Received: 29 June 2022 | Revised: 29 March 2023 | Accepted: 2 April 2023

DOI: 10.1111/manc.12437

ORIGINAL ARTICLE

The Manchester School WILEY

The effect of changes in the terms of trade on GDP and welfare: A Divisia approach to the System of National Accounts

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Abstract

What effect, if any, do changes in the terms of trade have on the level of output (GDP) or welfare? I examine this issue through two versions of a textbook, Heckscher-Ohlin-Samuelson (HOS), two-good model of a small, open economy. In the first version both goods are for final consumption. In the second, one good is an imported intermediate input into the other. In both versions, economic theory suggests that an improvement in the terms of trade raises welfare (consumption) but leaves aggregate output (GDP) unchanged. I then show that a national income accountant applying the principles of the 2008 System of National Accounts (SNA) would reach the same conclusions. This follows from a continuous-time analysis using Divisia index numbers. However in the case where imports are intermediate inputs and competition is imperfect, an improvement in the terms of trade does raise GDP: the size of the effect depends on the size of the markup of price over marginal revenue. I argue that the continuous time Divisia approach is the right framework for national income accounting, even though it can only be implemented approximately in practice. If the aim is to find the best approximation to the Divisia index, then the chained Fisher index (as used in the US and Canadian national accounts) or the chained Törnqvist are better approximations than is the chained Laspeyres (as used in Europe).

KEYWORDS
Divisia, GDP, Heckscher-Ohlin-Samuelson, SNA, terms of trade, welfare

JEL CLASSIFICATION
E01, F11, C43, D60

1 | INTRODUCTION[1]

In popular discussion GDP is often treated as a measure of welfare but national income accountants never tire of pointing out that it is designed to be a measure of output or income (e.g. European Commission et al., 2009). There are a number of well-known reasons why a measure of output may differ from one of welfare. For example, GDP is gross of capital consumption, and the position of the production boundary is somewhat arbitrary: the imputed rent of owner-occupiers is included while unpaid house work and child care are excluded. Moreover the treatment of environmental assets is unsatisfactory. But in this paper I am concerned with a much simpler issue: how should changes in the terms of trade be treated in the national accounts?

This issue has been debated by both national income accountants and economists for decades. It is discussed in the volumes setting out both the 1993 and the 2008 System of National Accounts (SNA): see Commission of the European Communities et al. (1993) and European Commission et al. (2009). But there is no agreement even within these manuals about which price index should be used to compute the real gain or loss from changes in the terms of trade, the so-called trading gain. Nonetheless both these versions of the SNA agree on the distinction between real GDP and real Gross Domestic Income (GDI): real GDI equals real GDP plus the trading gain. The SNA manuals are also clear that real GDI is a measure of welfare, or at least a step on the road to a more comprehensive measure of welfare, while GDP is a measure of output. On the other hand, Diewert and Morrison (1986) have questioned this distinction between welfare and GDP, suggesting that an improvement in the terms of trade should be treated as a form of technical progress; Fox and Kohli (1998) have applied their approach to Australia, 1960–1992. The distinction between real GDI and real GDP is empirically important at least for some countries, for example, Canada and Switzerland (Kohli, 2006, 2022). The allegedly declining terms of trade of primary producers in the 1950 and 1960s (the Prebisch thesis), the recently ended commodity price boom, the oil price shocks of the 1970 and 1980s, the gains to countries which can import ICT products at rapidly falling prices (Oulton, 2012b), and now the cost-of-living crisis, all these make changes in the terms of trade a subject of perennial interest.

The approach of this paper is to consider some very simple models of trading economies and calculate from first principles the changes in output and welfare which follow from a change in the terms of trade. The first such model, Model 1, the Heckscher-Ohlin-Samuelson (HOS) model of a small economy producing and trading consumption goods, predicts that (under certain assumptions) an improvement in the terms of trade, that is, an increase in the price of exports relative to that of

[1] This is a revised version of an earlier paper "GDP is a measure of output, not welfare. Or, HOS meets the SNA", Centre for Macroeconomics Paper No CFM-DP2019-06, March 2019.

imports, raises economic welfare. I then ask whether a national income accountant, applying the principles of the SNA to this theoretical model, would agree.

The second simple model, Model 2, also of the HOS form, again has two goods but now one of them, the imported good, is an intermediate input into the other. This is the type of model that has been used to analyse an oil price shock. Again I ask whether the theorist and the national income accountant would reach the same conclusions. Both these models are oversimplified and ignore many real world features. But considering them serves to illustrate the principles involved. And if we can't understand the relationship between GDP and welfare in these simple cases we will certainly fail to do so in more complicated ones.[2]

The national income accountant is assumed to use Divisia indices to calculate real GDP and consumption. Divisia indices have many desirable properties. One of their great advantages is that the product of the Divisia price index and the Divisia quantity index is the value index. Another is that they are consistent in aggregation (though this latter property is not used in the present paper).[3] Divisia price indices are also true cost-of-living (Konüs) indices when demand is homothetic.[4] However Divisia indices are defined in continuous time which may be thought a great drawback from a practical point of view. But it will be argued below (Section 7) that this drawback is much smaller than it first appears.

The rest of the paper is structured as follows. In the next Section 1 write down the economic relationships of Model 1 in mathematical form. In Section 3 I set out the national accounts of this textbook economy. I consider whether the national income accountant, with access to all the necessary data, would reach qualitatively the same conclusions as the theorist. If so, the national income accountant can go one better than the theorist by actually quantifying the changes in output and welfare following a change in the terms of trade. The conclusion is that theorist and accountant would agree that real consumption rises while real GDP is constant. Section 4 then goes on to consider the case where one of the goods is an intermediate input into the other (Model 2). Theorist and accountant are again in agreement that real consumption rises while real GDP is constant. But now there is an important qualification. GDP is constant under perfect competition. Under imperfect competition GDP increases when the terms of trade improve, an effect captured by the national income accountant but which might be missed by a theorist using the wrong model. For both Models 1 and 2 I employ a continuous time approach and use Divisia index numbers. Section 5 starts to get closer to real life measurement by comparing a discrete Laspeyres quantity index (a Eurostat requirement for EU member states) with a Divisia one. I show that a Laspeyres index is biased downwards when the terms of trade change, in both Models 1 and 2. Both these models are static: savings and investment have been excluded so the trade balance is always zero. So Section 6 discusses how the results might be affected when saving and investment are allowed. Section 7 discusses the pros and cons of a discrete versus a continuous approach as the conceptual framework behind national income accounting. It also compares US with European methods in the national accounts. Section 8 concludes.

[2]Reinsdorf (2010) considers a similar range of issues from the perspective of discrete index numbers rather than continuous (Divisia) ones as here. He does not emphasise the output versus welfare question.

[3]Consistency in aggregation means that a price (quantity) index composed out of prices (quantities) of goods and services at the lowest level yields the same result as a price (quantity) index composed in stages, first producing price (quantity) indices for sub-aggregates and then aggregating over the sub-aggregate indices; see Balk (2008), Section 3.7 for a theoretical analysis.

[4]They were originated by Divisia (1925-1926) and have been analysed by Hulten (1973) and Balk (2005). They were introduced into productivity analysis by Griliches and Jorgenson (1967). The relationship between Konüs and Divisia price indices is analysed in Oulton (2008) and (2012a). Jorgenson and Griliches (1971) argued that they should form the theoretical basis for economic measurement, the approach adopted here.

