

Equity Issuance Methods and Dilution

Mike Burkart

London School of Economics, UK, and Swedish House of Finance, Sweden, CEPR, UK, and ECGI, Belgium

Hongda Zhong

London School of Economics and CEPR, UK

We analyze rights and public offerings when informed shareholders strategically choose to subscribe. Absent wealth constraints, rights offerings achieve the full information outcome and dominate public offerings. When some shareholders are wealth constrained, rights offerings lead to more dilution of their stakes and lower payoffs, despite the income from selling these rights. In both rights and public offerings, there is a trade-off between investment efficiency and wealth transfers among shareholders. When firms can choose the flotation method, either all firms choose the same offer method or high and low types opt for rights offerings, while intermediate types select public offerings. (*JEL* G32)

Received September 23, 2021; editorial decision June 8, 2022 by Editor: Andrew Ellul.

Public companies undertake seasoned equity offerings (SEOs) to raise new equity capital from current shareholders and new investors. Broadly speaking, SEOs can be classified into two modes: public offerings and

The Review of Corporate Finance Studies 12 (2023) 78-130

© The Author(s) 2022. Published by Oxford University Press on behalf of The Society for Financial Studies. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (https://creativecommons.org/licenses/by/4.0/), which permits unrestricted reuse, distribution, and reproduction in any medium, provided the original work is properly cited.

https://doi.org/10.1093/rcfs/cfac029 Advance Access publication 22 August 2022

We thank Ulf Axelson, Jonathan Berk, Max Bruche, Andrew Ellul (editor), Julian Franks, Pedro Gete, Benjamin Hébert, Tingjun Liu, Evgeny Lyandres, Andrey Malenko, Martin Oehmke, and Giorgia Piacentino, and conference/seminar participants at 15th CSEF-IGIER Symposium on Economics and Institutions, China International Conference in Finance, Endless Summer Conference on Financial Intermediation and Corporate Finance, European Finance Association Annual Meeting, European Financial Intermediation Theory Conference, European Summer Symposium in Financial Markets, European Winter Finance Summit, LSE, NYU-Shanghai, Finance Theory Group (FTG) Summer Conference (LBS), FTG Early Idea Session (Northwestern), Olin/CFAR Corporate Finance Conference, SFS Cavalcade North America Conference, Tsinghua PBC, University of Colorado -Boulder, the Wellington Finance Summit, Norwegian School of Economics, the Chinese University of Hong Kong, the Chinese University of Hong Kong (Shenzhen), University of Florida, the University of Hong Kong, the University of North Carolina at Chapel Hill, and Duke University for valuable discussions and comments. Yingxuan Yang and Yang Guo provided excellent research assistance. Burkart acknowledges financial support from the Economic and Social Research Council [ESRC Research Grant ES/S016686/1]. Zhong gratefully acknowledges the support of the Economic and Social Research Council [ESRC New Investigator Grant: ES/T003758/1]. Send correspondence to Mike Burkart, m.c. burkart@lse.ac.uk.

rights offerings.¹ In public offerings, firms announce the issue size, and both current shareholders and new investors can subscribe.² In addition, the firm may offer current shareholders some guaranteed allocation of the newly issued shares up to their fractional ownership, which we refer to as dilution protection. In rights offerings, firms announce the issue size and offer short-term in-the-money call options, that is, rights, to current shareholders on a pro rata basis. Current shareholders receive the rights for free and decide whether to exercise them and receive new shares. Typically, rights can be sold to other investors who then exercise them. The total issue proceeds equal the strike price times the number of rights (or, equivalently, shares) issued.

Information asymmetries among the participants represent a major friction in capital markets, and they can lead to mispricing. Such mispricing is a particularly important concern for shareholders and investors at the time when new shares are issued (e.g., Myers and Majluf 1984). On the one hand, shareholders fear that their holdings get diluted by underpricing new shares. On the other hand, prospective investors worry that they may end up purchasing overpriced shares.

As we will show, there is a simple solution to the informational friction: a rights offering with a sufficiently low strike price, such that even the most pessimistic shareholders exercise their rights. If all current shareholders exercise their rights, their fractional ownership in the firm remains unchanged and no shares are issued to new investors. Accordingly, any dilution to the existing shares caused by the low strike price is exactly offset by the gains on the new shares. Consequently, all shareholders receive the full information payoff, regardless of any potential informational asymmetries among market participants or among shareholders. In contrast, a public offering always generates some wealth transfer among shareholders and investors because new shares are sold to investors at a premium or discount.

This suggests that rights offerings dominate public offerings in the sense that the former can avoid wealth transfers. In addition, rights offering have lower direct floating costs than public offerings (Smith 1977; Eckbo, Masulis, and Norli 2007). However, evidence shows that rights offerings are infrequent in the United States (e.g., Eckbo, Masulis, and Norli 2007). Outside of the United States, rights offerings are more common but are often not the predominate issue mode (Massa et al. 2016).

¹ A third way to raise equity financing is private placements in which new shares are sold to a small group of qualified investors. We are interested in equity issuance methods in which the share price is determined in competitive markets. Therefore, we do not analyze private placements. Though as we will briefly discuss in Section 6.5, private placements can be viewed as a special case of public offerings.

² In practice, issuing firms are typically assisted by underwriters who provide certification and possibly commitment to purchase all shares not taken up by investors (e.g., Eckbo and Masulis 1992). As will be discussed later, we abstract from underwriters.

This so-called "rights puzzle" has been explained with adverse selection problems, which are mitigated in public offerings through underwriter certification (e.g., Eckbo and Masulis 1992).³ Nonetheless, a fundamental question remains: why do firms bother with underwriter certification in public offerings given that rights offerings can circumvent the information problem?

In this paper, we relax one crucial assumption that allows rights offerings to resolve the information problem, namely, that all current shareholders have the (liquid) resources needed to exercise their rights. In other words, we assume that shareholders have different degrees of wealth constraints, and the combined (liquid) capital of informed as well as of all shareholders is insufficient to finance the entire issue. As a result, some uninformed new investors need to participate and the equity issue is plagued by adverse selection problems. The relevance of informational frictions in equity markets is widely accepted, in both the theoretical and empirical literatures (e.g., Myers and Majluf 1984; Rock 1986; Ritter 1987). To simplify the exposition, we assume that the more financially constrained shareholders have no (liquid) resources and therefore can neither subscribe to new shares in public offerings nor exercise their rights in rights offerings. Less-constrained shareholders can buy shares or exercise rights allocated to them on a pro rata basis. We call the former group constrained and the latter group unconstrained shareholders. The remaining shares are purchased by competitive uninformed new investors.

The aim of the paper is to compare public and rights offering in this setting with information asymmetries and some wealth-constrained shareholders. We intentionally do not engage with the security design problem of deriving the optimal selling procedure in this setting.⁴ Instead, we focus on two equity financing methods widely used in practice, and we explore whether they differ and how issue mode and terms affect wealth transfers among current shareholders as well as investment efficiency.

In the main model, we assume that there is asymmetric information only about the net present value (NPV) of the new investment project, while in an extension (Section 5) we consider the case with uncertainty about the assets in place. In the former setting, shareholders always benefit if the firm invests. This may not hold in the latter setting, which requires one to determine which firm types actually want to issue and invest as in Myers and Majluf (1984). We choose the former setting as

³ The related literature discusses other explanations for the choice of issue mode.

⁴ As in Myers and Majluf (1984), debt would be the optimal security in our setting. Still, the widely documented stock price reactions following equity issuances show that firms issue equity despite or also in the presence of asymmetric information problems.

our main one to focus on the novel feature of our model, namely, the strategic participation of informed unconstrained shareholders. We postpone the analysis of the underinvestment problem to Section 5, where we focus on our novel trade-off between investment efficiency and wealth transfers among shareholders.

The comparison of public and rights offerings in our main setting reveals a surprising results: constrained shareholders fare better in public offerings than in rights offerings, even though they obtain proceeds from selling their rights, but receive no (extra) compensation in a public offering. Intuitively, rights have a positive value only if the strike price is lower than the equilibrium price in a public offering. Such a lower strike price implies that more new shares must be issued in a rights offering to fund the investment. However, rights are priced at a discount on average because of the winner's curse problem, similar to Rock (1986). This implies more dilution to the existing holdings, which is not fully compensated by the proceeds from selling the rights.

Next, we analyze how firms choose offer mode and terms when knowing their type (Section 3). The choice of flotation method may therefore serve as a signal to uninformed investors. For this signaling game, we assume that firms maximize the total payoff to all current shareholders, or equivalently, minimize the payoff to new investors. As we show, only two kinds of equilibria can exist: The first kind is a pooling equilibrium in which all firms choose the same dilution protection in a public offer, or alternatively all firms choose the same strike price in a rights offering. In the second kind of equilibrium, a single rights and a single public offering coexist where high and low firm types opt for the rights offering, while intermediate firm types select the public offer.

The comparison between the pooling outcomes shows that public offers have smaller discounts, higher announcement returns, and are less underpriced than rights offerings. In the coexistence equilibrium outcomes the discount is also smaller in the public offering. Announcement returns and underpricing in the coexistence outcomes depend on the conditional means of the subset of firms choosing the rights, respectively, the public offer, making their ranking dependent on distributional assumptions. Numerical simulations suggest that announcement returns are higher and underpricing is less severe in the public offering than in the rights offering, in line with the predictions based on the comparison of the two pooling outcomes. There is some support in the empirical literature for these predictions as we will discuss in Section 4.

In Section 5, we analyze the implications of uncertainty about the assets in place in our setting with strategic participation of informed unconstrained shareholders. Replacing the project NPV with assets in place as the source of the asymmetric information introduces an under-investment problem without overturning the earlier analysis. In

particular, the above equilibrium outcomes become enriched by the Myers and Majluf (1984) feature that some firm types do not invest in equilibrium. That is, there are single public, respectively, single rights, offering equilibria in which all investing firms choose the same mode and terms, but the most highly valued firms do not invest. There may also exist equilibria in which a single public and a single rights offer coexist and in addition firm types with the highest value assets abstain from investing. Among the investing firms, the high and low firm types choose the rights offering, while intermediate firm types pick the public offer as above. As regards the underinvestment problem, we focus on the tradeoff between investment efficiency and redistribution among shareholders. In public offers, better dilution protection induces more firm types to invest but also increases redistribution among shareholders. In rights offerings, higher strike prices promote investment but also lead to more wealth transfers among shareholders. It has to be pointed out that these comparative static results hold for uniformly distributed assets in place.

Finally, we discuss the robustness of our results with respect to shareholder participation in rights offerings (Section 6.1), letting uninformed or constrained shareholders participate in the offers (Sections 6.2 and 6.3), or allowing unconstrained shareholders to purchase more shares than those allocated to them on a pro rata basis (Section 6.4). We also compare public offerings with private placements (Section 6.5).

We focus our discussion on papers that — like ours — consider asymmetric information problems the primary concern when raising equity financing. We only briefly discuss other explanations for the choice of issue method and also abstract from papers that analyze private placements or compare them with either public or rights offerings. The literature recognizes that rights offerings allow current shareholders to avoid — in principle — dilution. If all shareholders participate proportionally in a rights offering they maintain their fractional ownership. Consequently, there are neither adverse selection problems nor wealth transfers (e.g., Myers and Majluf 1984, p. 195 footnote 5; Berk and DeMarzo 2017, p. 856). However, as noted by, for example, Ursel (2006) or Wu, Wang and Yao (2016), if some shareholders sell their rights to investors, adverse selection problems arise as in the Myers and Majluf (1984) setting.⁵ Our analysis shows that the ensuing adverse selection problems are aggravated by the winner's curse problem when some current shareholders strategically decide whether to exercise or sell their rights.

⁵ If rights are nontradeable, wealth transfers between current shareholders and investors are eliminated, though not necessarily transfers among shareholders.

Eckbo and Masulis (1992) argue that underwriter certification and low shareholder take-up can explain why firms prefer public offerings. In their framework, underwritten offers are not direct sales as in Myers and Majluf (1984), but come with a noisy though informative certification of the firms' value. There is no such certification for uninsured rights offerings, and the fraction of the issue taken up by current shareholders is exogenously given. Clearly, undervalued firms experience a wealth loss, which increases as the shareholder take-up becomes smaller. Consequently, the choice of issue mode depends upon the shareholder take-up: if it is high (low), the uninsured rights offering entails less (more) wealth transfers to investors than the underwritten issue.⁶ Our framework differs along two important dimensions. First, public offers do not feature an underwriter who plays an informational role or guarantees the offer. Second, the take-up is not exogeneous but a strategic decision of unconstrained shareholders that affects the equilibrium offer and rights prices. Furthermore, we analyze the wealth transfers among current shareholders as well as the trade-off between such wealth transfers and investment efficiency.

Heinkel and Schwartz (1986) also consider an extended Myers and Majluf (1984) setting to examine the choice between fully underwritten public offers and uninsured rights offerings. In their model, firms differ in the probability distribution of their terminal stock price, and the distribution depends on a parameter that is private information to the firm. All firms want to raise the same amount of equity capital, and if the realized terminal share price is less than the subscription price, the offer fails and the firm incurs a fixed cost per share. Thus, issuing a larger number of shares — as a lower-quality firm must to raise the financing makes failure more costly. Failure (costs) are avoided by using an underwriter who guarantees the offer proceeds. The failure cost of the uninsured rights offer enables high-quality firms to use the subscription price to credibly reveal their types (expected terminal share price). Lowquality firms prefer to sell shares at a pooling price through an uninformed underwriter, because the (expected) failure cost of an uninsured rights offering exceed underwriter fees and possible undervaluation. Heinkel and Schwartz (1986) assume that firms choosing an underwritten offer sell their issue to the underwriter at the same price they would announce in a rights offer, if they were to choose that financing method. Therefore, the extent to which shareholders participate in the rights

⁶ Eckbo and Norli (2005) add more structure to the framework of Eckbo and Masulis (1992) to prove equilibrium. They also allow for a larger menu of flotation methods. As in Eckbo and Masulis (1992), the exogenous shareholder take-up is the crucial determinant for the issue choice.

offering plays no role.⁷ By contrast, our framework features a meaningful market for rights and current shareholders who strategically decide whether to participate, respectively, exercise or sell their rights. Furthermore, there is no failure risk because issue, respectively, rights prices adjust to allow competitive investors to break even in equilibrium.

Duong, Singh and Tan (2014) argue that failure risk can explain why firms prefer an underwritten public offer to a rights offer despite the higher direct flotation costs. The costs of (avoiding) failure are nonexistent in an underwritten offer but can be substantial in a rights offer. While firms can self-insure against failure through a sufficiently low subscription price, large discounts are disliked by managers and entail litigation risk and wealth transfers from passive shareholders to shareholders who exploit oversubscription provisions. Furthermore, the offer may still fail despite the discount leading to a delay or loss of the investment opportunity. As noted above, there is no failure risk in our setting. More importantly, we show that wealth transfers among shareholders already occur when some shareholders cannot exercise but only sell their rights. Thus, wealth transfers do not require some inattentive shareholders, that is, shareholders who neither exercise nor sell their rights.

Several other explanations exist for the choice of issuance methods that are not based on informational frictions. Smith (1977) attributes the prevalence of public offerings in the United States to agency conflicts among managers and shareholders. Hansen (1988) argues that shareholders face an additional flotation cost in the form of price concessions in rights offerings that are absent in public offerings. Hence, public offerings are more attractive even though the direct flotation costs are larger. Hansen and Pinkerton (1982) propose that differences in ownership structures account for the choice of flotation method. Firms with large blockholders opt for rights offering, whereas dispersedly held firms find public underwritten offer the more cost efficient way to raise new equity financing. Ursel (2006) argues that firms in poor financial condition with low net worth use rights offerings since current shareholders have larger incentives to inject new funds to keep the firm alive than outside investors. Thus, rights issues are a (equity) financing of last resort. Wu, Wang and Yao (2016) propose that the flotation choice is driven by rentprotection motives of controlling shareholders.⁸ In their model, the

⁷ In an extension, Heinkel and Schwartz (1986) introduce standby rights offers as a third issue mode. The underwriter promises to purchase any not taken-up shares in exchange for a fee and also learns the firm type at some cost. In equilibrium, the highest-quality firms choose the standby rights offer as reimburs-ing the underwriter for becoming informed is the less expensive option.

⁸ Exclusively focusing on rights offerings, Fried and Spamann (2020) show that preemptive rights do not protect minority shareholders against expropriation through an equity issue, so-called "cheap-stock tunneling."

controlling shareholder can maintain her fractional ownership in a rights offering, but her stake is diluted in a public offer, which is (more) costly when private benefits are large. In a cross-country study with a sample of share issues from 41 countries over 1990–2008, McLean, Zhang and Zhao (2013) find that the likelihood of public offerings relative to both private placements and rights offerings increase with investor protection. Finally, Holderness (2018) covers in his meta-analysis over 100 studies on equity issues in different countries. He argues that the flotation choice is driven by the presence or absence of mandatory shareholder approval. In countries in which shareholders must approve an issue, rights offerings are much more common, whereas public offers are more common in countries that allow management/boards to issue equity without shareholder approval.⁹

1. Model Setup and Benchmark

1.1 Model

Consider an economy populated by publicly traded firms with assets in place, *a*, and an investment opportunity that requires an outlay, *I*, and generates a payoff, I + b. For simplicity, we assume that both the value of the assets in place *a* and the investment cost *I* remain the same across all firms and are publicly known. By contrast, the net present value (NPV) of the investment b > 0 varies across firms and is distributed on $[\underline{b}, \overline{b}]$ according to the density function f(b), respectively, its distribution function F(b). As we will discuss in Section 5, conditional on investing, whether the information asymmetry concerns the assets in place *a* or the investment *b* is largely inconsequential. The number of existing shares is normalized to one. Since we want to compare equity flotation methods, we restrict firms to raise *I* by issuing new equity through either a public offering (PO) or a rights offering (RO), which we will describe later.

