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Preferences and Performance in Simultaneous First-Price Auctions: A Structural Analysis

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Motivated by the prevalence of simultaneous bidding across a wide range of auction markets, we develop and estimate a model of strategic interaction in simultaneous first-price auctions when objects are heterogeneous and bidders have non-additive preferences over combinations. We establish non-parametric identification of primitives in this model under standard exclusion restrictions, providing a basis for both estimation and testing of preferences over combinations. We then apply our model to data on Michigan Department of Transportation (MDOT) highway procurement auctions, quantifying the magnitude of cost synergies and evaluating the performance of the simultaneous first-price mechanism in the MDOT marketplace.

Key words: Auctions, Complementarities, Identification.

JEL Codes: D44, D47

1. INTRODUCTION

Simultaneous bidding in multiple first-price auctions is a commonly occurring but rarely discussed phenomenon in many real-world auction markets.[1] In environments where values over combinations are non-additive in the set of objects won, bidders must account for the possibility

1. To underscore the prevalence of simultaneous bidding in applications, note that many widely studied first-price marketplaces in fact exhibit simultaneous bids. Examples include markets for highway procurement (Li and Zheng, 2009; Krasnokutskaya, 2011; Groeger, 2014; Somaini, 2020, among others), snow-clearing (Flambard and Perrigne, 2006), recycling services (Kawai, 2011), oil and drilling rights (Hendricks, Pinkse and Porter, 2003), and to a lesser extent US Forest Service timber harvesting (Lu and Perrigne, 2008; Li and Zhang, 2010, Athey, Levin and Seira, 2011, many

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of winning multiple auctions at the time of bidding. This in turn substantially alters the strategic bidding problem compared to the standard first-price auction, with welfare implications depending on the nature and scope of non-additivities in preferences.

We develop a structural empirical model of bidding in simultaneous first-price auctions when objects are heterogeneous and bidders have non-additive preferences over combinations, to our knowledge the first in the literature. We represent the total value bidder i assigns to each combination as the sum of two components: the sum of i's *standalone valuations* for each object in the combination individually, plus a combination-specific *complementarity* (either positive or negative) capturing the incremental change in value i associates with winning the combination as a whole. We interpret standalone valuations as private information drawn independently across bidders conditional on observables, and complementarities as deterministic functions of observables.[2] We find this framework natural in a variety of procurement contexts—when, for instance, non-additivity in preferences can be represented as the expectation over a cost shock realized following a multiple win. Furthermore, and crucially, our framework reduces to the standard separable model when complementarities are zero, formally embedding this benchmark case in a richer model allowing synergies.[3]

Building on this framework, we make four main contributions. First, we establish a new set of identification results applicable even when complementarities are non-zero. We start by showing that optimal behaviour yields an inverse bidding system non-parametrically identified up to the unknown function describing complementarities, which in turn collapses to the standard inverse bidding function of Guerre, Perrigne and Vuong (2000) when complementarities are zero. Under natural exclusion restrictions—namely, that marginal distributions of standalone valuations are invariant either to characteristics of rival bidders or characteristics of other objects—we then translate this inverse bidding system into a system of linear equations in unknown bidder complementarities, with excludable variation in competition and other characteristics yielding non-parametric identification of these.

Second, we develop a two-step procedure by which to estimate complementarities in our model. First, in Step 1, we estimate the multivariate joint distribution of bids as a function of bidder- and auction-level characteristics. In Step 2, we use pairwise differencing and GMM estimation to estimate complementarities using moment conditions derived from our identification argument. Once complementarities are estimated, it is straightforward to use the inverse bidding system to estimate standalone project completion costs for each bidder.

Third, we apply our framework to analyse simultaneous bidding in Michigan Department of Transportation (MDOT) highway procurement markets. We view this market as typical of our target application: large numbers of projects are auctioned simultaneously (an average of 45 per letting round in our 2005–15 sample period), more than half of bidders bid on at least two projects simultaneously (with an average of 2.7 bids per round across all bidders in the sample), and combination and contingent bidding are explicitly forbidden. Within this marketplace, we show that factors such as the size of other projects, the number of bidders in other auctions, and the relative distance between projects have substantial impacts on i's bid in auction l, a finding hard to rationalize in standard separable models. We find substantial complementarities

others). Ausubel, Cramton, McAfee and McMillan (1997), Moreton and Spiller (1998), and Lunander and Lundberg (2013) provide evidence on importance of synergies in simultaneous auctions.

2. Note that this structure does not restrict dependence between i's standalone valuations for different objects in the market. We view this flexibility as critical, as in practice we expect i's standalone valuations to be positively correlated among each other even if independent from other bidders' valuations.

3. This article focuses on complementarities arising when auctions are run simultaneously. This complements the literature on potential linkages in valuations over time, e.g. Balat (2015), De Silva (2005), De Silva, Jeitschko and Kosmopolou (2005), Groeger (2014), and Jofre-Bonet and Pesendorfer (2003) among others.

in this application: comparing the 10th to the 90th percentile of estimated complementarities, we find that a combination win may generate anything from approximately 14% cost savings to approximately 12% cost increases depending on bidder and project characteristics, with large, heterogeneous, and overlapping projects all increasing joint completion costs.

Finally, we counterfactually compare the simultaneous first-price auction used in the MDOT marketplace to a simple efficient combinatorial benchmark: the Vickrey–Clarke–Groves (VCG) mechanism. By construction, the VCG auction yields lower social costs: our estimates suggest total social savings of approximately 7.4%. Interestingly, however, it also increases MDOT's payments to bidders slightly, by approximately 1.6%. In other words, from the procurer's perspective, even in the presence of substantial complementarities, the simultaneous first-price auction appears to perform very well.[4] This may help to explain the popularity of the simultaneous first-price format in practice.[5]

Although our work is the first to propose a general framework for identification and estimation in simultaneous first-price auctions, other prior studies have structurally analysed various types of synergy across auctions. Bajari and Fox (2013) estimate the deterministic component of bidder valuations in simultaneous ascending FCC spectrum auctions under the assumption that the allocation of licenses is pairwise stable in matches (i.e. the sum of valuations from two winning bidders must not be increased by swapping licenses), a condition which need not hold in the simultaneous first-price setting we consider here and which would have been very restrictive on the nature of the exposure problem. Kong (2021) studies identification and estimation in sequential auctions allowing for synergies and affiliation in each bidder's private valuations for different objects, complementing our analysis allowing for both features in simultaneous auctions.[6] Jofre-Bonet and Pesendorfer (2003) and subsequent studies have focused on sequential auctions of independent objects linked by bidder dynamics. Cantillon and Pesendorfer (2006) and Kim, Olivares and Weintraub (2014) analyse combinatorial auctions with the possibility of package bidding. They do not require a preference specification with regard to complementarities as bids are observed for each bundle and, thus, each bundle has its own first-order condition to which the identification strategy of Guerre *et al.* (2000) can be applied directly. In contrast, we have only an L-dimensional bid to learn an up to 2^L-dimensional vector of valuations for all possible bundles. Finally, there have been several theoretical studies analysing simultaneous first-price auctions, including Gentry, Komarova, Schiraldi and Shin (2019) who study the existence and properties of equilibrium in special cases of the model studied here.[7]

The rest of this article is organized as follows. Section 2 outlines the model, while Section 3 studies identification. Section 4 describes the Michigan Department of Transportation (MDOT) highway procurement marketplace, and Section 5 presents our estimation strategy and results. Section 6 compares MDOT's simultaneous first-price format with a combinatorial VCG mechanism. Finally, Section 7 concludes. Appendix A collects technical proofs, while

4. We also explored other leading combinatorial mechanisms, such as the descending proxy auction of Ausubel and Milgrom (2002). By construction, these lead to the same efficient allocation as the VCG auction, and in preliminary tests, they also led to very similar expected revenue. For this reason, we chose to focus for simplicity on the VCG auction.

5. More generally, we contribute to the growing literature that aims to understand the performance of different auction formats, e.g. Athey *et al.* (2011), Lewis and Bajari (2011), Decarolis (2017) among others.

6. Both we and Kong (2021) allow for affiliation in valuations across objects for each bidder but not affiliation in valuations across bidders. Both studies thus belong to the independent private values paradigm.

7. Tangentially related to our problem, there is also an empirical literature on *multi-unit auctions* for homogeneous, divisible goods like electricity and treasury bills. See e.g. Fevrier, Preget and Visser (2004), Chapman, McAdams and Paarsch (2007), Kastl (2011), Hortacsu and Puller (2008), Hortacsu and McAdams (2010), Wolak (2007), and Reguant (2014).

Supplementary Appendices B–G present extended identification, testing, and Monte Carlo simulation results.

2. EMPIRICAL FRAMEWORK

Consider a population of simultaneous first-price lettings. In each letting t, a set $\mathcal{N}_t = \{1,\ldots,N_t\}$ of risk-neutral bidders compete for (subsets of) a set $\mathcal{L}_t = \{1,\ldots,L_t\}$ of objects allocated via separate but simultaneous first-price auctions. Each bidder $i \in \mathcal{N}_t$ participates in a set of auctions, $\mathcal{L}_{it} \subset \mathcal{L}_t$, submitting a scalar bid b_{itl} in each auction l in which she participates. Bidding is simultaneous and objects are awarded auction by auction: the high bidder in auction l wins object l and pays her bid, with ties broken independently across bidders and auctions. Let L_{it} denote the number of auctions in which bidder i is participating, and $b_{it} \equiv (b_{itl})_{l\in\mathcal{L}_{it}}$ denote the $L_{it}\times 1$ vector of bids submitted by i in letting t.

For each letting t, the econometrician observes the following data. For each object $l = 1,\ldots,L_t$ auctioned in letting t, the econometrician observes a vector of characteristics X_{lt} describing this object. For each bidder i active in letting t, the econometrician observes bidder i's bid vector b_{it}, participation set $\mathcal{L}_{it}$, and a vector of bidder characteristics Z_{it}. In what follows, let $X_t \equiv (X_{1,t},\ldots,X_{L_t,t})$ describe characteristics of all objects auctioned in letting t, and $Z_t \equiv (Z_{1t},\ldots,Z_{N_t,t})$ describe characteristics of all active bidders.

Following Cantillon and Pesendorfer (2006) and Bajari and Fox (2013), we analyse bidding in the simultaneous first-price auction taking participation as given. That is, we take the endogenous outcome of interest to be the bid vectors $(b_{1t})_{i=1}^{N_t}$ submitted by each bidder, conditional on auction characteristics X_t, bidder characteristics Z_t, and participation sets $(\mathcal{L}_{it})_{i=1}^{N_t}$. We view this as a natural, and arguably necessary, the first step toward understanding simultaneous first-price auction markets: here, as elsewhere, one cannot analyse participation without understanding bidding. Importantly, however, as we show in Supplementary Appendix B, one can also view our analysis as applying to bidding within a two-stage entry and bidding model in which entry is interpreted as a process of value discovery. The key hypothesis in this case, following Levin and Smith (1994), Krasnokutskaya and Seim (2011), Moreno and Wooders (2011), Athey *et al.* (2011), Groeger (2014), and Li and Zhang (2015) among others, is that bidders discover private information about valuations only following costly entry.

For concreteness, we follow many prior studies on highway procurement auctions, e.g. Bajari and Ye (2003), Krasnokutskaya (2011), and Krasnokutskaya and Seim (2011) among many others, in assuming that bidders observe the participation structure $(\mathcal{L}_{it})_{i=1}^{N_t}$ at the time of bidding. We note, however, that our identification analysis applies equally when bidders observe only the set of *potential* participants in each auction; e.g. the set of planholders as in Li and Zheng (2009). In this case, one would simply reinterpret $\mathcal{L}_{it}$ as the set of auctions in which i is a potential participant, then proceed as we describe below.

In either case, to streamline notation, we adopt the convention that bidder i's characteristics Z_{it} include her participation set $\mathcal{L}_{it}$. From the perspective of both bidders and the econometrician, the common-knowledge observables (X_t, Z_t) fully characterize letting t.