2 | ECONOMIC RELATIONSHIPS IN MODEL 1

Figure 1a,b illustrate the first model and show the textbook analysis of the gain from an improvement in the terms of a trade in a small open economy. Figure 1a shows the original position and Figure 1b shows the position after the change in the terms of trade. In this simple form of the HOS model there are two goods and two factors of production which we can label land and labour, both inelastically supplied. Both goods are produced under constant returns to scale and are for final consumption. Hence there is a concave production possibility frontier or transformation curve showing possible combinations of output of each of the two goods given the factor endowments and the level of technology; this is the curve labelled PP′ in Figure 1a,b. All markets are perfectly competitive so production takes place on the frontier. The country is a price taker in international trade, shown by the initial terms of trade line TT′. The point P_0 marks the initial production point (the tangency of the production possibility curve with the terms of trade line), and the point C_0 the initial consumption point (the tangency of the terms of trade line with the highest available indifference curve, labelled U_0U_0'). The country has comparative advantage in good 1, exporting CD of good 1 and importing AB of good 2 in exchange.

Now there is an exogenous rise in the relative price of the export good (good 1): in Figure 1b the terms of trade line rotates from TT′ to T″T‴. So resources shift into good 1 and away from good 2; the production point moves from P_0 to P_1. Clearly this generates an improvement in *potential* welfare: The country can now consume at any point along the new terms of trade line. So potentially the country could choose a point like C_1 which lies to the north-east of the initial point C_0 and consume more of both goods. However to show that the change in the terms of trade generates an improvement in *actual* welfare requires more assumptions. The reason is that distributional issues cannot be ignored in the HOS model since goods prices determine factor prices. Suppose the export good is land-intensive. Then a rise in the relative price of good 1 raises the real rent on land and lowers the real wage (the Stolper-Samuelson theorem). To avoid these distributional issues assume that the population is composed of individuals who have equal shares in the endowments of land and labour. So they only care about their total income and not about factor prices per se. If all individuals have identical preferences and each maximises a conventional (strictly concave) utility function which depends on consumption of the two goods, we can draw representative indifference curves as in Figure 1a,b to indicate the actual level of welfare enjoyed before and after the change in the terms of trade. Clearly the country now enjoys a higher level of welfare since the representative consumer is now on a higher indifference curve, at point C_1 on the higher indifference curve U_1U_1' rather than at the initial point C_0.[5]

Figure 1 is an exercise in comparative statics. So the time period over which the terms of trade are supposed to change is left unclear. In the context of national income accounting, it is helpful to suppose that the change takes place continuously over a discrete time interval, 0 to T. As we shall see, this enables us to employ the powerful apparatus of Divisia indices to analyse the change.

Let us now write down the basic relationships of Figure 1 in mathematical terms.

(a) *The production possibility frontier*

The production possibility frontier (or transformation curve) can be defined implicitly as:

$$F(Y_1, Y_2;\ R, L, \tau) = 0 \tag{1}$$

[5]As drawn, consumers enjoy more of both goods at C_1. This is not a necessary outcome since the price of good 1 has risen. But even if consumption of good 1 fell there is still a rise in welfare on the assumptions made here.

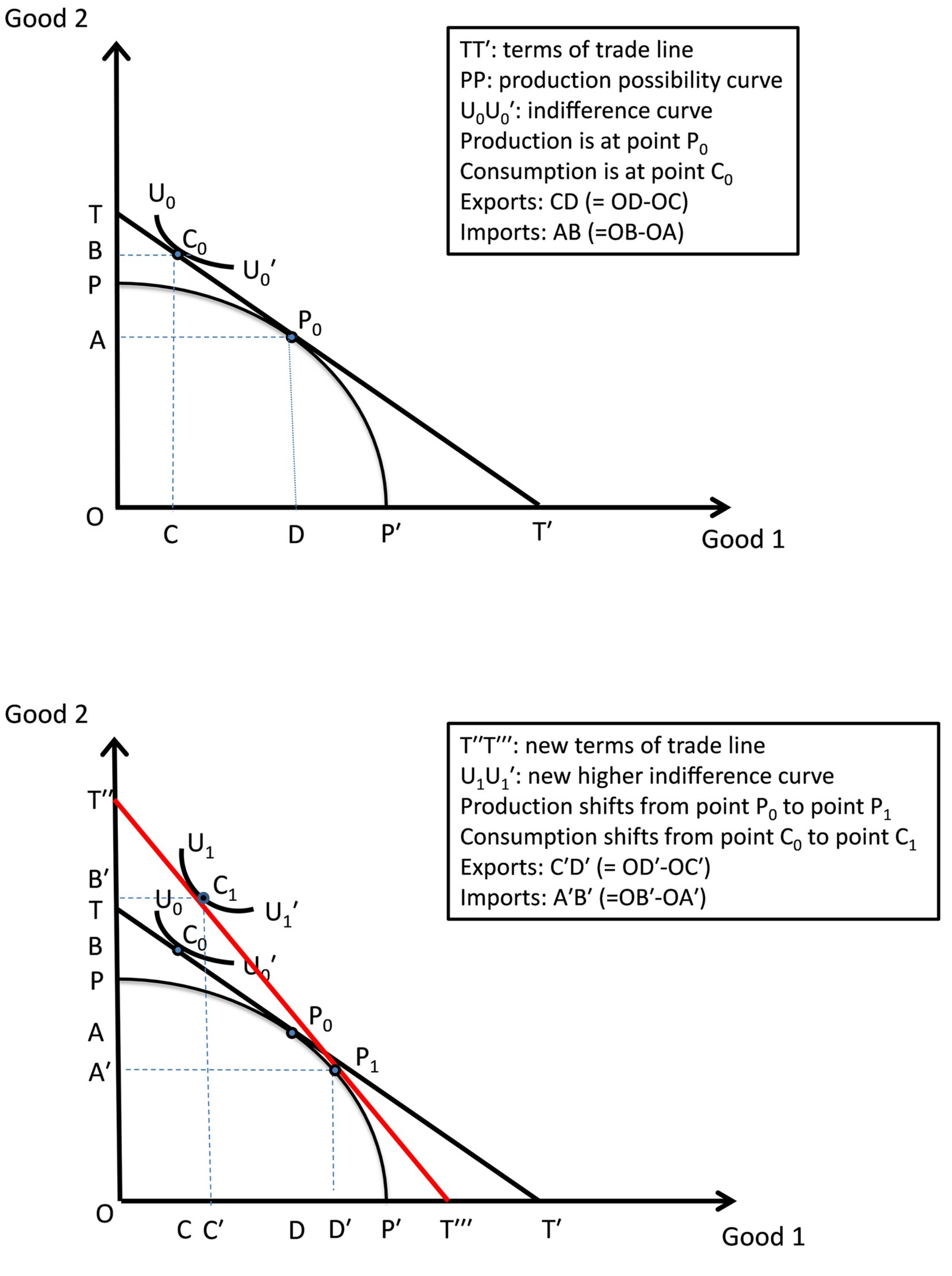

FIGURE 1 (a) Equilibrium in the HOS model of a small open economy, (b) The new equilibrium in the HOS model after an improvement in the terms of trade. The new terms of trade are given by the slope of the red line T″T‴.

