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Heterogeneous Global Booms and Busts

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We investigate the heterogeneous boom and bust patterns across countries that emerge as a result of global shocks. Our analysis sheds light on the emergence of core and periphery countries, and the joint determination of the depth of recessions and tightness of credit across countries. The model implies that interest rates are similar across core and periphery countries in booms, with larger credit and output growth in periphery countries. However, a common global shock that leads to a credit crunch across the globe gives rise to a sharper spike in interest rates and a deeper recession in periphery countries, while a credit flight to the core alleviates the adverse consequences in these countries. We explore the implication of the model about credit spreads, portfolio rebalancing, investment, non-performing debt and concentration of debt ownership during booms and busts, both in the time series and in the cross-section, and connect them to existing stylized facts. We further demonstrate how the anatomy of the global economy evolves as a result of aggregate demand and supply shocks to financing, such as a global saving glut.
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For decades before 2008, boom-bust patterns had been associated almost exclusively with emerging markets. The pattern — a boom phase started by poorly regulated financial liberalization leading to a surge in foreign capital, large credit flows to the non-financial sector, build-up of debt at low interest rates and rapidly increasing investment, abruptly turning to a bust phase where interest rates spike and credit flies to safety, triggering a collapse in output — has been connected to a large catalog of structural weaknesses in Latin American, East Asian, and Eastern European economies.

However, the global financial crisis in 2008 and especially the Eurozone crisis in 2010 have dramatically exposed similar vulnerabilities in a group of advanced economies. This has in turn led to a shift in focus on the role of increasingly

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globalized financial intermediation and the implied changes in global supply of capital.[1]

In this paper, we explore how the frictions in global supply of capital determine heterogeneous boom-bust cycles across the world. This analysis enables us to explain the emergence of core and periphery countries and to jointly determine the depth of recessions and tightness of credit markets across countries. Our model implies that interest rates are similar across core and periphery countries in booms, with larger credit and output growth in periphery countries. However, a common global shock that leads to a credit crunch across the globe gives rise to a sharper spike in interest rates and a deeper recession in periphery countries, while a credit flight to the core alleviates the adverse consequences in these countries. We derive further testable implications for portfolio rebalancing, investment, non-performing debt and concentration of debt ownership, both in the time series and in the cross-section. Finally, our model illustrates how the configuration of the global economy evolves as a result of aggregate demand and supply shocks to financing.

In our framework firms across countries compete for credit from international investors who cannot perfectly evaluate creditworthiness of firms. Firms in countries where investors have less trouble recognizing the creditworthy firms choose more stable investment levels, which in turn leads to less volatility in the credit market and an attenuated boom-bust cycle. In contrast, in countries where investors have a harder time identifying the creditworthiness of firms, firms choose a riskier strategy which exacerbates both the boom and the bust by tightening the credit market.

Our main observation is that the heterogeneity among boom-bust cycles arises as a consequence of the interaction between the disparity in firm credit worthiness within country and difference in effectiveness of investor expertise across countries. In particular, investors have different degrees of ability in identifying whether a firm is creditworthy or not, i.e. if the firm's collateral is good or bad. Investor expertise is more important for identifying creditworthy firms in certain countries, which we refer to as opaque countries.

As the first step of our analysis, we explore a global productivity shock that affects the proportion of creditworthy firms in every country symmetrically. We show how investors' *prudence*, the type of information that they choose to obtain about firms, is endogenously determined by the aggregate shock. When there are many creditworthy firms, investors are *bold*: the information they obtain identifies some firms who are not creditworthy, whereas some others are not distinguished from creditworthy ones. Thus investors can avoid missing out on any good investment opportunities at the expense of extending loans to some bad firms by mistake. On the contrary, with few creditworthy firms around, investors

[1]For instance, see Caballero et al. (2017) and citations therein on the role of global scarcity of safe assets, Caballero and Simsek (2016) on fickle capital flows, and Avdjiev et al. (2016) on how globalization has pushed decisions on credit supply outside the boundaries of affected countries, which is a new phenomenon for advanced economies.

are *cautious*: the information they obtain identifies every firm that is bad, i.e. not creditworthy, but it also pools some good firms with bad ones. This implies that investors can avoid financing any firm that may not repay, although doing so leads them to forgo some profitable investment opportunities. It then follows that investors lend to different firms in different countries in different aggregate states. That is, a global productivity shock is mapped to a change in investors' prudence, leading to heterogeneous lending choices across countries. With the microfoundation of investor prudence at hand, we then focus on the more comprehensive consequences of a change in this prudence.

We find that low-skilled investors, as a rational response to their imperfect information, heavily invest in opaque countries during booms and rebalance away towards more transparent countries in busts, consistent with the evidence presented in Gallagher et al. (2018).[2] As a result, more opaque countries are highly exposed to business cycles and suffer large busts, while more transparent ones have a much lower exposure and only experience a minor decline in output. Thus, *peripheral* and *core* countries emerge endogenously. During booms, firms in periphery countries enjoy large credit inflows at low interest rates and grow rapidly. However, during busts, the firms in these countries can obtain new credit only at high rates, if at all, and their output and credit flows collapse. In contrast, international capital floods a group of more transparent countries at low interest rates, their transparency effectively shielding them from negative exposure to global capital fluctuations.

As such, our model implies a qualitative difference in the functioning of credit markets between booms and busts. In booms, firms borrow at the same rate in core and periphery countries. In this state, credit quality is heterogeneous across investor portfolios, and highly skilled investors derive excess returns by extending credit to higher quality borrowers across all countries. In contrast, during busts there is a significant spread for borrowing between firms in core and periphery economies. In this state lenders are cautious, which implies the same high credit quality across their portfolios. As such, highly skilled investors derive excess returns by lending at higher rates to good firms in opaque, peripheral countries. This picture rationalizes the (sometimes puzzlingly) low premium on emerging market assets before the East Asian and Russian Crises and assets in the south of Europe before the Eurozone Crisis (Kamin and von Kleist, 1999; Duffie et al., 2003; Gilchrist and Mojon, 2018).

The real investment and output in each country is determined by how firms trade off investment and liquidity risk management, which is in turn driven by the credit market conditions they face. This trade-off leads to risky investment decisions by firms in the periphery: they produce at a high scale during booms (when credit is cheap), at the expense of abandoning production in busts (when

[2]Gallagher et al. (2018) find that a group of money market funds stopped lending only to European banks, and not to other banks with similar risk in 2011. Ivashina et al. (2015) find evidence that this led to significant disruption in the syndicated loan market. These facts are broadly consistent with our proposed mechanism.

credit is expensive). Therefore, when investors are bold, both core and periphery countries both enjoy a high output. However, when investors turn cautious, international credit markets become plagued by funding mismatch and the high exposure countries suffer a drastic output collapse. That is, investors' heterogeneous lending choices across countries invite differential strategies of firms, which in turn feeds into investors' decisions, and jointly shape heterogeneous global cycles.

Our model also suggests that most of the non-performing debt is issued in booms, in the periphery countries, and is financed by low skilled investors. Moreover, productivity dispersion among firms obtaining credit is larger in the periphery than in the core, consistent with the observed increase in the misallocation of capital during the pre-crisis years in the south of Europe (Reis, 2013; Gopinath et al., 2017).

The model provides further, yet-to-be-tested predictions about investor portfolio compositions throughout the cycle. It implies that ownership of debt is most concentrated during busts, especially in peripheral countries. In addition, the realized return on bonds issued in booms is higher in the periphery than in the core, and vice-versa in busts.

We then illustrate how the structure of the global economy evolves as a result of aggregate demand and supply shocks to financing. We first explore the effect of an aggregate credit demand shock, akin to the 2020-21 global pandemic. When credit demand increases in all countries, core countries absorb a larger share of the available capital. Not only is the boom less pronounced everywhere, but also the most opaque countries are more squeezed in a recession, which leads to a more dramatic collapse in output in the periphery.

Second, we analyze the consequences of increased capital supply by less sophisticated investors, which we believe sheds new light on the effects of the global saving glut on investment cycles as well as safe asset determination. Consistent with the literature on rising global imbalances, increased supply of capital by low-skilled investors decreases the yield on bonds in a boom (for a review, see Caballero et al., 2017). It also increases the supply of safe assets as defined by He et al. (2016), but not nearly enough to satisfy the increasing demand for safe assets in busts. As such, although all countries flourish more in the booms, the capital imbalance leads to an exacerbated cycle in periphery countries. An interesting commonality between the two scenarios is that they both have the same adverse effect on the core: the set of core countries shrinks.

Finally, our paper provides a novel equilibrium framework to study investment decisions and equilibrium pricing outcomes in an asymmetric information environment where there is two-sided heterogeneity. We believe that this is a parsimonious model well-suited to explore a broader set of questions concerning the interaction among financial institutions and the spillover to the real economy.

Related Literature.

Our paper is the first to show that frictions in the global supply of capital lead to an endogenous partitioning of countries into low and high exposure groups, creating heterogeneous global boom and bust patterns. It is related to a large and diverse body of work that studies international output and credit cycles.