The key frictions in the model are information asymmetry and heterogeneous wealth constraints. In particular, among current shareholders only a fraction $(1 - \eta)$ know the project's NPV *b*, while the remaining η shareholders merely know its distribution. Like the latter, investors only know the distribution of *b*. In addition, shareholders have different degrees of wealth constraint. We model such heterogeneous constraints in a simple binary fashion: a fraction π of shareholders are constrained in that they have no spare wealth nor can they borrow nor sell (part of) their current shares to participate in an offering. We relax this

⁹ He reports that shareholder-approved issues are associated with positive and higher announcement returns than managerial issues and that this holds across and within countries as well as for different issue methods.

assumption in Subsection 6.3. The remaining $(1 - \pi)$ shareholders are unconstrained and have financial slack to purchase those newly issued shares allocated to them on a pro rata basis. The essence of our assumptions is that neither the capital of the informed nor that of all shareholders is sufficient to fund the entire issue. Consequently, the participation of uninformed new investors is necessary to fund the issue. In practice, public offerings are targeted at new investors, while new investors indeed buy rights in countries in which rights offerings are common (Massa et al. 2016).¹⁰ In Section 6.4, we will ease this restriction and allow unconstrained shareholders to buy more, but not all, shares or rights and argue that our insights are robust.

Our assumption of some shareholders being wealth constrained should foremost be interpreted as them lacking liquid assets (cash) to invest rather than being literally wealth constrained. Liquidating other assets or borrowing on margin account can be costly making participating in the offer unattractive. Furthermore, borrowing to exercise the rights and immediately selling the underwritten shares to pay back the loan is equivalent to selling the rights directly, as we argue in Section 6.3. Finally, inattentive shareholders whose rights are sold by their brokers on their behalf are equivalent to constrained shareholders in the model. (See Section 6.1.)

Throughout the paper, we consider stylized versions of public and rights offerings. In public offerings, the firm issues new shares in the public market, but shareholders may receive some dilution protection. That is, shareholders are given priority over some fraction $\lambda \in [0, 1]$ of the new shares on a pro rata basis. Obviously, only the unconstrained shareholders can buy additional shares and possibly benefit from the dilution protection. Investors get to buy all $(1 - \lambda)$ nondilution protected shares and those dilution protected shares that shareholders do not wish to take up.¹¹ Shareholders and investors simultaneously decide whether to subscribe. Finally, investors' break-even condition determines the per share price P_{PO} and the number of newly issued shares such that $N_{PO} = \frac{I}{P_{PO}}$.¹²

¹⁰ Cronqvist and Nilsson (2005) report that new investors buy around 16% to 18% of the rights in Swedish rights offerings, and Bøhren, Eckbo and Michalsen (1997) find that new investors buy between 5.6% to 13.5% in Norwegian rights offerings. Unfortunately, no study providing the fraction of rights purchased by new investors for many other countries, notably the United States or United Kingdom, seems to exist.

¹¹ Dilution protection is very common in the United Kingdom. In countries in which shares are allocated on a pro rata basis by subscription, the parameter λ can be interpreted as the demand of the shareholders relative to that of the investors, similar to Rock (1986).

¹² Our stylized public offering is a direct share sale and resembles an at-the-market (ATM) offering, except that an ATM offering may split the total issuance into smaller quantities spread over some time period. In the United States, ATM offerings have recently become more popular, accounting for 40% of seasoned equity offerings in 2015 (Billett, Floros and Garfinkel 2019).

The payoff to shareholders in a public offering depends on the offer price P_{PO} , the number of shares issued N_{PO} , and their subscription decision. After issuing new shares and investing, the true firm value is equal to I + a + b. Given the number of shares is $N_{PO} + 1$, the true share value must equal $\frac{1}{N_{PO}+1}(I + a + b)$. If an unconstrained shareholder with β shares subscribes, she receives $\lambda\beta N_{PO}$ new shares in exchange for investing an amount $\lambda\beta N_{PO}P_{PO} = \lambda\beta I$. As a result, her final payoff as a function of the true firm type b is

$$\beta \left[\frac{\lambda N_{PO} + 1}{N_{PO} + 1} \left(I + a + b \right) - \lambda I \right]. \tag{1}$$

If a shareholder is constrained or chooses not to subscribe, her payoff is

$$\beta \frac{1}{N_{PO} + 1} (I + a + b).$$
 (2)

Unconstrained shareholders strategically subscribe to the newly issued shares to receive the higher of (1) and (2).

In rights offerings with a strike price P_S , $N_{RO} = \frac{1}{P_S}$, rights are issued to shareholders on a pro rata basis at no cost. Each right gives its owner the option to purchase a newly issued share at the strike price P_S . Unconstrained shareholders can choose between exercising the rights or selling them to investors. Simultaneously, investors decide whether to buy rights, which shareholders put up for sale. Since constrained shareholders can, by assumption, neither borrow nor sell their current shares, they have no choice but to sell their rights to new investors. In Subsection 6.3, we discuss the implications of relaxing this assumption. Doing nothing, that is, neither exercising nor selling their rights, is weakly dominated by selling the rights as long as the rights price P_R is weakly positive. Therefore, we rule out doing nothing as an option here, but discuss it in Section 6.1. The break-even constraint of the competitive investors determines P_R , and we exclude negative prices.

Following the rights offering the true firm value is I + a + b, and the number of shares is $N_{RO} + 1$. Hence, the true share value is $\frac{I+a+b}{N_{RO}+1}$. If an unconstrained shareholder with β shares exercise her rights, she receives βN_{RO} new shares and invests $\beta N_{RO} P_S = \beta I$. As a result, her payoff as a function of the true firm type *b* is equal to

$$\frac{a+I+b}{N_{RO}+1}(N_{RO}\beta+\beta)-\beta N_{RO}P_S=\beta(a+b).$$
(3)

If a shareholder sells her rights, her payoff is

$$\frac{a+I+b}{N_{RO}+1}\beta + P_R\beta N_{RO}.$$
(4)

Unconstrained shareholders strategically exercise their rights to receive the higher of (3) and (4). In Sections 1.2 and 2, we solve for the Perfect Bayesian Equilibrium outcomes when all firms adopt a given flotation method. That is, we derive the unconstrained shareholders' optimal participation decisions for a given dilution protection λ (strike price P_S) and the associated equilibrium price P_{PO} in a public offering (P_R in a rights offering). In Section 3, we let firms choose offer mode and terms to maximize the total payoffs to all shareholders.

1.2 Benchmark

A key assumption of our paper is heterogeneous wealth constraints of shareholders. As a benchmark, we temporarily abstract from such wealth constraints and only consider information asymmetries among shareholders. Specifically, we analyze the public offer and rights offer when all shareholders have wealth to participate in the offer ($\pi = 0$) but only some of them $(1 - \eta)$ know *b*, the NPV of the project. The η uninformed shareholders and the investors merely know the distribution of *b*.

As noted in the literature, rights offering can avoid wealth transfers between shareholders and new investors if "stockholders can be compelled to exercise their rights and hold the newly issued shares" (Myers and Majluf 1984, footnote 5). We extend this intuition by showing that rights offerings can resolve asymmetric information problems among current shareholders, ensuring that each and every shareholder receives the full information payoff a + b.

Proposition 1. Given all current shareholders are unconstrained, they all receive a net payoff of a+b in the unique Perfect Bayesian Equilibrium of a rights offering. Moreover, this equilibrium exists only if $P_S \leq a+\underline{b}$, and is implemented by all current shareholders exercising their rights.

When a shareholder exercises the rights allocated to her on a pro rata basis, her payoff does not depend on the strike price. Indeed, exercising the rights implies that her fractional ownership stake in the firm remains unchanged (see (3)). Therefore, any mispricing of the issue (strike price P_S) is fully offset by a corresponding value change of her "old" shares. However, informed shareholders of firms with low project values *b* may find it more profitable to sell their rights in the market. A sufficiently low strike price in combination with market beliefs that any rights sold would come from the worst firm type <u>b</u> make this an inferior option. As a result, informed as well as uninformed shareholders find it in their interest to exercise their rights. Consequently, they all receive a net payoff equal to

a+b, as they would under complete information.¹³ The proof in the appendix shows that this is the unique Perfect Bayesian Equilibrium. As shown by Myers and Majluf (1984), selling shares to investors in a public offering inevitably leads to wealth transfers. This holds true also in our setting.

Proposition 2. Any public offering with incomplete dilution protection $\lambda < 1$ leads to wealth transfers among shareholders and investors.

In public offerings without dilution protection $(\lambda = 0)$, new investors purchase all new shares in a successful offering, as in Myers and Majluf (1984). Since they are uninformed, the price P_{PO} must — in equilibrium — be the same for any and all firms, irrespective of the net present value of the investment opportunity. Moreover, investors only purchase shares if the price is such that they break even on average. Consequently, there is mispricing and redistribution across firm types: For firms whose investment project has a low (high) net present value, the new shares are overpriced (underpriced), and investors make a loss (profit). Accordingly, shareholders receive a payoff which is either larger or smaller than a + b, their full information payoff.

The asymmetric information problems are complicated by the dilution protection, since it adds a winner's curse problem when investors purchase shares in equilibrium (i.e., when $\lambda < 1$).¹⁴ Informed unconstrained shareholders take up their allocated quota $\lambda(1 - \eta)$ only if the issue is underpriced. As a result, investors end up buying more shares when a firm is overpriced. Hence, dilution protection leads to additional redistribution among shareholders and investors. As discussed in Subsection 6.2, uninformed shareholders benefit from taking up their allocated quota $\lambda\eta$.

Comparing Propositions 1 and 2, we establish the following benchmark: rights offerings dominate public offerings in the sense that the former, but not the latter, overcomes informational frictions and avoids redistribution both among shareholders and between shareholders and investors. Hence, the widespread use of public offerings cannot be attributed exclusively to asymmetric information problems. There must be at least one other friction. Subsequently, we (re)introduce wealth constraints (i.e., $\pi > 0$) and show how in this setting rights offers entail more redistribution among shareholders than public offers.

¹³ The \$1.3 billion rights offering by Wharf Ltd., a Hong-Kong-listed property developer, illustrates how rights offerings at low strike prices avoid shareholder dilution and secure full subscription (Chiu 2011).

¹⁴ In the limit ($\lambda = 1$), there cannot be an issue price P_{PO} in equilibrium at which shareholders strictly prefer not to subscribe to the offer, and the public offer becomes a *de facto* rights offering (see Subsection 2.3).

2. Offer Methods and Wealth Transfer

In this section, we characterize the shareholders' participation decisions in public offerings with any given dilution protection λ (Subsection 2.1) and in rights offerings with any given strike price P_S (Section 2.2). We then compare the wealth transfers among constrained and unconstrained shareholders across the different issue methods (Section 2.3). This analysis is a first step toward understanding the choices of flotation methods by different firm types, which will be examined in Section 3.

Given we formalize wealth constraints in a simple binary manner (unconstrained vs. unconstrained), the model (Subsection 1.1) allows for potentially four types of shareholders: constrained informed and uninformed ones and unconstrained informed and uninformed ones. To make the analysis more transparent, we abstract from information asymmetries among shareholders, which already have been analyzed in the benchmark case (Section 1.2). Specifically, all shareholders are informed $(1 - \eta = 1)$, a fraction $(1 - \pi)$ of shareholders is unconstrained, and the remaining fraction π is constrained. As we will discuss in Section 6.2, introducing uninformed constrained or unconstrained shareholders does not alter our qualitative results.

2.1 Public offerings

In a public offering, unconstrained shareholders subscribe to the new shares only if the payoff from subscribing (1) is higher than the payoff from abstaining (2).

Lemma 1. In a public offering with a given λ , unconstrained shareholders subscribe to the new shares if and only if

$$b \ge b_{PO}^* \equiv P_{PO} - a. \tag{5}$$

Unconstrained shareholders follow a simple threshold strategy and subscribe to an offer only if the sum of assets in place a and net present value of the investment b (weakly) exceed the price P_{PO} . That is, they subscribe only if the new shares are underpriced, similar to, for example, Rock (1986). Clearly, constrained shareholders have no choice but to abstain from the offer.

Since investors do not know the net present value of the investment, their participation in the offer must be unconditional, that is, cannot depend on the firm type b. At the same time, they anticipate that constrained shareholders never subscribe but that unconstrained ones only subscribe if the issue is not overpriced. Accordingly, the investors' collective payoff when subscribing is equal to

$$\begin{split} & \Pr(b \, < \, b_{PO}^*) \bigg[\frac{N_{PO}}{N_{PO} + 1} \big(a + I + E \big[b | b \, < \, b_{PO}^* \big] \big) - I \bigg] \\ & + \Pr(b \geq b_{PO}^*) [1 - (1 - \pi) \lambda] \bigg[\frac{N_{PO}}{N_{PO} + 1} \big(a + I + E \big[b | b \geq b_{PO}^* \big] \big) - I \bigg], \end{split}$$

In the above expression, the first line represents the case when the offer is overvalued and all N_{PO} new shares are purchased by the investors. They receive a fraction $\frac{N_{PO}}{N_{PO}+1}$ of the firm in exchange for contributing *I*. The second line reflects the case where unconstrained shareholders buy $(1 - \pi)\lambda N_{PO}$ shares. The remaining $[1 - (1 - \pi)\lambda]N_{PO}$ shares are purchased by the investors. Rearranging the terms in (6) using the fact that $N_{PO} = \frac{I}{P_{PO}}$ and factoring out $\frac{N_{PO}}{N_{PO}+1}$ yields

$$\begin{aligned} & \Pr(b \ge b_{PO}^*) [1 - (1 - \pi)\lambda] \big[a + E \big[b | b \ge b_{PO}^* \big] - P_{PO} \big] \\ & + \Pr(b < b_{PO}^*) \big[a + E \big[b | b < b_{PO}^* \big] - P_{PO} \big], \end{aligned}$$

which can be rewritten as

$$a + E(b) - P_{PO} - Pr(b \ge b_{PO}^*)(1 - \pi)\lambda [a + E[b|b \ge b_{PO}^*] - P_{PO}].$$
 (7)

The zero profit condition of the competitive investors, together with condition (5), determines the equilibrium issue price P_{PO} .

Proposition 3. For any given $\lambda \in [0, 1]$, there exists a Perfect Bayesian Equilibrium in which all firms raise *I*. The equilibrium price P_{PO} is decreasing in λ , and $a + b < P_{PO} \le a + E(b)$.

Since all firms issue and invest, the price P_{PO} in any equilibrium must exceed the value of the firm with the lowest net present investment (a + b)and can be at most equal to the unconditional mean (a + E(b)). Otherwise, investors would on average either earn a profit or not break even. In the limiting case of no dilution protection $(\lambda = 0)$, the information advantage of shareholders becomes irrelevant. They never receive any new shares, and there is no winner's curse problem. Hence, investors are willing to purchase the new shares at the unconditional average firm value, that is, $P_{PO} = a + E(b)$. Better dilution protection exacerbates the winner's curse problem, which decreases the equilibrium price P_{PO} . When shareholders enjoy better dilution protection, investors get to buy a smaller fraction $(1 - (1 - \pi)\lambda)$ of underpriced shares, while still buying all overpriced shares. Consequently, they can only break even if the equilibrium price is lower.

We now turn to the wealth transfers between constrained and unconstrained shareholders. Since investors break even on average, we can define the ex ante (prior to knowing b) wealth transfer from constrained to unconstrained shareholders as the difference between the expected actual payoff and the fair expected payoff (a + E(b)). Using the payoff of the constrained shareholders (2), we can express the wealth transfer as

$$WT_{PO} \equiv a + E(b) - \frac{1}{1 + \frac{I}{P_{PO}}} [I + a + E(b)].$$
 (8)

We next rank all public offerings with different dilution protections according to the extent of the ex ante wealth transfers among current shareholders.

Proposition 4. The ex ante wealth transfers from constrained to unconstrained shareholders in public offerings with a given dilution protection λ is equal to

$$WT_{PO} = \frac{I}{I + P_{PO}} [a + E(b) - P_{PO}] \ge 0,$$
(9)

and increases with the dilution protection λ .

Since a better dilution protection grants unconstrained shareholders the option to purchase more new shares, it exacerbates the winner's curse problem. The equilibrium price P_{PO} must decrease to allow investors to break even, which necessitates a larger issue (N_{PO}) . As a result, the unconstrained shareholders' ability to subscribe becomes more valuable. Hence, the wealth transfer increases with the dilution protection.

2.2 Rights offerings

In a rights offering with strike price P_S , $N_{RO} = I/P_S$ rights are issued. Unconstrained shareholders choose between exercising (payoff in (3)) or selling their rights (payoff in (4)), while investors simultaneously decide whether to purchase the rights put up for sale.

Lemma 2. In a rights offering, unconstrained current shareholders exercise their rights if and only if

$$b \ge b_{RO}^* \equiv P_S + P_R(N_{RO} + 1) - a.$$
 (10)

As in public offerings, unconstrained shareholders follow a simple threshold strategy. They prefer to sell their rights if the project return falls below the cutoff value b_{RO}^* . The expression of the cutoff value is more complicated than in the public offering because the "fair" value in a rights offering contains the bundle of strike price P_S and rights price P_R . Investors who purchase one right and exercise it have a payoff equal to

$$\frac{a+I+b}{N_{RO}+1} - P_S - P_R.$$

Rational investors also anticipate that constrained shareholders always sell their rights, whereas unconstrained shareholders sell them only if the project returns are low ($b < b_{RO}^*$). Consequently, the expected payoff to investors is

$$Pr(b \ge b_{RO}^{*})\pi N_{RO}\left(\frac{a+I+E(b|b \ge b_{RO}^{*})}{N_{RO}+1} - P_{S} - P_{R}\right) + Pr(b < b_{RO}^{*})N_{RO}\left(\frac{a+I+E(b|b < b_{RO}^{*})}{N_{RO}+1} - P_{S} - P_{R}\right)$$
(11)

Investors break even if the sum of strike and rights prices equals the conditional expected firm value (on a per share basis), taking into account when unconstrained shareholders subscribe.

$$P_{S}+P_{R} = \frac{1}{N_{RO}+1} \left[a + I + \frac{\pi Pr(b \ge b_{RO}^{*})E(b|b \ge b_{RO}^{*}) + Pr(b < b_{RO}^{*})E(b|b < b_{RO}^{*})}{\pi P(b \ge b_{RO}^{*}) + Pr(b < b_{RO}^{*})} \right].$$
(12)

Proposition 5. For any given $P_S \in (0, a + b_{RO}^*]$, there exists a Perfect Bayesian Equilibrium in which all firms issue rights with the same strike price P_S . The cutoff type b_{RO}^* solves

$$b_{RO}^* = \frac{(1-\pi)Pr(b < b_{RO}^*)E(b|b < b_{RO}^*) + \pi E(b)}{(1-\pi)Pr(b < b_{RO}^*) + \pi},$$
(13)

lies in (b, E(b)), and is independent of P_S .