Our model turns on two sets of structural assumptions: the first regarding bidder preferences and the second regarding equilibrium behaviour. We next describe each of these in turn.

2.1. *Bidder preferences*

For the next two sections, we suppress the letting subscript t for notational compactness. We reintroduce the letting subscript t when discussing estimation in Section 5.

If bidder $i=1,\dots,N$ participates in $L_i \geq 1$ auctions, then she may win any of 2^{L_i} combinations of objects. We index these combinations with an $L_i \times 1$ outcome vector ω, where $\omega_l = 1$ if object l is allocated to bidder i and $\omega_l = 0$ otherwise. We represent the set of all 2^{L_i} combinations for bidder i with a $2^{L_i} \times L_i$ outcome matrix Ω_i, where each row of Ω_i corresponds to a distinct outcome ω. For example, if $L_i = 2$, then Ω_i would satisfy

$$\Omega_i^T = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

Equivalently, one may view each outcome ω as the binary representation of some integer in the set $\{1,\dots,2^{L_i}\}$, with Ω_i collecting all such binary representations. With slight abuse of notation, we use the shorthand "$\omega \in \Omega_i$" to indicate that outcome ω is possible for bidder i.

To each outcome $\omega \in \Omega_i$, bidder i associates a *combinatorial valuation* Y_i^{ω}, which she receives in the event that this outcome is realized. Let $Y_i \equiv [Y_i^{\omega}]_{\omega \in \Omega_i}$, a $2^{L_i} \times 1$ vector, collect i's combinatorial valuations Y_i^{ω} for all possible outcomes $\omega \in \Omega_i$. For simplicity, and without loss of generality, we normalize the value of winning nothing to zero: $Y_i^0 = 0$.

Let bidder i's *standalone valuation* for object l, denoted V_{il}, be the valuation i assigns to the outcome "i wins object l alone." Let i's *standalone valuation vector*, denoted V_i, be the $L_i \times 1$ vector describing i's standalone valuations for each object in her participation set: $V_i \equiv [V_{il}]_{l=1}^{L_i}$. Finally, let K_i^{ω} denote i's *complementarity* between objects in combination $\omega \in \Omega_i$, defined as the difference between i's combinatorial valuation Y_i^{ω} and the sum of i's standalone valuations for objects won under ω:

$$K_i^{\omega} = Y_i^{\omega} - \omega^T V_i.$$

Let $K_i \equiv [K_i^{\omega}]_{\omega \in \Omega_i}$ be the $2^{L_i} \times 1$ vector containing the complementarities associated by i with each possible outcome $\omega \in \Omega_i$. Note that, by construction, we have

$$Y_i \equiv \Omega_i V_i + K_i.$$

We may thus equivalently represent bidder i's preferences in terms of the pair (V_i, K_i), where V_i describes i's valuations for each object individually, while K_i reflects departures from additivity in i's preferences over combinations. In particular, our model reduces to the canonical additively separable case if and only if $K_i = 0$ for all i.

As usual, we interpret standalone valuation vectors V_i as stochastic and private information for each bidder i. We further assume that standalone valuation vectors $(V_1,\dots,V_N)$ are distributed independently across bidders conditional on observables:

Assumption 1 (Independent private standalone valuations) *For each bidder $i=1,\dots,N$, standalone valuations V_i are distributed according to a joint c.d.f. $F_i(\cdot|X,Z)$, with V_i independent from V_j for all $j \neq i$, and $F_i(\cdot|X,Z)$ common knowledge.*

In Supplementary Appendix C, we allow for an additional auction-level characteristic A_l which additively shifts standalone valuations, with A_l common knowledge to bidders but unobserved to the econometrician, and valuations independent conditional on A_l and observables. This relaxes the requirement of independence conditional on observables, although our model is one of independent private valuations since A_l is known to bidders.[8]

8. Examples of situations when the independent private valuations paradigm would be violated include the case of bidders having only partial information regarding their own standalone valuations in the form of noisy signals and, thus, benefiting from information possessed by other bidders, or the case of externalities across bidders, or the situation when firm's utility when it loses depends on the winner's identity.

While standalone valuations are stochastic private information, we model complementarities K_i as determined by observables. We view this structure as natural in applications such as highway contracting, snow cleaning (Flambard and Perrigne, 2006), recycling (Kawai, 2011), and cleaning (Lunander and Lundberg, 2013), where factors such as capacity constraints, the distance between projects, the timing of projects, or types of work are the main considerations motivating analysis of complementarities. We emphasize, however, that insofar as our model interprets all complementarities across objects as arising through observables, the suitability of the model will inherently depend heavily on what observables are available.

Assumption 2 (Deterministic complementarities) *For all bidders $i=1,\ldots,N$, $K_i=\kappa_i(X,Z)$, where $\kappa_i(X,Z)$ is common knowledge.*

One may also interpret the complementarity function $\kappa_i(Z,X)$ as reflecting bidders' expectations, at the time of bidding, over *ex ante* unknown synergy effects associated with winning combination ω. The crucial hypothesis is that, at the time of bidding, this expectation depends only on common-knowledge observables. For example, if value discovery is costly, bidders may invest in learning standalone valuations prior to bidding, but invest in discovering idiosyncratic synergy effects only following a multiple win. In Supplementary Appendix D, we generalize our identification analysis to settings where complementarities additionally incorporate an *ex ante* unknown affine transformation of standalone valuations. This extension accommodates cases where, for instance, winning two auctions together increases or decreases i's valuation for one or both objects by a fixed percentage. It also allows complementarities to be stochastic private information, so long as the private information component of bidders' expected complementarities can be fully explained by standalone valuations.

Taken together, Assumptions 1 and 2 embed the canonical separable independent private values model within a richer framework allowing both flexible non-parametric complementarities and arbitrary dependence among elements of V_i for each bidder i. We view the latter as an essential empirical complement to the former, since correlation in i's bids could be driven either by complementarities in i's preferences or by dependence on i's valuations. By leaving such dependence unrestricted, we focus cleanly on the identification of non-additivities per se, even when bidder i's valuations exhibit statistical dependence across auctions.

2.2. *Equilibrium behaviour*

Let $\mathcal{V}_i \subset \mathbb{R}_+^{L_i}$ denote the support of the standalone valuation vector V_i for bidder $i=1,\ldots,N$, and let $\mathcal{B}_l \subset \mathbb{R}_+$ denotes the set of feasible bids in auction $l=1,\ldots,L$. Generically, one would define a pure strategy for bidder i as a mapping from the space of combinatorial valuation vectors Y_i to the space of feasible bids. Under Assumptions 1 and 2, however, each bidder's private information is fully described by their vector of standalone valuations V_i. To emphasize this point, in what follows we focus on $\mathcal{V}_i$ as the type space for bidder i. We define a pure strategy for bidder i given common-knowledge observables (X,Z) as a mapping $\sigma_i^{XZ}:\mathcal{V}_i \to \mathcal{B}_i$, where $\mathcal{B}_i \equiv \times_{l\in\mathcal{L}_i}\mathcal{B}_l$ denotes i's action space in the simultaneous bidding game.[9] Let $\sigma^{XZ}=(\sigma_1^{XZ},\ldots,\sigma_N^{XZ})$ denote a strategy profile for all bidders, and σ_{-i}^{XZ} denote a strategy profile for all rivals of bidder i.

9. We focus on pure strategies for expositional simplicity, but this is without essential loss of generality; all results below apply equally when bidders play mixed strategies.

Building on the first-order approach of Guerre *et al.* (2000), we base identification on necessary conditions for best-response behaviour in simultaneous first-price auctions. For this analysis to proceed, we require the following assumptions on bidder behaviour:

Assumption 3. *The distribution of bids observed at each market structure (X,Z) arises from play of a strategy profile σ^{XZ} which is a Bayesian Nash equilibrium of the simultaneous bidding game. Furthermore, for each (X,Z), only one strategy profile σ^{XZ} is played.*

When complementarities are zero, the existence of a pure strategy equilibrium is immediate and uniqueness follows under regularity conditions (Lebrun, 1999). More generally, with arbitrary complementarities, existence of a pure strategy equilibrium in any discrete bid space follows from results in Milgrom and Weber (1985). In continuous bid spaces with arbitrary complementarities, the theory provides little guidance regarding the existence or uniqueness of equilibrium in general; these are important open questions but beyond the scope of this article.[10] In this respect, our setting parallels other studies on complex auction games, in which either existence (Bajari and Fox (2013) on spectrum auctions, Ausubel and Milgrom (2002) on proxy auctions) or uniqueness (Jofre-Bonet and Pesendorfer, 2003; Roberts and Sweeting, 2013; Somaini, 2020, and references therein) is assumed as it cannot be guaranteed.[11]

To leverage necessary conditions for optimal behaviour, we require only Assumptions 1–3. For our analysis to yield point identification of model primitives, however, we further require equilibrium behaviour to satisfy the following additional conditions:

Assumption 4. *For each market structure (X,Z), the equilibrium strategy profile σ^{XZ} is such that (i) the joint cumulative distribution function of bids is absolutely continuous and (ii) for any auction $l=1,\ldots,L$ and any bidders i,j active in auction l, the marginal distributions of bids b_{il},b_{jl} have common infimums of support.*

As above, under the null of separability ($K_i=0$), these properties follow immediately from standard regularity conditions; when $K_i\neq 0$, we require them as assumptions. In practice, absolute continuity implies that marginal bid distributions are atomless, which in turn permits extension of the Guerre *et al.* (2000) first-order approach to settings with simultaneous auctions. Common infimums of support imply that bidders do not submit never-winning (or null) bids with positive probability. This may fail if, for example, bidders draw standalone valuations from distributions with asymmetric supports, or with binding public reserve prices. We emphasize that these assumptions apply to equilibrium objects, not to primitives; unfortunately, with non-zero

10. Gentry *et al.* (2019) establish equilibrium existence in some narrow special cases of the model here. Unfortunately, extending these to settings with general participation structures, asymmetries, and complementarities appears to be fundamentally beyond the scope of the present theory, in the sense that existing proof techniques appear inadequate to deliver these results. As in multi-unit auctions, the presence of both multidimensional bids and multidimensional types leads to the failure of classical differential-equations approaches to Bayes–Nash equilibrium. Monotonicity-based methods widely used in multi-unit auctions—e.g. Athey (2004), McAdams (2006), and Reny (2011)—can be applied in special cases, but do not apply at the level of generality we consider here. Other approaches—e.g. that of Jackson, Simon, Swinkels and Zame (2002) applied in Cantillon and Pesendorfer (2006)—deliver generalizations of Bayes–Nash equilibrium, but not Bayes–Nash equilibrium itself. See Gentry *et al.* (2019) for further discussion of these issues.

11. We note, however, that almost every real-world bid space is ultimately discrete. For instance, if bidders must bid in pennies, then existence is guaranteed as noted above. In this sense, we see existence as of more theoretical than a practical concern. In the main text, we follow the literature's convention of interpreting bid spaces as continuous and proceed to analyse identification. Supplementary Appendix E provides a more general partial identification analysis applicable in settings where discreteness is viewed as empirically important.

complementarities, the dearth of existing theory renders it unclear what conditions on primitives might guarantee these properties a priori. Importantly, however, in Supplementary Appendix E, we also derive identified sets for model primitives which are robust to arbitrary violations of Assumption 4. As we show, these identified sets can tightly bound primitives even in data generating processes where Assumption 4 fails, while continuing to yield point identification when Assumption 4 holds.

3. NON-PARAMETRIC IDENTIFICATION

We study identification based on a large number of simultaneous first-price auction markets. For each market, the econometrician observes the characteristics (X,Z), as well as the bid vectors $(b_i)_{i=1}^N$ submitted by all bidders. The identification problem is to recover the nonparametric primitives $F_i(\cdot|X,Z)$ and $\kappa_i(X,Z)$ for each bidder i.