First, our paper contributes to the extensive literature started by Kiyotaki and Moore (1997), which generates boom and bust patterns from financial frictions. In this line of work, a collapse in the value of collateral leads to a tightening of credit constraints in recessions (e.g. Kiyotaki and Moore, 1997; Lorenzoni, 2008; Mendoza, 2010; Gorton and Ordonez, 2014). Our mechanism does not operate via tighter collateral constraints in recessions. Instead, in the face of an adverse prudence shock, low-skilled investors find it optimal to rebalance their portfolios towards firms in more transparent countries. As a consequence, on top of the time-series pattern, we can derive predictions about the cross-sectional differences of real outcomes across countries.

There is also a group of papers that connect flight-to-quality episodes to international risk-sharing.[3] For instance, Gourinchas et al. (2017) and Maggiori (2017) argues that since the US financial sector is less risk averse or less constrained than others, it takes a leveraged position in the global risky asset in booms and deleverages during busts. Given the two-country representative agent approach of these papers, they are better suited to capture the characteristics of capital flows between the US and the rest of the world. Instead, we focus on the detailed interaction between heterogeneous global financial institutions and local firms. As such, our approach is more useful to explore other dimensions of the data, such as the real effects of the heterogeneous re-balancing of asset managers, the time-series and cross-sectional differences in returns, the distribution of non-performing debt, and the concentration of debt ownership. Therefore, we think of our modeling approach as being complementary to this literature.

Another stream of literature studies why sudden stops are more frequent in emerging market countries. Aguiar and Gopinath (2007) and Rey and Martin (2006) point to technological differences, while Eichengreen and Hausmann (1999), Caballero and Krishnamurthy (2003), and Broner and Ventura (2016) point to differential incentives for saving in foreign versus domestic currency, as a consequence of differences in country fundamentals. In contrast, we propose a mechanism which implies that heterogeneous patterns can arise within advanced economies as well, where the technology, the level of human and physical capital, and the legal-economic-political system are similar.

Turning to the Eurozone crisis, a series of papers emphasize a wide range of mechanisms including less stringent credit constraints for large and inefficient firms (Reis, 2013; Gopinath et al., 2017), the political connections of some banks and firms (Cuñat and Garicano, 2009), compromised structural reforms

[3] See Gourinchas and Rey (2014) for a detailed review.

(Fernandez-Villaverde et al., 2013), the role of downward wage rigidity (Schmitt-Grohé and Uribe, 2016), the role of perceived risk of a eurozone breakup (Battistini et al., 2014), the interaction between risk-shifting incentives of banks and sovereigns (Farhi and Tirole, 2016), coordination problems between monetary and fiscal policy (Aguiar et al., 2015), and the role of private debt expansion (Martin and Philippon, 2017). Our mechanism is complementary to these papers. Furthermore, while the loose financing conditions and the resulting exante expansion of debt in periphery countries is exogenous in this stream of papers, our model is able to generate this pattern endogenously.

We are also related to the literature which connects international capital flows to safe asset scarcity (He et al., 2016; Caballero et al., 2017; Farhi and Maggiori, 2017). Our model derives the equilibrium supply and demand for safe assets from informational frictions, a new element in this literature. As we demonstrate in Section IV.A, this approach generates novel predictions.

There is also a group of finance papers which rationalizes flight-to-quality in the financial markets using a Knightian-uncertainty shock (Caballero and Krishnamurthy, 2008), fund managers' incentives (Vayanos, 2004), or adverse-selection (Fishman and Parker, 2015). We add to this literature by explicitly modeling the interaction between flight-to-quality and real investment decisions, which contributes to the differences in credit cycles across countries.

Finally, we contribute to the large theoretical literature which studies trading under asymmetric information. The structure of the credit market in our model builds on Kurlat (2016), which we generalize in two directions. We first generalize this framework to allow for heterogeneous credit demand, and then embed the credit market into a macroeconomic environment to endogenize the credit demand curve. At the heart of our model is the feedback between the credit market and real investment, and firms' optimal resolution of the trade-off between investment and liquidity risk management across different states of the world. As such, both of these generalizations are crucial for our mechanism.[4]

The rest of the paper is organized as follows. Section I describes our model. Section II presents a simplified version of the model to analyze endogenous emergence of prudence as a function of an aggregate productivity shock. Section III characterizes the credit and real market equilibrium in the full model, while Section IV explores the testable implications and predictions of the model in the global economy. Section V concludes.

I. Model

Consider a three-period model, $t = 0, 1, 2$, with a single perishable good. There are two main types of agents in the model. First, there are firms who invest and produce. They are located across a continuum of countries. Second, there are

[4]Our follow up paper, Farboodi and Kondor (2021), presents a dynamic extension of our framework showing that endogenous prudence can lead to endogenous economic cycles.

international investors who provide financing for firms. There is also a third group of agents, bankers, whose only role is to provide a frictionless saving technology to firms. All agents are risk neutral, and there is no discounting. Agents maximize the expected sum of consumption across all periods.

We start this section with a description of the components of the model, and then proceed to the equilibrium definition. Certain modeling choices are discussed in Section III.B.

Shocks.

There is an aggregate shock θ that determines the aggregate state(s) with probability distribution $\pi(\theta)$. There is also an idiosyncratic liquidity shock ν at the firm level. Let ϕ $(1-\phi)$ denote the iid probability that a firm is (not) hit by a liquidity shock, $\nu = 1$ $(\nu = 0)$. Both shocks are publicly observable. Shocks are sequentially realized at $t = 1$, with the aggregate shock being realized first.

Firms and Production Technology.

There is a continuum of firms, indexed by $j = (\omega, \tau)$. Firms invest and produce, and are subject to liquidity shocks.

$\omega \in [0, 1]$ denotes the *opacity* of the firm, where $\omega = 0$ is the least and $\omega = 1$ is the most opaque firm. Firm opacity relative to the expertise of investors is the source of the information friction in our model. $\tau \in \{g, b\}$ denotes the *(pledgeability) type* of the firm, where g (b) is a good (bad) firm. The type of the firm determines whether investors can seize part of its output, if the firm is good, or not, if the firm is bad. Thus good firms are creditworthy. λ fraction of all firms are good $(1-\lambda$ are bad), and λ is drawn from distribution $\Xi(\lambda)$. Firms are distributed iid across opacity classes independent of their type.

Each firm is endowed with one unit of the good in period $t = 0$, and a technology akin to Holmström and Tirole (1998) and Lorenzoni (2008). From every unit of investment that pays off at $t = 2$, fraction β takes 2 periods to become productive. As such, bringing a unit of investment to completion in period $t = 2$ requires β initial investment into *scale of operation*, in $t = 0$, while the rest can be invested at $t = 1$ as well. $I(\omega, \tau)$ denotes the scale of operation firm $j = (\omega, \tau)$ chooses at date $t = 0$. The firm saves the rest of its endowment to period $t = 1$.

At $t = 1$, a fraction ϕ of firms are hit by a liquidity shock ν. The liquidity shock is observable and verifiable by all agents. A firm hit by the liquidity shock has to inject an extra ξ per unit of scale of operation it wants to maintain until $t = 2$. Any unit of investment that has not received the necessary liquidity injection fully depreciates. $i(\omega, \tau, \nu; \theta)$ denotes the investment that firm $j = (\omega, \tau)$ with idiosyncratic shock ν drives to completion in state θ. The scale requirement necessitates

$$\beta i(\omega, \tau, \nu; \theta) \leq I(\omega, \tau). \tag{1}$$

The firm finances the date $t = 1$ share of investment as well as liquidity injection (maintenance cost) from its savings and/or by issuing bonds to international investors.[5] At $t = 2$, each unit of completed investment produces ρ_τ units of good, with $\rho_g \geq \rho_b$. In line with Holmström and Tirole (1998), we make the following assumption on the production technology.

ASSUMPTION 1: *Continuing with full scale and abandoning production after a liquidity shock are socially positive NPV for both good and bad firms,*

$$\rho_\tau > \max(\xi\beta + (1-\beta), 1 + \phi\xi, \frac{1}{1-\phi}). \qquad \tau = g, b$$

Banks and Saving Technology.

When entrepreneurs save between $t = 0$ and $t = 1$, they do it using a state-contingent saving technology at actuarially fair terms through local banks. Bankers are competitive, deep-pocketed agents who do not have the expertise to seize any future income of firms. Thus, they cannot lend to firms. However, firms can save towards future aggregate or idiosyncratic states with bankers.

International Investors.

There is a continuum of investors, indexed by their *skill* level, $s \in [0, 1]$. Each investor chooses the measure of applications that she wants to consider for financing, δ. Investors have imperfect and heterogeneous information about firm types. Each investor has a prior λ that a given firm is good, and higher s investors can acquire higher quality information by conducting a test which we explain in detail below. Let $w(s)$ denote the mass of investors with skill $s \in [0, 1]$. We will refer to $w(s)$ as investor skill distribution. Each investor is endowed with one unit of the good in period $t = 1$. Participation in the international market costs κ per unit of applications an investor considers for financing, and she provides financing to (a selected subset of) firms who demand liquidity at her chosen interest rate. We call an investor who participates in the international market an *active* investor.