Unconstrained shareholders follow their threshold strategy (10), and the threshold b_{RO}^* is the solution to (13) in equilibrium. For any solution b_{RO}^* , the equilibrium rights price P_R is then uniquely given by (12). At first glance it may be surprising that the cutoff value b_{RO}^* does not depend on the strike price P_S . To understand this feature, it is perhaps best to consider the sell/exercise decision of unconstrained shareholders in firms with low project returns ($b < b_{RO}^*$). Clearly, exercising is not attractive if the strike price is overvalued, that is, if $P_S > a + b$. Exercising at low strike prices $P_S < a + b$ is attractive and resulting in a fair payoff of a + b to shareholders, but selling the rights is even more profitable because rights are priced by the conditional average firm value (b_{RO}^*). Since constrained investors in all firms sell their rights, the rights price for firms with low investment returns is effectively subsidized. Regardless of the strike price, firms above or below the conditional average belief are the same, and the cutoff value is therefore not affected by the strike price.

The strike price does, however, affect the wealth transfers from constrained to unconstrained shareholders. Given a strike price P_S , the payoff to constrained shareholders in a type-*b* firm is equal to

$$\frac{a+I+b}{N_R+1} + P_R N_R$$

which can be rewritten as

$$a+b+\frac{I}{I+P_S}[b_{RO}^*-b].$$

(See the proof of the subsequent proposition for details.) As in public offerings, we can define the ex ante wealth transfer among shareholders in rights offerings as the difference between the expected actual payoff and the fair expected payoff (a + E(b)):

$$WT_{RO} \equiv \frac{I}{I + P_S} \left[E(b) - b_{RO}^* \right]. \tag{14}$$

Proposition 6. The ex ante wealth transfers from constrained to unconstrained shareholders in rights offerings with a strike price P_S are equal to

$$\frac{I}{I+P_S} \left[E(b) - b_{RO}^* \right] > 0$$
 (15)

and decrease in P_S .

The intuition is similar to that of Proposition 4. Lower strike prices P_S require more rights N_{RO} to be issued. Since rights are on average underpriced because of the winner's curse problem, the ability to exercise the rights becomes more valuable when more rights are issued. For constrained shareholders who cannot exercise the rights, a lower strike price implies that a larger fraction of the company is sold at a discount on average. Hence, lower strike prices and larger numbers of rights N_{RO} lead to more wealth transfers from constrained to unconstrained shareholders.

Absent information asymmetries, exercising the rights and buying new shares or simply selling the rights yield the same payoff. Gains made from exercising are matched by the proceeds from the rights sale (Farinha, Mateus and Soares 2017). Proposition 6 implies that this does not hold in our setting with asymmetric information. Due to the winner curse problem, the rights price does not fully compensate for the dilution of existing share holdings. Therefore, the pricing of the new shares matters for constrained shareholders.

2.3 Comparing public and rights offerings

In rights offerings, nonparticipating shareholders (can) sell the rights instead of doing nothing in public offerings. In addition, investors are exposed to a larger extent to the winner's curse problem than in public offerings. Despite these two differences, one specific public offering (with full dilution protection) is equivalent to one specific rights offering (with zero rights price).

Proposition 7. Rights offerings with a strike price P_S such that the equilibrium rights price P_R equals 0 are equivalent to public offerings with full dilution protection ($\lambda = 1$).

 P_{PO} denotes the equilibrium issue price in a public offering with full dilution protection. Intuitively, in a rights offering with a strike price $P_S = P_{PO}$ the value of the rights is zero in equilibrium ($P_R = 0$), because such a right resembles an at-the-money option at expiration. Under this conjecture, the payoffs to the shareholders are the same in both issue modes. First, if unconstrained shareholders participate in the offering, they can maintain their fractional ownership at the same price in either offer. Conversely, if shareholders cannot or choose not to subscribe, their payoffs must again be the same in either offer. Their holdings are equally diluted since the prices P_{PO} and P_S are equal and hence also the number of newly issued shares. Moreover, selling the rights does not generate any income. Because all shareholders in the same firm types find it profitable to subscribe, respectively, to abstain, generating the same extent of the winner's curse problem in either offer.

The equivalence result allows to rank the flotation methods according to the extent to which they entail wealth transfers between constrained and unconstrained shareholders.

Corollary 1. Any rights offering entails more wealth transfers from constrained to unconstrained shareholders than public offerings.

On the one hand, wealth transfers increase in the dilution protection λ (Proposition 4). Therefore, a public offering with full dilution protection comes with the largest wealth transfers among all public offerings, whereas an offer without any dilution protection features no such wealth transfers. On the other hand, wealth transfers decrease with the strike price P_S in rights offerings (Proposition 6), which in turn is highest when rights have zero (resale) value. Hence, among all rights offerings the one with an equilibrium rights price P_R equal to 0 leads to the least wealth transfer. This least-wealth-transfer rights offering ($P_R = 0$) is equivalent to the aforementioned most-wealth-transfer public offering ($\lambda = 1$). Consequently, any rights offering generates (weakly) more wealth



Figure 1

Rank issue modes based on wealth transfers

transfer from constrained to unconstrained shareholders than any public offerings. Figure 1 depicts this result.

The corollary suggests one consideration that may affect firms' choice of issuance mode — wealth transfers among shareholders. Firms that aim to keep redistribution among shareholders down opt for public offerings with little or no dilution protection, whereas firms that want to favor their unconstrained shareholders choose rights offerings with low strike prices.

The corporate finance literature documents that shareholders have conflicting views, and that such shareholder heterogeneity affects corporate decisions.¹⁵ For example, tax clientele effects are known to be an important determinant in firms' payout policies, including dividends and share repurchases (see, e.g., Bhat and Pandey 1993; Lie and Lie 1999; Gaspar et al. 2013) and incorporation decisions (Babkin, Glover and Levine 2017). Anecdotal evidence indicates that some shareholders oppose a share issue because it would considerably dilute their holdings.¹⁶ To the best of our knowledge, no empirical study (yet) seems to explicitly explore how or whether redistribution among shareholders affects the choice between public and rights offerings. However, since the financing policy is conceptually similar to the payout policy (albeit the money flows in the opposite direction), there are reasons to believe that redistribution among shareholders is a consideration that may affect equity floatation methods.

¹⁵ For instance, shareholders have different tax positions (Eckbo and Verma 1994; Desai and Jin 2012), differ in their support for management (Matvos and Ostrovsky 2010), and have different investment horizons (Gaspar, Massa and Matos 2005; Chen, Harford and Li 2007). Further, shareholders may represent particular constituencies, such as unions (Matsusaka, Ozbas and Yi 2019; Agrawal 2012) or public pension funds (Romano 1993). Li, Maug and Schwartz-Ziv (2022) and Bolton et al. (2020) document how diverging opinions are reflected in the voting behavior of shareholders.

¹⁶ The French bank Natixis is an example of shareholder disagreement about an equity offering. After suffering heavy losses in the U.S. subprime market, Natixis opted for a rights issue to strengthen its capital base in the year 2008. Its two main shareholders, Caisse d'Epargne and Banque Populaire, were in favor of the issue, presumably because part of the proceeds were meant to repay their advancements to Natixis. By contrast, U.S. activist investors Greenlight Capital and Royal Capital Management were opposed (Daneshkhu 2008).

3. Choosing Offer Terms and Modes

We now examine the offer game when firms can strategically choose offer mode and terms, knowing their type b at the time of the issue decision. As in Section 2, there are $(1 - \pi)$ unconstrained shareholders and π constrained shareholders. After the firm and current shareholders learn its type, the firm decides whether to raise I and, if yes, chooses offer method (public or rights offer) and terms (dilution protection λ or strike price $P_{\rm s}$). Thereafter, shareholders and uninformed investors simultaneously decide to participate. More specifically, should the firm choose a public offer, unconstrained shareholders then choose whether to buy the λ dilution protected shares on the pro rata basis $(1 - \pi)$, and investors decide whether to buy all remaining shares. Should the firm choose a rights offering, constrained shareholders sell their rights, unconstrained shareholders choose whether to exercise or sell their rights, and the investors decide whether buy all rights offered by shareholders and if yes, to exercise them. Furthermore, investors update their beliefs about b based on firms' issue methods and break even on average, as in the baseline model.

When solving for the Perfect Bayesian Equilibria of this signaling game, we assume that firms choose the issue method to maximize current shareholder wealth. This objective function is less evident than it may appear at first sight since in our framework constrained and unconstrained shareholders typically disagree over the flotation method, the extent of dilution protection, or the strike price. In view of the diverging preferences among shareholders, some arbitrariness in choosing the firms' objective function is unavoidable. We opt for firms maximizing the weighted sum of the payoffs to the constrained and unconstrained shareholders because it is equivalent to minimizing new investors' payoffs, conditional on the firm's type b. The latter seems to us the least controversial objective function in our framework. When subscribing to a firm type b public offer with dilution protection λ , price P_{PO} , and number of new shares $N_{PO} = \frac{I}{P_{PO}}$, the investors realize a payoff equal to

$$\Pi_{PO}(b) = \begin{cases} [1 - \lambda(1 - \pi)] N_{PO} \left(\frac{a + I + b}{N_{PO} + 1} - P_{PO} \right) & \text{if } \frac{a + I + b}{N_{PO} + 1} \ge P_{PO} \\ \\ N_{PO} \left(\frac{a + I + b}{N_{PO} + 1} - P_{PO} \right) & \text{if } \frac{a + I + b}{N_{PO} + 1} < P_{PO}, \end{cases}$$
(16)

Similarly, in a rights offering with strike price P_S , number of rights $N_{RO} = \frac{I}{P_S}$, and rights price P_R , the investors' payoff is equal to

$$\Pi_{RO}(b) = \begin{cases} \pi N_{RO} \left(\frac{a+I+b}{N_{RO}+1} - P_S - P_R \right) & \text{if } \frac{a+I+b}{N_{RO}+1} \ge P_S + P_R \\ N_{RO} \left(\frac{a+I+b}{N_{RO}+1} - P_S - P_R \right) & \text{if } \frac{a+I+b}{N_{RO}+1} < P_S + P_R \end{cases}$$
(17)

We begin the analysis of possible equilibrium outcomes by establishing that offers of the same mode (PO or RO) with different terms cannot coexist in equilibrium.

Lemma 3. In any equilibrium, all public offerings have the same dilution protection λ , and all rights offerings have a common strike price P_S .

Intuitively, multiple public or rights offerings cannot co-exist because overvalued firms would invariably deviate to the offer with the highest price. Suppose to the contrary that there are two public offerings. Any low firm type with non participating unconstrained shareholders prefers the offer with the higher issue price, irrespective of whether the dilution protection is greater or smaller than that of the offer with the lower price. Hence, there can only be a single issue price in equilibrium. This in turn must imply a single dilution protection since unconstrained shareholders in undervalued firms strictly prefer more to less dilution protection.

The argument why multiple rights offerings do not co-exist is similar, though slightly more involved. Suppose that there are two rights offerings with different strike prices. High firm types must prefer the higher strike price, since dilution is more costly for those firms and the lower strike price leads to more dilution of the constrained shareholders. Further, for each of the two rights offerings there must be some undervalued and some overvalued firm types. Firms which are undervalued under the offer with the lower strike price prefer to deviate to the offer with the higher strike price and get subsidized by higher-valued types, rather than subsidizing lower-valued types in the rights offer with the low price.

Given Lemma 3, only two kinds of Perfect Bayesian Equilibrium can exist: pooling equilibria where all firms use the same issue method (Subsection 3.1), and a semi-pooling equilibrium where some firm types pool on a unique public offering and others pool on a unique rights offering (Subsection 3.2).

3.1 Single offer mode equilibria

When firms can strategically choose offer terms and mode, all firms choosing the same offer remains a Perfect Bayesian Equilibrium. The beliefs that any deviating firm is perceived to be the lowest firm type, b, supports these pooling outcomes. We first establish the pooling

equilibria for public offerings. $(P_{PO}(\lambda), N_{PO}(\lambda))$ denotes the public offering equilibrium outcome for any given dilution parameter λ , as characterized by Proposition 3.

Proposition 8. There exist pooling equilibria in which all firms choose some common dilution protection λ in a public offering. An issue with a given λ is such an equilibrium if and only if

$$[1 - \lambda(1 - \pi)]N_{PO}(\lambda)\left(\frac{a + I + \bar{b}}{N_{PO}(\lambda) + 1} - P_{PO}(\lambda)\right) \leq \pi I\left(\frac{\bar{b} - \underline{b}}{a + I + \underline{b}}\right).$$
(18)

Moreover, the condition always holds for any λ sufficiently close to 1.

Proposition 3 guarantees that $P_{PO}(\lambda)$ is indeed the equilibrium issue price associated with dilution protection λ . Condition (18) rules out any deviation to any other public or rights offerings given the investors' belief that any such firm would be the lowest type b.

Specifically, consider a deviation to another public offering $\hat{\lambda} > \lambda$. A deviating firm would have to sell its new shares at a price $\hat{P}_{PO} = a + b$, given the investors' off-equilibrium beliefs. Thus, any firm type (except type \underline{b}) deviating to $\hat{\lambda}$ would sell its shares at a discount. Clearly, this is not attractive for all low firm types $b \in [\underline{b}, b_{PO}^*(\lambda))$, which sell overpriced shares to investors in the pooling equilibrium. High firm types $b \in (b_{PO}^*(\lambda), \overline{b}]$ sell underpriced shares in the pooling offer or if they were to deviate, and in either case unconstrained shareholders would participate. By Proposition 3 the pooling price $P_{PO}(\lambda)$ is higher than \hat{P}_{PO} (price effect), but investors can purchase more shares since $\lambda < \hat{\lambda}$ (quantity effect). Clearly, the lower price (\hat{P}_{PO}) hurts the shareholders, but the better dilution protection $(\hat{\lambda})$ benefits them. Hence, undervalued firms prefer not to deviate if the price effect dominates, implying lower gains to investors from the pooling offer than from the deviating offer.

The condition for the price effect to prevail is determined by the highest firm type \bar{b} since it suffers most from selling underpriced shares. The left-hand side of condition (18) is the investors' equilibrium payoff when subscribing to firm \bar{b} . The right-hand side is their payoff when firm \bar{b} chooses full dilution protection $\hat{\lambda} = 1$. This is the best deviating public offer since it lets the firm sell as few underpriced shares as possible to investors. Depending on parameters, notably the support of firm types $[\underline{b}, \overline{b}]$, the condition may not hold for offerings with little dilution protection. In this case, current shareholders in firm \overline{b} prefer to sell fewer shares to new investors at the more deflated price \hat{P}_{PO} .

The advantage of selling fewer shares becomes increasingly smaller when the dilution protection λ of the pooling offer increases. In the limit when λ approaches 1, investors buy the same number of shares in the pooling and deviating offer. Once there is only the price effect, firms strictly prefer the pooling offer. By continuity, all firm types choose pooling offers with sufficiently good dilution protection.

Such pooling offers must also dominate deviations to any rights offering \hat{P}_S . Given the investors' beliefs, a weakly positive rights price \hat{P}_R must imply a strike price $\hat{P}_{S} \leq a+b$. Hence, buying and exercising the rights would be (weakly) profitable. Therefore, deviating to the rights offering cannot be attractive for low firm types $b \in [b, b_{PO}^*(\lambda))$. They prefer to sell overpriced shares to new investors. High firm types b $\in (b_{PO}^*(\lambda), b]$ would not want to switch to a rights offering with a zero strike price, that is, setting $\hat{P}_S = a + b$, since it is equivalent to a public offering with full dilution protection (Proposition 7). Furthermore, rights offerings with positive rights prices (but lower strike prices) dilute the stakes of the constrained shareholders more at deflated prices. Unconstrained shareholders can maintain their fractional ownership and are therefore indifferent across different combinations of strike and rights prices. Hence, high firm types (in fact, all firms) have no incentive to deviate to a rights offering with a lower strike price $(P_S < a+b).$

Next, we turn our attention to the possibility of a pooling rights offering. For any P_S , we write $(P_R(P_S), N_{RO}(P_S))$ as the rights offering outcome given by Lemma 2 and Proposition 5.

Proposition 9. There exist pooling equilibria in which all firms choose some common strike price P_S in a rights offering. An issue with a given P_S is such an equilibrium if and only if

$$\pi N_{RO}(P_S) \left(\frac{a+I+\bar{b}}{N_{RO}(P_S)+1} - P_S - P_R(P_S) \right) \le \pi I \left(\frac{\bar{b}-\underline{b}}{a+I+\underline{b}} \right)$$
(19)

Moreover, the condition always holds for any $P_S \in [a + \underline{b}, a + b_{RO}^*]$.