We analyse this problem using the following notation. For each bidder $i=1,\ldots,N$, let $G_i(\cdot|X,Z)$ be the joint cumulative distribution function of the $L_i\times 1$ bid vector b_i submitted by i conditional on characteristics (X,Z), and let $g_i(\cdot|X,Z)$ be the corresponding conditional joint density. For each auction $l\in\mathcal{L}_i$, let $P_{il}(b_i|X,Z)$ denote the marginal probability that bidder i wins auction l, and for each combination $\omega\in\Omega_i$, let $P_i^{\omega}(b_i|X,Z)$ denote the joint probability that bidder i wins combination ω, both interpreted as functions of i's bid vector $b_i\in\mathcal{B}_i$ taking rival strategies σ_{-i}^{XZ} as given. Finally, let $P_i^{\mathcal{L}}(b_i|X,Z)\equiv(P_{il}(b_i|X,Z))_{l\in\mathcal{L}_i}$, an $L_i\times 1$ vector, collect marginal win probabilities $P_{il}(b_i|X,Z)$ across auctions $l\in\mathcal{L}_i$, and let $P_i^{\Omega}(b_i|X,Z)\equiv[P_i^{\omega}(b_i|X,Z)]_{\omega\in\Omega_i}$, a $2^{L_i}\times 1$ vector, collect combinatorial win probabilities $P_i^{\omega}(b_i|X,Z)$ across combinations $\omega\in\Omega_i$. Note that, if there are no ties, then i's marginal probability of winning auction l, i.e. $P_{il}(b_i|X,Z)$, is simply the c.d.f. of the maximum rival bid in auction l, evaluated at i's bid b_{il}. Furthermore, by construction, marginal win probabilities $P_i^{\mathcal{L}}(b_i|X,Z)$ are related to combinatorial win probabilities $P_i^{\Omega}(b_i|X,Z)$ by the identity $P_i^{\mathcal{L}}(b_i|X,Z)\equiv\Omega_i^T P_i^{\Omega}(b_i|X,Z)$.

Under Assumption 3, $G_i(\cdot|X,Z)$ is identified directly for each $i=1,\ldots,N$, with identification of $(G_i(\cdot|X,Z))_{i=1}^N$ implying identification of $P_i^{\Omega}(\cdot|X,Z)$ and $P_i^{\mathcal{L}}(\cdot|X,Z)$ for all i. We first show that, given these directly identified objects, bidder i's primitives (F_i,κ_i) are identified up to κ_i. We then provide sufficient conditions for the identification of κ_i based on excludable variation in either the set of competitors faced or the characteristics of other objects.

3.1. *Non-parametric identification of F_i up to κ_i*

Consider the bidding problem faced by bidder $i=1,..,N$ with preferences (V_i,K_i) in market (X,Z), where standalone valuations V_i are drawn privately from $F_i(\cdot|X,Z)$ and complementarities $K_i=\kappa_i(X,Z)$ are common knowledge as described above. By hypothesis, taking rival strategies σ_{-i}^{XZ} as given, bidder i optimally submits the $L_i\times 1$ bid vector $b_i\in\mathcal{B}_i$ which maximizes her expected interim profit function

$$\pi_i(b_i;V_i,K_i|X,Z)=P_i^{\mathcal{L}}(b_i|X,Z)^T(V_i-b_i)+P_i^{\Omega}(b_i|X,Z)^T K_i, \tag{1}$$

where $P_i^{\mathcal{L}}(b_i|X,Z)^T(V_i-b_i)$ reflects the sum of bidder i's expected standalone payoffs in each auction, and $P_i^{\Omega}(b_i|X,Z)^T K_i$ reflects the change in i's expected payoffs induced by non-additivities in her preferences over combinations.

Under Assumption 4, one can show that the interim function profit function (1) is differentiable in b_i almost surely with respect to the measure on $\mathcal{B}_i$ induced by $G_i(\cdot|X,Z)$. Hence, under the hypothesis of equilibrium play, almost every bid $b_i\in\mathcal{B}_i$ submitted by i must satisfy the $L_i\times 1$

system of first-order necessary conditions:

$$\nabla_b P_i^{\mathcal{L}}(b_i|X,Z)(V_i-b_i)=P_i^{\mathcal{L}}(b_i|X,Z)-\nabla_b P_i^{\Omega}(b_i|X,Z)^T K_i, \tag{2}$$

where $\nabla_b P_i^{\mathcal{L}}(b_i|X,Z)$ is an $L_i \times L_i$ diagonal matrix and $\nabla_b P_i^{\Omega}(b_i|X,Z)$ is an $2^{L_i} \times L_i$ matrix.

Since the system (2) involves only L_i equations, we cannot uniquely solve (2) for (V_i, K_i) jointly. But if we fix any guess K_i for i's unknown complementarity vector $\kappa_i(X,Z)$, then for almost every bid b_i submitted by i there exists a unique, identified candidate $\xi_i(b_i|X,Z;K_i)$ for V_i at which b_i satisfies first-order necessary conditions for a best response:

$$\xi_i(b_i|X,Z;K_i) \equiv \Upsilon_i(b_i|X,Z)-\Psi_i(b_i|X,Z)\cdot K_i, \tag{3}$$

where $\Upsilon_i(b_i|X,Z)$ is an identified $L_i \times 1$ vector defined by

$$\Upsilon_i(b_i|X,Z) \equiv b_i+\nabla_b P_i^{\mathcal{L}}(b_i|X,Z)^{-1} P_i^{\mathcal{L}}(b_i|X,Z), \tag{4}$$

and $\Psi_i(b_i|X,Z)$ is an identified $L_i \times 2^{L_i}$ matrix defined by

$$\Psi_i(b_i|X,Z) \equiv \nabla_b P_i^{\mathcal{L}}(b_i|X,Z)^{-1} \nabla_b P_i^{\Omega}(b_i|X,Z)^T. \tag{5}$$

Note that $\xi_i(b_i|X,Z;K_i)$ is affine in K_i for all b_i and (X,Z). The additive term $\Upsilon_i(b_i|X,Z)$ is the standard auction-by-auction inverse bidding function of Guerre *et al.* (2000), vectorized over the L_i auctions played by i.[12] The multiplicative term $\Psi_i(b_i|X,Z)K_i$ adjusts for potential non-additivities in i's preferences, reflected in the conjectured complementarity vector K_i. The weights $\Psi_i(b_i|X,Z)$ on K_i correspond, intuitively, to the marginal effect of increasing each standalone bid b_{il} on i's probability of winning each higher-order combination, relative to the marginal effect of increasing b_{il} on i's probability of winning auction l.

Finally, since equilibrium bids must be optimal, if in fact $K_i=\kappa_i(X,Z)$, then we must have $V_i=\xi_i(b_i|X,Z;K_i)$ almost surely. Hence to each candidate K_i for $\kappa_i(X,Z)$, there corresponds a unique, identified candidate $\hat{F}_i(\cdot|X,Z;K_i)$ for the unknown c.d.f. $F_i(\cdot|X,Z)$:

$$\hat{F}_i(v|X,Z;K_i)=\int_{\mathcal{B}_i} 1[\xi_i(B_i|X,Z;K_i)\leq v]\, G_i(dB_i|X,Z). \tag{6}$$

We formalize these observations in the following proposition:

Proposition 1. *Suppose that Assumptions 1–4 hold. Then for almost every b_i drawn from $G_i(\cdot|X,Z)$, both $\Upsilon_{il}(b_i|X,Z)$ and $\Psi_i(b_i|X,Z)$ exist and are identified. Consequently, for all $K_i \in \mathbb{R}^{2^{L_i}}$, $\hat{F}_i(\cdot|X,Z;K_i)$ exists and is identified up to K_i. Furthermore, if $K_i=\kappa_i(X,Z)$, then (i) $V_i=\xi_i(b_i|X,Z;K_i)$ almost surely, and (ii) $F_i(\cdot|X,Z)=\hat{F}_i(\cdot|X,Z;\kappa_i(X,Z))$.*

Proof. See Appendix A. □

Identification thus reduces to recovery of the non-parametric function $\kappa_i(X,Z)$ describing i's complementarities, with both realizations and distributions of standalone valuations identified up to $\kappa_i(X,Z)$ through the inverse bidding function $\xi_i(b_i|X,Z;K_i)$.

12. To see this, recall that under Assumption 4 the lth element of $P_i^{\mathcal{L}}(b_i|X,Z)$ is simply the c.d.f. of the maximum bid among i's rivals in auction l, evaluated at b_{il}. Hence, the lth element of $\Upsilon_i(b_i|X,Z)$ reduces to

$$\Upsilon_{il}(b_i|X,Z)=b_{il}+\frac{P_{il}(b_{il}|X,Z)}{p_{il}(b_{il}|X,Z)} \quad \text{where} \quad p_{il}(b_{il}|X,Z)\equiv\frac{d}{db_{il}}P_{il}(b_{il}|X,Z),$$

i.e. the usual standalone inverse bid function of Guerre *et al.* (2000) in auction l.

3.2. *Non-parametric identification of complementarities based on variation in rival characteristics*

In view of Proposition 1, it is also clear that further structure is necessary for identification: under Assumptions 1–4, we can identify valuations only up to complementarities. But suppose that, to these assumptions, we add the hypothesis that bidder i's primitives (F_i,κ_i) depend only on bidder i's characteristics Z_i, not on the characteristics of rival bidders Z_{-i}:

Assumption 5. *For all bidders i, $F_i(\cdot|X,Z)=F_i(\cdot|X,Z_i)$ and $\kappa_i(X,Z)=\kappa_i(X,Z_i)$.*

Similar assumptions have been widely employed in the empirical auction literature; see e.g. Guerre, Perrigne and Vuong (2009) and Somaini (2020) among others. We will show that under Assumption 5, variation in competitor characteristics Z_{-i} induces a large (infinite) set of restrictions on the finite vector $\kappa_i(X,Z_i)$. Under mild conditions on variation in Z_{-i} made precise below, these restrictions will have the unique solution $K_i=\kappa_i(X,Z_i)$, leading to non-parametric identification of $\kappa_i(X,Z_i)$ and hence the model as above.

Toward this end, consider any bidder $i=1,\dots,N$. Fix any values of auction characteristics X and own characteristics Z_i. Let Z and Z' be any two vectors of bidder characteristics such that $Z_i=Z_i'$ but $Z_{-i}\neq Z_{-i}'$. Bids observed under market structures (X,Z) and (X,Z') will of course typically correspond to different realizations of i's standalone valuations V_i. But, under Assumption 5, V_i will be drawn from the same distribution $F_i(\cdot|X,Z_i)$ at both (X,Z) and (X,Z'). Furthermore, from Proposition 1, for each (X,Z) and each candidate complementarity vector K_i, there exists a unique, identified candidate $\hat{F}_i(\cdot|X,Z;K_i)$ for $F_i(\cdot|X,Z_i)$. Hence, if $K_i=\kappa_i(X,Z_i)$, then for almost every $v\in\mathbb{R}^{L_i}$, we must have

$$\hat{F}_i(v|X,Z;K_i)=F_i(v|X,Z_i)=\hat{F}_i(v|X,Z';K_i). \tag{7}$$

Clearly, if $\hat{F}_i(\cdot|X,Z;K_i)$ and $\hat{F}_i(\cdot|X,Z';K_i)$ coincide almost everywhere, then the expectations of random vectors drawn from these distributions must also coincide. But recall that, by definition, $\hat{F}_i(\cdot|X,Z;K_i)$ is the c.d.f. of the random vector $\xi_i(B_i|X,Z;K_i)$, where $B_i\sim G_i(\cdot|X,Z)$. Hence, if $K_i=\kappa_i(X,Z_i)$, then in view of (7) we must also have

$$\int_{\mathcal{B}_i}\xi_i(B_i|X,Z;K_i)G_i(dB_i|X,Z)=\int_{\mathcal{B}_i}\xi_i(B_i|X,Z';K_i)G_i(dB_i|X,Z'). \tag{8}$$