Investors can seize α per unit of investment maintained to date $t = 2$ only from good firms. The total credit a firm receives, $\ell(\omega, \tau, \nu; \theta)$, has to satisfy the pledgeability constraint

$$(1 + r(\omega, \tau, \nu; \theta))\, \ell(\omega, \tau, \iota, \nu; \theta) \leq \alpha i(\omega, \tau, \nu; \theta). \tag{2}$$

Information Friction and Prudence.

Each investor uses a test ι to gather evidence about the true (pledgeability) type of each firm that demands liquidity. The information that investor s receives

[5] In general, at $t = 1$ the firm needs to undertake $\max\{\min\{(1-\beta)i(\omega, \tau, 0; \theta), i(\omega, \tau, 0; \theta) - I(\omega, \tau)\}, 0\}$ additional investment if not hit by a liquidity shock, and $\xi\beta i(\omega, \tau, 1; \theta) + \max\{\min\{(1-\beta)i(\omega, \tau, \nu; \theta), i(\omega, \tau, 1; \theta) - I(\omega, \tau)\}, 0\}$ if hit by a liquidity shock.

about firm $j = (\omega, \tau)$ depends on her expertise level, s, opacity of the firm, ω, and the type of the test the investor uses, ι. Let $x(\tau; \omega, s, \iota)$ denote this information, given by

$$x(\tau; \omega, s, \iota) = \begin{cases} \tau & w.p.\ 1 & \text{if } s > \omega \\ g & w.p.\ \iota & \text{if } s \leq \omega \\ b & w.p.\ 1-\iota & \text{if } s \leq \omega \end{cases} \tag{3}$$

where $\iota \in [0, 1]$. That is, the investor observes perfectly whether a firm is good or bad, if her skill is higher than the opacity of the firm. However, an ι-test pools ι $(1-\iota)$ fraction of those applications which are too opaque for the given investor with good (bad) firms. Thus tests with larger ι imply more false positive and fewer false negative mistakes.

While there is a continuum of tests, for our purposes the two extreme ones, $\iota = 1$ and $\iota = 0$, are particularly interesting. We refer to the former as the *bold* test, and the latter as the *cautious* test. Sometimes we refer to the choice of investor's test as *prudence* of investor by the following intuition.

An investor who uses a bold test is a *bold* investor. Bold investors only make false positive mistakes: while $x = b$ is conclusive evidence that the firm is bad, $x = g$ might signal a good firm just as a relatively opaque bad firm. One interpretation is that bold investors are imprudent: they are interested in not missing out on any good firms, even at the expense of occasionally lending to bad firms by mistake.

Alternatively, an investor who uses a cautious test is a *cautious* investor. Cautious investors only make false negative mistakes: while $x = g$ is conclusive evidence that the firm is good, $x = b$ might signal a bad firm or a relatively opaque good firm. One can interpret this as cautious investors being prudent: they are interested in not lending to bad firms even at the expense of occasionally missing out on the good ones.[6]

After observing evidence $x(\tau; \omega, s, \iota)$, each investor picks an acceptance rule $\chi(\omega, \tau; s, \iota) \in \{0, 1\}$ she will use. The acceptance rule specifies the bonds the investor is willing to finance with each test, and has to be measurable with respect to her collected evidence $x(\tau; \omega, s, \iota)$. Investors cannot observe, and thus cannot condition their decisions, on the total amount of credit that a given firm takes on. Let X denote the set of all possible acceptance rules, and X_s the set of acceptance rules that are feasible for investor s.

Liquidity Shock and Financing

At $t = 1$, a firm $j = (\omega, \tau)$ can obtain credit by issuing bonds on the international market to a subset of investors who are willing to lend to it. The firm receives one unit of financing per bond and promises to pay back $1 + r(\omega, \tau; \theta)$ at date $t = 2$. The repayment is subject to the pledgeability constraint (2). The interest rate $r(\omega, \tau; \theta)$ is determined in equilibrium.

[6]See Section III.B for a discussion.

Market Structure.

At $t = 1$, many markets open for issuing bonds. Each market m is defined by an interest rate $\tilde{r}(m)$ and it can be active or inactive in equilibrium. The set of all markets is denoted by M. A market is active if both firms and investors are present in that market.[7]

Firms can go to as many markets as they desire, and demand σ units of credit for the corresponding interest rate $\tilde{r}(m)$ in each market m, such that

$$\sigma(m, \omega, \tau, \nu; \theta) \leq \bar{\sigma}_\theta, \tag{4}$$

where $\bar{\sigma}_\theta$ is the common upper bound for all markets.[8] We refer to $\sigma(m, \omega, \tau, \nu; \theta)$ as credit demand of firm (ω, τ) in market m in state θ.

Each investor s chooses (at most) one market m to lend in, the measure of applications she wants to consider to finance (test) δ, and her acceptance rule $\chi(\omega, \tau; s, \iota)$. If there are investors of multiple skills who offer credit at a given m, the transactions of the least selective investors, i.e., those with least informative evidence, clears first.

Markets do not have to clear. In particular, firms understand that in each state θ, for each firm $j = (\omega, \tau)$, and each market m, there is an equilibrium measure $\eta(m, \omega, \tau, \nu; \theta)$ such that a firm j demanding $\sigma(m, \omega, \tau, \nu; \theta)$ credit at market m can raise only $\eta(m, \omega, \tau, \nu; \theta)\sigma(m, \omega, \tau, \nu; \theta)$. We call $\eta(m, \omega, \tau, \nu; \theta)$ the rationing function. As such, we define $\ell\,(\omega, \tau, \nu; \theta)$, the total amount of credit raised by firm j, and j's effective interest $r(\omega, \tau; \theta)$ as follows:

$$\ell\,(\omega, \tau, \nu; \theta) \equiv \int_M \sigma(m, \omega, \tau; \theta) d\eta(m, \omega, \tau, \nu; \theta), \tag{5}$$

$$r(\omega, \tau, \nu; \theta) \equiv \frac{\int_M \tilde{r}(m)\sigma(m, \omega, \tau, \nu; \theta) d\eta(m, \omega, \tau, \nu; \theta)}{\ell\,(\omega, \tau, \nu; \theta)}. \tag{6}$$

Thus, an investor s who chooses market m with interest rate $\tilde{r}(m)$, finances a representative pool of firms (1) who demand credit in market m, (2) who satisfy investor s acceptance rule based on her evidence about the firms, and (3) whose demand is not exhausted by investors less selective than s.

Finally, equilibrium supply and demand determines the allocation function A, which assigns a measure $A(.; \chi, m, \theta)$ to bonds of firms financed by acceptance rule-market pair (χ, m) in aggregate state θ. International investor s choice of how many applications she wants to test, δ, has to satisfy her budget constraint given the allocation function. That is, the total credit extended by an investor

[7] The structure of the credit market generalizes Kurlat (2016) by introducing quantities. We provide more details on the formalization and on the methodological contribution in the Online Appendix.

[8] We set the upper bound $\bar{\sigma}_\theta$ just sufficiently high that it is not a binding constraint for the most transparent good firms in state θ. This assumption is intuitive since markets should allow for bond issuance that is consistent with that of the most transparent good firms.

has to at most equal her endowment.

This market structure allows for many-to-many matching. A given firm might obtain credit from a group of heterogeneous investors (as described by the rationing function η), and a given investor might finance a pool of heterogeneous firms (as described by the allocation function A).

Countries.

Firms are distributed among a unit mass of countries, $c \in [0,1]$. We assume that the distribution is iid with respect to the firm pledgeability type τ, but not with respect to the opacity ω. Let $\omega_c = \mathbb{E}_c[\omega]$ denote the implied opacity of country c, which is defined as the average opacity of the firms in that country.

To isolate our main mechanism, we assume that each country is populated with firms of a single opacity and no other firms. Thus, we can index the countries by ω_c up to a random permutation. Since all the firms in country ω_c have the same opacity level, to save on notation in the reminder of the paper we will use ω to index country opacity as well.

Furthermore, we assume that the mapping between opacity classes and country names is random, and investors have an uninformative prior about this mapping. Thus, from investor perspective, all countries are exante identical.[9]

Firm Problem.

At $t = 0$, each firm chooses at what scale to operate and how much to save in order to invest later and/or insure the risk of liquidity shock. At $t = 1$, after the realization of the aggregate state and the idiosyncratic liquidity shock, each firm can invest further subject to (1) or submit demand for bond issuance to a subset of markets, taking each market's interest rate and rationing function as given. The problem of a firm $j = (\omega, \tau)$ can be written as

$$\max_{\substack{I(\omega,\tau),\{i(\omega,\tau,\nu;\theta)\}_{\theta,\nu},\\ \{\sigma(m,\omega,\tau,\nu;\theta)\}_{\theta,\nu,m}}} \mathbb{E}_{\theta,\nu}\left[\rho_\tau i(\omega,\tau,\nu;\theta) - \mathbb{1}_{\tau=g}\ell(\omega,\tau,\nu;\theta)\right] - 1 \tag{7}$$

subject to constraints (1)-(2), (4)-(6), and firm resource constraint(s).

