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Who's got the power? Wage determination and its resilience in the Great Recession

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Abstract
Whereas wage inequality has risen markedly in most OECD countries in recent decades, it has fallen in several Southern European economies. To shed light on this phenomenon, we embed sectoral bargaining, which is common in Southern European economies, in a dynamic search and matching model. We estimate the model using comprehensive employer-employee data from Portugal for the last two decades and its data on collective bargaining agreements in different sectors, which allows us to assess the evolution of rent sharing. We find that since the mid-2000s, worker bargaining power has grown slightly at the bottom of the skill distribution while shrinking at the middle and top, contributing to the compression of the wage distribution. These changes, which persisted even during the Great Recession, increased the importance of sectoral bargaining in wage determination, weakened the relationship between wages and firm productivity, and reduced the assortative matching of workers to firms.

Key words: search and matching, wage determination, collective bargaining and trade unions, rent sharing, bargaining power, assortative matching, wage inequality
JEL codes: C55; C61; C62; C78; J31; J51; J53

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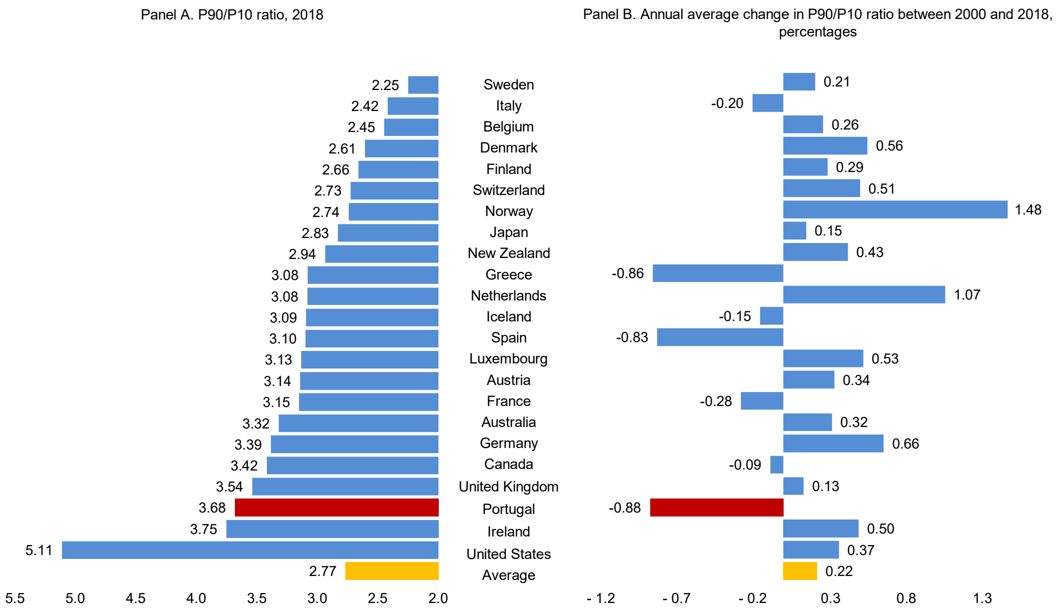
1 Introduction

The erosion of unions' influence, the rise of product and monopsony powers, and technological progress have contributed over the last two decades to a structural change in the labour markets of many economies. This transition has affected the way wage determination shares rents between workers and firms, influencing trends in inequality, labour shares, corporate profitability and wage (and inflation) dynamics.

There is substantial heterogeneity amongst labour market institutions in Continental Europe that results in several institutional clusters. Among these France, Spain, Italy, Greece and Portugal form the Southern European grouping, the markets of which share a number of similarities (see Botero et al., 2004 and Boeri, 2011), including high employment protection and generous employment benefits. Their main common feature in wage determination is the key role of sector level collective bargaining in which unions bargain for most contracts (high coverage), despite their modest numbers of membership (low union density) (see Addison et al. (2021), for further details).

While wage inequality has been rising in most of the OECD, it has fallen in Southern Europe (see Figure 1). This discrepancy seems to be unrelated with the inequality level or natural geographical boundaries. Portugal and Italy, for example, are at polar opposites of the inequality grid but have identical dynamics, whereas France and Germany are very close in inequality levels but on clearly different paths. Can the evolution of wage determination in Southern Europe be compatible with such an anomalous inequality trend?

Figure 1: Inequality of Gross Earnings of Full-Time Dependent Employees



Notes: Every OECD country that adhered before the early 1990's are included, excluding Turkey due to data limitations. When the data for the date were unavailable, the closest available date was used. *Source: OECD.*

This paper develops a multiple worker-occupation-firm dynamic search and matching model in which the wage determination takes place at both sectoral and firm levels. We estimate the equilibrium wage resorting to the Portuguese employer-employee administrative data and the information about the positioning of each worker in the sector level collective bargaining agreements between unions and firms.[1] This *positioning* yields a ranking that is bargained for decades, directly covering around 85 percent of the private sector labour contracts without any *opt-out* possibility. The outcome is a granular system of around 30,000 sector-occupation job descriptions, which allows us to adopt a novel identification approach, benefiting from the proper bargaining dynamics of the market to define outside options and types of workers.[2] The resulting unified and consistent framework allows us to estimate the evolution of several wage determination measures as the worker bargaining power, the elasticity of wages to quasi-rents, the average passthrough of bargained wages to actual wages, and the degree of assortativeness in the market.

We find that the wage determination has been undergoing structural change over the last two decades. Firms have likely increased their monopsony power, and workers have lost bargaining power. Their wages have become more determined by collectively bargained wage floors and less linked to productivity developments. However, this erosion of bargaining, of around one third, has been confined to the top and middle of the skill distribution, with the bottom seeing its bargaining position reinforced by around 7 percent. The consequent bargaining power compression has led to a wage compression, which is associated with an increase in mismatch, as measured by a significant fall in assortative matching. Despite economic tribulations, this structural change has demonstrated considerable resilience.

On average, our findings conform with those of the literature. We estimate the worker bargaining power at 20 percent for the entire economy. The average level of the elasticity of wages to quasi-rents is 0.062, within the 0.05-0.15 most accepted interval (see Card et al., 2018). The average of our assortative matching measure is estimated at 44.1%, while the average passthrough of changes in bargained wages into changes of total wages is estimated at 44.8%, in line with Card and Cardoso (2021).

This paper contributes to the debate on fall (rise) of worker (firm) bargaining power. Since the 1980s, corporate profitability and firm markups have recorded increasing trends, while the labour share has been in decline. Gutiérrez and Philippon (2017), Farhi and Gourio (2018) and De Loecker et al. (2020) have linked these trends to the rise of product market power. Krueger (2018) noted in his *2018 Jackson Hole* address, that the

[1]See Appendix A for a detailed description of the dataset used in this study.

[2]As noted by Card and Cardoso (2021), this structure exists in countries like Portugal, Belgium, France, Italy, Netherlands and Spain. See Cardoso and Portugal (2005), Martins (2014), Addison et al. (2017) and Card and Cardoso (2021) for further detail about the ranking for Portugal.

evolution of labour market practices has not only enhanced monopsony power, but also weakened worker bargaining powers. Following his line, Stansbury and Summers (2020) provides a case for a relevant (if not leading) role of bargaining power trends in explaining those macroeconomic dynamics, and Lombardi et al. (2020) linked a weakening of worker bargaining power with an abatement of inflation dynamics, and an amplification of employment adjustments over the business cycle. We, too, detect overall worker bargaining power erosion, and specifically in the case of Portugal.

Our paper also contributes to the wage inequality literature by providing bargaining power trends for different skill groups that are compatible with the wage compression witnessed in Southern Europe. Since the 1980s skill-biased technological change, and then job polarization have been pointed to as the leading causes of the growing wage inequality.[3] However, from the outset of the debate several developed economies such as France, Japan, and Germany were not revealing the same wage inequality trend, encouraging several authors to focus on the mediating role of "institutions" (Freeman and Katz, 1995).[4] Amongst these institutions, different degrees of unionization, wage determination structures and minimum wage policies have been the most studied.[5] We reinforce the importance of the role of the labour market institutions (and its dynamics) in shaping wage inequality and sorting outcomes, on an equal footing with product market dynamics. More recently, in a precise mirror to our findings, Card et al. (2013) and Song et al. (2019) empirically linked the increase in sorting and wage inequality for Germany and the US.

Methodologically, we propose a wage determination process with a key role for sectoral bargaining. The model entails: (i) firms having a hierarchical occupational structure with worker-firm heterogeneity; (ii) on-job-search à la Roy (1951); (iii) binding firing taxes as in Boeri (2011); (iv) a union that is unable to cause an *hold-up problem* in the event of a bargaining breakdown, as in Dobbelaere and Luttens (2016); and (v) the adoption of a two-step wage determination that unfold first through bargaining at sector level between firms and a representative union, and then at firm level either with or without bargaining with unions. The resulting equilibria allows a straightforward comparison between our proposition and several other canonical models (i.e. Pissarides (2000), Acemoglu and Hawkins (2014), Cahuc et al. (2008) and Mortensen (2009)).

Empirically, we estimate the wage floors that are compatible with the collective bargaining ranking and we follow the model's postulate that those coincide with the workers' outside options. While, its empirical use is not novel, as Card et al. (2014) use an identical

[3]See Autor et al. (2003) and Acemoglu and Autor (2011).

[4]The literature has also focused on top income inequality (Piketty and Saez, 2003).

[5]See DiNardo et al. (1996), Card and DiNardo (2002), Dickens and Manning (2004) and Autor et al. (2008) for evidence on the influence of these institutional settings on wage inequality.

concept, we present a theoretical framework that sustains such a reduced-form prediction, thereby reinforcing the relevance of measures of bargained wages as an alternative to estimate outside options. Among the indicators employed, Caldwell and Harmon (2019) use past co-workers' job movements as a measure of the value of the social network of the worker. Schubert et al. (2020) analyse the workers' job histories, and Caldwell and Danieli (2020) implements a sufficient statistic that assesses the supply of jobs in the area and the workers' flexibility to take them.

Our approach relies on firms and unions being capable of assessing each worker's worth, thus establishing a credible and consistent administrative ranking. We turn to this ranking to enhance the identification of the marginal products of the match, as it allows us to identify types of workers. This option is an alternative to the use of statistical rankings of workers and/or firms, an approach that has received great attention. Bonhomme et al. (2019) and Lentz et al. (2018) employ a two-step algorithm in which either firms or workers (or both) are classified in the first step into categories by using a *k-means clustering algorithm*. Sorkin (2018) resorts to *Google's PageRank* algorithm to rank firms based on revealed preference, thereby identifying the value of compensating differentials; and Hagedorn et al. (2017) present the classic *Kemeny-Young rank aggregation* algorithm as a way to rank workers and then firms based on the worker's ranking.

The final contribution of the paper resides in its approach to estimating the bargaining power. Traditionally, the literature has two strands: (a) a reduced-form approach using to proxies of quasi-rents, potentially supplemented with either instrumental variables (see Card et al. (2018) for an extensive overview), or an AKM formulation following Abowd et al. (1999); and (b) a structural approach with a full definition of every object of the model, such as the production and matching functions (see Cahuc et al. (2006) and Mortensen et al. (2010) for prime examples). We take an intermediate approach. Our model structure allows for an estimation that does not rely on worker/firm fixed effects, resulting in a parsimonious parameter set when compared with an AKM strategy. At the same time, we abstain from fully defining matching and production functions.

The paper is organized as follows. Section 2 presents the main features of the labour market and the way they are introduced into the model. Sections 3 and 4 present the model and the identification strategy of the wage equation, respectively. Section 5 discusses the empirical results. Section 6 concludes.

2 The Wage Determination in Continental Europe

The wage determination process has been more centralized and/or coordinated in continental Europe when compared with the Anglo-Saxon counterparts. The European group is still largely dominated by industry or sector level labour agreements in which trade unions play a considerable role, whereas in the Anglo-Saxon economies those agreements are often signed at firm, plant, or even individual level (see Table 1). Even within Continental Europe labour market institutions that differ significantly co-exist, leading their analysis to be developed according to *countries' institutional clusters*. As illustrated in Figure 2, Boeri (2011) divides Europe into four clusters - *Continental*, *Nordic*, *Southern* and *Anglo-Saxon* - reflecting the contemporary balance of each labour market in the use of employment protection, unemployment benefits policies, and active labour market policies. Botero et al. (2004) partitions the Continent into *French*, *German*, *Socialist* and *Anglo-Saxon* based on legal ancestry.

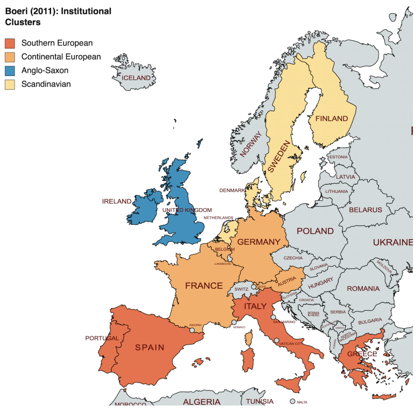
Table 1: Average Collective Bargaining Indicators by Decade

Countries:	Level of Bargaining		Bargaining Centralization		Union Density (%)		Union Coverage (%)		Single Employer Bargaining (%)	
	2000's	2010's	2000's	2010's	2000's	2010's	2000's	2010's	2000's	2010's
Portugal	3	3	2.8	2.6	18.4	17.4	80.5	75.5	3	5
Spain	3	3	2.52	2.17	19.3	18.9	70.9	70.5	7.5	5.9
France	3	3	2.4	2.3	10.7	10.9	98	94.2	3.6	3.8
Italy	3	3	2.59	2.41	33.6	35.5	80	80	-	-
Greece	3.9	2.4	3.6	1.3	27.75	21.07	100	57.1	8.2	15
Germany	3	3	2.2	2.2	21.7	17.7	65	57.1	8.5	8.1
Austria	3	3	2.29	2.28	33.3	27.6	78.5	78.5	3	3
Netherlands	3.4	3	2.59	2.19	20.4	18.2	82	83.1	10.1	7.6
Belgium	4.5	4.6	4.09	4.14	54.9	53.6	94.6	92	10	-
Switzerland	3	3	2.49	2.39	74.5	67	91.9	89.6	8.3	5.6
Denmark	3	3	2.34	2.3	70.4	67.4	77	79.3	22	-
Sweden	3	3	2.49	2.39	74.5	67	92	89.6	8.3	5.6
Norway	3.2	3	2.57	2.39	54.3	52.3	73.7	71	-	-
Finland	3.65	3.67	3.06	3.06	69.2	63.45	86.2	89.6	9	9
Poland	1	1	0.96	0.9	17.1	16.9	20.9	17.9	15.4	14.8
Czech Republic	2	2	1.8	1.8	23	13.3	27.9	33.1	27.9	33.1
United Kingdom	1	1	1	1	28.6	24.9	34.6	28.3	29.4	27.5
United States	1	1	1	1	12.2	10.7	13.4	11.9	-	-

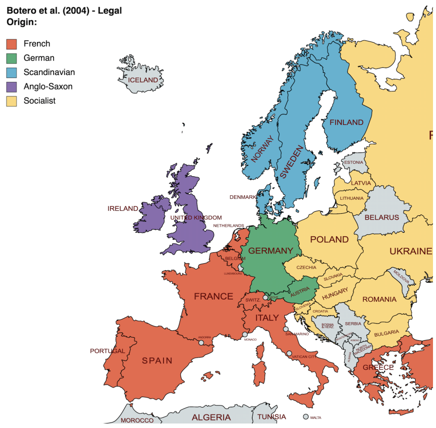
Notes: Level of bargaining takes the values: (5) central or cross-industry level bargaining; (4) alternating between central and industry bargaining; (3) sector or industry bargaining; (2) sector or company bargaining; (1) company bargaining. Bargaining Centralization is a measure created by ICTWSS, ranging between 1 and 5, with 5 being the highest level of centralization. *Source: ICTWSS*, version 6.1, 1960-2018.

While these groups of labour markets have recorded a common structural change in their institutions since the 1980s, with indirect measures displaying a detrimental evolution to workers bargaining powers, the measures' magnitudes have evolved differently across geographies (Figure 2). The Anglo-Saxon markets have witnessed a strong co-movement of de-unionization and fall in collective bargaining coverage, de-emphasizing the role of trade unions and largely side-lining them. The Nordics have maintained a high degree of unionization and collective bargaining coverage. Others, like Germany, have followed the Anglo-Saxon paradigm although to a lesser degree. There is yet another group, the Southern European and French based markets, which have coordinated a sub-

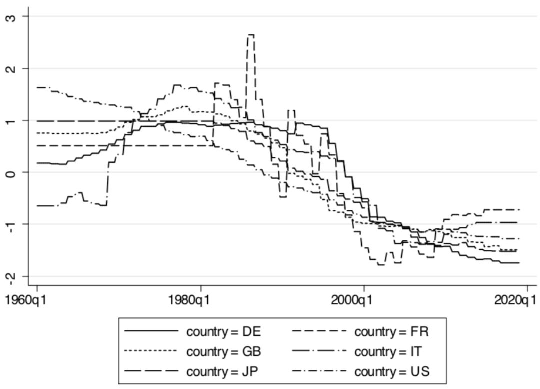
Figure 2: Labour Markets in Europe and Indirect Measures of Worker Bargaining Power Evolution



(a) **Institutional Clusters**, Boeri (2011).



(b) **Legal Origins**, Botero et al. (2004).



(c) **Synthetic Indicator of Worker Bargaining Power**, Viviano et al. (2020).

Note: In panel C, each line represents their synthetic indicator of the worker's bargaining power, by using the first principal components of several OECD indicators, including union density, union coverage, employment protection indexes, and coverage of collective agreements. *Sources:* Botero et al. (2004), Boeri (2011), and Lombardi et al. (2020).

stantial, even leading, fall in union membership with an untouched, and nearly universal, level of collective bargaining coverage, implying that unions directly bargain the large majority of contracts, despite their meagre membership.[6] This last group and feature is precisely the focus of the current article.[7]

In a shorter time horizon, particularly since the 2010's, Figure 2 points to a divergent path of the institutional dynamics. France and Italy, both in the Southern European group, witnessed reinforcements of the worker position; the Anglo-Saxon recorded a broad stability; whereas Germany presents a continuing decline. Together, these facts reinforce the interest of analysing the contemporary consequences of such paths.

[6]See Bryson et al. (2011), Pontusson (2013), and Visser (2016).

[7]Considering France and the Southern Europe as a consistent group for wage determination is not necessarily at odds with Boeri (2011). He acknowledges the heterogeneity of his Continental group, and that France is likely closer to Spain than to Germany.

Modelling the Continental European Labour Market Wage Determination

In continental Europe, the standard wage determination consists of a multi-stage process in an environment where there are severance payments in the event of employer-initiated worker dismissal that is not prompted on disciplinary grounds. In Southern Europe, this process starts with a bargaining between trade unions and employers' associations about sector/industry tables of wage floors (or sometimes floors for wage dynamics). Then, the tables (or dynamics) are potentially improved upon at firm level, and worker placement according to those agreed tables, (especially of new hires) is performed.

We introduce this wage determination, firing taxes, and a description of the behaviour of trade unions in our model.[8] This explicit incorporation serves two purposes. First, it allows an enhanced mapping between the available data and the model moments we seek to estimate. Second, it highlights the impact and interplay of these features, while preserving straightforward theoretical links with some of the most canonical models in the search and matching literature (see online appendix A for further details). The proposed framework can be implemented for other cases in which some of these components are either absent or simplified.

The modelled wage determination has two stages. In the first - *the collective bargaining* - we have a Nash bargaining regarding the wage floor of each type of worker that takes place between a trade union and the average firm of the economy. In the second stage there is a wage setting at firm level, constrained by the existence of the applicable wage floor. This wage setting can be either an *ex-post bargaining*, with a Nash bargaining between the same trade union and the specific firm, or an *ex-ante wage posting* in which the firm defines the wage.

This modelling choice is made for three reasons. First, the two approaches will be *structurally isomorphic*, particularly given the characteristics of our trade union.[9] The sole difference between the equilibrium wage equations is in the interpretation of their parameters. Second, while interpretation matters, the focus of our work is to measure the evolution of worker (firm) strength in the rent splitting process, not to directly search for their precise drivers. We show that for these purposes we do not need to select which procedure is behind the wage setting at firm level, and can instead take a broader definition of worker/firm power encompassing both procedures, as in Stansbury and Summers

[8]Concretely for these countries, Booth (2014) recommends the explicit modelling of the behaviour of trade unions given their importance in collective bargaining. See Pissarides (1986) for the first inclusion of unions in a search and matching framework, and Bauer and Lingens (2010) and Dobbelaere and Luttens (2016) for recent treatments with an approach closer to ours. Finally, also see Abowd and Lemieux (1993) for a mainstream treatment, and Krusell and Rudanko (2016), who revisits the question in the context of a frictional labour market in a macroeconomic perspective.

[9]See Manning (2011) for a discussion about this isomorphism in a parsimonious setting.

(2020). Finally, the literature has acknowledged the simultaneous existence of posting and bargaining in the labour market (see Hall and Krueger (2010) for an example), with the use depending on several characteristics such as worker's education and occupation and the presence of unions, among others. Thus, in this paper, we show the isomorphism and measure the worker/firm strength in splitting rents in a framework that is compatible with both wage setting mechanisms.

The bargaining follows a *right-to-manage* perspective. The trade union bargains over wages whenever bargaining takes place, allowing firms to choose employment. Given the prevalence of collective bargaining and the decline in union membership, the trade union is assumed to have universal coverage (including the unemployed) with potentially different welfare weights for different groups.[10] However, the union will lack the ability to force a full lockdown of production when a bargaining process breaks down. As in Dobbelaere and Luttens (2016) *gradual collective bargaining* structure, if negotiations break down the parties reach stalemate until one of the workers in that contract leaves the firm without any severance payment being levied.[11] Then, both sides restart bargaining every contract aiming at unlocking the stalemate, with the process unfolding gradually, as before, until a full simultaneous agreement is reached. In this process, unions represent each *type* of worker without realizing the potential general equilibrium effects each decision could have on the employment and wages of other types.

In case of disagreement, the existence of a bargaining *stalemate* instead of a *full lockdown* leads to a firm level Nash bargaining that is identical to individual bargaining, as in Cahuc et al. (2008) or Acemoglu and Hawkins (2014). The value of our trade union lies on its ability to have more accurate information than isolated workers and to politically impose a set of implicit clauses on the wage determination, rather than coordinating worker's actions to impose costs on firms via lockdowns and strikes.[12] Concretely, the sides commit to four clauses: (i) the sides settle a collective bargaining wage floor that is

[10]Pissarides (1986) shows that if the trade union has the monopoly to bargain wages in the spirit of Dunlop (1944) and values only the situation of the unemployed, the model equilibrium will be efficient with the search externality fully internalized. Our model will be able to have this as a specific result.

[11]Dobbelaere and Luttens (2016) justifies the discarding of this mainstream prior due to the near lack of empirical evidence of such an event, with the sole exception of the *Ronald Reagan and the air traffic controllers* case in 1981, which they argue was political. Holden (1988) resorts to the Nordic *peace-clause* to also avoid a *full-lockdown assumption*.

