Proof of Proposition 3.1 Using (4) and (8), the first-order condition for an informed agent of type i is $\theta_i - p_i^I(q) - \phi_i^{-1}q - kq = 0$. The second-order condition, $k + 2\phi_i^{-1} > 0$, is satisfied. Noting that $p_i^I(q_i^I) = p$, we obtain the optimal portfolio:

$$q_i^I = \frac{\phi_i}{k\phi_i + 1}(\theta_i - p). \tag{38}$$

Comparing this expression for q_i^I with (1), we see that $\mu_i = \alpha_i = \phi_i/(k\phi_i + 1)$. From (5),

$$\Phi = \phi_i + \alpha_i = \phi_i + \frac{\phi_i}{k\phi_i + 1}.$$

Since the right-hand side of this equation is increasing in ϕ_i, and is equal to the same value Φ for all i, ϕ_i must be the same for all i, and so must α_i. Letting $\phi_i = \phi$ and $\alpha_i = \alpha$ for all $i \in L_I$ gives us (12), from which (17) also follows.

Similarly, using (6) and (9), we can derive the optimal portfolio for an uninformed agent of type i:

$$q_i^U = \frac{\delta_i}{k\delta_i + 1}\left[\mathbb{E}(\theta_i|p) - p\right] = -\frac{\delta_i}{k\delta_i + 1}\left[1 - \frac{\sigma_{\theta_i p}}{\sigma_p^2}\right]p, \tag{39}$$

thus establishing (14). Equation (13) follows from (7). Using the market-clearing condition $D(p, \theta) = 0$, and noting that $\mu_i = \alpha$, we have $p = \Phi^{-1}\alpha \sum_{i \in L_I} N_i\theta_i =$

$\Phi^{-1}\alpha\gamma^\top\theta$. The price function given by (16) now follows from (12) and (17). Using this price function, we have $\sigma_{\theta_i p} = \sigma_\theta^2(k\phi+2)^{-1}R_i^\top\gamma$, and $\sigma_p^2 = \sigma_\theta^2(k\phi+2)^{-2}\gamma^\top R\gamma$, so that $\sigma_{\theta_i p}/\sigma_p^2 = (R_i^\top\gamma/\gamma^\top R\gamma)(k\phi+2)$. Substituting this expression into (14) gives us (15). □

Proof of Proposition 3.2 Equation (20) can be written as

$$k\delta_i^2 + b_i\delta_i - \phi\frac{k\phi+2}{k\phi+1} = 0, \tag{40}$$

where

$$b_i := 2 - \frac{R_i^\top\gamma}{\gamma^\top R\gamma}(k\phi+2) - k\phi\frac{k\phi+2}{k\phi+1}, \tag{41}$$

for $i \in L_U$. Since ϕ and $\{\delta_i\}_{i\in L_U}$ must be strictly positive, the only admissible solution to (40) is

$$\delta_i = g_i(\phi) := \frac{-b_i + \sqrt{b_i^2 + 4k\phi\frac{k\phi+2}{k\phi+1}}}{2k}. \tag{42}$$

Substituting for δ_i in (19), we get an equation which involves only the variable ϕ:

$$f(\phi) := \frac{\phi}{k\phi+1}\left[(k\phi+2) - N\right] + \sum_{i\in L_U} M_i\left[g_i(\phi) - \phi\frac{k\phi+2}{k\phi+1}\right] = 0. \tag{43}$$

We have

$$g_i(0) = 0, \tag{44}$$

$$g_i'(0) = \left[1 - \frac{R_i^\top\gamma}{\gamma^\top R\gamma}\right]^{-1}, \tag{45}$$

$$\lim_{\phi\to\infty}\frac{g_i(\phi)}{\phi} = 1 + \frac{R_i^\top\gamma}{\gamma^\top R\gamma}, \tag{46}$$

and consequently, $f(0) = 0$, and

$$f'(0) = -(N-2) + \sum_{i\in L_U} M_i\left[\left(1 - \frac{R_i^\top\gamma}{\gamma^\top R\gamma}\right)^{-1} - 2\right],$$

$$\lim_{\phi\to\infty}\frac{f(\phi)}{\phi} = 1 + \sum_{i\in L_U} M_i\frac{R_i^\top\gamma}{\gamma^\top R\gamma}.$$

Since $\lim_{\phi\to\infty} f(\phi)/\phi > 0$, we have $\lim_{\phi\to\infty} f(\phi) = \infty$. Moreover, since $R_i^\top\gamma/\gamma^\top R\gamma \leq 1/2$ for all $i \in L_U$, and $N \geq 3$,

$$f'(0) \leq -(N-2) + \sum_{i\in L_U} M_i\left[\left(1 - \frac{1}{2}\right)^{-1} - 2\right] = -(N-2) < 0.$$

Therefore, by the continuity of f, there exists $\phi > 0$ such that $f(\phi) = 0$. Substituting this ϕ into (42), we get a positive solution $g_i(\phi)$ for δ_i, $i \in L_U$. It is apparent from (40) that g_i is strictly increasing in ϕ. $\square$

Proof of Lemma 3.3 We have

$$\frac{R_i^\top \gamma}{\gamma^\top R \gamma} = \frac{R_i^\top \gamma}{N_i R_i^\top \gamma + \sum_{j \neq i} N_j R_j^\top \gamma}. \tag{47}$$

We will invoke our standing assumptions that $L_I \geq 2$, and $R_j^\top \gamma \geq 0$ for all j.

Condition (i): The sufficiency of this condition is immediate from (47).

Condition (ii): If $N_i \geq 2$, condition (i) applies. If $N_i = 1$, and all correlations are nonnegative, we have

$$\begin{aligned}
\frac{R_i^\top \gamma}{\gamma^\top R \gamma} &= \frac{1 + \sum_{j \neq i} \rho_{ij} N_j}{\left[1 + \sum_{j \neq i} \rho_{ij} N_j\right] + \sum_{j \neq i} N_j \left[N_j + \sum_{\ell \neq j} \rho_{j\ell} N_\ell\right]} \\
&\leq \frac{1 + \sum_{j \neq i} \rho_{ij} N_j}{\left[1 + \sum_{j \neq i} \rho_{ij} N_j\right] + \sum_{j \neq i} N_j [N_j + \rho_{ij}]} \\
&= \frac{1 + \sum_{j \neq i} \rho_{ij} N_j}{2\left[1 + \sum_{j \neq i} \rho_{ij} N_j\right] + \sum_{j \neq i} N_j^2 - 1} \\
&\leq \frac{1}{2}.
\end{aligned}$$

Condition (iii): If all pairwise correlations are equal to ρ, we have

$$\frac{R_i^\top \gamma}{\gamma^\top R \gamma} = \frac{(1-\rho) N_i + \rho N}{(1-\rho) \sum_j N_j^2 + \rho N^2}.$$

If $N_i \geq 2$, condition (i) applies. If N_i is equal to 0 or 1, we have

$$\frac{R_i^\top \gamma}{\gamma^\top R \gamma} \leq \frac{1}{N} \frac{(1-\rho) N_i + \rho N}{(1-\rho) + \rho N} \leq \frac{1}{N} \leq \frac{1}{2}. \quad \square$$

Proof of Proposition 4.2 *Proof of (i):* From (20),

$$\delta_i \frac{k\delta_i + 2}{k\delta_i + 1} - \phi \frac{k\phi + 2}{k\phi + 1} = \frac{R_i^\top \gamma}{\gamma^\top R \gamma} (k\phi + 2) \frac{\delta_i}{k\delta_i + 1}. \tag{48}$$

Since $x(kx+2)/(kx+1)$ is strictly increasing in x, the result follows.

Proof of (ii): We can rewrite (48) as

$$k\delta_i + 2 - (k + \delta_i^{-1}) \phi \frac{k\phi + 2}{k\phi + 1} = \frac{R_i^\top \gamma}{\gamma^\top R \gamma} (k\phi + 2).$$

The left-hand side of this equation is strictly increasing in δ_i, for given ϕ. The result follows.