International Investor Problem.

International investors, or experts, arrive in $t = 1$, once both the aggregate and idiosyncratic shocks are realized. They have unit wealth and consume in $t = 1, 2$. Each investor picks a market, m, and submits her acceptance rule χ, and supply of credit δ. Furthermore, each investor uses a test ι. In Section II we allow the

[9] See Section III.B for a discussion.

investor to choose the test, while in Section III and for the the following analysis each agent is endowed with a test ι depending on the aggregate state. Thus, the problem of investor s in aggregate state θ can be written as

(8)

$$\max_{m,\chi,\delta,\iota} \delta \left[(1+\tilde{r}(m;\theta)) \int_{(\omega,\nu)} dA(\omega,g,\nu;\chi,m,\theta) - \int_{(\omega,\tau,\nu)} dA(\omega,\tau,\nu;\chi,m,\theta) - \kappa \right] + 1$$

s.t.

$$\chi \in X_s$$

$$\delta \int_{(\omega,\tau,\nu)} dA(\omega,\tau,\nu;\chi,m,\theta) \leq 1.$$

We end this section with the formal definition of equilibrium.

DEFINITION 1 (Equilibrium Definition): *A global equilibrium is a set of the firm's investment plan $I(\omega,\tau)$, $\{i(\omega,\tau,\nu;\theta)\}_{\theta=H,L}$ and demand function for credit $\{\sigma(m,\omega,\tau,\nu;\theta)\}_{\theta=H,L}$, investor's choice of test, $\iota(s,\theta)$, interest rate $\tilde{r}(m;\theta)$, and acceptance rule $\chi(\omega,\tau;s,\iota)$, along with a rationing function $\{\eta(m,\omega,\tau,\nu;\theta)\}_{\theta=H,L}$, allocation function $A(\omega,\tau;\chi,m,\theta)$, and interest rate schedule $\{\tilde{r}(m;\theta)\}_{\theta=H,L}$ and the corresponding equilibrium credit allocation and interest rate $\{\ell(\omega,\tau,\nu;\theta), r(\omega,\tau,\nu;\theta)\}_{\theta=H,L}$, such that*

(i) each firm's investment plan and demand function are optimal for the firm, given the rationing function and the interest rate schedule;

(ii) each investor's test, interest rate and the corresponding acceptance rule are optimal for the investor, given the allocation function and the interest rate schedule;

(iii) rationing function, allocation function, and interest rate schedule are consistent with investor and firm optimization.

II. Endogenous Prudence

In this section we characterize the economy when investors choose their test endogenously in response to an exogenous productivity shock. The characterization includes the implied information regime, as well as the credit market conditions associated with each information regime. We show how a relatively small productivity shock can trigger a fragmentation of the credit market, leading to the dry-up of credit even for good firms in some countries but not in others.

The exogenous aggregate shock θ determines the fraction of good firms, λ. In particular, the aggregate shock takes two values, $\theta = \lambda_L, \lambda_H$ ($\lambda_L < \lambda_H$), with probability $\pi(\lambda_L) = \pi_L$ and $\pi(\lambda_H) = \pi_H = 1 - \pi_L$ respectively.

In order to keep the argument focused, we make a few simplifying assumptions in this section. Assume the skill distribution of investors described by $w(s)$ consists of only three mass points $s_0, s_1, 1$, where $0 < s_0 < s_1 < 1$. For every other s, $w(s) = 0$. We refer to the group with skill $s_0, s_1, 1$ as unskilled, moderately skilled, and highly skilled, respectively. Furthermore, assume that production does not involve any time to build, $\beta = 0$; there is no liquidity shock, $\phi = 0$; investors can seize a unit per investment from good firms, $\alpha = 1$; and bankers can provide only non-state-contingent saving technology between periods $t = 0$ and $t = 1$.

Since there is no liquidity shock, argument ν drops out of all functions, and will be suppressed throughout the rest of the section.

With no time to build, $\beta = 0$, and no state-contingent saving each firm chooses to save its unit endowment to period $t = 1$ and maximize profits state by state subject to a state-by-state resource constraint. At $t = 1$, in each state the firm borrows from international investors and invests $i(\omega, \tau; \theta)$ to produce in $t = 2$. Optimization problem (7) simplifies to

(9)

$$\max_{i(\omega,\tau;\theta),\{\sigma(m,\omega,\tau;\theta)\}_m} \quad \rho_\tau i(\omega, \tau, \theta) - \mathbb{1}_{\tau=g}\left(1 + r(\omega, \tau; \theta)\right)\ell(\omega, \tau; \theta) - 1 \qquad \forall\, \theta$$

$$\text{(10)} \qquad \text{s.t.} \qquad i(\omega, \tau; \theta) = 1 + \ell(\omega, \tau; \theta),$$

$$\text{(11)} \qquad \ell(\omega, \tau; \theta) \leq \frac{1}{r(\omega, \tau; \theta)},$$

and constraints (4)-(6). Equation (11) represents the *credit capacity* of each firm at $t = 1$. Intuitively, in period $t = 2$, investors can seize one unit per unit of investment from good projects only. As this only covers the principal of the loan, the interest rate repayment has to be covered by the firm's initial endowment, which implies a down-payment of $r(\omega, \tau; \theta)$ for every unit of credit that the firm receives.

For good firms, the profitability of implementing their project using borrowed funds is decreasing in the interest rate $r(\omega, \tau; \theta)$. Furthermore, there is an interest rate, $\bar{r}$, at which the project becomes zero NPV for a good firm. Thus for $\tilde{r}(m) < \bar{r}$, good firms demand the maximum allowed as long as their credit constraint (11) is not violated given the equilibrium function $\eta(m, \omega, \tau; \theta)$. Since $\rho_b > 1$ and bad firms do not pay back, they demand the maximum allowed at any interest rate $\tilde{r}(m)$, again as long as their credit constraint is not violated.

Since investors have imperfect information about the firm type and cannot seize assets from bad firms, they might not be willing to extend as much credit as firms desire, or might require a high interest rate which would in turn limit the credit capacity of the firms. In fact, credit market outcomes critically depends on the information regime implied by investors' choice of the test, which we characterize next.

To ease the exposition we make the following assumption:[10]

ASSUMPTION 2: *Assume*

(i) $(1-\lambda_H) < \frac{\kappa}{s_1} < \frac{\kappa}{s_0} < (1-\lambda_L)$.

(ii) $w(s_0)$ *is sufficiently large and* $w(s_1)$ *and* $w(1)$ *are sufficiently small such that some unskilled investors participate in the credit markets in both aggregate states.*

(iii) $\rho_g \geq 1 + \frac{\kappa}{\lambda_L s_0}$, *i.e. productivity of good firms is sufficiently high compared to the cost of testing,.*

(iv) $s_0 \leq (1-(s_1-s_0))(s_1-s_0)$.

The first proposition characterizes investors' choice of test and choice of lending, for each aggregate shock.

PROPOSITION 1 (Information Choice and Lending Standards): *In both aggregate states* $\theta = \lambda_H, \lambda_L$ *all investors in groups* $s = s_1, 1$ *and a fraction of investors in group* $s = s_0$ *are active.*

(i) *If* $\theta = \lambda_H$, *all active investors choose the bold test,* $\iota = 1$, *and finance the applications for which* $x(\tau;\omega,s,1) = g$. *Therefore for* $s_i = s_0, s_1$,

$$\chi(\omega,g;s_i,1) = 1 \quad \forall\omega$$
$$\chi(\omega,b;s_i,1) = 1 \quad \forall\omega > s_i, \quad \chi(\omega,b;s_i,1) = 0 \quad \forall\omega \leq s_i,$$

and for $s = 1$,

$$\chi(\omega,g;1,1) = 1 \quad \text{and} \quad \chi(\omega,b;1,1) = 0 \quad \forall\omega.$$

(ii) *If* $\theta = \lambda_L$, *all investors choose the cautious test,* $\iota = 0$, *and finance the applications for which* $x(\tau;\omega,s,0) = g$. *Therefore for* $s_i = s_0, s_1$,

$$\chi(\omega,g;s_i,0) = 1 \quad \forall\omega \leq s_i, \quad \chi(\omega,g;s_i,0) = 0 \quad \forall\omega > s_i$$
$$\chi(\omega,b;s_i,0) = 0 \quad \forall\omega,$$

and for $s = 1$,

$$\chi(\omega,g;1,0) = 1 \quad \text{and} \quad \chi(\omega,b;1,0) = 0 \quad \forall\omega.$$

Lemma 1 reflects the quality-quantity trade-off which is at the core of investors' informational choice. For a fixed measure of bad firms, the bold test has a low

[10] The results for other parameters are available form the authors upon request.

rejection rate and thus a high lending probability. However, it involves a higher expost default rate which increases the lending cost. As such, the bold test leads to high quantity, low quality lending. Therefore, it is only optimal for investors to be bold if the fraction of bad firms in the economy is sufficiently small.