[12]The ability of unions to have more accurate information than isolated workers, and to coordinate actions of workers to enforce state-contingent actions or implicit clauses in the contract is precisely at the core of Hogan (2001) analysis. As a matter of fact, the dataset on workers' characteristics was created so that unions could inspect and monitor firms' behaviour. Created by law in 1976, *Quadros de Pessoal* were mandatorily sent by firms to the Ministry, and posted in a visible place in each establishment, with every relevant employment characteristic, including the wages and the worker's position.

compatible with the worker's *least viable match value*, where no value beyond the average firm and worker's opportunity costs would be generated; (ii) they assume ex-ante that neither party will exert their at-will option to dissolve that match for wage determination purposes - the *match stability principle*; (iii) whenever wage posting takes place, the firm posts wages assuming the workforce they have when they start the wage determination process - the *workforce stability principle*; and (iv) no worker is unfairly treated, so that his type is correctly assessed and the placement occurs accordingly. Finally, we complement the bargaining apparatus with employment protection, which notably translates into the existence of firing costs/taxes, in the spirit of Bentolila and Bertola (1990), Bertola and Caballero (1994) and Boeri (2011), when the employer has the initiative to dissolve the match.

A reasonable outcome of these contract foundations is the nonexistence of sequential bargaining in case of poaching, contrary to Postel-Vinay and Robin (2002). There, firms bid for worker's services, creating an enforceable link between the current value of the worker's outside option and the history of his past job offers while employed. This link has been disputed due to the empirical rarity of a sequential bidding in defining wages, and the predictable lack of enforceability of incumbent firm - individual worker promises.[13] Barron et al. (2006) highlights the implications of this process on the co-workers' contracts, and thus define as theoretically reasonable the existence of at most a selective counter-offer policy. Empirically, evidence points to its limited use if there is transparent pay, which in our case trade unions are assumed to reinforce (see Cullen and Pakzad-Hurson (2021)). Altogether, we assume that unions and firms predict the value of outside options for each type of worker, and those predictions are enforced in the collective bargaining. If a worker receives a beneficial proposal, he leaves the match.

As the model equilibrium conditions will elicit, the collective bargaining wage floor will be the worker's outside option value assuming match stability (or his *fire-sale*). It corresponds to the worker's value if he becomes exogenously and unexpectedly unemployed. While the existence of the first stage bargaining will not change the equilibrium wages, its modelling allows for the use of data to improve identification, even if one has access to only the wage ranks and job descriptions of the workers in the collective bargaining, as is our case.[14] We will establish the observed minimum base wage in each position of the collective bargaining wage table as the proxy of the wage floor, and resort to the very granular sector-occupation wage ranks of the tables to build a comparable measure of the

[13]See Pissarides (1994), Shimer (2006), and Dolado et al. (2008) for examples of studies resorting to this set of arguments.

[14]Card and Cardoso (2021) were capable to collect a dataset with the agreed minimum base wage by collecting those directly from the wages tables. However, due to limitations in linking their collected data and the administrative data, they just linked roughly half of the workers for their 2008-2016 period.

outside option value. Card et al. (2014) implements an identical choice for the Veneto region of Italy. We strengthened the case for its use to identify rent-sharing parameters by presenting the mapping between the data and the wage determination we propose.

Stylized Facts on the Portuguese Wage Determination

In Portugal, a representative design of Southern Europe, the collective agreements are overwhelmingly negotiated by employer's associations and unions linked to the two union confederations. They are signed without any *opt-out* possibility for a firm in an industry or sector covered and reach much further than the workforce of the initial signatory parties. That is due to the subsequent and highly anticipated government-led administrative extensions of the initial agreements to either other similar and initially uncovered sectors, or to an entire sector when the initial coverage was reduced to some employers. Even the fringe of tailored firm level agreements is greatly influenced by the closest existing sector agreements.[15] This constitutionally protected part of the wage determination process is independent of any union membership consideration, resulting in a collective agreement coverage of more than 85 percent of the private sector workforce.

The agreements set a substantive array of rules on working conditions and a system of wage floors or *bargained wages* for detailed categories of workers. As presented in figure 3, those wage floors are defined based on the firm's sector and the worker's wage rank (or category) within a given occupation (i.e. senior manager, junior manager, and so on). As an example, this system functions in a way similar to that of the organization of the armed forces. There is a hierarchy composed by groups of ranks or occupations (i.e. generals, senior officers, junior officers and enlisted grades), arguably comparable across branches/sectors (i.e. army, air force and navy), with each occupation having a plethora of ranks (i.e. field marshall, general, brigadier, captain, and so on). The vast number of sectoral agreements and their extensions accounts for the fact that Portugal has no fewer than 300 wage tables, with 30,000 job descriptions and around 5,000 wage bargained wages (see Martins (2014) and Card and Cardoso (2021)).

This *bargained wage* sets the minimum wage conditions of each labour relationship, but it does not necessarily correspond to the actual wage of the worker, as the latter results from the proper wage bargaining dynamics at firm level. As seen by the *wage cushion* measure, Figure 4a, it is extremely common, and a stable feature of the market, to see firms paying above the minimum condition.[16] However, as seen in Figures 4b,

[15]There exist a fringe of 4 percent of the workforce covered by tailored firm-level agreements often signed for large firms. However, those agreements can only exist if they improve upon the labour conditions of the most suitable sector level preset.

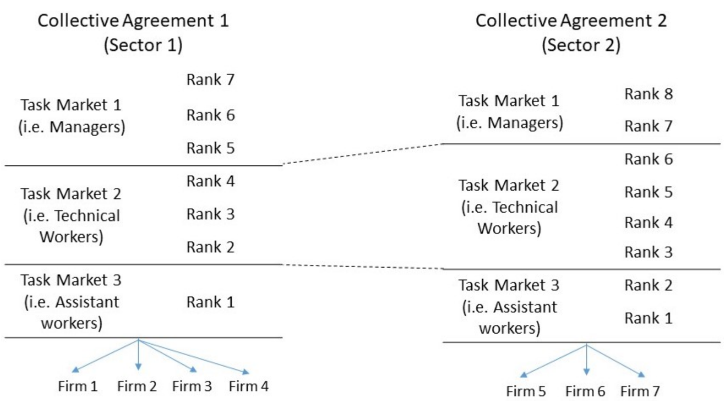
[16]The term *wage cushion* was proposed in Cardoso and Portugal (2005), corresponding to the difference in the levels of the bargained wage and the base wage actually paid to the worker. Note that while Cardoso and Portugal (2005) and Card and Cardoso

Figure 3: The Collective Bargaining System

(a) Example of a Bargained Wage Table for Hospitality Sector

Occupation	Wage Rank	Job Description	Minimum Base Wage
Managers	13	General Manager	1,515
	12	Board Assistant; Commercial Manager; Service Manager; Human Resources Manager; Technical Manager	1,240
	11	Head of Department; Head of Division; Head of Services; Nutrition Technician 1st Class	1,018
Skilled Workers	10	Head of section (office); Head of Sales; Inspector; Board Secretariat officer; Nutrition Technician 2nd Class	898
	9	Administrative; Head of Cafeteria; Head of Purchases; Head of Kitchen; Head of Pastries; Head of Storage; Head of Dinning Room; Inspector of Sales	808
	8	Cashier; Head of Preparation Room; Controller; Cook of 1st Class; Sub-Head of Dinning Room; Administrative Assistant; Pastry Cook; Sales Technician	771
	7	Driver of Heavy Vehicles; Storage Keeper; Polyvalent Worker	716
	6	Driver of Non-heavy Vehicles; Administrative Assistant 2nd Class; Pastry Cook 2nd Class; Sub-Head of Dinning Room 2nd Class; Sales Representative	700
Unskilled Workers	5	Cook 2nd Class; Controller of Balcony; Controller of Bar; Controller of Storage and packing; Admin. Assistant 3rd Class	629
	4	Head of Copa; Cook of 3rd Class; Packing worker; Storage Worker	582.50
	3	Controller cashier; Storage worker; Bar Worker; Balcony Worker 1st Class; Distribution Handler;	570
	2	Balcony Worker 2nd Class; Admin. Intern; Hospitality Assist.	562
	1	Driver Assistant; Distribution Assistant; Barman Intern (1 year); Cook Intern (1 year); Pastry Intern (1 year); Cleaning Worker; Dining room Employer	557
National Minimum Wage			**557**

(b) System of Tables of Bargained Wages.



Notes: In panel A the amounts are in euros and correspond to the monthly payment. The table in panel A is extracted from the sector agreement signed on 22^{nd} April 2017, between *AHRESP* (e.g. the association of employers of hospitality and similar) and *SITESE* (e.g. the union of workers and technicians of services, commerce and hospitality). *Sources: Boletim do Trabalho e do Emprego, 2017; authors.*

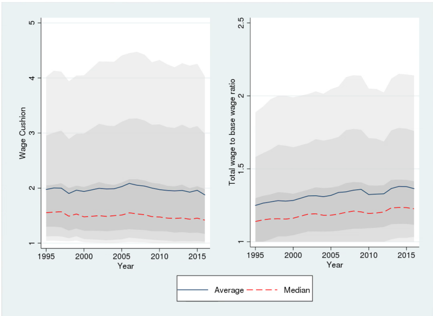
4c and 4d, the wage cushion is binding for a sizable proportion of the market in every occupation. It is not rare that it corresponds to a binding constraint in the wage paid to the worker.

Also, in Figure 4a, another stylized fact is clear: the base wage does not often match

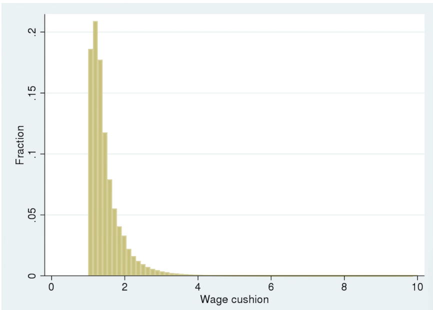
(2021) assess the difference between the workers' bargained and base wages, we assess the difference between bargained and total wages. Further, this concept differs from the *wage drift* which assesses minimum wage changes versus actual wage changes. See Holden (1988) for treatments of this concept for the Nordic countries.

Figure 4: Wage Cushion, Total Wage, and Base Wage

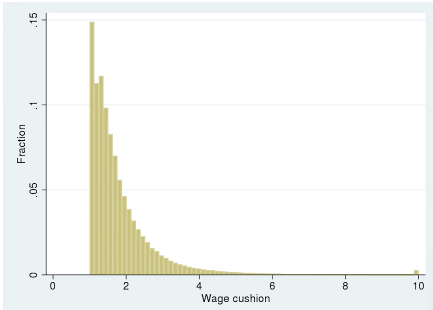
(a) Evolution of the Wage Cushion and the Total-to-Base Wage Ratio



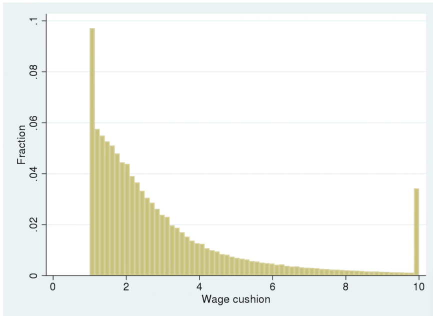
(b) Wage Cushion Distribution for Unskilled Workers



(c) Wage Cushion Distribution for Skilled Workers



(d) Wage Cushion Distribution for Managers



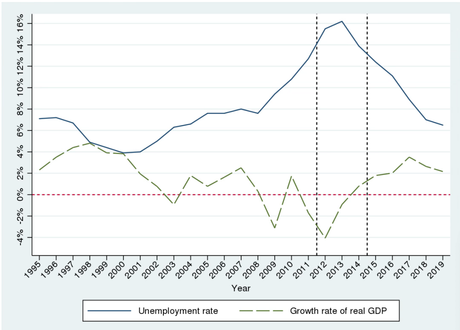
Notes: The wage cushion is calculated as the ratio $wage_{cushion} = \frac{wage_{total}}{wage_{bargained}}$. In panel A, the fading grey shades correspond, from the darker to the lighter respectively, to (a) 75^{th}-25^{th} percentile range; (b) $90^{th} - 10^{th}$ percentile range; and (c) $95^{th} - 5^{th}$ percentile range. In the histograms, the right tail is censored at a wage cushion of 10. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

the total compensation of the workers. That is because the proper institutional setting often determines the existence of several supplements, such as meal subsidies or even tenure-related payments. This distinction confers different degrees of future enforceability among types of pay, with some other regular compensation supplements, such as shift subsidies or availability supplements, being in *de-jure* temporary.

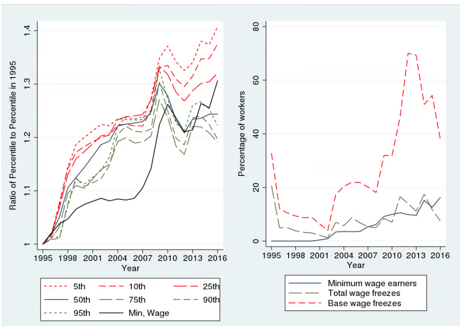
This system has been tested repeatedly by the turbulent Portuguese economic performance over the last two decades. As discussed in Blanchard and Portugal (2017), the last period of consistent growth ended by 2001, being followed by a long slump, two geminated crises, and a modest recovery from 2014 on. During these troubled times, the Financial and European debt crises struck the Portuguese economy severely. In the labour market the entropy was evident by 2013, with the unemployment rate skyrocketing to record highs, in a phenomenon coined by Carneiro et al. (2014) as *a catastrophic job destruction* (see figure 5). The dramatic surge in unemployment was largely fuelled by very low job creation dynamics, and a stunning and unique flood of bankruptcies and firms exiting the market.

Figure 5: The Labour Market Dynamics

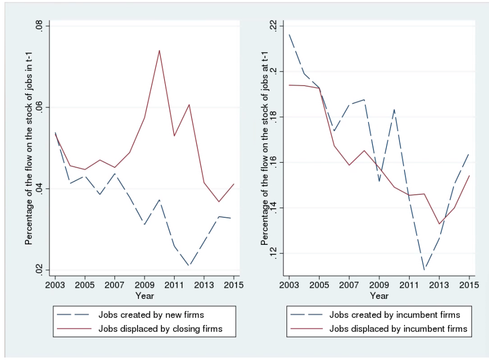
(a) The the real GDP growth and the unemployment rate.



(b) The evolution of the wage distribution in the labour market.



(c) The evolution of the job flows in the labour market.



Notes: In the graph the two vertical black dashed lines identify the period of *Financial Assistance Program* with the ECB, the IMF and the European Commission, namely between 7 April 2011 and 30 June 2014. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016; Pordata website.

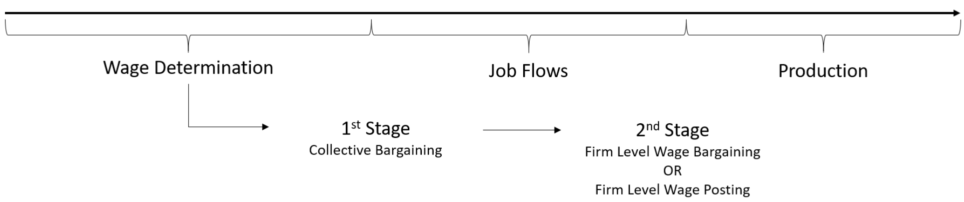
However, as discussed in Addison et al. (2017), the Portuguese labour market has not witnessed substantial structural alterations in either the explicit or implicit rules governing the wage determination. Even at the peak of the Great Recession - a markedly low inflation recessionary period - the collective bargaining and the wage determination simply entered into a period of stall. At the time safeguarded by downward nominal wage rigidity clauses, trade unions did not find room to bargain improvements on current contracts; or conditions to agree on sharp reductions potentially proposed by firms. Therefore, a process of downward real wage adjustment with significant unemployment growth unfolded, while nominally constrained by law. Once completed, the sides resumed the usual bargaining dynamics. At the end of the day, the traditional wage setting coordination of the Southern European labour markets, achieved through a system of industry/sector collective agreements, remained intact.

3 The Model

The model corresponds to a workplace level (firm/occupation/time partition) search and matching, with firing taxes, on-job search, collective bargaining, and either intra-firm wage bargaining or ex-ante wage posting. In this section we focus our description of the model on the components required to obtain dynamic equilibrium wages.[17]

Labour market structure. Each period t is decomposed in three hypothetical moments (Figure 6) according with wage determination happening before the job flow decisions take place.

Figure 6: Structure of the Model in each Moment t



Consider an economy with a *numeraire* good sold under perfectly competitive conditions, and produced by a unit measure of large firms. Each firm employ multiple workers, from the available pool $i \in \{1, \ldots, \aleph\}$, with each worker specializing in one of the available occupations, $j \in \{1, \ldots, J\}$. Workers sell their work to a single firm exclusively. Time is continuous, and workers, the union, and firms discount time at rate $r \geq 0$.

The labour market is assumed to be frictional. Firms are required to post vacancies to hire workers, and pay a cost γ_j per vacancy posted in occupation j. Workers, either employed or unemployed, direct their search at occupation level by selecting the occupation they are willing to perform. They incur the search cost c_j if they search, and then meet firms following a random search process within the occupation.

The flow of worker-firm meets in occupation j is determined by a constant returns to scale matching function, $M(u_j(t)+e_j(t), \bar{V}_j(t))$, where $u_j(t)$ is the measure of unemployed workers searching for a job in occupation j at moment t, $e_j(t)$ is the measure of employed workers searching in market j at time t, and $\bar{V}_j(t)$ is the measure of vacancies. The market tightness is given by:

$$\theta_j(t) = \frac{\bar{V}_j(t)}{u_j(t) + e_j(t)}, \tag{1}$$

and

$$\theta_j(t)q(\theta_j(t)) = \frac{M(u_j(t) + e_j(t), \bar{V}_j(t))}{\bar{V}_j(t)} \tag{2}$$

[17] The online Appendix A provides: (i) the conditions under which the model became isomorphic to several search and matching models in the literature; (ii) remainder components of the model to ensure the existence and uniqueness of equilibrium; (iii) the derivation of the equilibrium wages; and (iv) the definition and properties of the dynamic equilibrium and the steady state equilibrium.

represents the Poisson rate at which a worker, either employed or unemployed meets a firm. Further, $q(\theta_j(t))$ is the Poisson rate at which a firm meets a candidate, per vacancy posted. For notational ease, we often write simply θ_j, u_j, e_j or $q(\theta_j)$, omitting its time dependence.

Matches are dissolved due to one of four reasons: (a) a bargaining breakdown in the wage negotiation; (b) a termination exogenous shock, representing reasons beyond the control of workers and firms, which occurs with probability $\bar{s}$; (c) a successful on-job-search of a worker; or (d) the decision of the firm to fire the worker at will, which may be triggered after the firm pays a firing tax given by S.

Description of Market Agents. Firms employ a J dimensional vector of workers, $\mathbf{N}$, resort to an exogenously predetermined capital input, K, whose rental cost, $I(K)$, is considered to be fixed and sunk, and implement the available homogeneous production function $F(\mathbf{N}, K)$.[18] Moreover, firms bargain with a representative union, with $I - \beta$ representing the firm's bargaining power vector - a $[J \times 1]$ dimensional vector - implying heterogeneous bargaining powers across occupations. The firm exogenous heterogeneity is captured in the two dimensional tuple $\{K, \beta\}$.[19] The cumulative distribution of the firm's types in each moment is given by $\Gamma(K, \beta)$.

Workers may suffer a death shock with a constant hazard rate δ, and new workers arrive at the market at the same rate. In each period, each worker has an exogenous level of skill - a - which evolves through a stationary and invariant Markov process. Then, firms incur in an operating cost per employed worker, $A(j, a) = \omega_j(a)$, dependent on the worker's skill and occupation, otherwise the worker becomes fully unproductive. The function $G_{j,t}(a|K, \beta)$, conditional on the firm's characteristics, represents the number of workers with at most skill a employed in occupation j at moment t. In the process of matching in the labour market we critically assume that a hiring firm acquires knowledge about a only after the hiring is completed. Nevertheless, the function $G_{j,t}(a|K, \beta)$ is assumed to be common knowledge at each moment t.

The *right-to-manage* union fully represents the workforce in the wage bargaining, while employment decisions and wage posting (when they occur) are left to firms. The

[18]The firm's production function is continuous at all arguments, concave, with constant returns to scale, and infinitely differentiable for all positive arguments. As will be clear in our identification strategy, the adoption of a homogeneous production function is taken for exposition purposes, and does not constrain our empirical environment.

[19]In describing the model, we present β as a scalar, so that we ease the notational burden. When pertinent, we present the implied differences. Further, we assume that while the agents are forward looking, they assume that $\{K, \beta\}$ will be stable, so that any future change in firm's fundamentals is fully unexpected, when bargaining takes place.

goal of this utilitarian union is to maximize the workforce value given by:

$$W_t = \sum_{j=1}^{J} \int_a \varphi(a) \int_K \int_\beta \Xi_{j,t}(a|K,\beta) dG_{j,t}(a|K,\beta) d\Gamma(K,\beta) da + \int_a [1-\varphi(a)] Out(a) dU_t(a) da, \tag{3}$$

where: (i) $\varphi(a)$ is the relative welfare weight of an employed worker with skill a versus an unemployed worker of the same skill; (ii) $U_t(a)$ is the number of unemployed with at most skill a; (iii) $\Xi_{j,t}(a|K,\beta)$ is the value of a worker of type a conditional on being in a firm of type $\{K,\beta\}$; and (iv) $Out(a)$ is the value of the outside option of the worker of type a.

Value functions.[20] The profit of a firm with fundamentals $\{K,\beta\}$ is assumed to be strictly concave and twice continuously differentiable in employment. It is given by:

$$\begin{aligned} r\Pi(K,\beta) - \frac{\partial \Pi(K,\beta)}{\partial t} &= \underbrace{F(\mathbf{N}(K,\beta);K)}_{\text{Production}} - \underbrace{\sum_{j=1}^{J} \int_a w_j(a|K,\beta) dG_j(a|K,\beta) da}_{\text{Wage Bill}} \\ &- \underbrace{\sum_{j=1}^{J} \int_a A(j,a) dG_j(a|K,\beta) da}_{\text{Operating Cost Bill}} - \underbrace{\sum_{j=1}^{J} \int_a s_j(a|K,\beta) J_j(a|K,\beta) dG_j(a|K,\beta) da}_{\text{Cost of Displacement}} - \underbrace{I(K)}_{\text{Sunk Cost}} \\ &- \underbrace{\sum_{j=1}^{J} \int_a \tilde{s}_j(a|K,\beta) S dG_j(a|K,\beta) da}_{\text{Exp. firing tax}} + \underbrace{\sum_{j=1}^{J} \max_{V_j(K,\beta)} \Big\{ -\gamma_j V_j(K,\beta) + V_j(K,\beta) q(\theta_j) J_j^R(K,\beta) \Big\}}_{\text{Value of the Hiring Policy}}, \end{aligned} \tag{4}$$

where $\tilde{s}_j(a|K,\beta)$ corresponds to the probability that the firm (K,β) fires at will the worker of type a paying in that event a firing tax of S, and $s_j(a|K,\beta)$ corresponds to the probability that the match (a,K,β) is dissolved. The intuition of equation (4) is standard in the models of this type (see Cahuc et al. (2008)). Accordingly, profit of a firm $\{K,\beta\}$ accounts for: (a) the output of the firm; (b) the firm expenditure in the wages of the employed workers; (c) the firm expenditure with operating costs; (d) expected firing taxes; (e) the sunk cost related to the capital input; (f) the firm losses due to the separation shock; and (g) the proceeds of the firm's optimal vacancy posting behaviour (i.e. $V_j(K,\beta)$), considering the probability that the firm meets a candidate, the cost of creating a vacancy (i.e. γ_j), and $J_j^R(K,\beta)$ the firm's expectation about the marginal profit obtained with a new hire.