Finally, recall that $\xi_i(\cdot|X,Z;K_i)$ is affine in K_i. Hence, we may equivalently rewrite each integral in (8) as an identified affine function of K_i as follows:

$$\begin{aligned}\int_{\mathcal{B}_i}\xi_i(B_i|X,Z;K_i)G_i(dB_i|X,Z)&=\int_{\mathcal{B}_i}[\Upsilon_i(B_i|X,Z)-\Psi_i(B_i|X,Z)\cdot K_i]\,G_i(dB_i|X,Z)\\&\equiv\bar{\Upsilon}_i(X,Z)-\bar{\Psi}_i(X,Z)\cdot K_i,\end{aligned} \tag{9}$$

where $\bar{\Upsilon}_i(X,Z)$, an identified $L_i\times 1$ vector, and $\bar{\Psi}_i(X,Z)$, an identified $L_i\times 2^{L_i}$ matrix, denote the expectations of the functions $\Upsilon_i(\cdot|X,Z)$ and $\Psi_i(\cdot|X,Z)$ with respect to bids drawn from i's equilibrium bid distribution $G_i(\cdot|X,Z)$:

$$\bar{\Upsilon}_i(X,Z)\equiv\int_{\mathcal{B}_i}\Upsilon_i(B_i|X,Z)G_i(dB_i|X,Z),$$

$$\bar{\Psi}_i(X,Z)\equiv\int_{\mathcal{B}_i}\Psi_i(B_i|X,Z)G_i(dB_i|X,Z).$$

Substituting (9) into (8) under the hypothesis $K_i = \kappa_i(X,Z_i)$, we obtain a system of L_i linear restrictions on the unknown vector $\kappa_i(X,Z_i) \in \mathcal{K}_i$:

$$\left[\bar{\Upsilon}_i(X,Z) - \bar{\Upsilon}_i(X,Z')\right] - \left[\bar{\Psi}_i(X,Z) - \bar{\Psi}_i(X,Z')\right] \cdot \kappa_i(X,Z_i) = 0. \tag{10}$$

Recall that the first $L_i + 1$ elements of $\kappa_i(X,Z_i)$ are zero by construction. Hence, (10) is a system of L_i equations in $2^{L_i} - L_i - 1$ unknowns. When $L_i > 2$, we have $2^{L_i} - L_i - 1 > L_i$, hence the system (10) alone will be insufficient to identify $\kappa_i(X,Z_i)$. But recall that (10) must hold for *any* Z, Z' such that $Z_i = Z_i'$. In other words, for given (X,Z_i), every distinct realization of rival characteristics Z_{-i} generates a set of L_i linear restrictions parallelling (10), all of which must hold simultaneously at $K_i = \kappa_i(X,Z_i)$. Pooling such linear restrictions across many markets with varying rival characteristics Z_{-i}, we ultimately conclude:

Proposition 2. *Maintaining Assumptions 1–5, consider any bidder $i = 1,\ldots,N$ and any values of observed auction and bidder characteristics X and Z_i. Suppose there exists a collection of rival type realizations $\{Z_{-i}^j\}_{j=1}^J$ in the support of $Z_{-i}|X,Z_i$, such that the sub matrix formed by the last $(2^{L_i} - L_i - 1)$ columns of the $J(J-1)L_i \times 2^{L_i}$ matrix*

$$\left[\bar{\Psi}_i(X,Z_i,Z_{-i}^j) - \bar{\Psi}_i(X,Z_i,Z_{-i}^k)\right]_{j,k\in\{1,\ldots,J\}}$$

has rank $2^{L_i} - L_i - 1$. Then $\kappa_i(X,Z_i)$ is identified.

Recall that the identification criterion (10) exploits only invariance of *first moments* of $F_i(\cdot|X,Z_i)$, even though the underlying distributional restriction (7) implies that relations analogous to (8) hold for the whole characteristic function. The system of equations in Proposition 2 merely provides a simple sufficient condition under which the full characteristic system has a unique solution. Note also that variation in, e.g. number of rivals in each auction will produce exactly the kind of variation required in Proposition 2: non-linear changes in the Jacobian Ψ_i of probabilities of winning different combinations, which map into bidding as weights on the unknown vector $\kappa_i(X,Z_i)$. Even discrete variation in Z_{-i} thus naturally gives rise to the rank condition required for non-parametric identification of $\kappa_i(X,Z_i)$, which in turn implies identification of $F_i(\cdot|X,Z_i)$ through Proposition 1.

3.3. *Non-parametric identification of complementarities based on variation in characteristics of other objects*

While the restriction that own primitives are invariant to competitor characteristics is both natural and widely employed, one could also consider identification based on other exclusion restrictions. For example, one could assume that standalone valuations for each object and complementarities for each combination depend only on the characteristics of the objects in question, not on those of other objects. Letting X^ω be the sub-vector of X describing objects in the combination $\omega \in \Omega_i$, we formalize this idea as follows:

Assumption 6. *For each bidder i, $F_{il}(\cdot|X,Z) = F_{il}(\cdot|X_l,Z)$ for each object $l \in \mathcal{L}_i$, and $\kappa_i^\omega(X,Z) = \kappa_i^\omega(X^\omega,Z)$ for each combination $\omega \in \Omega_i$.*

Importantly, this assumption allows both $F_i(\cdot|Z,X)$ and $\kappa_i(X,Z)$ to depend on Z_{-i}. Any variation in X which does not affect either X_l or X^ω would then give rise to a system of identifying equations paralleling Proposition 2. Sources of such variation include, for example, variation in bidder i's distance to other projects holding distance between projects constant, or in characteristics of projects bid by i's rivals but not by i. Obviously, where appropriate, Assumptions 5 and 6 can also be maintained jointly, as we do in our application.

4. APPLICATION: MICHIGAN HIGHWAY PROCUREMENT

MDOT allocates contracts for a wide range of highway construction and maintenance services via low-price sealed-bid auctions. The vast majority of MDOT projects are allocated via large simultaneous letting rounds, which take place on average every 3 weeks. There are an average of 45 auctions per letting round and more than half (56%) of bidders submit bids on multiple contracts within a letting.[13] A bid is an itemized description of unit costs for each line item specified in contract plans; bids are submitted to MDOT project by project, with the winner of each project the bidder submitting the bid involving the lowest total project costs. Contracts are advertised up to 10 weeks prior to letting, with the closing deadline for submitting, amending, or withdrawing bids typically 10 am on the letting date. MDOT then publicly opens bids and allocates contracts, with winning bidders held liable for the completion of contracts won. We expect factors such as capacity constraints, project proximity, project types, and scheduling overlap to induce substantial non-additivities in bidder payoffs across auctions. We focus on potential complementarities across auctions within a MDOT letting round, abstracting from complementarities across letting rounds.[14]

4.1. *Data*

MDOT provides detailed records on contracts auctioned, bids received, and letting outcomes on its letting website (https://www.bidx.com/mi/main). Drawing from these records, we observe data on (almost) all contracts auctioned by MDOT over the sample period January 2005 to March 2014.[15] Our sample includes 8224 auctions, where for each auction we observe the project description, project location, pre-qualification requirements, the MDOT engineer's estimate of total project cost, and the list of participating firms and their bids. We classify projects into five types: bridge work, major construction, paving (primarily hot-mix asphalt), safety (e.g. signing and signals), and miscellaneous. Roughly 80% of contracts are for bridge work, major construction, and paving, with the remainder split between safety and other miscellaneous construction.

The data contain information on 726 unique bidders active in the MDOT marketplace over our sample period, which we classify by size and scope of activity as follows. We define a bidder as "regular" if it submitted more than 100 bids in the sample period, and "fringe" otherwise. This yields a total of 36 regular bidders, with all other bidders classified as "fringe." For the subsample of bidders who have submitted more than 50 bids, we also collect data on the number and location of plants owned by the firm. This data is derived from a variety of sources: OneSource North America Business Browser, Dun and Bradstreet, Hoover's, Yellowpages.com and firms' websites.

13. To ensure the quality of work, MDOT runs a pre-qualification process, which involves a check on the financial status of the firm and its backlog from all construction activities. A bid submission includes a detailed break down of all costs involved in the contract. The winner is the bidder submitting the lowest tabulated bid.

14. A formal analysis of both static and dynamic complementarities is beyond the scope of the current paper, although it would be a very interesting avenue for future research.

15. MDOT records for a small number of contracts are incomplete. Although we have data from October 2002 to March 2014, we have discarded the first few years (from October 2002 to December 2004) as we use lettings from these years to construct bidder backlog variables.

TABLE 1
Auction-level summary statistics

	Mean	St. Dev.	Min	Max
Auctions per round	45.19	35.67	1	133
Total bids per round	228.2	180.9	1	669
Distinct bidders per round	83.99	57.08	1	207
Number of bidders per auction	5.049	3.186	1	28
Large regular bidders per auction	0.766	1.016	0	4
Other regular bidders per auction	1.787	1.911	0	11
Fringe bidders per auction	2.496	2.402	0	20
Engineer's estimate (in thousands)	1,521	4,754	4.412	165,313
Project duration (in days)	175.8	205.1	2	1,838
Money left on the table	0.075	0.104	0	3.538

TABLE 2
Bidder-level summary statistics

	Mean	*St.Dev.*	Min	Max
Bids by round	2.716	2.786	1	33
Bids by round if large	5.717	5.160	1	33
Bids by round if regular	4.780	3.956	1	33
Backlog (in millions)	5.790	19.01	0	275.5

We then further classify bidders as "large" or "small" based on this data, with "large" bidders those owning at least 6 plants in Michigan. We thus obtain a final classification of 8 large regular bidders, 28 small regular bidders, and 686 fringe bidders (of which 4 are large bidders) in the MDOT marketplace.

Table 1 surveys the auction side of the MDOT marketplace. The first key feature emerging from this table is the large number of contracts auctioned simultaneously in the market: a mean of 45 per letting, with a maximum of 133 on a single letting date.[16] On average about five bids are received per contract, which is small relative to the average number of bidders (approximately 84) active in any given letting. For each contract, MDOT prepares an "Engineer's Estimate" of expected procurement cost which is released to bidders before bidding; as evident from the dispersion in this estimate, projects vary substantially in size and complexity. The statistic "Money Left on the Table" measures the percent difference between the lowest and second-lowest bids. On average, this is 7.4%, or roughly \$112,000 per contract, suggesting the presence of substantial uncertainty over rival bids.

Table 2 summarizes bidder behaviour in the MDOT marketplace. Consistent with Table 1, the average bidder competes in roughly 2.7 auctions per round, with large and regular bidders competing in substantially more. The variable "backlog" provides a bidder-specific measure of capacity utilization.[17] Note that number of bids submitted by any given bidder is small relative to the number of auctions in the marketplace, with even large bidders competing in less than fifteen percent of total auctions on average.

Finally, Figure 1 plots the histogram (over all bidders i and lettings t) of the number of bids submitted by bidder i in letting t. More than 55% of active bidders submit multiple bids in the

16. Note that smaller supplemental lettings are occasionally held 2 or 3 weeks after the main letting in a given month.

17. We define backlog for bidder i at date t as the sum of work remaining among projects l won by i up to t, where work remaining on project l at date t is defined as total project size (measured by the engineer's estimate) times the proportion of scheduled project days remaining at date t.

