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Testing explosive bubbles with time-varying volatility: The case of Spanish public debt

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A B S T R A C T

In this paper the dynamics of the Spanish public debt-GDP ratio is analysed during the period 1850–2021. We use recent procedures to test for explosive bubbles in the presence under time-varying volatility (Harvey et al., 2016; Harvey et al., 2019, 2020; Kurozumi et al., 2022) in order to test the explosive behavior of Spanish public debt over this long period. We extend previous analysis of Esteve and Prats (2022) where assume constant unconditional volatility in the underlying error process.

1. Introduction

Questions such as the balancing of budget deficits, the interactions between monetary and fiscal policies, and the fiscal discipline required in monetary unions, have been intensively discussed over the last decades. In particular, one of the main problems concerning fiscal authorities is the sustainability of government deficits, which is related to the issue of long-run solvency.

Fiscal policy is regarded as sustainable when, if maintained in the indefinite future, it does not violate the solvency constraint; and a government is said to be solvent if the present value budget constraint, i.e., its intertemporal budget constraint (IBC) holds. In other words, the public deficit can be sustainable if the government can borrow. However, if the interest rate on the government debt exceeds the growth rate of the economy, debt dynamics would lead to an ever-increasing ratio of debt to GDP. The dynamics of debt accumulation could only be stopped only if the ratio of the primary budget deficit to GDP would turn to be a surplus, or if seigniorage were allowed for.

The condition for fiscal sustainability implies that initial public debt equals the expected present value of future primary public surpluses, commonly known as the Intertemporal Budget Constraint (IBC) if and only if discounted future public debt converges to zero (Transversality Condition (TC) of the intertemporal decision problem of the government). The TC, rules out a Ponzi scheme (whereby debt is perpetually rolled over) as the necessary condition for lenders to hold government bonds.[1] There is a large literature

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[1] Over the arithmetic of deficit and debt sustainability, see Esteve and Prats (2022) for details.

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on the topic, though empirical tests of solvency (or fiscal sustainability), have gone through different stages.[2] Below are shown several methods that have been used in empirical applications to test whether this TC is fulfilled.[3] The Spanish case can be of interest given the permanent difficulties experienced when balancing the government budget across those years. Furthermore, the Spanish economy seems to be an interesting case study because it has been characterized by chronic government deficits and episodes with high levels of public debt. In a recent paper of Esteve and Prats (2022) the dynamics of the Spanish public debt-GDP ratio is analysed during the period 1850–2020. They use tests for recurrent explosive bubbles (Phillips et al., 2011, 2015a,b) in order to identify episodes of explosive dynamic of public debt, which can be attributed to active budget policies (unsustainables) that ran in the past. These tests assume constant unconditional volatility in the underlying error process, and recently Harvey et al. (2016) and Harvey et al. (2019) demonstrated that the asymptotic null distribution of the Phillips et al. (2011), Phillips et al. (2015a,b) test depends on the nature of the volatility through the variance profile under the existence of heteroskedasticity, so if the test is compared to critical values derived under a homoskedastic error assumption, its size is not controlled under time-varying volatility. This lack of size control typically leads to serious over-sizing, and consequently frequent spurious identification of a bubble.

However, a general decline in the unconditional volatility of the shocks driving macroeconomic series has been a commonly observed phenomenon. Furthermore, time-varying volatility is a well-known stylized fact observed in economic time series and specially in financial data (see, for example, Rapach et al., 2008. The aim of this paper is to analyze the dynamics of the Spanish public debt-GDP ratio during the period 1850–2021 through a new methodology that allows the existence of explosive bubbles under the presence of time-varying volatility.

To overcome the problem identified in Harvey et al. (2016) and Harvey et al. (2019), in this article we extend previous analysis of Esteve and Prats (2022) using recent procedures to test for explosive bubbles in the presence under time-varying volatility (Harvey et al., 2016, 2019, 2020; Kurozumi et al., 2022) in order to identify the explosive behavior of Spanish public debt during the period 1850–2021. To our knowledge, this paper contributes to the empirical literature for several reasons: it is the first time that this novel methodology has been applied to debt sustainability; the methodology used is very robust and therefore allows more reliable results to be obtained than with other methodologies; the tests used allow correcting the bias of other tests that do not take into account excessive volatility; a very large sample of data is used, which makes the methodology work correctly; finally, the results allow corroborating the existence of a high explosive behavior of the series of the Spanish public debt-GDP which can be attributed to active budget (unsustainable) policies that ran in the past.