Similar to (18) in the pooling public offering equilibrium, condition (19) rules out any deviation to any other public or rights offerings. Consider an initial rights offering with some strike price $P_S \in [a + \underline{b}, a + b_{RO}^*]$. A firm may deviate either to another rights offering or switch to a public offer. In either deviation the financing terms are set by the investors' belief that the firm is of type \underline{b} . Hence, if a firm were to choose another rights offering, the strike price would be (weakly) lower ($\hat{P}_S \leq a + \underline{b}$) and the number of new shares (weakly) larger $(I/\hat{P}_S \geq I/P_S)$. Clearly, all low firm types $b \in [\underline{b}, b_{RO}^*)$ prefer the initial rights offering since they sell overpriced shares to investors and the stakes of their shareholders get less diluted. High firm types $b \in (b_{RO}^*, \overline{b}]$ sell underpriced shares whether they adhere to the initial rights offering or deviate. In either case unconstrained shareholders subscribe, thereby maintaining their fractional ownership. That is, their payoff is a + b, regardless of the strike price,

and they are indifferent. By contrast, constrained shareholders in high firm types prefer the initial rights offering as it dilutes their ownership stake (weakly) less. Moreover, any possible difference in rights price $(\hat{P}_R(\hat{P}_S) - P_R(P_S))$ never fully compensates them for being more diluted. The reason is that the sum of \hat{P}_R and \hat{P}_S are such that investors break even for the lowest firm type <u>b</u>, whereas the sum of P_R and P_S is based on all firm types (taking into account the winner's curse problem).

As in Proposition 8, the highest-value firm type \bar{b} suffers most from issuing underpriced rights (shares). The left-hand side of Equation (19) is the investors' payoff from purchasing and exercising the rights of firm \bar{b} in the initial offering. If firm \bar{b} were to deviate to another rights offering, it would set $\hat{P}_S = a + \underline{b}$, which in turn implies $\hat{P}_R = 0$, since it dilutes the ownership stakes of its constrained shareholders the least. The righthand side of condition (19) is the corresponding payoff to the investors. Such a deviation can be attractive to firm type \bar{b} only if the strike price in the initial rights offering is lower and therefore were to dilute its constrained shareholders more. Hence, a pooling equilibrium in which all firms choose the same strike price P_S always exists for $P_S \in [a + \underline{b}, a + b_{RO}^*]$.

As argued in the discussion of Proposition 8, the best deviating public offers for high-value firms is full dilution protection $(\hat{\lambda} = 1)$, again because it lets the firm sell as few underpriced shares as possible to investors. Since this public offer is equivalent to the rights offering with $\hat{P}_R = 0$ (Proposition 7), neither the highest-value firm \bar{b} nor any other undervalued type $b \in (b_{RO}^*, \bar{b})$ would want to deviate.

3.2 Coexistence of rights and public offers

Lemma 3 rules out equilibria with multiple rights offerings, respectively, multiple public offerings, so the only possibility left is that the two offer modes coexist in equilibrium — a case described by the following proposition.

Proposition 10. Any coexistence equilibrium is characterized by three cutoffs $\underline{b}^{\dagger} < \overline{b}^{\dagger} < \overline{b}^{\dagger}$. Low-value firms $b \in (\underline{b}, \underline{b}^{\dagger})$ and high-value firms $b \in (\overline{b}^{\dagger}, \overline{b})$ choose rights offering and intermediate types $b \in (\underline{b}^{\dagger}, \overline{b}^{\dagger})$ choose public offerings. Furthermore, in all firm types $b > b^{\dagger}$, unconstrained shareholders participate.

Figure 2 plots the investors' payoff as a function of the true firm type b, with the red curve representing the payoff from the public offering and the blue one the payoff from the rights offering. Investors buying shares from overvalued firms ($b < b_{PO}^*$ and $b < b_{RO}^*$) realize a loss, depicted by the parts of the curves below the horizontal axis. Shareholders of these



Figure 2 Payoff to new investors in a coexistence equilibrium

firms do not purchase any new shares, leaving the entire issue to the investors. By contrast, unconstrained shareholders take-up their shares in undervalued offers ($b \ge b_{PO}^*$ or $b \ge b_{RO}^*$), and the investors can buy only a fraction of the new shares. The resultant positive payoffs are the parts of the curves above the horizontal axis. Since the slope of the curves is equal to the investors' fractional ownership, the curves have a kink at zero.

Since firms maximize the total payoff to shareholders, or equivalently minimizing the payoff to investors, they choose the lower contour of the respective payoff curves in Figure 2. The key feature is that the payoff curve in the negative region ($b < b_{RO}^*$) is steeper for the rights offering than for the public offering. The reason is that the strike price P_S must be smaller than the public offering price P_{PO} , and hence more shares are being issued and sold to investors. As a result, the lowest-quality firms choose the rights offering to sell more overvalued shares to investors. In a coexistence equilibrium, some high-quality firms must also choose the rights offering. Since the rights offering provides full dilution protection to unconstrained shareholders, investor can buy fewer shares of undervalued firms in the rights offering. Consequently, the payoff curve from the public offer has a steeper slope above the horizontal axis. Being able to sell fewer shares to the investors is more valuable to the highest-quality firms, which therefore choose rights offerings.¹⁷

Such coexistence equilibria require that the issue price P_{PO} exceeds the strike price P_S . Intuitively, if P_{PO} would be smaller than P_S , all overvalued firm types whose unconstrained shareholders do not subscribe (to

¹⁷ For undervalued firms the benefit of the public offer is the higher price, while its cost is the lesser extent to which it protects shareholders from dilution. As the dilution cost increases in the firm type, the higher types among all undervalued firms opt for the rights offering.



Figure 3 Numerical example for the coexistence equilibrium

either offer) would prefer the rights offering. It would entail less dilution and (possibly) some revenues from the rights sale. This condition is merely necessary, but not sufficient, to ensure the existence of these equilibria. Since all our attempts to characterize a general sufficient condition have been in vain, we use a numerical example to show that such an equilibrium can exist. The equilibrium payoffs depicted in Figure 3 are based on an investment cost I=1, assets in place a=0.2, a fraction of constrained investor $\pi = 0.2,$ four equally likely firm types $b \in \{1, 3, 6, 12\}$, dilution protection $\lambda = 0.4$, and a strike price $P_S = 1$. The equilibrium issue price in the public offering is $P_{PO} = 4.41$, and the three cutoffs of Proposition 10 are $b^{\dagger} = 2.02$, $b^{\dagger} = 4.21$, and $\bar{b}^{\dagger} = 9.61$.¹⁸

For a sample of 85 U.S. rights offerings and 85 matched public offerings Kothare (1997) documents that bid-ask spreads increase after rights issues but decrease after public offerings. Since bid-ask spreads also reflect the extent of information asymmetries (Glosten and Milgrom 1985), these changes are consistent with the coexistence equilibrium outcome: Firms opting for a right offering are more heterogeneous, and the associated larger adverse selection problem implies larger bid-ask spreads.

⁸ This example with four discrete firm types can easily be expanded to one with a continuum of types without affecting the equilibrium outcome and the shape of Figure 3. In fact, any continuous distribution of *b* with equal (i.e., a $\frac{1}{4}$) probabilities on each of the intervals $[0.5, \underline{b}^{\dagger}], [\underline{b}^{\dagger}, b^{\dagger}], [\underline{b}^{\dagger}, \overline{b}^{\dagger}], [\overline{b}^{\dagger}, 13]$ and conditional means of 1, 3, 6, 12 (i.e., the four firm types in the discrete example), respectively, generates the same figure and equilibrium outcome.

Relatedly, Ursel (2006) reports that firms with more volatile stock prices tend to use rights offering. To the extent that volatility is also affected by (uncertainty about) the underlying firm values, this finding is in support of more diverse firms choosing rights offerings.

4. Discounts, Underpricing, and Announcement Returns

In this section, we explore empirical implications of our model. Prior to choosing the flotation method, firm types are indistinguishable to the (uninformed) market participants and therefore trade at a common initial share price P_0 . The offer discount in a public offering (rights offering) can be expressed as the difference between initial price and public offering price (strike price), normalized by the initial price:

$$\frac{P_0 - P_{PO}}{P_0} \quad \text{and} \quad \frac{P_0 - P_S}{P_0}$$

We choose to be agnostic about the extent to which market participants anticipate the investment and the offering mode and hence do not pin down the price P_0 at which shares initially trade. Consequently, we cannot make predictions about the size or sign of the discount in either flotation mode but merely rank the discount across the two modes. As discussed in Subsection 2.3, the strike price P_S must be smaller than the issue price P_{PO} in a public offering when all firms choose the same issue mode. Otherwise the rights price would be negative. In any equilibrium where a public offering and a rights offering coexist (Proposition 10), the strike price also must be smaller than the issue price. Hence, our model implies that discounts are larger in rights offerings than in public offerings.

Following the IPO literature, we define underpricing as the (1-day) return on shares purchased in an issue. That is, underpricing is the difference between the postissue share price and the public offering price P_{PO} (the strike price P_S), normalized by the public offering price P_{PO} (the strike price P_S). Since all firm types invest, the expected firm value and hence the postissue price is equal to $E \frac{(I+a+b)}{1+I/P_{PO}}$ following a public offer and equal to $E \frac{(I+a+b)}{1+I/P_S}$ following a rights offering. Formally, the underpricing in a public offering and a rights offering is

$$\frac{\frac{I+a+E(b|PO)}{1+I/P_{PO}} - P_{PO}}{P_{PO}} \quad \text{and} \quad \frac{\frac{I+a+E(b|RO)}{1+I/P_S} - P_S}{P_S}, \tag{20}$$

which, with some manipulation, can be expressed as

$$\frac{a + E(b|PO) - P_{PO}}{P_{PO} + I} \quad \text{and} \quad \frac{a + E(b|RO) - P_S}{P_S + I}.$$

The comparison of underpricing in public and rights offerings is more involved.¹⁹ When comparing underpricing across the pooling equilibria where all firms either opt for the same rights or for the same public offering, the conditional expectations E(b|RO) and E(b|PO) reduce to the unconditional mean E(b). Hence, the comparison is solely driven by the strike and issue prices P_S and P_{PO} . Since, the strike price is (weakly) smaller than the issue price ($P_S \leq P_{PO}$), underpricing is more severe in rights offerings.

In the equilibrium in which a rights and a public offer coexist (Proposition 10), underpricing is determined by the prices P_{PO} and P_S as in the pooling equilibria and, in addition, by the conditional expectations E(b|RO) and E(b|PO), which are in general not identical. Unfortunately, we cannot analytically establish generic (qualitative) results. Though, numerical simulations suggest that firms that issue rights are the types with a higher average project NPV, b. In the numerical example in Section 3.2, the average project NPV of firms issuing rights is $\frac{1+12}{2} = 6.5$, whereas the average is $\frac{3+6}{2} = 4.5$ for the firms using the public offering. Still, the price effect (P_{PO} vs. P_S) clearly dominates the firm-quality effect (E(b|RO) vs. E(b|PO)), and underpricing (given by (20)) is 2.85 in the rights offering and 0.0528 in the public offering. Trusting that this example (and our other simulations) are reasonably representative, we argue that there is less underpricing in public offerings.

Finally, the announcement return is the difference between postissue share price and the initial share price, normalized by the initial price:

$$\frac{E\frac{(I+a+b)}{1+I/P_{PO}} - P_0}{P_0} \quad \text{and} \quad \frac{E\frac{(I+a+b)}{1+I/P_S} - P_0}{P_0}.$$

Like underpricing, the announcement returns across the two pooling equilibria are purely driven by the price effect since both initial price and postissue price are the same when all firms either opt for a public or a rights offer. Hence, announcement returns are lower following rights offering since the strike price is weakly smaller than the issue price $(P_S \leq P_{PO})$. In the coexistence equilibria, there are again the opposing price and firm quality effects at work. Our numerical simulations suggest that overall the price effect dominates the firm quality effect. Hence, we are inclined to argue that announcement returns are higher in public offerings.

¹⁹ The underpricing in rights offering is defined for shareholders rather than investors as the formula does include the rights price P_R .

Our predictions receive some support in the empirical literature, though few studies compare public and rights offerings within a country. Armitage (2007) studies discounts in rights offers and open offers (similar to public offerings) in the United Kingdom. Consistent with our prediction, he documents that rights are often issued at a discount of 15% to 20% relative to the market price, whereas open offers are usually discounted by less than 10%.²⁰

In terms of announcement return, Slovin, Sushka and Lai (2000) and Barnes and Walker (2006) find that in the United Kingdom, abnormal returns are significantly more negative for rights offerings (on average -3.1% announcement return) than for placements (3.3%).²¹ For Hong Kong, Wu, Wang and Yao (2005) report that rights issues have on average a significant 3-day announcement return of -8.0%, while public offers and private placements are associated with significantly positive announcement returns. In France where rights and public offerings are both common, Gajewski and Ginglinger (2002) report significant 2-day average excess returns of -1.28% for standby rights issues, -2.84% for uninsured rights issues, and an insignificant negative return for public offerings. The proportion of public offerings increases from 4.84% over the 1986–1989 period to 16.84% over the 1990–1996 period. Finally, Veld, Verwijmeren and Zabolotnyuk (2020) find in their meta-analysis covering 199 studies that rights issues are associated with more negative abnormal returns.

5. Investment Efficiency and Wealth Transfers

In the main model investors and shareholders are asymmetrically informed about the NPV of the project b, whereas the value of the asset in place a is common knowledge. Here we consider the reverse case when shareholders have private information about the assets in place a, while the NPV of the project b is common knowledge. The first observation is that, conditional on issuing shares, the premoney total firm value a + b is the crucial term incorporating the informational friction. Consequently, it is irrelevant whether the information asymmetry is about the NPV of the project or the value of the assets in place. In fact, the term a + b

²⁰ In the United Kingdom, a firm is not permitted to offer shares to the public without initially making an offer to existing qualifying shareholders (Barnes and Walker 2006). In a rights issues, shareholders who do not wish to take-up their rights can sell them. In an open offer, the new shares are offered pro rata to the existing shareholders, but shareholders cannot sell their entitlements. Instead, the placees commit to take the remaining shares. To the extent that the shareholders do not receive any compensation should they not participate in the issue, an open offer is similar to a public offering in our model.

²¹ U.K. placings are a nonrights method of flotation in which an underwriter buys an equity offering and sells the shares to clients, typically institutions, and other outside investors. A placing is not a private placement, but a form of public securities issuance comparable to a firm commitment offering in the United States (Slovin, Sushka and Lai 2000).

jointly appears in all payoff expression throughout the analysis. Hence, one merely needs to replace all the project net payoffs b with the asset values a (and vice versa) in the lemmata and propositions, and all the results carry over to the case where the information asymmetry is about the assets in place instead of the project NPV.

The more interesting observation is that asymmetric information about assets in place — though not about project NPV — can lead some firm types to abstain from investing as in Myers and Majluf (1984). When firms can choose issue mode as in Section 3 as well as whether to issue at all, the firm types with the most valuable assets in place may prefer to forgo the investment because issuing shares to investors may dilute the stake of the shareholders too much.²² Our main insight is that offer methods that lead to more wealth transfers among shareholders also result in more investment.

More specifically, assume that the investment cost *I* and the NPV of the project *b* are the same for all firms, while the value of the assets in place is now distributed on $[\underline{a}, \overline{a}]$ with $\underline{a} \ge 0$ according to the density function f(a), respectively, its distribution function F(a). As before, only current shareholders are informed and know the realization of $a \in [\underline{a}, \overline{a}]$. As in Section 3, we assume that firms maximize the weighted sum of the payoffs to constrained (π) and unconstrained $(1 - \pi)$ shareholders, or equivalently, minimize the investors' payoff. Rather intuitively, the equilibrium outcomes combine features of Myers and Majluf (1984) and of Proposition 8, respectively, Propositions 9 and 10. To avoid repetition, we relegate the analysis and the formal statement of the results to the appendix. Here, we discuss the equilibrium outcomes informally.

First, there exist semipooling equilibria in which all firm types above the cutoff type $\hat{a}_{PO} \in (\underline{a}, \overline{a})$ abstain from investing, while all other types $a \in [\underline{a}, \hat{a}_{PO}]$ invest and choose a public offer with some common dilution protection λ . Similar to Proposition 8, unconstrained shareholders in firms which invest only subscribe if the assets in place are (at or) above the threshold value $a_{PO}^* \in (\underline{a}, \hat{a}_{PO})$. Thus, unconstrained shareholders in investing firms only subscribe if the new shares are under-priced $(a \in [a_{PO}^*, \hat{a}_{PO}])$, and firm types with the highest value assets in place $(a \in (\hat{a}_{PO}, \overline{a}_{PO}])$ do not invest since the discount would exceed the project NPV and shareholders would be worse off.

Second, there exist semipooling equilibria in which all firm types above the cutoff type $\hat{a}_{RO} \in (\underline{a}, \overline{a})$ abstain from investing, while all other types $a \in [\underline{a}, \hat{a}_{RO}]$ invest and choose a rights offer with some common strike price P_S . The underlying logic is the same as for the semipooling public

²² Asymmetric information about both assets in place and project NPV does not add any qualitative new feature relative to asymmetric information only about the assets in place since the crucial term incorporating the informational friction is the sum of a + b.

offer equilibria: firm types with the highest asset values $(a \in (\hat{a}_{RO}, \bar{a}])$ do not issue any rights since the equity stakes of the shareholders would be diluted by more than the project NPV. Among the investing firms $(a \in [\underline{a}, \hat{a}_{RO}])$ unconstrained shareholders exercise their rights only if the right issue is underpriced $(a \in [a_{RO}^*, \hat{a}_{RO}])$.

Finally, there may also exist an equilibrium in which a public offer and a rights offer coexist and in addition firm types with the highest value assets abstain from investing.

In the remainder of this section, we want to explore whether offer terms, that is, dilution protection λ and strike price P_S , that promote investment efficiency entail more redistribution among shareholders. In general, the investment cutoff types (\hat{a}_{PO} and \hat{a}_{RO}) and the subscription cutoff types (a_{PO}^* and a_{RO}^*) not only depend on the respective offer terms λ and P_S but also on the distribution function F(a) and its support. As a result, clear-cut comparative static results require to impose distributional assumptions. For simplicity, we assume that the assets in place are uniformly distributed on $[\underline{a}, \overline{a}]$.