Here Y_1, Y_2 are the outputs of the two goods, considered to be functions of time (t); the endowments of land (R) and labour (L) and the level of technology (τ) are assumed constant. Differentiating Equation (1) with respect to time (t):

$$\frac{dY_2}{dt} = -\left(\frac{\partial F/\partial Y_1}{\partial F/\partial Y_2}\right)\frac{dY_1}{dt} \tag{2}$$

Here $(\partial F/\partial Y_1)/(\partial F/\partial Y_2)$ is the marginal rate of transformation between goods 1 and 2 and so in a perfectly competitive economy this equals the relative price of the two goods, P_1/P_2. With a little bit of algebra, including dividing through by the total value of output $(P_1Y_1 + P_2Y_2)$, Equation (2) becomes

$$s_{Y_1}^{GDP}\hat{Y}_1 + s_{Y_2}^{GDP}\hat{Y}_2 = 0 \tag{3}$$

where

$$s_{Y_1}^{GDP} := \frac{P_1Y_1}{P_1Y_1 + P_2Y_2}, s_{Y_2}^{GDP} := \frac{P_2Y_2}{P_1Y_1 + P_2Y_2}$$

are the shares of each product in the total value of output (nominal GDP), a hat (^) denotes a growth rate, for example, $\hat{Y}_1 = d\log Y_1/dt$, and the symbol ": = " means "is defined to be".

(b) *Utility and welfare*

Let the representative consumer's expenditure function be given by

$$x = c(P_1, P_2)u \tag{4}$$

where x is expenditure, $c(\cdot)$ is a strictly concave function of relative prices and u is utility. Here I am going a bit beyond what is strictly implied by Figure 1 since I am assuming that consumer demand is homothetic, in which case the expenditure function can be written in multiplicative form as in (4). Using (4) and selecting any arbitrary level of utility $\bar{u}$, a true cost-of-living (Konüs) index P_C at time t relative to time 0 is

$$\frac{P_C(t)}{P_C(0)} = \frac{c(P_1(t), P_2(t))\bar{u}}{c(P_1(0), P_2(0))\bar{u}} = \frac{c(P_1(t), P_2(t))}{c(P_1(0), P_2(0))} \tag{5}$$

In this case the Konüs price index is independent of the chosen level of utility.[6] Applying Shephard's Lemma (Varian, 1992, p. 74), the growth rate of this price index at any point t in the time interval (0, T) is

$$\hat{P}_C(t) = s_1^C(t)\hat{P}_1(t) + s_2^C(t)\hat{P}_2(t) \tag{6}$$

where s_1^C, s_2^C are the shares of goods 1 and 2 in the value of consumption:

$$s_1^C := \frac{P_1C_1}{P_1C_1 + P_2C_2}, s_2^C := \frac{P_2C_2}{P_1C_1 + P_2C_2} \tag{7}$$

[6]The true cost-of-living index was introduced by Konüs (1939). On the relationship between homotheticity and true cost-of-living indices see Hulten (1973), Samuelson and Swamy (1974), and Deaton and Muellbauer (1980), chapter 7.

So in this case the Konüs price index is also a Divisia index. The growth of the corresponding Divisia quantity index of consumption is

$$\hat{C}(t) = s_1^C(t)\hat{C}_1(t) + s_2^C(t)\hat{C}_2(t) \tag{8}$$

The total change in welfare over the period (0, T) can then be measured by the change in real consumption, that is, nominal consumption deflated by the price index:

$$\log\left[\frac{C(T)}{C(0)}\right] = \log\left[\frac{P_1(T)C_1(T) + P_2(T)C_2(T)}{P_1(0)C_1(0) + P_2(0)C_2(0)}\right] - \log\left[\frac{P_C(T)}{P_C(0)}\right] \tag{9}$$

where from (6)

$$\log\left[\frac{P_C(T)}{P_C(0)}\right] = \int_0^T \left[s_1^C(t)\hat{P}_1(t) + s_2^C(t)\hat{P}_2(t)\right] dt \tag{10}$$

Alternatively the total change in real consumption can be expressed directly in terms of the quantity index:

$$\log\left[\frac{C(T)}{C(0)}\right] = \int_0^T \left[s_1^C(t)\hat{C}_1(t) + s_2^C(t)\hat{C}_2(t)\right] dt \tag{11}$$

Note that all the prices and quantities in Equations (9), (10) and (11) are observable.[7]

So far we have viewed the Konüs price index as just an ideal cost-of-living index. We may note in passing that there is also an interpretation in terms of the compensating variation: the amount that a household must be paid (or taxed) after some change in prices to give it the same utility level as the one it started with. Following Hicks (1945-46) and Hausman (2003) the representative consumer's compensating variation (CV) between two periods 0 and T for a given utility level $\bar{u}$ is:

$$CV = e\left(\mathbf{P}(T), \bar{u}\right) - e\left(\mathbf{P}(0), \bar{u}\right)$$

where $e(\mathbf{P}(t), u)$ is the expenditure function, which may be non-homothetic, and $\mathbf{P}(t)$ is the price vector at time t. The close connection with the Konüs price index is clear since the latter measures the price level in period T relative to period 0 by

$$P_C(T) = \frac{e\left(\mathbf{P}(T), \bar{u}\right)}{e\left(\mathbf{P}(0), \bar{u}\right)}$$

and the discrete growth rate of the Konüs price index between periods 0 and T is

$$P_C(T) - 1 = \frac{e\left(\mathbf{P}(T), \bar{u}\right)}{e\left(\mathbf{P}(0), \bar{u}\right)} - 1 = \frac{CV}{e\left(\mathbf{P}(0), \bar{u}\right)}$$

[7]If demand is non-homothetic then Konüs and Divisia indices are not identical. Welfare measures now depend on the viewpoint: whose utility is to be the reference point when welfare changes are to be measured in monetary terms?
See Oulton (2008) and (2012a) for practical ways in which Konüs indices can be estimated from real world data.
Non-homotheticity is also a problem for discrete index numbers such as chained Laspeyres or chained Fisher, making their interpretation problematic. The counterpart to non-homotheticity on the output side is non-constant returns to scale, assumed away in the models discussed here. Again, non-constant returns are a problem for both continuous (Divisia) and discrete index numbers.

That is, the growth rate of the Konüs index over a discrete period of time is the CV generated by the price change as a proportion of the original expenditure level. This shows that, contrary to a common view, the growth of a Konüs price index, and also that of a Divisia index when demand is homothetic, measures the change in consumer surplus resulting from price changes. So although the value of consumption does not include the level of consumer surplus, *changes* in real consumption (when measured by a Divisia index) do include *changes* in consumer surplus.

3 | NATIONAL INCOME ACCOUNTING IN MODEL 1

(a) *The national accounts in Model 1*

In this Section 1 set out the national accounts of the HOS economy depicted in Figure 1. Here we must be careful to distinguish between relationships which derive entirely from the principles of the SNA and those which also rest on particular empirical features of the HOS model, such as that trade always balances.