Proof of (iii): If $M = 0$ ($L_U = \varnothing$), then $k\phi + 2 = N$ from (18). Now suppose $M \geq 1$ ($L_U \neq \varnothing$). From (12), (13), and part (i) of this proposition, we have

$\beta_i \le \alpha$ for all $i \in L_U$, and $\beta_i = \alpha$ if and only if $\mathcal{V}_i = 0$. Hence, from (12) and (18), $\alpha[(k\phi + 2) - N] = \sum_{i \in L_U} M_i \beta_i \le M\alpha$, with equality if and only if $\mathcal{V}_i = 0$ for all $i \in L_U$. This proves the result. $\square$

Proof of Proposition 4.3 *Proof of (i):* This is immediate from (12).

Proof of (ii) and (iii): From (12) and (13), $\alpha - \beta_i = \delta_i - \phi$. Hence, statements (ii) and (iii) are equivalent to statements (i) and (ii) of Proposition 4.2, respectively.

Proof of (iv): Suppose $L_U = L$. If $\beta_i = 0$ for all i, then $R_i^\top \gamma$ must be the same for all i from (15). It follows that $\mathcal{V}_i$ is the same for all i. Conversely, if $\mathcal{V}_i$ is the same for all i, then so is δ_i from Proposition 4.2 (ii), and hence β_i due to (13). We denote the common value of β_i across all types i by β. Since $\mathcal{V}_i$ is the same for all i, so is $R_i^\top \gamma$, and hence $R_i^\top \gamma / \gamma^\top R \gamma = 1/N$. From (15), $\beta \propto 1 - (1/N)(k\phi + 2) \propto N - (k\phi + 2)$. On the other hand, from (18), $\beta \propto (k\phi + 2) - N$. It follows that $\beta = 0$ and $k\phi + 2 = N$.

Proof of (v): Suppose $L_I = L_U = L$. Let i_0 and j_0 be types with the lowest and highest price informativeness, respectively, i.e. $R_{i_0}^\top \gamma = \min_{i \in L} R_i^\top \gamma$ and $R_{j_0}^\top \gamma = \max_{i \in L} R_i^\top \gamma$. Since price informativeness is not the same for all types, $R_{i_0}^\top \gamma < R_{j_0}^\top \gamma$. Using the result in part (iii), $\beta_{i_0} = \max_{i \in L} \beta_i > \min_{i \in L} \beta_i = \beta_{j_0}$. If $R_{i_0}^\top \gamma > 0$, then using the assumption that $L_I = L$, and hence $N_i \ge 1$ for all $i \in L$,

$$\frac{R_{i_0}^\top \gamma}{\gamma^\top R \gamma} = \frac{R_{i_0}^\top \gamma}{\sum_i N_i R_i^\top \gamma} < \frac{R_{i_0}^\top \gamma}{\sum_i N_i R_{i_0}^\top \gamma} = \frac{1}{N}.$$

It follows that, whether $R_{i_0}^\top \gamma$ is positive or equal to zero,

$$\frac{R_{i_0}^\top \gamma}{\gamma^\top R \gamma} < \frac{1}{N}. \tag{49}$$

Using an analogous argument,

$$\frac{R_{j_0}^\top \gamma}{\gamma^\top R \gamma} > \frac{1}{N}. \tag{50}$$

It suffices to show that it is impossible that $\beta_i \ge 0$ for all i, or that $\beta_i \le 0$ for all i. We establish this by contradiction. Suppose $\beta_i \ge 0$ for all i. Then, from (18), $k\phi + 2 \ge N$. Consequently, using (50), $1 - (R_{j_0}^\top \gamma / \gamma^\top R \gamma)(k\phi + 2) < 0$. Hence, from (15), $\beta_{j_0} < 0$, a contradiction. Similarly, if $\beta_i \le 0$ for all i, then $k\phi + 2 \le N$. Using (49), $1 - (R_{i_0}^\top \gamma / \gamma^\top R \gamma)(k\phi + 2) > 0$. Hence, from (15), $\beta_{i_0} > 0$, a contradiction. $\square$

Proof of Proposition 5.1 Solving for agents' portfolio choices, analogous to (38) and (39) but with zero price impact, we obtain the slope coefficients $\hat{\alpha}$ and $\hat{\beta}_i$ given by (26) and (27), respectively. Using the market-clearing condition,

$$\sum_{i \in L_I} N_i \hat{\alpha}(\theta_i - \hat{p}) - \sum_{i \in L_U} M_i \hat{\beta}_i \hat{p} = 0,$$

the equilibrium price is

$$\hat{p} = \hat{\Phi}^{-1}\hat{\alpha}\gamma^\top\theta, \tag{51}$$

where

$$\hat{\Phi} := N\hat{\alpha} + \sum_{i\in L_U} M_i\hat{\beta}_i. \tag{52}$$

Therefore, $\sigma_{\theta_i\hat{p}}/\sigma_{\hat{p}}^2 = (R_i^\top\gamma/\gamma^\top R\gamma)\hat{\Phi}\hat{\alpha}^{-1}$, so that, from (26) and (27),

$$\hat{\beta}_i = k^{-1}\left[1 - \frac{R_i^\top\gamma}{\gamma^\top R\gamma}\hat{\Phi}\hat{\alpha}^{-1}\right]. \tag{53}$$

Plugging this expression into (52), and using (26),

$$\hat{\Phi}\hat{\alpha}^{-1} = N + \sum_{i\in L_U} M_i\left[1 - \frac{R_i^\top\gamma}{\gamma^\top R\gamma}\hat{\Phi}\hat{\alpha}^{-1}\right],$$

which gives us $\hat{\Phi}\hat{\alpha}^{-1} = \lambda$, where λ is defined by (25). Equations (24) now follows from (51). Also, from (53),

$$\hat{\beta}_i = k^{-1}\left[1 - \frac{R_i^\top\gamma}{\gamma^\top R\gamma}\lambda\right]. \tag{54}$$

Finally, we verify that the slope parameters satisfy all the properties in Proposition 4.3. Parts (i), (ii) and (iii) are obvious. For part (iv), observe that if $\hat{\beta}_i = 0$ for all i, then $\mathcal{V}_i = \mathcal{V}_j$ for all i, j, by part (iii). Conversely, if $\mathcal{V}_i = \mathcal{V}_j$ for all i, j, then $R_i^\top\gamma/\gamma^\top R\gamma = N^{-1}$ for all i, so that $\lambda = N$ from (25). Plugging these values into (54), we see that $\hat{\beta}_i = 0$ for all i. For part (v), we use the same argument as in the proof of Proposition 4.3 (v). Equations (49) and (50) still apply. If $\hat{\beta}_i \geq 0$ for all i, then, from (54), $R_i^\top\gamma/\gamma^\top R\gamma \leq \lambda^{-1}$ for all i. Hence, from (25), $\lambda^{-1} \leq N^{-1}$. Taken together, we have $R_i^\top\gamma/\gamma^\top R\gamma \leq N^{-1}$ for all i, contradicting (50). A similar argument shows that we cannot have $\hat{\beta}_i \leq 0$, for all i, either. □

Proof of Proposition 5.2 From (43), ϕ satisfies the following equation for all ξ (for notational ease, we suppress the dependence of ϕ on ξ):

$$\frac{k\phi+2}{k\phi+1} - \frac{\xi N}{k\phi+1} + \xi\sum_{i\in L_U} M_i\left[\frac{g_i(\phi;\xi)}{\phi} - \frac{k\phi+2}{k\phi+1}\right] = 0, \tag{55}$$

where

$$g_i(\phi;\xi) = \frac{-b_i(\phi;\xi) + \sqrt{b_i^2(\phi;\xi) + 4k\phi\frac{k\phi+2}{k\phi+1}}}{2k}, \tag{56}$$

$$b_i(\phi;\xi) = 2 - \frac{R_i^\top\gamma}{\xi\gamma^\top R\gamma}(k\phi+2) - k\phi\frac{k\phi+2}{k\phi+1}. \tag{57}$$

It is convenient to think of ξ taking integer values, $\{1, 2, \ldots\}$. We claim that $\lim_{\xi\to\infty}\phi = \infty$. Suppose not. Then $\{\phi(\xi)\}$ is a bounded sequence, which we can assume to be convergent without loss of generality (since we can always consider a convergent subsequence). From (13), (15) and (17),

$$\frac{k\phi+2}{k\phi+1}-\frac{g_i(\phi;\xi)}{\phi}=\frac{g_i(\phi;\xi)/\phi}{kg_i(\phi;\xi)+1}\left[1-\frac{R_i^\top\gamma}{\xi\gamma^\top R\gamma}(k\phi+2)\right].$$

Taking ξ to be large enough so that the term in square brackets on the right-hand side is positive, we have

$$\frac{g_i(\phi;\xi)}{\phi}=\frac{k\phi+2}{k\phi+1}\left(1+\frac{1}{kg_i(\phi;\xi)+1}\left[1-\frac{R_i^\top\gamma}{\xi\gamma^\top R\gamma}(k\phi+2)\right]\right)^{-1}.$$

Since $\lim_{\xi\to\infty} g_i(\phi;\xi)\in[0,\infty)$, $\lim_{\xi\to\infty} g_i(\phi;\xi)/\phi<\lim_{\xi\to\infty}(k\phi+2)/(k\phi+1)$. Therefore (55) cannot hold for sufficiently large ξ (this is true even if L_U is empty). This is a contradiction. Hence we must have $\phi\to\infty$.