[20] For simplicity, we drop the subscript t in the remainder of this section and in appendices, except when the expression is dynamic.

The corresponding HJB equation of the marginal profit of a worker is given by:

$$
\begin{aligned}
rJ_j(a|K,\beta) - \frac{\partial J_j(a|K,\beta)}{\partial t} &= \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a|K,\beta) - A(j,a) \\
&- \underbrace{\sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da}_{\text{Employment effect on wages of other occupations}} - \underbrace{s_j(a|K,\beta)J_j(a|K,\beta)}_{\substack{\text{Expected value loss of}\\ \text{dissolving the match}}} - \underbrace{\tilde{s}_j(a|K,\beta)rS}_{\text{Firing tax}} + \\
&+ \underbrace{\sum_{l=1}^{J} \left\{ y_l(K,\beta)V_l(K,\beta) - \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da \right\} \frac{\partial J_j(a,K,\beta)}{\partial N_l(K,\beta)}}_{\text{Impact on value of the job of hiring and firing policies of the firm}}.
\end{aligned} \tag{5}
$$

Altogether, the value function of a filled job in firm (K,β) by a worker a can be described as the match marginal productivity discounting the value of the worker's wage, the impact of the marginal hiring on the wages set in the other occupations, the loss inherent to the dissolution of the match, potentially including a firing tax if the dissolution was a firm at-will decision, and lastly the impact of the firm's hiring and firing decisions on other occupations on the value of the filled job.

Regarding the unemployed worker, we have that his HJB equation is given by:

$$
\begin{aligned}
rOut(a) - \frac{\partial Out(a)}{\partial t} &= \underbrace{b}_{\substack{\text{Unemployment}\\ \text{benefit}}} + \\
&+ \underbrace{\sum_{j=1}^{J} \xi_j^o(a) \left\{ \theta_j q(\theta_j) \frac{\int_K \int_\beta \Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a) - c_j(a) \right\}}_{\text{Expected value of searching for a job}},
\end{aligned} \tag{6}
$$

where $\xi_j^o(a)$ corresponds to an indicator function being 1 if the unemployed is searching in occupation j, and zero otherwise.[21] By the same token, the corresponding value function for the employed worker in the match with fundamentals $\{a,K,\beta\}$ is given by:

$$
\begin{aligned}
r\Xi_j(a|K,\beta) - \frac{\partial \Xi_j(a|K,\beta)}{\partial t} &= w_j(a|K,\beta) + \underbrace{\bar{s}\left(Out(a) - \Xi_j(a|K,\beta) \right)}_{\text{Value loss of losing the job}} + \\
&+ \underbrace{\sum_{l=1}^{J} \xi_l^{\Xi}(a|K,\beta) \left\{ \theta_l q(\theta_l) \frac{\int_K \int_\beta \mathbf{1}[\Xi_l(a|K,\beta) > \Xi_j(a|K,\beta)]\Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a) - c_j(a) \right\}}_{\text{Value of searching for a job while employed}} \\
&+ \underbrace{\sum_{l=1}^{J} \left[y_l(K,\beta)V_l(K,\beta) - s_l(K,\beta)N_l(K,\beta) \right] \frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}}_{\text{Impact of hiring and firing policy of the firm in the value of the employment}}.
\end{aligned} \tag{7}
$$

[21]It is assumed that the unemployed worker searches only in the occupation that maximizes his expected value of search. The same happens in the case of the employed worker. The unemployment benefits - b - are independent of a merely for exposition purposes. Considering it dependent on a, i.e. $b(a)$, it will not affect our identification strategy.

$\mathbf{1}[\Xi_l(a|K,\beta) > \Xi_j(a|K,\beta)]$ represents an indicator function equal to 1 if the value in alternative match is greater than the current value. $\xi_l^{\Xi}(a|K,\beta)$ is an indicator function equal to 1 if the employed worker is searching in occupation l, and zero otherwise.

Collective Bargaining Protocol. Collective bargaining unfolds through wage bargaining. The trade union and the average firm follow bilateral bargaining protocols, with a system of offers and counter-offers in the spirit of Rubinstein (1982) and Brügemann et al. (2018).[22] The sides will bargain binding wage floors, under *the principle of match stability*, which is algebraically translated into:

$$\tilde{s}_l(a|K,\beta) = \xi_l^{\Xi}(a) = 0, \forall l \in \{1,\ldots,J\}, \forall \{a,K,\beta\}. \tag{8}$$

Note that match stability implies that neither side is ex-ante considering that the other will dissolve the match at-will. The union will represent each type-a worker without realizing the potential effects that the wage for that type may have on the wages and employment of other types.

The average firm and the trade union bargain the wage floor, which is compatible with the lowest surplus viable match, namely the match that generates a zero expected quasi-rent. Note that the level of the expected quasi-rent of the match of a worker of type a with the average firm in the bargaining corresponds to:

$$\begin{aligned} E_{K,\beta}[QR_j(a,K,\beta)] =& E_{K,\beta}\left[\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}\right] - A(j,a) \\ & - E_{K,\beta}\Bigg[\underbrace{\sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da}_{\text{Employment effect on wages of other occupations}}\Bigg] - rOut(a) - \frac{\partial Out(a)}{\partial t}. \end{aligned} \tag{9}$$

Altogether, the aggregate bargaining solves the axiomatic constrained Nash bargaining, considering the Dobbelaere and Luttens (2016) proposition, and thus that the disagreement points are given by the loss of one match with a type-a worker without the existence of side payments. Consequently:

$$\begin{aligned} & w_{j,t}^{MIN}(a) = argmax_w \Big\{E_{K,\beta}[\Xi_{j,t}(a|K,\beta)] - Out(a)\Big\}^{\beta}\Big\{E_{K,\beta}[J_{j,t}(a|K,\beta)]\Big\}^{1-\beta} \\ & \text{subject to:} \\ & \tilde{s}_l(a|K,\beta) = \xi_l^{\Xi}(a|K,\beta) = 0, \quad \textbf{(match stability)} \\ & E_{K,\beta}[QR_j(a,K,\beta)] = 0, \quad \textbf{(No quasi-rent condition)} \\ & \forall l \in \{1,\ldots,J\}, \forall \{a,K,\beta\}. \end{aligned} \tag{10}$$

[22] Brügemann et al. (2018) found that the ordering at which the contracts are bargained does not influence the outcome of the bargaining.

The solution of the collective bargaining problem is given by:

$$w_{j,t}^{MIN}(a) = rOut(a) - \frac{\partial Out(a)}{\partial t}, \tag{11}$$

which precisely defines the minimum wage at aggregate level for type-a worker, i.e. the bargained wage of type-a worker, as the level of his outside option.[23]

Bargaining or Posting. Once collective bargaining is settled, there is a Bernoulli random shock dependent on characteristics of the firm defining if the actual wages at firm level will be settled through bargaining or posting. With probability $\vartheta(K, \beta) \in [0, 1]$, the firms of that type have the opportunity to post wages instead of bargaining.

Firm level Wage Bargaining. Considering the wage floor bargained for each type-a worker, the firm-level bargaining takes place, with the union and each firm bargaining contracts for each match fundamentals $\{a, K, \beta\}$. In case of a full bargaining breakdown one of the matches, with match fundamentals as $\{a, K, \beta\}$, is expected to be dissolved without the existence of side payments among the market actors involved.

Accordingly, the wage of a match with fundamentals $\{a, K, \beta\}$ is obtainable by solving an axiomatic generalized Nash bargaining as:

$$\begin{aligned}
&w_{j,t}(a|K,\beta) = argmax_w \Big\{ \Xi_{j,t}(a|K,\beta) - Out(a) \Big\}^{\beta} \Big\{ J_{j,t}(a|K,\beta) \Big\}^{1-\beta} \\
&\text{subject to:} \\
&\tilde{s}_l(a|K,\beta) = \xi_l^{\Xi}(a|K,\beta) = 0, \qquad \textbf{(match stability)} \\
&w_{j,t}(a|K,\beta) \geq w_{j,t}^{MIN}(a) = rOut(a) - \frac{\partial Out(a)}{\partial t} \qquad \textbf{(col. bargaining constraint)} \\
&\forall l \in \{1, \ldots, J\}, \forall \{a, K, \beta\}.
\end{aligned} \tag{12}$$

The unique solution of the bargained equilibrium wages is given by:

$$w_j(a|K,\beta) = \begin{cases} \underbrace{(1-\beta_j)\underbrace{rOut(a) - \frac{\partial Out(a)}{\partial t}}_{Out^{\star}(a)} + \underbrace{\int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz}_{\substack{\textit{Perturbed marginal productivity} \\ \textit{of the worker in the workplace (f,j,t)}}} - \beta_j \underbrace{A(j,a)}_{\text{Op. Cost}}}_{\text{Idiosyncratic Surplus of the Match } \{a,K,\beta\}}, & \text{if } w_j(a|K,\beta) \geq w_{j,t}^{MIN}(a) \\ w_{j,t}^{MIN}(a) & \text{, if otherwise.} \end{cases} \tag{13}$$

[23] The solution of the aggregate bargaining entails that the worker's bargaining power does not directly influence the bargained wage of type-a worker, but it influences the outside option through the worker's bargaining powers of the expected potential offers of the worker. Thus aggregate movements of bargaining powers affect the level of the outside option and the bargained wage, while idiosyncratic movements of bargaining powers do not.

The expression of the idiosyncratic surplus of the match in the interior solution has a *perturbed marginal productivity of the worker*, influenced by the heterogeneity in the bargaining powers across occupations, as in Cahuc et al. (2008). Critically, notice that this term is invariant within the workplace (f, j, t).[24]

Wage Posting. If the firm receives the opportunity of posting wages instead of bargaining, it will decide the actual wages alone. The firm will do so considering: (i) the collective bargaining constraint; (ii) its labour supply for each occupation - $\varrho\left[w(a|K,\beta) - \left(Out(a) - \frac{\partial Out(a)}{\partial t}\right)\right]$ - is a function of the difference between the actual wage and the worker's outside option; (iii) it will hold both match and workforce stability, so that the firm holds the principle assumed with the trade union and sets the wage considering the workforce it had when the wage determination process started.[25]

Following Manning (2011), the firms take the labour supply as:

$$\varrho[w_j(a|K,\beta) - Out(a)] = \left[w_j(a|K,\beta) - \left(Out(a) - \frac{\partial Out(a)}{\partial t}\right)\right]^{\tilde{\epsilon}_j(K,\beta)}, \tag{15}$$

where $\tilde{\epsilon}_j(K,\beta)$ corresponds to the elasticity of labour supply to the difference between the wage and the worker's outside option. Then, the firm maximizes the product between the discounted marginal profit and the supply of labour, given match stability, worforce stability, and the collective bargaining constraint:

$$\begin{aligned}
&w_{j,t}(a|K,\beta) = argmax_w[r + s(a|K,\beta)] \times \left[J_{j,t}(a|K,\beta) - \frac{\partial J_j(a|K,\beta)}{\partial t}\right] \times \varrho[w_j(a|K,\beta) - Out(a)] \\
&\text{subject to:} \\
&\tilde{s}_l(a|K,\beta) = \xi_l^{\Xi}(a|K,\beta) = 0, \qquad \textbf{(match stability)} \\
&w_{j,t}(a|K,\beta) \geq w_{j,t}^{MIN}(a) = rOut(a) - \frac{\partial Out(a)}{\partial t} \qquad \textbf{(col. bargaining constraint)} \\
&y_l(K,\beta)V_l(K,\beta) = \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da, \qquad \textbf{(workforce stability condition)} \\
&\forall l \in \{1,\dots,J\}, \forall\{a,K,\beta\}.
\end{aligned} \tag{16}$$

[24]Technically, we refer to a workplace as the combination of worker-observations that share (f, j, t) dimensions - intuitively, the workers that at moment t are in firm f in occupation j. Additionally, note that the average wage in each workplace in the absence of corner solutions is given by:

$$w_j(K,\beta) = (1-\beta_j)E[Out^{\star}(a)|K,\beta] + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz - \beta_j E[A(j,a)|j,K,\beta]. \tag{14}$$

[25]Dynamically, any subsequent workforce adjustments during the job flow moment will have an impact only on future wage determination processes.

If one considers that the *worker's bargaining power* in the wage posting model is given by $\tilde{\beta}^P_j = \frac{\tilde{\epsilon}_j(K,\beta)}{1+\tilde{\epsilon}_j(K,\beta)}$, then the equilibrium wages in the posting process become:

$$w_j(a|K,\beta) = \begin{cases} \underbrace{(1-\beta^P_j)\underbrace{rOut(a) - \frac{\partial Out(a)}{\partial t}}_{Out^\star(a)} + \underbrace{\int_0^1 z^{\frac{1-\tilde{\beta}^P_j}{\tilde{\beta}^P_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz}_{\substack{\textit{Perturbed marginal productivity} \\ \textit{of the worker in the workplace (f,j,t)}}} - \tilde{\beta}^P_j \underbrace{A(j,a)}_{\text{Op. Cost}}}_{\text{Idiosyncratic Surplus of the Match } \{a,K,\beta\}}, & \text{if } w_j(a|K,\beta) \geq w^{MIN}_{j,t}(a) \\ w^{MIN}_{j,t}(a), & \text{if otherwise.} \end{cases} \tag{17}$$

Two points are noteworthy. First, if for a given firm $\tilde{\beta}^P_j = \beta_j$ that implies that the solution of posting and bargaining will coincide. That occurs if $\tilde{\epsilon}_j(K,\beta) = \frac{\beta^P_j}{1-\beta^P_j}$, namely the bargaining strength of union and firm stems from a direct relationship with labour supply as defined. Second, contrary to Manning (2011), the outside options in posting and bargaining are exactly the same, as they are both anchored on the outcome of collective bargaining.[26]

4 Identification of the Wage Equation

In the empirical implementation we consider only interior solutions, as we assume that either the worker or the firm will pull out at-will from any match that generates less than the least viable match.[27] Thus, for our purposes, the difference between the average wage within the workplace, in equation (14), and individual wages, in the top branch of equation (13), is given by:

$$w_j(a|K,\beta) = w_j(K,\beta) + (1-\beta^\star_j)\underbrace{\Big[Out^\star(a) - E[Out^\star(a)|K,\beta]\Big]}_{\substack{\text{Diff. in outside options=} \\ \Delta Out^\star(a,K,\beta)}} + \beta^\star_j \underbrace{\Big\{E[A(j,a)|K,\beta] - A(j,a)\Big\}}_{\text{Diff. in Op. Costs}=\Delta A_j(a,K,\beta)}. \tag{18}$$

There are two terms influencing the difference, namely: (i) the differences in the level of outside options; and (ii) the heterogeneity in the level of operating costs. Notice that with a slight abuse of notation, we will consider hereinafter that $\beta^\star_j = \{\beta_j; \tilde{\beta}^{POST}_j\}$ depending on which firm level wage determination process is taking place.

[26]In a simpler model without collective bargaining, the outside option in a wage bargaining model is typically the value of the unemployed, often measured as a general value, while in the posting model it would be the specific outside wage of the worker.

[27]See Haanwinckel and Soares (2020) for an analysis of binding minimum wages in a wage bargaining setting close to ours, and the consequences for the wage setting. They assume that an arbitrary minimum wage is set, and thus may destroy viable matches, while we assume that the minimum wage results from bargaining, and thus matches that do not survive are by definition not viable.

Empirical Outside Options. While we know the wage rank (i.e. $rank_{i,t}$) of each worker, we lack direct information about the collectively bargained wage floors (i.e. $w^{MIN}_{j,t}(a)$). For example, we know that all workers are covered by the bargained wage table presented in Figure 3 and we know their specific wage rank, but we do not know for example that the cashiers, which have a wage rank 8, have a collectively bargained wage floor of 771 euros in 2017. In the spirit of Card et al. (2014), we take the empirical minimum wage observed amongst the workers in each of the collective bargaining wage rank cells - $rank_{i,t}$ - as a proxy of the bargained wage - $w^{MIN}_{i,t}$. In our cashiers' example, we assess the observed wages of all wage rank 8 workers and take the minimum base wage paid in that rank as a proxy of the collectively bargained wage floor of 771 euros.[28]

While in theory we could resort directly to our proxy of $w^{MIN}_{i,t}$, we use the insight of Pei et al. (2018) to minimize the potential impact of measurement error arising from the way we defined it. If there is measurement error in the proxy and it is classic, it will raise only efficiency concerns, but not bias the wage equation estimates, if we consider that the outside options can be appropriately described by the following empirical model:

$$ln[w^{MIN}_{i,t}] = \underbrace{\lambda_{j,t} + \psi[rank_{i,t}, age_{i,t}, female_i]}_{ln[Out^{\star}(a_t)]=ln[w^{MIN}_{j,t}(a)]} + v_{i,t}, \tag{19}$$

where $\lambda_{j,t}$ is an occupation-time fixed effect, $v_{i,t}$ corresponds to a disturbance, and $\psi(.)$ is a fairly flexible function. It accounts for: (a) the 3^{rd} order polynomial on age of the worker (i.e. $age_{i,t}$), (b) a set of dummies accounting for the current wage rank cell/positioning in the table of the worker (i.e. $rank_{i,t}$), and (c) a gender dummy.[29]

Given $\lambda(j,t)$, we allow the average outside option to evolve freely between occupations, but holding a parallel trends assumption within each occupation j. Then, we create gender-age profiles for the evolution of the outside option, and we make such profiles heterogeneous and dependent on each position of the wage table - $rank_{i,t}$ - in which the worker is currently placed. So in our example in Figure 3, we first consider that the average outside options for the wage ranks of skilled workers (i.e. ranks 6 to 10) evolve holding parallel trends among them and freely relatively to other ranks. Then, we create two 3^{rd} order polynomial age profiles for each wage rank (i.e., heterogeneous at $rank_{i,t}$ level) of evolution of the outside option depending on gender. Thus, we allow the outside options to evolve in tandem within occupations, but heterogeneously for workers

[28] Cardoso and Portugal (2005) resort to the mode of the base wage as proxy of the bargained wage. While it can work in linear models, as such a proxy correlates closely with actual bargained wages, in the case of non-linear models and structural equations it distorts the differences between wage and bargained wage, leading to a significant number of wages being below the bargained wage.

[29] The derivation details of this functional form, and the insight of Pei et al. (2018) applied to this case are provided in online Appendix B.

depending on their gender, age, and current position in the wage tables.

Functional form of Operating Costs. The operating cost, $A(j, a_t)$, correspond to the idiosyncratic marginal productivity of the worker-workplace match. For estimation purposes, we assume an additive functional form as:

$$A(j, a_t) = \underbrace{\xi_{i,t} + \mathbf{X}_{i,t}\zeta_j}_{\text{Worker Characteristics}}, \tag{20}$$

where $\xi_{i,t}$ corresponds to time-varying unobservable characteristics of the worker, and $\mathbf{X}_{i,t}$ to a vector of k observable characteristics of the worker. Notice that $\xi_{i,t}$ would translate into a very high-dimensional parameter set, which ordinarily is beyond identification capabilities of ordinarily implemented models.

In our approach we do not directly estimate the individual operating costs. Rather, we assume that unions and firms have an estimate of these operating costs, and consequently use it to place the worker in the wage ranks. They do so under two conditions:

Condition 1. ***Fair placement.*** *This condition has two properties:*

1. *If a worker is placed in a given wage rank, it can only be because the union and the firm agree that he has a level of operating cost that is compatible with that rank.*
2. *For every group of workers with identical operating cost level a wage rank is created.*

Consequently, the worker's operating cost is approximately equal to the average operating cost of his wage rank:

$$A(j, a_t) \approx E[A(j, a_t)|rank_{i,t}] = E[\mathbf{X}_{i,t}|rank_{i,t}]\zeta_j + E[\xi_{i,t}|rank_{i,t}]. \tag{21}$$

Condition 2. ***Average wage-rank heterogeneity only on observables.*** *The expected value of the unobserved characteristics of operating costs at wage-rank level is the same within each occupation.*

$$E[\xi_{i,t}|rank_{i,t}] = E\left[E[\xi_{i,t}|rank_{i,t}]\middle|j, t\right], \forall \ rank \in j. \tag{22}$$

Intuitively, the idiosyncratic characteristics of a worker, either observable or unobservable, contribute to his career path through the wage ranks, but neither alter the definition of the ranks, which are based on average operating cost levels. Moreover, the expected value of the wage ranks' unobserved component (i.e. $E[\xi(i,t)|rank(i,t)]$) is the same in the corresponding occupation j of the relevant collective agreement. Wage ranks are differentiated based on average observable characteristics of their respective workforce, allowing unions and firms to verify and agree upon those differences.[30]

[30] Note that we are not fully excluding idiosyncratic pay to a given characteristic of the

Given the equation (20) and conditions 1 and 2, the difference in operating cost between the average worker in the workplace f, j, t and the worker's rank (i.e. $rank_{i,t}$) is:

$$\Delta A(j, a_t) = \underbrace{\left\{ E\left[E\left[\mathbf{X}_{i,t} \middle| rank_{i,t} \right] \middle| f, j, t \right] - E\left[\mathbf{X}_{i,t} \middle| rank_{i,t} \right] \right\}}_{\text{Diff. in workplace versus rank on time-varying observables}} \zeta_j = \Delta E[\mathbf{X}_{i,t} | rank_{i,t}, f, j, t] \zeta_j, \tag{23}$$

as the unobservable components cancel out. Notice that the *ceteris paribus* interpretation of empirical marginal effects of any of the variables in matrix $E[\mathbf{X}_{i,t}|rank_{i,t}]$ is equivalent to the interpretation of a change in $\mathbf{X}_{i,t}$ in terms of wage change. Mechanically, one expects that the change in operating costs changes the rank of the worker, implying that the worker gets promoted. Equation (23) holds for that same worker, but in a different wage rank that the worker was then assigned to.[31]

The estimation procedure. Considering equation (18), and the described behaviour of outside options and operating costs, we have that the log of actual wages corresponds to:

$$ln[w_j(a_t|K_t, \beta)] = ln\left[w_j(K_t, \beta_{j,t}) + (1 - \beta^\star_{j,t}) \Delta Out^\star(a_t, K_t, \beta_t) + \beta^\star_{j,t} \Delta E[\mathbf{X}_{i,t}|rank_{i,t}, f, j, t] \zeta_j \right]. \tag{24}$$

The logarithm of the wage has three components: (i) the average wage in the workplace; (ii) the worker-workplace difference in the outside option, weighted by the firm's bargaining power; and (iii) the wage rank-workplace difference in the operating cost observables, weighted by the worker's bargaining power.

The estimation of equation (24) follows two steps. The first deals with potential measurement error in our proxy of outside option values and estimates in equation (19):

$$ln[w^{MIN}_{i,t}] = \underbrace{\lambda_{j,t} + \psi[rank_{i,t}, age_{i,t}, female_i]}_{ln[Out^\star(a_t)]} + v_{i,t}, \tag{\textbf{1}^{\textbf{st}} \textbf{ Step}}$$

While this first step estimates a large number of parameters, due to the ψ term, it is much more parsimonious than a model that resorts to worker and/or firm effects. In the second step, with the predicted outside option value - $\widehat{Out^\star(a_t)}$, we estimate the actual wage empirical model of equation (24), which corresponds to:

$$ln[w_{i,t}] = ln\left[\bar{w}_{f,j,t} + (1 - \beta^\star_{j,t}) \Delta \widehat{Out^\star(a_t, K, \beta)} + \beta^\star_{j,t} \zeta_j (\bar{\mathbf{X}}_{fjt} - \bar{\mathbf{X}}_{rank_{i,t}}) \right] + \epsilon_{i,t}. \tag{\textbf{2}^{\textbf{nd}} \textbf{ Step}}$$

worker *vis-à-vis* the remuneration in the corresponding rank, as long as such payment is performed by resorting to bonuses, or irregular compensation policies.