A national income accountant measuring this economy would note the following supply-use relationships[8]

$$P_1 Y_1 = P_1 C_1 + P_1 X_1 \tag{12}$$

$$P_2 Y_2 = P_2 C_2 - P_2 M_2 \tag{13}$$

The accountant would then go on to define nominal GDP from the expenditure and output sides as follows:

$$GDP(E) := P_E E \equiv P_1 C_1 + P_2 C_2 + P_1 X_1 - P_2 M_2 \tag{14}$$

$$GDP(O) := P_Y Y \equiv P_1 Y_1 + P_2 Y_2 \tag{15}$$

Here GDP(E) is conceived of as a price index (P_E) times a quantity index (E) and similarly GDP(O) is conceived of a price index (P_Y) times a quantity index (Y). Adding Equations (12) and (13) shows that GDP(E) = GDP(O) or

$$P_E E = P_Y Y \tag{16}$$

National accountants are interested in growth rates as well as levels. So to obtain Divisia price and quantity indices, totally differentiate Equations (14) and (15) with respect to time:

$$\begin{aligned} \hat{P}_E + \hat{E} = & \left[s_1^{GDP} \hat{P}_1 + s_2^{GDP} \hat{P}_2 + s_X^{GDP} \hat{P}_1 - s_M^{GDP} \hat{P}_2) \right] \\ & + \left[s_1^{GDP} \hat{C}_1 + s_2^{GDP} \hat{C}_2 + s_X^{GDP} \hat{X}_1 - s_M^{GDP} \hat{M}_2) \right] \end{aligned} \tag{17}$$

$$\hat{P}_Y + \hat{Y} = \left[s_{Y_1}^{GDP} \hat{P}_1 + s_{Y_2}^{GDP} \hat{P}_2 \right] + \left[s_{Y_1}^{GDP} \hat{Y}_1 + s_{Y_2}^{GDP} \hat{Y}_2 \right] \tag{18}$$

Here s_1^{GDP}, s_2^{GDP} are the shares of consumption of the two goods in nominal GDP and $s_{Y_1}^{GDP}, s_{Y_2}^{GDP}$ are the shares of output of the two goods in nominal GDP. Identifying terms in prices with the price indices and terms in quantities with the quantity indices we have:

[8] In principle the accountant would allow for the possibility that the country also exports good 2 and imports good 1 (two-way trade). For simplicity I ignore this since in the model this cannot happen as the goods are assumed to be homogeneous,

$$\hat{P}_E = \left[s_1^{GDP}\hat{P}_1 + s_2^{GDP}\hat{P}_2 + s_X^{GDP}\hat{P}_1 - s_M^{GDP}\hat{P}_2\right] \tag{19}$$

$$\hat{E} = \left[s_1^{GDP}\hat{C}_1 + s_2^{GDP}\hat{C}_2 + s_X^{GDP}\hat{X}_1 - s_M^{GDP}\hat{M}_2)\right] \tag{20}$$

$$\hat{P}_Y = \left[s_{Y_1}^{GDP}\hat{P}_1 + s_{Y_2}^{GDP}\hat{P}_2\right] \tag{21}$$

$$\hat{Y} = \left[s_{Y_1}^{GDP}\hat{Y}_1 + s_{Y_2}^{GDP}\hat{Y}_2\right] \tag{22}$$

Taking account again of Equations (12) and (13) we conclude that

$$\hat{P}_E = \hat{P}_Y \tag{23}$$

and

$$\hat{E} = \hat{Y} \tag{24}$$

Since the growth rates are always equal the levels of E and Y are always equal too provided that we choose the same reference period for the price indices (i.e. $P_E(r) = P_Y(r) = 1$ in some reference period r). In other words, real GDP(E) equals real GDP(O): $E(t) = Y(t)$, and $P_E(t) = P_Y(t)$, all t.[9]

The national income accountant would also wish to calculate the growth of real consumption which can be measured as nominal consumption deflated by the Consumer Price Index or directly by an index of real consumption. The CPI can be expressed as a Divisia price index and real consumption can be measured as a Divisia quantity index: These results have already been derived: see Equations (6) and (8) above. Note that since the trade balance is zero in this model, real consumption corresponds to real Gross Domestic Income (GDI) which is considered to be a measure of welfare in the SNA (see the Supplementary Information S1 online Annex).

(b) *The trade balance*

In the textbook model of Figure 1 trade always balances as there is no saving or investment:

$$B := P_1X_1 - P_2M_2 = 0 \tag{25}$$

Here B is the trade balance, X_1 is exports of good 1 and M_2 is imports of good 2. So differentiating with respect to time and dividing through by the value of output (GDP):

$$\frac{1}{GDP}\frac{dB}{dt} = \left[\frac{P_1X_1}{GDP}\frac{1}{P_1}\frac{dP_1}{dt} - \frac{P_2M_2}{GDP}\frac{1}{P_2}\frac{dP_2}{dt}\right] + \left[\frac{P_1X_1}{GDP}\frac{1}{X_1}\frac{dX_1}{dt} - \frac{P_2M_2}{GDP}\frac{1}{M_2}\frac{dM_2}{dt}\right] = 0$$

Defining $s_X^{GDP} := \frac{P_1X_1}{GDP}, s_M^{GDP} := \frac{P_2M_2}{GDP}$ as the shares of exports and imports in nominal GDP, and noting from (25) that $s_M^{GDP} = s_X^{GDP}$, the last equation can be rearranged as

[9]The equality of real GDP(O) and real GDP(E) when Divisia indices are employed was proved in the more general case with many goods and with intermediate consumption in Oulton (2004b). Both there and here the same price was assumed to apply for a given product whatever the use to which the product was put (e.g. exports, consumption or investment) and each industry was assumed to produce only one product. The more realistic case where industries and products are distinguished and where there is price discrimination or product heterogeneity is examined in Oulton et al. (2018) and the equality of real GDP(O) and real GDP(E) is shown to still hold.

$$\left(\hat{P}_1 - \hat{P}_2\right) = -\left(\hat{X}_1 - \hat{M}_2\right) \tag{26}$$

In words: with balanced trade, if the terms of trade improve, then import volumes rise faster than export volumes. This relationship connects changes in volumes and prices and will be used below.

(c) *The national income accountant's conclusions*

If the national income accountant carried out these calculations for the economy of Figure 1 over the interval (0, T), what conclusions would he or she reach? Consider first GDP. The Divisia index of aggregate output (Y) can be written as

$$\hat{Y} = s_{Y_1}^{GDP}\hat{Y}_1 + s_{Y_2}^{GDP}\hat{Y}_2 \tag{27}$$

and we have already seen from (3) that the right hand side of (27) is zero. Hence

$$\hat{Y}(t) = 0, 0 \leq t \leq T \tag{28}$$

in this model economy. So the total change in output over the interval [0, T] is also zero:

$$\log\left[\frac{Y(T)}{Y(0)}\right] = \int_o^T \hat{Y}(t)dt = 0 \tag{29}$$

Equations (3) and (27) say that a reallocation of factors, raising the output of one industry while reducing that of another, with endowments and technology held constant, leaves aggregate output unchanged. This makes perfectly good sense economically: only an increase in the endowment of one or both factors or an improvement in technology can increase aggregate output. In other words we are identifying an outward movement in the production possibility frontier (due say to technical progress, land reclamation or population growth) with an increase in aggregate output. But this does have an important implication: in the economy of Figure 1 welfare can increase while output (GDP) remains the same. Consequently, GDP must be interpreted as a measure of output but not of welfare.[10]

Second, the accountant would note that real consumption has increased. Empirically trade is balanced so from (14) and (15) $P_1C_1 + P_2C_2 = P_1Y_1 + P_2Y_2$. Hence

$$s_1^C = s_1^{GDP}, s_2^C = s_2^{GDP}, \text{and } s_M^{GDP} = s_X^{GDP}$$

Now from (20) and (24) and applying the definition of real consumption growth in Equation (8),

$$\begin{aligned}\hat{Y} &= \hat{C} + s_M^{GDP}\left(\hat{X}_1 - \hat{M}_2\right)\\ &= \hat{C} - s_M^{GDP}\left(\hat{P}_1 - \hat{P}_2\right)\end{aligned} \tag{30}$$

where use is made of (26). But as we have just seen, $\hat{Y} = 0$ so

$$\hat{C} = s_M^{GDP}\left(\hat{P}_1 - \hat{P}_2\right) > 0 \tag{31}$$

[10]Reinsdorf (2010), who employs a figure similar to Figure 1 by way of illustration, concludes too that aggregate output is constant in this case and for the same reason: there is a movement along the production possibility frontier but no shift in the frontier. Kohli (2022) reaches a similar conclusion.