Since $\delta_i\geq\phi$ (Proposition 4.2), $\delta_i\to\infty$ as well, for all $i\in L_U$. From (12), $\alpha\to k^{-1}=\hat\alpha$. In order to calculate the limits of p and β_i, we need to compute the rate at which ϕ increases relative to ξ. From (55), we have

$$\begin{aligned}0&=\frac{k\phi+2}{k\phi+1}-\frac{\xi(N+M)}{k\phi+1}+\xi\sum_{i\in L_U}M_i\left[\frac{g_i(\phi;\xi)}{\phi}-1\right]\\&=\frac{k\phi+2}{k\phi+1}-\frac{\xi(N+M)}{k\phi+1}+\sum_{i\in L_U}M_i\big[h_i(\phi;\xi)-\xi\big],\end{aligned}\tag{58}$$

where

$$h_i(\phi;\xi):=\frac{g_i(\phi;\xi)\xi}{\phi}=-\frac{b_i(\phi;\xi)\xi}{2k\phi}+\sqrt{\left[\frac{b_i(\phi;\xi)\xi}{2k\phi}\right]^2+\frac{k\phi+2}{k\phi(k\phi+1)}\xi^2}.\tag{59}$$

Note that $h_i(\phi;\xi)$ is strictly positive and satisfies

$$\begin{aligned}0&=[h_i(\phi;\xi)]^2+\frac{b_i(\phi;\xi)\xi}{k\phi}h_i(\phi;\xi)-\frac{k\phi+2}{k\phi(k\phi+1)}\xi^2\\&=h_i(\phi;\xi)\left[h_i(\phi;\xi)-\xi-\frac{R_i^\top\gamma}{\gamma^\top R\gamma}\left(1+\frac{2}{k\phi}\right)+\frac{2\xi}{k\phi}-\frac{\xi}{k\phi+1}\right]-\frac{k\phi+2}{k\phi(k\phi+1)}\xi^2\\&=h_i(\phi;\xi)\left[h_i(\phi;\xi)-\xi-\frac{R_i^\top\gamma}{\gamma^\top R\gamma}\left(1+\frac{2}{k\phi}\right)\right]+\frac{k\phi+2}{k\phi(k\phi+1)}\left[h_i(\phi;\xi)-\xi\right]\xi.\end{aligned}$$

Dividing both sides of this equation by $h_i(\phi;\xi)$, and noting that $\xi/h_i(\phi;\xi)=\phi/g_i(\phi;\xi)$, we obtain

$$h_i(\phi;\xi)-\xi=\left[1+\frac{k\phi+2}{k\phi(k\phi+1)}\frac{\phi}{g_i(\phi;\xi)}\right]^{-1}\frac{R_i^\top\gamma}{\gamma^\top R\gamma}\left(1+\frac{2}{k\phi}\right).\tag{60}$$

We have already established that $\phi\to\infty$ as $\xi\to\infty$. From (59),

$$\frac{g_i(\phi;\xi)}{\phi}=-\frac{b_i(\phi;\xi)}{2k\phi}+\sqrt{\left[\frac{b_i(\phi;\xi)}{2k\phi}\right]^2+\frac{k\phi+2}{k\phi(k\phi+1)}}.$$

Since $b_i(\phi;\xi)/\phi \to -k$, we see that $g_i(\phi;\xi)/\phi \to 1$, so that $h_i(\phi;\xi) - \xi \to R_i^\top\gamma/\gamma^\top R\gamma$ from (60). It follows from (58) that $k\phi/\xi \to \lambda$, where λ is defined by (25). Therefore, from (15),

$$\beta_i = \frac{\delta_i}{k\delta_i+1}\left[1 - \frac{R_i^\top\gamma}{\xi\gamma^\top R\gamma}(k\phi+2)\right] = \frac{\delta_i}{k\delta_i+1}\left[1 - \frac{R_i^\top\gamma}{\gamma^\top R\gamma}\left(\frac{k\phi}{\xi}+\frac{2}{\xi}\right)\right], \quad (61)$$

which converges to $\hat{\beta}_i$, given by (54). Similarly, from (16),

$$p = (k\phi+2)^{-1}(\xi\gamma)^\top\theta = \left(\frac{k\phi}{\xi}+\frac{2}{\xi}\right)^{-1}\gamma^\top\theta, \quad (62)$$

which converges to $\hat{p}$, given by (24). Finally, we show that ϕ and δ are monotonic in ξ. From (43), $\phi(\xi)$ solves

$$f(\phi(\xi);\xi) := \frac{\phi}{k\phi+1}\left[(k\phi+2) - \xi N\right] + \xi\sum_{i\in L_U} M_i\left[g_i(\phi;\xi) - \phi\frac{k\phi+2}{k\phi+1}\right] = 0, \quad (63)$$

where $g_i(\phi;\xi)$ is given by (56), and $b_i(\phi;\xi)$ by (57); we suppress the dependence of ϕ on ξ to economize on notation. For given ϕ, $b_i(\phi;\xi)$ is increasing in ξ, and hence from (40), $g_i(\phi;\xi) = \delta_i(\phi;\xi)$ is decreasing in ξ. We have

$$\begin{aligned}\frac{\partial f(\phi(\xi);\xi)}{\partial\xi} &= -\frac{\phi}{k\phi+1}N + \sum_{i\in L_U} M_i\left[g_i(\phi;\xi) - \phi\frac{k\phi+2}{k\phi+1}\right] + \xi\sum_{i\in L_U} M_i\frac{\partial g_i(\phi,\xi)}{\partial\xi} \\ &= -\frac{\phi}{\xi}\left[\frac{k\phi+2}{k\phi+1}\right] + \xi\sum_{i\in L_U} M_i\frac{\partial g_i(\phi,\xi)}{\partial\xi},\end{aligned}$$

which is negative (the second equality follows from (63)). Hence, for any $\xi \geq 1$, there exists $\epsilon > 0$ such that for all $\xi' \in (\xi, \xi+\epsilon)$, $f(\phi(\xi), \xi') < 0$. Since $\phi(\xi)$ is defined as the highest solution to $f(\phi;\xi) = 0$, $\phi(\xi') > \phi(\xi)$. It follows that ϕ is strictly increasing in ξ.

Assuming that L_U is nonempty, we have (from (43)),

$$\delta(\phi(\xi);\xi) := M^{-1}\sum_{i\in L_U} M_i g_i(\phi;\xi) = \frac{\xi M - 1}{\xi M}\phi\frac{k\phi+2}{k\phi+1} + \frac{N}{M}\frac{\phi}{k\phi+1}.$$

Since $\delta(\cdot;\cdot)$ is strictly increasing in both arguments, and ϕ is strictly increasing in ξ, it follows that δ is strictly increasing in ξ. □

Proof of Proposition 5.3 *Proof of (i):* The depth parameter ϕ satisfies the following equation:

$$\frac{k\phi+2}{k\phi+1} - \frac{\xi N}{k\phi+1} + \sum_{i\in L_U} M_i\left[\frac{g_i(\phi;\xi)}{\phi} - \frac{k\phi+2}{k\phi+1}\right] = 0, \quad (64)$$

which is the same as (55), except that ξ does not multiply M_i. The proof that $\phi \to \infty$, $\delta_i \to \infty$, $\alpha \to \hat{\alpha}$, and $g_i(\phi;\xi)/\phi \to 1$, is identical to that in Proposition

5.2. Using the last of these results, it follows from (64) that $k\phi/\xi \to N$. Therefore, from (61) and (62),

$$\lim_{\xi\to\infty} \beta_i = k^{-1}\left[1 - \frac{R_i^\top \gamma}{\gamma^\top R\gamma} N\right], \qquad \text{and} \qquad \lim_{\xi\to\infty} p = N^{-1}\gamma^\top \theta.$$

From (25), for the economy parametrized by $(\xi\gamma, \nu)$,

$$\lambda = \frac{\xi N + M}{1 + \sum_{i\in L_U} M_i \frac{R_i^\top \gamma}{\xi\gamma^\top R\gamma}},$$

so that $\lambda/\xi \to N$. Using (24) and (54), we conclude that $\lim_{\xi\to\infty}\beta_i = \lim_{\xi\to\infty}\hat{\beta}_i$ and $\lim_{\xi\to\infty} p = \lim_{\xi\to\infty}\hat{p}$.