As in Section 2, here we analyze the redistribution from an ex ante point of view, that is, before the firm type *a* has been realized. Since asymmetric information about the assets in place results in types with the highest assets in place abstaining from investing, the ex ante wealth redistribution among shareholders is now the product of two terms, the probability of being a type that invests times the redistribution conditional on the firm investing. As for the conditional redistribution equation, (8) respectively (14), still apply except that expectations are taken with respect to assets in place rather than project NPVs. The probability of investing is simply the probability that a firm type is smaller or equal to the investment cutoff type \hat{a}_{PO} , respectively, \hat{a}_{RO} . Thus, the ex ante wealth transfer in public offers, \hat{WT}_{PO} , is equal to

$$\hat{WT}_{PO} \equiv Pr(a \le \hat{a}_{PO}) \times \left\{ E(a|a \le \hat{a}_{PO}) + b - \frac{1}{1 + \frac{I}{P_{PO}}} [I + E(a|a \le \hat{a}_{PO}) + b] \right\},$$
(21)

and in rights offers the ex ante wealth transfer, \hat{WT}_{RO} , is equal to

$$\hat{WT}_{RO} \equiv Pr(a \le \hat{a}_{RO}) \times \left\{ \frac{I}{I + P_S} \left[E(a|a \le \hat{a}_{RO}) - a_{RO}^* \right] \right\}.$$
 (22)

Proposition 11. Given $a \sim U(\underline{a}, \overline{a})$, better dilution protection in public offers increases investment efficiency and wealth transfers among shareholders, and higher strike prices in rights offer increase investment efficiency and wealth transfers among shareholders.

There is a trade-off between wealth transfer among shareholders and investment efficiency. Better dilution protection λ in public offers results in a higher investment cutoff \hat{a}_{PO} : more firm types invest when dilution protection is better since (unconstrained) shareholders can purchase more shares in underpriced issues, in turn increasing the asset cutoff value, \hat{a}_{PO} , at which the entire NPV accrues to the investors. As in the baseline case (Proposition 4), redistribution among shareholders, conditional on investing, increases with better dilution protection since it exacerbates the winner's curse problem. As a result, there are more wealth transfers among shareholders (because investors on average must break even). Thus, both terms increase in dilution protection, and the ex ante wealth transfers among shareholders also increase.

Higher strike prices P_S in rights offers lead to more investment (a higher cutoff value of \hat{a}_{RO}) because fewer new shares have to be issued to finance the investment I. That is, the equity stakes of the shareholders become less diluted, and the cutoff asset value \hat{a}_{RO} at which investors extract the entire NPV of the investment increases. The impact of higher strike prices P_S on redistribution among shareholders, conditional on investing, is more intricate than in the baseline case (Proposition 6). On the one hand, higher strike prices lead to less dilution and less wealth transfers. On the other hand, the value of the average asset in place of the investing firms is higher when strike prices are higher. As a result, the winner's curse problem becomes larger, and there is more redistribution among shareholders. As it turns out, these two effects cancel each other out in the case of uniformly distributed assets in place. Thus, wealth transfers, conditional on investing, remain constant as the strike price increases, and the ex ante wealth transfer is solely driven by more firms investing.

To conclude, the better dilution protection in public offers increase wealth transfers among shareholders in the main model with uncertainty about the project NPV and in the setting with asymmetric information about the assets in place when investing is not profitable for highest firm types ($a \in (\hat{a}_{PO}, \bar{a}]$). By contrast, higher strike prices in rights offer have opposite effects on wealth transfers in these two settings. Consequently, Corollary 1 does no longer hold with asymmetric information about the assets in place, even though the equivalence result (Proposition 7) remains true.

6. Extensions and Discussion

In this section we discuss the robustness of our results with respect to the shareholder participation, in particular, allowing constrained or

uninformed shareholders to participate in the offers as well as unconstrained shareholders to purchase more shares than those allocated to them on a pro rata basis. Last, we compare public offerings to private placements.

6.1 Shareholder participation in rights offerings

Our model assumes a functioning rights market where shareholders and investors trade without frictions other than the adverse selection problem.²³ Therefore, all nonexercised rights are in equilibrium sold to investors, and rights offerings do not face a subscription risk. This requires that rights are in fact tradable, which holds true in most countries (Holderness and Pontiff 2016). Furthermore, many countries offer protection to shareholders who do not to respond to a rights offering by either having brokers sell the rights on their behalf (e.g., Italy or Sweden) or by having an investment bank sell all nonexercised rights and credit the proceeds to the nonparticipating shareholders (e.g., Australia). In some countries, most notably the United States, firms can choose whether or not to make rights transferable.²⁴ In the study of Holderness and Pontiff (2016) about 50% of the firms in their U.S. sample opt for transferable rights, while in the international sample of Massa et al. (2016) more than 60% of the rights offering have tradable rights in countries that do not make transferability mandatory. Thus, shareholders do indeed either exercise or sell their rights (or have them sold on their behalf) in many countries as our model assumes. Still, in practice, rights do at times lapse because of inattention, wealth constraints, or restricted transferability. When valuable rights expire those shareholders who hold these rights lose out even more than the constrained shareholders in our analysis.²⁵

6.2 Uninformed current shareholders

With the exception of the benchmark in Subsection (1.2), all current shareholders are assumed to be informed about the true quality of the project *b*, respectively, the assets in place *a* in Section 5, abstracting from information asymmetries among shareholders. We choose this setup to highlight our novel channel of strategic participation by some unconstrained informed shareholders. A more general setting would comprise four types of shareholders: constrained informed and uninformed ones,

²³ In their international study, Massa et al. (2016) find that rights are typically less liquid than the underlying shares and often undervalued. The latter feature is consistent with a winner's curse problem in the rights market.

²⁴ In the United Kingdom, Singapore, and Hong Kong offers without tradeable rights are called open offers and are separately regulated (Massa et al. 2016).

²⁵ In their study of 179 rights offerings in the United States, Holderness and Pontiff (2016) find that shareholder nonparticipation leads to wealth transfers of around 7% of the raised capital, and that these transfers are typically at the expense of small individual shareholders.

plus unconstrained informed and uninformed ones. Our (qualitative) results carry over to such a richer setting for two reasons. First, whether or not constrained shareholders are informed is immaterial since they can, by assumption, not act strategically. Either they do nothing in public offerings or mechanically sell their rights. Hence, it is without loss of generality to assume that all constrained shareholders are informed, respectively, uninformed. Second, uninformed unconstrained shareholders cannot purchase more new shares than those allocated to them on a pro rata basis, that is, $(1 - \pi)\eta\lambda N_{PO}$ in a public offer and $(1 - \pi)\eta N_{RO}$ in a rights offer. Consequently, they are not directly exposed to the winner's curse problem. Since the equilibrium prices are set such that new investors break even, uninformed unconstrained shareholders therefore make — on average — a gain from participating and always subscribe to new shares, respectively, exercise their rights. In equilibrium, these shareholders therefore always take up the same fixed fraction of shares $(1-\pi)\eta\lambda$ in public offerings, respectively, $(1-\pi)\eta$ in rights offerings. Hence, one can abstract from these and apply the analysis of the main model to the remaining shares $[1 - (1 - \pi)\eta\lambda]N_{PO}$, respectively, $[1 - (1 - \pi)\eta]N_{RO}$, generating the same results qualitatively.

6.3 Margin borrowing and trading of shares

Relaxing the assumption that constrained shareholders can neither borrow nor trade their shares to participate in an offer seems an obvious extension. In a public offering, the strategy of selling existing shares to buy newly issued ones is futile given both shares are traded simultaneously at the same market clearing price. Still, when shareholders can trade their existing shares, investors are confronted with a winner's curse problem in both the primary and secondary market. While this exacerbates the extent of the adverse selection, it does not qualitatively change the nature of the winner's curse problem.

In rights offerings, constrained shareholders can sell part of their shares to have the funds to exercise the remaining rights. A complete analysis requires a fully specified trading environment, for example, whether only rights or also shares can be traded separately. Regardless of the chosen model setup, the constrained shareholders always need to sell some shares or rights or both to participate in the rights offering, which again changes the extent, but not the qualitative nature, of the winner's curse problem.

Instead of selling some of their shares, constrained shareholders could borrow to participate in an offer. For example, they could borrow on their margin accounts and exercise undervalued rights. However, if the newly acquired shares are sold immediately to cover the margin loan, the payoff is the same as if the rights were sold instead.²⁶ Exercising the rights and getting the full information payoff (3) requires shareholders to hold the shares sufficiently long until the true firm value is realized. In practice, this may take a long time, making borrowing on margin accounts prohibitively expensive or even infeasible.

6.4 Buying more shares

Key features of our model are that some shareholders strategically participate in the offer, but that the share, respectively, the rights, price is determined by uninformed competitive investors. For simplicity, we assume a fraction of unconstrained shareholders who can buy at most those newly issued shares or rights that are allocated to them on a pro rata basis. Since the constrained shareholders cannot purchase any new shares, competitive investors always buy some — or all — shares (rights) in equilibrium, and their break-even constraint determines the price. We are confident that our qualitative results carry over to more general settings, meaning we can relax these assumptions as, for example, introducing uninformed unconstrained shareholders (Section 6.2). Similarly, we could allow for the possibility that unconstrained shareholders can buy more rights or shares than those allocated to them on a pro rata basis, $(1 - \pi)$, respectively, $\lambda(1 - \pi)$.²⁷ This would reduce the shares or rights that investors can purchase when the issue is underpriced, and therefore exacerbate the winner's curse problem. Since investors must break even on average, the constrained shareholders would ultimately bear the cost of the aggravated winner's curse problem. That is, there would be more redistribution among shareholders but the results would not change qualitatively. Obviously, this does not hold if the unconstrained shareholders could purchase all shares or rights since it would fully resolve the adverse selection problem.²⁸ Hence, our results crucially hinge on the assumptions that informed capital is scarce and that shareholders do not have sufficient available wealth to absorb the entire offering and cannot borrow sufficient amounts to do so.²⁹ The latter can be motivated by limits of arbitrage arguments (e.g., Barberis and Thaler 2003). As such limits are orthogonal to our analysis we believe that a

²⁶ A simple example illustrates this point. Suppose the market price after a rights offering is £10 and the strike price is £4. If a shareholder exercises her right and immediately sell the share at the market price, the payoff is £10 – £4, which is exactly the price the right commands in the market.

²⁷ Formally, one merely needs to reinterpret $(1 - \pi)$, respectively, $\lambda(1 - \pi)$, as the proportion of shares that the unconstrained shareholders can at most purchase.

²⁸ For instance, we intentionally do not allow firms to issue $N_{RO} = I/\pi P_S$ nontransferable rights such that the take-up by unconstrained shareholders would be sufficient to fund the investment, thereby resolving all information problems.

²⁹ Otherwise, the equity market would be free from informational frictions — a highly unlikely scenario in view of the plentiful evidence on share prices reactions to corporate decisions and announcements.

limits-to-arbitrage-based micro foundation of our purchase limits $(1 - \pi)$, respectively, $\lambda(1 - \pi)$ would neither affect our main results nor generate substantive new insights.

6.5 Private placement

In a private placement, the issuing firm negotiates a share sale to a small group of qualified investors who may be shareholders or investors. Since most — if not all — shareholders do not qualify, private placements can be viewed as being similar to public offerings with zero dilution protection in our model. The key difference is the pricing mechanism. The public offering price P_{PQ} is the market clearing price set by competitive investors, whereas the issue price in a private placement is the outcome of the bargaining between firm and qualified investors. In practice, private placements are sold at a discount relative to the current share price (e.g., Eckbo, Masulis, and Norli 2007). The discount may reflect qualified investors' strong bargaining position or may be compensation for the costs of investigating the firm or for valuable monitoring. In either case, wealth is transferred between shareholders and qualified investors. Shareholders are treated equally and typically have no available action to take. In this sense, they are similar to the constrained shareholders in our model.

7. Conclusion

We analyze seasoned equity offerings where some shareholders are informed and can strategically choose to participate. When all shareholders have wealth to participate in the issue, right offerings achieve the full information outcome while public offerings necessarily generate wealth transfers. In contrast, we show that rights offerings generate more wealth transfers among existing shareholders when some of them are constrained. In rights offerings, investors must purchase the rights to buy the underlying shares, rather than only buying these shares as in a public offering. Hence, a positive right price implies a discount in the strike price relative to the public offering price. Therefore, constrained shareholders become more diluted in a rights offering, and lower strike prices increase the wealth transfer from them to unconstrained shareholders. More generally, constrained and unconstrained shareholders have diverging preference over flotation methods and terms. Moreover, there is a trade-off between investment efficiency and wealth transfers among shareholders in both rights and public offerings.

When firms choose the flotation mode and terms to maximize the total payoff to all shareholders, there are only two kinds of equilibria. On the one hand, there exist pooling equilibria in which all firms choose the same public offering, or alternatively all firms choose the same rights offering. On the other hand, there exist equilibria with a single rights and a single public offering. In such an equilibrium, high- and low-quality firms opt for the rights offering, while intermediate firm types choose the public offer. Low-quality firms prefer a rights offering to sell a larger fraction of their overvalued firms. High quality firms favor a rights offering because it allows unconstrained shareholders to maintain their fractional ownership, thereby selling fewer undervalued shares to investors.

Appendix

A Proofs for Sections 2 to 4

Proof of Proposition 1: In any equilibrium, the informed shareholders can secure a net payoff of at least a + b by exercising the rights. Similarly, the uninformed shareholders must receive at least a net payoff a + E(b). New investors must on average at least break even. Because the total firm value net of investment *I* is a + b, the above payoffs are exactly the equilibrium payoffs for investors, informed and uninformed shareholders.

Next, we will show that uninformed shareholders receive exactly a + b net of investment in equilibrium as well. Suppose otherwise, then some uninformed shareholders can earn a net payoff strictly larger than a+b. The informed shareholders in the same firm would deviate to this strategy to earn a strictly larger payoff, a contradiction. Thus, all shareholders receive exactly a+b, which can be implemented by exercising the rights. We now prove that no equilibrium exists for strike prices $P_S > a + \underline{b}$. Consider the informed shareholders of a firm type $b \in [\underline{b}, P_S - a)$, which implies $P_S > a + b$. If they choose not to exercise, their payoff is

$$\frac{I+a+b}{1+\frac{I}{P_s}} > \frac{I+a+b}{1+\frac{I}{a+b}} = a+b.$$

Thus, these informed shareholders' equilibrium payoff must be strictly higher than a+b. A contradiction and therefore, no such equilibrium exists.

To complete the proof, we show that when the strike price $P_S \le a + \underline{b}$, all shareholders have an incentive to exercise their rights. The equilibrium is supported by the investors' belief that any sold rights come from the worst firm type \underline{b} . The price of the rights is therefore

$$P_R = \frac{I+a+\underline{b}}{1+\frac{I}{P_S}} - P_S.$$

The payoff to shareholders of a type b firm should they choose to sell the rights is

$$\frac{I+a+b}{1+\frac{I}{P_S}} + P_R N_{RO}$$

Since $P_R \leq \frac{I+a+b}{1+\frac{J}{P_c}} - P_S$ for any $b \geq \underline{b}$, the above payoff is bounded by

$$\frac{I+a+b}{1+\frac{I}{P_S}} + \left(\frac{I+a+b}{1+\frac{I}{P_S}} - P_S\right)\frac{I}{P_S} = a+b,$$

which can be achieved by exercising the rights. Therefore, all shareholders have an incentive to exercise their rights. ■

Proof of Proposition 2: Given $\lambda < 1$, investors must purchase some shares in equilibrium. Since their purchase decision cannot depend on *b*, their break-even condition implies a unique P_{PO} . The per-share payoff to investors is

$$\frac{I+a+b}{1+N_{PO}} - P_{PO}$$

which is linear in b and has a unique root at $b = P_{PO} - a$. Hence, the investors payoff is nonzero for any firm type $b \neq P_{PO} - a$, implying wealth transfers. **Proof of Lemma 1:** Unconstrained shareholders subscribe if and only if

$$(\lambda N_{PQ} + 1)(I + a + b) - \lambda I(N_{PQ} + 1) > (I + a + b)$$

which is equivalent to

$$\lambda N_{PO}(I+a+b) \geq \lambda I(N_{PO}+1)$$

which in turn is equivalent to

$$N_{PO}(a+b) > I.$$

Together with the fact that $P_{PO}N_{PO} = I$, we have condition (5). **Proof of Proposition 3:** When $P_{PO} = a + \underline{b}$, condition (5) always holds, and unconstrained shareholders subscribe. In this case, $Pr(b \ge b_{PO}^*) = 1$ and $E(b|b \ge b_{PO}^*) = E(b)$. Therefore, investor payoff (7) becomes

$$[1 - (1 - \pi)\lambda][a + E(b) - P_{PO}],$$

which is strictly positive.

For any $P_{PO} \ge a + E(b)$, it follows by definition that $E(b|b \ge P_{PO} - a) \ge P_{PO} - a$, with strict inequality for $P_{PO} < a + \overline{b}$. Investor payoff (7) for such prices is strictly negative. By continuity, investor payoff (7) as a function of P_{PO} has a root and all roots lie in (a + b, a + E(b)).