So as long as the terms of trade are improving, real consumption is rising. More generally, we conclude that consumption (welfare) is rising faster than GDP if the terms of trade are improving:

$$\hat{C} > \hat{Y} \text{ if } \hat{p} > 0 \tag{32}$$

A subtle point here is that, empirically, the value of consumption is always equal to the value of GDP: $P_C C = P_Y Y$ because the balance of trade is zero. Nonetheless, the volume of consumption is growing faster than the volume of output and this is made possible by the improvement in the terms of trade.

Summing up we have.

Proposition 1. *In the HOS model with two consumption goods, an improvement in the terms of trade increases consumption and welfare but leaves GDP unchanged.*

In other words the theorist and the national income accountant would be in agreement about the effect of an improvement in the terms of trade in the HOS model. The fact that at a point in time output (GDP) is not growing (equation (28)) while consumption is growing (equation (32)) is a local, first order result for a small change in the term of change. But it also gives the correct result for a large change, and a way of calculating it, by integrating over the small changes as in equation (29).

4 | TERMS-OF-TRADE EFFECTS WHEN IMPORTS ARE NOT CONSUMER GOODS

4.1 | Imported intermediate inputs

What difference would it make if one of the goods served as an input into the production of the other? Let us consider the simplest possible case of an intermediate input. Suppose that the country is completely specialised in the production of good 1, part of whose output is exported to pay for imports of good 2 which is used as an intermediate input, say energy. This corresponds to the much analysed case of a country which imports but does not produce energy products like oil or gas. We continue to consider an improvement in the terms of trade (a fall in the relative price of energy).

Consider first the national accounts. Supply and use of good 1 must be equal:

$$P_1 Y_1 = P_1 C_1 + P_1 X_1 \tag{33}$$

and nominal GDP is now

$$\begin{aligned} GDP(E) &:= P_E E \equiv P_1 C_1 + P_1 X_1 - P_2 M_2 \\ GDP(O) &:= P_Y Y \equiv P_1 Y_1 - P_2 M_2 \end{aligned} \tag{34}$$

Using (33) we see that $GDP(E) = GDP(O)$. Furthermore, if trade is balanced (Equation (25)) then GDP(E) equals nominal consumption which also equals nominal value added or nominal GDP(O):

$$GDP(E) = P_E E = P_1 C_1 = P_1 Y_1 - P_2 M_2 = P_Y Y = GDP(O) \tag{35}$$

By totalling differentiating the relationships in Equation (34) with respect to time, and separating terms in prices and quantities, we obtain

$$\hat{P}_E = \left[s_1^{GDP} \hat{P}_1 + s_X^{GDP} \hat{P}_1 - s_M^{GDP} \hat{P}_2 \right] = \hat{P}_1 + s_M^{GDP} \left(\hat{P}_1 - \hat{P}_2 \right) \tag{36}$$

$$\hat{E} = \left[s_1^{GDP} \hat{C}_1 + s_X^{GDP} \hat{X}_1 - s_M^{GDP} \hat{M}_2) \right] = \hat{C}_1 + s_M^{GDP} \left(\hat{X}_1 - \hat{M}_2 \right) \tag{37}$$

$$\hat{P}_Y = \left(1 + s_M^{GDP}\right)\hat{P}_1 - s_M^{GDP}\hat{P}_2 \tag{38}$$

$$\hat{Y} = \left(1 + s_M^{GDP}\right)\hat{Y}_1 - s_M^{GDP}\hat{M}_2 \tag{39}$$

using the fact that $s_M^{GDP} = s_X^{GDP}$ from (25). These last equations may be compared with (19)-(22). As in the previous model

$$\hat{P}_Y = \hat{P}_E \text{ and } \hat{Y} = \hat{E} \tag{40}$$

For the price indices this follows directly from (36) and (38). The equality of the growth rates of the volume indices then follows since *GDP(E)* = *GDP(O)* from (35) (The equality of the volume indices can also be seen as a consequence of double deflation, implicit in equation (39)). These results rest solely on national income accounting principles together with the empirical facts that in this model economy good 1 is exported, good 2 imported, and trade is balanced. They make no use of economic theory.

However, if we want to answer substantive questions, such as, what is the effect of a fall in the price of imported energy on GDP? then we need to invoke some theory. So assume a neo-classical production function for good 1:

$$Y_1 = Y_1(R, L, M_2, \tau) \tag{41}$$

Here as before τ indexes the level of technology. Dual to this is a price (or cost) function:

$$P_1 = P_1(P_R, P_L, P_2, \tau) \tag{42}$$

Suppose as before that R and L are fixed in supply and technology is constant. Then a lower price for energy encourages producers to move down the demand curve and increase energy input so that

$$\dot{Y}_1 = \frac{\partial Y_1}{\partial M_2}\dot{M}_2$$

or

$$\hat{Y}_1 = \left(\frac{s_M^{GDP}}{1 + s_M^{GDP}}\right)\hat{M}_2 > 0 \tag{43}$$

assuming inputs are paid their marginal products, that is, that in this case $\partial Y_1/\partial M_2 = P_2/P_1$, and noting that $P_2 M_2/P_1 Y_1 = s_M^{GDP}/\left(1 + s_M^{GDP}\right)$. Plugging (43) into (39) we find

$$\hat{Y} = 0 \tag{44}$$

that is, real GDP is unchanged even though gross output of good 1 has risen. By differentiating the price function (42) and from (38) we see that the GDP deflator is constant too (relative to trend) while the price of good 1 falls: $\hat{P}_Y = 0$ and $\hat{P}_1 < 0$.

What about welfare? This is measured by the growth of consumption of good 1 which from (37) and (40) is

$$\hat{C}_1 = \hat{Y} - s_M^{GDP}\left(\hat{X}_1 - \hat{M}_2\right) > \hat{Y} \tag{45}$$

since from (26) $\hat{X} - \hat{M} < 0$ when the terms of trade are improving ($\hat{p} > 0$). Using (26) again and (44) this last equation can be written as

$$\hat{C}_1 = s_M^{GDP} \hat{p} > 0 \tag{46}$$

So consumption rises even though GDP is constant.

The marginal products of labour and land in terms of gross output of the consumption good have risen, assuming (as is usual) a positive relationship between the marginal products of the domestic inputs and the volume of the imported input, that is, that $(\partial/\partial M_2)(\partial Y_1/\partial R) > 0$ and $(\partial/\partial M_2)(\partial Y_1/\partial L) > 0$. In other words, the real consumption wage (the money wage divided by the price of consumption) has risen and so has the real consumption rent on land. But the real product wage (the money wage divided by the price of value added) and the real product rent are both unchanged since (relative to trend) the GDP deflator is unchanged.