We establish the monotonicity properties in the same way as in the proof of Proposition 5.2. Here we have

$$f(\phi(\xi);\xi) := \frac{\phi}{k\phi+1}\left[(k\phi+2) - \xi N\right] + \sum_{i\in L_U} M_i \left[g_i(\phi;\xi) - \phi\frac{k\phi+2}{k\phi+1}\right] = 0,$$

so that

$$\frac{\partial f(\phi(\xi);\xi)}{\partial \xi} = -\frac{\phi}{k\phi+1}N + \sum_{i\in L_U} M_i \frac{\partial g_i(\phi,\xi)}{\partial\xi} < 0,$$

implying that ϕ is strictly increasing in ξ. Also,

$$\delta(\phi(\xi);\xi) = \frac{M-1}{M}\phi\frac{k\phi+2}{k\phi+1} + \frac{\xi N}{M}\frac{\phi}{k\phi+1},$$

from which we can conclude that δ is strictly increasing in ξ.

Proof of (ii): We need to assume that $R_\ell \geq 0$, otherwise our standing assumption that $R_i^\top\gamma \geq 0$ for all i will be violated for large N_ℓ. We can write

$$\frac{R_i^\top\gamma}{\gamma^\top R\gamma} = \frac{\rho_{i\ell}N_\ell + \sum_{j\neq\ell}\rho_{ij}N_j}{N_\ell\left[N_\ell + \sum_{j\neq\ell}\rho_{\ell j}N_j\right] + \sum_{j\neq\ell}N_j\left[\rho_{j\ell}N_\ell + \sum_{m\neq\ell}\rho_{jm}N_m\right]}. \tag{65}$$

Hence, if $N_\ell \to \infty$,

$$\frac{R_i^\top\gamma}{\gamma^\top R\gamma} \to 0. \tag{66}$$

By exactly the same arguments as in the proof of part (i), we can show that $\phi\to\infty$, $\delta_i \to \infty$ for all $i \in L_U$, $\alpha \to \hat{\alpha}$, and

$$\frac{k\phi}{N_\ell} \to 1. \tag{67}$$

Using (65) and (67),

$$\frac{R_i^\top\gamma}{\gamma^\top R\gamma}k\phi = \frac{N_\ell\left[\rho_{i\ell}N_\ell + \sum_{j\neq\ell}\rho_{ij}N_j\right]}{N_\ell\left[N_\ell + \sum_{j\neq\ell}\rho_{\ell j}N_j\right] + \sum_{j\neq\ell}N_j\left[\rho_{j\ell}N_\ell + \sum_{m\neq\ell}\rho_{jm}N_m\right]}\left(\frac{k\phi}{N_\ell}\right),$$

which converges to $\rho_{i\ell}$. From (15) and (66), we conclude that $\beta_i \to k^{-1}(1-\rho_{i\ell})$ which, from (54), is equal to $\lim_{N_\ell\to\infty}\hat{\beta}_i$. Finally, from (16) and (67),

$$p = (k\phi+2)^{-1}\sum_i N_i\theta_i = \left(\frac{k\phi}{N_\ell}+\frac{2}{N_\ell}\right)^{-1}\left(\theta_\ell + \sum_{i\neq\ell}\frac{N_i}{N_\ell}\theta_i\right),$$

which converges to θ_ℓ as $\xi\to\infty$. From (24) and (25), $\hat{p}$ converges to θ_ℓ as well. $\square$

Proof of Proposition 5.4 *Proof of (i):* We need to check that the depth parameters are bounded, as well as bounded away from zero, as the number of uninformed agents of any type goes to infinity. From (43), ϕ satisfies

$$\frac{k\phi+2}{k\phi+1} - \frac{N}{k\phi+1} + \sum_{i\in L_U} M_i\left[\frac{g_i(\phi)}{\phi} - \frac{k\phi+2}{k\phi+1}\right] = 0. \tag{68}$$

Suppose $M_i\to\infty$ for some i and consider the sequence $\{\phi(M_i)\}$. If $\phi\to\infty$, then from (46), and the assumption that $R_i^\top\gamma > 0$, $\lim_{\phi\to\infty} g_i(\phi)/\phi > 1$, and hence (68) cannot be satisfied for large M_i. It follows that $\{\phi(M_i)\}$ is bounded. We assume without loss of generality that it is convergent (as otherwise we can choose a convergent subsequence). If $\phi(M_i)\to 0$, then using (44) and (45), and the assumptions that $N_j\geq 2$ and $R_j^\top\gamma>0$ for all j,

$$\lim_{\phi\to 0}\frac{g_i(\phi)}{\phi} = \left[1-\frac{R_i^\top\gamma}{\gamma^\top R\gamma}\right]^{-1} < \left[1-\frac{1}{N_i}\right]^{-1} \leq 2.$$

Again, (68) cannot hold for large M_i, and consequently $\{\phi(M_i)\}$ must be bounded away from zero. In addition, for every $j\in L_U$, $\{\delta_j(M_i)\}$ is a bounded sequence due to (42), and bounded away from zero since $\delta_j\geq\phi$.

Proof of (ii): From (15) and (18), and recalling that $\delta_i = g_i(\phi)$, $\phi(M_i)$ solves

$$f(\phi(M_i);M_i) := \frac{\phi}{k\phi+1}\left[(k\phi+2)-N\right] - \sum_{j\in L_U} M_j\beta_j(\phi) = 0, \tag{69}$$

where

$$\beta_j(\phi) = \frac{g_j(\phi)}{kg_j(\phi)+1}\left[1-\frac{R_j^\top\gamma}{\gamma^\top R\gamma}(k\phi+2)\right], \tag{70}$$

and we suppress the dependence of ϕ on M_i. Consider M_i and M_i' satisfying $M_i > M_i' \geq 1$. Three cases arise depending on the sign of $\beta_i(\phi(M_i'))$.

Case 1: $\beta_i(\phi(M_i')) > 0$. From (69), $f(\phi(M_i');M_i) < f(\phi(M_i');M_i') = 0$. Since $\phi(M_i)$ is defined as the highest solution to $f(\cdot;M_i)=0$, and $\lim_{\phi\to\infty} f(\phi;M_i)=\infty$ for given M_i, we have $\phi(M_i) > \phi(M_i')$.

Case 2: $\beta_i(\phi(M_i')) < 0$. From (70), $\beta_i(\phi) < 0$ for all $\phi\geq\phi(M_i')$. Hence, from (69), $f(\phi;M_i) > f(\phi;M_i') \geq f(\phi(M_i');M_i') = 0$ for all $\phi\geq\phi(M_i')$. Hence, $\phi(M_i) < \phi(M_i')$.

Case 3: $\beta_i(\phi(M_i')) = 0$. From (69), $f(\phi(M_i'); M_i) = f(\phi(M_i'); M_i') = 0$. Moreover, for any $\phi > \phi(M_i')$, $\beta_i(\phi) < 0$, so that $f(\phi; M_i) > f(\phi; M_i') > f(\phi(M_i'); M_i') = 0$. Hence $\phi(M_i')$ is the highest zero of $f(\cdot; M_i)$, i.e. $\phi(M_i) = \phi(M_i')$.

Putting these three cases together, we see that $\phi(M_i) - \phi(M_i') \propto \beta_i(\phi(M_i'))$.