In this context, the main economic policy implication of our paper points to the need to strengthen the sustainability of Spanish public finances in the medium term. To do this, a multi-year fiscal consolidation plan will have to be carefully designed and rigorously applied once the Spanish economy is firmly on the path to recovery, as recently proposed by the European Commission and the IMF. The sooner such a comprehensive plan is designed and announced, the better, as it would help boost confidence and reduce uncertainty about Spanish economic policies.

The scheme of the paper is as follows. Section 2 we introduce the econometric methodology. Section 3 presents and discusses the main empirical results. Section 4 draws the main conclusions.

2. Econometric methodology

2.1. The heteroskedastic bubble model

Kurozumi et al. (2022) consider the time series process $\{y_t\}$ generated according to the following DGP that allows one explosive regime with a subsequent collapsing regime,

$$y_t = \eta + u_t \tag{1}$$

$$u_t = \left\{ \begin{array}{ll} u_{t-1} + \varepsilon_t, & t = 1, \dots, \lfloor \tau_{1,0} T \rfloor, \\ (1+\delta_1) u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{1,0} T \rfloor + 1, \dots, \lfloor \tau_{2,0} T \rfloor, \\ (1-\delta_2) u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{2,0} T \rfloor + 1, \dots, \lfloor \tau_{3,0} T \rfloor, \\ u_{t-1} + \varepsilon_t, & t = \lfloor \tau_{3,0} T \rfloor + 1, \dots, T, \end{array} \right\} \tag{2}$$

$$\varepsilon_t = \sigma_t e_t \tag{3}$$

where $\delta_1 \geqslant 0$, $\delta_2 \geqslant 0$, $0 \leq \tau_{1,0} < \tau_{2,0} \leq \tau_{3,0} \leq 1$. The process $\{y_t\}$ evolves as a unit root process, but a bubble possible emerges at $\lfloor \tau_{1,0} T \rfloor + 1$ with the explosive AR(1) coefficient given $1 + \delta_1$ followed by the collapsing regime from $\lfloor \tau_{2,0} T \rfloor + 1$ to $\lfloor \tau_{3,0} T \rfloor$ generated as a stationary process, which is interpreted as the return to the normal time series behavior. The magnitude of δ_2 specifies the extend of the collapse of the bubble with the duration between $\lfloor \tau_{2,0} T \rfloor + 1$ to $\lfloor \tau_{3,0} T \rfloor$

In the presence of heteroskedastic the volatility of the innovations is given by σ_t in (3) and it can be non-stationary, whilst the conventional homoskedasticity assumption, as employed in PWY and PSY and others papers, implies that $\sigma_t = \sigma$ for all t.

[2] A very good, updated and clarifying study on the different approaches to evaluate this question is the one by D'Erasmo et al. (2016), where the authors identify the more important defaults in the traditional approach to evaluate debt sustainability, and examine three alternative approaches that provide useful econometric and model-simulation tools to analyze debt sustainability.

[3] For a recent review of empirical applications, see Beqiraj et al. (2018) and the references therein.

On the other hand, the time series process $\{y_t\}$ can simply rewrite as,

$$y_t = (1 + \delta_t)y_{t-1} + \varepsilon_t \tag{4}$$

or

$$\Delta y_t = \delta_t y_{t-1} + \varepsilon_t \tag{5}$$

The null hypothesis, H_0 is that no bubble is present in the series and y_t follows a unit root process throughout the sample period, i.e., $\delta_t = 0$ in expression (4).[4] The alternative hypothesis H_1 is that a bubble is present in the series, which corresponds to the case where δ_t in (4) is not stable at 1 and the model is given by (1)–(3) with $\delta_1 > 0$.

2.2. Test for explosive bubbles under stationarity volatility

Phillips et al. (2011) and Phillips et al. (2015a,b) proposed test for explosive bubbles based on recursive right-tailed Dickey–Fuller-type unit root tests which can detect evidence of the explosive behavior of a time series $\{y_t\}$.

Phillips et al. (2011) proposed the maximum of the ADF test statistics constructed using subsamples. The testing procedure developed from a regression model of the form,

$$\Delta y_t = \mu + \delta y_{t-1} + \varepsilon_t \tag{6}$$

for $t = \lfloor \tau_1 T \rfloor + 1$ to $\lfloor \tau_2 T \rfloor$.

