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INEQUALITY MEASUREMENT AND THE RICH: WHY INEQUALITY INCREASED MORE THAN WE THOUGHT

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To compare income and wealth distributions and to assess the effects of policy that affect those distributions require reliable inequality-measurement tools. However, commonly used inequality measures such as the Gini coefficient have an apparently counter-intuitive property: income growth among the rich may actually reduce measured inequality. We show that there are just two inequality measures that both avoid this anomalous behavior and satisfy the principle of transfers. We further show that the recent increases in US income inequality are understated by the conventional Gini coefficient and explain why a simple alternative inequality measure should be preferred in practice.

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1. INTRODUCTION

Policy-makers who are concerned about inequality need reliable indicators to target taxes and other distributional programs. However, do commonly used inequality measures behave in the way that an intuitive understanding of inequality suggests that they ought to behave? For example, when the rich get richer does measured inequality go up? Perhaps. Standard inequality measures, such as the Gini and Theil indices, sometimes indicate the opposite. The reason for this behavior is that these and many other relative inequality indices are expressed as ratios, where the numerator is an indicator of dispersion and the denominator is the mean. The indicator of dispersion in these indices ensures that they satisfy the principle of transfers; division by the mean ensures that they are independent of income scale. However, they have a property that may seem unattractive as a guide to policy.

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To see why this is so, consider what happens when the income of just one person is changed in a direction "away from equality" so that, if the person's income is above the point representing equality, the income is increased; if it is below this point, it is decreased. In such situations we might want to appeal to the following *principle of monotonicity in distance*:

> *if two distributions differ only in respect of one individual's income, then the distribution that registers greater distance from equality for this individual's income is the distribution that exhibits greater inequality.*

The principle appears to be attractive as a practical criterion and a guide to policy, but it is evidently possible that the Gini, the Theil index, and other standard inequality indices might behave in a problematic fashion. When the rich get richer, both the numerator and the denominator of the index will change in the same direction and, as a result, the value of the index could fall. To illustrate what can happen, suppose income distribution $\mathbf{x}$ changes to $\mathbf{x}'$ where:

$$\mathbf{x} = \{1, 2, 3, 4, 5, 9, 10\} \quad \text{and} \quad \mathbf{x}' = \{1, 2, 3, 4, 7, 9, 10\}. \tag{1}$$

Given that the mean of $\mathbf{x}$ is 4.857 and the mean of $\mathbf{x}'$ is 5.143, it is clear that the fifth person's income increase from 5 to 7 represents a move away from equality. However, computing the Gini and the Theil indices, we find *more* inequality in $\mathbf{x}$ than in $\mathbf{x}'$:

$$\text{Gini}(\mathbf{x}) = 0.361 > \text{Gini}(\mathbf{x}') = 0.357 \tag{2}$$

$$\text{Theil}(\mathbf{x}) = 0.216 > \text{Theil}(\mathbf{x}') = 0.214. \tag{3}$$

Therefore, indeed, the Gini and the Theil indices do not respect the principle of monotonicity in distance. More generally, the numerator and the denominator change in the same direction for any income lying above the point representing equality; if an income below that point is reduced, the numerator increases and the denominator decreases. Therefore, variations away from equality are attenuated (amplified) when someone rich (poor) gets richer (poorer).

In this paper, we develop a median-normalized class of inequality measures that respects the principle of monotonicity in distance, based on a few elementary principles. We show that it is closely related to the generalized-entropy class of inequality measures, where the mean in the denominator is replaced by the median. However, with one important exception, members of this class do not respect the principle of transfers.

We further show that the mean logarithmic deviation (MLD) index, or second Theil index, which is a limiting case of the mean-normalized generalized-entropy class of inequality measures, is also a limiting case of our median-normalized class of measures. Thus, it shares properties of both mean- and median-normalized inequality measures. In other words, the MLD index is the only relative inequality measure that respects *both* the principle of transfers *and* the principle of monotonicity in distance. Indeed, for the example in (1) earlier, we find less inequality in $\mathbf{x}$ than in $\mathbf{x}'$ with the MLD index:

$$\text{MLD}(\mathbf{x}) = 0.254 < \text{MLD}(\mathbf{x}') = 0.263. \tag{4}$$

Why does this matter? The lack of adherence to the principle of monotonicity in distance may have strong implication for empirical studies. Examining inequality in Great Britain and in the United States over recent decades, we show that the conventional Gini index under-records variations of inequality and the MLD index should be preferred in practice.

The paper is organized as follows. Section 2 sets out an approach to inequality measurement based on principles that accord with intuition; it characterizes the inequality measures that are consistent with these principles and compares the two core principles—the principle of monotonicity in distance and the principle of transfers. Sections 3 and 4 discuss the behavior of the inequality measures introduced in Section 2 in terms of their sensitivity to different parts of the income distribution and decomposability by population subgroups. Section 5 shows how the alternative approach to inequality developed here affects the conclusions on inequality comparisons in the United States and in Great Britain. Section 6 concludes.

2. Inequality: An Approach

What is an inequality measure, and what should it do? Technically, inequality measurement can be seen as a way of ordering distributions for a particular type or types of data. The mention of data types is important because in economics we often need to consider several different types—for example, where the data can be represented as cardinal, non-negative numbers (perhaps wages), where they can be represented as numbers without sign restriction (wealth?), or where the data are categorical.

There is potentially a large collection of statistical tools that may appear to do the job of inequality comparisons. The fact that two different inequality measures could rank a pair of distributions in opposite ways may not matter—each of the two measures may respect the same underlying principles, but give different weight to information in different parts of the distribution. What may indeed matter is when two different measures contradict each other in practice because they are founded on different, potentially conflicting, economic principles. We take the position that the economic principles to be invoked should be clear and should apply to all relevant types of data.

We will first describe a core set of principles that capture three key aspects of inequality measurement. We then show how these principles can be used to characterize a family of inequality measures. Using this family we show how two fundamental principles of inequality may be in conflict.

2.1. *Reference Point and Principles*

The meaning of inequality comparisons can be expressed concisely by adopting three elementary principles. To discuss these we introduce the concept of a *reference* *point*, a particular value used as the basis for assessing changes in inequality. In principle there are several possibilities for specifying this reference point, and which of them seems reasonable may depend on the data type (Cowell and Flachaire, 2021) and on the specific inequality measure. It could be the mean—what everyone would have if there were to be perfect equality and if lump-sum transfers were possible. It could be the median, arguably a more satisfactory way of characterizing an "equality reference point." For some data types it could be a value that is independent of the distributions being compared. We use the reference point concept to give meaning to the first of the three principles, the one already mentioned in the introduction.

The three principles can be summarized as follows:

- The principle of *monotonicity in distance* means that any movement away from the reference point should be regarded as an increase in inequality. Therefore, if two distributions differ only in respect of x_i (person i's income or other measure of status for i), then the distribution that registers greater individual distance from equality for i is the distribution that exhibits greater inequality.
- The second principle, *independence*, ensures the following. Suppose there is some particular i for which the value of x_i is the same in both distribution $\mathbf{x}$ and distribution $\mathbf{x}'$. Identical small variations in this common value should leave the inequality ranking of distributions $\mathbf{x}$ and $\mathbf{x}'$ unchanged. This principle provides the basis for decomposing inequality by subgroup of the population as discussed in Section 4 later.
- The third principle, *scale invariance*, refers to the rescaling of values in each distribution and encapsulates two ideas: First, rescaling all values—the observations in the distributions $\mathbf{x}$ and $\mathbf{x}'$ as well as the reference point—should leave inequality comparisons unaltered. Second, the meaning of inequality comparisons should not change if all the observations in the distributions were to grow proportionately, while the reference point remained unchanged. This does not mean that inequality levels remain unchanged under rescaling, just that the answer to the question "is $\mathbf{x}$ more unequal than $\mathbf{x}'$?" is not reversed under either form of rescaling.