Proof of (iii): Since ϕ is bounded, so are Φ, α, and $\{\beta_i\}_{i\in L_U}$, from Proposition 3.1. From (2), $\Phi = N\alpha + \sum_{j\in L_U} M_j\beta_j$. Therefore, as M_i goes to infinity, $M_i\beta_i$ remains bounded. This in turn implies that β_i converges to zero. $\square$

Proof of Proposition 5.5 Let $\mathbf{1}$ denote the L-vector each element of which is 1. Then price informativeness for type i is

$$\mathcal{V}_i = \frac{(R_i^\top \mathbf{1})^2}{\mathbf{1}^\top R\mathbf{1}} = \frac{[1+(L-1)\bar{\rho}]^2}{L[1+(L-1)\bar{\rho}]} = \frac{1+(L-1)\bar{\rho}}{L}.$$

Writing $\bar{\rho}(L)$ as $\bar{\rho}_L$, we have

$$\begin{aligned}\Delta\bar{\mathcal{V}} &= \frac{1+L\bar{\rho}_{L+1}}{L+1} - \frac{1+(L-1)\bar{\rho}_L}{L} \\ &\propto \left[L + L^2\bar{\rho}_{L+1}\right] - \left[L+1+(L^2-1)\bar{\rho}_L\right] \\ &= L^2(\bar{\rho}_{L+1}-\bar{\rho}_L) - (1-\bar{\rho}_L),\end{aligned}$$

which gives us the desired result. $\square$

Proof of Lemma 6.1 From (8), (10) and (12), we see that

$$\begin{aligned}\mathcal{U}_i^I &= \mathbb{E}\big[\mathbb{E}(W_i|\theta_i, p)\big] \\ &= \mathbb{E}\left[(\theta_i - p)q_i^I - \frac{k}{2}(q_i^I)^2\right] \\ &= \mathbb{E}\left[\frac{k\phi+2}{2\phi}(q_i^I)^2\right] \\ &= \frac{k\phi+2}{2\phi}\alpha^2\sigma^2_{\theta_i-p} \qquad (71) \\ &= \frac{k\phi+2}{2\phi}\left[\frac{\phi}{k\phi+1}\right]^2\sigma^2_{\theta_i-p}.\end{aligned}$$

Using (28) and (29), we obtain (30). Similarly, from (9), (11), (15) and (16),

$$\begin{aligned}\mathcal{U}_i^U &= \mathbb{E}\big[\mathbb{E}(W_i|p)\big] \\ &= \mathbb{E}\left[\big[\mathbb{E}(\theta_i|p) - p\big]q_i^U - \frac{k}{2}(q_i^U)^2\right] \\ &= \mathbb{E}\left[\frac{k\delta_i+2}{2\delta_i}(q_i^U)^2\right] \\ &= \frac{k\delta_i+2}{2\delta_i}\beta_i^2\sigma_p^2 \qquad (72)\end{aligned}$$

$$= \frac{k\delta_i + 2}{2\delta_i}\left[\frac{\delta_i}{k\delta_i+1}\right]^2\left[1-\frac{R_i^\top\gamma}{\gamma^\top R\gamma}(k\phi+2)\right]^2\frac{\sigma_\theta^2\gamma^\top R\gamma}{(k\phi+2)^2}.$$

Using (29) and (32), we obtain (31). □

Proof of Proposition 6.2 We fix an equilibrium of a given economy (in particular, we fix ϕ, α, σ_p^2 and $\gamma^\top R\gamma$), and consider the utilities of agents of type i for different hypothetical values of $R_i^\top\gamma$, and hence of ψ_i, given by (23). There is a one-to-one correspondence between ψ_i, $\mathcal{V}_i$, β_i and δ_i; a higher value of ψ_i is associated with a higher value of $\mathcal{V}_i$ and δ_i, and a lower value of β_i.

From (30) and (32), the utility of an informed agent can be written as

$$\mathcal{U}_i^I = \frac{\sigma_\theta^2}{2}F(\phi)\left[1+\frac{\gamma^\top R\gamma}{(k\phi+2)^2}(1-2\psi_i)\right], \tag{73}$$

which is linear and strictly decreasing in ψ_i. From (13) and (72),

$$\mathcal{U}_i^U = \frac{k\delta_i+2}{2\delta_i}(\delta_i-\Phi)^2\sigma_p^2.$$

Differentiating with respect to δ_i, we obtain

$$\frac{\partial\mathcal{U}_i^U}{\partial\delta_i} = (\delta_i-\Phi)(k+\delta_i^{-1}+\delta_i^{-2}\Phi)\sigma_p^2, \tag{74}$$

$$\frac{\partial^2\mathcal{U}_i^U}{\partial\delta_i^2} = (k+2\delta_i^{-3}\Phi^2)\sigma_p^2. \tag{75}$$

From (40) and (41),

$$\frac{\partial\delta_i}{\partial\psi_i} = \frac{\delta_i}{2k\delta_i+b_i}, \tag{76}$$

$$\frac{\partial^2\delta_i}{\partial\psi_i^2} = \frac{(2k\delta_i+b_i)\frac{\partial\delta_i}{\partial\psi_i}-\delta_i\left[2k\frac{\partial\delta_i}{\partial\psi_i}-1\right]}{(2k\delta_i+b_i)^2} = \frac{2\delta_i(k\delta_i+b_i)}{(2k\delta_i+b_i)^3}. \tag{77}$$

From (17) and (40),

$$\delta_i(k\delta_i+b_i) = \Phi. \tag{78}$$

Differentiating $\mathcal{U}_i^U$ with respect to ψ_i, and noting that $2k\delta_i+b_i>0$ (from (42)), we have

$$\frac{\partial\mathcal{U}_i^U}{\partial\psi_i} = \frac{\partial\mathcal{U}_i^U}{\partial\delta_i}\frac{\partial\delta_i}{\partial\psi_i} \propto \delta_i-\Phi. \tag{79}$$

Using (74)–(79),

$$\frac{\partial^2\mathcal{U}_i^U}{\partial\psi_i^2} = \frac{\partial^2\mathcal{U}_i^U}{\partial\delta_i^2}\left[\frac{\partial\delta_i}{\partial\psi_i}\right]^2 + \frac{\partial\mathcal{U}_i^U}{\partial\delta_i}\frac{\partial^2\delta_i}{\partial\psi_i^2}$$

$$\propto \left[k+2\delta_i^{-3}\Phi^2\right]\frac{\delta_i^2}{(2k\delta_i+b_i)^2} + (\delta_i-\Phi)\left[k+\delta_i^{-1}+\delta_i^{-2}\Phi\right]\frac{2\delta_i(k\delta_i+b_i)}{(2k\delta_i+b_i)^3}$$

$$
\begin{aligned}
&\propto \left[k\delta_i^3 + 2\Phi^2\right]\delta_i(2k\delta_i + b_i) + 2(\delta_i - \Phi)\left[k\delta_i^2 + \delta_i + \Phi\right]\delta_i(k\delta_i + b_i) \\
&= \left[k\delta_i^3 + 2\Phi^2\right]\left[k\delta_i^2 + \Phi\right] + 2(\delta_i - \Phi)\left[k\delta_i^2 + \delta_i + \Phi\right]\Phi \\
&= k^2\delta_i^5 + 3k\delta_i^3\Phi + 2\delta_i^2\Phi,
\end{aligned}
$$

which is positive. Hence, $\mathcal{U}_i^U$ is strictly convex in ψ_i, achieving its minimum value of 0 at $\psi_i = 1$, which corresponds to $\beta_i = 0$ or $\delta_i = \Phi$.