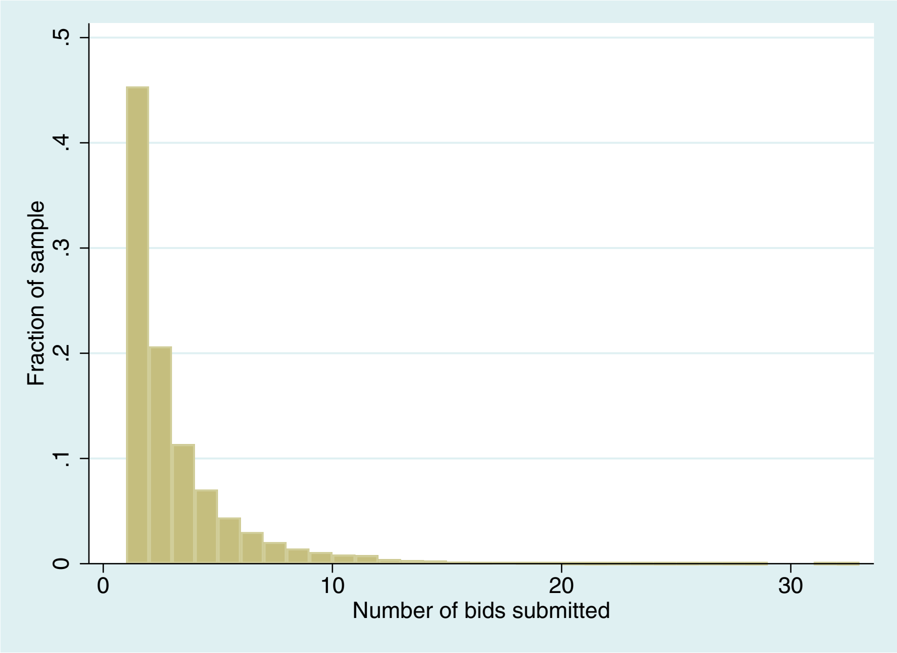

FIGURE 1
Number of simultaneous bids submitted per bidder by letting

same letting. Despite this, it is relatively uncommon for a typical bidder to compete in a large number of auctions; roughly 92% of bidders in our sample bid in 6 or fewer auctions and only 2.5% bid in more than 10.

4.2. *Descriptive regressions*

We next present a series of regressions exploring simultaneous bidding in the MDOT marketplace. The unit of analysis in these regressions is a bidder-auction-round combination. The dependent variable is the log of the bid submitted by bidder i in auction l in letting t, regressed on a vector of covariates intended to capture the effect of own- and cross-auction characteristics on i's bid in the auction l.

4.2.1. Regression specification. As usual, we control for a number of auction-level characteristics which we expect to be key direct determinants of i's bid in the auction l: the size of auction l, captured by the MDOT engineer's estimate of project cost, the level of competition i faces in the auction l, and the distance between project l and i's base of operations.[18] To control for the direct cost effects of capacity usage, we also include a standardized bidder-level backlog variable, derived from the backlog measure described above by subtracting the mean and dividing by the standard deviation of backlogs for each bidder over time.

18. For each bidder–project pair, we construct the minimum straight-line distance (in miles) between any of i's plants and the centroid of the county in which project l is located. We take the shortest distance if bidder i owns multiple plants.

To explore cross-auction interaction in the MDOT marketplace, we also include a set of covariates relevant for combination payoffs but irrelevant for standalone valuations after conditioning on characteristics of the auction l. To control for cross-auction competition which may shift combination win probabilities, we consider the total number of rivals across all auctions played by bidder i. To capture the presence of capacity constraints or diseconomies of scale, we consider two variables: the (log of) the sum of engineer's estimates across all auctions in which i is bidding, and the fraction of days overlapping among projects for which i bids.[19] To account for the possibility that complementarities between similar projects may differ from those between different projects, we include an index of concentration for the types of projects for which i is bidding, defined as a Herfindahl index over shares of each project type in i's participation set. Finally, to measure potential economies or diseconomies induced by the distance between projects, we consider the (log of) total distance between the current project and each other project for which i bids, normalized by the total distance between each of these projects and the closest plant owned by bidder i.

4.2.2. Regression results. Table 3 reports OLS estimates for our baseline regression specifications: log bids on the own- and cross-auction characteristics defined above. We include a full set of bidder type, project type, and letting date indicators, with standard errors clustered by bidder to allow for correlation in elements of b_{it}. We also consider a specification with bidder identity rather than bidder type fixed effects.

Estimated effects of own-auction characteristics correspond closely both to our prior and to findings elsewhere in the literature. Bids are increasing almost one for one in project size, with the coefficient on log engineer's estimate exceeding 0.97. Bidders facing more competition bid more aggressively, with one additional competitor associated with a 4–5% decrease in average bids. Finally, a 1% increase in i's distance to the project leads to about a 2% increase in i's bid on average. More importantly, estimated cross-auction effects are also significant, with magnitudes stable across specifications and signs broadly consistent with our prior expectations. The positive coefficient on the log sum of engineer's estimates suggests that competing for many large projects leads to a substantial decrease in aggressiveness by bidder i in auction l, with the negative coefficient on same-type projects suggesting that this effect is ameliorated when the two projects are of the same type. Similarly, the positive sign on log distance among projects suggests that increasing distance to other projects leads to less aggressive bids. Finally, the significant negative coefficient on total number of rivals in auctions participated by i suggests that facing more competition across auctions leads bidder i to bid more aggressively in the auction l. Taken together, these results corroborate the hypothesis that simultaneous bidding induces strategic spillovers across auctions.

5. ESTIMATION OF COMPLEMENTARITIES

We now turn to this article's primary interest: estimation of the function $\kappa_i(\cdot)$ describing complementarities. In principle, the results in Section 3 support fully non-parametric estimation of κ_i. In practice, the dimensionality of the problem renders this infeasible. We therefore implement estimation in two steps. First, we estimate a parametric approximation to the equilibrium distribution G_{it} of bids submitted by each bidder i in letting t. Second, we map observed bids through the inverse bidding function (3) implied by these first-step estimates to obtain a set of

19. That is, the total number of overlapping days for projects for which i submits bids, scaled by the sum of days scheduled for each of these projects.

TABLE 3
OLS estimates of cross-auction effects

$y = ln(bid)$	1	2
Log engineer's estimate	0.9708***	0.9763***
	(0.0021)	(0.0020)
Log number of rivals	−0.0496***	−0.0355***
	(0.0081)	(0.0076)
Log distance to project	0.0213***	0.0135***
	(0.0019)	(0.0022)
Log days to project start	0.0040***	0.0036***
	(0.0013)	(0.0013)
Standardized backlog	0.0023*	0.0024**
	(0.0012)	(0.0012)
Log number of big rivals faced	0.0016	0.0101
	(0.0063)	(0.0070)
Log number of regular rivals faced	0.0238***	0.0229***
	(0.0039)	(0.0046)
Multiple-bid indicator	−0.0897***	−0.1750***
	(0.0249)	(0.0353)
Log sum engineer's estimate across played auctions	0.0058***	0.0114***
	(0.0018)	(0.0025)
Log sum number of rivals across played auctions	−0.0146***	−0.0123***
	(0.0035)	(0.0046)
Log distance across played projects	0.0037*	0.0037
	(0.0021)	(0.0029)
Fraction overlapping time across projects	0.0189***	0.0148***
	(0.0044)	(0.0056)
Same-type-auctions concentration index	−0.0110**	−0.0284***
	(0.0049)	(0.0079)
Large bidder		0.0093
		(0.0151)
Regular bidder		−0.0031
		(0.0075)
Year FE, Month FE, Auction type FE	YES	YES
Bidder type FE	NO	YES
Bidder ID FE	YES	NO
R^2	0.9736	0.9778

Notes: Unit of analysis is bidder-auction-round, with standard errors clustered by bidder. There are 41,524 observations. Variables *log engineer's estimate*, *log number of rivals* of each type, and *log of distance to project* measure size, strength of competition, and distance to project l, respectively. Remaining variables capture cross-auction characteristics: number of rivals in other auctions, sum engineer's estimate, distance to auctions scaled by distance to project l in which i is competing and number of overlapping days among projects scaled by the total number of days to completion.

moment conditions based on the exclusion restrictions in Section 3, which we then use to estimate parameters in κ_i. Following Groeger (2014), we assume there is no binding reserve price.[20]

5.1. *First step: estimation of G_{it}*

The first step in our procedure is to estimate the conditional joint distribution G_{it} of bids submitted by each bidder i in letting t. In view of the dimensionality of this problem, we follow Cantillon and Pesendorfer (2006) and Athey *et al.* (2011) in estimating a parametric approximation to this joint distribution, which we specify as follows. We model the $L_{it} \times 1$ bid vector b_{it} as drawn from a multivariate log-normal distribution characterized by mean vector μ_{it}

20. When a bidder is a sole participant (which happens only 136 times out of 8824 auction analysed), they will face MDOT that draws a completion cost from a fringe bidder's bid distribution.

and variance–covariance matrix Σ_{it}:

$$\ln(b_{it}) \sim g(\mu_{it}, \Sigma_{it}).$$

In theory, each bidder's equilibrium bid function depends not only on the bidder's own characteristics and the characteristics of the projects for which it bids but also on competitors' characteristics and the characteristics of all the auctions where they participate. In practice, it will be impossible to condition on all theoretically relevant variables, so we propose a parsimonious specification where we choose variables in each category guided by the reduced form analysis in Section 4. Thus, we allow the parameters μ_{it} and Σ_{it} to depend on a vector of observables including i's characteristics Z_{it}, project characteristics X_{lt}, characteristics of the combination of objects for which i bids, and the number and types of rivals i faces within and across auctions. Specifically, for each auction $l = 1, \ldots, L_{it}$ played by i, we model the mean and variance of $\ln(b_{it,l})$ as $\mu_{it,l} = \alpha \cdot M^{\mu}_{it,l}$ and $\sigma^2_{it,l} = \exp(\beta \cdot M^{\sigma}_{it,l})$ respectively, where $M^{\mu}_{it,l}$ and $M^{\sigma}_{it,l}$ are vectors of covariates specified in Panels A and B of Table 4, and α and β are parameter vectors to be estimated. Meanwhile, we model the covariance $\rho_{it,kl}$ between distinct elements $\ln(b_{it,k})$ and $\ln(b_{it,l})$ of $\ln(b_{it})$ as

$$\rho_{it,kl} = \frac{\exp(\gamma \cdot M^{\rho}_{it,kl}) - 1}{\exp(\gamma \cdot M^{\rho}_{it,kl}) + 1},$$

where $M^{\rho}_{it,kl}$ is a vector of interactions between observable characteristics of projects k and l specified in Panel C of Table 4, and γ is a vector of parameters to be estimated.[21]

We estimate the parameters (α, β, γ) in this first-step model by maximum likelihood, pooling data from bidders that participate in different numbers of auctions, with results reported in Table 4. Not surprisingly, mean parameters $\hat{\alpha}$ are very similar to coefficients in our descriptive regressions. Variance parameters $\hat{\beta}$ suggest that bidders competing in multiple auctions and for larger projects submit less dispersed bids.[22] Finally, correlation parameters $\hat{\gamma}$ suggest at least two broad patterns in bidding behaviour across auctions. First, bidders tend to bid more similarly for projects in the same county or of the same type. Second, if competing for two projects whose schedules overlap, bidders tend to bid for one relatively more aggressively than the other. This is consistent with our prior that overlapping schedules exacerbate diseconomies of scale.

To evaluate the goodness of fit of this first-step model, Figure 2 plots the observed distribution of log bids across all auctions and bidders, together with the predicted distribution of log bids implied by the estimates in Table 4. As can be seen in Figure 2, the fit of our parametric approximation appears excellent, reinforcing confidence in the first-step estimates above.

5.2. *Second step: estimation of complementarities*

In view of our low-bid procurement application, we translate the general model in Section 2 into low-bid terms as follows. Let V_{itl} be i's private standalone cost for completing project $l \in \mathcal{L}_{it}$, and $\kappa_i(X_t, Z_{it})$ be the vector of cost complementarities associated by bidder i with each combination

21. Since at this stage we model the distribution of bids conditional on observables as continuous with respect to continuous characteristics, we implicitly assume continuity of the equilibrium selection mechanism, which represents a strengthening of Assumption 3. For further discussion on the continuity of equilibrium selection in games with multiple equilibria, see de Paula (2013) and Aguirregabiria and Mira (2008).

22. While the parametrization of Σ_{ilt} does not imply its positive semi-definitiveness, the estimated variance-covariance matrix is positive semi-definite.