2.2. *Inequality Measures*

The three principles just outlined lead directly to a characterization of a specific class of inequality measures.[1] In an n-person society with equality reference point r, the inequality index takes either the form

$$G(\mathbf{x}) := \frac{1}{2n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| x_i - x_j \right| \tag{5}$$

[1]The three principles described in Section 2.1 are made precise in Axioms 1–3 set out in the Technical Appendix. The formal result establishing (5) and (6) is also in the Technical Appendix. The result also uses the axiom of continuity of the inequality ordering and the axiom of anonymity. Anonymity means that if all information relevant to inequality is embodied in the income or status measure, then switching the labels on the individuals must leave inequality unchanged.

(the "Absolute Gini"), or the form

$$I_\alpha(\mathbf{x}; r) := \frac{1}{\alpha[\alpha - 1]}\left[\frac{1}{n}\sum_{i=1}^{n} x_i^\alpha - r^\alpha\right], \tag{6}$$

where α is a sensitivity parameter, or by some strictly increasing transformation $\Psi(\cdot; r)$ of (5) or (6).

To develop the formulations (5) and (6) into practical inequality indices, two further things need to be done.

First, the reference point r needs to be specified in (6). When analyzing income inequality, it is often assumed to be natural to take mean income as the reference point, $r = \mu$, in that $(\mu, \mu, \ldots, \mu)$ represents a perfectly equal distribution in the case where lump-sum transfers of x are assumed to be possible; but other specifications of r may also make sense in the case of the family (6). However, in the definition of the absolute Gini (5) there is already an embedded reference point, the median; for this measure it is always the case that $r = m$. To see this note that (5) can be written as

$$G(\mathbf{x}) = \sum_{i=1}^{n} \kappa_i x_{(i)},$$

where $x_{(i)}$ denotes the ith component of $\mathbf{x}$ if it is arranged in ascending order and

$$\kappa_i := \frac{2}{n^2}\left[i - \frac{n+1}{2}\right].$$

This implies that the sign of any change in $x_{(i)}$ is determined by $i - \frac{n+1}{2}$. If $x_{(i)}$ alone is increased, then $G(\mathbf{x})$ increases or decreases according as $x_{(i)} \gtrless m$. Therefore, for the inequality measure G, the monotonicity criterion applies specifically to the median as a reference point.

Second, the expression (6) is defined only up to an increasing transformation: we need to determine what type of normalization is appropriate to make it into a practical index. Two small aspects of this normalization have already been incorporated in (6), the division by the constant $\alpha[\alpha - 1]$ and the division by population size to ensure that the index is independent under population replication.

There remains a third normalization step. This concerns the way that the index should behave when all incomes change proportionately (the principle of scale invariance on inequality comparisons ensures that inequality *comparisons* remain unaffected by such income changes, but says nothing about inequality *levels*). It is often assumed that inequality should remain constant under such proportional changes. However, there are several ways of doing this. One could divide through by the reference point—perhaps the mean—but it could be some other function of incomes. Here we investigate the use of the median m as an alternative normalization criterion instead of the mean μ, as in the following examples. Note that this is essentially a question of implementation, rather than the introduction of a new principle.

The conventional ("relative") Gini index (Yitzhaki and Schechtman, 2013) is found from the absolute Gini (5) after normalizing by the mean:

$$G\left(\frac{\mathbf{x}}{\mu}\right) = \frac{E_{x,x'}\left([x - x']^2\right)}{2\mu} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|}{2\mu n^2}. \tag{7}$$

As an alternative to the conventional Gini index, Gastwirth (2014) proposed replacing the mean by the median in the standard definition of the Gini index:

$$G\left(\frac{\mathbf{x}}{m}\right) = \frac{E_{x,x'}\left([x-x']^2\right)}{2m} = \frac{\mu}{m}\,G\left(\frac{\mathbf{x}}{\mu}\right). \tag{8}$$

Now consider the family (6). If we use the mean as the reference point and also normalize by the mean we find that this yields the generalized-entropy class of measures given by[2]

$$I_\alpha\left(\frac{\mathbf{x}}{\mu};\mu\right) = \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)^{\alpha} - 1\right], \quad \alpha \neq 0, 1, \tag{9}$$

$$I_0\left(\frac{\mathbf{x}}{\mu};\mu\right) = -\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\mu}\right), \tag{10}$$

$$I_1\left(\frac{\mathbf{x}}{\mu};\mu\right) = \sum_{i=1}^{n}\frac{x_i}{\mu}\log\left(\frac{x_i}{\mu}\right). \tag{11}$$

If instead we use the mean as the reference point, but normalize by the median, we find that (6) yields the following:

$$I_\alpha\left(\frac{\mathbf{x}}{m};\mu\right) = \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{m}\right)^{\alpha} - \left(\frac{\mu}{m}\right)^{\alpha}\right], \tag{12}$$

$$= \left(\frac{\mu}{m}\right)^{\alpha} I_\alpha\left(\frac{\mathbf{x}}{\mu};\mu\right), \quad \text{for all } \alpha \in \mathbb{R}. \tag{13}$$

2.3. *Monotonicity in Distance and the Transfer Principle*

There is an obvious difficulty with the type of normalization that we have just discussed. If we normalize by an expression that involves the income vector, then the behavior of the resulting inequality measure may be affected by the specific form of income-normalization that is adopted.

Monotonicity in Distance

This point is easily seen for the Gini index (7) in the case of normalization by the mean: if one income is increased the mean increases and, as a result, the

[2]The limiting form as $\alpha \to 0$, the MLD (10), follows from (9) using l'Hôpital's rule. The limiting form for $\alpha \to 1$, the Theil index (11), follows from (9) by expressing it in the equivalent form $\frac{1}{\alpha[\alpha-1]}\sum_{i=1}^{n}\frac{x_i}{n\mu}\left[\left(x_i/\mu\right)^{\alpha-1}-1\right]$ and again applying l'Hôpital's rule.

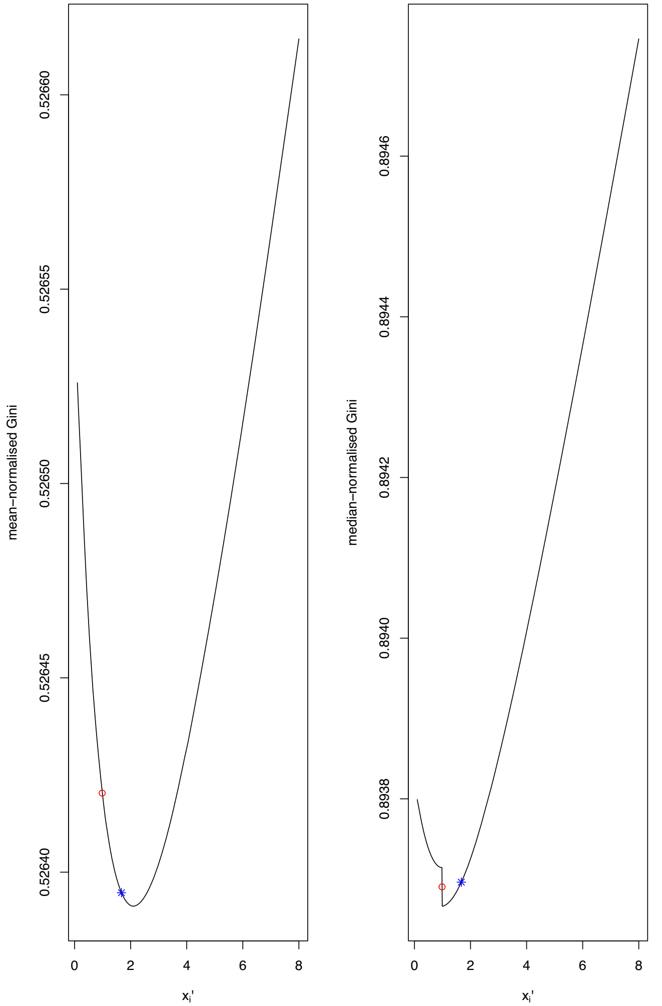

Figure 1. Principle of Monotonicity in Distance: Mean-Normalized Gini (Left) and Median-Normalized Gini (Right) Indices, Computed from 5,000 Observations Drawn from a Lognormal Distribution, and 1 Additional Observation x_i', where $x_i' \in]0, 8]$ [Colour figure can be viewed at wileyonlinelibrary.com].