[31]Noteworthy, implicit in this ceteris paribus analysis, we are referring to the change in operating costs assuming that it is not affecting the outside option. To refer to the marginal effects of a covariate on wages one would have to estimate the impact of the change in such covariate both on operating costs and outside options.

We use a non linear least squares, as:

$$\hat{\Theta} = argmax_{\theta \in \Theta} \sum_{i}^{N} \left\{ ln[w_{i,t}] - f\Big(\theta, \mathbf{X}_{i,t}, \bar{w}_{f,j,t}, \Delta \widehat{Out^{\star}(a_t}, K, \beta)\Big) \right\}^2, \tag{25}$$

where $\theta = \theta(\beta^{\star}_{j,t}, \zeta_j)$ is the parameter vector, and $\epsilon_{i,f,j,t}$ corresponds to a disturbance.[32]

This section presented the identification strategy to pinpoint the structural parameters of the wage determination. While we will not estimate every model parameter, particularly those on job flows, their inclusion allows to understand how those interact with equilibrium wages, ensuring that our identification strategy is valid in their presence.

5 Empirical Results

Trends in the Wage Determination Process

Figure 7a presents a wage determination in structural change. In the 1995-2016 period, we observe substantial erosion in worker bargaining power at the middle and the top of the skill distribution. At the bottom, the initial erosion until the mid-2000's was more than fully reversed thereafter. By 2016 managers and skilled workers had lost around 29 and 33 percent of their bargaining power, whereas the unskilled had gained around 7 percent (Figure 7b).

Market-wide, it is not surprising that *workers are losing bargaining power*. Our findings are consistent with the perceptions in the literature that highlight the sizable fall in trade union membership and the rise of monopsonistic practices as relevant drivers of this bargaining fallout, the consequences of which might already be echoing through rising corporate profitability, sluggish wage growth, declining labour share of income, and

[32]In the estimation, we adopted robust standard errors instead of clustering at any dimension. Abadie et al. (2017) advocate the absence of clustering in the presence of a fixed effect specification when there are homogeneous treatment effects within the cluster formed at the level of the fixed effect. We assume such homogeneity by design as the workplace heterogeneity arises solely from the heterogeneity in worker's characteristics and not from the valuation of their characteristics. Moreover, the use of average real hourly wage, i.e. $w(K, \beta)$, approximates our setting to the fixed effect setting.

The time variation of bargaining powers will be modelled in two alternative specifications. Firstly, we will consider

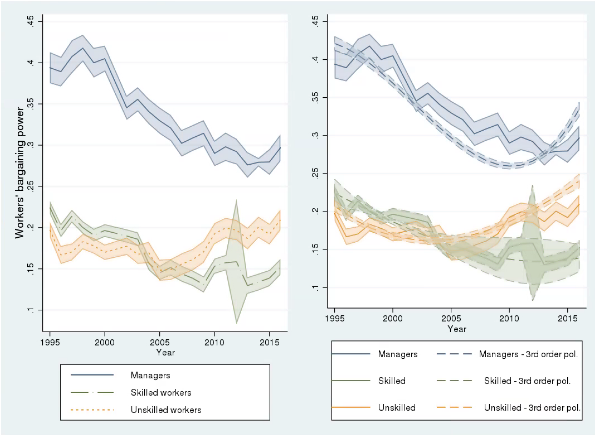
$$\beta^{\star}_{j,t} = \mathbf{D}'\tilde{\beta}^{\star}, \tag{26}$$

where $\mathbf{D}$ is a $[(J \times T) \times 1]$ vector of year-occupation dummies, and $\tilde{\beta}$ is the corresponding vector of parameters. Alternatively, we will assume sufficient smoothness of the time series of bargaining powers, and consequently fit a polynomial approximation as:

$$\beta^{\star}_{j,t} \approx b_0 + b_{1,j} \times t + b_{2,j} \times t^2 + b_{3,j} \times t^3. \tag{27}$$

Figure 7: Estimated Workers' Bargaining Power per Occupation

(a) Estimated Workers' Bargaining Power per Occupation



(b) Average Predicted Change in Bargaining Powers Implied by Best Fitting 3rd Order Polynomial Trend to the Dummy Series

Period	Dates	Ratio $\beta_{final}/\beta_{initial} - 1$		
		Managers	Skilled workers	Unskilled workers
The Boom	1995-2000	-4.44%	-12.96%	-11.95%
The Slump	2000-2008	-17.98%	-20.58%	0.39%
Financial Crisis	2008-2011	-7.69%	-6.23%	5.77%
Euro Crisis	2011-2014	-3.43%	-0.89%	8.00%
Timid Recovery	2014-2016	1.78%	3.54%	6.04%
Overall	1995-2016	-28.89%	-33.48%	7.08%
β_{1995}		40.30%	21.76%	18.99%
β_{2016}		28.66%	14.48%	20.34%

Notes: The fading shades correspond to the $95^{th} - 5^{th}$ confidence interval range, using clustered standard errors at collective bargaining level. The 3rd order polynomial best fitting curve to the dummy series of bargaining powers is used to avoid the over-influence of any transitory fluctuation. The periods reported are collected from Blanchard and Portugal (2017), who outline a detailed macroeconomic analysis of the Portuguese Economy. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

reduced unemployment and inflation (see Stansbury and Summers, 2020 and Krueger, 2018). Our findings further align with indirect measures of wage determination momentum, such as the synthetic indicator of Lombardi et al. (2020) in Figure 2. Our findings are also broadly consistent with those in one of the few works for Continental Europe estimating time paths of bargaining powers; Hirsch and Schnabel (2011) resort to a right-to-manage model and report a one-third decline in worker bargaining power in the 2002-2009 period for Germany.[33]

This decline in worker bargaining power translates into *the weakening of the link between productivity and wages.* As noted by Card et al. (2018), the evolution of the elasticity of wages to exogenous changes in the quasi-rents provides a succinct description of the link between productivity heterogeneity and wage inequality.[34] In our context, this elasticity is given by:

$$\epsilon_t^{QR}(K,\beta) = \frac{\partial w_j(a|K,\beta)}{\partial QR_j(K,\beta)} \frac{QR_j(K,\beta)}{w_j(a|K,\beta)} = \beta^{\star}_{j,t} \frac{QR_j(K,\beta)}{w_j(a|K,\beta)}. \tag{28}$$

Figure 8a presents its average evolution for the entire economy, confirming a significant

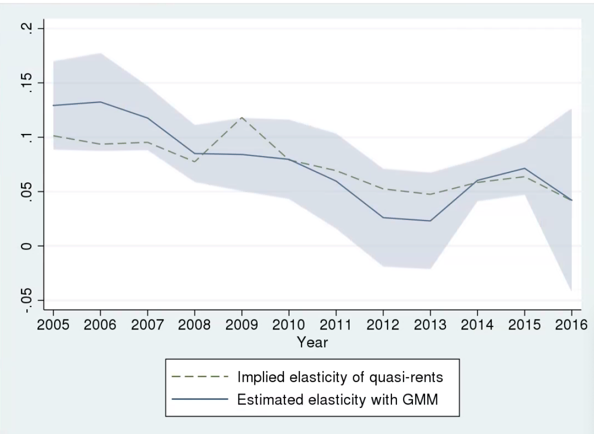
[33]For the preceding period though, i.e. 1995-2000, the cited authors find a stable trend, whereas we record a decreasing path in every occupation.

[34]See Garin and Silvério (2019) for a compatible theory review of the difference in changes in quasi-rents which are: (a) idiosyncratic to the match; and (b) general to the relevant portions of the labour market. The major difference is due to the presence of feedback effects on the outside options. We analyze option (a) in this section.

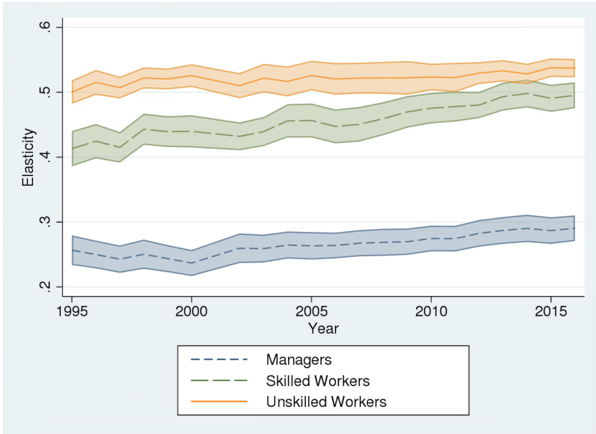
downward trend, with its magnitude falling roughly by half during the period of analysis.[35]

Figure 8: Elasticity of Wages to an Exogenous Change in the Quasi-Rents and Average Passthrough

(a) Estimated Average Elasticity of Wages to an Exogenous Change in the Quasi-Rents



(b) Estimated Average Passthrough



Notes: The fading shades correspond to the $95^{th} - 5^{th}$ confidence interval range, using clustered standard errors at collective bargaining level. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

While loosening its connection to productivity, *wages has seen its link to collectively bargained wage floors being reinforced*, particularly for the groups that have lost bargaining power. These dynamics are inferred by estimating the passthrough rate of wage floors, which broadly displays the influence of collective bargaining on wages. In detail, this measure informs about how changes in bargained wages translates into changes in total wages, thus being a key measure for the analysis of the wage determination whenever collective bargaining is present.[36] In our findings of Figure 8b, the workers' passthrough

[35]We follow the insight of Card et al. (2014). Thus, we compute the quasi-rents as:

$$E_{ft}[\widehat{QR_j(K,\beta)}] = GVA_{ft} - 0.1K - \widehat{Out(a)}, \tag{29}$$

where GVA_{ft} is the gross value added per hour, K is the level of firm's assets, and 10% the considered costs of capital. To estimate the average elasticity per year for the entire economy, we used both the direct computation of its implied value, and a two-step GMM with the logarithm of sales as instrument (i.e. $ln[F(K,\beta)]$). Notice that while for external validity we resorted here to the gross value added data of SCIE 2005-2016, we have not used gross value added up to this point, thus rendering our identification of bargaining powers not reliant on its existence, or any of its proxies.

[36]In our setting, the average passthrough per year and occupation, i.e. the average elasticity of the total wage to the wage floor, is given by:

$$\tau(j,t) = E\left[(1-\beta^{\star}_{j,t})\frac{Out(a)}{w(a|K,\beta)}\right]. \tag{30}$$

rate has recorded an upward trend for every skill type, but it has been more pronounced for the managers and particularly for the skilled workers. Managers went from 25 percent passthrough at the beginning of the period to almost 30 percent, while the skilled workers went from around 41 percent to 50 percent in the same period.[37] This finding also provides support to the claim that the fall of bargaining powers was not coincidental with an increase in quasi-rents capable of off-setting its compression effect on the wage distribution. If that were the case, we would find a stable passthrough, as the influence of the outside option component of wages in the actual wages would not increase vis-á-vis the quasi-rent component, which was not verified in our estimates.

The overall downward trend in the worker bargaining power coexists with a *worker bargaining power compression* across skill groups. These dynamics are consistent with an environment in which trade unions are still able to reflect their classic wage compression priority in the bargaining process, while they are unable to stop their bargaining power drainage.[38] This could be particularly the case given the increasing misalignment between union density and collective bargaining coverage, implying an ever present but weakened trade union. Moreover, these dynamics are to be encouraged at least partially by a heterogeneous evolution of the elasticity of the supply of workers, with a greater decrease at the top and middle of the skill distribution, coupled with a lesser reduction (or even increase) at the bottom. This latter process could be obtained by either the proper evolution of the supply of workers of the different groups, or the *fragmentation* of the wage determination process whereby firm-level wage posting, coupled with lower implied worker bargaining powers, has become more frequent in firms that have more skilled workers.

One may think that the surge in bargaining powers of the unskilled was mainly due to the national minimum wage evolution, especially in the 2005-2010 period (see Figure 5). However, it is unlikely that a wage floor that has evolved by less than the 5^{th} percentile of the wage distribution for the large majority of our period is determining the recorded hikes in worker bargaining power. In our wage determination process the national minimum wage, $\underline{\mathrm{w}}_t$, corresponds to the lowest collectively bargained wage floor. For a worker in a wage rank whose floor is the national minimum wage, his wage is determined as

$$w_{i,t} = \underbrace{\underline{\mathrm{w}}_t}_{=Out(a_t,K,\beta)} + \beta^{\star}_{unskilled,t}[QR(a_t, K, \beta) - \underline{\mathrm{w}}_t], \tag{31}$$

implying that national minimum wage hikes do not imply changes in the worker bar-

[37]The results are unlikely to result from a more tightly controlled implementation of collective bargaining. By 1995, the Portuguese labour market was already stable with the current bargaining system running for around a decade.

[38]See Freeman (1980), Card et al. (2003) or Aidt and Tzannatos (2002) for studies establishing the influence of trade unionism on wage inequality.

gaining power - a structural parameter determining the share of the match value *above* the wage floor that the worker is able to capture. We interpret the evolution of the minimum wage as an integral part of the collective bargaining process from which it arises. Rather than being an exogenous value, we consider that it simply corresponds to a notable collective bargained wage floor among 5,000 - the minimum among the minima.

In general, *these trends are notably resilient*. They are not a virtue of a particular phase of the economic cycle, as presented in Figure 7b. They result instead from a gradual process that has compressed the relative bargaining positions of the groups, even in the midst of severely adverse conditions, notably throughout the Great Recession. Furthermore, this bargaining power compression leads to *a more compressed wage distribution* if the underlying productivity distribution does not sufficiently compensate the shift in bargaining momentum. Our results point to such case, as does Portugal's slow growth of productivity in the period analysed (see Blanchard and Portugal (2017)).

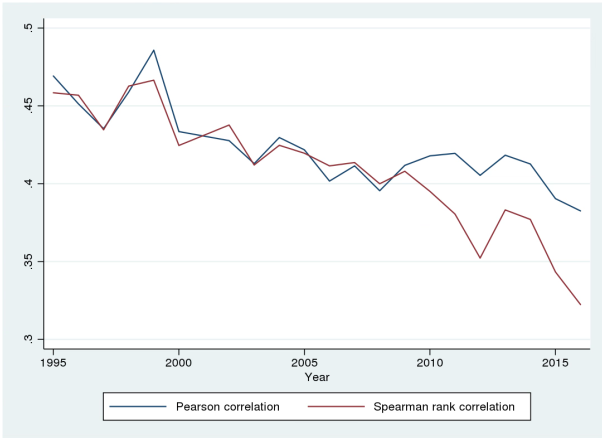
Throughout the Great Recession, our estimates and the severe downward real wage adjustment showed in Figure 5 could only take place jointly with a substantial underlying reduction of the real value of outside options (or bargained wages) and of quasi-rents. This type of adjustment is not at the odds with the findings of reduced-form decomposition methods for that period in Portugal, as reported in Card and Cardoso (2021). Those authors find that the 2010-2016 real wage adjustment in Portugal was positively influenced by a better-educated pool of workers, but negatively influenced by real wage floor reductions, real wage cushion reductions, and reallocation of workers over the wage floors. In our model, real wage floor (wage cushion) reductions corresponds to a reduction of the real value of outside options (quasi-rents) for staying workers. The reallocation of workers over wage floors relate with the real outside option, both for staying workers, and movers. Stayers may have not been promoted as often, because the evolution of their outside option did not achieve the required improvement, or was even depreciated. The market value of new hires, could also have been reduced, resulting in them entering at lower positions in the wage tables. Finally, the positive education effect those authors found, everything else being equal, would have increased wage cushions and outside options due to the impact of education on productivity. However, this effect was surpassed by the others.

The dynamics of the wage determination process have revealed significant resilience, and that is not at odds with a significant downward real wage adjustment. In this, we add to the work of Card and Cardoso (2021) by displaying how one could reason the adjustment in the light of a search and matching model. If someone were inclined to believe that the adjustment was driven by a temporary fall in bargaining positions of the workers (which would rapidly resume once the crisis fade) a rude awakening is in store.

Wage Determination and Assortative Matching

The study of the link between the worker and firm quality is long-lived. In the presence of complementarities between the quality of the firm and the worker (i.e. the production function is super modular), the optimal allocation of workers and firms would result in perfect positive assortative matching. Alternatively, if the two dimensions are substitutes the optimal result is perfect negative assortative matching. Labour market frictions are suspected to play a role in causing mismatch from the optimal matching in the economy. With regard to this, we assess the degree of assortative matching as the correlation between worker and firm quality, and take its evolution as a sign of the strength of mismatch as proposed by Eeckhout and Kircher (2011).[39] We find a downward trend over the period of analysis, independently of the type of correlation. The earlier years show assortative matching measures in the range of roughly 0.45, and the latter years around 0.40 or even lower.

Figure 9: Measure of Assortative Matching



Sources: Quadros de Pessoal and Relatório Único, 1995-2016.

The bargaining power and wage compression combined with the decoupling of the worker's wage from the firm productivity may trigger numerous effects, working as an

[39]In our setting, a measure of this class is given by:

$$AM_t = corr\left\{ \underbrace{ln\left(E\left[\frac{1}{\beta^{\star}_{j,t}}\int_0^1 z^{\frac{1-\beta^{\star}_{j,t}}{\beta^{\star}_{j,t}}}\frac{\partial F(\mathbf{Q}_{j,t}(z)\mathbf{N}(K_t,\beta_t),K_t)}{\partial N_j(K_t,\beta_{j,t})}dz\middle| f,t\right] - E[A(j,a_t)|f,t]\right)}_{\text{firm component}};\ \underbrace{ln[Out^{\star}(a_t)]}_{\text{worker component}}\right\}, \tag{32}$$

where intuitively the worker component is given by his outside option value, while the firm component is composed by the average real productivity in the firm. Note that we do not disagree with Eeckhout and Kircher (2011), who state that with typical information on wages alone it is impossible to determine the sign of the assortative matching, nor with the literature that criticizes the use of fixed effects to infer it. We rely on the collective bargaining ranking to effectively estimate assortative matching measures.

additional wedge especially if amplified by progressive income taxation. Among them, these trends may degrade the incentives for skill acquisition, and on-the-job training. The alignment of worker and firm incentives may be reduced, and the incentive for firms and workers to search for the best pairing given their types may be laxed. These mechanisms underlying both wage inequality and assortative matching have been analysed. For Germany, Card et al. (2014) link the increase of wage inequality and assortative matching, while for Portugal, Torres et al. (2018) record, as we do, a decline in both.

In a broader literature, the often mentioned higher degree of wage compression in continental Europe is recurrently linked with higher unemployment levels among low-skilled workers - the ones who are most likely to benefit from such compression while employed (see Bertola and Ichino (1995) and Siebert (1997)). In a structural approach, Heckman and Jacobs (2010) remarks that an higher wage compression reduces the returns to skill acquisition, increasing the odds of the dropouts' mass to face unemployment. Our findings are in line with this strand of the literature by witnessing that the increased wage compression and decoupling of wages and productivity have likely contributed to the reduced quality of the matching process.

Parameter Estimates and the Literature

The estimate for the average bargaining power of the entire economy is around 20% conforming with many previous empirical studies. For the Veneto region of Italy, Card et al. (2014) find a reduced form coefficient of the outside option (i.e. $(1-\beta)$) of 80%, when using sector minimum wage as its proxy, in both OLS and IV within spell models of rent sharing. For Germany, Hirsch and Schnabel (2011) estimate bargaining powers between 11% and 18%, for the years 1992-2009. For France, Cahuc et al. (2006) structurally estimate bargaining powers mostly in the range between 0 and 38% depending on the occupation.[40] For Denmark, resorting to a structural model, Bagger et al. (2014) estimate an average workers' bargaining power of around 30 percent, and while Mortensen et al. (2010) match that empirical estimate for the same dataset, they further presents sectoral heterogeneity, ranging from 7-61 percent. In contrast, Dumont et al. (2012) presents higher workers' bargaining power estimates for Belgium of between 45 and 71 percent depending on the sector under analysis.

In a related dimension, the literature presents mixed results regarding the relative bargaining position of the occupation groups. Some studies report greater bargaining power at the bottom of the wage table (e.g., Dumont et al. (2012)). Other findings, depending on the analysis performed, include broad monotonicity between wage tables

[40] The sole exception of that range is 98% for managers in the construction sector. Their partition of occupations is identical to ours, but they have 4 categories. Their two top categories (i.e. 1 and 2) are condensed in our 1^{st} category.

and bargaining powers; very nearly identical bargaining powers across occupations; U-shaped results with the middle part having less bargaining power; or even mixed results depending on the sector of activity under analysis (see Cahuc et al. (2006), Mortensen et al. (2010) or Bagger et al. (2014) for some of these results).

Regarding the average magnitude of the elasticity of wages to quasi-rents, our average result is 0.062, with a standard error of 0.009. Our result is aligned with those of several comparable studies for the Portuguese labour market.[41] Alternatively, Card et al. (2018) presents a more extended review covering 22 different studies including for several European countries and the U.S., using a wide-range of different methodologies. They locate the reasonable estimates for this elasticity in the range between 0.05-0.15, and find for Portugal an average estimate nearly identical to ours. Given this proliferation of estimates and identification strategies for this elasticity, our estimates, implied by our identification strategy, suggest the external validity of our approach.

We estimate the average passthrough of bargained wages into total wages at 44.8 percent, while Card and Cardoso (2021) estimates this measure at about 50 percent for Portugal. Our finding implies that a 10 percent increase of bargained wages translates into a 4.5 percent increase in total wages, supporting the classic continental European feature of imperfect passthrough. Changes in wage floors are associated to meaningful changes in total wages, but increases (decreases) in the former correspond to a shrinkage (boost) of the wage cushion. Moreover, in our results the link between wage floors and total wages is ordered, with the lowest occupations showing the strongest link (see Figure 8b). This finding also aligns with that of Card and Cardoso (2021).

The empirical estimates of assortative matching locate the Pearson correlation in the range 0.38-0.48, with an average over the period of 44.14% (see Figure 9). These findings are in line with the most recent developments on this specific literature, and very different from the downward biased estimates typically obtained through the AKM framework (in our sample the result would be 0.1797). For example, our findings conform with those of Bonhomme et al. (2019) for Sweden, Borovicková and Shimer (2017) for Austria, and are somewhat higher than Kline et al. (2019) for Veneto, Bonhomme et al. (2020) for a range of European countries and the US, and Lentz et al. (2018) for Denmark.

The way our model relates with the AKM model in terms of assessing wage differentials is relevant to assess how we are attaining the fit of our model in comparison with that benchmark. Consequently, we perform a covariance decomposition assessment. Then,

[41] Using the same dataset: Card et al. (2018) find 0.056 (with s.e. of 0.016) for the period 2005-2009; Card et al. (2016) find 0.14-0.16 for males and 0.04-0.05 for females, for the period 2002-2009; Martins (2009) find 0.03-0.05 for the period 1993-1995; Garin and Silvério (2019) find .15 (with s.e. of 0.066) for the period 2005-2013; and Cardoso and Portela (2009) find 0 for transitory shocks and 0.09 for permanent shocks for the period 1991-2000.

we decompose the main components of the equilibrium wage equation into worker, job description, and firm and time dimensions through an AKM model.[42] Using the same dataset, we then resort to a typical covariance decomposition of three different AKM based strategies, namely with: (a) workplace-worker fixed effects; (b) worker-firm-time-wage rank fixed effects; and (c) (b) worker-firm-time fixed effects.