TABLE 4
First-step MLE estimates of parameters in G_i

Mean μ_{ilt}	$\hat{\alpha}$	MLE SEs	95% CI	
Auction l and bidder characteristics				
Constant	0.3766	0.0158	0.3456	0.4076
Log engineer's estimate	0.9769	0.0008	0.9753	0.9785
Log rivals in auction	−0.0352	0.0027	−0.0405	−0.0299
Log distance to project	0.0129	0.0009	0.0111	0.0147
Log days to the start	0.0039	0.0008	0.0023	0.0055
Standardize backlog	0.0024	0.001	0.0004	0.0044
Big bidder	0.0023	0.0044	−0.0063	0.0109
Regular bidder	−0.0023	0.0025	−0.0072	0.0026
Log number of big rivals faced	0.0104	0.003	0.0045	0.0163
Log number of regular rivals faced	0.0237	0.0021	0.0196	0.0278
Bidder Type FE	YES	–	–	–
Auction Type FE	YES	–	–	–
Other auctions characteristics				
Multiple bids dummy	−0.1728	0.0206	−0.2132	−0.1324
Same-type-auctions index	−0.0292	0.005	−0.039	−0.0194
Fraction overlapping time	0.0148	0.0035	0.0079	0.0217
Log sum engineer's (across l)	0.0113	0.0014	0.0086	0.014
Log sum rivals (across l)	−0.012	0.002	−0.0159	−0.0081
Log distance across played projects	0.0032	0.0017	−0.0001	0.0065
Year FE	YES	–	–	–
Month FE	YES	–	–	–
Variance σ^2_{ilt}	$\hat{\beta}$	MLE SEs	95% CI	
Constant	0.0652	0.0723	−0.0765	0.2069
Multiple bids dummy	−0.2211	0.0189	−0.2581	−0.1841
Log engineer's estimate	−0.2582	0.0053	−0.2686	−0.2478
Covariance ρ_{iklt}	$\hat{\gamma}$	MLE SEs	95% CI	
Constant	0.0052	0.0005	0.0042	0.0062
Same county projects	0.0033	0.0006	0.0021	0.0044
Same-type projects	0.0019	0.0004	0.0011	0.0027
Fraction overlapping time	−0.001	0.0005	−0.002	0

$\omega \in \Omega_{it}$. We adopt the convention that $\kappa_i^{\omega}(X_t, Z_{it}) > 0$ means that winning combination ω increases bidder i's joint completion costs, while $\kappa_i^{\omega}(X_t, Z_{it}) < 0$ means that winning combination ω decreases bidder i's joint completion costs.

While our identification argument allows $\kappa_i^{\omega}(X_t, Z_{it})$ to be non-parametric, in practice the high dimensionality of κ_i^{ω} renders non-parametric inference infeasible. We therefore adopt a parsimonious parametric structure in which the complementarity i associates with combination ω is modelled as a linear index of a $1 \times Q$ vector of the bidder and combination-level observables M_{it}^{ω} which includes the total combination size and its interaction with bidder size, the distance among projects, the overlapping time between projects, the Herfindahl index of project types in combination ω, and a set of dummies for type of bidder i:

$$\kappa_i^{\omega}(X_t, Z_{it}) = M_{it}^{\omega}\theta_0, \tag{11}$$

where $\theta_0 \subset \Theta$ is a $Q \times 1$ vector of parameters to be estimated. Let M_{it}^{κ} be the $2^{L_{it}} \times Q$ matrix whose rows collect covariate vectors M_{it}^{ω} describing each combination $\omega \in \Omega_{it}$.[23] By construction, under (11), we then have $\kappa_i(X_t, Z_{it}) = M_{it}^{\kappa}\theta_0$.

23. If ω contains only one object, then of course M_{it}^{ω} and $\kappa_i^{\omega}(X_t, Z_{it})$ are taken to be zero.

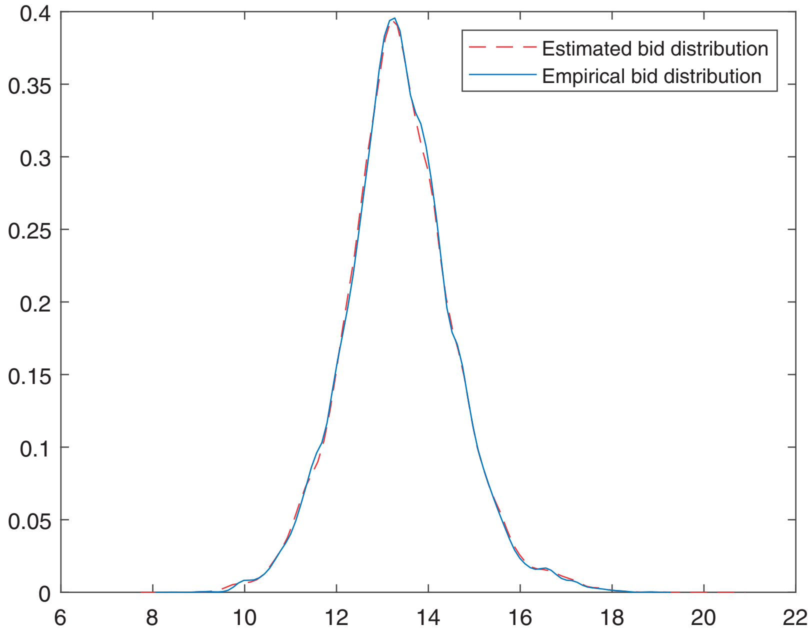

FIGURE 2

Predicted vs. actual distribution of log bids

We aim to estimate the parameters θ_0 governing complementarities. Toward this end, we maintain Assumptions 5 and 6. We further assume that any bidders i,j with the same observed type have the same distribution of standalone valuations: i.e. that $Z_i = Z_j$ implies $F_i(\cdot|X,Z_i) = F_j(\cdot|X,Z_j)$. We decompose bidder i's standalone cost realization for project l as $v_{itl} = E[V_{itl}|X_t,Z_t] + \epsilon_{itl}$, where $E[V_{itl}|X_t,Z_t]$ is the *ex ante* unknown mean of V_{itl} conditional on observables, and ϵ_{itl} is an unobserved cost residual which by definition satisfies $E[\epsilon_{itl}|X_t,Z_t]=0$. Assumptions 5 and 6 imply that $E[V_{itl}|X_t,Z_t] = E[V_{itl}|X_{lt},Z_{it}]$. Recalling that $\kappa_i(X_t,Z_{it}) = M_{it}^{\kappa}\theta_0$, we may thus re-express the inverse bid function (3) as

$$\Upsilon_{itl}(b_{itl}|X_t,Z_t) = E[V_{itl}|X_{lt},Z_{it}] + \Psi_{itl}(b_{it}|X_t,Z_t)M_{it}^{\kappa}\cdot\theta_0 + \epsilon_{itl}, \quad E[\epsilon_{itl}|X_t,Z_t]=0. \tag{12}$$

In applications, one may be willing to model $E[V_{itl}|X_{lt},Z_{it}]$ as a parametric function of X_{lt} and Z_{it}: e.g. $E[V_{itl}|X_{lt},Z_{it}] = X_{lt}\theta_X + Z_{it}\theta_Z$, with θ_X and θ_Z to be estimated.[24] After plugging in first-step estimates for $\Upsilon_{itl}(b_{itl}|X_t,Z_t)$ and $\Psi_{itl}(b_{it}|X_t,Z_t)$, equation (12) would then reduce to a linear-in-parameters estimating equation. The "regressors" $\Upsilon_{itl}(b_{itl}|X_t,Z_t)M_{it}^{\kappa}$ multiplying θ_0 in this equation would be endogenous, since i's bid vector b_{it} is a function of the cost residual ϵ_{itl}. From above, however, we know that $E[\epsilon_{itl}|X_t,Z_t]=0$, which implies that any function of (X_t,Z_t) is mean independent of ϵ_{itl}. Further, observe that the covariates M_{it}^{κ} shift $[\Psi_{itl}(b_{it}|X_t,Z_t)M_{it}^{\kappa}]$ directly, while other elements of $(Z_{-i,t},X_{-l,t})$ shift $[\Psi_{itl}(b_{it}|X_t,Z_t)M_{it}^{\kappa}]$ through the function $\Psi_{itl}(b_{it}|X_t,Z_t)$. Any element of $(Z_{-i,t},X_{-l,t})$ which enters either M_{it}^{κ} or $\Psi_{itl}(b_{it}|X_t,Z_t)$ is therefore a valid excluded instrument for the endogenous "regressors" $[\Psi_{itl}(b_{it}|X_t,Z_t)M_{it}^{\kappa}]$ multiplying θ_0. Estimation of $(\theta_0,\theta_X,\theta_Z)$ could then proceed via linear GMM, using elements of M_{it}^{κ} and other

24. Of course, more flexible parameterizations of $E[V_{itl}|X_{lt},Z_{it}]$ could also be considered.

functions of (X_t, Z_t) as instruments for $[\Psi_{itl}(b_{it}|X_t, Z_t)M_{it}^{\kappa}]$, and moments based on X_{lt} and Z_{it} to pin down θ_X and θ_Z.

We aim, however, to estimate without parameterizing $E[V_{itl}|X_{lt}, Z_{it}]$. Toward this end, we use a matched pairwise differencing strategy in the spirit of Honoré and Powell (2005) and Aradillas-Lopez, Honoré and Powell (2007) to eliminate $E[V_{itl}|X_{lt}, Z_{it}]$ from (12). Specifically, for any distinct bidder-auction observations itl and $j\tau s$, define the differenced residual

$$\begin{aligned}\eta_{itl,j\tau s} = & \Upsilon_{itl}(b_{il}|X_t, Z_t) - \Upsilon_{j\tau s}(b_{js}|X_\tau, Z_\tau) \\ & - \Big[\Psi_{itl}(b_{il}|X_t, Z_t)M_{it}^{\kappa} - \Psi_{j\tau s}(b_{j\tau}|X_\tau, Z_\tau)M_{j\tau}^{\kappa}\Big] \cdot \theta_0. \end{aligned} \tag{13}$$

Then for any two observations itl, $j\tau s$ matched such that $X_{tl} = X_{\tau s}$ and $Z_{it} = Z_{j\tau}$, we have $\eta_{itl,j\tau s} = \epsilon_{itl} - \epsilon_{j\tau s}$, and therefore $E[\eta_{itl,j\tau s}|X_t, Z_t, X_\tau, Z_\tau] = 0$. At the same time, non-matched variables $(Z_{-it}, X_{t,-l})$ will typically differ from $(Z_{-j,\tau}, X_{\tau,-s})$, with all of these variables relevant instruments for the endogenous difference term multiplying θ_0 in (13).

In implementing this pairwise differencing strategy, we match on both a set of discrete covariates denoted by y_{itl}^d and a set of continuous covariates denoted by y_{itl}^c. Discrete covariates y_{itl}^d include year, month, regular, bidder type, project type, number of plants owned by each bidder, and a dummy indicating whether the project starts in the next 180 days. Meanwhile, continuous covariates y_{itl}^c are size and distance, both standardized to have mean zero and standard deviation one. For discrete covariates y^d, we employ exact matching, which effectively splits the whole dataset into a finite number of subgroups, among which we form all non-redundant matches. Let $\hat{D}_n$ be a subgroup defined by the discrete covariates and $\hat{\mathcal{D}} = \{\hat{D}_1, \ldots, \hat{D}_n, \ldots, \hat{D}_{|\hat{\mathcal{D}}|}\}$ the collection of these subgroups. Within each subgroup of discrete matches, we then use a Gaussian product kernel to assign weights to each potential match on the basis of differences in their continuous covariates y^c, scaling bandwidths for each covariate proportionally to Scott's rule of thumb based on the size of each subgroup.