contribution of each distance between all other incomes in (7) decreases.[3] This problem was illustrated by the example in the introduction (see equations 1 and 2); it is further illustrated by the example depicted in Figure 1. This shows the values of the mean-normalized (7) and median-normalized (8) Gini coefficient for a sample of 5,000 observations, drawn from a standard lognormal distribution, and one additional observation x_i', where $x_i' \in]0, 8]$. Recall that the reference point for the Gini coefficient has to be the median, m: this is indicated in each panel by the

[3]When only $x_k \uparrow$, we have $\mu \uparrow$ and so $|x_i - x_j|/\mu \downarrow$ for all $i, j \neq k$.

point marked ∘ on the graph; the mean, marked ∗, is shown just for information. The left-hand panel of Figure 1 confirms that the conventional (mean-normalized) Gini is not consistent with the principle of monotonicity in distance: if x_i' is very low—below the median (marked ∘)—then increasing x_i' reduces inequality; if x_i' lies well above the median, then increasing x_i' increases inequality; both these things seem to accord with common sense—see also Lambert and Lanza (2006). However, there is a part of the curve just to the right of ∘ where an increase in an above-median income *reduces* inequality: a violation of monotonicity. However, the median-normalized Gini satisfies monotonicity for all $x_i' < m$ and for all $x_i' > m$, as can be seen from the right-hand panel of Figure 1. Exactly at $x_i' = m$ there is a sudden decrease in the graph: this arises from a tiny change in the median value while preserving rank; this feature occurs only in finite sample and disappears as n becomes large.[4]

As a result of this sudden decrease, $G\left(\frac{\mathbf{x}}{m}\right)$ does not satisfy monotonicity everywhere in finite sample because it is multivalued at the point where $x_i' = m$; but it satisfies a weaker version of this principle for perturbations of x_i' that are strictly below or above the median.

Now consider this issue for the family of inequality measures defined in (9). If we set $r = \mu$ in the non-normalized (6) and differentiate with respect to x_i, we find the following impact on inequality:

$$\delta_i(\mathbf{x}) := \frac{\partial I_\alpha(\mathbf{x};\mu)}{\partial x_i} = \frac{1}{n}\,\frac{x_i^{\alpha-1} - \mu^{\alpha-1}}{\alpha - 1}. \tag{14}$$

The expression $\delta_i(\mathbf{x})$ is positive/negative according as $x_i \gtrless \mu$, for all values of α—a property directly inherited from monotonicity. However, if we normalize by the mean to obtain (9) and differentiate, we have:

$$\frac{\partial I_\alpha\left(\frac{\mathbf{x}}{\mu};\mu\right)}{\partial x_i} = \mu^{-\alpha}\delta_i(\mathbf{x}) - \frac{\alpha}{\mu n} I_\alpha\left(\frac{\mathbf{x}}{\mu};\mu\right). \tag{15}$$

Clearly, if $\alpha > 0$ and $\delta_i(\mathbf{x}) > 0$, then, for some $\mathbf{x}$, expression (15) will be negative; likewise, if $\alpha < 0$ and $\delta_i(\mathbf{x}) < 0$, then, for some $\mathbf{x}$, expression (15) will be positive. In sum, for mean-normalized inequality measures and any $\alpha \neq 0$, there will always

[4]In the neighborhood of the median, there is a large change in $G\left(\frac{\mathbf{x}}{m}\right)$: this is due to a change in the median, which affects every term in the median-normalized Gini. For $n = 5001$:

$$m = \begin{cases} x_{(2500)} & \text{if } x_i' \le x_{(2500)} \\ x_i' & \text{if } x_i' \in [x_{(2500)}, x_{(2501)}] \\ x_{(2501)} & \text{if } x_i' \ge x_{(2501)}, \end{cases}$$

where $x_{(k)}$ denotes the k^{th}-order income. The median varies in a narrow interval: its values are bounded by two adjacent mid-rank incomes. In our example $m \in [0.9647, 0.9659]$. As n becomes large, this interval will become infinitesimal and the right-hand panel of Figure 1 would take the form of a simple U-shaped graph of $G\left(\frac{\mathbf{x}}{m}\right)$, with its minimum at the median.

be some distribution for which the anomalous behavior illustrated in the introduction will emerge: the normalized indices must violate the property of monotonicity in distance. By contrast, consider normalizing by the median. Differentiating (12), we have

$$\frac{\partial I_\alpha\left(\frac{\mathbf{x}}{m};\mu\right)}{\partial x_i} = m^{-\alpha}\delta_i(\mathbf{x}) - \frac{\alpha}{m} I_\alpha\left(\frac{\mathbf{x}}{m};\mu\right)\frac{\partial m}{\partial x_i}. \tag{16}$$

For all values of α, the first term on the right-hand side takes the sign of $\delta_i(\mathbf{x})$. The second term is zero if inequality is zero, if $\alpha = 0$, or if the change in x_i does not change the median; otherwise it takes the sign of $-\alpha$. Therefore, if $\alpha > 0$ and $x_i < \mu$, both $\delta_i(\mathbf{x})$ and expression (16) are negative. If $\alpha \geq 0$ and $x_i > \mu$ and if we confine attention to distributions for which $\mu \geq m$, then the second term in (16) is zero, so $\delta_i(\mathbf{x})$ determines the sign of the whole expression (16). Therefore, for right-skew distributions—such as income or wealth distributions—it is true that the principle of monotonicity in distance is satisfied for the median-normalized inequality indices (12), for $\alpha \geq 0$. Figure 2 is similar to Figure 1, but drawn for mean-normalized (9) and median-normalized (12) inequality measures with $\alpha = 1.1$. The left-hand panel shows that the minimum of the mean-normalized inequality index is not where $x_i' = \mu$ (marked $*$), but where $x_i' = 3.05$: so for any $\mu \leq k_1 < k_2 < 3.05$, the index will exhibit more inequality with $x_i' = k_1$ than with $x_i' = k_2$, which is inconsistent with the principle of monotonicity in distance. The right-hand panel shows that the median-normalized inequality index is at a minimum when x_i' is equal to the mean and it increases when x_i' moves away from the mean: the principle of monotonicity in distance is respected.

One might wonder whether measures of the form $I_\alpha(\mathbf{x}/m; m)$—where the median is both the reference point and the scaling factor—are worthy of consideration. To address this, it is worth considering a more general form of normalizing (6). Let r be the reference point, as before, and λ the scale factor: in principle both could depend on $\mathbf{x}$. Then a generalized version of (12) can be written as:

$$I_\alpha\left(\frac{\mathbf{x}}{\lambda};r\right) = \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\lambda}\right)^\alpha - \left(\frac{r}{\lambda}\right)^\alpha\right],. \tag{17}$$

First consider the limit case, as $\alpha \to 0$. Then (17) becomes

$$I_0\left(\frac{\mathbf{x}}{\lambda};r\right) = \log\left(\frac{r}{\lambda}\right) - \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\lambda}\right) = \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{r}{x_i}\right). \tag{18}$$

Notice that this implies that I_0 is independent of λ for any r. Now let x_i vary in (18):

$$\frac{\partial}{\partial x_i} I_0\left(\frac{\mathbf{x}}{\lambda};r\right) = \frac{1}{r}\frac{\partial r}{\partial x_i} - \frac{1}{n x_i}. \tag{19}$$

To respect the principle of monotonicity in distance, we must have (19) negative (positive) as $x_i < r$ ($x_i > r$). Evaluate (19) at $x_i = r$. We get

$$\left.\frac{\partial}{\partial x_i} I_0\left(\frac{\mathbf{x}}{\lambda};r\right)\right|_{x_i=r} = \frac{1}{r}\left[\frac{\partial r}{\partial x_i} - \frac{1}{n}\right], \tag{20}$$