Our covariance decomposition results are in Table 2. The major drivers of wage differentials reside at workplace level, which explains around 63 percent of the overall heterogeneity in wages, while the components attributed to the worker, namely the level of the outside options and operating cost components, contribute around 22 percent. In general, the AKM literature has seen a sizable proliferation of variance decompositions. Typically, those (as in Card et al. (2014) for Germany, Torres et al. (2018) for Portugal, and Song et al. (2019) for the US) attribute a leading role to the worker dimension, which can be perceived as disagreeing with our findings.

However, the workplace consists of an occupation-firm-time cell, and thus can not be equated to a firm. The workplace is worth roughly 60 percent of the heterogeneity in both a workplace-worker AKM and our approach. The difference between those approaches resides in the worker and residual components, with the residuals considerably reduced in the AKM approach - a consequence of its very large parameter set.[43] As one moves towards more time invariant formulations of the firm side, the worker fixed effect is largely able to absorb the *leftovers* of the variation that was previously enclosed in the workplace definition, with the job-descriptions acquiring a peripheral role.

Regarding the remainder of the model, the estimates of the operating costs are displayed in Figure 10a. There, we have the total effect of education, tenure, age, and female proportions at the wage rank on the profile of real productivity of the match per hour,

[42]In detail, the proposed variance-covariance decomposition is based on:

$$\begin{aligned} ln[w_j(a_t|K_t,\beta_t)] = ln\Bigg[& \underbrace{(1-\beta^{\star}_{j,t})Out^{\star}(a_t)}_{\text{Out. option component}} + \underbrace{\int_0^1 z^{\frac{1-\beta^{\star}_{j,t}}{\beta^{\star}_{j,t}}} \frac{\partial F(\mathbf{Q}_{j,t}(z)\mathbf{N}(K_t,\beta_t),K_t)}{\partial N_j(K_t,\beta_{j,t})} dz}_{\text{Real productivity per hour of the match}} \\ & - \underbrace{\beta^{\star}_{j,t}\mathbf{X}_{i,t}\zeta_j}_{\text{Obs. operating costs comp.}} + \underbrace{\beta^{\star}_{j,t}[E[E(\xi(i,t)|rank(i,t))|j,t]] - \beta^{\star}_{j,t}\xi(i,t)}_{\text{Unobservable operating costs comp.}} + \underbrace{\epsilon_{i,f,j,t}}_{\text{residual}}\Bigg]. \end{aligned} \tag{33}$$

with the covariance decomposition components obtained as:

$$\mathbf{\Gamma}(n) = \frac{cov\Big(ln[w_j(a_t|K_t,\beta_t)];\Gamma_{i,f,j,t}(n)\Big)}{var(ln[w_j(a_t|K_t,\beta_t)])}, \quad \text{with} \sum_{n=1}^{6}\mathbf{\Gamma}(n) = 1 \tag{34}$$

with $\Gamma_{i,f,j,t}(n)$ representing each of the components of equation (33).

[43]See online Appendix B for a comparison of parameter dimensionality and goodness-of-fit between our approach and these AKM methodologies.

Table 2: Variance Decomposition of the Logarithm of Real Hourly Wages - Comparison with AKM Model

Dimension	Components	Our methodology		AKM		
		Aggregate	Detailed	Workplace Worker	Firm Worker Job-title Year	Firm Worker Year
Worker	Worker attributes	21.27%	10.60%	34.37%	38.06%	41.42%
	Job-title FE		6.06%	-	-	-
	Generic time FE		2.49%	-	-	-
	Within residual		2.12%	-	-	-
Workplace	Firm FE	60.99%	37.59%	58.16%	35.60%	39.02%
	Job-Title FE		9.35%		-	-
	Generic time FE		2.68%		-	-
	Within residual		11.37%		-	-
Year effects FE		-	-	-	6.91%	7.68%
Job-Title FE		-	-	-	7.93%	-
Residual		17.74%	17.74%	7.47%	11.46%	11.88%

Notes: The detailed decomposition consists of resorting to AKM models to decompose the worker and workplace components, namely: $(1-\beta^{\star}_{j,t})Out^{*}(a_t) - \beta^{\star}_{j,t}[\mathbf{X}_{i,t}\zeta_j + \xi(i,t)] + \beta^{\star}_{j,t}E[E(\xi(i,t)|rank(i,t))|j,t] = \mathbf{X}_{i,t}\tilde{\zeta} + \alpha_i + \delta_j + \chi_t + \epsilon_t$, and $\int_0^1 z^{\frac{1-\beta^{\star}_{j,t}}{\beta^{\star}_{j,t}}} \frac{\partial F(\mathbf{Q}_{j,t}(z)\mathbf{N}(K_t,\beta_t),K_t)}{\partial N_j(K_t,\beta_{j,t})} dz - \beta^{\star}_{j,t}E[E(\xi(i,t)|rank(i,t))|j,t] = \alpha_i + \gamma_f + \delta_j + \chi_t + \epsilon_t$, respectively. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

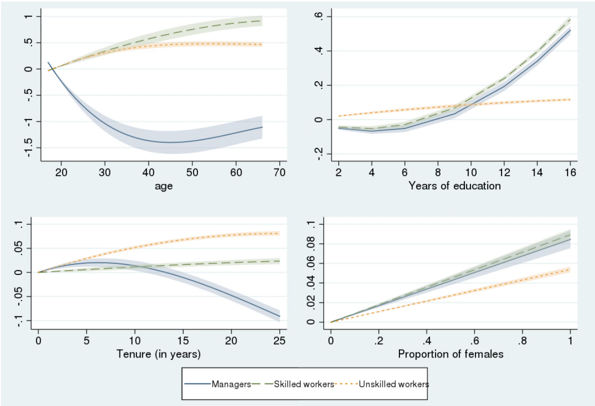
as well as some of the distributional features on those dimensions. Critically, the profiles presented relate with operating costs, and should not be interpreted as the effect of a given variable on wages, because the dimensions considered also affect the outside option of the worker. For instance, the overall effect of an additional year of education on wages should combine the effect it has in the productivity of the current match of the worker (which Figure 10a addresses) with the effect it has on the worker's outside option. This latter effect is beyond the scope of this paper, however.

As a general remark, the estimated behaviour of the productivity is consistent with the broad literature on wage determination, in which the contribution of the individual characteristics become more important for higher skill level occupations. It also confirms in our model that education is the most powerful productivity enhancer only when the job can benefit from the proceeds of schoolwork.

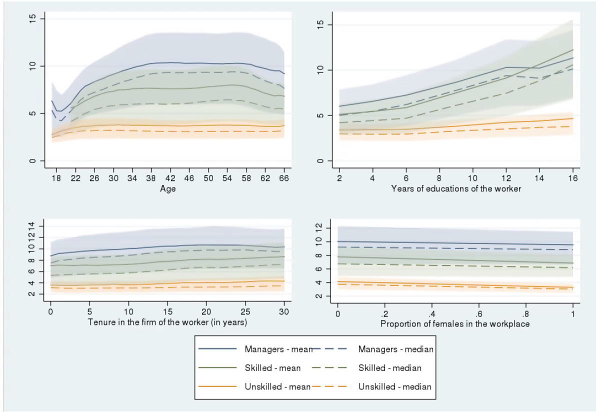
Differently, the tenure on the firm, although displaying lower effects when compared with education, presents a reversed relationship. The lower parts of the wage tables benefit the most from the permanence at the firm, with a stable monotonic relationship, even for very long tenures. In a different archetype, while increasing more at early stages of the worker-firm match, the productivity gains for managers fade after the first five

Figure 10: The Operating Cost Estimates

(a) Estimated total effect on real productivity per hour



(b) Distributional features of the real productivity per hour



Notes: In panel A the fading grey shades correspond to the $95^{th} - 5^{th}$ confidence interval. In panel B, the fading shades correspond to the $75^{th} - 25^{th}$ percentile range of the implied distribution. *Sources:* Quadros de Pessoal and Relatório Único, 1995-2016.

years, becoming increasingly detrimental for very long tenures. In the middle of the wage distribution the profile is positive but weak.

Regarding the age profiles, the gains for skilled and unskilled workers are increasing with the wage tables, though ceasing to further accrue earlier in the career the lower the position of the worker on those wage tables. Relatively to managers, increases in experience translates into consistent real productivity gains at a later stage in their careers, generally after around 40 years of age, which coincides with the most sizable mass of existing managers.[44] Before the cited threshold, the younger the manager the higher is his real productivity, ceteris paribus. Tentatively, their appointment includes relevant traits that make them particularly productive, as it is likely to be linked with an entrepreneurial/family-owned business perspective - the ideal and perhaps only green-way to obtain a managerial position so early in their careers (see Blanchflower and Oswald (2007) for an analysis on the traits of young entrepreneurs).

6 Final Remarks

Wage inequality has been on the rise almost everywhere, with the main exception of the Southern European countries. Those countries have a significant collective bargaining apparatus, as does all of continental Europe, but in their case it has been evolving in a singular way. Their labour union membership has fallen substantially, but the proportion of contracts that are directly bargained by unions have remained very high.

[44] In the period of analysis, 90 percent of the managers are older than 28 years, and 75 percent are older than 33.

Thus, the analysis of the wage determination mechanisms in those countries, and how its structural change has influenced wage inequality is the focal point of this paper. For this purpose, we develop an empirically implementable dynamic search and matching model with a collective bargaining apparatus, and we implement it in the Portuguese labour market using data from 1995 until 2016. The proposed model has the convenience of discipline regarding the use of data about the characteristics of the placement of the worker-firm match on the collective bargaining wage tables - the most perennial and comparable characterization of the labour relationships.

Consequently, our empirical identification does not rely on the mobility of workers across firms, or on the definition and estimation of a production function or marginal product. Despite such flexibility, our framework allows for the consistent and straightforward estimation of bargaining powers, elasticities of wages to quasi-rents, the passthrough rate of bargained wages onto actual wages, as well as the assortative matching of the market. The average results in each of those dimensions are consistent with findings reported in the literature, including that which is focused on Portugal. The underlying estimated heterogeneity across the different components of the wage determination equation resemble the partition across market dimensions that an identical AKM model would attain. More importantly, our model and identification strategy allow us to depict flexible trends on those measures.

We find that wage determination is indeed witnessing a structural change. It has synchronized a notable stability of worker bargaining power at the bottom of the skill distribution, with a perennial erosion at the middle and the top. Wages have become more reliant on sectoral bargaining, increasing their decoupling from firm productivity. This transition has contributed to a compression of the wage distribution, and to a downward trend of assortative matching in the market.

The structural slow paced continuous erosion of bargaining powers for managers and skilled workers warns of potential future productivity hazards, especially if amplified by progressive income taxation. In the absence of wage differentials across skill groups, and the increasing decoupling of wages from firm productivity levels, the sorting in the economy may be degraded, the incentives for skill acquisition and on-the-job training may be abated, and the alignment of worker and firm incentives may be reduced. Any of these endanger future productivity levels.

Throughout the timeframe of this study, the macroeconomic context of Portugal has been especially turbulent, with the Great Recession triggering a significant wage adjustment. However, our estimated trends maintained their dynamics, thus displaying a sound resilience of the wage determination process even through those difficult times. The wage adjustment in Portugal reflects the evolution of the valuation of the real quasi-rents of the worker-firm match and of the worker's real outside options. In a reduced-form analysis, such as the one presented by Card and Cardoso (2021), it would translate into reductions in real wage floors and real wage cushions, and workers entering the market at relatively lower wage ranks. This evolution was not due to union or worker concessions during a particularly turbulent time, which would result in a sudden fall in worker bargaining power. Accordingly, wages will not rapidly resume once the crisis fades. Their evolution will largely depend on the evolution of productivity, and the functioning of the labour market in establishing the share of the match productivity that accrues to workers.

All in all, we have shown that labour market institutions, particularly the way wage determination takes place, and with whom, play a key role in shaping wage dynamics and inequality. These institutions have been in a structural change across many economies. In continental Europe trade unions are still a relevant player. In the case we studied they retained the ability to bargain on behalf of most of workers, but they lost a large share of the membership they once had. It seems that they might have kept their inequality concerns, but they lost the ability to claim for the most skilled their customary share of the pie. An alternative interpretation is that firms are being able to exert their monopsonistic power more often, or the evolution of labour market frictions and employer concentration have allowed them to find greater such powers. While apparently divergent, those explanations are often strongly complementary. Finding paths to understand such a transformation, its routes, and their relative importance is still a research challenge and a policy need.

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Appendix A - Dataset

The data sources of this study comprises information about the balance sheet and income statement of the firms obtained from SCIE - *Sistema de Contas Integradas das Empresas*, for the years 2005-2016, and information about the characteristics of the workers and of their employment relationship, provided in *Quadros de Pessoal* (Personnel Tables) for the years 1986-2009, and *Relatório Único* (Single Report), for the years 2010-2016.[45]

The three datasets are fully matched for the common years of the data. Consequently, for each year from 1986 to 2016, we have a longitudinal matched employee-employer-contract-job-title database, which from 2005 is supplemented with matched information at firm level about the balance sheet and income statement of those firms.

The *Quadros de Pessoal* and the *Relatório Único* datasets are recorded by the Ministry of Employment and Social Security and correspond to a mandatory survey on an annual basis for all establishments with at least one wage earner. In this survey all workers employed in October of each year are reported, although civil servants and workers in domestic service are not covered. Therefore, the dataset covers the entire population of workers of private-sector firms in manufacturing and services. Further, the long-lived requirement of the information to be published at establishment level, ordinarily at the door of the establishment ensures greater validity.

The dataset reports the firm's location, industry, employment, sales, ownership, and legal basis. Worker information includes gender, age, skill, occupation, schooling completed, starting date at the firm, earnings, and working hours. In addition, the survey also records the collective bargaining arrangement and the specific job-title held by the worker under collective agreement, which is of particular importance for this study.

In these datasets the following restrictions were applied: (a) we only consider full-time employers in receipt of what is contractually defined for the reporting month; (b) we exclude workers from agriculture, fisheries, and energy products/extraction sectors; (c) we exclude workers aged less than 18 years or greater than 65 years; (d) we exclude workers earning less than 80 percent of the minimum wage[46]; and (e) we only consider firms that have on average 10 workers in the years they existed in the dataset. A significant array of descriptive statistics on this matched dataset is provided in table 3.

Then we combine the established matched dataset with the SCIE dataset, with a coverage of the match above 97 percent. The SCIE dataset is managed by National Statistics Institute, which provides a unified survey system. Its reporting is mandatory for the universe of registered firms operating in Portugal, including those with no employees. This dataset has a vast array of accounting information, namely with detailed information about every entry of the balance sheet and income statement legally required

[45] For the year 1990 and 2001, the survey was either not administered, or not digitized.

[46] Corresponding to the lowest admissible wage in the case of apprendiceships.

for accounting purposes under the *SNC - Sistema Normalização de Contas.* Among the information provided, one has access to the level of assets, liabilities and equity, and its typical accounting partitions, as well as profits, output value, value-added, payroll, purchase of intermediate goods, investment levels, service of debt among others.

Table 3: Descriptive Statistics

(a) Descriptive Statistics on the Dataset Dimmensions

Variables:	Percentiles			Mean	Standard	Skweness	Kurtosis	Total
	5	50	95		Deviation			Number
Observations	-	-	-	-	-	-	-	29,560,504
Per Year	1,305,735	1,391,557	1,474,118	1,386,460	-	-	-	-
Workers characteristics								
Age (years)	22	38	57	38.46	10.87	0.26	2.23	-
Education (years)	4	9	16	8.56	3.97	0.38	2.01	-
Tenure in the Firm (years)	0	6	23	8.7	9.09	1.26	4	-
Female (perc./year)	39.91	41.89	46.04	42.4	0.03	0.25	1.73	-
Mover to the Firm (perc./year)	15.96	22.77	27.08	22.81	4.89	1.20	5.96	-
Duration of Spells (years)								
Workers	1	4	14	6.04	5.29	1.03	3.07	4,816,652
Worker-Firm	1	2	10	3.92	4.09	1.86	6.16	7,436,538
Worker-Occupation	1	2	10	3.88	3.91	1.80	6.01	7,513,257
Worker-Rank	1	1	3	1.27	0.68	2.14	5.59	22,946,436
Firms								
Workforce Size	6	13	55	33.84	187.13	53.28	4,488.52	1,139,819
Workplace Size	1	3	15	8.9	58.36	70.69	7,894.81	4,309,996
Collective agreements								
Agreements-Year	477	536	764	574	108	1.02	2.46	-
Ranks in Agreement-Year	2	25	113	48.43	75.59	5.07	47.31	-
Firms-Year	1	6	164	82.55	350.89	12.54	224.98	-
Workers-Year	9	260	4,599	2,455.2	9,073.7	8.34	93.09	-
Years	1	5	21	8.47	7.45	1.26	3.64	-

(b) Descriptive Statistics on Worker's Wages

Variables:	Percentiles					Mean	Standard	Skweness	Kurtosis
	5	25	50	75	95		Deviation		
Log of Nominal Monthly Wages									
Total Wages	5.89	6.30	6.62	7.06	7.86	6.72	0.61	0.90	4.31
Base Wages	5.77	6.14	6.36	6.76	7.55	6.49	0.54	1.15	4.95
Bargained Wages	5.44	5.81	6.01	6.21	6.81	6.04	0.40	1.26	6.63
Wage cushion $\left(\frac{w_{total}}{w_{base\|barg.}}\right)$	1	1.23	1.5	2.07	4.25	1.98	3.66	324.94	194,821.4
Base Wage Ratio $\left(\frac{w_{base}}{w_{total}}\right)$	0.49	0.73	0.84	0.93	1	0.81	0.16	-1.07	4.15
Hours Worked	152	168	173	173	180	169.5	9.16	-1.20	5.93

Notes: There are 21 years in the dataset. The year 2001 is missing from the dataset as it was not recorded. *Source: Quadros de Pessoal, 1995-2009 and Relatório Único, 2010-2016.*

Online Appendix

Who's Got the Power? Wage Determination and its Resilience in the Great Recession

Hugo de Almeida Vilares, Hugo Reis

A - Theory Appendix

In this appendix we present: (A1) the relationship of the model with the search and matching literature; (A2) additional Conditions of the Model (A3) the derivation of equilibrium wages for both types of equilibria; (A4) the description of the dynamic and steady state equilibria and their properties.

A1 - The Model and the Search and Matching Literature

Bridging the Model and some Canonicals of the Search and Matching Literature. Our model was constructed so it can be easily reshaped as several canonical models in the literature of search, mostly resorting to: (a) a wise choice of parameters, and (b) variations of the modelling of the choice of capital.

Our model becomes the model of Acemoglu and Hawkins (2014) if: (a) we set $J = 1$, as they abstain from modelling different occupations; (b) $\xi^{\Xi}(a|K, \beta) = 0, \forall\{a, K, \beta\}$, as they don't consider a model with on job search; (c) $\{K\}$ to be relabelled as z their idiosyncratic firm's productivity parameter; (d) $\beta = \bar{\beta}$ so there is homogeneous bargaining power in the economy; (e) $a = \bar{a}$ and known by workers and firms; (f) $\tilde{s}(a|K, \beta) = 0$ as there is no firing at-will; and (g) $\vartheta(K, \beta) = 0, \forall\{K\beta\}$, so no wage posting in the model.

Further, in the model of Acemoglu and Hawkins (2014) consider $\gamma(V)$ to be linear and not strictly convex as presented. The fundamental implication of this deviation is that instead of having a growth path of each firm, which is pivotal in the author's analysis, we assume a firm can immediately attain its optimal scale without incurring in further costs due to simultaneous hiring.[1] Closely related with this deviation, we do not model entry and exit of firms, and thus in their model consider $FC_t = \infty$ - no entry of new firms, and $\delta = 0$ - no exit of firms.[2]

[1] The extension to allow the incorporation of their class of vacancy costs increases the complexity of the model by some degrees, given one is required to keep track of the history of firms.

[2] This deviation allows for notation simplicity, and given the intention to focus on optimal scale, comes without further implication.

Secondly, one can also adapt this model to resemble Cahuc et al. (2008), which models a representative firm. For this purpose, consider: (a) $\xi^{\Xi}(a|K,\beta) = 0, \forall\{a, K, \beta\}$, so there is no on job search; (c) $A(j,a) = 0, \forall j, a$ so that operating costs are fully neglectful; (d) there is no firing at will so $\tilde{s}(a|K,\beta) = 0, \forall\{a, K, \beta\}$; (e) $\vartheta(K,\beta) = 0, \forall\{K\beta\}$, so no wage posting in the model; and either (f1) $\{K,\beta\} = \{k, \bar{\beta}\}, \forall\{K,\beta\}$, where k is a given constant, and β is homogeneous across firms, but potentially different across occupations, representing a $J \times 1$ vector. Therefore, firms do not have any heterogeneity arising from capital or bargaining powers, and we follow the most restrictive version of their model, without capital; or (f2) consider that capital is also chosen optimally *ex-ante* to the wage determination and job flows decisions and thus add the following condition to our equilibrium:[3]

$$\frac{\partial F(\mathbf{N}, K)}{\partial K} = r + d + \int_0^1 \sum_{j=1}^{J} N_j \pi^{\frac{1-\beta_j}{\beta_j}} \frac{\partial^2 \tilde{F}(\mathbf{N}\mathbf{A_j}(\pi), K)}{\partial N_j \partial K} d\pi, \tag{1}$$

where d is the depreciation rate, which we have abstracted in our model formulation, and $\mathbf{A_j}(\pi)$ is identical to equation (41) of appendix A2.

Thirdly, resorting to the insights of Cahuc and Wasmer (2001), the model can also be translated to the large firm version of the matching model of Pissarides (1990). For that purpose, we have: either (a1) $J = 1$ so that there is only one type of occupation; or (a2) $J > 1$, but the types of occupations are perfect substitutes; (c) $\xi^{\Xi}(a|K,\beta) = 0, \forall\{a, K, \beta\}$, so no on job search; (d) $a = \bar{a}, \forall j, a$, so that the worker heterogeneity is fully neglectful for equilibrium purposes; (e) $\vartheta(K,\beta) = 0, \forall\{K\beta\}$, so no wage posting in the model; and (f) perfect capital markets, and simultaneous decision of labour and capital so that the capital stock of the firm becomes a function of employment ($K_t(\mathbf{N})$), and the following condition hold:[4]

$$\frac{\partial F(\mathbf{\Phi}_t, K_t)}{\partial K_t} = r + d. \tag{2}$$

Fourthly, the model would mimic Mortensen (2009) if we consider: (a) there is not entry or exit of firms (i.e. $\delta = 0$ and $FC_f = \infty$); (b) $J = 1$ so that there is only one type of occupation; (c) $a = \bar{a}, \forall j, a$ so that operating costs are fully neglectful; (d) K is constant overtime, and represents the idiosyncratic productivity of the firms,

[3] See Cahuc et al. (2008) to the details on how to obtain this expression from our model under this set of assumptions. Further notice that if one considers optimal choice of capital, *ex-ante* to the wage determination and job flows decisions, and fully neglects the firm heterogeneity arising from bargaining powers, then we are in an environment of a representative firm.