Finally, we show that P_{PO} is decreasing in λ . Suppose $\lambda_1 < \lambda_2$. $P_{PO,i}$ denotes the corresponding solution to (7) for λ_i (i=1, 2). Also, $Pr_i(b|b \ge b_{PO,i}^*)$ and $E_i(b|b \ge b_{PO,i}^*)$ denote the corresponding values for λ_i , where $b_{PO,i}^*$ is given by (5). Since $P_{PO,i} < a + E(b) < a + E_i(b|b \ge b_{PO,i}^*)$, we have

$$a + E(b) - P_{PO,1} - Pr_1(b|b \ge b^*_{PO,1})(1 - \pi)\lambda_2 \Big[a + E\Big[b|b \ge b^*_{PO,1}\Big] - P_{PO,1} \Big].$$

$$< a + E(b) - P_{PO,1} - Pr_1(b|b \ge b^*_{PO,1})(1 - \pi)\lambda_1 \Big[a + E\Big[b|b \ge b^*_{PO,1}\Big] - P_{PO,1} \Big]$$

$$= 0.$$

Hence, for $\lambda = \lambda_2$, (7) is negative when $P_{PO} = P_{PO,1}$. Since (7) is positive for $P_{PO} = a + \underline{b}$, there must exist a $P_{PO,2} \in (a + \underline{b}, P_{PO,1})$, completing the proof. **Proof of Proposition 4:** From (8), we have

$$WT_{PO} = [a + E(b)] \left(1 - \frac{P_{PO}}{I + P_{PO}} \right) - \frac{P_{PO}}{I + P_{PO}} I$$
$$= [a + E(b)] \frac{I}{I + P_{PO}} - \frac{I}{I + P_{PO}} P_{PO}$$
$$= [a + E(b) - P_{PO}] \frac{I}{I + P_{PO}},$$

which establishes (9). Since $P_{PO} \le a + E(b)$ from Proposition 3, the wealth transfer $WT_{PO} \ge 0$, which clearly is decreasing in P_{PO} . Furthermore, the issue price P_{PO} is decreasing in λ (Proposition 3). Hence, WT_{PO} is increasing in λ .

Proof of Lemma 2: Unconstrained shareholders exercise their rights if

$$a+b \ge \frac{I+a+b}{N_{RO}+1} + P_R N_{RO},$$

which implies

$$N_{RO}(a+b) \ge I + P_R N_{RO}(N_{RO}+1),$$

which in turn implies

$$a+b \ge \frac{I}{N_{RO}} + P_R(N_{RO}+1).$$

Using the fact that $\frac{1}{N_{RO}} = P_S$, condition (10) follows immediately. **Proof of Proposition 5:** Rewriting (10) as

$$P_{R} = \frac{1}{N_{RO} + 1} \left[I + \frac{(1 - \pi)P(b < b_{RO}^{*})E(b|b < b_{RO}^{*}) + \pi E(b)}{(1 - \pi)P(b < b_{RO}^{*}) + \pi} + a \right] - P_{S}$$

and using to substitute P_R in (12), we have

$$\begin{split} b_{RO}^* &= P_S + \left[\frac{1}{(N_{RO}+1)} \left(I + \frac{(1-\pi)P(b < b_{RO}^*)E(b|b < b_{RO}^*) + \pi E(b)}{(1-\pi)P(b < b_{RO}^*) + \pi} + a \right) - P_S \right] (N_{RO}+1) - a \\ &= P_S + \left(I + \frac{(1-\pi)P(b < b_{RO}^*)E(b|b < b_{RO}^*) + \pi E(b)}{(1-\pi)P(b < b_{RO}^*) + \pi} + a \right) - P_S (N_{RO}+1) - a \\ &= P_S + I + \frac{(1-\pi)P(b < b_{RO}^*)E(b|b < b_{RO}^*) + \pi E(b)}{(1-\pi)P(b < b_{RO}^*) + \pi} + a - I - P_S - a \\ &= \frac{(1-\pi)P(b < b_{RO}^*)E(b|b < b_{RO}^*) + \pi E(b)}{(1-\pi)P(b < b_{RO}^*) + \pi}, \end{split}$$

which is the expression in the statement.

Next, we will show that $b_{RO}^* \in (\underline{b}, E(b))$. At $b_{RO}^* = \underline{b}$, the right-hand side of (13) is $E(b) > \underline{b}$. Since $E(b|b < b_{RO}^*) < E(\overline{b})$ whenever $b_{RO}^* < \overline{b}$, the right-hand side of (13) is in turn dominated by E(b). Therefore, b_{RO}^* must exist and lie in $(\underline{b}, E(b))$. \blacksquare **Proof of Proposition 6:** The payoff to constrained shareholders and to shareholders in firms of

Proof of Proposition 6: The payoff to constrained shareholders and to shareholders in firms of type $b < b_{RO}^*$ is

$$\frac{I+a+b}{N_R+1} + P_R N_{RO}$$

Using (10) and the definition of b_{RO}^* (Proposition 5) this payoff can be rewritten as

$$\frac{I+a+b}{N_{RO}+1} + \frac{N_{RO}}{N_{RO}+1}(I+a+b_{RO}^*) - N_{RO}P_S$$

$$= \frac{a+b+N_{RO}b_{RO}^* + N_{RO}a}{N_{RO}+1}$$

$$= a+b + \frac{N_{RO}}{N_{RO}+1}(b_{RO}^*-b)$$

$$= a+b + \frac{I}{I+P_S}(b_{RO}^*-b)$$

Since investors break even, the ex ante wealth transfer among shareholders is therefore

$$a + E(b) - \left\{ a + E(b) + E\left[\frac{I}{I + P_S} \left(b_{RO}^* - b\right)\right] \right\} = \frac{I}{I + P_S} \left[E(b) - b_{RO}^*\right].$$

Proof of Proposition 7: For $\lambda = 1$ Equation (7) and $b_{PO}^* = P_{PO} - a$ imply the following condition for b_{PO}^* :

$$E(b) - b_{PO}^* - P(b \ge b_{PO}^*)(1 - \pi) \left[E(b|b \ge b_{PO}^*) - b_{PO}^* \right] = 0.$$

Solving for b_{PO}^* , we have

$$b_{PO}^* = \frac{E(b) - (1 - \pi)P(b \ge b_{PO}^*)E(b|b \ge b_{PO}^*)}{1 - P(b \ge b_{PO}^*)(1 - \pi)}.$$
 (A1)

Together with the fact that

$$E(b) = P(b < b_{PO}^*)E(b|b < b_{PO}^*) + P(b \ge b_{PO}^*)E(b|b \ge b_{PO}^*),$$

condition (A1) is equivalent to the condition for b_{RO}^* in a rights offering (13). Therefore, $b_{PO}^* = b_{RO}^*$, which implies that $P_{PO} = P_S$ and $N_{PO} = N_{RO}$ as well. In both types of offerings, existing shareholders receive a + b if they participate (subscribe or exercise the rights) and the same payoff $\frac{L+a+b}{N_{RO}+1}$ if they do not participate. Overall, everyone's payoff in a public offering is exactly the same as in a rights offering.

Proof of Proposition 8: Consider a public offering equilibrium with issue price $P_{PO}(\lambda)$ and number of new shares $N_{PO}(\lambda)$, as characterized in Proposition 3. Suppose further that investors perceive any firm that deviates as being of type \underline{b} . Hence, if a firm deviates to another public offering with $\hat{\lambda} \neq \lambda$, it has to sell its shares at $\hat{P}_{PO} = a + \underline{b}$ with $\hat{N}_{PO} = \frac{I}{P_{PO}}$. The payoff (for investors and unconstrained shareholders) from buying these shares is

$$\frac{I+a+b}{\hat{N}_{PO}+1} - \hat{P}_{PO} = \frac{b-\underline{b}}{\hat{N}_{PO}+1} \ge 0.$$

As a result, no firm of type $b \le b_{PO}^*$ wants to deviate to $\hat{\lambda}$ because $\prod_{PO}(b|\lambda) \le 0$. Firms of type $b > b_{PO}^*$ do not deviate either if

$$[1 - \lambda(1 - \pi)]N_{PO}\left(\frac{I + a + b}{N_{PO} + 1} - P_{PO}\right) \le \left[1 - \hat{\lambda}(1 - \pi)\right]\hat{N}_{PO}\left(\frac{I + a + b}{\hat{N}_{PO} + 1} - \hat{P}_{PO}\right)$$
(A2)

holds. Since both sides are linear in b, it is sufficient to show that the inequality is satisfied at the endpoints b_{PO}^* and \bar{b} . By definition of b_{PO}^* (Lemma 2), the left-hand side equals 0 for $b = b_{PO}^*$, while the right- hand side is

$$\left[1-\hat{\lambda}(1-\pi)\right]\hat{N}_{PO}\left(\frac{b_{PO}^*-\underline{b}}{\hat{N}_{PO}+1}\right)>0.$$

For $b = \overline{b}$, the left-hand side of (A2) is the left-hand side of (18). The right-hand side of (A2) can be rewritten as

$$\left[1-\hat{\lambda}(1-\pi)\right]\left(\frac{I+a+\bar{b}}{1+\hat{P}_{PO}/I}-I\right) = \left[1-\hat{\lambda}(1-\pi)\right]I\left(\frac{\bar{b}-\underline{b}}{I+a+\underline{b}}\right),$$

which is (weakly) larger than the right-hand side of (18) since $\hat{\lambda} \leq 1$. Hence, condition (18) implies that (A2) holds for $b = \bar{b}$.

If a firm deviates to a rights offering with \hat{P}_S , the associated rights price \hat{P}_R is given by

$$\hat{P}_R = \frac{I+a+b}{\hat{N}_{RO}+1} - \hat{P}_S.$$
(A3)

For $\hat{P}_R \ge 0$, it must be that $\hat{P}_S \le a + b$. Condition (A3) and Lemma 2 imply that

$$b \ge \hat{P}_S + \hat{P}_R(\hat{N}_{RO} + 1) - a = \underline{b}$$

Hence, the payoff from exercising (and buying) rights is (weakly) positive. Consequently, no firm of type $b \le b_{PO}^*$ wants to deviate to a rights offering since $\prod_{PO}(b|\lambda) \le 0$. Firms of type $b > b_{PO}^*$ do not deviate either if

$$[1 - \lambda(1 - \pi)]N_{PO}\left(\frac{I + a + b}{N_{PO} + 1} - P_{PO}\right) \le \pi \hat{N}_{RO}\left(\frac{I + a + b}{\hat{N}_{RO} + 1} - \hat{P}_{S} - \hat{P}_{R}\right).$$
(A4)

As above, both sides are linear in b, and it is sufficient to show that the inequality is satisfied at the endpoints. For $b = b_{PO}^*$, the left-hand side equals 0, while the right-hand side is

$$\pi \hat{N}_{PO}\left(\frac{b_{PO}^* - \underline{b}}{\hat{N}_{PO} + 1}\right) > 0.$$

For $b = \overline{b}$, the right-hand side of (A4) is equal to

$$\pi \hat{N}_{RO} \left(\frac{\bar{b} - \underline{b}}{\hat{N}_{PO} + 1} \right) = \pi I \left(\frac{\bar{b} - \underline{b}}{I + \hat{P}_S} \right)$$

Since $\hat{P}_S \leq a + \underline{b}$, it must be that

$$\pi I\left(\frac{\bar{b}-\underline{b}}{I+\hat{P}_S}\right) \ge \pi I\left(\frac{\bar{b}-\underline{b}}{I+a+\underline{b}}\right)$$

holds. Hence, condition (18) implies that (A4) holds for $b = \overline{b}$.

Finally, as $\lambda \to 1$, the left-hand side of condition (18) becomes

$$\pi N_{PO}(1) \left(\frac{I+a+\bar{b}}{N_{PO}(1)+1} - P_{PO}(1) \right) = \pi \left(\frac{I+a+\bar{b}}{1+\frac{P_{PO}(1)}{I}} - I \right).$$

Since $P_{PO}(1) > a + \underline{b}$ (Proposition 3),

$$\pi \left(\frac{I+a+\bar{b}}{1+\frac{P_{PO}(1)}{I}} - I \right) < \pi \left(\frac{I+a+\bar{b}}{1+\frac{a+\bar{b}}{I}} - I \right) = \pi I \left(\frac{\bar{b}-\underline{b}}{I+a+\underline{b}} \right).$$

Thus, condition (18) is satisfied in the limit and in continuity also holds when λ is sufficiently close to one.

Proof of Proposition 9: Consider a rights offering equilibrium with strike price P_S , number of new shares N_{RO} , and associated rights price P_R , as characterized in Proposition 5. Furthermore, investors perceive any firm that deviates from this equilibrium rights offering as being of type *b*. Parallel to the proof of Proposition 8, it suffices to establishes that

$$\pi N_{RO}(P_S) \left(\frac{I+a+\bar{b}}{N_{RO}(P_S)+1} - P_S - P_R(P_S) \right) \le \pi \hat{N}_{RO} \left(\frac{I+a+b}{\hat{N}_{RO}+1} - \hat{P}_S - \hat{P}_R \right)$$
(A5)

for any deviating rights offering with \hat{P}_S and that

$$\pi N_{RO}(P_S) \left(\frac{I+a+\bar{b}}{N_{RO}(P_S)+1} - P_S - P_R(P_S) \right) \le \left[1 - \hat{\lambda}(1-\pi) \right] \hat{N}_{PO} \left(\frac{I+a+b}{\hat{N}_{PO}+1} - \hat{P}_{PO} \right)$$
(A6)

for any deviating public offering with $\hat{\lambda}$. The right-hand sides of (A5) and (A6) are the same as those in (A4) and (A2), which are both weakly positive. Since by Lemma 2

$$\pi N_{RO}(P_S) \left(\frac{I+a+\bar{b}}{N_{RO}(P_S)+1} - P_S - P_R(P_S) \right) \le 0$$

for any firm type $b \leq b_{RO}^*$, conditions (A5) and (A6) hold for these types. As for the firm types $b > b_{RO}^*$, it suffices to establishes that (A5) and (A6) hold for $b = \overline{b}$ because of the linearity in b. For $b = \overline{b}$ the left-hand sides of (A5) and (A6) are the left-hand side of (19). As shown in the proof of Proposition 8, the right-hand sides of (A5) and (A6) are weekly larger than $\pi I\left(\frac{\bar{b}-\bar{b}}{I+a+\bar{b}}\right)$, the right-hand side of (19). Hence, condition (19) implies that (A5) and (A6) hold for $b = \overline{b}$.

Given $P_R(P_S) \ge 0$, the left-hand side of (19) is bounded by

$$\pi N_{RO}(P_S) \left(\frac{I+a+\bar{b}}{N_{RO}(P_S)+1} - P_S \right) = \pi \left(\frac{I+a+\bar{b}}{1+\frac{P_S}{I}} - I \right).$$

Since $P_S \ge a + b$,

$$\pi\left(\frac{I+a+\bar{b}}{1+\frac{P_s}{I}}-I\right) \leq \pi\left(\frac{I+a+\bar{b}}{1+\frac{a+\bar{b}}{I}}-I\right) = \pi I\left(\frac{\bar{a}-\underline{a}}{I+a+\underline{b}}\right).$$

Thus, condition (19) holds for any $P_S \in [a + b, a + b_{RO}^*]$. **Proof of Lemma 3:** Suppose there were two public offerings with corresponding $\lambda_2 > \lambda_1$ both adopted by some firms. $P_{PO,i}$ and $b_{PO,i}^*$ (i=1, 2) denote the corresponding issue price and cutoff type in each offering. Lemma 1 and Proposition 3 imply that some firms in each offering must weakly lie below the respective cutoff type $b_{PO,i}^*$. Let $b_i \leq b_{PO,i}^*$ (i=1, 2) be two such firms. For these firms all N_i new shares are issued to the investors. From (16), each firm's optimal choice of issue terms implies

$$N_{i}\left[\frac{I+a+b_{i}}{N_{i}+1}-P_{PO,i}\right] \leq N_{-i}\left[\frac{I+a+b_{i}}{N_{-i}+1}-P_{PO,-i}\right].$$

Since $N_i P_{PO,i} = I$, we have

$$N_i \frac{I+a+b_i}{N_i+1} \le N_{-i} \frac{I+a+b_i}{N_{-i}+1}.$$

Hence, $\frac{N_i}{N_i+1} \leq \frac{N_{-i}}{N_{-i}+1}$ for i=1, 2, which implies $N_1 = N_2$ and as a result, $P_{PO,1} = P_{PO,2}$ and $b_{PO,1}^* = b_{PO,2}^*$. Finally, consider a different pair of firms in each issue mode, with their firm types above the respective cutoff types: $b_i > b_{PO,i}^*$. Using the fact that $b_{PO,i}^* = P_{PO,i} - a$, we have

$$\frac{I+a+b_i}{N_i+1} > \frac{I+a+b_{PO,i}^*}{\frac{I}{P_{PO,i}}+1} = \frac{I+P_{PO,i}}{\frac{I}{P_{PO,i}}+1} = P_{PO,i}.$$

In equilibrium, unconstrained shareholders subscribe, and new investors receive $[1 - \lambda_i(1 - \pi)]N_i$ new shares. The optimal choice of issue terms implies

$$[1 - \lambda_i(1 - \pi)]N_i \left[\frac{I + a + b_i}{N_i + 1} - P_{PO,i}\right] \le [1 - \lambda_{-i}(1 - \pi)]N_{-i} \left[\frac{I + a + b_i}{N_{-i} + 1} - P_{PO,-i}\right].$$

Using the fact that N_i and $P_{PO,i}$ are the same across i = 1, 2, the above expression simplifies to

$$1 - \lambda_i(1 - \pi) \leq 1 - \lambda_{-i}(1 - \pi),$$

for i = 1, 2. Hence, $\lambda_1 = \lambda_2$. There is at most one public offering in equilibrium. Suppose there were two rights offerings with strike prices $P_{S,2} > P_{S,1}$, both adopted by some firms. We write the corresponding rights price as $P_{R,i}$ (i = 1, 2). By Proposition 5, some unconstrained shareholders must choose to (or not to) exercise the rights in each offer. $b_{I,e}$ ($b_{i,ne}$) i = 1, 2 denotes the type of firms that issue rights with a strike price of $P_{S,i}$, and the unconstrained shareholders (do not) exercise their rights. The optimal choice of offer terms states

$$\pi N_i \left[\frac{I+a+b_{i,e}}{N_i+1} - P_{S,i} - P_{R,i} \right] \le \pi N_{-i} \left[\frac{I+a+b_{i,e}}{N_{-i}+1} - P_{S,-i} - P_{R,-i} \right],$$

which, combined with the fact that $P_{S,i}N_i = I$, implies

$$N_{i}\left[\frac{I+a+b_{i,e}}{N_{i}+1}-P_{R,i}\right] \leq N_{-i}\left[\frac{I+a+b_{i,e}}{N_{-i}+1}-P_{R,-i}\right]$$

Since $N_1 = \frac{I}{P_{S,1}} > \frac{I}{P_{S,2}} = N_2$, the above condition implies

$$b_{1,e} \leq b^{cf} \equiv \frac{P_{R,1}N_1 - P_{R,2}N_2}{\frac{N_1}{N_1 + 1} - \frac{N_2}{N_2 + 1}} - (I+a),$$

and

$$b_{2,e} \ge b^{cf} \ge b_{1,e}$$
.