Given the sample of weighted matched pairs thus constructed, we proceed as follows. For each bidder in the estimation sample, we construct empirical analogues $\hat{\Upsilon}_{it}$ and $\hat{\Psi}_{it}$ of the equilibrium objects $\Upsilon_i(b_{itl}|X_t, Z_t)$ and $\Psi_i(b_{it}|X_t, Z_t)$ from our first-step bid distribution estimates $(\hat{G}_i(\cdot|X_t, Z_t))_{i=1}^{N_t}$, approximating gradients using finite differences.[25] Plugging in these first-step estimates $\hat{\Upsilon}_{it}$ and $\hat{\Psi}_{it}$ into (13), we obtain an estimated residual $\hat{\eta}_{itl,j\tau s}$ for each pair of bidders in our matched sample. We form moments based on interactions between these weighted matched differenced residuals $\hat{\eta}_{itl,j\tau s}$ and a vector of instruments $I_{itl,j\tau s}$ (at least Q-dimensional) formed from (X_t, Z_t) and (X_τ, Z_τ).[26] This yields a vector of sample moments $\hat{m}(\theta)$ which, omitting the

25. In practice, a small number of estimated $\hat{\Psi}_{it}$ and $\hat{\Upsilon}_{itl}$ are either very small or very large. To prevent bias from these outliers, we trim the top and bottom 2.5% of values in each of $\hat{\Psi}_{it}$ and $\hat{\Upsilon}_{itl}$.

26. The instruments used are of three types: Z_i/X^{ω} type instruments such as the individual characteristics (big, regular, bidder-type dummies), the number of auctions bid, the sum and the average of the combinatorial-auction characteristics for all possible combinations, the log sum of engineer estimates interacted with regular- and big-bidder dummies, the sum of backlog interacted with big-bidder dummies, log sum of distance and log overlapping time across the maximum number of auctions. Second, we use Z_{-i}-type instruments such as the total number of rivals across auctions in the log. Finally, we use X_{-l}-type instruments such as the log sum of engineer estimates and distance across all **other** auctions.

TABLE 5
Estimated complementarity parameters θ_0

Combination characteristics (Elements of M_{it}^{κ})	$\hat{\theta}$	SE
Fraction overlapping time across projects	0.1099*	0.0567
Distance across played projects	−0.00005	0.0001
Sum engineer's estimate in millions	0.0884***	0.0232
Same-type-auctions index	−0.1833	0.1272
Regular bidder	−0.3834**	0.1513
Big bidder	0.5339**	0.2561
Big bidder × size	−0.1461**	0.0611
Bidder type FE	YES	–

Notes: Coefficient magnitudes are in millions of dollars, positive coefficients imply higher completion costs associated with a combination win. ***, **, and * correspond to 1%, 5%, and 10% significance level, respectively.

normalization factor for simplicity, we may express as

$$\widehat{m}(\theta) = \sum_{\hat{D}_n \in \hat{\mathcal{D}}} \sum_{\substack{x \in \hat{D}_n \\ k \neq x \in \hat{D}_n}} \frac{1}{h^{(1)}(y_x^d) \cdot h^{(2)}(y_x^d)} \times R\left(\frac{y_x^{c,(1)} - y_k^{c,(1)}}{h^{(1)}(y_x^d)}, \frac{y_x^{c,(2)} - y_k^{c,(2)}}{h^{(2)}(y_x^d)}\right) I'_{x,k} \hat{\eta}_{x,k}, \tag{14}$$

where $x \equiv itl$ and $k \equiv j\tau s$ denote distinct bidder-letting-auction observations, $y_x^d = y_k^d$ for all $x \in D_n$ and $k \neq x \in D_n$, $h^{(1)}(y_x^d)$ and $h^{(2)}(y_x^d)$ are the bandwidths and R is a bivariate Gaussian product kernel defining continuous matching weights.[27]

Finally, we estimate θ_0 using a two-step efficient GMM procedure based on the moment restrictions $E[\hat{m}(\theta_0)] = 0$. The standard errors incorporate two-way clustering on i and j to account for correlation generated by the pairwise-differencing strategy, and are adjusted to account for the first step estimation as in Newey and McFadden (1994).[28]

5.3. *Main results: estimated complementarities*

Table 5 reports estimates of θ_0 derived from the two-step matched-difference GMM procedure outlined above. Coefficient magnitudes are in millions of dollars, with negative signs reflecting lower costs and positive signs reflecting higher costs.

The variable "Sum of engineer's estimates" reflects the total size of projects in a combination, with a positive coefficient suggesting that more total work renders a joint win less valuable, as we would expect in the presence of capacity constraints. The coefficient on "Fraction overlapping time" suggests that perfect schedule overlap increases average completion costs by about $\$109,900$. Although not statistically significant, the point estimate on "Same-type auction index" suggests that more homogeneous combinations are less costly; a 0.1 change in the Herfindahl index of project types reduces costs by $\$18,330$. Regular bidders have lower costs for winning

27. Supplementary Appendix G reports the results of two Monte Carlo simulation studies exploring this weighted matched-difference GMM procedure. These confirm that our matching procedure can recover complementarities even in moderately sized samples. We also compare our preferred two-step procedure with a theoretically more efficient one-step procedure, finding that two-step estimation involves negligible efficiency losses.

28. As noted above, one could alternatively parameterize $E[V_{itl}|X_{lt}, Z_{it}]$, either parsimoniously or flexibly, and estimate based on (12) directly. This would simplify inference as one would no longer need to account for the two-way clustered errors which matching introduces. On the other hand, one would need either a parametric form for $E[V_{itl}|X_{lt}, Z_{it}]$ or to introduce many auxiliary parameters. While both approaches have important practical advantages, we have elected to focus on the matching approach here.

TABLE 6
Empirical distribution of normalized complementarities across bidders

Decile rank	Decile of normalized complementarities
10th	−0.1428
20th	−0.0498
30th	−0.0132
40th	0.0190
50th	0.0413
60th	0.0603
70th	0.0792
80th	0.0932
90th	0.1178

Notes: The “normalized complementarity” for bidder i is the estimated complementarity $\kappa_i^{\omega}(Z_i, W_i; \hat{\theta})$ corresponding to the outcome that i wins all projects bid, divided by the sum of engineer’s estimates for these projects. Deciles are evaluated over the distribution of (Z_i, W_i) across bidders. Negative numbers mean lower costs.

combinations which leads to a per-bundle cost advantage of \$383,400. Big bidders have higher costs for small combinations than other regular bidders, but also experience cost advantages for larger combinations, consistent with these bidders having both higher fixed costs and greater economies of scale. With the exception of the coefficient on the distance between projects, which is negative but small and insignificant, these effects are all natural and consistent with our priors. While not reported in Table 5, we also include a vector of bidder type dummies in $\kappa_i(\cdot)$; none of these are statistically significant.

We next translate the parameter estimates $\hat{\theta}$ in Table 5 into estimates for underlying bidder complementarities. Specifically, we first construct, for each bidder i, the estimated complementarity associated with i winning all projects for which they bid. We then normalize this complementarity by the total size of projects in this combination and analyse the deciles of these normalized complementarities across bidders.

Results of this procedure are reported in Table 6. As evident from Table 6, there is substantial heterogeneity in complementarities across bidders in the MDOT sample, with a joint win leading to cost savings of approximately 14% of combination size at the 10th quantile of normalized complementarities, transitioning to cost increases of approximately 12% at the 90th quantile. Recalling the parameter estimates in Table 5, we view these patterns as consistent with an underlying U-shaped cost curve, with completion costs falling until firm resources are fully employed and rising thereafter.

We conclude this section with a note on interpretation of Tables 5 and 6 under endogenous entry. In Supplementary Appendix B, we embed our bidding model within a fully specified entry and bidding game, showing that our estimation strategy is robust to this extension. Hence the parameter estimates reported in Table 5 remain valid even under entry. In interpreting Table 6, however, it is important to note that the distribution of complementarities among projects *in which bidders enter* will differ from that which would arise if projects were randomly assigned. In particular, insofar as bidders tend to bid for combinations involving cost synergies, we would expect the distribution in Table 6 to be negatively skewed.

6. COUNTERFACTUAL: VICKREY–CLARKE–GROVES AUCTION

While the simultaneous first-price auction is clearly inefficient when bidders have combinatorial preferences, little is known about the magnitude of these inefficiencies in practice. Furthermore, little is known either theoretically or empirically about the revenue properties of the simultaneous first-price auction (FPA) mechanism relative to other feasible multi-object mechanisms such as

TABLE 7
Combinatorial VCG outcomes vs. simultaneous FPA outcomes

Mechanism	Outcome	Estimate	Std Err
FPA, estimated κ	Completion costs per auction (in dollars)	1,397,326	2,559
	MDOT payments per auction (in dollars)	1,599,995	–
VCG, estimated κ	Completion cost per auction (in dollars)	1,294,185	15,376
	MDOT payments per auction (in dollars)	1,625,111	7,794
VCG, if instead $\kappa=0$	Completion cost per auction (in dollars)	1,305,311	13,129
	MDOT payments per auction (in dollars)	1,616,155	1,234

Notes: Results are based on the self-contained sample of 5481 auctions such that no bidder in any auction competes against any rival bidding in more than 12 auctions.

the VCG auction. As a first step toward answering these questions, we compare revenue and efficiency under MDOT's actual simultaneous low-price auction with counterfactual outcomes which would have arisen under a combinatorial VCG auction.

Since both the number of combinations and the number of potential allocations increase exponentially in the number of auctions played, it is unfortunately infeasible to solve for VCG outcomes on the full MDOT sample. We therefore focus on the subsample of 5481 self-contained auctions such that no bidder is competing against a rival bidding in more than 12 auctions.[29] For this counterfactual sample, we consider $R=100$ simulation replications. In each replication, we draw a new set of parameters from their asymptotic distribution, then estimate standalone costs for each bidder i and letting t in the counterfactual sample by mapping i's observed bid b_{it} through the inverse bid function (2), given the relevant complementarity estimates.[30] We simulate allocations, costs of project completion, and payments to bidders under both the baseline simultaneous FPA and the counterfactual combinatorial VCG, computing final completion costs inclusive of complementarities in both cases.[31] Finally, we take means and standard deviations of per-auction payments and costs across replications to obtain our final counterfactual results, reported in Table 7.

Two patterns emerge from this exercise. First, as expected, the simultaneous first-price mechanism is socially inefficient, generating expected social costs of roughly \$1.397 million per auction, vs. \$1.294 million per auction for the VCG mechanism. In other words, within our counterfactual sample, per-auction completion costs are roughly \$100,000 lower under the VCG mechanism than under the simultaneous FPA mechanism. In both level and percentage terms, this efficiency gain is non-trivial, suggesting that switching mechanisms could lower social costs by approximately 7.4%. Second, although leading to substantially lower social costs, the VCG mechanism in fact increases MDOT's payments to bidders by about 1.6 percent: from \$1.600 million per auction under the simultaneous FPA to \$1.625 million per auction under the

29. To construct this self-contained sample, we first drop all bidders competing in more than 12 auctions. We then drop any bidder facing a rival (in any auction) who is dropped, and proceed recursively in this fashion until no further bidders are dropped. This recursive procedure alleviates the curse of dimensionality inherent in the VCG allocation problem while ensuring that counterfactual VCG outcomes are comparable to actual FPA outcomes, in the sense that every bidder in the VCG counterfactual is bidding in the same auctions against the same rivals as in the actual data. The resulting counterfactual sample consists of 5481 of our original 8,224 auctions, representing approximately 24,000 of our original 41,000 bid-level observations.

30. In practice, a small fraction of estimated standalone costs are either negative or implausibly large. To prevent bias from these outliers, we windsorize standalone costs at thresholds derived from the 5th and 95th percentiles of relative standalone costs among single-auction bidders.