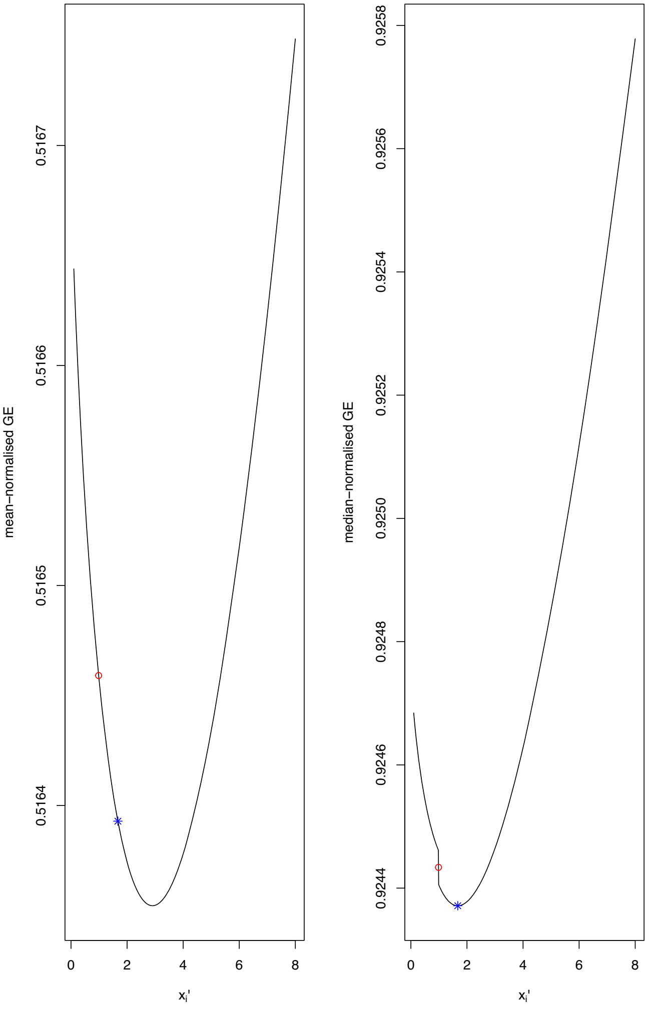

Figure 2. Principle of Monotonicity in Distance: Mean-Normalized (Left) and Median-Normalized (Right) Inequality Measures I_α, Computed for $\alpha = 1.1$ Using the Same Data as in Figure 1 [Colour figure can be viewed at wileyonlinelibrary.com].

which is zero only if $\frac{\partial r}{\partial x_i} = 1/n$. This will be true only if r is the mean. Substituting $r = \mu$ in (19), we get:

$$\frac{\partial}{\partial x_i} I_0\left(\frac{\mathbf{x}}{\lambda}; r\right) = \frac{1}{n}\left[\frac{1}{\mu} - \frac{1}{x_i}\right], \tag{21}$$

which is negative (positive) as $x_i < r$ ($x_i > r$).

We can immediately see two things. First, the equations (19)–(21) show that I_0 has an embedded reference point, the mean, μ (contrast this with the absolute Gini where the embedded reference point is the median, m): so, it would make no sense to try to use m as a reference point for I_0. Second, in the cases where $\alpha \neq 0$ in (17), the counterpart to the analysis in (19) to (21) would be less transparent, because a variation in x_i would potentially affect both r and λ. In particular, if $r = m$ it would be possible to have situations where $x_i < r$ and, if the median remains unchanged, the increase in x_i increases measured inequality: the principle of monotonicity is violated.

Principle of Transfers

The principle of transfers—that a transfer from a poorer person to a richer person should always increase inequality—has long been regarded as the cornerstone of inequality analysis.

As is well known, the absolute Gini coefficient (5) and the regular Gini (7) both satisfy the principle of transfers. The median-normalized Gini (8) will satisfy the principle if the transfers take place strictly above or strictly below the median; in other cases the median may shift and the principle may be violated.

What of the family (6)? Let the x-value of person i change by an amount $\Delta x > 0$ and that for person $j \neq i$ by an amount $-\Delta x$. Denote the consequent change in the median by Δm; the change in the mean is zero, by construction. Differentiating (9) and (12), we obtain, respectively:

$$\Delta I_\alpha \left(\frac{\mathbf{x}}{\mu}; \mu \right) = \mu^{-\alpha} \left[\delta_i (\mathbf{x}) - \delta_j (\mathbf{x}) \right] \Delta x \tag{22}$$

$$\Delta I_\alpha \left(\frac{\mathbf{x}}{m}; \mu \right) = m^{-\alpha} \left[\delta_i (\mathbf{x}) - \delta_j (\mathbf{x}) \right] \Delta x - \frac{\alpha}{m} I_\alpha \left(\frac{\mathbf{x}}{\mu}; \mu \right) \Delta m, \tag{23}$$

where δ_i is defined in (14). The transfer principle requires that each of these expressions be positive/negative according as $x_i \gtrless x_j$. This is obviously true in the case of (22), but in the case of (23), this can only be true for arbitrary $\mathbf{x}$ if $\alpha = 0$.

Therefore, median-normalized inequality measures in (8) and (12) do not respect the principle of transfers, but a weaker version of this principle. They respect this principle for any transfers *strictly* above/below the median, that is, as long as the median is unchanged. However, we can easily find a counterexample, where transfer from the median individual to a poorer individual increases $I_\alpha(\mathbf{x}/m; \mu)$. For instance, let us consider the following distributions:

$$\mathbf{x}'' = \{1, 2, 3, 5, 10\} \qquad \text{and} \qquad \mathbf{x}''' = \{1, 2.5, 2.5, 5, 10\}. \tag{24}$$

From the principle of transfers, mean-normalized inequality measures always exhibit more inequality in $\mathbf{x}''$ than in $\mathbf{x}'''$. By contrast, a median-normalized inequality measure may exhibit less inequality in $\mathbf{x}''$ than in $\mathbf{x}'''$: for instance, we have $G(\mathbf{x}'', m) < G(\mathbf{x}''', m)$ and $I_1(\mathbf{x}''/m; \mu) < I_1(\mathbf{x}'''/m; \mu)$,[5] which is *not* consistent with the principle of transfers—see Gastwirth (2014).

In sum, the main difference between the two types of normalization for I_α is that the median-normalized class of inequality measures respects the principle of

[5] $G(\mathbf{x}'', m) = 0.560$ and $G(\mathbf{x}''', m) = 0.656$. $I_1(\mathbf{x}''/m; \mu) = 0.3745$, $I_1(\mathbf{x}'''/m; \mu) = 0.4414$.

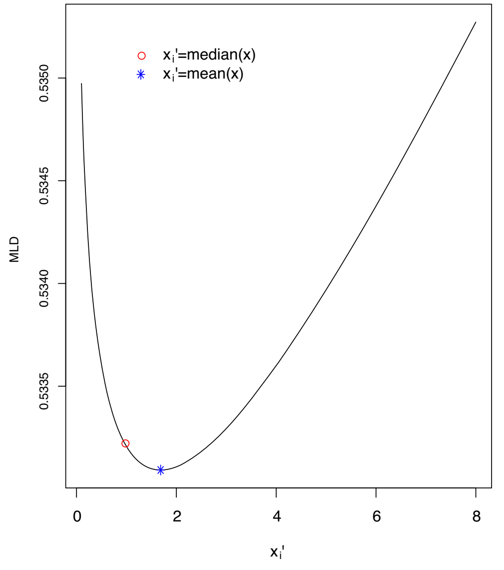

Figure 3. Principle of Monotonicity in Distance: The Mean Logarithmic Distance I_0, Computed for the Same Data as in Figure 1 [Colour figure can be viewed at wileyonlinelibrary.com].

monotonicity in distance when $\alpha \geq 0$, while mean-normalized class of inequality measures in (9) respects the (Pigou-Dalton) principle of transfers.

2.4. *Two Principles in One*

It is clear from equations (15), (21), and (23) that, apart from the absolute Gini (5) (which satisfies monotonicity if the embedded reference point, the median, is used) there is exactly one case where both the principle of monotonicity in distance and the principle of transfers are respected. This is where $\alpha = 0$, corresponding to the MLD index that has the mean as an embedded reference point.

From equation (18) we know that in this case $I_0\left(\frac{\mathbf{x}}{m};\mu\right) = I_0\left(\frac{\mathbf{x}}{\mu};\mu\right)$. Figure 3 illustrates the principle of monotonicity using the same data as in Figures 1 and 2. We can see that the index is minimum when x_i' is equal to the mean ($*$) and it changes smoothly as x_i' moves away from the mean.