Formally, abundant unskilled investor wealth implies that investors compete for projects and the entrepreneurs accept the lowest interest rate on the credit market that is offered to them. In other words, unskilled investors choose the test that lets them offer the lowest interest rate without making negative profits. This choice is governed by the quality-quantity trade-off. We further show that the optimal test is never an interior one, i.e. the unskilled investors choose either to be bold or cautious.

The quality-quantity trade-off reflected in Lemma 1 also determines the credit market outcome for different levels of the aggregate shock, summarized in the next proposition.

PROPOSITION 2 (Credit Market Outcome):

(i) If $\theta = \lambda_H$, there is a single common prevailing interest rate, $r_H \equiv \frac{\kappa+(1-\lambda_H)(1-s_0)}{\lambda_H} < \bar{r}$. Good firms raise credit up to their capacity, $\ell(\omega, g; \lambda_H) = \frac{1}{r_H}$. The credit allocated to bad firms is strictly positive, and strictly below their capacity, $0 < \ell(\omega, b; \lambda_H) < \frac{1}{r_H}$, and increasing in their opacity.

(ii) If $\theta = \lambda_L$, credit markets are fragmented. There is an interest rate schedule $r_L(\omega)$ at which good firms raise credit. This interest rate is weakly increasing in firm opacity,

$$r_L(\omega) = \begin{cases} \underline{r} & \text{if } \omega \in [0, s_0] \\ \hat{r} & \text{if } \omega \in [s_0, s_1] \\ \bar{r} & \text{if } \omega \in [s_1, 1] \end{cases}$$

where $\underline{r} \equiv \frac{\kappa}{\lambda_L s_0} \leq \hat{r} \leq \bar{r} = \rho_g - 1$. Bad firms do not raise any credit, $\ell(\omega, b; \lambda_L) = 0$, and credit allocated to good firms $\ell(\omega, g; \lambda_L)$ is decreasing in their opacity.

When there are many good firms, investors are bold. Each investor provides loans for those firms without conclusive bad evidence. These include all good firms in every country and bad firms which are sufficiently opaque relative to the investor's skill. In this case there is a single prevailing interest rate r_H at which all bonds are traded, determined by bold unskilled investors' zero profit condition.

At this rate, unskilled investors are indifferent whether or not to enter the credit market given the fraction of defaults in their bond portfolio implied by their bold test. In equilibrium, just enough of them enter to satisfy the credit demand of all good firms along with the more skilled investors. For moderately and highly skilled investors, the equilibrium interest rate provides an information rent as they obtain a better quality loan portfolio than the unskilled investors at the same rate.

In this equilibrium, good firms demand credit only at interest rates not higher than r_H while bad firms demand credit at every interest rate. This is rational as good firms can borrow up to their capacity at r_H, they are not willing to pay a higher rate. It is also the reason why no investor wants to deviate to advertise a higher interest rate than r_H: they would receive applications only from bad firms. Nevertheless, some bad firms obtain credit at interest rate r_H as with the bold test, sufficiently opaque bad firms do not produce conclusive evidence that they are bad for unskilled and moderately skilled investors.

When only a small fraction of firms are good, all investors choose to be cautious, hence bad firms cannot raise any financing. At the same time, each investor can only identify good firms from sufficiently transparent countries relative to her skill. Thus supply of funding varies for good firms from different countries. It follows that investors with different skills finance not only a different set of firms, but they do so at different interest rates.

More precisely, investors with skill levels $s = s_0, s_1$ and 1 lend at interest rates $\underline{r}, \hat{r}$ and $\bar{r}$ to good firms with opacity $\omega \in [0, s_0]$, $\omega \in (s_0, s_1]$ and $\omega \in (s_1, 1]$ respectively, where $\underline{r} \leq \hat{r} \leq \bar{r}$ and $\underline{r} < \bar{r}$. As a result, credit markets are fragmented in this aggregate state, and countries are partitioned into three groups: $\omega \in [0, s_0]$ are the most transparent countries whose firms produce conclusive evidence for being good for all investors. $\omega \in [s_0, s_1]$ are moderately transparent countries. For these countries, investors have to be at least moderately skilled to be able to identify the good firms. $\omega \in [s_{1,}1]$ are the most opaque countries, and their firms can only be identified by highly-skilled investors.

Good firms from the most transparent countries are financed at a relatively low interest rate $\underline{r}$ by unskilled investors only. In fact, this is the minimal interest rate that cautious unskilled investors are willing to accept given the cost of testing κ. Any higher rate would be eliminated by investors' free-entry condition as unskilled capital is abundant. In stark contrast, good firms from the most opaque countries can only raise financing from highly-skilled investors. As capital of these investors is in short supply, these investors offer the highest rate which is accepted by any good firm, $\bar{r}$. Thus good firms in these countries are squeezed and typically cannot sell bonds up to their credit capacity. Lastly, the following Lemma explains the credit conditions faced by firms in moderately transparent countries.

LEMMA 1: *Good firms in moderately transparent countries $\omega \in [s_0, s_1]$ in state $\theta = \lambda_L$ face the following interest rate $\hat{r}$:*

$$\hat{r} = \begin{cases} \underline{r} & \text{if } \frac{w(s_1)}{s_1 - s_0} \geq \frac{\lambda_L}{\underline{r}} \\ \frac{\lambda_L(s_1 - s_0)}{w(s_1)} & \text{if } \frac{w(s_1)}{s_1 - s_0} \in \left[\frac{\lambda_L}{\bar{r}(r_H)}, \frac{\lambda_L}{\underline{r}}\right] \\ \bar{r}(r_H) & \text{if } \frac{w(s_1)}{s_1 - s_0} < \frac{\lambda_L}{\bar{r}(r_H)} \end{cases}$$

The interest rate for bonds issued by countries of intermediate opacity is intuitive. These countries have to be served by moderately skilled investors. If these investors have excess cash, then these bonds pay the same low interest rate as the most transparent bonds, $\underline{r}$. On the other extreme, if funding is in short supply, the intermediate firms can only raise financing at the maximum interest rate $\bar{r}$. In the intermediate range, there is cash-in-the-market pricing.

Assumption 2.(i) implies that if $\theta = \lambda_L$, $\underline{r} < r_H$. That is, when $\theta = \lambda_L$ an unskilled investor choosing a bold test – offering the minimal interest rate, r_H, at which she makes zero profits – could be undercut by a cautious unskilled investor offering an interest rate in the range of $[\underline{r}, r_H)$. This is why, $\theta = \lambda_L$ is consistent only with an equilibrium where unskilled investors are cautious. Alternative, if $\theta = \lambda_H$ then $\underline{r} > r_H$, which implies that in equilibrium, all unskilled investors choose to be bold.[11]

Proposition 2 and Lemma 1 together enable us to consider the heterogeneity across countries in recessions with different degrees of severity. A recession that corresponds to a medium adverse fundamental shock (λ_L not too low) leads to a wide degree of heterogeneity across countries in the recession: firms in more opaque countries are squeezed while firms in less opaque countries face much lower interest rates. However, a large drop in fundamentals (small λ_L) increases the rate at which firms in less opaque countries can borrow, $\underline{r}$, and decreases the spread across countries, $\bar{r} - \underline{r}$, making the recession more severe and similar across the spectrum of countries.[12]

Finally, it is insightful to examine the output in this economy. Equation (10) implies that the output of a firm (ω, τ) is simply $y(\omega, \tau; \theta) \equiv \rho_\tau(1 + \ell(\omega, \tau; \theta))$. Therefore, aggregate output in a country with transparency ω in aggregate state θ is

$$Y(\omega; \theta) = \theta \rho_g(1 + \ell(\omega, g; \theta)) + (1 - \theta)\rho_b(1 + \ell(\omega, b; \theta)).$$

Then the following corollary follows directly from Proposition 2.

COROLLARY 1 (Heterogeneous Downturns):

(i) Aggregate output is higher in the the high state than the low state in every country,

$$Y(\omega; \lambda_H) > Y(\omega; \lambda_L) \qquad \forall\, \omega.$$

(ii) The drop in aggregate output between the high and low state is increasing in

[11] Assumption 2.(i) also requires $\frac{\kappa}{s_0}, \frac{\kappa}{s_1} \in (1 - \lambda_L, 1 - \lambda_H)$. This implies that whenever unskilled investors choose a given test, moderately skilled investors choose the same one. For skilled investors, any choice leads to identical signals, so their choice is irrelevant. We abstract away from the case where this condition does not hold, as it complicates the exposition without providing additional economic insights.