Proof of (i): We will show that $\Delta\mathcal{U}_i := \mathcal{U}_i^I - \mathcal{U}_i^U$ is strictly decreasing in ψ_i. We have already established that $\mathcal{U}_i^I$ is linear in ψ_i, and $\mathcal{U}_i^U$ is strictly convex in ψ_i, so that $\Delta\mathcal{U}_i$ is strictly concave in ψ_i. Hence it suffices to show that $\partial(\Delta\mathcal{U}_i)/\partial\psi_i < 0$ at $\psi_i = 0$. From (73), (74), (76), and the relations $\sigma_p^2 = \sigma_\theta^2(k\phi + 2)^{-2}\gamma^\top R\gamma$, $\delta_i|_{\psi_i=0} = \phi$, and $(2k\delta_i + b_i)|_{\psi_i=0} = \sqrt{4 + \left(k\phi\frac{k\phi+2}{k\phi+1}\right)^2}$, we have

$$
\begin{aligned}
\left.\frac{\partial(\Delta\mathcal{U}_i)}{\partial\psi_i}\right|_{\psi_i=0} &\propto \left.\left[\frac{\delta_i}{2k\delta_i + b_i}(\Phi - \delta_i)(k + \delta_i^{-1} + \delta_i^{-2}\Phi) - F(\phi)\right]\right|_{\psi_i=0} \\
&= \frac{\phi}{\sqrt{4 + \left(k\phi\frac{k\phi+2}{k\phi+1}\right)^2}}(\Phi - \phi)(k + \phi^{-1} + \phi^{-2}\Phi) - F(\phi) \\
&\propto (k\phi + 1)^2 + (k\phi + 2) - (k\phi + 2)\sqrt{4 + \left(k\phi\frac{k\phi + 2}{k\phi + 1}\right)^2} \\
&\propto \left[(k\phi + 1)^2 + (k\phi + 2)\right]^2(k\phi + 1)^2 \\
&\quad - (k\phi + 2)^2\left[4(k\phi + 1)^2 + (k\phi)^2(k\phi + 2)^2\right] \\
&= -2(k\phi)^3 - 8(k\phi)^2 - 12k\phi - 7,
\end{aligned}
$$

which is negative.

Proof of (ii): We will show that $\mathcal{U}_i^I(\psi_i) - \mathcal{U}_i^U(\psi_i) > 0$ if and only if $\psi_i \in [0, \psi^*)$, for some cutoff value $\psi^* > 1$. This cutoff value corresponds to a slope parameter $\beta^* < 0$ for uninformed agents, and price informativeness $\mathcal{V}^*$.

If $\psi_i = 1$, then $\beta_i = 0$, and hence $\Delta\mathcal{U}_i = \mathcal{U}_i^I > 0$. From part (iii), $\Delta\mathcal{U}_i$ is strictly decreasing in ψ_i, implying that $\Delta\mathcal{U}_i(\psi_i) > 0$ for all $\psi_i \in [0, 1]$. If $\Delta\mathcal{U}_i(\psi_i) > 0$ for all ψ_i, we can pick $\psi^* = \max_i \psi_i + \epsilon$, for some small $\epsilon > 0$, and we are done. If not, $\Delta\mathcal{U}_i(\psi^*) = 0$ for some $\psi^* > 1$. Since $\Delta\mathcal{U}_i$ is strictly decreasing in ψ_i, it is positive for $\psi_i < \psi^*$ and negative for $\psi_i > \psi^*$.

Proof of (iii): The utility calculations are as in the imperfectly competitive case (see the proof of Lemma 6.1), but with zero price impact. From (26), (27), (71) and (72), we have

$$
\hat{\mathcal{U}}_i^I = \frac{k}{2}\hat{\alpha}^2\sigma_{\theta_i - \hat{p}}^2 = \frac{1}{2k}\sigma_{\theta_i - \hat{p}}^2, \tag{80}
$$

$$
\hat{\mathcal{U}}_i^U = \frac{k}{2}\hat{\beta}_i^2\sigma_{\hat{p}}^2 = \frac{1}{2k}\left[1 - \frac{\sigma_{\theta_i\hat{p}}}{\sigma_{\hat{p}}^2}\right]^2\sigma_{\hat{p}}^2. \tag{81}
$$

Therefore,

$$\Delta \hat{\mathcal{U}}_i = \frac{1}{2k}\left(\sigma_\theta^2 + \sigma_{\hat{p}}^2 - 2\sigma_{\theta_i\hat{p}} - \left[\sigma_{\hat{p}}^2 + \frac{\sigma_{\theta_i\hat{p}}^2}{\sigma_{\hat{p}}^2} - 2\sigma_{\theta_i\hat{p}}\right]\right) = \frac{\sigma_\theta^2}{2k}(1-\mathcal{V}_i), \tag{82}$$

where the second equality follows from (21) and (24).

Using Lemma 6.1, (29) and (82), and recalling that $\delta_i \geq \phi$, and hence $F(\delta_i) \geq F(\phi)$, we have

$$\Delta \mathcal{U}_i = \frac{\sigma_\theta^2}{2}\Big[F(\delta_i)(1-\mathcal{V}_i) - [F(\delta_i) - F(\phi)]\, G_i\Big] \leq \frac{\sigma_\theta^2}{2} F(\delta_i)(1-\mathcal{V}_i) < \frac{\sigma_\theta^2}{2k}(1-\mathcal{V}_i).$$

The result follows from (82). □

Proof of Lemma 7.1 Using the same argument as in the proof of Proposition 5.4, we see that the sequences $\{\phi(M_\ell)\}$ and $\{\delta_\ell(M_\ell)\}$ are both bounded and bounded away from zero. Since we only require these results for $i = \ell$ rather than for all i, the conditions on $\{N_i, R_i^\top \gamma\}_{i\in L}$ (given in Definition 7.1) are weaker than those in Proposition 5.4.

From Proposition 5.4 (iii), $\beta_\ell = 0$. It follows from (11) that $p = \mathbb{E}(\theta_\ell|p) = \psi_\ell p$, and hence $\psi_\ell = 1$. Equation (34) follows from (23), and equations (16) and (34) together give us the right-hand side of (33). □

Proof of Proposition 7.2 Taking the limit of (24) as $M_\ell \to \infty$, we see that the price function of the competitive $\mathcal{F}_\ell$-economy coincides with that of the $\mathcal{F}_\ell$-economy, which is given by (33). Moreover, it is clear from (33) that this price function is invariant with respect to ξ.

From (34), it is immediate that ϕ is strictly increasing in ξ, and $\lim_{\xi\to\infty} \phi = \infty$. From (12) and (26), α is also strictly increasing in ξ and converges to $\hat{\alpha}$.

From (17), (34), (40) and (41), δ_i solves

$$k\delta_i^2 + b_i\delta_i - \Phi = 0, \tag{83}$$

where

$$b_i := 2 - \frac{R_i^\top \gamma}{R_\ell^\top \gamma} - k\Phi. \tag{84}$$

Note that

$$\frac{\partial \Phi}{\partial \phi} = 1 + \frac{1}{(k\phi+1)^2} > 0. \tag{85}$$

From (83), the derivative of δ_i with respect to any variable x satisfies $2k\delta_i\delta_i' + b_i'\delta_i + b_i\delta_i' - \Phi' = 0$ (denoting derivatives with respect to x by a prime), which gives us

$$\delta_i' = \frac{\Phi' - b_i'\delta_i}{2k\delta_i + b_i}. \tag{86}$$

Note that, from (83), $2k\delta_i + b_i > k\delta_i + b_i = \Phi/\delta_i > 0$. Now taking x to be the variable ξ, we have $\Phi' > 0$ due to (85), and $b_i' = -k\Phi' < 0$. Therefore, $\delta_i' > 0$. Moreover, since $\delta_i \geq \phi$, $\lim_{\xi\to\infty} \delta_i = \infty$.

From (15) and (34),

$$\beta_i = \frac{\delta_i}{k\delta_i + 1}\left[1 - \frac{R_i^\top \gamma}{R_\ell^\top \gamma}\right].$$

It is clear that if $\beta_i = 0$, it is invariant with respect to ξ, and if $\beta_i \neq 0$, it depends on ξ only through δ_i. Moreover, in the latter case, $|\beta_i|' \propto \delta_i'$. Hence, the stated properties of $|\beta_i|$ follow from the corresponding properties of δ_i.

From (30), $\mathcal{U}_i^I = (\sigma_\theta^2/2)F(\phi)G_i$. Since $p = \hat{p}$ for all ξ, G_i is invariant with respect to ξ. Since $F'(\phi) > 0$, and ϕ is strictly increasing in ξ, $\mathcal{U}_i^I$ is strictly increasing in ξ. It converges to $(2k)^{-1}\sigma_\theta^2 G_i$, which is equal to $\hat{\mathcal{U}}_i^I$, from (80). From (33), $\sigma_p^2 = \sigma_\theta^2 \mathcal{V}_\ell$, which is invariant with respect to ξ. Hence, the statements about $\mathcal{U}_i^U$ follow from (72) and (81).