3. Sensitivity of the Inequality Measures

Section 2 gives us two main families of inequality measures: the mean-normalized and the median-normalized. We have seen some of the advantages of each of the two families. Now it is time to consider how individual members of each family compare with each other: which family member appears particularly attractive? We can do this by considering how sensitive any family member is in different regions of the distributions being compared.

An inequality measure implicitly puts different weight on different parts of a distribution. In the case of the class (13), we can see that this sensitivity is captured by the parameter α as follows:

$\alpha > 1$ puts more weight on high values of x (where $x_i \gg m$),
- $\alpha < -1$ puts more weight on low values of x (where $x_i \ll m$),
- $-1 \leq \alpha < 0$ puts more weight on values of x close to and above the median,
- $0 < \alpha \leq 1$ puts more weight on values of x close to and below the median.

When $\alpha \in [-1, 1]$, the index puts more weight on values of x in the middle of the distribution, rather than in the tails.

The limiting case $\alpha = 0$ is *non-directional*, in the sense that it does not put more weight on values of x above/below the median or above/below the mean. This can be seen using (12) to rewrite (10) as follows:

$$I_0\left(\frac{\mathbf{x}}{m}; \mu\right) = \log\left(\frac{\mu}{g}\right), \tag{25}$$

where g is the geometric mean of the distribution. Therefore, the MLD index is the log difference between the arithmetic mean and the geometric mean, and it is clear that in computing arithmetic and geometric means, every value of x has the same weight.

The relationship between median- and mean-normalized inequality measures is given by (13). For income distributions, skewed to the right ($m < \mu$), median-normalized inequality measures are always greater (less) than mean-normalized inequality measures for $\alpha > 0$ ($\alpha < 0$). To illustrate this feature, Figure 4 plots values of median- and mean-normalized inequality measures, as defined in (12) and (9), for different values of α, using a sample of 5,000 observations drawn from the standard lognormal distribution. In this example, the ratio $\mu/m \approx 1.648$ and it is clear from Figure 4 that median-normalized indices are greater (less) than mean-normalized indices when $\alpha > 0$ ($\alpha < 0$). The two curves intersect at $\alpha = 0$.

Calculating median-normalized inequality measures does not require microdata: as (13) shows, knowing the mean-normalized measure along with the mean and the median is enough.

Now consider the Gini family. There is a link between the median-normalized Gini (8) and the median-normalized generalized-entropy measures in (12). Indeed, the median-normalized measure in (12) with $\alpha = 2$ is equal to

$$I_2(\mathbf{x}/m; \mu) = \frac{\sigma^2}{2m^2} = \frac{E([x - \overline{x}]^2)}{2m^2} = \frac{E_{x,x'}\left([x - x']^2\right)}{4m^2}. \tag{26}$$

We can see that $G(\mathbf{x}; m)$ and $[I_2(\mathbf{x}/m; \mu)]^{1/2}$ are two very similar measures; both are ratios of a dispersion measure on twice the median. For the Gini, the dispersion measure is based on Manhattan L_1-distance, while for the generalized-entropy measure it is based on Euclidean L_2-distance. As a consequence, $I_2(\mathbf{x}/m; \mu)$ puts more weight on high values of x, compared with the Gini. It is also true for mean-normalized inequality measures $G(\mathbf{x}; \mu)$ and $[I_2(\mathbf{x}/\mu; \mu)]^{1/2}$, because both are ratios of a dispersion measure on twice the mean.

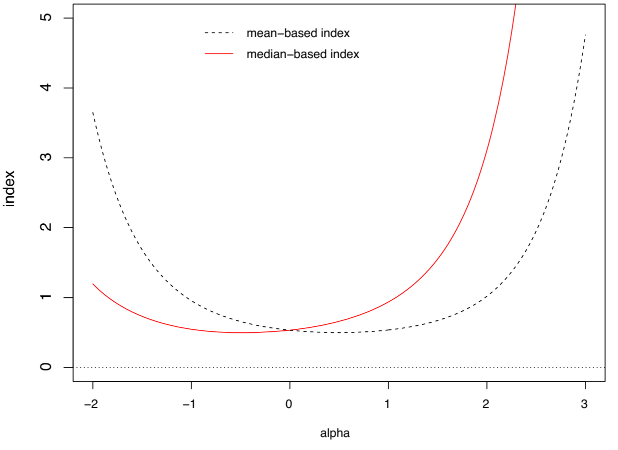

Figure 4. Median- and Mean-Normalized Inequality Measures, for $\alpha \in [-2; 5]$ [Colour figure can be viewed at wileyonlinelibrary.com].

Using L_1-distance, it is clear that the Gini index does not put higher weight on high/low values of x. In the class of inequality measures defined in (12), it is the limiting case $\alpha = 0$ which does not put more weights on high/low values of x. The Gini and MLD indices are quite similar in that they put similar weights on each value of x.

4. Decomposability

A further way of assessing members of the mean-normalized and the median-normalized families is by looking at the relation between inequality in the population overall and inequality in the collection of subgroups of the population.

Let the population be divided into K mutually exclusive and exhaustive groups, and let the proportion of population falling in group k be p_k; furthermore, let m_k and μ_k denote, respectively, the median and the mean in group k and m and μ denote, as before, the corresponding population median and mean. There are two types of decomposability by groups that are of interest: (1) "*non-overlapping*" decomposability where we impose an additional condition that the groups can be unambiguously ordered by income (e.g., the richest person in group k has an income that is less than the poorest person in group $k+1$ for all $0 < k < K$); (2) general decomposability, where no additional conditions are imposed.

If the grouping is chosen such that the non-overlapping property is respected, then the absolute Gini, the regular Gini, and the median-Gini coefficient, respectively, can be decomposed as follows (Cowell, 2016):

$$G(\mathbf{x}) = \sum_{k=1}^{K} p_k^2 G(\mathbf{x}_k) + G^{\text{btw}}(\mathbf{x}) \tag{27}$$

$$G\left(\frac{\mathbf{x}}{\mu}\right) = \sum_{k=1}^{K} p_k^2 \frac{\mu_k}{\mu} G\left(\frac{\mathbf{x}_k}{\mu_k}\right) + G^{\text{btw}}\left(\frac{\mathbf{x}}{\mu}\right) \tag{28}$$

$$G\left(\frac{\mathbf{x}}{m}\right) = \sum_{k=1}^{K} p_k^2 \frac{m_k}{m} G\left(\frac{\mathbf{x}_k}{m_k}\right) + G^{\text{btw}}\left(\frac{\mathbf{x}}{m}\right), \tag{29}$$

where G^{btw} is evaluated by assuming that, in each group k, all incomes are concentrated at the group mean μ_k (for equations 27 and 28) or the group median (for equation 29). In all cases the non-overlapping property of the grouping is important: for other types of grouping the Gini is not decomposable; this also applies to the median-normalized Gini index in (8).

No restriction on grouping is necessary for the class of inequality measures given by (9) or by (12). In the case of scaling by the mean, and scaling by the median, respectively, decomposition can be expressed as:

$$I_\alpha\left(\frac{\mathbf{x}}{\mu};\mu\right) = \sum_{k=1}^{K} p_k \left[\frac{\mu_k}{\mu}\right]^\alpha I_\alpha\left(\frac{\mathbf{x}_k}{\mu_k};\mu_k\right) + \frac{1}{\alpha^2-\alpha}\left(\sum_{k=1}^{K} p_k \left[\frac{\mu_k}{\mu}\right]^\alpha - \left[\frac{\mu}{\mu}\right]^\alpha\right) \tag{30}$$

$$I_\alpha\left(\frac{\mathbf{x}}{m};\mu\right) = \sum_{k=1}^{K} p_k \left[\frac{m_k}{m}\right]^\alpha I_\alpha\left(\frac{\mathbf{x}_k}{m_k};\mu_k\right) + \frac{1}{\alpha^2-\alpha}\left(\sum_{k=1}^{K} p_k \left[\frac{\mu_k}{m}\right]^\alpha - \left[\frac{\mu}{m}\right]^\alpha\right). \tag{31}$$

In particular, the MLD index, the limiting where case $\alpha = 0$, can be decomposed as follows:

$$I_0\left(\frac{\mathbf{x}}{m};\mu\right) = I_0\left(\frac{\mathbf{x}}{\mu};\mu\right) = \sum_{k=1}^{K} p_k\, I_0\left(\frac{\mathbf{x}_k}{\mu_k};\mu_k\right) - \sum_{k=1}^{K} p_k \log\left(\frac{\mu_k}{\mu}\right). \tag{32}$$

Taking a natural special case as an example, this means that we may partition the population into a group of females F and a group of males M and, using an obvious notation, express overall inequality as

$$I_\alpha = w^{\text{F}} I_\alpha^{\text{F}} + w^{\text{M}} I_\alpha^{\text{M}} + I^{\text{btw}}, \tag{33}$$

where the weights w^{F}, w^{M}, and the between-group inequality component I^{btw} are functions of the income mean (or of the median) for each of the two groups and overall; comparing I_α^{F} and I_α^{M} enables one to say precisely where changes in inequality have taken place.