The key parameter of interest is δ. We want to test the null hypothesis of a unit root, $H_0 : \delta = 1$, against the right-tailed alternative, $H_1 : \delta > 1$, al least in some subsample. The model is estimated by Ordinary Least Squares (OLS) and the t-statistics associated with the estimated δ is referred to as ADF statistic.

The $SADF$ test is then a supremum statistic based on the forward recursive regression and is simply defined as,

$$SADF(r_0) = \sup_{r_2 \in [r_{0,} 1]} ADF_0^{r_2} \tag{7}$$

where the right-tail is the rejection region. This test can be used for testing for a unit root against explosive behavior in some subsample.

Second, Phillips et al. (2015a,b) proposed a generalized version of the $\sup ADF$ ($SADF$) test of Phillips et al. (2011). Their Generalized Supremum ADF ($GSADF$) test is,

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1], r_1 \in [0, r_2 - r_0]} ADF_0^{r_2} \tag{8}$$

The statistic (8) is used to test the null of a unit root against the alternative of recurrent explosive behavior, as the statistic (7).

Note that the $SADF$ test previously proposed by Phillips et al. (2011) is a special case of $GSADF$ test, obtained by setting $r_1 = 0$ and $r_2 = r_\omega \in [r_0, 1]$.[5]

The $SADF$ and $GSADF$ assume constant unconditional volatility in the underlying error process, and recently Harvey et al. (2016) and Harvey et al. (2019) demonstrated that the asymptotic null distribution of the Phillips et al. (2011), Phillips et al. (2015a,b) test depends on the nature of the volatility through the variance profile $\eta(s)$ under the existence of heteroskedasticity, so if the test is compared to critical values derived under a homoskedastic error assumption, its size is not controlled under time-varying volatility. This lack of size control typically leads to serious over-sizing, and consequently frequent spurious identification of a bubble.[6]

2.3. Test for explosive bubbles under time-varying volatility

To take into account this issue, a several number of recently test for explosive bubbles has been proposed under assumption of time-varying volatility:

- Harvey et al. (2016), Harvey et al. (2019) and Kurozumi et al. (2022) developed a wild bootstrap algorithm for $SADF$ test and $GSADF$ test. They propose use this bootstrap scheme, applied to the first differences of the data, in order to replicate in the bootstrap data the pattern of non-stationarity volatility present in the original innovations. We call these test as $SADF_b$ and $GSADF_b$.
- Harvey et al. (2019) proposed two test:

[4] The null hypothesis can be expressed using (2) in several ways such that $\tau_{1,0} = 1$, $\delta_1 = 0$, $\tau_{2,0} = 1$, or $\delta_1 = \delta_2 = 0$.

[5] Phillips and Shi (2018) showed that although $GSADF$ procedure is designed to detect the bubble behaviour, it can also detect crisis periods (see also Phillips and Shi, 2019, Phillips and Shi, 2020) which are often observed in empirical applications, for example, Esteve and Prats (2022).

[6] Some classical unit root tests are severely oversized because their limiting distributions depend on a particular function, the so-called variance profile, of the underlying volatility process (see Cavaliere, 2004, Cavaliere and Taylor, 2007a,b, 2008, 2009 and references therein).

Table 1
Descriptive statistics, Spanish public debt-GDP ratio.

Statistics	1850–2021
Mean	67.3
Minimum	11.2
Maximum	169
Standard deviation	35.289
Variance	1245
Skewness	0.636
Kurtosis	0.293

  - A weighted least squares (WLS) modification of Phillips et al. (2011) test. Their supremum based test is,

$$SBZ(r_0) = \sup_{r\in[r_{0},1]} BZ_r \tag{9}$$

  - A union U test of rejections testing strategy because none of the test, SBZ and $SADF$ dominate each other across all volatility specifications. We call these test as SBZ_u.

- Harvey et al. (2020) proposed another method which control size under time-varying volatility. They proposed two test:
  - A sign-based variant of the Phillips et al. (2015a,b) test for explosive behavior, the supremum sign-based test is,

$$sGSADF(r_0) = \sup_{r_2\in[r_0,1],r_1\in[0,r_2-r_0]} sADF_{r_1}^{r_2} \tag{10}$$

  - A union U test of rejections testing strategy with wild bootstrap implementation with $GSADF$ and $sGSADF$ tests as the same approach by Harvey et al. (2019). We call this test as $sGSADF_u$ test (and $sSADF_u$).