[4] See Cahuc and Wasmer (2001) for the specific details about this equivalence.

presented in the paper as $p(x)$; (e) $\vartheta(K,\beta) = 0, \forall\{K\beta\}$, so no wage posting in the model; and (f) $\beta = \bar{\beta}$ is constant for every firm and in every occupation.

In a nutshell, our model is isomorphic to a wide range of standard search and matching models. Mainly that is attainable by: (a) sufficiently restricting the parameterization of the model (i.g. heterogeneity); (b) considering alterations of the capital allocation mechanism, whose implications for our modelling objectives are fairly minor; and (c) considering the dynamics of vacancy costs and entry and exit of firms.

A2 - Additional Conditions of the Model

In this appendix we present the derivation of additional relevant conditions to fully describe both the dynamic and the steady state equilibria of the model.

Worker Skill. Each worker is exogenously endowed with an initial generic training and ability, whose stock is given by $a(0)$ extracted from the distribution $\Psi_0(a) = N(\mu_0, \Sigma_0)$. Then, the worker develops skill through a stationary and invariant process with the Markov property, so that the transitions are described by the cumulative distribution function

$$\begin{aligned} \Psi(a'|a) &= Prob(a_i(t+1) \leq a'|a_i(t) = a), \\ \psi(a'|a) &= \frac{d}{da'}\Psi(a'|a) \sim N(B_0 a, CC'), \end{aligned} \tag{1}$$

and accordingly, the density over the history of the worker $a^t = [a(t), a(t-1), \dots, a(0)]$ corresponds to:

$$\psi(a^t) = \psi[a(t)|a(t-1)] \dots \psi[a(1)|a(0)]\psi_0[a(0)], \tag{2}$$

with the unconditional invariant distribution given by:

$$\psi(a') = \int_a \psi(a'|a)\psi(a)da. \tag{3}$$

The operating cost, corresponds to the flow:[5]

$$A(j,a) = \omega_j(a). \tag{5}$$

[5]Technically, assume that $\omega_j(x) > \omega_l(x), \forall x \in [0, \bar{A}], \forall j > l$ due to the increasing complexity of the occupation. Further $\omega_j(x)$ is strictly convex and holds $\lim_{x\to 0^+} \omega(x) = \bar{A}$, $\lim_{x\to\infty} \omega(x) = 0$. The use of a Markov process in this context is classical. Bonhomme et al. (2017) uses a Markovian process to describe earnings directly, whereas we adopt a Markovian process in skill, which allow for dynamics to be treated in a slightly different angle. Jointly, the operating cost function and the

Consistently, the economy pool of workers is given by:

$$\aleph = \int_a d\aleph(a,t)da = \sum_{j=1}^{J} \int_a \int_K \int_\beta dG_{j,t}(a|K,\beta)d\Gamma(K,\beta)da + \int_a dU_t(a)da, \tag{6}$$

where $\aleph(a,t)$ consists in the number of available workers with at most an estimated operated cost a in period t.[6]

Firm's Hiring Policy. The profit of the firm with fundamentals $\{K,\beta\}$ is assumed to be strictly concave and twice continuously differentiable in employment. It is given, as in equation (4) of the text. Solving for the optimal vacancy policy, we obtain first order conditions as:[7]

$$J_j^R(K,\beta) = \begin{cases} \frac{\gamma_j}{q(\theta_j)} & \text{if } V_j(K,\beta) > 0 \\ \\ \left[\underline{J}_j^R(K,\beta); \frac{\gamma_j}{q(\theta_j)}\right] & \text{if } V_j(K,\beta) = 0 \end{cases} . \tag{8}$$

Firm's Firing Policy. Apart from the hiring policy, firms also define their firing policy. Firms, may fire at-will any worker, as long as they pay a corresponding exogenous

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skill acquisition can be represented by a linear state-space system, as:

$$\begin{aligned} a(t) =& B_0 a(t-1) + Ce(t) \\ A(j,a) =& \omega_j(a(t)), \end{aligned} \tag{4}$$

with $e(t) \sim N(0,I)$. See Ljungqvist and Sargent (2012) for further details of this process.

[6]Notice that $\aleph(a,t)$ unfolds according with:

$$d\aleph\Big(a,t\Big) = \int_a d\aleph\Big(a,t-\epsilon\Big)\psi(a'|a)da. \tag{7}$$

Further $\aleph$ is exogenous and fixed. Then given $\Gamma(K,\beta)$ and an initial distributions $dG_{j,0}(a|K,\beta)$ and $d\aleph(a,0)$ the distribution $dU_0(a)$ is identified, and given the dynamics of the former distributions the dynamics of the latter is equally identified. The dynamics of $dG_{j,t}(a|K,\beta)$ are described latter.

[7]The considered corner solution exists due to the impossibility of costless firing at will, as motivated in Bertola and Caballero (1994). Further we assume $\underline{J}_j^R(K,\beta) > -\infty$, so it is bounded from below.

firing tax, given as S. Therefore, the decision of fire at-will is given by:[8]

$$\tilde{s}_j(a|K,\beta) = argmax_{\tilde{s}_j(a|K,\beta)\in\{0,1\}} \sum_{j=1}^{J} \int_a \Bigg[\underbrace{\Big(1 - \tilde{s}_j(a|K,\beta)\Big) J(a,K,\beta)}_{\text{Value of not firing}} +$$

$$\underbrace{+ \tilde{s}_j(a|K,\beta)\Bigg(- J_j(a,K,\beta) - S + \underbrace{\max\Big\{ - e^{-r\Delta t}\gamma + e^{-r\Delta t} q(\theta_j) J_j^R(K,\beta); 0 \Big\}}_{\text{Optimally decide if replace or not the fired worker}} \Bigg)}_{\text{Value of Firing the worker}} \Bigg] \times \quad (9)$$

$$\times\, dG_j(a|K,\beta)da,$$

where the firm balances the option of keeping the worker, against the options to firing him, and subsequently either replace or not replace him in the following period. Notice the firm considers a consistent policy for its entire workforce due to the linkages between the marginal values of the jobs across workers. The maximand function $\tilde{s}_j(a|K,\beta)$ corresponds to a threshold function as:

$$\tilde{s}_j(a|K,\beta) = \begin{cases} 1 & \text{if } a \leq a_j(K,\beta) \\ 0 & \text{if } a > a_j(K,\beta) \end{cases} . \quad (10)$$

Beyond a potential bargaining breakdown with the union, and its decision of firing a given worker, the firm is also subjected to displacement due to an exogenous shock, for example due to the death of a worker, which happens with probability $\bar{s}$, and a successful outcome of the on-job-search of its employed worker, given by the function $m_j(a|K,\beta)$. The displacement rate function is given by:

$$s_j(a|K,\beta) = \begin{cases} 1 & \text{if } a \leq a_j(K,\beta) \\ \bar{s} + m_j(a|K,\beta) & \text{if } a > a_j(K,\beta). \end{cases} \quad (11)$$

Altogether, we have defined the behaviour of firm's (K,β) type in managing its workforce.

Job Search. Synchronously, both workers and the unemployed have the option to search for jobs. For that purpose, when they decide to search, they do so in the occupation which yields the most expected return, and in searching they incur in a cost given by $c_j(a)$, dependent on the occupation they intend to search for, and their

[8]The operator Δt represents a time lag. $-e^{-r\Delta t}$ represents a discount factor, where (1) $\lim_{\Delta t\to 0^+} -e^{-r\Delta t} = 1$, and (2) $\lim_{\Delta t\to\infty} -e^{-r\Delta t} = 0$.

type a. For an employed worker, he solves his search problem as:

$$\xi_j^{\Xi}(a|K,\beta) = argmax_{\xi_l^{\Xi}\in\{0,1\},\sum_{l=1}^{J}=\{0,1\}} \sum_{l=1}^{J}\Bigg[\xi_l^{\Xi}(a|K,\beta)\Bigg\{\theta_l q(\theta_l)\times$$
$$\times\left(\frac{\int_K\int_\beta \mathbf{1}[\Xi_l(a|x,y)>\Xi_j(a|K,\beta)]\Xi_l(a|x,y)V_l(x,y)d\Gamma(x,y)}{\int_K\int_\beta V_l(x,y)d\Gamma(x,y)} - \Xi_j(a|K,\beta)\right) - c_l(a)\Bigg\}\Bigg], \tag{12}$$

where $\mathbf{1}[\Xi_l(a|x,y)>\Xi_j(a|K,\beta)]$ is an indicator function being 1 if the potential offer $\Xi_l(a|x,y)$ provides an higher value than $\Xi_j(a|K,\beta)$.

The unemployed solves a similar problem as:

$$\begin{aligned}\xi^u(a) = argmax_{\xi_l^u\in\{0,1\},\sum_{l=1}^{J}=1} \sum_{l=1}^{J}\Bigg[\xi_l^o(a)\Bigg\{\theta_l q(\theta_l)\times \\ \times\left(\frac{\int_K\int_\beta \Xi_l(a|x,y)V_l(x,y)d\Gamma(x,y)}{\int_K\int_\beta V_l(x,y)d\Gamma(x,y)} - Out(a)\right) - c_l(a)\Bigg\}\Bigg],,\end{aligned} \tag{13}$$

where $Out(a)$ is the unemployed value, given his type, and ξ are indicator functions, displaying the selected occupation of the worker, given his context. We assume that: (i) $\theta_j q(\theta_j)\int_K\int_\beta \Xi_j(a|x,y)V_l(x,y)d\Gamma(x,y) - Out(a) - c_j(a) > 0$ for at least one occupation j, so that the unemployed always search in some market; (ii) $\Xi_j(a|x,y) > Out(a)\ \forall\{x,y\}$, so that after they decide to search and after paying the search cost, which becomes sunk, an unemployed worker will always prefer to work;[9] (iii) $c_j(a)$ is differentiable, convex, strictly decreasing in a, holds $\lim_{x\to 0} c_j(x) = \bar{C} > 0$ and $\lim_{x\to\infty} c_j(x) = 0$; and (iv) $c_j(a) > c_l(a)\ \forall l > j$. Altogether, the optimal search behaviour for the unemployed follows a system of threshold rules. Thus, the vector of search choices becomes:

$$\xi^u(a) = \Big[1(l=1, a>a_1),\ldots,1(l=j, a_{l-1}>a>a_l),\ldots,1(j=J, a<a_J)\Big] \tag{15}$$

implying that unemployed perfectly segment across occupations accordingly to their type. Furthermore, given the search behaviour of an unemployed, we have that:

$$\xi_j^{\Xi}(a|K,\beta) \in \{\mathbf{0}, \xi^u(a)\}, \tag{16}$$

[9]Further, we also assume that:

$$\lim_{N_j\to\infty}\frac{\partial F(\mathbf{N},K)}{\partial N_j} < b, \tag{14}$$

where b stands for the unemployment benefit, so that eventually a firm shall not grow indefinitely.

so that an employed worker if he searches, he does so in the occupation he would search if he was unemployed. Concretely, given $\Xi_l(a|K,\beta) > Out(a), \forall l \in \{0,\dots,J\}$, the worker may eventually decide not to search, when he would do so if unemployed. Therefore, the worker's type, a, is a sufficient statistic of occupation choice in on-job search, conditional on searching.

As a reference, this behaviour of the agents regarding employment flows and selection reproduces the behaviour of the classical selection model of Roy (1951).[10]

Probability of successful on-job-search. Following this structure, the probability of a success on-job-search for a worker with match fundamentals (a, K, β) in occupation j is given by:

$$m_j(a|K,\beta) = \sum_{l=1}^{J} \xi_l^{\Xi}(a|K,\beta)\, \underbrace{\theta_l q(\theta_l)\Big[1 - D_l(\Xi_j(a|K,\beta))\Big]}_{\text{Prob. the worker accepts a job in } j} \tag{17}$$

where $D_l(w(a|K,\beta))$ is the distribution of wage offers in occupation j.[11] The expectation of the marginal profit of a new hire to be given by:

$$J_j^R(K,\beta) = \underbrace{\frac{1}{u_j}\int_a \xi_j^u(a)J(a|K,\beta)dU(a)}_{\substack{\text{Expected value of a job}\\ \text{when unemployed is hired}}} +$$
$$+\underbrace{\int_a \frac{\sum_{l=1}^J \int_K \int_\beta \xi_j^{\Xi}(a|x,y) 1\Big(\Xi_j(a|K,\beta) > \Xi_l(a|x,y)\Big) J_j(a|K,\beta)dG_l(a|x,y)d\Gamma(x,y)}{\sum_{l=1}^J \int_K \int_\beta \xi_j^{\Xi}(a|x,y)dG_l(a|x,y)d\Gamma(x,y)} da}_{\text{Expected value of a job when a employed searching at occupation } j \text{ is hired}}. \tag{18}$$

Market tightness and vacancy yield. Given the search behaviour of workers and unemployed, the labour market tightness of occupation j is also given by:

$$\theta_j = \frac{\int_K \int_\beta V_j(K,\beta)d\Gamma(K,\beta)}{\int_a \xi_j^u(a)dU(a)da + \sum_{l=1}^J \int_a \int_K \int_\beta \xi_j^{\Xi}(a|K,\beta)dG_l(a|K,\beta)d\Gamma(K,\beta)da}, \tag{19}$$

where: (i) $e_j = \sum_{l=1}^J \int_a \int_K \int_\beta \xi_j^{\Xi}(a|K,\beta)dG_l(a|K,\beta)d\Gamma(K,\beta)da$ represents the number

[10] Notice that in this framework, theoretically our results will not be plagued by endogenous mobility conditional on the described behaviour.

[11] The presented distribution is given by:

$$D_j(\Xi_j(a|K,\beta)) = \frac{\int_K \int_\beta V_l(x,y) 1\Big(\Xi_j(a|K,\beta) > \Xi_l(a|x,y)\Big) d\Gamma(x,y)}{\int_K \int_\beta V_l(K,\beta)d\Gamma(x,y)}.$$

of workers performing on-job-search in occupation j; (ii) $u_j = \int_a \xi_j^u(a)dU(a)da$ gives the number of unemployed searching for a job in the occupation j; and (iii) the number of vacancies are obtained as $\int_K \int_\beta V_j(K,\beta)d\Gamma(K,\beta)$.

Given this market structure, the probability of a firm of type $\{K,\beta\}$ to find a worker of type a in occupation j corresponds to:

$$y_j(a|K,\beta) = q(\theta_j)\frac{\frac{\partial u_j}{\partial a} + \frac{\partial e_j}{\partial a}X_j^-(\Xi_j(a|K,\beta)}{\frac{\partial uj}{\partial a} + \frac{\partial e_j}{\partial a}}, \tag{20}$$

where $X_j^-(w(a|K,\beta)) = lim_{x\uparrow w(a|K,\beta)}X_j(x)$ is the distribution of wages that employed workers which are searching in occupation j are receiving. Notice, as typical in these type of models we assume workers do not move to a worse paying match.[12] Accordingly, the vacancy yield of a firm of type (K,β), i.e. the probability of firm of type $\{K,\beta\}$ to hire a worker, is given by:

$$y_j(K,\beta) = \int_a y_j(a|K,\beta)da. \tag{21}$$

Evolution of workforce composition. Finally, the expected evolution of the workforce composition of a firm of type (K,β) is then given by:[13]

$$\begin{aligned}\frac{\partial dG_{j,t}(a|K,\beta)}{\partial t} = & \underbrace{-dG_{j,t-\epsilon}(a|K,\beta)}_{\text{Workers of type } a \text{ at period } t-\epsilon} + \underbrace{y_{j,t}(a|K,\beta)V_{j,t}(K,\beta)}_{\text{Prob. hiring worker of type } a} \\ & + \underbrace{\int_{a''}\Big(1 - s_{j,t-\epsilon}(a''|K,\beta)\Big)\psi(a|a'')dG_{j,t-\epsilon}(a''|K,\beta)da''}_{\text{Prob. that incumbent workforce at firm develops into workforce of type } a},\end{aligned} \tag{22}$$

where: (i) the probability that a worker to keep his operational cost fixed from a period to another is zero so the workforce in a at period $t-\epsilon$ will not be in a at period t; (ii) $y_{j,t}(a|K,\beta)V_{j,t}(K,\beta)$ represents the probability of hiring a worker of precisely operating cost a per vacancy posted (i.e. $V_j(K,\beta)$; and (iii) the third term consider, from the workers that have not left the firm of type $\{K,\beta\}$, those whose skill acquisition process leave them precisely at operating cost level a, where $\psi(a|a'')$ is the probability distribution function of the random component of the skill acquisition process, from previous period a'' to current period a.

[12]The distribution presented is given by: $X_j(w) = \frac{\sum_{l=1}^J \int_K \int_\beta \xi_j^{\Xi}(a|x,y)1\Big(\Xi_j(a|K,\beta) > \Xi_l(a|x,y)\Big)dG_l(a|x,y)d\Gamma(x,y)}{\sum_{l=1}^J \int_K \int_\beta \xi_j^{\Xi}(a|x,y)dG_l(a|x,y)d\Gamma(x,y)}$.

[13]For notation clarity, in this equation, we refer to a'' as the skill stock in the previous period, and a as the skill process in the current period.

A3 - Derivation of Equilibrium Wages

On the heterogeneity of bargaining powers. The derivation of the interior solution of the dynamic equilibrium wages follows closely the steps considered in Acemoglu and Hawkins (2014) and Cahuc et al. (2008). In this derivation, we will assume that β is a vector of bargaining powers, implying instead of a common bargaining power for every occupation within the firm, the existence of heterogeneous bargaining powers per occupation, i.e. $\beta = [\beta_1, \ldots, \beta_j, \ldots, \beta_J]$. We will start by derivating the equilibrium wages for the ex-post bargaining regime, and then show the isomorphism of the ex-ante posting regime.

System of differential equations for equilibrium wages in the ex-post bargaining regime. Consider the equation (7) of the text:

$$r\Xi_j(a|K,\beta) - \frac{\partial \Xi_j(a|K,\beta)}{\partial t} = w_j(a|K,\beta) + \underbrace{\bar{s}\Big(Out(a) - \Xi_j(a|K,\beta)\Big)}_{\text{Value loss of losing the job}} +$$
$$\underbrace{+\sum_{l=1}^{J}\xi_l^{\Xi}(a|K,\beta)\left\{\theta_l q(\theta_l)\frac{\int_K\int_\beta \mathbf{1}[\Xi_l(a|K,\beta) > \Xi_j(a|K,\beta)]\Xi_l(a|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a) - c_j\right\}}_{\text{Value of searching for a job while employed}} \quad (1)$$
$$\underbrace{+\sum_{l=1}^{J}\Big[y_l(K,\beta)V_l(K,\beta) - s_l(K,\beta)N_l(K,\beta)\Big]\frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}}_{\text{Impact of hiring and firing policy of the firm in the value of the employment}},$$

At this stage we impose the assumption that both parties bargain under the assumption of *match stability*, i.e. no party, at-will, will dissolve the match, implying the parties believe, for wage bargaining purposes, that $\xi_l^{\Xi}(a|K,\beta) = 0, \forall l \in \{1,\ldots,J\}$, and $\tilde{s} = 0$. Thus equation (1) becomes:

$$\Big(r+\bar{s}\Big)\Big(\Xi_j(a|K,\beta) - Out(a)\Big) - \left[\frac{\partial \Xi_j(a|K,\beta)}{\partial t}\right] = w_j(a|K,\beta) - rOut(a) +$$
$$+\sum_{l=1}^{J}\left[y_l(K,\beta)V_l(K,\beta) - \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da\right]\frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}. \quad (2)$$

Given the bargaining arrangement, expressed in equation (12) of the text, assuming the match stability condition, and that the aggregate bargaining constraint is not binding, we have:

$$\Big(1-\beta_j\Big)\Big(\Xi_j(a|K,\beta) - Out(a)\Big) = \beta_j\Big(J_j(a|K,\beta)\Big). \quad (3)$$

In addition, considering that the outside option bargained between the parties is not affected by changes in firm's employment, given the presence of a large number of firms, i.e.:

$$\frac{\partial Out(a)}{\partial N_j(K,\beta)} = 0, \ \ \forall j \in [1,\ldots,J], \tag{4}$$

we have that:

$$\Big(1-\beta_j\Big)\Big(\frac{\partial \Xi_j(a|K,\beta)}{\partial t} - \frac{\partial Out(a)}{\partial t}\Big) = \beta_j\Big(\frac{\partial J_j(a|K,\beta)}{\partial t}\Big), \tag{5}$$

$$\Big(1-\beta_j\Big)\Big(\frac{\partial \Xi_j(a|K,\beta)}{\partial N_j(K,\beta)}\Big) = \beta_j\Big(\frac{\partial J_j(a|K,\beta)}{\partial N_j(K,\beta)}\Big). \tag{6}$$

Using the result of equation (6) with equation (1), we have:

$$\begin{aligned}
&\sum_{l=1}^{J}\Bigg[\Bigg\{y_l(K,\beta)V_l(K,\beta) - \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da\Bigg\}\frac{\partial J_j(a|K,\beta)}{\partial N_l(K,\beta)}\Bigg] = \\
&= \frac{1-\beta_j}{\beta_j}\sum_{l=1}^{J}\Bigg[\Bigg\{y_l(K,\beta)V_l(K,\beta) - \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da\Bigg\}\frac{\partial \Xi_j(a|K,\beta)}{\partial N_l(K,\beta)}\Bigg] = \\
&= \frac{1-\beta_j}{\beta_j}\Bigg[\Big(r+\bar{s}\Big)\Big(\Xi_j(a|K,\beta) - Out(a)\Big) - \frac{\partial \Xi_j(a|K,\beta)}{\partial t} - w_j(a|K,\beta) + rOut(a)\Bigg].
\end{aligned} \tag{7}$$

Moreover, resorting to equation (5) of the text, under the assumption of match stability, we have:

$$\begin{aligned}
&rJ_j(a|K,\beta) - \frac{\partial J_j(a|K,\beta)}{\partial t} = \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a|K,\beta) \\
&- \sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)}dG_l(a|K,\beta)da - A(j,a) \\
&- \bar{s}J_j(a|K,\beta) + \sum_{l=1}^{J}\Bigg\{y(l|K,\beta)V(l|K,\beta) - \int_a s_l(a|K,\beta)G_l(a|K,\beta)da\Bigg\}\frac{\partial J_j(a,K,\beta)}{\partial N_l(K,\beta)}.
\end{aligned} \tag{8}$$

and together with equation (7), one obtains:

$$\begin{aligned}
&\beta_j(r+\bar{s})J_j(a|K,\beta) - \beta_j\frac{\partial J_j(a|K,\beta)}{\partial t} = \beta_j\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \beta_j w_j(a|K,\beta) \\
&- \beta_j\sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)}dG_l(a|K,\beta)da - \beta_j A(j,a) + \\
&+ (1-\beta_j)\Bigg[\Big(r+\bar{s}\Big)\Big(\Xi_j(a|K,\beta) - Out(a)\Big) - \frac{\partial \Xi_j(a|K,\beta)}{\partial t} - w_j(a|K,\beta) + rOut(a)\Bigg].
\end{aligned} \tag{9}$$

Incorporating equations (3) and (5), and simplifying the resulting equation, we obtain a system of differential equations governing the equilibrium wages for each match with fundamentals $\{a, K, \beta\}$. Such system is given by:

$$\begin{aligned} w_j(a|K,\beta) &= (1-\beta_j)\Big(rOut(a) - \frac{\partial Out(a)}{\partial t}\Big) \\ &+ \beta_j\Big(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da - A(j,a)\Big). \end{aligned} \tag{10}$$