On the other hand, the optimal offer term choice for the $b_{i,ne}$ firm implies

$$N_{i}\left[\frac{I+a+b_{i,ne}}{N_{i}+1}-P_{R,i}\right] \leq N_{-i}\left[\frac{I+a+b_{i,ne}}{N_{-i}+1}-P_{R,-i}\right],$$

which following the same logic, implies $b_{1,ne} \leq b_{2,ne}$, $b_{1,ne} \leq b^{cf}$, and $b_{2,ne} \geq b^{cf}$. Hence, combined with Lemma 2, we have

$$b_{1,ne} < b_{1,e} \le b^{cf} \le b_{2,ne} < b_{2,e}$$

However, this relation cannot hold in equilibrium, because $b_{1,e}$ firm has an incentive to deviate to the rights offering with strike price $P_{S,2}$. With $P_{S,1}$, investors in the $b_{1,e}$ firm collectively receive

$$\pi N_1 \left[\frac{I+a+b_{1,e}}{N_i+1} - P_{S,1} - P_{R,1} \right],$$

which is weakly positive because of Lemma 2 and the fact that the unconstrained shareholders exercise their rights, that is, $b_{1,e} \ge b_{RO,1}^*$. On the other hand, with $P_{S,2}$, investors in the $b_{1,e}$ firm collectively receive

$$N_2\left[\frac{I+a+b_{1,e}}{N_i+1}-P_{S,2}-P_{R,2}\right] < N_2\left[\frac{I+a+b_{2,ne}}{N_i+1}-P_{S,2}-P_{R,2}\right],$$

which is in turn weakly negative because the unconstrained shareholders in $b_{2,ne}$ firms choose to sell the rights, that is, $b_{2,ne} \leq b_{RO,2}^*$. Consequently,

$$\pi N_1 \left[\frac{I+a+b_{1,e}}{N_i+1} - P_{S,1} - P_{R,1} \right] \ge 0 > N_2 \left[\frac{I+a+b_{1,e}}{N_i+1} - P_{S,2} - P_{R,2} \right].$$

Hence, the $b_{1,e}$ firm has an incentive to deviate to using a rights offering with strike price $P_{S,2}$. Contradiction! Concluding the proof.

Proof of Proposition 10: We begin with the following lemma that establishes the existence of b^{\dagger} .

Lemma 4.Suppose b_1 (b_2) is any firm type where the unconstrained shareholders (do not) subscribe to the new shares. Then $b_1 > b_2$.

Proof of Lemma 4: Suppose instead that $b_1 < b_2$. By Lemma 1 and 2 relatively better firms see shareholders participate. Therefore, b_1 and b_2 firms must have different offering modes. Without loss of generality assume b_1 adopts a public offering with dilution protection λ and b_2 adopts a rights offering with strike price P_S and rights price P_R . Because current shareholders participate, Lemma 1 states that

$$b_1 > P_{PO} - a$$

Hence, the investor payoff in firm b_1 is

$$[1 - \lambda(1 - \pi)]N_{PO}\left[\frac{I + a + b_1}{N_{PO} + 1} - P_{PO}\right] > 0.$$

Similarly, Lemma 2 implies the investor payoff in firm b_2 is

$$N_{RO}\left[\frac{I+a+b_2}{N_{RO}+1}-P_S-P_R\right] < 0$$

However, b_1 firm then has an incentive to deviate to the rights offerings because

$$N_{RO}\left[\frac{I+a+b_1}{N_{RO}+1} - P_S - P_R\right] < N_{RO}\left[\frac{I+a+b_2}{N_{RO}+1} - P_S - P_R\right] < 0.$$

The contradiction establishes the lemma.

As a consequence of Lemma 4, there is a cutoff type, b^{\dagger} , below which current shareholders do not participate in the offering. The next lemma establishes the existence of b^{\dagger} . Lemma 5. There exist $\underline{b}^{\dagger} < b^{\dagger}$ such that all firms with $b \in (\underline{b}^{\dagger}, b^{\dagger})$ (resp. $b < \overline{b}^{\dagger}$) choose public (resp. rights) offerings.

Proof of Lemma 5: By Lemma 4, the set of firms in $(\underline{b}, \underline{b}^{\dagger})$ can be partitioned into two subsets B_{RO} and B_{PO} , conducting rights offerings and public offerings respectively. Lemma

1 and Lemma 4 imply that both sets are nonempty. For any firm type $b_1 \in B_{PO}$, the IC condition suggests

$$N_{RO}\left[\frac{I+a+b_1}{N_{RO}+1} - P_S - P_R\right] \ge N_{PO}\left[\frac{I+a+b_1}{N_{PO}+1} - P_{PO}\right].$$
 (A7)

Proposition 7 states that a rights offering with $P_R = 0$ is equivalent to a public offering with $\lambda = 1$, which in turn implies two public offerings cannot coexist. Thus, it must be that $P_R > 0$. Using the fact $N_{RO}P_S = N_{PO}P_{PO} = I$ and $P_R > 0$, condition (A7) implies

$$N_{RO}\frac{I+a+b_1}{N_{RO}+1} > N_{RO}\left[\frac{I+a+b_1}{N_{RO}+1} - P_R\right] \ge N_{PO}\frac{I+a+b_1}{N_{PO}+1}.$$

Hence, $N_{RO} > N_{PO}$. This condition in turn implies

$$b_1 \geq \frac{N_{RO}P_R}{\frac{N_{RO}}{N_{RO}+1} - \frac{N_{PO}}{N_{PO}+1}} - a - I \equiv \underline{b}^{\dagger}.$$

Completing the proof of the lemma. ■

Finally, we are ready to establish the existence of \vec{b}' . From Lemma 5, (16), (17), and the issue mode being chosen optimally, we have

$$\begin{cases} \Pi_{PO}(b) > \Pi_{RO}(b) & \text{ for } b < \underline{b}^{\dagger} \\ \Pi_{PO}(b) < \Pi_{RO}(b) & \text{ for } \underline{b}^{\dagger} < b < b^{\dagger} \end{cases}$$

Clearly, both Π_{PO} and Π_{RO} are increasing, concave, and piece-wise linear functions. The only kink is the unique root of each respective function. Therefore, the graph of Π_{PO} and Π_{RO} must intersect exactly once at \underline{b}^{\dagger} when $\Pi < 0$ and once when $\Pi > 0$. Write the intersection as \overline{b}^{\dagger} . If such an \overline{b}^{\dagger} does not exist, only one issue mode exists for $b > b^{\dagger}$, when shareholders subscribe, a contradiction.

Since Π_{RO} is steeper than Π_{PO} when $\Pi < 0$ ($N_{RO} > N_{PO}$), the reverse must be true when $\Pi > 0$ to generate an intersection. Hence, we must have

$$\begin{cases} \Pi_{PO}(b) > \Pi_{RO}(b) & \text{ for } b > \bar{b}^{^{\dagger}} \\ \Pi_{PO}(b) < \Pi_{RO}(b) & \text{ for } b^{^{\dagger}} < b < \bar{b}^{^{\dagger}} \end{cases}$$

This establishes the proposition. \blacksquare

B Section 5: Single-Offer Mode Equilibrium Outcomes

Proposition 12. When the support of $[a, \bar{a}]$ is large enough and

$$[1 - \lambda(1 - \pi)]N_{PO}\left(\frac{\hat{a}_{PO} + I + b}{N_{PO} + 1} - P_{PO}\right) \le \pi I\left(\frac{\hat{a}_{PO} - \underline{a}}{\underline{a} + I + b}\right).$$
(A8)

holds, there exist semipooling equilibria in which firm types $a \in (\hat{a}_{PO}, \bar{a}]$ do not issue shares, firm types $a \in [\underline{a}, \hat{a}_{PO}]$ all choose a public offer with a common λ , and only unconstrained

shareholders in firms $a \in [a_{PO}^*, \hat{a}_{PO}]$ subscribe to the offer. The two cutoff types a_{PO}^* and \hat{a}_{PO} solve

$$a_{PO}^{*} = \frac{(1 - \lambda(1 - \pi))Pr(a_{PO}^{*} \le a < \hat{a}_{PO})E(a|a_{PO}^{*} \le a < \hat{a}_{PO}) + Pr(a < a_{PO}^{*})E(a|a < a_{PO}^{*})}{(1 - \lambda(1 - \pi))Pr(a_{PO}^{*} \le a < \hat{a}_{PO}) + Pr(a < a_{PO}^{*})}$$
(A9)

and

$$(1 - \lambda(1 - \pi))\frac{N_{PO}}{1 + N_{PO}}(I + \hat{a}_{PO} + b) - (1 - \lambda(1 - \pi))I = b.$$
(A10)

Moreover, condition (A8) holds for $a \sim U(\underline{a}, \overline{a})$ and either λ or π sufficiently close to one.

Proof: Given unconstrained shareholders subscribe for $a \ge a_{PO}^* \equiv P_{PO} - b$, investors break even when

$$\begin{aligned} & Pr(a < a_{PO}^{*}) \left(\frac{N_{PO}}{N_{PO} + 1} \left(b + I + E(a|a < a_{PO}^{*}) \right) - I \right) \\ & + Pr(a_{PO}^{*} \le a < \hat{a}_{PO}) \times (1 - \lambda(1 - \pi)) \times \left(\frac{N_{PO}}{N_{PO} + 1} \left(b + I + E(a|a_{PO}^{*} \le a < \hat{a}_{PO}) \right) - I \right) = 0 \end{aligned}$$

Solving the break-even condition for P_{PO} yields

$$P_{PO} = \frac{(1 - \lambda(1 - \pi))Pr(a_{PO}^* \le a < \hat{a}_{PO})(b + E(a|a_{PO}^* \le a < \hat{a}_{PO})) + Pr(a < a_{PO}^*)(b + E(a|a < a_{PO}^*))}{(1 - \lambda(1 - \pi))Pr(a_{PO}^* \le a < \hat{a}_{PO}) + Pr(a < a_{PO}^*)}$$

and allows us to express $a_{PO}^* = P_{PO} - b$ as

$$a_{PO}^{*} = \frac{(1 - \lambda(1 - \pi))Pr(a_{PO}^{*} \le a < \hat{a}_{PO})E(a|a_{PO}^{*} \le a < \hat{a}_{PO}) + Pr(a < a_{PO}^{*})E(a|a < a_{PO}^{*})}{(1 - \lambda(1 - \pi))Pr(a_{PO}^{*} \le a < \hat{a}_{PO}) + Pr(a < a_{PO}^{*})}$$

For the cutoff type \hat{a}_{PO} shareholders are indifferent between investing or not and therefore

$$(1 - \lambda(1 - \pi))\frac{N_{PO}}{1 + N_{PO}}(I + \hat{a}_{PO} + b) - (1 - \lambda(1 - \pi))I = b.$$

Solving this equality for \hat{a}_{PO} gives

$$\hat{a}_{PO} = (1 + N_{PO}) \left[\frac{b}{(1 - \lambda(1 - \pi))N_{PO}} + P_{PO} \right] - I - b$$

$$= \left(1 + \frac{I}{a_{PO}^* + b} \right) \left[\frac{b(a_{PO}^* + b)}{(1 - \lambda(1 - \pi))I} + a_{PO}^* + b \right] - I - b$$

$$= \frac{b(a_{PO}^* + b)}{(1 - \lambda(1 - \pi))I} + \frac{b}{(1 - \lambda(1 - \pi))} + a_{PO}^* + b + I - I - b$$

$$= a_{PO}^* + \frac{b(a_{PO}^* + b + I)}{(1 - \lambda(1 - \pi))I}.$$
(A11)

To establish the existence of a_{PO}^* , we use (A11) to substitute \hat{a}_{PO} in (A9) and define

$$f(x) \equiv x - \frac{(1 - \lambda(1 - \pi))Pr\left(x \le a < x + \frac{b(x + b + I)}{(1 - \lambda(1 - \pi))I}\right)E\left(a|x \le a < x + \frac{b(x + b + I)}{(1 - \lambda(1 - \pi))I}\right) + Pr(a < x)E(a|a < x)}{(1 - \lambda(1 - \pi))Pr\left(x \le a < x + \frac{b(x + b + I)}{(1 - \lambda(1 - \pi))I}\right) + Pr(a < x)}$$

Existence of a_{PO}^* is equivalent to a root of f(x). At $x = \underline{a}$

$$f(\underline{a}) = \underline{a} - E\left[a|\underline{a} \le a < \underline{a} + \frac{b(\underline{a} + b + I)}{(1 - \lambda(1 - \pi))I}\right] < 0$$

and at $x = \bar{a}$

$$f(\bar{a}) = \bar{a} - E(a|a < \bar{a}) > 0$$

In continuity, a root of f(x) exists in $[\underline{a}, \overline{a}]$ and we define it as a_{PO}^* . The definition of f is independent of \overline{a} , and so is a_{PO}^* . Therefore, as long as \overline{a} is large enough, $\hat{a}_{PO} = a_{PO}^*$ + $\frac{b(a_{PO}^* + b+I)}{(1-\lambda(1-\pi))I} > a_{PO}^*$ also must be strictly less than \overline{a} , which implies that \hat{a}_{PO} exists in $(a_{PO}^*, \overline{a}]$.

Condition (A8) ensures that no type has an incentive to deviate. Conditional on investing $a \in [\underline{a}, \hat{a}_{PO}]$, condition (A8) is equivalent to (18). Proposition 8 therefore implies that no issuing types has an incentive to deviate to another issue method. Since type \hat{a}_{PO} is indifferent between investing and not, all types $a < \hat{a}_{PO}$ are better off investing. Conversely, all types $a > \hat{a}_{PO}$ prefer doing nothing to the equilibrium issue mode, which in turn dominates any other issue method for these types.

Finally, the equilibrium condition (A10) implies that the left-hand side of condition (A8) is equal to b and hence simplifies to

$$b \leq \pi I \left(\frac{\hat{a}_{PO} - \underline{a}}{\underline{a} + I + b} \right).$$

With $a \sim U(a, \bar{a})$ we can analytically solve for

$$a_{PO}^* = \underline{a} + \frac{\left(b + I + \underline{a}\right)b}{I\sqrt{1 - \lambda(1 - \pi)} - b},$$

and

$$\hat{a}_{PO} = a_{PO}^* + \frac{b(a_{PO}^* + b + I)}{(1 - \lambda(1 - \pi))I}$$

As $\lambda \to 1 \ a_{PO}^*$ approach $a_{\lambda}^{\lim} \equiv \underline{a} + \frac{(b+I+\underline{a})b}{I\sqrt{\pi}-b} > \underline{a}$, and for or $\pi \to 1 \ a_{PO}^*$ approaches $a_{\pi}^{\lim} \equiv \underline{a} + \frac{(b+I+\underline{a})b}{I-b} > \underline{a}$.

$$\pi I \left(\frac{\hat{a}_{PO} - \underline{a}}{\underline{a} + I + b}\right) > \pi I \left(\frac{b(a_{PO}^* + b + I)}{(1 - \lambda(1 - \pi))I}\right)$$
$$= b\left(\frac{a_{PO}^* + b + I}{\underline{a} + I + b}\right) \left(\frac{\pi}{(1 - \lambda(1 - \pi))}\right)$$
$$\to b\frac{a_{\lambda}^{\lim} + b + I}{\underline{a} + I + b} > b,$$

respectively, $b \frac{d_n^{lm} + b + I}{a + I + b} > b$ in the final step for $\pi \to 1$. Hence, condition (A8) holds for either λ or π sufficiently close to one.

Proposition 13. When the support of $[a, \bar{a}]$ is large enough and

$$\pi N_{RO} \left(\frac{\hat{a}_{RO} + I + b}{N_{RO} + 1} - P_S - P_R \right) \le \pi I \left(\frac{\hat{a}_{RO} - a}{\underline{a} + I + b} \right)$$
(A12)

holds, there exist semipooling equilibria in which firm types $a \in (\hat{a}_{RO}, \bar{a}]$ do not issue any rights, firm types $a \in [\underline{a}, \hat{a}_{RO}]$ all choose a rights offer with a common P_S , and only unconstrained shareholders in firms $a \in [a_{RO}^*, \hat{a}_{RO}]$ subscribe to the offer. The two cutoff types a_{RO}^* and \hat{a}_{RO} solve

$$a_{RO}^{*} = \frac{\pi Pr(a_{RO}^{*} \le a < \hat{a}_{RO}) E(a | a_{RO}^{*} \le a < \hat{a}_{RO}) + Pr(a < a_{RO}^{*}) E(a | a < a_{RO}^{*})}{\pi Pr(a_{RO}^{*} \le a < \hat{a}_{RO}) + Pr(a < a_{RO}^{*})}$$
(A13)

and

$$\pi N_{RO} \left(\frac{I + \hat{a}_{RO} + b}{1 + N_{RO}} - P_R - P_S \right) = b.$$
(A14)

Moreover, condition (A12) always holds for any $P_S \ge a + b$.