Summing up we have.

Proposition 2. *Suppose a 2-good HOS model where the country specialises in good 1 and imports good 2, and good 2 is an input into good 1. Then a fall in the relative price of good 2 raises consumption and welfare but leaves GDP and the GDP deflator (relative to trend) unchanged.*

This conclusion is exactly the same as in the earlier model of two final consumption goods. So whether or not one of the goods is an intermediate input makes no difference. This may seem surprising given the considerable debate in the past about the effect of oil price rises on GDP and inflation, starting with Bruno and Sachs (1985) who argued that an oil price rise is a supply shock. In fact, Barsky and Kilian (2002) in re-visiting the Bruno-Sachs analysis reached the same conclusion as we have here, but they did so by making the restrictive assumption that the aggregate production function is separable into value added and energy. The argument of the present paper shows that this assumption is not necessary. Barsky and Kilian went too far however in claiming that an oil price rise is not a supply shock, that is, it cannot change real GDP under any circumstances. Their model like the present one is static with fixed input supplies. Once we introduce the possibility of growth, that is, if we drop the assumption of a fixed supply of land and allow capital to be accumulated, then effects on GDP are likely. The increase in energy input following an energy price fall raises the marginal product of both labour and capital. So an expansion of the capital stock is warranted together perhaps with an increase in labour supply (A further qualification is that the model has nothing to say about any effects on GDP via aggregate demand but only considers aggregate supply.)

Blinder and Rudd (2008) disputed the Barsky-Kilian conclusion. They based their analysis on an equation on page 13 of their paper (which they attribute to Bruno and Sachs (1985)). The right hand side of this equation can be written in my notation as $Y_1 - P_2 M_2 / P_1$. They claimed that the left hand side measures real GDP. They then show that a rise (fall) in the price of the imported input would lower (raise) what they call GDP. But reference to my Equation (34) shows that the left hand side is $(P_Y Y / P_1)$ which is not equal to real GDP (Y); in fact, the left hand side is nominal GDP deflated by the price of consumption (more generally, the price of expenditure), not by the GDP deflator. Hence their conclusion is incorrect.[11]

Kehoe and Ruhl (2008) also studied the same model as here but using discrete time. They show that if GDP is calculated by a chain-weighted Fisher index, then the first order effect of a change in the terms of trade on real GDP is zero (their Equation (41)). This leaves open the possibility that the

[11]Their analysis actually misinterprets Bruno and Sachs (1985). The latter distinguish carefully between three concepts: (1) what they call "real income", which is the right hand side of the equation labelled real GDP by Blinder and Rudd; (2) "double deflated value added", which is a fixed base index; and (3) a Divisia index of value added which is the same as the one I use here (see their chapter 2, Appendix 2B). They show that the fixed base index is biased by comparison to the true index, the Divisia.

second order effect could be significant when the change in the terms of trade is large so generating a significant effect on GDP. Other recent macro work emphasises the importance of second order effects (e.g. Baqaee & Farhi, 2019). In contrast my results for a continuous time model are more general, covering both large and small changes, and are unambiguous: zero effect on GDP.[12]

An important qualification to Proposition 2 is if there is imperfect competition in the domestic economy. Then profit-maximising firms set the price of the imported input equal to the marginal revenue product, not the value of its marginal product: $\partial Y_1/\partial M_2 = P_2/MR_1$ where MR_1 is marginal revenue in good 1. Hence (43) becomes

$$\hat{Y}_1 = \left(\frac{P_1}{MR_1}\right)\left(\frac{s_M^{GDP}}{1+s_M^{GDP}}\right)\hat{M}_2 \tag{47}$$

and plugging this into (39) we now find

$$\hat{Y} = s_M^{GDP}\left(\frac{P_1}{MR_1} - 1\right)\hat{M}_2 > 0$$

Summing up, we have:

Proposition 3. *In Model 2 under imperfect competition an improvement in the terms of trade (a fall in the relative price of the imported input), with other inputs held constant, raises real GDP and real consumption. The statistician can still use Equation (39) to measure real GDP and Equation (45) to measure real consumption, in both cases correctly.*

So in the presence of imperfect competition an improvement in the terms of trade does act like a productivity shock, raising real GDP even with other inputs held constant.[13] This is because under imperfect competition the quantity of the imported input is too low (its marginal product is too high) so an increase in its use raises output, holding constant the other inputs. In this model it is still reasonable to use output prices to measure real GDP since the terms of trade are exogenous. So the statistician using equations (39) and (45) still gets the right answer. But the perfect competition model makes the wrong prediction.[14] Despite assuming perfect competition in their theoretical model Kehoe and Ruhl (2008) actually found some empirical support for my Proposition 3, rather than my Proposition 2.

4.2 | Imported capital goods

For the sake of completeness it is worth briefly discussing too the case where the imported input is a capital good. This has been analysed by Oulton (2012b) who shows via a two-sector growth model

[12]Kehoe and Ruhl (2008) do consider the effect of large changes in the terms of trade but in a more restrictive model with a constant elasticity of substitution between the imported input and domestic inputs (their Section 6). And here they use a fixed-base, not chain-weighted, measure of real GDP. They find that a large deterioration of the terms of trade reduces GDP. But this finding is not in conflict with mine because of the fixed-base assumption. In fact it emphasises the danger of the fixed-base assumption.

[13]Gopinath and Neiman (2014) make this point explicitly. The basic idea goes back to Hall (1988); see also Basu and Fernald (2002).

[14]Proposition 2 analyses the effect of what might be called a pure change in the terms of trade. The analysis of trade liberalisation leading to changes in tariff revenues and trading costs is more complex. Now a change in domestic trading costs can have a first order effect on output and productivity as wells as on welfare (Burstein & Cravino, 2015).

that a continuing fall in the relative price of the imported capital good raises the growth rate of the stock of this good which in turn leads to faster growth of both GDP and consumption. So in contrast to the two models above an improvement in the terms of trade boosts GDP since it leads to faster capital accumulation. Welfare rises too but this is a result of the rise in GDP. These results are consistent with how the SNA would handle this case.

5 | DOWNWARD BIAS OF A LASPEYRES QUANTITY INDEX OF GDP

The analysis above vindicates the practices of national income accountants who employ real world approximations to Divisia indices (see Section 7 below). But some national statistical agencies such as EU countries and the UK employ chained Laspeyres indices which are not guaranteed to be good approximations to Divisia indices. This is problematic as it can be shown that chained Laspeyres indices are biased downwards when the terms of trade change.