Finally we show that $\Delta\mathcal{U}_i$ is strictly increasing in ξ. Note that $\mathcal{V}_i$ and G_i do not depend on ξ, $\partial\phi/\partial\xi > 0$, δ_i depends on ξ only through ϕ (see (83) and (84)), and

$$F'(x) = 2(kx+1)^{-3}, \tag{87}$$

from (29). Hence, from (30), (31), (42), (84), (85) and (86), we have

$$\begin{aligned}
\frac{\partial(\Delta\mathcal{U}_i)}{\partial\xi} &\propto G_i F'(\phi) - \big[G_i - (1-\mathcal{V}_i)\big]F'(\delta_i)\frac{\partial\delta_i}{\partial\phi} \\
&\geq G_i\left[F'(\phi) - F'(\delta_i)\frac{\partial\delta_i}{\partial\phi}\right] \\
&\propto F'(\phi) - F'(\delta_i)\frac{\partial\delta_i}{\partial\phi} \\
&= \frac{2}{(k\phi+1)^3} - \frac{2}{(k\delta_i+1)^3}\frac{(k\delta_i+1)\frac{\partial\Phi}{\partial\phi}}{2k\delta_i+b_i} \\
&\geq \frac{2}{(k\phi+1)^3} - \frac{2}{(k\phi+1)^2(2k\delta_i+b_i)}\frac{\partial\Phi}{\partial\phi} \\
&\propto (2k\delta_i+b_i) - (k\phi+1)\frac{\partial\Phi}{\partial\phi} \\
&\propto (2k\delta_i+b_i)^2 - (k\phi+1)^2\left(\frac{\partial\Phi}{\partial\phi}\right)^2 \\
&= b_i^2 + 4k\phi\frac{k\phi+2}{k\phi+1} - (k\phi+1)^2\left(1 + \frac{1}{(k\phi+1)^2}\right)^2 \\
&= 4 + \left(k\phi\frac{k\phi+2}{k\phi+1}\right)^2 + \left(\frac{R_i^\top\gamma}{R_\ell^\top\gamma}\right)^2 \\
&\qquad - 2\frac{R_i^\top\gamma}{R_\ell^\top\gamma}\left(2 - k\phi\frac{k\phi+2}{k\phi+1}\right) - \left(k\phi+1+\frac{1}{k\phi+1}\right)^2 \\
&= \left(\frac{R_i^\top\gamma}{R_\ell^\top\gamma}\right)^2 - 2\frac{R_i^\top\gamma}{R_\ell^\top\gamma}\left(2 - k\phi\frac{k\phi+2}{k\phi+1}\right).
\end{aligned}$$

By the definition of an $\mathcal{F}_\ell$-economy, $R_j^\top\gamma > 0$ for all $j \in L_I$. Hence, $\partial(\Delta\mathcal{U}_i)/\partial\xi > 0$

if

$$k\phi \frac{k\phi + 2}{k\phi + 1} \geq 2. \tag{88}$$

Using (34),

$$k\phi + 2 = \xi \frac{\gamma^\top R\gamma}{R_\ell^\top \gamma} \geq \frac{\gamma^\top R\gamma}{R_\ell^\top \gamma} > N_\ell.$$

Since $k\phi(k\phi + 2)/(k\phi + 1)$ is strictly increasing in ϕ,

$$k\phi \frac{k\phi + 2}{k\phi + 1} > (N_\ell - 2)\frac{N_\ell}{N_\ell - 1}.$$

The right hand side of this inequality is increasing in N_ℓ. Therefore, if $N_\ell \geq 4$, (88) is satisfied. This completes the proof. □

Proof of Proposition 7.3 *Proof of (i):* From (34), we obtain

$$\frac{\partial \phi}{\partial N_i} = k^{-1} \frac{2(R_\ell^\top \gamma)(R_i^\top \gamma) - \rho_{i\ell}\gamma^\top R\gamma}{(R_\ell^\top \gamma)^2}. \tag{89}$$

Therefore,

$$\frac{\partial^2 \phi}{\partial N_i^2} \propto (R_\ell^\top \gamma)^2 - 2\rho_{i\ell}(R_\ell^\top \gamma)(R_i^\top \gamma) + \rho_{i\ell}^2 \gamma^\top R\gamma = (R_\ell^\top \gamma - \rho_{i\ell} R_i^\top \gamma)^2 + \rho_{i\ell}^2 \gamma^\top R\gamma(1 - \mathcal{V}_i),$$

which is positive.

Proof of (ii): From (89),

$$\frac{\partial \phi}{\partial N_\ell} = k^{-1}(2 - \mathcal{V}_\ell^{-1}), \tag{90}$$

from which the result is immediate.

Proof of (iii): Suppose all pairwise correlations are equal to ρ. From (89), $\partial\phi/\partial N_i > 0$ if $\rho < 0$. For $\rho \geq 0$, using the condition that $N_\ell + N_i \geq N_j$ for all $j \neq \ell, i$, we have

$$\begin{aligned}
\frac{\partial \phi}{\partial N_i} &\propto 2(R_\ell^\top \gamma)(R_i^\top \gamma) - \rho\gamma^\top R\gamma \\
&> (R_\ell^\top \gamma)(R_i^\top \gamma) - \rho\gamma^\top R\gamma \\
&= \left[N_\ell + \rho N_i + \rho \sum_{j \neq \ell, i} N_j\right]\left[N_i + \rho N_\ell + \rho \sum_{j \neq \ell, i} N_j\right] \\
&\qquad - \rho(1-\rho)\left[N_\ell^2 + N_i^2 + \sum_{j \neq \ell, i} N_j^2\right] - \rho^2 \left[N_\ell + N_i + \sum_{j \neq \ell, i} N_j\right]^2 \\
&= (1-\rho^2)N_\ell N_i + \rho(1-\rho)(N_\ell + N_i)\sum_{j \neq \ell, i} N_j - \rho(1-\rho)\sum_{j \neq \ell, i} N_j^2 \\
&\geq (1-\rho^2)N_\ell N_i,
\end{aligned}$$

which is positive. $\square$

Proof of Lemma 7.4: Using (32) and (34),

$$G_i = 1 + \frac{(R_\ell^\top \gamma)(R_\ell^\top \gamma - 2R_i^\top \gamma)}{\gamma^\top R \gamma} = 1 + \frac{(R_\ell^\top \gamma - R_i^\top \gamma)^2 - (R_i^\top \gamma)^2}{\gamma^\top R \gamma}, \tag{91}$$

which gives us the desired expression for G_i. $\square$

Proof of Proposition 7.5 By the definition of an $\mathcal{F}_2$-economy, $\mathcal{U}_2^U = 0$ for all N_2. We will show that $\partial \mathcal{U}_1^I / \partial N_2 < 0$, $\partial \mathcal{U}_1^U / \partial N_2 < 0$, and $\partial \mathcal{U}_2^I / \partial N_2 < 0$. Utilities are given by Lemma 6.1, where we recall that $F' > 0$. It is easy to check that $\partial \mathcal{V}_1 / \partial N_2 < 0$. Hence it suffices to show that $\partial \phi / \partial N_2 \le 0$, $\partial \delta_1 / \partial N_2 \le 0$, $\partial G_1 / \partial N_2 < 0$, and $\partial G_2 / \partial N_2 < 0$.

(i) Proof of $\partial \phi / \partial N_2 \le 0$: In view of (90), we can equivalently show that $\mathcal{V}_2 \le 1/2$. It is easy to check that

$$\mathcal{V}_2 = \frac{(\rho N_1 + N_2)^2}{N_1^2 + N_2^2 + 2\rho N_1 N_2} \le \frac{1}{2}$$

if and only if $N_2 \le (\sqrt{1-\rho^2} - \rho) N_1$. For $\rho \in (-1, 1/2]$, the function $\sqrt{1-\rho^2} - \rho$ is minimized at $\rho = 1/2$, and the minimum value is greater than $1/3$. Hence, given our assumptions that $\rho \le 1/2$ and $N_2 \le N_1/3$, we have $\mathcal{V}_2 \le 1/2$.