[12] The 2008 Great Recession can be an example of the latter scenario where credit spreads spiked all over the world.

country opacity. In particular, if $\Delta Y(\omega) = Y(\omega;\lambda_H) - Y(\omega;\lambda_L)$, *then*

$$\Delta Y(\omega)|_{\omega\in[0,s_0]} < \Delta Y(\omega)|_{\omega\in[s_0,s_1]} < \Delta Y(\omega)|_{\omega\in[s_1,1]}$$

The first part of the corollary is as expected: $\theta = \lambda_H$ ($\theta = \lambda_L$) corresponds to a boom (downturn) in all countries. The second part is especially central to our analysis. It shows that while the adverse fundamental shock is the same across countries, its effect is amplified in opaque countries by the credit market. In particular, while in transparent countries good firms can still borrow a lot at relatively low rates when fundamentals deteriorate, in opaque countries all firms are severely squeezed. Even the best of the firms can borrow only limited quantities at very high rates. This makes the recession much more severe in these countries.

This simplified economy illustrates several key features of our mechanism. Although a common global shock governs the aggregate state of the economy, different countries can experience widely heterogeneous outcomes within the same state. A good aggregate shock corresponds to a global boom where output is high in every country. Investors are bold and choose to extend loans widely, even to low quality firms. Thus, there is abundant credit available at similar low interest rates to firms in all countries. On the other hand, a bad aggregate shock leads to a global bust. The global downturn corresponds to a credit crunch across the world as investors turn cautious about the quality of the loans that they extend. Investors' cautiousness leads to a spike in global interest rates, amplified in some countries. These countries are affected more than others by the adverse global shock and experience a deeper output collapse. That is, frictions in global supply of capital create heterogeneous boom-bust episodes around the globe.

The full model generalizes this economy and explores the two-way interaction between real investment and credit market outcomes. It provides a novel perspective on why countries with ex ante similar investment levels end up having radically different output, and experience dissimilar boom-bust episodes. Furthermore, it allows us to investigate the determinants of set of core and periphery countries and how they evolve as a consequence of global shocks.

III. Global Equilibrium

The rest of the paper focuses on the two-way feedback between the credit market and real investment. In order to study this interaction, we modify the economy to have the firm's scale matter, by setting $\beta = 1$. Furthermore, we generalize the skill distribution of investors to any differentiable function $w(s)$ with $w(s) > 0$ and $w'(s) < 0$ over the full support $s \in [0, 1]$.[13]

To keep the model tractable, instead of deriving investor information choice from first principles as in the previous section, we introduce the concept of pru-

[13] Note that $w(s)$ is not a probability density function as $\int_0^1 w(s)ds$ is not required to be 1.

dence shock as follows. Motivated by Proposition 1, we assume that in aggregate state $\theta = H$ all investors are bold, $\iota(s, H) = 1$, and in aggregate state $\theta = L$ all investors are cautious, $\iota(s, H) = 0$. To separate the effect of the prudence shock, we assume that this is the only aggregate shock in our economy, hence $\lambda_H = \lambda_L = \lambda$. For simplicity, we also normalize the cost of tests to zero, $\kappa = 0$.

$\beta = 1$ means that no extra investment at $t = 1$ pays off in the final date, which in turn implies that firms with no liquidity shock do not participate in the credit market at date $t = 1$, $i(\omega, \tau, 1; \theta) = I(\omega, \tau)$. Alternatively, firms hit by a liquidity shock need to raise $\alpha i(\omega, \tau, 1; \theta)$ in order to have $i(\omega, \tau, \nu; \theta)$ units of investment producing at $t = 2$. For analytical tractability, let $\alpha = \xi$. It follows that with perfect information, all good firms could borrow at 0 interest rate, implying that investors could provide full financing for a liquidity shock to good firms. Finally, since every participant in the credit market has $\nu = 1$, we will suppress ν from all of the functions throughout the rest of the paper.

Firm optimization problem (7) can be written as

(12)
$$\max_{I(\omega,\tau),\{i(\omega,\tau;\theta)\}_\theta,\{\sigma(m,\omega,\tau;\theta)\}_{\theta,m}} \sum_\theta \pi_\theta \Big[(1-\phi)\rho_\tau I(\omega,\tau) + \phi\big(\rho_\tau - \mathbb{1}_{\tau=g}\xi\big) i(\omega,\tau;\theta)\Big] - 1$$

(13)
$$\text{s.t.} \qquad I(\omega,\tau) + \phi\xi \sum_\theta \pi_\theta \frac{r(\omega,\tau;\theta)}{1+r(\omega,\tau;\theta)} i(\omega,\tau;\theta) = 1,$$

and constraint (1) with $\beta = 1$, constraint (2) holding with equality, and constraints (4)-(6).

At $t = 0$, each firm chooses how much to invest and how much to save in order to insure the risk of liquidity shock. Firms not hit by the shock do not demand credit while those who are hit borrow the maximum possible without violating the pledgeability constraint (2), as long as the interest rate is not prohibitively high. In order to sustain this borrowing, firms save enough down payment from their own endowment through state-contingent saving.[14] Putting these together, Equation (13) gives firm's exante budget constraint.

At $t = 1$, after the realization of the aggregate state and the idiosyncratic liquidity shock, each firm submits its demand for bond issuance to a subset of markets, taking each market's interest rate and rationing function as given.

Note that in $t = 0$, conditional on type τ, firms face a different problem only to the extent that they expect to face a different interest rate $r(\omega, \tau; \theta)$ in $t = 1$. Therefore, heterogeneous decisions about scale of operation and continued investment are solely driven by the differences in financing conditions that the firm expect in credit markets.

The international investor problem is given by problem (8), with two simplifi-

[14]See Online Appendix C for detail.

cations: $\kappa = 0$ and ι is no longer a choice since it is determined by the aggregate state.

A. *Simple Global Equilibrium*

In order to highlight the main mechanism of the model we restrict attention to a "*simple global equilibrium*", the simplest variant of equilibrium. In a simple global equilibrium, no investor chooses to finance bonds independent of her signal. Online Appendix B discusses other variants of the credit market equilibrium that arise depending on the choice of parameters.

To characterize the equilibrium, we proceed by backward induction. We start by analyzing the credit market outcome at $t = 1$, taking scale of operation choices as given, and then characterize the equilibrium in real quantities determined at $t = 0$.

Equilibrium Interest Rates and Credit Allocation

In this section, we characterize the credit market outcome at $t = 1$ for each aggregate state. The argument generalizes Proposition 2 from Section II into two separate results on interests rates and credit allocation.

PROPOSITION 3 (International Interest Rates):

(i) If $\theta = H$, there is a single common prevailing interest rate, r_H. Moreover, there is a threshold skill level, $s_H \in [0, 1)$, such that only investors who are more skilled than this threshold, $s \geq s_H$, participate in the credit market.

(ii) If $\theta = L$, there is a weakly increasing, continuous interest rate schedule $r_L(\omega)$ at which good firms with opacity ω raise credit. This schedule is characterized by endogenous thresholds $0 \leq \omega_1 \leq \omega_2 \leq 1$, and a continuous increasing function $\hat{r}(\omega)$, such that

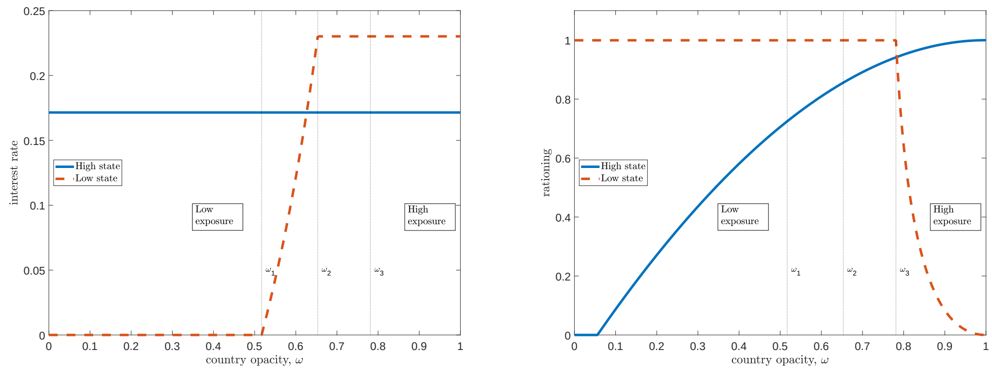
$$r_L(\omega) = \begin{cases} 0 & \text{if } \omega \in [0, \omega_1] \\ \hat{r}(\omega) & \text{if } \omega \in (\omega_1, \omega_2] \\ \bar{r}(r_H) & \text{if } \omega \in (\omega_2, 1] \end{cases} \tag{14}$$

and $\hat{r}(\omega_1) = 0$, $\hat{r}(\omega_2) = \bar{r}(r_H)$.

PROPOSITION 4 (International Credit Allocation):

(i) If $\theta = H$, a good firm that is hit by the liquidity shock uses its full scale as collateral and obtains $\ell(\omega, g; H) = \xi I(\omega, g)/(1 + r_H)$ credit.

Bad firms demand $\bar{\sigma}_H$ credit, but they are rationed. Specifically, there is a weakly decreasing function, $\eta_H(\omega)$, such that a bad firm with opacity ω obtains $\ell(\omega, b; H) = \eta_H(\omega)\bar{\sigma}_H$ credit with $\eta_H(1) = 1$ and $\eta_H(\omega) = 0$ for all $\omega < s_H$.