Further, we have that the average wage per workplace - $w_j(K,\beta)$ - is given by:

$$w_j(K,\beta) = \frac{1}{N_j(K,\beta)}\int_a w_j(a|K,\beta)dG_j(a|K,\beta)da, \tag{11}$$

and consistently with equation (10), becomes:

$$\begin{aligned} w_j(K,\beta) &= (1-\beta_j)E\Big(rOut(a) - \frac{\partial Out(a)}{\partial t}\Big|K,\beta\Big) + \\ &+ \beta_j\Big(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J}\frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)}N_l(K,\beta) - E(A(j,a)|K,\beta)\Big). \end{aligned} \tag{12}$$

Further, given equation (11), the proper wage for the match fundamentals $\{a, K, \beta\}$ is given by:

$$\begin{aligned} w_j(a|K,\beta) &= (1-\beta_j)\Big(rOut(a) - \frac{\partial Out(a)}{\partial t}\Big) + \\ &+ \beta_j\Big(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J}\frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)}N_l(K,\beta) - A(j,a)\Big). \end{aligned} \tag{13}$$

Solving the system of differential equations for the partial equilibrium wages To solve the system of differential equations of equation (12), we follow the insight of Cahuc et al. (2008). Thus, take the partial derivative of the average wages, $w_j(K,\beta)$, with respect to employment in another occupation $N_l(K,\beta)$, $l \neq j$, given the difference between any $w_j(a|K,\beta)$ and $w_j(K,\beta)$ is based, in the moment of the bargaining of prices within the firm, on the exogenous values, i.d. (a) $Out(a)$ versus $E[Out(a)|K,\beta]$; and (b) $E[A(j,a)|K,\beta]$ and $A(j,a)$. Thus:

$$\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} + \beta_j\frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)} = \beta_j\Big[\frac{\partial^2 F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)\partial N_l(K,\beta)} - \sum_{k=1}^{J}\frac{\partial^2 w_k(K,\beta)}{\partial N_j(K,\beta)\partial N_l(K,\beta)}N_k(K,\beta)\Big] \tag{14}$$

which yields second-order differential equation as:

$$(1-\beta_j)\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \beta_j \frac{\partial^2}{\partial N_j(K,\beta)\partial N_l(K,\beta)}\Big[F(\mathbf{N}(K,\beta),K) - \sum_{j=1}^{J} w_k(K,\beta)N_k(K,\beta)\Big]. \tag{15}$$

Further, given the equality of second-order cross derivatives, one can also infer that:

$$\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \underbrace{\frac{\beta_j}{1-\beta_j}\frac{1-\beta_l}{\beta_l}}_{\chi_{jl}}\frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)}, \tag{16}$$

and:

$$\sum_{j=1}^{J} N_j(K,\beta)\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \sum_{j=1}^{J}\chi_{lj}N_j(K,\beta)\frac{\partial w_l(K,\beta)}{\partial N_j(K,\beta)}. \tag{17}$$

Jointly, this allows to write equation (12) as:

$$\begin{aligned} w_j(K,\beta) &= (1-\beta_j)E\left(rOut(a) - \frac{\partial Out(a)}{\partial t}\middle| K,\beta\right) + \\ &+ \beta_j\left(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J}\chi_{j,l}\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)}N_l(K,\beta) - E(A(j,a)|K,\beta)\right). \end{aligned} \tag{18}$$

The case of homogeneous β at firm level. At this stage let us first assume that $\beta_j = \beta$ - homogeneous bargaining power at firm level so that $\chi_{jl} = 1, \forall\{l,j\}$. Considering the generalized spherical coordinates $\iota, \omega_1, \ldots, \omega_{J-1}$, where ι is the distance to the origin such that $\sum_{j=1}^{J} N_j(K,\beta)^2 = \iota^2$, where ω_j are angles of projection in different subplanes, one can write:

$$\begin{aligned} N_1(K,\beta) &= \iota cos\omega_1 \ldots cos_{J-2}cos_{J-1} \\ N_2(K,\beta) &= \iota cos\omega_1 \ldots cos_{J-3}sin_{J-2} \\ N_2(K,\beta) &= \iota cos\omega_1 \ldots cos_{J-2}sin_{J-3} \\ &\ldots \\ N_{J-1}(K,\beta) &= \iota cos\omega_{f1t}sin\omega_{f2t} \\ N_J(K,\beta) &= \iota sin\omega_{f1t}, \end{aligned} \tag{19}$$

and with such coordinates, using the notation $\omega = (\omega_1, \ldots, \omega_J)$, one writes:

$$\sum_{l=1}^{J} N_l(K,\beta)\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \iota\frac{\partial w_j(\iota,\omega,K,\beta)}{\partial\iota}, \tag{20}$$

where ι is the scale of use of occupations, and ω reflects the proportions in which the different types of occupations are used. $\omega = (0, \ldots, 0)$ means that firm only employ workers in the first occupation. Then equation (12) reads as:

$$\beta \frac{\partial w_j(\iota, \omega, K, \beta)}{\partial \iota} + w_j(\iota, \omega, K, \beta) = (1-\beta) E\left[rOut(a) - \frac{\partial Out(a)}{\partial t} \middle| K, \beta \right] + \\ + \beta \left[\frac{\partial F(\iota, \omega, K, \beta)}{\partial N_j(K, \beta)} \right] - \beta E[A(j,a)|K, \beta]. \tag{21}$$

Notice that given the exogeneity of: (1) $(1-\beta)\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\right]$; and (2) $A(j,a)$, we can drop it and thus we have:

$$\frac{dw_j(\iota, \omega, K, \beta)}{d\iota} + \frac{w_j(\iota, \omega, K, \beta)}{\beta \iota} - \frac{\partial F(\iota, \omega, K_{ft})}{\partial \phi_{fjt}} \frac{1}{\iota} = 0. \tag{22}$$

Notice that the solution of the homogeneous equation $\frac{\partial w_j(\iota, \omega, K, \beta)}{\partial \iota} + \frac{w_j}{\beta \iota} = 0$ is given by:

$$w_j(\iota, \omega, K, \beta) = C \iota^{-\frac{1}{\beta}} \tag{23}$$

and thus derivating it towards ι, while assuming C depends on ι, one obtains:

$$\frac{dw_j(\iota, \omega, K, \beta)}{d\iota} = \frac{dC}{d\iota} \iota^{\frac{-1}{\beta}} - \frac{1}{\beta} C \iota^{-1-\frac{1}{\beta}} \tag{24}$$

which plugging back (24) and (23) in equation (22), one obtains:

$$\frac{dC}{d\iota} \iota^{\frac{-1}{\beta}} - \frac{1}{\beta} C \iota^{-1-\frac{1}{\beta}} + \frac{C \iota^{-\frac{1}{\beta}}}{\beta \iota} - \frac{\partial F(\iota, \omega, K, \beta)}{\partial N_j(K, \beta)} \frac{1}{\iota} = 0 \tag{25}$$

and simplifying one obtains:

$$\frac{dC}{d\iota} = \iota^{\frac{1-\beta}{\beta}} \frac{\partial F(\iota, \omega, K, \beta)}{\partial N_j(K, \beta)}, \tag{26}$$

and through integration one gets:

$$C_j(\omega, K, \beta) = \int_0^{\iota} z^{\frac{1-\beta}{\beta}} \frac{\partial F(z, \omega, K, \beta)}{\partial N_j(K, \beta)} dz + D, \tag{27}$$

where D is the constant of integration. Given the property that $\lim_{\iota \to 0^+} \iota w_j(\iota, \omega, K, \beta) = 0$, we have that the constant D is identically equal to zero. Therefore the solution to equation (21) satisfies:

$$w_j(\iota, \omega, K, \beta) = (1-\beta) E\left[rOut(a) - \frac{\partial Out(a)}{\partial t} \middle| K, \beta \right] + \\ + \iota^{\frac{-1}{\beta}} \left(\int_0^{\iota} z^{\frac{1-\beta}{\beta}} \frac{\partial F(z, \omega, K, \beta)}{\partial \phi_{fjt}} dz \right) - \beta E[A(j,a)|K, \beta]. \tag{28}$$

Notice that if $\mathbf{N}(K,\beta) = (\iota,\omega)$, then $(z\iota,\omega) = [zN_1(K,\beta), zN_2(K,\beta), \dots, zN_J(K,\beta)] = z\mathbf{N}(K,\beta)$, one can turn equation (28) in:

$$\begin{aligned} w_j(K,\beta) =& (1-\beta)E\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\middle| K,\beta\right] + \\ &+ \int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial F(z\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - \beta E[A(j,a)|K,\beta], \end{aligned} \tag{29}$$

and by doing so fully eliminating the spherical coordinates, which results in the solution of the system of differential equations in equation (12).

Further, from equation (29), one can infer that the equilibrium wages, and the solution of the system of differential equations in equation (13) is given by:

$$\begin{aligned} w_j(a|K,\beta) =& (1-\beta)\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\right] + \\ &+ \int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial F(z\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - \beta A(j,a), \end{aligned} \tag{30}$$

The case of heterogeneous β at firm level. It is helpful to consider a new variable, as does Cahuc et al. (2008). Accordingly, define $\mathbf{M}_j(K,\beta) = \{M_{j,1}(K,\beta), M_{j,2}(K,\beta), \dots, M_{j,J}(K,\beta)\}$, such that:

$$\sum_{l=1}^{J} M_{j,l} \frac{\partial v_l(\mathbf{M}_j,K)}{\partial M_{j,l}} = \sum_{l=1}^{J} \chi_{jl} N_l(K,\beta) \frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)}, \tag{31}$$

with $v_j[M_{j,j}(K,\beta),K] = w_j(K,\beta)$. Also, we assume it holds:

1. $G(\mathbf{M}_j,K) = F(\mathbf{N}(K,\beta),K)$;
2. $M_{j,l} = M_{j,l}(N_l(K,\beta))$;
3. $\frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)} = \frac{\partial v_l(\mathbf{M}_j)}{\partial M_{j,l}} \frac{dM_{j,l}}{dN_l(K,\beta)}$.

For equation (31) to hold it suffices that the following equation to hold:

$$M_{j,l} \frac{\partial v_l(\mathbf{M}_j,K)}{\partial M_{j,l}} = \chi_{jl} N_l(K,\beta) \frac{\partial w_j(K,\beta)}{\partial N_l(K,\beta)}. \tag{32}$$

Given property (3), one obtains a differential equation for $M_{j,l}$, which is given by:

$$M_{j,l} = \chi_{jl} N_l(K,\beta) \frac{dM_{j,l}}{dN_l(K,\beta)}. \tag{33}$$

One feasible solution, not necessarily the only one, corresponds to:

$$M_{j,l} = N_l(K,\beta)^{\frac{1}{\chi_{jl}}} = N_l(K,\beta)^{\chi_{lj}} \tag{34}$$

given $\chi_{lj} = \frac{1}{\chi_{jl}}$. Considering that the mapping between notations, and properties (1) and (2), we have:

$$\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_l(K,\beta)} = \chi_{lj} N_l(K,\beta)^{\chi_{lj}-1} \frac{\partial G(\mathbf{M}_j,K)}{\partial M_{j,l}}, \tag{35}$$

and concretely,

$$\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} = \chi_{jj} N_j(K,\beta)^{\chi_{jj}-1} \frac{\partial G(\mathbf{M}_j,K)}{\partial M_{j,j}} = \frac{\partial G(\mathbf{M}_j,K)}{\partial M_{j,j}}, \tag{36}$$

since $\chi_{jj} = 1$. The system in equation (12) can be expressed as:

$$\begin{aligned} v_j(\mathbf{M}_j,K) = (1-\beta_j)E\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\middle|K,\beta\right] + \\ + \beta_j\left(\frac{\partial G(\mathbf{M}_j,K)}{\partial M_{j,j}} - \sum_{l=1}^{J} M_{j,l}\frac{\partial v_j(\mathbf{M}_j,K)}{\partial M_{j,l}}\right) - \beta_j E[A(j,a)|K,\beta], \end{aligned} \tag{37}$$

which is identical to equation (18). Therefore, following the procedure explained for identical β's, one obtains:

$$\begin{aligned} v_j(\mathbf{M}_j,K) = (1-\beta_j)E\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\middle|K,\beta\right] + \\ + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial G(z\mathbf{M}_j,K)}{\partial M_{j,j}}, K)dz - \beta_j E[A(j,a)|K,\beta] \end{aligned} \tag{38}$$

and translating the transformed variables in the initial notation variables, one realizes equation (38) becomes:

$$\begin{aligned} w_j(K,\beta) =& (1-\beta_j)E\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\middle|K,\beta\right] + \\ &+ \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta} dz - \beta_j E[A(j,a)|K,\beta] \end{aligned} \tag{39}$$

Then, given the definition of the wages for each match fundamentals $\{a,K,\beta\}$, and namely that the heterogeneity arises in operating costs and outside options only, one realize that:

$$\begin{aligned} w_j(a|K,\beta) =& (1-\beta_j)\left[rOut(a) - \frac{\partial Out(a)}{\partial t}\right] + \\ &+ \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - \beta_j A(j,a), \end{aligned} \tag{40}$$

where the matrix $\mathbf{Q}_j(z)$ is a diagonal matrix of the shape:

$$\mathbf{Q}_j(z) = \begin{bmatrix} z^{\frac{\beta_{f1t}}{1-\beta_{f1t}}\frac{1-\beta_{fjt}}{\beta_{fjt}}} & 0 & \dots & 0 \\ 0 & \dots & \dots & 0 \\ 0 & \dots & z^{\frac{\beta_{flt}}{1-\beta_{flt}}\frac{1-\beta_{fjt}}{\beta_{fjt}}} & 0 \\ \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & z^{\frac{\beta_{fJt}}{1-\beta_{fJt}}\frac{1-\beta_{fJt}}{\beta_{fjt}}} \end{bmatrix}. \tag{41}$$

As one can notice, considering heterogeneous β fundamentally change the calculus of the relevant marginal product of labour, and the portion of the idiosyncratic surplus that the worker is capable to extract.

Ex-ante wage posting regime isomorphism. Let us consider the the wage setting problem faced by the firm in equation (16) of the text. For the interior solution this problem can be re-written as:

$$\begin{aligned} &\max_{w} \; [r + s(a|K,\beta)] \times \left[\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a|K,\beta) - A(j,a)\right. \\ &\left.\sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da\right] \times \left[w_j(a|K,\beta) - \left(Out(a) - \frac{\partial Out(a)}{\partial t}\right)\right]^{\tilde{\epsilon}_j(K,\beta)} \end{aligned} \tag{42}$$

which has a first order solution as:

$$\begin{aligned} &-\left[w_j(a|K,\beta) - \left(Out(a) - \frac{\partial Out(a)}{\partial t}\right)\right]^{\tilde{\epsilon}_j(K,\beta)} + \tilde{\epsilon}_j(K,\beta)\left[\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a|K,\beta) - A(j,a)\right. \\ &\left.-\sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da\right]\left[w_j(a|K,\beta) - \left(Out(a) - \frac{\partial Out(a)}{\partial t}\right)\right]^{\tilde{\epsilon}_j(K,\beta)-1} = 0 \end{aligned} \tag{43}$$

which becomes:

$$\begin{aligned} &w_j(a|K,\beta) = \frac{1}{1+\tilde{\epsilon}_j(K,\beta)}\left[Out(a) - \frac{\partial Out(a)}{\partial t}\right] + \frac{\tilde{\epsilon}_j(K,\beta)}{1+\tilde{\epsilon}_j(K,\beta)}\left[\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - A(j,a)\right. \\ &\left.-\sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da\right]. \end{aligned} \tag{44}$$

If one performs a maping between $\tilde{\epsilon}_j(K,\beta)$ and $\tilde{\beta}^P_j$, so that $\tilde{\beta}^P_j = \frac{\tilde{\epsilon}_j(K,\beta)}{1+\tilde{\epsilon}_j(K,\beta)}$, then one has:

$$
\begin{aligned}
w_j(a|K,\beta) = (1-\tilde{\beta}_j^P)\left[Out(a) - \frac{\partial Out(a)}{\partial t}\right] + \tilde{\beta}_j^P \Bigg[\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - A(j,a) \\
- \sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da\Bigg].
\end{aligned}
\tag{45}
$$

This equation is the same as equation (13), thus proving the claimed isomorphism. To identify the equilibrium wage equation, one can follow the procedure already explained in the previous subsections to find the equilbrium wages under ex-post bargaining.

The solution of the aggregate bargaining. Consider the solution of the firm level bargaining, as provided in equation (13) of the text. Accordingly, the average wage for a type a worker is given by:

$$
\begin{aligned}
&E_{K,\beta}[w_j(a|K,\beta)] = (1-\beta_j)\left(rOut(a) - \frac{\partial Out(a)}{\partial t}\right) \\
&+ \beta_j \left[E_{K,\beta}\left(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da\right) - A(j,a)\right],
\end{aligned}
\tag{46}
$$

and therefore, we have that:

$$
\begin{aligned}
E_{K,\beta}[w_j(a|K,\beta)] = \left(rOut(a) - \frac{\partial Out(a)}{\partial t}\right) + \beta_j \Bigg[E_{K,\beta}\Bigg(\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} \\
- \sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da\Bigg) - A(j,a) - \left(rOut(a) - \frac{\partial Out(a)}{\partial t}\right)\Bigg],
\end{aligned}
\tag{47}
$$

$$
E_{K,\beta}[w_j(a|K,\beta)] = \left(rOut(a) - \frac{\partial Out(a)}{\partial t}\right) + \beta_j E_{K,\beta}[QR(a)], \tag{48}
$$

and consequently, once we impose the constraint of zero expected quasi-rents, we have:

$$
w_{j,t}^{MIN}(a) = rOut(a) - \frac{\partial Out(a)}{\partial t}. \tag{49}
$$

Therefore, the solution of the aggregate bargaining in equation (10) of the text is equation (55). Notice, that the key to identify this solution is to realize that an identical derivation to the one presented in equations (1) to (10) of this appendix can be easily computed for the average firm in the market. Indeed if equation (10) holds for every single firm, it also holds on average for each type a worker. Then one needs just to consider the additional constraint presented in the aggregate bargaining versus the firm bargaining, namely the absence of quasi-rents.

A4 - The Dynamic and Steady State Equilibria.

Additional Assumptions for dynamic equilibrium. At this stage we consider two additional technical assumptions, as follows:

- **Bounded expression** - $\lim_{N_j \to} w_j(a|K,\beta)N_j = 0$;
- **Smoothness of Production Function** - $F(\mathbf{N}(K,\beta),K)$ is continuous for all $N_j > 0$, and infinitely differentiable for all $N_j > 0$. Further, $N_j \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j}$, and the quantity $N_1^{m_1} \dots N_J^{m_J} \frac{\partial^2 F}{\partial N_1^{m_1} \partial \dots} N_J^{m_J}$, or simply $\mathbf{N^m F^{(m)} N}(N,\beta)$, with $\bar{m} \sum_{j=1}^{J} m_j$ is continuous at zero.

These assumptions are fairly technical to ensure there exists an equilibrium wage function that is smooth in all $N_j > 0$, and unique. Altogether, the dynamic equilibrium of the model is defined as follows:

Theorem 1 (Dynamic Equilibrium). *A tuple*

$$\Big\{\theta_j(t), Out(a), G(a|K,\beta), dG(a|K,\beta), J(a|K,\beta), \Xi(a|K,\beta), w_j(a|K,\beta), \xi_j^u(a), \\ \xi^{\Xi}(a|K,\beta), s_j(a|K,\beta), m_j(a|K,\beta), y_j(a|K,\beta), V_j(K,\beta), d\aleph(a)\Big\} \tag{1}$$

is a dynamic equilibrium if for all t, the following statements are jointly satisfied:

- *$J(\cdot)$, $Out(\cdot)$ and $\Xi(\cdot)$ satisfy HJB equations (5), (6) and (7) of the text;*
- *Vacancy Posting is optimal so it holds equation (8) and equation (18) of A2;*
- *$G(a|K,\beta)$ has a density $dG(a|K,\beta)$ satisfying equation (22) of A2;*
- *Job search is optimal so it solves the problems in equations (12) and (13) of A2;*
- *$s_j(a|K,\beta)$ holds equation (11) and $m_j(a|K,\beta)$ holds equation (17) of A2;*
- *The vacancy yield holds equations (20) and (21) of A2;*
- *The market tightness hold equation (19) of A2, and equation (1) of the text;*
- *The unemployed distribution $dU(a)$ and the distribution of workers $d\aleph(a)$ follow equations (7) of A2;*
- *The equilibrium wage satisfies equations (10), (12) and (16) of the text.*

Core definitions and assumptions of the steady state equilibrium. In the model one can define a steady state equilibrium where all aggregate variables are constant over time, and where wages and the vacancy-posting strategies of firms depend only on firm's fundamentals (K, β). Let us define a level a^R such that:

$$J_j^R(K, \beta) = J_j(a^R|K, \beta), \tag{2}$$

so that it is the level of skill that is compatible with the expected marginal profit profit of the firm with fundamentals (K, β). Thus consistent with equation (16) of the text, we have:

$$\begin{aligned} rJ_j(a^R|K,\beta) - \frac{\partial J_j(a^R|K,\beta)}{\partial t} &= \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a^R|K,\beta) - A(j,a^R) \\ - \sum_{l=1,l\neq j}^{J} \int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da &- s_j(a^R|K,\beta)J_j(a^R|K,\beta) + \\ + \sum_{l=1}^{J} \left\{ y_l(K,\beta)V_l(K,\beta) - \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da \right\} &\frac{\partial J_j(a^R,K,\beta)}{\partial N_l(K,\beta)}. \end{aligned} \tag{3}$$

Given the steady state equilibrium imposes stability of aggregate variables, there is stability of the workforce, namely:

$$y_l(K,\beta)V_l(K,\beta) = \int_a s_l(a|K,\beta)dG_l(a|K,\beta)da, \tag{4}$$

and, also consider that:

$$\frac{\partial J_j(a^R|K,\beta)}{\partial t} = \frac{\partial \Xi_j(a^R|K,\beta)}{\partial t} = \frac{\partial Out(a^R)}{\partial t} = 0. \tag{5}$$

Through a process identically presented in *The system of Differential equations for equilibrium wages in the ex-post bargaining regime* part of appendix A3, we have:

$$\begin{aligned} (r+\bar{s})J_j(a^R|K,\beta) &= \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - w_j(a^R|K,\beta) \\ - \sum_{l=1,l\neq j}^{J} \int_a &\frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da - A(j,a^R). \end{aligned} \tag{6}$$

The Vacancy Curve. Given by assumption $\bar{s} > 0$, then we have that $V(K, \beta) > 0$ in a steady state equilibrium. Thus, given equation (10) of appendix A2, we have:

$$J_j^R(K,\beta) = \frac{\gamma_j}{q(\theta_j)}. \tag{7}$$

Consequently:

$$\underbrace{\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - A(j,a^R)}_{\text{Marginal Product net of op. costs}} = \underbrace{w_j(a^R|K,\beta)}_{\text{Wage}} + \underbrace{(r+\bar{s})\frac{\gamma_j}{q(\theta_j)}}_{\text{Turnover Costs}} + \\ + \underbrace{\sum_{l=1,l\neq j}^{J}\int_a \frac{\partial w_l(a|K,\beta)}{\partial N_j(K,\beta)} dG_l(a|K,\beta)da}_{\text{Employment effect on wages}}. \tag{8}$$

This result is typical in steady-state equilibria of search and matching models, and intuitively entails that the expected marginal worker produces on the margin the value of the cost of hiring such worker.[14] Following similar steps to the ones presented to solve this system, we have:

$$\underbrace{\frac{\int_0^1 \frac{1}{\beta} z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz}{\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}}}_{\text{Overemployment Effect - } OE_j(K,\beta)} \frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - A(j,a^R) = \underbrace{w_j(a^R|K,\beta) + (r+\bar{s})\frac{\gamma_j}{q(\theta_j)}}_{\text{Labour costs}}$$

$$OE_j(K,\beta)\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - A(j,a^R) = w_j(a^R|K,\beta) + (r+\bar{s})\frac{\gamma_j}{q(\theta_j)}. \tag{9}$$

Considering the wage equation with the assumptions identified in equations (5) and (6), and the definition of employment effect, we have:

$$w_j(a^R|K,\beta) = (1-\beta_j)\sum_{j=1}^{J}\xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\} \\ + \int_0^1 z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - \beta A(j,a^R) \\ w_j(a^R|K,\beta) = (1-\beta_j)\sum_{j=1}^{J}\xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\} \\ + \beta OE_j(K,\beta)\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \beta A(j,a^R). \tag{10}$$

Joining equations (9) and (10) so that one eliminates wages, we have:

$$OE_j(K,\beta)\frac{\partial F(\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - A(j,a^R) = \\ \sum_{j=1}^{J}\xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\}, \tag{$VC_j(K,\beta)$}$$

[14]For instance, Cahuc et al. (2008) finds an identical equation in their equation (9).

and equivalently:

$$\int_0^1 \frac{1}{\beta_j} z^{\frac{1-\beta_j}{\beta_j}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} dz - A(j,a^R) =$$
$$\sum_{j=1}^{J} \xi_j^u(a) \Bigg\{ b + \theta_j q(\theta_j) \Bigg(\frac{\int_K \int_\beta \Xi_l(a^R|K,\beta) V_l(K,\beta) d\Gamma(K,\beta)}{\int_K \int_\beta V_l(K,\beta) d\Gamma(K,\beta)} - Out(a^R) \Bigg) - c_l \Bigg\}. \tag{$VC_j(K,\beta)$}$$

Those equations corresponds to the vacancy curves of the firm with fundamentals (K,β) - $VC_j(K,\beta)$.[15] Notice that the right-hand side of the vacancy curve is unambiguously increasing in θ.