- Kurozumi et al. (2022) proposed a test based on the sup-type t-statistics expanded under the null hypothesis, using the time transformed data based on the variance profile, $\eta(s)$. They consider the $SADF$ and $GSADF$ test statistics with a version of the GLS-type demeaning. Their test statistics based on the time-transformed ADF test statistics is,

$$SADF = \sup_{r_2\in[r_0,1]} TADF_{r_0}^{r_2} \tag{11}$$

  and

$$GSTADF = \sup_{r_2\in[r_0,1],r_1\in[0,r_2-r_0]} TADF_{r_1}^{r_2} \tag{12}$$

3. Empirical application

We consider a long historical time series in which many fiscal crisis events are known to have occurred. The length of this database makes it particularly suitable for the econometric approach adopted in this paper (172 years).

The data and sources are:

– 1850–2000: (a) public debt, total outstanding liabilities, B_t, from Carreras and Tafunell (2005), Table 12.34, serie 2895; (b) nominal GDP, Y_t, from Carreras and Tafunell (2005), Table 17.7, serie 4744; (c) the public debt-to-GDP ratio, $b_t = B_t/Y_t$. – 2001–2021; (d) public debt, general government, debt compiled according to Excessive Deficit Procedure (EDP), from Banco de España (2021), Table, 2.15.a, and Banco de España (2022), Table 11.B; (e) nominal GDP, Y_t, from Banco de España (2022), Table 23.a; (f) the public debt-to-GDP ratio (EDP). (f) the public debt-to-GDP ratio, $b_t = B_t/Y_t$. Some descriptive statistics for both series are shown in Table 1.[7]

In our empirical analysis, we use annual data of the Spanish public debt-GDP ratio, b_t, for the period 1850–2021. We can broadly follow dynamics of the path the Spanish public debt, as % of GDP, between 1850 and 2021 in Fig. 1, where the expansions of public debt are markedly visible in it. A more detailed account of the evolution of the Spanish public finances and the historical public debt cycle over this period can be found in Esteve and Prats (2022).

Fig. 2 contains plot of the first differences of b_t. A simple visual analysis of this plot suggests that the assumption of stationarity unconditional volatility of Esteve and Prats (2022) could be unrealistic for this time series, with volatility appearing over the sample period in some subperiod. Fig. 3 shows the estimated variance profile of b_t, which is defined as $\hat{\eta}(s)$. We construct the estimator of the variance profile using the approach suggest by Cavaliere and Taylor (2007b), Harvey et al. (2022) and Kurozumi et al. (2022). They use the kernel-type local least squares method to estimate the time varying parameter δ_t in (4) and (5).

From Fig. 3 note that the variance profile if this time series show that exist three regimes in which volatility moves from high (1874–1880) to low (1881–2003) and finally to high (2004–2021). These two regimes in which volatility is high are the same

[7] Data avalaible on request from the authors.

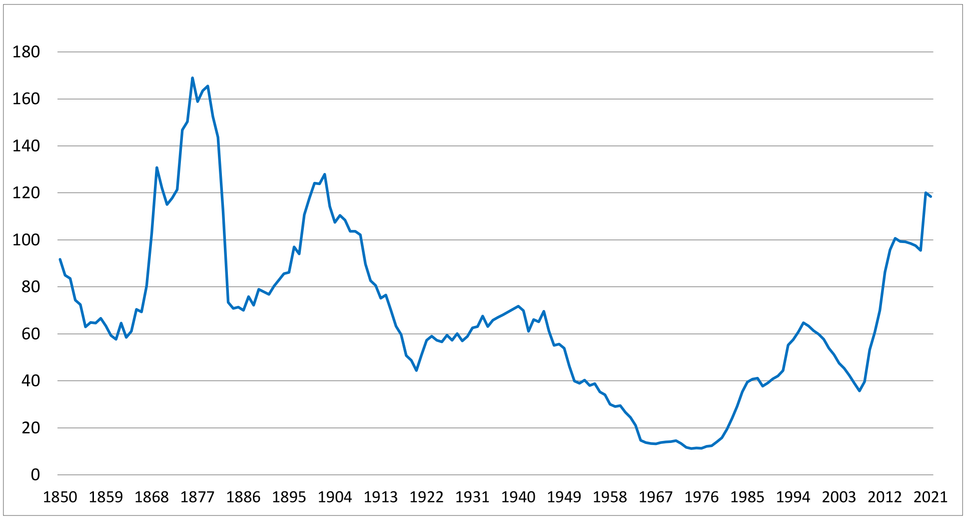

Fig. 1. Spanish public debt as % of GDP 1850–2021.