(a) Interest Rate Schedule (b) Rationing Function

Figure 1. : Panel (a) plots the interest rate schedule. Panel (b) plots the rationing function for bad firms of opacity ω in the high state (solid), and good firms of opacity ω in the low state (dashed). Parameters are $\lambda = 0.85$, $\xi = 5.9$, $\phi = 0.7$, $\pi_H = 0.75$, $w(s) = 5(1-s)$.

(ii) If $\theta = L$, bad firms do not obtain any credit; $\ell(\omega, b; L) = 0$.

There is a threshold $0 < \omega_3 < 1$ such that good firms with opacity $\omega \in [0, \omega_3]$ fully pledge their investment in their scale of operation as collateral and obtain $\ell(\omega, g; L) = \xi I(\omega, g)/(1 + r_L(\omega))$ credit. In contrast, good firms with opacity $\omega \in [\omega_3, 1]$ are only partially financed. Specifically, there is a decreasing function $\eta_L(\omega)$ such that a firm with $\omega \in [\omega_3, 1]$ obtains $\ell(\omega, g; L) = \eta_L(\omega)\xi I(\omega_3, g)/(1 + r_L(\omega))$ credit with $\eta_L(\omega_3) = 1$ and $\eta_L(1) = 0$.

Credit market outcomes share some common aspects across the simplified and more general model. In the high state, there is a single prevailing interest rate r_H, at which all good and some bad firms raise credit, represented by the solid blue horizontal line on the left panel of Figure 1. In the low state, credit markets are fragmented. Good firms face an interest rate schedule $r_L(\omega)$ which is increasing in the opacity of their country of origin, while bad firms cannot raise any credit. In both states, investors who participate in the credit market accept any application for which their test generates a signal $x = g$.

In the high state, the continuous investor distribution leads to a marginal investor, s_H who is the least skilled investor participating in the credit market. In this state, the credit allocation is similar to the simplified economy: every active investor participates at market with interest rate r_H. All good firms are fully financed while bad firms are rationed and obtain credit only from investors who mistake them to be good firms. The corresponding rationing function $\eta_H(\omega)$ is the solid curve on the right panel of Figure 1.

As such, the interest rate r_H and the marginal type s_H are pinned down by two conditions. First, the interest rate has to compensate the marginal investor for the defaults she experiences in her portfolio due to loans that are extended to bad firms. Second, the wealth of all participating investors has to be sufficient to cover the aggregate credit demand by good firms.

In the low state, the continuous investor distribution results in a continuous interest rate schedule $r_L(\omega)$, as illustrated by the dashed curve on left panel of Figure 1. Thus, there is a continuum of active markets. Investors of all skill levels participate in the credit market, albeit at different interest rates. Lowest skilled investors with $s \leq \omega_1$ lend to the good firms from the least opaque countries $\omega < \omega_1$ at interest rate $r_L = 0$. On the opposite end of the spectrum, highest skilled investors $s > \omega_3$ lend at the high interest rate $\bar{r}$ to good firms from the most opaque countries, $\omega > \omega_3$. In between, a moderately skilled investor with skill level $s = \omega$ participates in the market with interest rate $r = r_L(\omega)$ and lends to good firms of country with opacity ω.

Similar to the simplified model, firms in the most transparent countries $\omega \in [0, \omega_1]$ face the minimal rate at which any cautious investor is willing to lend. Excess supply of capital from low-skilled investors relative to the demand from sufficiently transparent countries, limited by their scale, forces the low-skilled investors to their outside option of not lending, i.e. zero interest rate.

By contrast, the most opaque good firms $\omega \in (\omega_2, 1]$ can only raise financing from highly-skilled investors, and only at the maximal interest rate $\bar{r}$. In fact, due to the short supply of capital from highly skilled investors relative to the corresponding demand, the most opaque firms $\omega \in (\omega_3, 1]$, are only partially financed. The corresponding rationing function $\eta_L(\omega)$ is the dashed curve on the right panel of Figure 1. Lastly, firms in the intermediate opacity group $\omega \in (\omega_1, \omega_2]$ are financed at intermediate interest rates by investors with intermediate skill.

The key difference between the simplified and generalized setting in the credit market constitutes the main technical contribution of the paper. In the general model, not only is the distribution of the supply of financing subject to minimal restrictions as in Kurlat (2016), but also the heterogeneous demand for financing across different firms is accommodated. This generalization not only addresses a realistic phenomena, but also paves the way to analyze the observed heterogeneity in real outcomes across countries.

Beyond the technical challenges, the critical economic difference in the credit market is that the heterogeneity across countries, which was due to the exogenous three-point investor distribution in the simplified model, arises endogenously here. In other words, countries are endogenously partitioned into *exposure groups* to the aggregate shock. As a general picture, we argue that the partition defined by $(\omega_1, \omega_2, \omega_3)$ is the main determinant of the exposure to credit cycles across countries. Countries with $\omega \in (\omega_3, 1]$ and $\omega \in [0, \omega_1)$ constitute the *high exposure* and *low exposure* group, respectively, as illustrated in Figure 1.

Section IV explores this endogenous heterogeneity in the credit market as well as in the real economy, across different countries.

Equilibrium in Real Investment

In this section we characterize the equilibrium in the real economy, where we analyze how a firm $j = (\omega, \tau)$ chooses its investment plan, $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_\theta$, foreseeing the equilibrium in the credit market described by Propositions 3 and 4. This is where our generalized framework helps the most to generate new insights as it embeds a two-way interaction between credit market and real outcomes, which we abstracted away from in Section II.

The first proposition describes the optimal investment plan for good firms.

PROPOSITION 5 (Good Firm Investment):
In a simple global equilibrium, a good firm chooses

$$I(\omega, g) = \frac{1 + (1 - \eta_L(\omega)) \frac{\phi \xi \pi_L \frac{\bar{r}(r_H)}{1+\bar{r}(r_H)}}{1 + \phi \xi \pi_H \frac{r_H}{1+r_H}}}{1 + \phi \xi \left(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r_L(\omega)}{1+r_L(\omega)} \right)}, \tag{15}$$

$$i(\omega, g; H) = I(\omega, g) \tag{16}$$

$$i(\omega, g; L) = \eta_L(\omega) I(\omega, g). \tag{17}$$

where $0 \le \eta_L(\omega) \le 1$ *is a weakly increasing function.*[15]

It is intuitive to analyze a firm's optimal investment and continuation decision backwards. First, consider a good firm hit by a liquidity shock at $t = 1$, which has already chosen its scale. (16) and (17) imply that all good firms fully maintain their scale when investors are bold. Moreover, most good firms also fully maintain their investment when investors are cautious, except those in the high exposure region. In this region, credit (partially) dries up even for good firms because of the scarcity of capital of high-skilled investors, as explained in Section III.A.

At $t = 0$ firms foresee this and choose their scale accordingly. Given a firm's future investment policy and the market conditions it expects to face, its scale of operation is determined by constraint (13). For good firms, this constraint encapsulates a key yet simple trade-off. The fraction of the maintenance cost covered by the firm's saving limits the scale it can afford.

Bad firms' investment plan choice differs from that of good firms because they face different conditions in the market for credit. Bad firms understand that they will not be able to obtain any credit when investors are cautious and that they are rationed when investors are bold. The next proposition describes their optimal choice.

[15] $\eta_L(\omega)$ is defined in Online Appendix D, Equation (D.21).

PROPOSITION 6 (Bad Firm Investment): *In a simple global equilibrium, bad firms choose the following investment plan:*

$$
\begin{aligned}
I\left(\omega, b\right) &= 1 - r_H \phi \pi_H \eta_H(\omega) \bar{\sigma}_H \\
i(\omega, b; H) &= \frac{(1 + r_H)\eta_H(\omega)\bar{\sigma}_H}{\xi} \\
i(\omega, b; L) &= 0.
\end{aligned}
\tag{18}
$$

where $0 \leq \eta_H(\omega) \leq 1$ *is a weakly decreasing function.*[16]

Bad firms' choice of their scale is determined by the trade-off embedded in the financing constraint (13). Opaque bad firms can obtain more credit in the high state, which they do not plan to pay back.

The last result in this section provides sufficient conditions for existence of a simple global equilibrium. The precise characterization of the equilibrium and exact sufficient conditions are provided in the Online Appendix.

PROPOSITION 7 (Existence): *There exist* $\underline{\lambda}$ *and a weakly increasing function* $\Lambda(\omega)$ *such that if* $\lambda > \underline{\lambda}$, $\Lambda(\omega) \leq \frac{1-\lambda}{1-\lambda\omega}$ $\forall\omega$, $\xi \geq \frac{1}{1-\phi}$, $w(0) \geq \phi\lambda\xi$ *and* $\lim_{s\to 1} w(s) = 0$*, then there exists a simple global equilibrium.*

We end this section by discussing certain modeling assumptions. In the next section, we provide a detailed discussion of how our model predicts the emergence of core and periphery economies.