31. In these simulations, we set MDOT's effective reserve price for each project equal to 200% of the MDOT engineer's cost estimate; other plausible values generate very similar results. We use the Gurobi solver to find VCG allocations: Gurobi Optimization, LLC (2021).

combinatorial VCG. Insofar as MDOT's objective is to minimize its payments, the simultaneous FPA therefore appears to perform well relative to leading combinatorial alternatives such as VCG.[32]

The fact that the VCG mechanism leads to gains in terms of efficiency but not in terms of payments is not necessarily surprising. Prior work has shown that VCG may exhibit poor revenue performance in the presence of synergies; see e.g. Ausubel and Milgrom (2006). Other relevant features of the auction environment, such as bidder asymmetry, may also lead VCG to exhibit poor revenue performance, see e.g. Krishna (2009). To explore how synergies and asymmetry interact to shape VCG payment performance, we also re-simulated VCG outcomes using the estimated standalone costs above, but setting complementarities to zero. Comparing VCG outcomes with and without complementarities, we find that complementarities reduce per-auction social costs by about \$11,000 but increase per-auction VCG payments by about \$9,000. Therefore, up to \$20,000 of the change in bidders' per-auction margins can thus be explained by failure to pass through efficiency gains from complementarities under VCG. While non-negligible, this is small relative to the approximately \$130,000 increase in per-auction margins observed when moving from FPA to VCG, suggesting that the latter is driven more by standalone cost asymmetries than by complementarities.[33]

7. CONCLUSION

Motivated by an institutional framework common in procurement applications, we develop and estimate a structural model of bidding in simultaneous first-price auctions. We analyse the identification of this model, showing that excluded variation in either characteristics of rival bidders or characteristics of other auctions supports non-parametric identification of cross-object complementarities. Finally, we apply this model to data on MDOT highway construction and maintenance auctions. Our estimates suggest the presence of both positive and negative synergies among projects, with magnitudes sufficient to induce non-trivial efficiency losses. Nevertheless, we find that switching to an efficient VCG mechanism would slightly increase MDOT's expected procurement costs. We view this as evidence that simultaneous FPA can perform well even in environments with economically important complementarities, a finding which may help to rationalize the widespread popularity of the simultaneous FPA mechanism in practice.

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32. This analysis is only partial in that we hold entry behaviour fixed across mechanisms. Since the VCG auction reduces social costs but not MDOT payments, it must generate greater profit to bidders. This may translate into greater entry, which could in turn reduce procurement costs. In contrast, since new entrants are by definition marginal, we expect efficiency gains net of entry to be similar to those reported above.

33. To gain further insight on factors affecting VCG vs. FPA revenue performance, we also conducted several simple numerical simulations in a setting where two asymmetric bidders compete in two auctions, with one or both bidders having a positive complementarity. The results, reported in Supplementary Appendix G.3, confirm that either revenue ranking is possible depending on the interaction between asymmetry and complementarities, with asymmetry alone typically favouring FPA, and the effects of complementarities varying depending on whether these are assigned to the strong bidder, the weak bidder, or both.

Supplementary Data

Supplementary data are available at *Review of Economic Studies* online. And the replication packages are available at https://dx.doi.org/10.5281/zenodo.6304625.

Data Availability Statement

The data and code base underlying this article are available in the Zenodo digital repository, at https://dx.doi.org/10.5281/zenodo.6304625.

Appendix A: Proof of Proposition 1

The proof of Proposition 1 rests on two key claims. First, the first-order system (2) must be well defined for almost every b_i submitted by i, i.e. almost everywhere with respect to the measure induced by $G_i(\cdot|X,Z)$. Second, at almost every b_i at which first-order conditions hold, the matrix $\nabla P_i^{\mathcal{L}}$ must be invertible. We establish each claim in turn.

First show that the first-order system (2) is well defined for almost every b_i submitted by i. Recall that we can write bidder i's objective as

$$\pi(b;v_i,K_i|X,Z) = (\Omega v_i + K_i - \Omega b)^T P_i^{\Omega}(b|X,Z),$$

where v_i and K are given at the time of maximization. Note that the system (2) necessarily holds at any best response where $\pi(\cdot;v_i,K_i|X,Z)$ is differentiable and that Assumption 3 implies that each observed b_i is the best response. Hence, the system (2) will be well defined for almost every b_i submitted by i if and only if $\pi(\cdot;v_i,K_i|X,Z)$ is differentiable almost everywhere with respect to the measure on B_i induced by $G_i(\cdot|X,Z)$. But under Assumption 4, $G_i(\cdot|X,Z)$ is absolutely continuous. To establish the claim, it thus suffices to show differentiability of $\pi(\cdot;v_i,K_i|X,Z)$ a.e. with respect to Lebesgue measure on $\mathcal{B}_i$.

Clearly $(\Omega v_i + K - \Omega b)$ is differentiable in b. Thus, differentiability of $\pi(\cdot;v_i,K_i|X,Z)$ at b is equivalent to differentiability of $P_i^{\Omega}(\cdot|X,Z)$ at b. Let B_{-i} be the $L_i \times 1$ random vector describing maximum rival bids in the set of auctions in which i participates. Again applying Assumption 4 to rule out ties, the probability i wins combination ω at bid b is

$$P_i^{\omega}(b|X,Z) = \Pr(\{\cap_{\{l:\omega_l=1\}} 0 \le B_{-i,l} \le b_l\} \cap \{\cap_{\{l:\omega_l=0\}} b_l \le B_{-i,l} < \infty\}|X,Z).$$

For each $\omega \in \Omega_i$, let b^{ω} be the $(\sum \omega) \times 1$ sub-vector of b describing i's bids for objects in ω, B_{-i}^{ω} be the $(\sum \omega) \times 1$ sub-vector of B_{-i} describing maximum rival bids for objects in ω, and $G_{-i}^{\omega}(b^{\omega}|X,Z)$ be the equilibrium joint c.d.f. of B_{-i}^{ω}. Applying the formula for a rectangular probability and simplifying, we can then represent $P_i(\cdot|X,Z)$ in the form

$$P_{-i}^{\omega}(b|X,Z) = \sum_{\omega' \in \Omega} a_{\omega'}^{\omega} G_{-i}^{\omega'}(b^{\omega'}|X,Z),$$

where each $a_{\omega'}^{\omega}$ is a known scalar (determined by ω, ω') taking values in $\{-1,0,1\}$. But by absolute continuity each c.d.f. $G_{-i}^{\omega}(\cdot|X,Z)$ is differentiable a.e. (Lebesgue) in its support, and interpreted as a function from $\mathcal{B}_i$ to $\mathbb{R}^{L_i}$, each $b^{\omega'}$ is continuously differentiable in b. Thus interpreted as a function from $\mathcal{B}_i$ to $\mathbb{R}$, each $G_{-i}^{\omega'}(b^{\omega'}|X,Z)$ is differentiable on a set of full Lebesgue measure in B_{-i}. The set of points in $\mathcal{B}_i$ at which all $G_{-i}^{\omega'}(b^{\omega'}|X,Z)$ are differentiable is the intersection of points in $\mathcal{B}_i$ at which each $G_{-i}^{\omega'}(b^{\omega'}|X,Z)$ is differentiable, i.e. the intersection of a finite collection of sets of full Lebesgue measure in $\mathcal{B}_i$. But from above differentiability of $G_{-i}^{\omega'}(b|X,Z)$ for all ω' implies differentiability of $P_{-i}^{\omega}(b|X,Z)$. Hence $P_{-i}^{\omega}(\cdot|X,Z)$ is differentiable on a set of full Lebesgue measure in $\mathcal{B}_i$. This in turn implies differentiability of $\pi(\cdot;v_i,K_i|X,Z)$ a.e. with respect to the measure on $\mathcal{B}_i$ induced by $G_i(\cdot|X,Z)$, as was to be shown.

We next establish that the first-order system (2) must yield a unique solution $\tilde{v}$ for almost every b_i submitted by i. Let $\tilde{B}_i$ be the set of points in $\mathcal{B}_i$ at which $\pi(\cdot;v_i,K_i|X,Z)$ is differentiable in b; from above, $\tilde{B}_i$ has full Lebesgue measure in $\mathcal{B}_i$. Choosing any $b \in \tilde{B}_i$ and rearranging (2) yields

$$\nabla_b P_i^{\mathcal{L}}(b|X,Z)\tilde{v} = \nabla_b P_i^{\mathcal{L}}(b|X,Z)b + P_i^{\mathcal{L}}(b|X,Z) - \nabla_b P_i^{\Omega}(b|X,Z)^T K_i.$$

Hence, uniqueness of $\tilde{v}$ is equivalent to invertibility of the $L_i \times L_i$ matrix $\nabla_b P_i^{\mathcal{L}}(b|X,Z)$. Recall that $P_i^{\mathcal{L}}(b|X,Z)$ is an $L_i \times 1$ vector whose lth element describes the probability that bid vector b wins auction l. Note that $b \in \tilde{B}_i$ rules out ties at b. Thus for $b \in \tilde{B}_i$ the mth element of $P_i^{\mathcal{L}}(b|X,Z)$ is the marginal c.d.f. of the maximum rival bid $B_{-i,m}$ in auction m, from which it follows that $\nabla_b P_i^{\mathcal{L}}(b|X,Z)$ is a diagonal matrix whose m,mth element is the marginal p.d.f. of $B_{-i,m}$. Denote this p.d.f. by $g_{-i,m}(b|X,Z)$; recall that by absolute continuity this p.d.f. is well defined. Then $\nabla_b P_i^{\mathcal{L}}(b|X,Z)$ will be invertible at b if and only if $g_{-i,m}(b|X,Z) > 0$ for all $m = 1,\ldots,L_i$.

We aim to show that this latter property is an implication of equilibrium bidding under Assumption 4. Toward this end, recall that by hypothesis of equilibrium play, each submitted bid b_i is a best response to rival play at (X,Z) for some (v,K). Suppose that there exists an $\epsilon > 0$ such that $g_{-i,m}(\cdot|X,Z) = 0$ on $(b_{im} - \epsilon, b_i]$. Then player i could infinitesimally reduce b_{im} without affecting either $P_i^{\mathcal{L}}$ or P_i^{Ω}. Furthermore, if $P_{im}(b_i|X,Z)>0$, so that bidder i wins auction m with strictly positive probability at bid b_i, this deviation will *strictly* increase bidder i's profits. Hence we must have either $g_{-i,m}(\cdot|X,Z)>0$ or $P_{im}(B_i|X,Z)=0$ almost everywhere (Lebesgue) in the support of B_i. By absolute continuity of G_i, this in turn implies we must have either $g_{-i,m}(\cdot|X,Z)>0$ or $P_{im}(B_i|X,Z)=0$ for almost every b_i submitted by i. Furthermore, absolute continuity and common lower support jointly imply that we can have $P_{im}(B_i|X,Z)=0$ for at most a set of bids of G_i-measure zero. Hence, we must have $g_{-i,m}(\cdot|X,Z)>0$ for G_i-a.e. bid b_i submitted by i.

Since m was arbitrary, $\nabla_b P_i^{\mathcal{L}}(b_i|X,Z)$ must be invertible for G_i-a.e. bid b_i submitted by i. Hence for almost every b_i submitted by i there will exist a unique $\tilde{v}$ satisfying (2) at b_i, given by

$$\tilde{v}=b_i+\nabla_b P_i^{\mathcal{L}}(b_i|X,Z)^{-1}P_i^{\mathcal{L}}(b_i|X,Z)+\nabla_b P_i^{\mathcal{L}}(b_i|X,Z)^{-1}\nabla_b P_i^{\Omega}(b_i|X,Z)^T K.$$

For the set of b_i at which $\tilde{v}$ is not unique, which is measure zero with respect to G_i, we may take any $\tilde{v}$ solving (2). Regardless of the solutions chosen on this set of G_i-measure zero, equation (6) will uniquely define $\hat{F}_i(\cdot|X,Z;K_i)$. Moreover, if $K_i=\kappa_i(X,Z_i)$, then by hypothesis b_i is a best response for bidder i given v_i. Thus for every b_i where (2) has the unique solution $\xi(b_i|X,Z;K_i)$, we must have $v_i=\xi(b_i|X,Z;K_i)$ when $K_i=\kappa_i(X,Z_i)$. Since the set of b_i where (2) does not have a unique solution is of measure zero with respect to G_i, it thus follows that $F_i(\cdot|X,Z_i)=\hat{F}(\cdot|X,Z;\kappa_i(X,Z_i))$.

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