Shorrocks (1980, p. 625) argued that the MLD index is the "most satisfactory of the decomposable measures," because it unambiguously splits overall inequality into the contribution due to inequality within subgroups and that due to inequality between subgroups, for arbitrary partitions of the population. This property, called *path independent decomposability* by Foster and Shneyerov (2000), is not shared by

(30) and (31), because the weights in the within-subgroup terms are not independent of the between-group term. Indeed, changing the income subgroup means, μ_k, will also affect the within-subgroup contribution through μ_k/μ in (30), but not in (32). It follows that the inequality that would result from removing differences between subgroups, the inequality within subgroups being unchanged, is given by the first term in (32), not that in (30) or (31).

5. Application

Let us look at how some of the measures we have discussed perform in practice. To do this, we choose measures that give similar weighting schemes to different parts of the distribution (see Section 3), which means using either the MLD index (25) or one of the Gini variants. Recall that, while there are separate variants of the Gini for normalization by the mean (7) or by the median (8), the MLD is the same whether one normalizes by the mean or by the median: this issue of normalization will prove to be crucial in the empirical application. In addition, while the MLD satisfies both the principle of monotonicity in distance and the principle of transfers, the mean-normalized Gini satisfies the principle of transfers, but not the principle of monotonicity in distance.

We use these inequality measures to compare the recent history of income inequality in Great Britain and the United States.

5.1. *Inequality in Great Britain*

First, we examine inequality in Great Britain, from 1961 to 2015. The values of Gini and MLD indices are given by the Institute for Fiscal Studies (*Tools and resources: "Incomes in the UK"*).[6] They are based on the Family Expenditure Survey up to and including 1992, and the Family Resources Survey thereafter. We use inequality indices computed on disposable income before housing costs. The median-normalized Gini is calculated using the formula on the right-hand side of (8).

Figure 5 (top plot) shows values of the regular Gini, the MLD, and the median-Gini indices, defined in (7) and (25). The series for the three inequality measures describe similar patterns. We can see that inequality increased considerably during the 1980s and it appears to have fallen slightly during the 1990s, as suggested by Jenkins (2000). However, we can see that inequality is relatively stable from 1990s onwards: if we estimate linear regressions of the log of each index against time, over the period 1994–2015, we find slope coefficients not significantly different from zero.[7]

Figure 5 (bottom plot) shows variations in inequality between two successive years, in percentage, for the three indices. We can see that variations from the MLD

[6] See https://www.ifs.org.uk/tools_and_resources/incomes_in_uk.

[7] We obtain: $\widehat{\log \text{Gini}}_t = \underset{(1.512)}{-2.871} + \underset{(0.00075)}{0.00090}t$, $\log \widehat{\text{MLD}}_t = \underset{(3.130)}{-7.578} + \underset{(0.00157)}{0.00298}t$, and $\log \widehat{\text{med-Gini}}_t = \underset{(2.077)}{-4.778} + \underset{(0.00104)}{0.00195}t$.

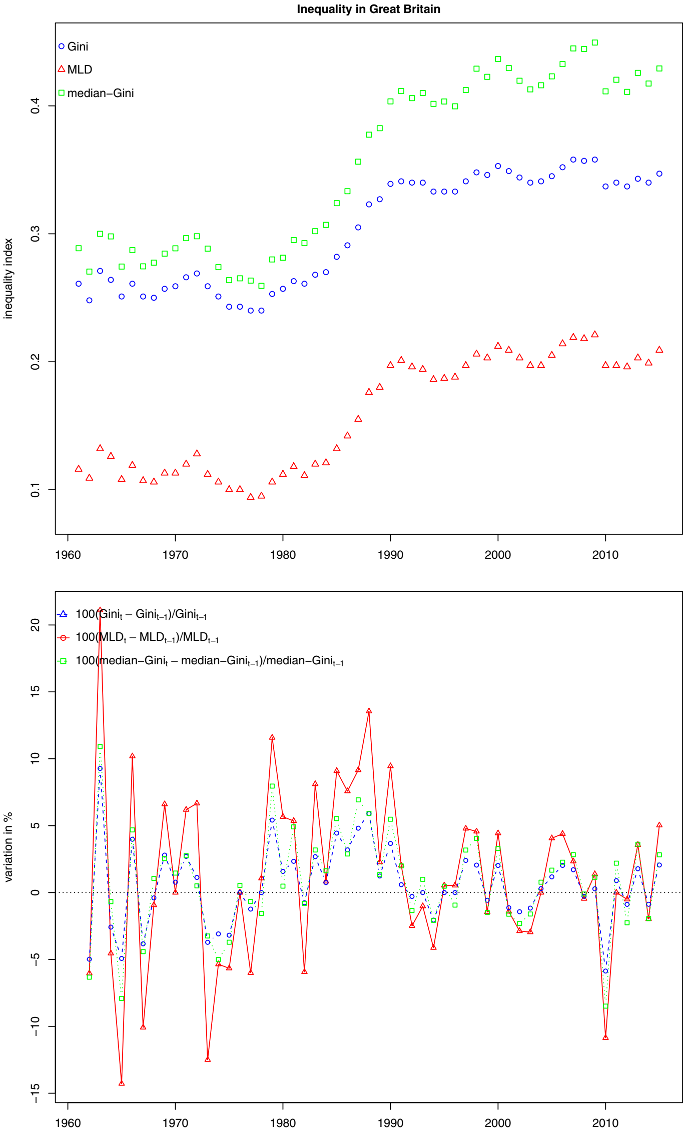

Figure 5. Inequality in Great Britain from 1961 to 2014 [Colour figure can be viewed at wileyonlinelibrary.com].

index are always greater than variations from the Gini index, with large differences. For instance,

- between 1962 and 1963, inequality increases by 9.274 percent using the Gini, by 10.914 percent using the median-Gini, and by 21.101 percent using the MLD;
- between 1982 and 1983, inequality increases by 2.682 percent using the Gini, by 3.182 percent using the median-Gini, and by 8.108 percent using the MLD;

- between 2009 and 2010, inequality decreases by 5.866 percent using the Gini, by 8.487 percent using the median-Gini, and by 10.86 percent using the MLD.

Such discrepancies are explained by the fact that both the numerator and the denominator of the Gini coefficient will vary in the same direction for any variations in incomes above the mean: any variations in incomes above the mean will be attenuated in terms of the impact on measured inequality. Underlying this behavior is the fact that the Gini coefficient does not respect the principle of monotonicity in distance. The same is true for any mean-normalized inequality index, such as the generalized-entropy measures (9), with the exception of the MLD index. As we have seen, the MLD index is the limiting case $\alpha = 0$ of both mean-normalized indices (9) and median-normalized indices (12), and it respects the principle of monotonicity in distance. By contrast to the regular Gini, neither the MLD nor the median-normalized Gini attenuates variations in income toward the top of the income distribution.

5.2. *Inequality in the United States*

Second, we examine inequality in the United States, from 1967 to 2016. The values of Gini and MLD indices are given by the U.S. Census Bureau, in the report *Income and Poverty in the United States: 2016* (Table A-3)[8]; the median-Gini is calculated as before. The sample survey was redesigned in 1994; it is thus not fully comparable over time: for more on data quality issues, see McGuinness (1994) and Burkhauser et al. (2012).