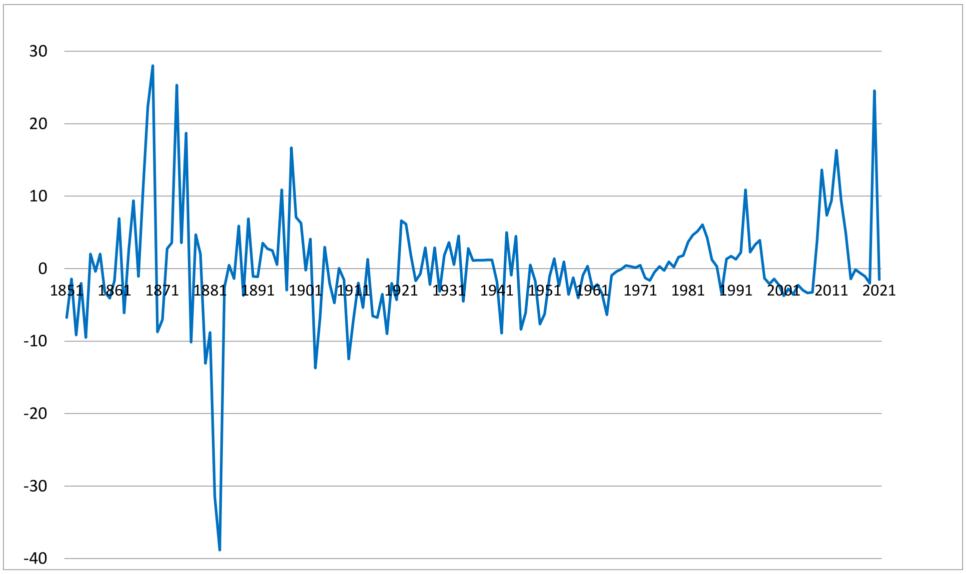

Fig. 2. Spanish public debt as % of GDP: first-differences 1851–2021.

subperiods identified with an explosive bubble in the Spanish public debt-to-GDP ratio by Esteve and Prats (2022). However, it is important to highlight that these two regimes in which volatility is high are of short duration and only represent 28% of the total sample.

Table 2 present the results of tests for explosive bubbles under stationarity volatility and the tests for explosive bubbles under time-varying volatility presented in the previous section: the standard $SADF$ and $GSADF$ tests, the wild bootstrap $SADF$ and $GSADF$ tests ($SADF_b$ and $GSADF_b$), a union of rejections of the $SADF_b$ and SBZ tests, and $GSADF_b$ and SBZ tests (SBZ_u), a union of rejections of the $SADF_b$ and sign-based tests $sSADF$, and $SGADF_b$ and sign-based tests $sGSADF$ ($sSADF_u$ and $sGSADF_u$) and the $STADF$ and $GSTADF$ tests. We show the bootstrap p-values associated with the different tests, $SADF$ tests (Panel A) and $GSADF$ tests (Panel B).[8]

[8] Following Kurozumi et al. (2022) for the wild bootstrap p -values, B= 999 bootstrap replications were used. For the standard $SADF$ and $GSADF$ tests and the time-transformed tests $STADF$ and $GSTADF$, the p-values are obtained by simulations of the asymptotic distributions of the test statistics under homoskedasticity. We use $r_0 = \lfloor\ 0.01{+}1.8/\sqrt{T}\rfloor$ for calculations of the p-values.