(ii) Proof of $\partial \delta_1 / \partial N_2 < 0$: From (84) and (86),

$$\frac{\partial \delta_1}{\partial N_2} \propto \frac{\partial \Phi}{\partial \phi} \frac{\partial \phi}{\partial N_2} + \left[\frac{\partial \frac{R_1^\top \gamma}{R_2^\top \gamma}}{\partial N_2} + k \frac{\partial \Phi}{\partial \phi} \frac{\partial \phi}{\partial N_2} \right] \delta_1. \tag{92}$$

From (85), $\partial \Phi / \partial \phi > 0$. Also,

$$\frac{\partial \frac{R_1^\top \gamma}{R_2^\top \gamma}}{\partial N_2} = \frac{\rho R_2^\top \gamma - R_1^\top \gamma}{(R_2^\top \gamma)^2} = -\frac{(1-\rho^2) N_1}{(R_2^\top \gamma)^2},$$

which is negative. Hence, from (92), $\partial \delta_1 / \partial N_2 < 0$.

(iii) Proof of $\partial G_1 / \partial N_2 < 0$: From (91),

$$\begin{aligned}
\frac{\partial G_1}{\partial N_2} &= \frac{(\gamma^\top R \gamma)\big[R_2^\top \gamma - 2R_1^\top \gamma + (1-2\rho) R_2^\top \gamma\big] - 2(R_2^\top \gamma)(R_2^\top \gamma - 2R_1^\top \gamma)(R_2^\top \gamma)}{(\gamma^\top R \gamma)^2} \\
&\propto (1 - \mathcal{V}_2)(R_2^\top \gamma - 2R_1^\top \gamma) + R_1^\top \gamma - \rho R_2^\top \gamma \\
&= \frac{(1-\rho^2) N_1^2}{N_1^2 + N_2^2 + 2\rho N_1 N_2} \big[(\rho - 2) N_1 + (1-2\rho) N_2\big] + (1-\rho^2) N_1 \\
&\propto N_1 \big[(\rho - 2) N_1 + (1-2\rho) N_2\big] + (N_1^2 + N_2^2 + 2\rho N_1 N_2) \\
&= N_2^2 + N_1 N_2 - (1-\rho) N_1^2,
\end{aligned}$$

which is negative if and only if $N_2 < \frac{\sqrt{5-4\rho}-1}{2} N_1$. This condition is satisfied for $\rho \leq 1/2$ and $N_2 \leq N_1/3$.

(iv) Proof of $\partial G_2/\partial N_2 < 0$: This follows from (22) and (36). □

Proof of Lemma 8.1 Equation (37) follows from (30) and (36). Differentiating, we obtain

$$\frac{\partial \mathcal{U}_\ell^I}{\partial N_\ell} \propto (1-\mathcal{V}_\ell)F'(\phi)\frac{\partial \phi}{\partial N_\ell} - F(\phi)\frac{\partial \mathcal{V}_\ell}{\partial N_\ell}. \tag{93}$$

Differentiating (21) gives us

$$\frac{\partial \mathcal{V}_\ell}{\partial N_\ell} = \frac{2R_\ell^\top \gamma \left[\gamma^\top R\gamma - (R_\ell^\top \gamma)^2\right]}{(\gamma^\top R\gamma)^2} = \frac{2R_\ell^\top \gamma(1-\mathcal{V}_\ell)}{\gamma^\top R\gamma}.$$

The derivative $\partial\phi/\partial N_\ell$ is given by (90). Therefore, from (93),

$$\frac{\partial \mathcal{U}_\ell^I}{\partial N_\ell} \propto -2F(\phi)\frac{R_\ell^\top \gamma}{\gamma^\top R\gamma} + k^{-1}F'(\phi)(2-\mathcal{V}_\ell^{-1}).$$

The function F is given by (29), and its derivative F' by (87). Using (34),

$$\begin{aligned}
\frac{\partial \mathcal{U}_\ell^I}{\partial N_\ell} &\propto -\phi(k\phi+1)(k\phi+2)\frac{R_\ell^\top \gamma}{\gamma^\top R\gamma} + k^{-1}(2-\mathcal{V}_\ell^{-1}) \\
&= -\phi(k\phi+1) + k^{-1}(2-\mathcal{V}_\ell^{-1}) \\
&\propto -\left[\frac{\gamma^\top R\gamma}{R_\ell^\top \gamma} - 2\right]\left[\frac{\gamma^\top R\gamma}{R_\ell^\top \gamma} - 1\right] + \left[2 - \frac{\gamma^\top R\gamma}{(R_\ell^\top \gamma)^2}\right] \\
&= -\mathcal{V}_\ell^{-1}(\gamma^\top R\gamma - 3R_\ell^\top \gamma + 1) \\
&= -\mathcal{V}_\ell^{-1}\left[(N_\ell - 3)R_\ell^\top \gamma + \sum_{j\neq\ell} N_j R_j^\top \gamma + 1\right],
\end{aligned}$$

which is negative if $N_\ell \geq 3$. Moreover, from (37), $\lim_{N_\ell\to\infty} \mathcal{U}_\ell^I = 0$ (since F is bounded and $\lim_{N_\ell\to\infty} \mathcal{V}_\ell = 1$). □

Proof of Proposition 8.2 We define $\bar{c}$ as the lowest possible value of $\mathcal{U}_\ell^I$ when $N_\ell = 3$. More precisely, using (37),

$$\bar{c} := \inf_X \mathcal{U}_\ell^I = \frac{\sigma_\theta^2}{2} \inf_X \left[F(\phi)(1-\mathcal{V}_\ell)\right], \tag{94}$$

where

$$X := \left\{\{N_i\}_{i\in L} \,\middle|\, N_\ell = 3, N_i \geq 1 \text{ for some } i \neq \ell\right\}.$$

Note that $\gamma^\top R\gamma/R_\ell^\top \gamma > N_\ell$. If $N_\ell = 3$, then $\phi > k^{-1}$ from (34), and hence $F(\phi) > (3/4)k^{-1}$ from (29). Moreover, for any $i \neq \ell$, $\lim_{N_i\to\infty} \mathcal{V}_\ell = \rho_{i\ell}^2 < 1$, and hence $\inf_X(1-\mathcal{V}_\ell) > 0$. It follows that $\bar{c} > 0$. Now we see from Lemma 8.1 that $\partial N_\ell^e/\partial c_\ell < 0$ and $\lim_{c_\ell\to 0} N_\ell^e = \infty$. Proposition 6.2 (iii) implies that

$N_\ell^e < \hat{N}_\ell^e$. $\square$

Proof of Proposition 8.3 *Proof of (i):* We need to assume that $R_i \geq 0$ in order to ensure that our standing assumption that $R_j^\top \gamma \geq 0$ for all j is not violated for large N_i. Let $N_i \to \infty$, and suppose the sequence $\{N_\ell^e(N_i)\}$ is bounded. Then $\phi \to \infty$ from (34), and hence $F(\phi) \to k^{-1}$ from (29). Also, $\mathcal{V}_\ell \to \rho_{i\ell}^2$ from (21), so that, from (37), $\lim_{N_i \to \infty} \mathcal{U}_\ell^I = \sigma_\theta^2 (2k)^{-1}(1 - \rho_{i\ell}^2)$. Moreover, the same limit is obtained if N_ℓ is fixed at 3. Hence, due to the definition of $\bar{c}$, given in (94), $\lim_{N_i \to \infty} \mathcal{U}_\ell^I \geq \bar{c} > c_\ell$. This is a contradiction. Hence $\lim_{N_i \to \infty} N_\ell^e = \infty$.

Proof of (ii): Now we specialize to the case where all pairwise correlations are the same, and $N_\ell + N_i \geq N_j$ for all $j \neq \ell, i$. From Proposition 7.3 (iii), $\partial \phi / \partial N_i > 0$. From (21),

$$\frac{\partial \mathcal{V}_\ell}{\partial N_i} = \frac{2\rho(R_\ell^\top \gamma)(\gamma^\top R \gamma) - 2(R_\ell^\top \gamma)^2 (R_i^\top \gamma)}{(\gamma^\top R \gamma)^2} \propto \rho \gamma^\top R \gamma - (R_\ell^\top \gamma)(R_i^\top \gamma),$$

which is negative (see the proof of Proposition 7.3 (iii)). Consequently $\partial \mathcal{U}_\ell^I / \partial N_i > 0$ from (37). Using Proposition 8.2, it follows that $\partial N_\ell^e / \partial N_i > 0$. $\square$

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