Figure 6 (top plot) shows values of the regular Gini, the MLD, and the median-Gini indices. The MLD and the median-Gini reveal quite *different* patterns from that of the regular Gini. The increase in inequality since the 1980s is much higher using the MLD index or the median-Gini than would appear using the Gini index. If we estimate linear regressions of Gini, median-Gini, and MLD indices in log against time, over the period 1994–2016, we find slope coefficients significantly different from zero and equal to 0.00264, 0.00517, and 0.01292, respectively.[9] It means that inequality increases at an average annual rate of 0.264 percent using the Gini index and 0.517 percent using the median-Gini index, and at an average annual rate of 1.292 percent using the MLD index. Thus, over the period since 1994, the rate of growth of inequality using the MLD index is between four and five times that of the Gini index.[10]

[8]See http://www.census.gov/data/tables/2017/demo/income-poverty/p60-259.html

[9]We obtain: $\widehat{\log \text{Gini}}_t = \underset{(0.378)}{-6.064} + \underset{(0.000188)}{0.00264}t$, $\widehat{\log \text{MLD}}_t = \underset{(1.35)}{-26.54} + \underset{(0.00067)}{0.01292}t$, and $\widehat{\log \text{med-Gini}}_t = \underset{(0.925)}{-10.83} + \underset{(0.00046)}{0.00517}t$. For the contrast between the regular Gini and the median-normalized Gini over the period, see Gastwirth (2014), page 314.

[10]Clearly growth/change comparisons of inequality depend on the cardinalization of the inequality indices. In principle any cardinalization could be used, but it makes sense in practice to confine attention to those that are used in practice. There are no alternative cardinalizations of the Gini coefficient that are used in the literature. However, in the case of the MLD, there is an alternative cardinalization in current use: the Atkinson inequality index with parameter 1 is given by $A_1(\mathbf{x}) = 1 - \frac{g}{\mu}$, where g is the geometric mean. Using (9), it is clear that $A_1(\mathbf{x}) = 1 - \exp(-I_0)$, and so growth $(A_1) = \lambda$growth (I_0),

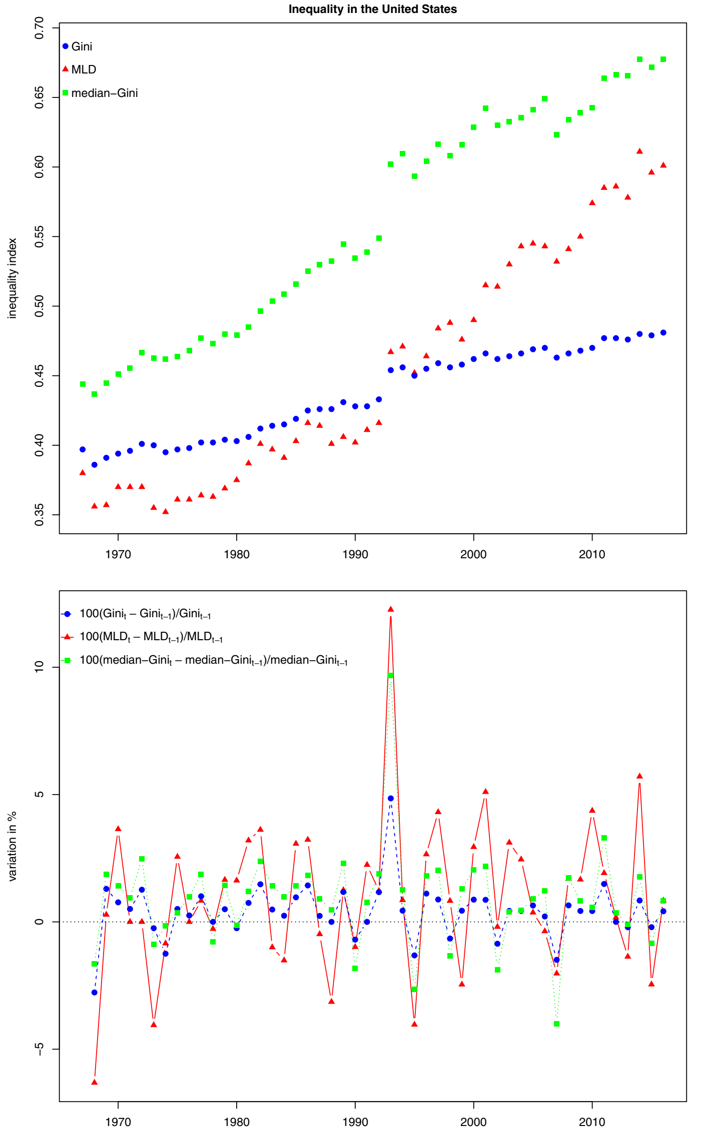

Figure 6. Inequality in the United States from 1967 to 2015 [Colour figure can be viewed at wileyonlinelibrary.com].

Figure 6 (bottom plot) shows variations in inequality between two successive years, in percentage, for the Gini and MLD indices. We can see that variations in the

where $\lambda = I_0\left[1 - A_1\right]/A_1 = I_0/\left[\exp(I_0) - 1\right] = 1/\left[\frac{1}{2!}I_0 + \frac{1}{3!}I_0^2 + ...\right] < 1$. Therefore, the proportional changes in A_1 will be less than those of I_0. However, for our data, this change in growth rate attributable to the change in cardinalization is relatively modest, as λ ranges from 0.71 to 0.89. In no case is the conclusion that the Gini understates the changes in inequality reversed.

Table 1
Inequality in the United States: A Five-Income Parable

	Incomes			Base Year 1994			Base Year 2016	
	1994	2016		Gini	MLD		Gini	MLD
mean	*\$68,138*	*\$83,143*		*0.4222*	*0.3523*		*0.4452*	*0.3999*
x_5	\$167,364	\$213,941	$x_5\uparrow$	0.4676	0.4314	$x_5\downarrow$	0.4004	0.3302
x_4	\$79,610	\$95,178	$x_4\uparrow$	*0.4212*	0.3613	$x_4\downarrow$	*0.4469*	0.3975
x_3	\$51,160	\$59,149	$x_3\uparrow$	0.4125	0.3465	$x_3\downarrow$	0.4539	0.4095
x_2	\$30,368	\$34,504	$x_2\uparrow$	0.4123	0.3389	$x_2\downarrow$	0.4537	0.4155
x_1	\$12,186	\$12,943	$x_1\uparrow$	0.4195	0.3425	$x_1\downarrow$	0.4475	0.4102

MLD index are mostly greater than variations in the Gini index, with a few results in the opposite direction. For instance,

- between 1972 and 1973, inequality decreases by 0.249 percent using the Gini, by 0.882 percent using the median-Gini, and by 4.054 percent using the MLD;
- between 1992 and 1993, inequality increases by 4.85 percent using the Gini, by 9.67 percent using the median-Gini, and by 12.26 percent using the MLD;
- between 2005 and 2006, inequality increases by 0.213 percent using the Gini and by 1.221 percent using the median-Gini, it decreases by 0.367 percent using the MLD.

Such opposite results could be explained by the fact that a shift to the right of incomes greater than the mean can lead to a decrease in mean-normalized inequality measures, such as GE indices and the Gini. It is because these measures do not respect the principle of monotonicity in distance (see Sections 1 and 2.3).[11]

A Parable

What is going on can be illustrated in the simple parable depicted in Table 1. The scene is set in the first group of three columns: this shows the incomes $(x_1, \dots, x_5)$ corresponding to the mean incomes of the five quintile groups in 1994 and 2016, arranged in descending order[12] at the top are the mean incomes in the two years. The x_4 is a visual reminder of the position of the mean in this income distribution.

The second group of columns in Table 1 takes the five-income distribution in 1994 as a base case and, at the top, presents the estimates inequality according to the Gini coefficient and the MLD index (these are estimates of between group inequality in the United States, when we decompose by the five quintile groups). Further down, we set out the results of five simple experiments that address the following question: how would inequality change if we increased just x_i from its base (1994) value to its 2016 value? The intuitive monotonicity principle suggests that if x_i is above the mean, such a change should increase inequality and, if x_i is below the

[11]The shift in incomes toward top incomes in the United States over the last decades is well documented—see, for example, Atkinson and Piketty (2010), Krueger (2012), and Piketty (2014).

[12]Again see http://www.census.gov/data/tables/2017/demo/income-poverty/p60-259.html.