A Laspeyres index of GDP between periods t and t-1 in Model 1 can be written as

$$Q^L_{t,t-1} = \frac{P_{1,t-1}Y_{1t} + P_{2,t-1}Y_{2t}}{P_{1,t-1}Y_{1,t-1} + P_{2,t-1}Y_{2,t-1}}$$

that is, all outputs are valued at the prices of t-1. Dividing numerator and denominator by $P_{1,t-1}$ and defining the terms of trade as $p_{t-1} = P_{1,t-1}/P_{2,t-1}$, this becomes

$$Q^L_{t,t-1} = \frac{p_{t-1}Y_{1t} + Y_{2t}}{p_{t-1}Y_{1,t-1} + Y_{2,t-1}}$$

Now introduce the GDP function (Diewert & Morrison, 1986; Woodland, 1982) which is defined in the two-goods case as

$$\pi(p) = \max_{Y_1, Y_2}(pY_1 + Y_2)$$

assuming natural resources, labour and technology are held constant. As these authors show, if the assumptions of the HOS model are satisfied the economy behaves as if it is maximising the GDP function. Assume the maximum is unique. Now consider the numerator of the Laspeyres quantity index. At prices p_{t-1} the pattern of output which maximises GDP is $\left(Y_{1,t-1}, Y_{2,t-1}\right)$, not (Y_{1t}, Y_{2t}). Hence $p_{t-1}Y_{1t} + Y_{2t} < p_{t-1}Y_{1,t-1} + Y_{2,t-1}$ and so

$$Q^L_{t,t-1} < 1$$

That is, the Laspeyres quantity index predicts that in Model 1 GDP falls following a change in the terms of trade. But we have already seen that a Divisia index predicts zero change of GDP in this model. Hence the Laspeyres index is biased downwards.

By a similar argument we can show that a Paasche quantity index, $Q^P_{t,t-1}$, is biased upwards in Model 1:

$$Q^P_{t,t-1} = \frac{p_tY_{1t} + Y_{2t}}{p_tY_{1,t-1} + Y_{2,t-1}} > 1$$

So a Fisher index which is the geometric mean between Laspeyres and Paasche is likely to be a good approximation to the Divisia index, which is equal to 1 in this case.

Notice that this result, a downwards bias in the Laspeyres, holds for *any* change in the terms of trade, favourable or unfavourable. So the error in the Laspeyres index is not cancelled out if a favourable change is reversed in the subsequent period. On the contrary the errors cumulate. So even changes in the terms of trade which are reversed over time will lead to a systematic underprediction of GDP growth.

This argument obviously generalises to the *N*-good case. An analogous argument applies to Model 2: a Laspeyres quantity index of GDP is biased downwards and a Paasche quantity index biased upwards.[15]

6 | EXTENSION TO THE DYNAMIC CASE

6.1 | Terms of trade effects when capital can be accumulated

The models analysed in this paper have been static: savings and investment have been excluded so the trade balance is always zero. The only exception is that, as discussed in Section 4.2, a continuous fall in the price of imported capital goods increases the growth rate of real GDP and real consumption. How would the conclusions be affected if savings and investment were introduced into Models 1 and 2? To illustrate the possibilities, consider first Model 1 but now extended to allow capital to be accumulated. Specifically, assume that capital is not produced at home but can be imported, that is, there are still two consumer goods produced at home but now there is a third good, imported capital. Assume that TFP growth is zero in both domestic industries. Also assume for concreteness that the export good is relatively capital intensive. Consider as before a one-off rise in the price of the export good, so resources shift to this good. So far the conclusion is the same as in the static Model 1. There is no effect on the level of GDP from the terms of trade improvement, but real consumption is higher. But now there is a further, secondary effect. Initially we may suppose that the return to capital was the same at home and abroad. But now the return to capital is higher at home since it increases with the shift in resources to the capital-intensive good. So capital flows in to equalize the rate of return once more at home and abroad (The production possibility frontier shifts out, asymmetrically not uniformly). After this transitional process is completed, capital per worker will be higher at home than before, so GDP per worker will be higher too. Higher GDP per worker suggests higher long run consumption per worker though this is moderated by a higher depreciation requirement. The transition may require (depending on savings preferences) a temporary balance of trade deficit, offset by a later surplus. Whether welfare is higher along the transitional path as well as when long run equilibrium is restored depends on how the prospect of higher future consumption is valued along that path (see below). If the export good is labour intensive then this secondary effect is in the opposite direction.

The long run growth rate is zero both before and after the change in the terms of trade. Introducing TFP growth would allow for long run growth but would lead to further complications if it is growing at different rates in the two domestic industries.

Model 2 can be similarly extended by introducing an imported capital good alongside the imported intermediate input (energy). Capital is now variable while labour is still fixed. TFP growth is assumed given exogenously. On standard assumptions there is a balanced growth path proportional to the TFP growth rate. In reaction to a fall in the price of energy there will be a change in the desired capital-energy and capital-labour ratios, whose sign depends on the pattern of substitution and complementarity between the three inputs. Consequently there will be a transitional period in which

[15]This conclusion was already reached by Bruno and Sachs (1985).

capital is accumulated at a different rate from on the balanced growth path. At the end of the transition the growth rate of GDP and of consumption will be the same as before. Whether the level of real consumption will be higher along the new balanced growth path will depend on whether capital per worker is higher or not.

In summary, the main insight of the two static models — a zero effect on real GDP under perfect competition plus a rise in real consumption — survives as the impact effect from a change in the terms of trade. But in the long run this result may have to be qualified if there are induced changes in capital per worker.

6.2 | Measuring welfare in the presence of saving and investment

In a dynamic context GDP and GDI now include expenditure on investment goods plus the trade balance. So a related issue is how should these be deflated to obtain real measures? In the case of GDP the SNA has always been clear that each component should be deflated by an appropriate price index, so prices for consumption goods, investment goods, exports and imports are all required. This creates no difficulties for a Divisia index of GDP. But things are not quite so clear for a welfare measure. Up to now, welfare has been measured by real consumption. But future consumption is now affected by consuming less today, that is, by investment, in order to consume more in the future. So how should this be taken into account? Weitzman (1976) argued that we should be concerned with permanent income, that is, the maximum sustainable level of consumption, suggesting this should be measured by what I call Weitzman's Net Domestic Product[16] (WNDP). This is defined in nominal terms as NDP, that is, GDP minus depreciation (capital consumption). It is converted to real terms by deflating all components, including investment expenditure and the current account balance (equal to net acquisition of foreign assets), by a consumer price index; the latter should be wider than the CPI and should cover public as well as private consumption. The argument for deflating investment by a consumer price index is that investment is only relevant to welfare insofar as it raises future consumption.

More formally, Weitzman's NDP can be defined in nominal per capita terms as

$$Y = P_C c + P_I \Delta k$$

Here Y is nominal NDP, c is real consumption, Δk is real *net* investment (k is the capital stock), all in per capita terms, and P_C, P_I are the prices of consumption goods and investment goods respectively. Net investment now includes the accumulation of foreign assets via the trade balance. A heuristic argument for deflating everything by the price index for consumption goods is as follows. Assuming $c = f(k)$ is the production function for consumption goods, the increment to future consumption made possible by net investment today is

$$\begin{aligned}\Delta c &= f'(k)\Delta k \\ &= (r+\delta)\left(\frac{P_I}{P_C}\right)\Delta k\end{aligned}$$

where r is the real rate of return, δ is the depreciation rate, so $(r+\delta)P_I$ is the rental price of capital. The second line uses the fact that under competition the marginal product of capital is set equal to the real rental price of capital. Dividing both sides by $r+\delta$,

[16]Weitzman called his concept Net *National* Product but I am ignoring net income from foreign assets here. Also, NNP is not used now as an official term in the SNA.

$$\left(\frac{P_I}{P_C}\right)\Delta k = \frac{\Delta c}{r+\delta}$$

The left hand side is net investment revalued to consumption units while the right hand side can be interpreted as the present value of the future stream of consumption made possible by that net investment. Hence we can deflate nominal NDP by the consumption price to get Weitzman's real NDP (real WNDP):

$$y = c + \Delta k\left(\frac{P_I}{P_C}\right) = c + \frac{\Delta c}{r+\delta}$$

which can be considered a measure of permanent income.