Lemma on Profit of firms and wages. The flow profit

$$r\Pi(K,\beta) - \frac{\partial \Pi(K,\beta)}{\partial t} = \underbrace{F(\mathbf{N}(K,\beta);K)}_{\text{Production}} - \underbrace{\sum_{j=1}^{J} \int_a w_j(a|K,\beta) dG_j(a|K,\beta) da}_{\text{Wage Bill}}$$
$$- \underbrace{\sum_{j=1}^{J} \int_a A(j,a) dG_j(a|K,\beta) da}_{\text{Operating Cost Bill}} - \underbrace{I(K)}_{\text{Sunk Cost}} - \underbrace{\sum_{j=1}^{J} \int_a s_j(a|K,\beta) J_j(a|K,\beta) dG_j(a|K,\beta) da}_{\text{Cost of Displacement}} +$$
$$+ \underbrace{\sum_{j=1}^{J} \max_{V_j(K,\beta)} \Big\{ -\gamma_j V_j(K,\beta) + V_j(K,\beta) q(\theta_j) J_j^R(K,\beta) \Big\}}_{\text{Value of the Hiring Policy}}. \tag{12}$$

is continuous, strictly concave and satisfies:

$$\lim_{\mathbf{N}\to\mathbf{0}^+} \Pi(K,\beta) = 0. \tag{13}$$

Then, given:

$$\lim_{N_j(K,\beta)\to\infty} \frac{\partial F(\mathbf{N}(K,\beta))}{\partial N_j(K,\beta)} < b, \tag{14}$$

assumed in the model, it implies that:

$$\lim_{N_j(K,\beta)\to\infty} \Pi(K,\beta) = -\infty, \tag{15}$$

[15] Notice that as typically we have:

$$\begin{aligned} &\lim_{x\to 0^+} q(x) = \infty; \;\; \lim_{x\to\infty} q(x) = 0; \;\; \frac{\partial q(\theta)}{\partial \theta} < 0 \\ &\lim_{x\to 0^+} xq(x) = 0; \;\; \lim_{x\to\infty} xq(x) = \infty; \;\; \frac{\partial \theta q(\theta)}{\partial \theta} > 0. \end{aligned} \tag{11}$$

implying that the optimal workforce size vector $\mathbf{N}(K,\beta)$ is finite in every occupation for every firm. Further notice that

$$\lim_{N_j(K,\beta)\to\infty} w_j(a^R|K,\beta) = \sum_{j=1}^{J}\xi_j^u(a)\Big\{b+\theta_j q(\theta_j)\Big(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\Big) - c_l\Big\}+ \\ + \lim_{N_j(K,\beta)\to\infty}\int_0^1 z^{\frac{1-\beta_j}{\beta_j}}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz - \beta_j A(j,a^R)$$

$$\lim_{N_j(K,\beta)\to 0^+} w_j(a^R|K,\beta) = \sum_{j=1}^{J}\xi_j^u(a)\Big\{b+\theta_j q(\theta_j)\Big(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\Big) - c_l\Big\}+ \\ + \lim_{N_j(K,\beta)\to 0^+}\int_0^1 z^{\frac{1-\beta_j}{\beta_j}}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)}dz - \beta_j A(j,a^R) \tag{16}$$

is identical to

$$\lim_{N_j(K,\beta)\to\infty} w_j(a^R|K,\beta) = \sum_{j=1}^{J}\xi_j^u(a)\Big\{b+\theta_j q(\theta_j)\Big(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\Big) - c_l\Big\}+ \\ + \lim_{N_j(K,\beta)\to\infty}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \beta_j A(j,a^R)$$

$$\lim_{N_j(K,\beta)\to 0^+} w_j(a^R|K,\beta) = \sum_{j=1}^{J}\xi_j^u(a)\Big\{b+\theta_j q(\theta_j)\Big(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\Big) - c_l\Big\}+ \\ + \lim_{N_j(K,\beta)\to 0^+}\frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K)}{\partial N_j(K,\beta)} - \beta_j A(j,a^R). \tag{17}$$

Therefore, given the production function is strictly concave and displaying decreasing returns to scale, one concludes, given

$$\sum_{j=1}^{J}\xi_j^u(a)\Big\{b+\theta_j q(\theta_j)\Big(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\Big) - c_l\Big\} > 0, \tag{18}$$

by assumption, that wages are strictly positive and strictly decreasing with firm size. As noted by Acemoglu and Hawkins (2014), given

$$\lim_{N_j(K,\beta)\to 0^+} w_j(a^R|K,\beta) = +\infty \tag{19}$$

the level of employment that maximizes the flow profit is strictly positive.

Considerations on steady-state equilibrium. Given the shape of the profit function of a firm, precisely: (a) $\lim_{\mathbf{N}\to\mathbf{0}^+}\Pi(K,\beta)=0$; (b) $\Pi(K,\beta)$ is strictly concave on employment; and (c) $J_j^R(K,\beta)$ is strictly decreasing in employment, and given

$$J_j^R(K,\beta) = \frac{\gamma_j}{q(\theta_j)}, \tag{20}$$

then equation (20) has a unique vector of employment $\mathbf{N}(K,\beta)$ conditional on the values of the endogenous variables.

The steady-state condition The steady state equilibrium concept offers a greater simplification to our framework, which arises from imposing stability of aggregate flows at firm level. Thus, the stability of aggregate flows, equation (23) of appendix A2, and equation (5) jointly yield:

$$\begin{aligned} 0 = & - dG_j(a|K,\beta) + y_j(a|K,\beta)V_j(K,\beta) + \\ & + \int_{a''} \Big(1 - s_j(a''|K,\beta)\Big)\psi(a|a'')dG_j(a''|K,\beta)da'' \end{aligned} \tag{21}$$

which after further simplification becomes:

$$dG_j(a|K,\beta) = \frac{\int_{a''} \Big(1 - s_j(a''|K,\beta)\Big)\psi(a|a'')dG_j(a''|K,\beta)da''}{1 - s_j(a|K,\beta)} \tag{22}$$

Accordingly, in the steady state equilibrium we hold that:

$$G_j(a|K,\beta) = \int_{-\infty}^{a} \frac{\int_{a''} \Big(1 - s_j(a''|K,\beta)\Big)\psi(a|a'')dG_j(a''|K,\beta)da''}{1 - s_j(a|K,\beta)} da. \tag{23}$$

Given the stability of the workforce in each workplace economy-wide, we therefore also can hold that:

$$\int_K \int_\beta y_j(K,\beta)V_j(K|\beta)dKd\beta = \int_K \int_\beta \int_a s_j(a|K,\beta)dG_j(a|K,\beta)dadKd\beta. \tag{24}$$

Steady state equilibrium description. Therefore, the steady-state equilibrium is a specialization of the dynamic equilibrium presented with the following properties:

Theorem 2 (Steady-State Equilibrium)**.** *A tuple*

$$\begin{aligned} &\Big\{\theta_j(t), Out(a), G(a|K,\beta), dG(a|K,\beta), J(a|K,\beta), \Xi(a|K,\beta), w_j(a|K,\beta), \xi_j^u(a), \\ &\xi^{\Xi}(a|K,\beta), s_j(a|K,\beta), m_j(a|K,\beta), y_j(a|K,\beta), V_j(K,\beta), d\aleph(a)\Big\} \end{aligned} \tag{25}$$

is a steady state equilibrium if for $q(\theta) > 0$ and $\bar{s} > 0$, the following statements are jointly satisfied:

- *$J(\cdot)$, $Out(\cdot)$ and $\Xi(\cdot)$ satisfy HJB equations (5), (6) and (7) of the text;*
- *Vacancy Posting is optimal so it holds equation (8) and equation (18) of A2;*
- *$G(a|K,\beta)$ has a density $dG(a|K,\beta)$ satisfying equation (22) of A2, and equation (23);*

- *Job search is optimal so it solves the problems in equations (12) and (13) of A2;*

- $s_j(a|K,\beta)$ *holds equation (11), and* $m_j(a|K,\beta)$ *holds equation (17) of A2;*

- *The vacancy yield holds equations (20), and (21) of A2;*

- *The market tightness hold equation (19) of A2, and equation (1) of the text;*

- *The unemployed distribution* $dU(a)$ *and the distribution of workers* $d\aleph(a)$ *follow equation (7) of A2;*

- *The equilibrium wage satisfies equations (10), (12) and (16) of the text;*

- *The steady state conditions of equation (4) and equation (24);*

- *The stability of expectations of HJB functions in equation (5).*

Note that the the distribution of skill within workplaces satisfies an ergodicity condition and thus $G_j(a|K,\beta)$ is unique. So there is no loss of generality to apply such distribution which is assumed to be continuously differentiable. Note that $\bar{s} > 0$, which is partially justified by the death and birth shocks d. Note, that to ease the technical explanation on ergodicity, one can reason such shocks as a massive destructive shock on skill, which leads the worker to have his skill extracted from $\Psi_0(a)$, rather than having the worker dying and a new worker entering the market. Thus, we arive at the uniqueness of the invariant distribution through Theorem 11.9 of Stokey et al. (1989).[16]

B - Identification Strategy Appendix

B1 - First Step of the Empirical Model

The equilibrium wage expression with heterogeneous bargaining powers is given by:

$$w_j(a|K,\beta) = (1-\beta_j)Out^{\star}(a) + \int_0^1 z^{\frac{1-\beta}{\beta}} \frac{\partial F(\mathbf{Q}_j(z)\mathbf{N}(K,\beta),K}{\partial N_j(K,\beta)} dz - \beta_j A(j,a). \tag{1}$$

Let us focus in the dynamic behaviour of $Out^{\star}(a)$ function.

The time effect on outside Options. Notice that we have that the outside option is given by:

$$Out(a) = E_t[Out(a)] + \Big[Out(a) - E_t[Out(a)]\Big], \tag{2}$$

[16]An identical argument is used in Acemoglu and Hawkins (2014).

where $E_t[Out(a)]$ is the expected outside option of worker with skill level a. With standard algebraic manipulations one obtains:

$$Out(a) = E_t[Out(a)]\left[1 + \frac{Out(a) - E_t[Out(a)]}{E_t[Out(a)]}\right], \tag{3}$$

and considering a first order Taylor approximation, we have:

$$ln[Out(a)] = ln\left[E_t[Out(a)]\right] + \frac{Out(a) - E_t[(Out(a)]}{E_t[Out(a)]}. \tag{4}$$

Inside the expected value of the outside option. We have that the expected value of the outside option of the worker is given by:

$$ln\left[E_t(Out(a))\right] = ln\left\{E_t\sum_{j=1}^{J}\xi_j^u(a)\left\{b + \theta_j q(\theta_j)\left(\frac{\int_K\int_\beta \Xi_l(a^R|K,\beta)V_l(K,\beta)d\Gamma(K,\beta)}{\int_K\int_\beta V_l(K,\beta)d\Gamma(K,\beta)} - Out(a^R)\right) - c_l\right\}\right\}. \tag{5}$$

At this point, we consider a first order Taylor approximation around the initial value of a for each worker. Thus:

$$E_t\left(Out(a)\right) = E_t\left(Out(a_{\tau_0(i)})\right) + \frac{\partial E_t(Out(\tilde{a}))}{\partial a}\bigg|_{\tilde{a}\in[a_{\tau_0(i);a]}}\left(a - a_{\tau_0(i)}\right), \tag{6}$$

where $a_{\tau_0(i)}$ represents the skill value of the worker in the moment he enters the labour market.

Moreover, let us consider the expected value of skill a worker with a in the current period t should have had in the first period of her current contract - $E_t[a_{\tau_0(i)}|a]$. Consequently, equation 6 becomes:

$$\begin{aligned} E_t\left(Out(a)\right) = {} & E_t\left(Out(a_{\tau_0(i)})\right) + \frac{\partial E_t(Out(\tilde{a}))}{\partial a}\bigg|_{\tilde{a}\in[a_{\tau_0(i);a]}}\left(a - E_t[a_{\tau_0(i)}|a]\right) + \\ & + \frac{\partial E_t(Out(\tilde{a}))}{\partial a}\bigg|_{\tilde{a}\in[a_{\tau_0(i);a]}}\left(E_t[a_{\tau_0(i)}|a] - E_t[a_{\tau_0(i)}]\right), \end{aligned} \tag{7}$$

and after a first order Taylor approximation around the logarithm of expected value of outside option, one obtains:

$$\begin{aligned} ln\left[E_t\left(Out(a)\right)\right] \approx {} & ln\left[E_t\left(Out(a_{\tau_0(i)})\right)\right] + \\ & + \frac{1}{E_t[Out(a_{\tau_0(i)})]}\frac{\partial E_t(Out(\tilde{a}))}{\partial a}\bigg|_{\tilde{a}\in[a_{\tau_0(i);a]}}\left(a - E_t[a_{\tau_0(i)}|a]\right) + \\ & + \frac{1}{E_t[Out(a_{\tau_0(i)})]}\frac{\partial E_t(Out(\tilde{a}))}{\partial a}\bigg|_{\tilde{a}\in[a_{\tau_0(i);a]}}\left(E_t[a_{\tau_0(i)}|a] - E_t[a_{\tau_0(i)}]\right). \end{aligned} \tag{8}$$

Functional form of empirical model for outside options. Combining equations

(4) and (8), we have:

$$
\begin{aligned}
ln[Out(a)] = & \underbrace{\frac{Out(a) - E_t[Out(a)]}{E_t[Out(a)]}}_{\text{Occupation-market-time effect - } \lambda_{j,t}} \\
& \underbrace{+ ln\left[E_t\left(Out[a_{\tau 0(i)}]\right)\right] + \frac{1}{E_t[Out(a_{\tau 0(i)})]} \frac{\partial E_t(Out(\tilde{a}))}{\partial a}\bigg|_{\tilde{a} \in [a_{\tau 0(i);a}]} \left(a - E_t[a_{\tau 0(i)}|a]\right) +}_{\substack{\text{expected contract profile -} \\ \psi(age_{i,t}, rank_{i,t}, female_i)}} \\
& \underbrace{+ \frac{1}{E_t[Out(a_{\tau 0(i)})]} \frac{\partial E_t(Out(\tilde{a}))}{\partial a}\bigg|_{\tilde{a} \in [a_{\tau 0(i,f);a}]} \left(E_t[a_{\tau 0(i)}|a] - E_t[a_{\tau 0(i)}]\right)}_{\text{disturbance - } v^S_{i,t}} .
\end{aligned}
\tag{9}
$$

where we explicitly introduce a parallel trend assumption, namely:

$$
\frac{Out(a) - E_t[Out(a)]}{E_t[Out(a)]} = \frac{Out - E_t[Out]}{E_t[Out]}(j,t) = \lambda_{j,t}. \tag{10}
$$

Intuitively, one is assuming that the evolution of outside option value of every worker of a given type a within a occupation j is identical.

The reduced form of Outside Options and measurement error. Altogether, we therefore have that the reduced form representation, as presented in equation (19) of the text, is given by:

$$
ln[w^{MIN,\star}_{i,t}{}_{i,t}] = \underbrace{\lambda_{j,t} + \psi(age_{i,t}, rank_{i,t}, female_i)}_{\widehat{Out(a)}} + v^S_{i,t}. \tag{11}
$$

Moreover, following the insight of Pei et al. (2018), notice that in the case of existence of a classical measurement error in $w^{MIN}_{i,t}$, so that:

$$
w^{MIN}_{i,t} = w^{MIN,\star}_{i,t}{}_{i,t} + \eta_{i,t}, \tag{12}
$$

where $\eta(i,t)$ is a classical measurement error, with the following properties:

1. $E[\eta_{i,t}] = 0$;
2. $E[\lambda_{j,t}\eta_{i,t}] = 0$;
3. $E[\psi(age_{i,t}, rank(i,t), female_i)\eta_{i,t}] = 0$;
4. $E[v^S_{i,t}\eta_{i,t}] = 0$.

Consequently:

$$ln[w_{bargain}] = \underbrace{\lambda_{j,t} + \psi(age_{i,t}, rank_{i,t}, female_i)}_{ln[\widehat{Out^{\star}}(a)]} + \underbrace{v^S_{i,t} - \eta_{i,t}}_{v_{i,t}}, \tag{13}$$

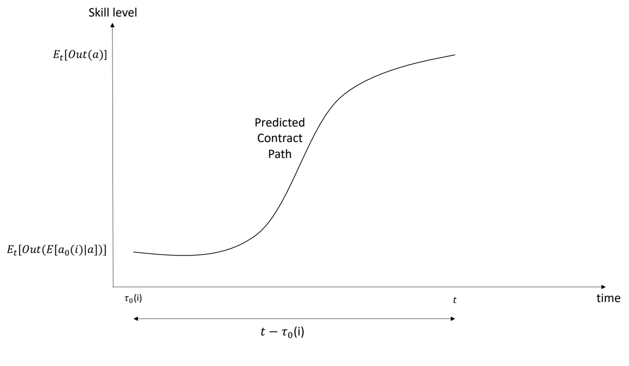
with $v_{i,t}$ corresponding to the composite error term.

In a nutshell, the first stage of our empirical implementation, beyond providing empirical structure to our estimation, also provides relevant answer to the existence of measurement error, particularly given $w_{bargain}$ corresponds to a proxy. As long as the measurement error is classical, it only has efficiency impacts, and not on the consistency of the estimates. Given the high dimensionality of our data, naturally efficiency of the estimator does not lie in the top of our priorities.

Intuition on the expected contract profile. We take advantage of the knowledge of: (i) the actual rank of the worker, which is linked with $E_t[Out(a)]$, apart from the trend behaviour; (ii) the experience of the worker, given by $age_{i,t}$, so that we are capable to estimate the predicted contract path of each worker; and (iii) we allow for heterogeneous contract profiles by gender.

The identification of the predicted contract path enables the estimation of a time-occupation effect, so that it controls for any time trend. Altogether, we are bunching the information of the workers sharing the same contract at collective agreement level (i.e. experience, actual rank and gender), and thus we improve our position to better value the individual-occupation effect. Accordingly our identification follows the intuition of figure B1.

Figure B1: Structure of the Model in each moment t.



B2 - Heuristic Comparison with AKM on Parameter Dimmensionality, Sample and Mobility Restrictions

In this appendix, we perform an heuristic comparison between our empirical model with standard AKM models, over two dimensions: (a) parameter dimensionality; (b) sample and mobility restrictions.

We use the collective bargaining ranking to discipline our empirical knowledge of workers' types, and the average wage in the workplace, i.e. firm-occupation-time, to discipline our empirical knowledge of the workplace type. This represents a deviation to an AKM approach, and results in a model that do not need to impose any largest connected set requirement. It also do not rely on workplace movers to estimate coefficients, and will not estimate an high dimensional fixed effect parameter set, thus resulting in a much more parsimonious empirical setting.

Table 1: Goodness-of-Fit and Parameter Dimensionality Comparison

Panel A: Our model - 1^{st} Stage

Heterogeneous Slopes and Intercepts	Number of Parameters
Wage Rank - Age function - Female	606,696
Occupation - Year	63
Adjusted R^2	0.9371
N. Obs.	29,586,448

Panel B: Our model - 2^{nd} Stage

Heterogeneous Slopes and Intercepts	Number of Parameters
Bargaining power	63
Operating costs	9
Adjusted R^2	0.8675
N. Obs.	29,586,448

Panel C: AKM with firm-worker-time FE

Heterogeneous Slopes and Intercepts	Number of Parameters
Worker	3,662,504
Firm	127,930
Year	20
Tenure and age polynomials	6
Adjusted R^2	0.8626
N. Obs.	28,725,252

Panel D: AKM with workplace-worker-time FE

Heterogeneous Slopes and Intercepts	Number of Parameters
Worker	3,608,164
Workplace (firm, occupation, time)	1,803,878
Year	20
tenure and age polynomials	6
Adjusted R^2	0.9068
N. Obs.	28,060,307

Sources: Quadros de Pessoal, 1995-2009 and Relatório Único, 2010-2016.

To have a grasp on the referred differences, we implement a parameter dimensionality and goodness-of-fit comparison of our model and two classical AKM alternatives. In detail, the AKM models are: (a) a firm-worker-year fixed effect formulation, in the spirit of Card et al. (2014) and Song et al. (2019); and (b) the worker-workplace two dimensional fixed effect. Notice that the workplace dimension corresponds to firm, occupation and time cells so that it resembles the dimension of analysis we use in this study. Algebraically, one can represent both models as:

$$ln(w_{ift}) = \alpha_i + \phi_t + \psi_f + \mathbf{Q}\delta_1 + \epsilon_{ift}, \tag{1}$$

$$ln(w_{ift}) = \alpha_i + \eta_{f,j,t} + \mathbf{Q}\delta_2 + \epsilon_{ift}. \tag{2}$$

We will use a $3rd$ order polynomial on tenure and the second and third powers of age as controls in matrix $\mathbf{Q}$, as gender and education are enclosed in the worker fixed effect, and age is not separately identified from the worker fixed effect and the time effect, as referred by Card et al. (2018).

In table 1, we present the results for the same dataset composed of 29,586,448. First, notice that largest connected set requirements cost around 3 percent of the sample in the case of standard AKM and around 5 percent in the case of the workplace-worker AKM. Further, in the combination of both steps, we have estimated 16 and 11 percent of the parameters in those AKM models, respectively.

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