TABLE 2
RELATIVE IMPACT OF x_i ON INEQUALITY, BASED ON 1994 DISTRIBUTION

	Percentage Change in …		
	(a) Gini	(b) MLD	ratio (b)/(a)
x_5 ↑	10.76	22.43	2.08
x_4 ↑	−0.23	2.54	−11.07
x_3 ↑	−2.29	−1.66	0.72
x_2 ↑	−2.34	−3.82	1.64
x_1 ↑	−0.64	−2.79	4.35

mean, this change should decrease inequality. In the case of the MLD this is exactly what happens: increasing any of the lower incomes 1, 2, or 3 reduces MLD below its base value of 0.3523 and increasing either of the upper incomes 4, 5 increases the MLD. However, the Gini coefficient behaves oddly: increasing the above-mean income 4 reduces inequality below its base value of 0.4222.

The third group of columns in Table 1 presents the complementary set of five experiments, taking the five-income distribution in 2016 as a base case and examining the impact of reducing x_i to its 1994 level, for any value of i. Again the MLD accords with intuition: reducing incomes 4 or 5 (above the line) reduces inequality below the base value of 0.3999, and reducing incomes 1, 2, or 3 (below the line) increases inequality. Again the Gini behaves in an apparently anomalous fashion for income 4: reducing an above-mean income increases inequality.

Also revealed by the parable is the relative size of the impact on inequality in the United States of any variation in incomes $\left(x_1, \ldots, x_5\right)$. Table 2 is derived from the "base year 1994" columns in Table 1 and measures the percentage change in inequality (Gini or MLD) caused by raising income x_i to its 2016 level. Clearly the impact of raising the topmost income has double the impact on MLD, compared to Gini; but the effect of raising the bottom income has an impact on MLD that is 4.35 times the impact on Gini.

In sum, the parable shows us two things about inequality in a time of rising incomes. First, in the recent history of the United States, the increase in some upper incomes did indeed lower the Gini coefficient, exactly as we argued in the context of an artificial example in Section 2.3 (but the effect on the MLD was in the expected direction). Second, in the context of the United States during this period, the MLD shows a larger response to income changes at the top and the bottom of the distribution than does the Gini coefficient.

5.3. *United States vs. Great Britain*

Finally, we compare inequality in Great Britain and in the United States with the two variants of the Gini and the MLD indices. Although the income definitions in the two countries are different, comparisons of inequality changes are instructive.

Figure 7 —top plot—shows values of the Gini index. It is clear that inequality is always much higher in the United States compared to Great Britain—a point that would remain true under alternative definitions of income. However, when we compare trends over recent years, we can see that the increase in inequality is not

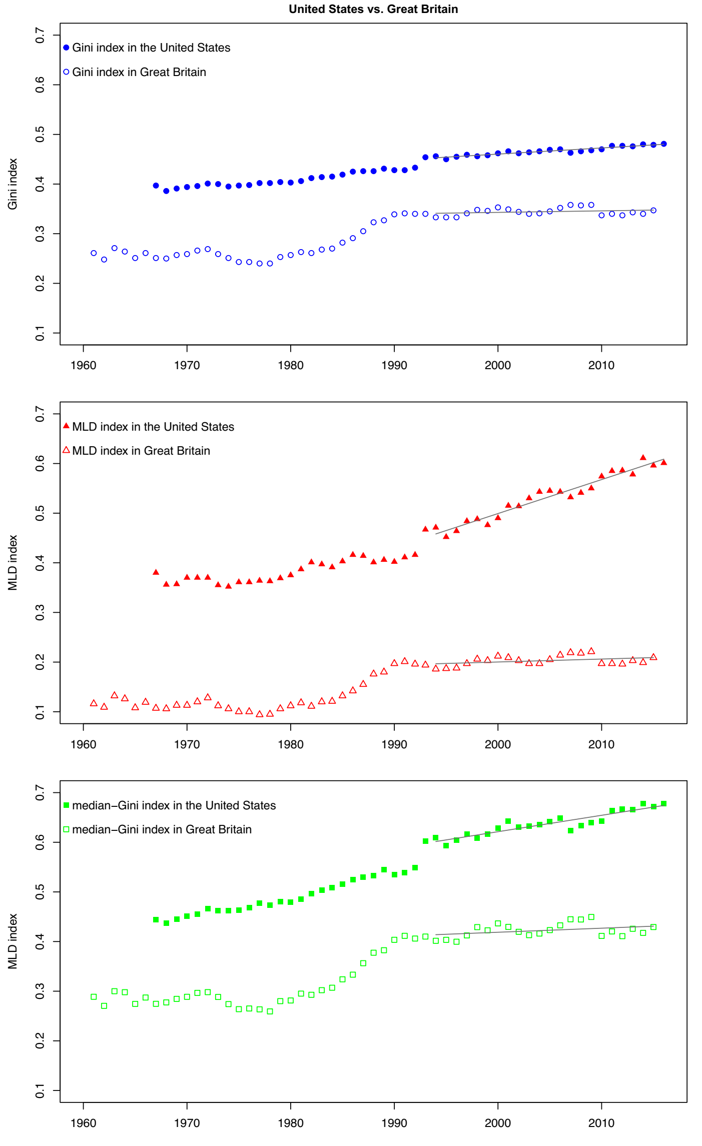

Figure 7. Inequality in the United States and in Great Britain [Colour figure can be viewed at wileyonlinelibrary.com].

very different. We have seen that, from 1994 to 2016, the annual rate of growth is not significantly different from zero in Great Britain and is equal to 0.3273 percent in the United States (see footnotes 7 and 9).

Figure 7 —middle and bottom plots—shows values of the MLD and the median-Gini indices. Now we find a quite different picture: inequality increases are much higher in the United States than in Great Britain since 1994. Indeed, for the MLD we have seen that while the annual rate of growth is not significantly different

from zero in Great Britain, it is equal to 1.5427 percent in the United States (see footnotes 7 and 9)

Overall, comparisons based on the Gini index suggest that the increase in inequality since 1994 is not very different between Great Britain and the United States, while it is found to be remarkably different based on comparisons of the MLD index.

As we noted at the beginning of Section 2, if two inequality measures give different weight to information in different parts of the distribution, it is not concerning to find that they rank some distributions in opposite ways; but what could be concerning is "when two different measures contradict each other in practice because they are founded on different, potentially conflicting, economic principles." The empirical application here illustrates exactly this point. The inequality indices have similar weighting schemes across the income distributions. Where they differ crucially is in terms of the principles on which they are founded (principle of transfers, principle of monotonicity in distance, or both). We have seen that we indeed find indices that contradict each other, because they are founded on different principles, not because they put different weights on different part of the distribution.

6. Conclusion

For assessing the distributional impact of policies—or for just making international comparisons of inequality—practical measurement tools are needed. For a non-technical policy-maker or observer, one might think that the choice of practical measurement tool is very obvious. The Gini index is probably the most widely used inequality index in the world and is often published by statistical agencies as part of their data reporting. This index has a simple weighting scheme on individual incomes, it respects the principle of transfers, it is independent of income re-scaling, and it is also decomposable (but only for groupings that are "non-overlapping" in terms of income). However, as we discussed, the Gini coefficient does not satisfy the monotonicity in distance property, which can result in some strange behavior. Measures that are closely related to the Gini also have drawbacks. Although the absolute Gini satisfies both the monotonicity principle (if the median is the reference point) and the transfer principle, its value is not independent of income scale—double all the incomes and you double the inequality index. The median-normalized Gini does not satisfy the principle of transfers.

By contrast the MLD index has all of the attractive properties of the Gini coefficient and more: it also respects the principle of monotonicity in distance and is decomposable for arbitrary partitions with the path-independence property.

As we have seen, the lack of the principle of monotonicity in distance has strong implications in empirical studies. Indeed, the Gini coefficient and other indices may understate variations in inequality and, as a consequence, may be poor indicators of the effectiveness of redistribution policies. Our application suggests that the increase in inequality in the United States over recent years is significantly understated by the Gini index. By contrast, the MLD index has more desirable properties, estimates variations in inequality more accurately, and should be preferred in practice.

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Supporting Information

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Appendix S1: Supporting Information.