Weitzman (1976) showed that this last measure can also be considered a measure of the return on wealth (the present value of current and future consumption). However this is only the case when the real rate of return is constant over time and when there is no contribution to future consumption from TFP growth. Oulton (2004a) shows that under certain assumptions the level of real WNDP is a constant fraction of the true level which takes future TFP growth into account, so the growth rate of WNDP is still correctly measured[17]; these issues are discussed further by Sefton and Weale (2006), Duernecker et al. (2021), and Durán and Licandro (2022). Despite these difficulties with Weitzman's measure, Sefton and Weale (2006) still recommend using a consumer price index to deflate NDP in a welfare context.[18]

How does the SNA recommend that the trade balance should be deflated? As discussed above, the SNA makes a distinction between the concepts of output and welfare. This distinction is supported by economic theory as we have seen. The SNA sees the issue as arising only for real measures, encapsulated in the difference between real GDP and real GDI (which are equal in nominal terms). The only issue is how the trade balance should be deflated to get real GDI as opposed to real GDP. Unfortunately in my view, SNA 2008 makes no firm recommendation as to how this should be done, though it favours the deflator for total final expenditure rather than the deflator for consumption as argued for here.[19] The absence of a firm recommendation in SNA 2008 has allowed the Eurostat version of the SNA, ESA 2010 (Eurostat, 2013), to recommend an average of export and import price indices as the deflator, an inferior alternative.[20]

7 | DISCRETE VERSUS CONTINUOUS APPROACHES TO ECONOMIC MEASUREMENT

In practice Divisia indices cannot be calculated since data are only available at discrete intervals rather than continuously. But they can be approximated by chained indices of which the most commonly used for volume changes are the annually chained Laspeyres, Fisher or Törnqvist. Economic modellers and productivity analysts (following Griliches & Jorgenson, 1967 and Jorgenson et al., 1987) often use the Törnqvist. National income accountants generally use either the chained Laspeyres (mandated by Eurostat for EU countries and still used in the UK) or the chained Fisher (as in Canada and the US).

[17] Oulton (2004a) presents estimates of Weitzman's NDP for the US over 1973–2000.

[18] For other approaches to measuring welfare in a dynamic context see Basu and Fernald (2002), Jorgenson and Schreyer (2017), and Jorgenson (2018).

[19] Kohli (2022) also favours the deflator for total final expenditure.

[20] See the Supplementary Information S1 online Annex for further discussion.

The 2008 SNA has a whole chapter devoted to price and volume measures (European Commission et al., 2009, Chapter 15). Unfortunately nowhere does it mention Divisia index numbers. Despite this I am arguing that real world price and volume indices are best thought of as (more or less good) approximations to the ideal, the Divisia index. This approach, originally advocated by Jorgenson and Griliches (1971), enables us to link economic theory to the practice of national income accounting without having to assume particular functional forms for the underlying relationships like utility functions or production functions. As has been shown above, the Divisia approach enables one to prove intuitively plausible propositions which one would otherwise struggle to establish. Large changes can be handled as well as small ones.

The alternative approach is to assume that economic behaviour can be explained exactly in discrete time by utility or production functions which take the form of a "quadratic mean of order r". These functional forms are second order approximations to any functions acceptable to economic theory. Then there is a superlative index number (dependent on r) which is exact for this particular functional form (Diewert, 1976; Mizobuchi & Zelenyuk, 2021). Furthermore this index number measures large changes correctly as well as small ones. The drawbacks to this approach are that the results are dependent on the choice of the parameter r, and that the attractive properties of the Divisia index – price index times volume index equals value index and consistency in aggregation—are either lost, or compel the choice of a particularly value for r. For example, setting $r = 2$ results in the Fisher index which satisfies the first of these properties but not the second, consistency in aggregation. Setting $r = 0$ results in the Törnqvist index which satisfies neither property. I am not aware of any superlative index number which satisfies both properties.[21] Under the Divisia approach superlative index numbers are just used as approximations to the true Divisia index.

In practice the discrete approach assuming flexible forms is used by statistical agencies in chained form. Then one is allowing the parameters of the quadratic mean, apart from r, to change from period to period. If the parameters were unvarying then the chained index between say 0 and T would yield the same result as the non-chained index which uses just the weights from the two endpoints 0 and T, provided that the assumptions, such as constant returns to scale, are satisfied (Diewert, 1976). This is not generally found to be the case. So changes over time in the parameters of the quadratic mean can justify the use of chaining by statistical agencies.

In the light of the finding above, that a Laspeyres quantity index is biased downwards in both Models 1 and 2, the decision by Eurostat to mandate chained Laspeyres indices for use in the national accounts of EU member states looks like a mistake, at least to the extent that changes in the terms of trade are empirically important. But at the time of writing the cost of living crisis engulfing Europe make changes in the terms of trade difficult to ignore.

8 | CONCLUSIONS

This paper has argued first, that the conclusions of economic theory about the effect of some exogenous change, like a change in the terms of trade, on aggregates like output, consumption or welfare can be translated into statements about the effects on Divisia index numbers. Second, Divisia index numbers provide a clear conceptual foundation for national income accounting. Third, the System of

[21] Hill (2006) has shown that the empirical results can be quite sensitive to the choice of the parameter r, even though the results for $r = 0$ and $r = 2$ are usually similar. But Hill's results are for direct, not chained, index numbers. Oulton (2022) using US data on value added in 63 industries over 1987–2019 shows that sensitivity to the r parameter is greatly reduced for chained index numbers.

National Accounts provides the means to measure the effects studied in economic models, at least approximately. Fourth, I have shown that there is no conflict, at least in principle, between the conclusions from textbook models about the effects of changes in the terms of trade and what a national income accountant would conclude by applying the rules of the SNA.

The SNA provides a practical approach to measuring output and welfare. It can be viewed as providing approximations to theoretical concepts like Divisia index numbers which cannot be measured exactly. Of course, some approximations are better than others. In particular chained Fisher indices as used in the US and Canada are better approximations than chained Laspeyres indices as used in the UK and the EU.

Finally, some of the results here, in particular Proposition 2, rest on the assumption of perfect competition. But much of modern macroeconomics is built on the contrary assumption, imperfect competition, at least for short run analysis.[22] Extracting estimates of productivity and welfare from the national accounts is a much more challenging task under imperfect competition since it requires the estimation of margins which are not directly observed.

ACKNOWLEDGEMENTS

I am grateful to two anonymous referees of an earlier version which appeared in the ESCoE discussion paper series, and to Marshall Reinsdorf, Kevin Fox and Francesco Caselli for helpful comments. This paper benefited from being presented at an ESCoE seminar hosted by the ONS on 26 February 2019, at the Sixth World KLEMS Conference in March 2021, and at the 36th IARIW General Conference in August 2021, where my discussant was Robert Inklaar to whom I owe thanks. I am also grateful to Jonathan Haskel who drew my attention to conflicting results in the literature on what is called here Model 2.

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[22]See Basu (2019) for a survey of margin estimates in the United States which vary widely though are generally positive. Macroeconomists of the real business cycle school hold to the perfect competition assumption (price equals marginal cost) but they seem to be in the minority.

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How to cite this article: Oulton, N. (2023). The effect of changes in the terms of trade on GDP and welfare: A Divisia approach to the System of National Accounts. *The Manchester School*, 1–22. https://doi.org/10.1111/manc.12437