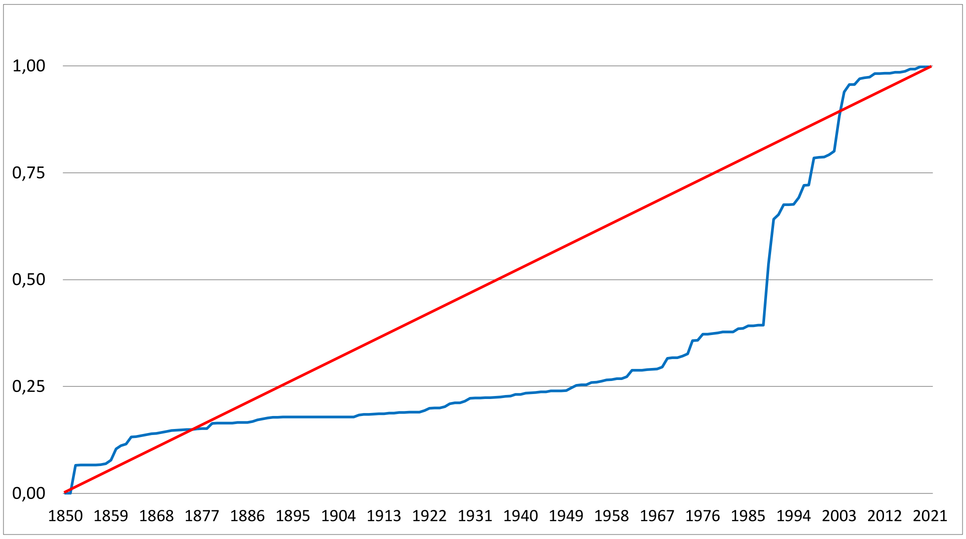

Fig. 3. Spanish public debt as % of GDP: the estimated variance profile, $\eta(s)$ 1850–2021.

Table 2
Test for explosive bubbles under non-stationarity volatility in Spanish public debt-GDP ratio, *p*-values.

Panel (a) $SADF$ tests				
$SADF$	$SADF_b$	SBZ_u	$sSADF_u$	$STADF$
0.4695	0.4034	0.2565	0.6649	0.1071
Panel (b) $GSADF$ tests				
$GSADF$	$GSADF_b$	$GSBZ_u$	$sGSADF_u$	$GSTADF$
0.000*	0.0788***	0.1239	0.1514	0.3083

Note: *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively.

First we observe that the standard $SADF$ statistic does not reject the null in favour of explosive behavior at conventional significance levels.[9] This is the same result obtained in Esteve and Prats (2022). This behavior not explosive is preserved when we use all test $SADF$ for explosive bubbles under time-varying volatility. Second, we find evidence of explosive behavior with the standard $GSADF$ statistic at the 0.01-level in this period, as in Esteve and Prats (2022). Finally, this pattern of result is also obtained when considering the wild bootstrap $GSADF_b$ test at the 0.10-level, providing evidence for the presence of a speculative bubble, under the new assumption of time-varying volatility of time series.

Overall, these findings corroborate those presented in Esteve and Prats (2022) but show also that the behaviour explosive of the Spanish public debt-GDP ratio during the period 1850–2021 can also be partly explained by volatility changes.

4. Conclusions

This paper analyzes the dynamics of the Spanish public debt GDP ratio over the period 1850–2021. We use recently developed procedures to test for explosive bubbles under time-varying volatility (Harvey et al., 2016, 2019, 2020; Kurozumi et al., 2022) in order to test for explosive behavior of the Spanish public debt during this long period. We extend the previous analysis of Esteve and Prats (2022) where evidence of explosive behavior of the ratio of the Spanish public debt GDP ratio during the period 1850–2020 is presented under the assumption of constant unconditional volatility in the underlying error process. Now, our empirical application demonstrates that is important to take non-stationary volatility into account when testing for a bubble in a time series with unstable volatility. The methods used in our work are significantly more robust because they allow for episodes of explosivity and heteroskedasticity.

In general, the results corroborate those presented in Esteve and Prats (2022) identifying the same four periods: the first between 1874–1880 related to the first and second Cuban wars and the budget efforts made in this period; the second episode, occurred in 1917–1920, related to the fiscal adjustment that occurred after an explosive debt path associated with Cuban war; the third episode, dated between 1951 and 1981, is associated to another fiscal adjustment during the period of Franco's regime until the arrival of democracy in 1979; and the fourth episode, between 1982 and 2002, was the result of chronic government deficits.

[9] Note that the original SADF test suffers from severe size distortion under nonstationary volatility, as is observed in the existing literature.

But also, the results demonstrate that the explosive behavior of the Spanish public debt GDP ratio during the period 1850–2021 can also be partly explained by changes in volatility.

CRediT authorship contribution statement

Vicente Esteve: Data curation, Software, Conceptualization, Methodology, Formal Analysis, Writing. **María A. Prats:** Data curation, Conceptualization, Methodology, Writing.

Data availability

Data will be made available on request.

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