Proof: Given unconstrained shareholders subscribe for $a \ge a_{RO}^* \equiv P_S + P_R(N_{RO} + 1) - b$, investors break even when

$$P_{R}+P_{S} = \frac{1}{N_{RO}+1} \left(b + I + \frac{\pi Pr(a_{RO}^{*} \le a < \hat{a}_{RO})E(a|a_{RO}^{*} \le a < \hat{a}_{RO}) + Pr(a < a_{RO}^{*})E(a|a < a_{RO}^{*})}{\pi Pr(a_{RO}^{*} \le a < \hat{a}_{RO}) + Pr(a < a_{RO}^{*})} \right)$$

Using the break-even condition to substitute P_R in the definition of a_{RO}^* gives

$$a_{RO}^* = \frac{\pi Pr(a_{RO}^* \le a < \hat{a}_{RO}) E(a|a_{RO}^* \le a < \hat{a}_{RO}) + Pr(a < a_{RO}^*) E(a|a < a_{RO}^*)}{\pi Pr(a_{RO}^* \le a < \hat{a}_{RO}) + Pr(a < a_{RO}^*)}$$

For the cutoff type \hat{a}_{RO} shareholders are indifferent between investing or not and therefore

$$\pi N_{RO}\left(\frac{I+\hat{a_{RO}}+b}{1+N_{RO}}-P_R-P_S\right)=b.$$

Using Equations (A13) and (A14), the cutoff type \hat{a}_{RO} can be written as

$$\hat{a}_{RO} = (1 + N_{RO}) \left(\frac{b}{\pi N_{RO}} + P_R + P_S \right) - I - b$$

= $(1 + N_{RO}) \left[\frac{b}{\pi N_{RO}} + \frac{1}{N_{RO} + 1} \left(b + I + a_{RO}^* \right) \right] - I - b$
= $\frac{b(1 + N_{RO})}{\pi N_{RO}} + b + I + a_{RO}^* - I - b = a_{RO}^* + \frac{b}{\pi} \left(\frac{P_S}{I} + 1 \right).$ (A15)

We now establish the existence of a_{RO}^* and \hat{a}_{RO} . We use (A15) to substitute \hat{a}_{RO} in (A13), and define

$$f(x) = x - \frac{\pi Pr(x \le a < x + \frac{b}{\pi} \left(\frac{P_S}{I} + 1\right)) E(a|x \le a < x + \frac{b}{\pi} \left(\frac{P_S}{I} + 1\right)) + Pr(a < x) E(a|a < x)}{\pi Pr(x \le a < x + \frac{b}{\pi} \left(\frac{P_S}{I} + 1\right)) + Pr(a < x)}.$$

Existence of a_{RO}^* is equivalent to a root of f(x). At x = a,

$$f(\underline{a}) = \underline{a} - E\left[a|\underline{a} \le a < \underline{a} + \frac{b}{\pi}\left(\frac{P_S}{I} + 1\right)\right] < 0$$

and at $x = \bar{a}$,

$$f(\bar{a}) = \bar{a} - E(a|a < \bar{a}) > 0.$$

In continuity, a root of f(x) exists in $[\underline{a}, \overline{a}]$ and we define it as a_{RO}^* . The definition of f is independent of \overline{a} , and so is a_{RO}^* . Therefore, as long as \overline{a} is large enough, $\hat{a}_{RO} = a_{RO}^* + \frac{b}{\pi} (\frac{P_s}{I} + 1)$ also must be strictly less than \overline{a} , which implies that \hat{a}_{RO} exists in $(a_{RO}^*, \overline{a}]$.

Condition (A12) ensures that no type has an incentive to deviate. Conditional on investing $a \in [a, \hat{a}_{RO}]$, condition (A12) is equivalent to (19). Proposition 9 therefore implies that no issuing type has an incentive to deviate to another issue method. Since type \hat{a}_{RO} firm is indifferent between investing and not, all types $a < \hat{a}_{RO}$ are better off investing. Conversely, all types $a > \hat{a}_{RO}$ prefer doing nothing to the equilibrium issue method, which in turn dominates any other methods for these types.

Finally, the equilibrium condition (A14) implies that the left-hand side of condition (A12) is equal to b and hence the condition simplifies to

$$b \leq \pi I\left(\frac{\hat{a}_{RO} - \underline{a}}{\underline{a} + I + b}\right)$$

Since $\hat{a}_{RO} = a_{RO}^* + \frac{b}{\pi} \left(\frac{P_S}{I} + 1 \right) \ge \underline{a} + \frac{b}{\pi} \left(\frac{P_S}{I} + 1 \right)$ and by assumption $P_S \ge \underline{a} + b$,

$$\pi I\left(\frac{\hat{a}_{RO}-\underline{a}}{\underline{a}+I+b}\right) \ge \pi I\left(\frac{\underline{b}_{\pi}\left(\frac{P_{S}}{I}+1\right)}{\underline{a}+I+b}\right) \ge \pi I\left(\frac{\underline{b}_{\pi}\left(\frac{\underline{a}+b}{I}+1\right)}{\underline{a}+I+b}\right) = b.$$

C Section 5: Coexisting Equilibrium Outcomes

There are the cutoff types \underline{a}^{\dagger} , a^{\dagger} , and \overline{a}^{\dagger} parallel to those in terms of project NPV in Proposition 10. When the support of *a* is large enough, there exists an additional cutoff type, $\hat{a} \in (\overline{a}^{\dagger}, \overline{a})$, which satisfies

$$\pi N_{RO} \left(\frac{I + \hat{a} + b}{1 + N_{RO}} - P_{RO} - P_S \right) = b.$$
 (A16)

Since $\hat{a} > \overline{a}^{\dagger} > a^{\dagger}$, type \hat{a} firm uses rights offering if it issues new equity, and the unconstrained shareholders exercise their rights. The left-hand side of condition (A16) is the payoff to the investors from exercise the rights sold by the constrained shareholders. Condition (A16) implies that when type \hat{a} -firm conducts a rights offering, the investors extract the entire NPV *b*, leaving the shareholders indifferent about investing or not. Hence, all firm types below \hat{a} issue and those above do not.

D Section 5: Proof of Proposition 11

With $a \sim U(\underline{a}, \overline{a})$ the cutoff type a_{PO}^* as given in equation A9 becomes

$$a_{PO}^{*} = \frac{(1 - \lambda(1 - \pi))\frac{(\hat{a}_{PO} - a_{PO}^{*})(a_{PO}^{*} + \hat{a}_{PO})}{2(\bar{a} - \underline{a})} + \frac{(a_{PO}^{*} - \underline{a})(a_{PO}^{*} + \underline{a})}{2(\bar{a} - \underline{a})}}{(1 - \lambda(1 - \pi))\frac{\hat{a}_{PO} - a_{PO}^{*}}{\overline{a} - \underline{a}} + \frac{a_{PO}^{*} - \underline{a}}{\overline{a} - \underline{a}}}$$
(A17)

Using (A11) to substitute \hat{a}_{PO} in (A17) yields

$$a_{PO}^{*} = \frac{(b+I)b + I\underline{a}\sqrt{1-\lambda(1-\pi)}}{I\sqrt{1-\lambda(1-\pi)} - b} = \underline{a} + \frac{(b+I+\underline{a})b}{I\sqrt{1-\lambda(1-\pi)} - b}$$

Hence, a_{PO}^* is increasing in λ , and therefore also $\hat{a}_{PO} = a_{PO}^* + \frac{b(a_{PO}^*+b+I)}{(1-\lambda(1-\pi))I}$ and $Pr(a < \hat{a}_{PO})$. With $a \sim U(\underline{a}, \overline{a})$ the wealth transfer conditional on investing

$$E(a|a < \hat{a}_{PO}) + b - \frac{1}{1 + \frac{I}{P_{PO}}} [I + E(a|a < \hat{a}_{PO}) + b]$$

becomes

$$\begin{split} &= \frac{I}{I+P_{PO}} [E(a|a < \hat{a}_{PO}) + b - P_{PO}] = \frac{I}{I+b+a_{PO}^*} [E(a|a < \hat{a}_{PO}) - a_{PO}^*] \\ &= \frac{I}{I+b+a_{PO}^*} \left[E \left(a|a < a_{PO}^* + \frac{b(a_{PO}^* + b + I)}{(1-\lambda(1-\pi))I} \right) - a_{PO}^* \right] \\ &= \frac{b}{2[1-\lambda(1-\pi)]} + \frac{I(\underline{a} - a_{PO}^*)}{2(I+b+a_{PO}^*)} \\ &= \frac{b}{2[1-\lambda(1-\pi)]} + \frac{I\left(\underline{a} - \frac{(b+I)b+I\underline{a}\sqrt{1-\lambda(1-\pi)}}{I\sqrt{1-\lambda(1-\pi)} - b} \right)}{2\left(I+b+\frac{(b+I)b+I\underline{a}\sqrt{1-\lambda(1-\pi)}}{I\sqrt{1-\lambda(1-\pi)} - b} \right)} \\ &= \frac{b}{2[1-\lambda(1-\pi)]} - \frac{b}{2\sqrt{1-\lambda(1-\pi)}} = \frac{b}{2\sqrt{1-\lambda(1-\pi)}} \left(\frac{1}{\sqrt{1-\lambda(1-\pi)}} - 1\right), \end{split}$$

and is increasing in λ , like $Pr(a < \hat{a}_{PO})$. Hence, \hat{WT}_{PO} is increasing in λ .

With $a \sim U(\underline{a}, \overline{a})$ the cutoff type a_{RO}^* as given by Equation (A13) becomes

$$a_{RO}^{*} = \frac{\pi \frac{(\hat{a}_{RO} - a_{RO}^{*})(a_{RO}^{*} + \hat{a}_{RO})}{2(\hat{a} - a)} + \frac{(a_{RO}^{*} - \underline{a})(a_{RO}^{*} + \underline{a})}{2(\hat{a} - \underline{a})}}{\pi \frac{\hat{a}_{RO} - a_{RO}^{*}}{\hat{a} - \underline{a}} + \frac{a_{RO}^{*} - \underline{a}}{\hat{a} - \underline{a}}}$$
(A18)

Using (A15) to substitute \hat{a}_{PO} in (A18) yields

$$a_{RO}^* = \frac{b}{\sqrt{\pi}} \left(\frac{P_S}{I} + 1\right) + \underline{a}$$

Hence, a_{RO}^* is increasing in P_S , and therefore also $\hat{a}_{RO} = a_{RO}^* + \frac{b}{\pi} \left(\frac{P_S}{I} + 1\right)$ and $Pr(a < \hat{a}_{RO})$. With $a \sim U(a, \bar{a})$ the wealth transfer conditional on investing

$$\frac{I}{I+P_S} \left[E(a|a \le \hat{a}_{RO}) - a_{RO}^* \right]$$

becomes

$$= \frac{I}{P_{S}+I} \left(\frac{\underline{a} + a_{RO}^{*} + \frac{b}{\pi} \left(\frac{P_{S}}{I} + 1 \right)}{2} - a_{RO}^{*} \right) = \frac{b}{2\pi} + \frac{I}{P_{S}+I} \left(\frac{\underline{a} - a_{RO}^{*}}{2} \right)$$
$$= \frac{b}{2\pi} + \frac{I}{P_{S}+I} \left(\frac{\frac{b}{\sqrt{\pi}} \left(\frac{P_{S}}{I} + 1 \right)}{2} \right) = \frac{b}{2\pi} + \frac{b}{2\sqrt{\pi}},$$

and is independent of P_S . Since $Pr(a < \hat{a}_{RO})$ is increasing in P_S , so is \hat{WT}_{RO} .

References

Agrawal, A. 2012. Corporate governance objectives of labor union shareholders: Evidence from proxy voting. *Review of Financial Studies* 25:187–226.

Armitage, S. 2007. Discounts in placing pre-renounced shares in rights issues. *Journal of Business Finance & Accounting* 34:1345–69.

Babkin, A., B. Glover, and O. Levine. 2017. Are corporate inversions good for shareholders? *Journal of Financial Economics* 126:227–51.

Barberis, N., and R. Thaler. 2003. A survey of behavioral finance. *In Handbook of the economics of finance*, eds. G. Constantinides, M. Harris, and R. Stulz, 1053–128. Amsterdam, the Netherlands: Elsevier North Holland.

Barnes, E., and M. Walker. 2006. The seasoned-equity issues of UK firms: Market reaction and issuance method choice. *Journal of Business Finance & Accounting* 33:45–78.

Berk, J., and P. DeMarzo. 2017. Corporate Finance, 4th ed. Harlow, UK: Pearson.

Bhat, R., and I. Pandey. 1993. Dividend decision: A study of managers' perception. *Working Paper*, Indian Institute of Management Ahmedabad.

Billett, M., I. Floros, and J. Garfinkel. 2019. At-the-market offerings. *Journal of Financial and Quantitative Analysis* 54:1263–83.

Bøhren, Ø., B. Eckbo, and D. Michalsen. 1997. Why underwrite rights offerings? Some new evidence. *Journal of Financial Economics* 46:223–61.

Bolton, P., T. Li, E. Ravina, and H. Rosenthal. 2020. Investor ideology. *Journal of Financial Economics* 137:320–52.

Chen, X., J. Harford, and K. Li. 2007. Monitoring: Which institutions matter? *Journal of Financial Economics* 86:279–305.

Chiu, J. 2011. Wharf seeks \$1.3 billion in rights offering. *Wall Street Journal*, February 11. https://www.wsj.com/articles/SB10001424052748703745704576136833521169042

Cronqvist, H., and M. Nilsson. 2005. The choice between rights offerings and private equity placements. *Journal of Financial Economics* 78:375–407.

Daneshkhu, S. 2008. Heavy discount on Natixis rights issue. *Financial Times*. https://www.ft.com/content/1ffSeebe-7aa7-11dd-adbe-000077b07658

Desai, M., and L. Jin. 2012. Institutional tax clienteles and payout policy. *Journal of Financial Economics* 100:68–84.

Duong, T., R. Singh, and E. Tan. 2014. What's Wrong with Rights? Working Paper, Iowa State University.

Eckbo, B., and R. Masulis. 1992. Adverse selection and the rights offer paradox. *Journal of Financial Economics* 32:293–332.

Eckbo, B., R. Masulis, and Ø. Norli. 2007. Security offerings. In *Handbook of Corporate Finance: Empirical Corporate Finance 1*, ed. B. Eckbo, 233–373. Amsterdam: Elsevier North Holland.

Eckbo, B., and Ø. Norli. 2005. The equity flotation method pecking order. Working Paper, Tuck Business School, Dartmouth College.

Eckbo, B., and S. Verma. 1994, Managerial share ownership, voting power, and cash dividend policy. *Journal of Corporate Finance* 1:33–62.

Farinha, J., C. Mateus, and N. Soares. 2017. Price discounts in rights issues: Why do Managers insist on what investors hate? *European Business Review* 29:457–75.

Fried, J., and H. Spamann. 2020. Cheap-stock tunneling around preemptive rights. *Journal of Financial Economics* 137:353–70.

Gajewski, J., and E. Ginglinger. 2002. Seasoned equity issues in a closely held market: Evidence from France. *Review of Finance* 6:291–319.

Gaspar, J., M. Massa, and P. Matos. 2005. Shareholder investment horizons and the market for corporate control. *Journal of Financial Economics* 76:135–65.

Gaspar, J., M. Massa, P. Matos, R. Patgiri, and Z. Rehman. 2013. Payout policy choices and shareholder investment horizons. *Review of Finance* 17:261–320.

Glosten, L., and P. Milgrom. 1985. Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics* 14:71–100.

Hansen, R. 1988. The demise of the rights issue. Review of Financial Studies 1: 289-309.

Hansen, R., and J. Pinkerton. 1982. Direct equity financing: A resolution of a paradox. *Journal of Finance* 37:651–65.

Heinkel, R., and E. Schwartz. 1986. Rights versus underwritten offerings: An asymmetric information approach. *Journal of Finance* 41:1–18.

Holderness, C. 2018. Equity issuances and agency costs: The telling story of shareholder approval around the world. *Journal of Financial Economics* 129:415–39.

Holderness, C., and J. Pontiff. 2016. Shareholder nonparticipation in valuable rights offerings: New findings for an old puzzle. *Journal of Financial Economics* 120:252–68.

Kothare, M. 1997. The effects of equity issues on ownership structure and stock liquidity: A comparison of rights and public offerings. *Journal of Financial Economics* 43:131–48.

Li, S., E. Maug, and M. Schwartz-Ziv. 2022. When shareholders disagree: Trading after shareholder meetings. *Review of Financial Studies* 35:1813–67.

Lie, E., and H. Lie. 1999. The role of personal taxes in corporate decisions: An empirical analysis of share repurchases and dividends. *Journal of Financial and Quantitative Analysis* 34:533–52.

Massa, M., V. Mataigne, T. Vermaelen, and M. Xu. 2016. *Choices in equity finance. A global perspective. Working Paper*, INSEAD.

Matsusaka, J., O. Ozbas, and I. Yi. 2019. Opportunistic proposals by union shareholders. *Review of Financial Studies* 32:3215–65.

Matvos, G., and M. Ostrovsky. 2010. Heterogeneity and peer effects in mutual fund proxy voting. *Journal of Financial Economics* 98:90–112.

McLean, R., T. Zhang, and M. Zhao. 2013. Investor protection and choice of share issuance mechanism. Working Paper, Georgetown University. Myers, S., and N. Majluf. 1984. Corporate financing and investment decisions when firms have information that investors do not have. *Journal of Financial Economics* 13:187–221.

Ritter, J. R. 1987. The costs of going public. Journal of Financial Economics 19:269-81.

Rock, K. 1986. Why new issues are underpriced. Journal of Financial Economics 15:187-212.

Romano, R. 1993. Public pension fund activism in corporate governance reconsidered. *Columbia Law Review* 93:795–853.

Slovin, M., M. Sushka, and K. Lai. 2000. Alternative flotation methods, adverse selection, and ownership structure: evidence from seasoned equity issuance in the UK. *Journal of Financial Economics* 57:157–90.

Smith, C., 1977. Alternative methods for raising capital: Rights versus underwritten offerings. *Journal of Financial Economics*5:273–307.

Ursel, N. 2006. Rights offerings and corporate financial condition. Financial Managment 35:31-52.

Veld, C., P. Verwijmeren, and Y. Zabolotnyuk. 2020. Wealth effects of seasoned equity offerings: A metaanalysis. *International Review of Finance* 20:77–131.

Wu, X., Z. Wang, and J. Yao. 2005. Understanding the positive announcement effects of private equity placements: New insights from Hong Kong data. *Review of Finance* 9:385–414.

———. 2016. A rent protection explanation for SEO flotation method choice. *Journal of Financial and Quantitative Analysis* 51:1039–69.