B. Comments

In this section, we remark on the interpretation of agents and markets, as well as some of the assumptions.

The focus of our analysis is the capital flows that are channeled through global financial institutions towards local firms. In reality, multiple channels serve these flows with potentially several layers of intermediation. For instance, a large fraction of European firms finance themselves using loans from local banks. Banks in turn often fund these loans by selling commercial paper to money market funds (Ivashina et al., 2015; Gallagher et al., 2018). Larger firms can also raise capital on the corporate bond market directly from bond mutual funds and other asset managers (Gilchrist and Mojon, 2018). We expect the predictions of our model to hold in a variety of these contexts, with the labels adjusted appropriately. For instance, applied to the commercial paper market, local banks are the firms and money market funds are the international investors.

As our focus is on the interaction of international investors and local firms, our choice for modeling countries is decidedly simplistic: a country comprises a set of firms. For our mechanism to be relevant, we need two weak requirements

[16] $\eta_H(\omega)$ is defined in Online Appendix D, Equation (D.22).

related to the allocation of firms across countries. First, this allocation cannot be uniform in opacity, i.e., $\mathbb{E}_c[\omega]$ has to differ across countries. Second, investors' prior on the allocation has to be coarse. For simplicity, we push both of these requirements to the extreme. We assume that each country is populated with firms of a single opacity, and furthermore, that investors have an uninformative prior about the mapping between country name and opacity level. To show that this latter assumption is stronger than needed, in Online Appendix E.2 we generalize our framework allowing for partially informative priors and show that our results are robust to this generalization. For instance, in the European context, investors can recognize that it is harder to learn about Italian and Spanish firms compared to German ones, as long as their prior is uninformative about how Spanish and Italian firms compare to each other.

Intuitively, we think of the coarseness of prior information on opacity of a country, ω, as an assumption which captures the fact that boom-bust patterns are often preceded by major changes in the countries of interest, contributing to investors' uncertainty about ω. For example, the introduction of the European Monetary Union, or the major economic reforms preceding the fast growth of East Asian countries perhaps led investors to rely less on their existing knowledge of these markets.

Furthermore, to emphasize that our mechanism relies only on the informational frictions vis-a-vis the international capital supply (supply side), we suppress difference in production fundamentals (demand side) across countries. In particular, we assume that every country has the same composition of good and bad firms. We make this assumption solely for expositional purposes. However, we do not doubt that fundamental differences across emerging and developed countries, or core and periphery countries exist.

IV. Emergence of Core and Periphery Economies

We have shown that firms which are exante identical in production fundamentals but different in opacity face radically different credit conditions. Firms foresee this and choose their scale of operation accordingly. Their operating scale, in turn, effect their liquidity demand, feeding back to the equilibrium outcome of credit markets.

In this section, we focus on the aggregate implications of this two-way interaction of firms' investment decisions and credit market outcomes at the country level. We illustrate how the heterogeneity in credit conditions implies heterogeneity in the composition of investment across countries, which in turn leads to heterogeneous boom and bust patterns across countries. A group of countries are highly exposed to business cycles and suffer large busts, while a second group have a much lower exposure and only experience a minor decline in output. Thus, *peripheral* and *core* countries emerge endogenously.

In Section IV.A we explore the cross-country implications of the model in a global economy where the set of core and peripheral countries are fixed. As such, we focus on the differences in real and credit market outcomes across countries over boom and bust periods. In Section IV.B we expand this analysis to consider the evolution of the structure of the global economy in the face of various shocks. There we analyze how spillover effects in demand and supply of capital among countries can lead to an expansion of the periphery and higher exposure to shocks.

A. *Heterogeneous Booms and Busts*

We first study the cross-section of real outcomes including output, debt and default across countries, and then investigate the model implications about credit market outcomes such as credit spreads, ownership of debt and portfolio returns. Lastly, we examine the determination of *safe assets* in the context of our model.

Real Economy: Output, Credit, and Default

The total output $Y(\omega, \theta)$ in country ω and state θ can be calculated by aggregating across all firms within the country,

$$Y(\omega,\theta) \equiv \begin{cases} \rho_g \lambda I(\omega,g) + \rho_b\,(1-\lambda)\,((1-\phi)I(\omega,b) + \phi i(\omega,b,H)) & \text{if } \theta = H \\ \rho_g \lambda((1-\phi) + \phi\eta_L(\omega))I(\omega,g) + \rho_b\,(1-\lambda)\,(1-\phi)I(\omega,b). & \text{if } \theta = L \end{cases}$$

As depicted in Figure 2b, output is higher in the high state compared to the low state in every country. As such, we refer to the high and low aggregate state in our model as the boom and bust phase of the global cycle, with the output as the critical statistic.[17]

Similarly, the total debt outstanding by all firms in country ω and state θ is given by

$$C(\omega,\theta) \equiv \begin{cases} \phi\xi\,(\lambda I(\omega,g) + (1-\lambda)\,i(\omega,b,H)) & \text{if } \theta = H \\ \phi\xi\,(\lambda\eta_L(\omega)I(\omega,g)). & \text{if } \theta = L \end{cases}$$

We next use these definitions to explore the cross-country predictions of the model in different dimensions. It is most insightful to interpret the cross-country pattern in output along with that of credit, Figures 2b and 2c respectively. While the peripheral economies enjoy higher credit and output growth compared to all other countries during booms, they experience the largest credit dry-up during busts. Note that the cross-country variation of output in boom is relatively small. On the contrary, during the bust, the output dramatically collapses in peripheral economies while the drop is much less pronounced in the core. The wide degree of dispersion is particularly striking given that the cross-country variation of investment, as illustrated in Figure 2a, is an order of magnitude smaller than that of output collapse. Notably, while the scale initially falls as the countries'

[17]Given that in our model each country has the same initial endowment, for empirical purposes, levels of output, credit and debt in Figure 2 can be interpreted as output, credit and debt growth.

exposure to global cycles increases, it is non-monotone and the most peripheral countries have more similar level of investment to core countries.

The non-monotonicity of the exante choice of scale of operation follows from a gambling behavior by peripheral firms in response to the credit conditions that they face. The good firms in the periphery know that they cannot raise much financing in busts, so if they want to continue production in the bust they only have one option: they have to save enough to do the required liquidity injection on their own, which in turn hurts their scale of investment in both boom and busts. Instead, they follow a risky strategy: they invest a lot and do not save enough to ensure against liquidity shocks. If there is a boom, they produce at very high scale and borrow in the bold credit market if necessary, while in the bust they abandon production if they need liquidity. This trade-off is embodied in the investment-saving constraint (13).[18]

The risk taking behavior of bad firms further amplifies this pattern. While all bad firms have to abandon all production in bust, the ones in the periphery know that they can free ride on good firms in booms when investors are bold, invest, and do not pay back. Thus they choose a scale that they know they have to give up in bad times. All peripheral firms partially or fully abandon production in a bust, leading to a collapse in output. Figures 2c and 2b outline the cross-section of credit and output across the spectrum of countries in each state, highlighting the plunge in the periphery in the bust. In Section IV.B we discuss how this risky strategy of the peripheral firms leads to a negative spillover from boom to bust and underlies the grim consequences of a saving glut.

One interpretation of the collapse in real output that follows credit rationing is capacity underutilization, which has been documented in the recessions (e.g. Ragan, 1976; Shapiro et al., 1989; Fernald, 2015). Our simplified model of production in Section II cannot accommodate this phenomena since there is no choice of the scale of operation.

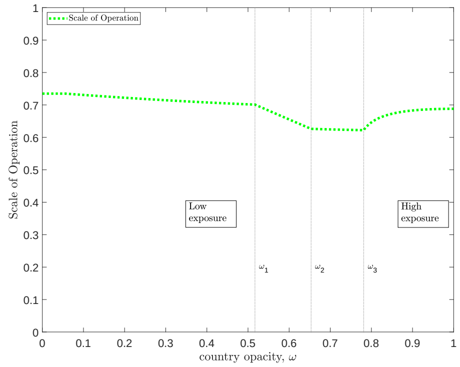
These observations lead to the first testable prediction of the model.

COROLLARY 2 (Testable Predictions I: Real Economy):
Total output and debt to output ratio are more cyclical in peripheral versus core countries.

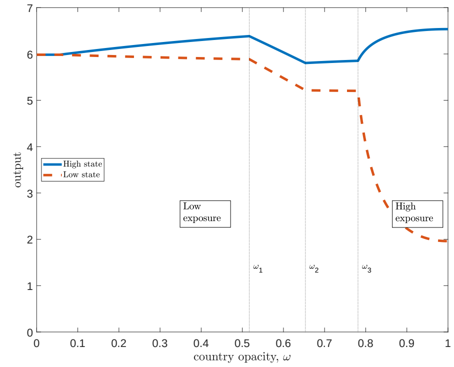
These patterns are consistent with the stylized facts of sudden stop crises in emerging markets in general (Calvo et al., 2004), and with the experience of periphery countries during the European sovereign debt crisis (Lane, 2013; Martin and Philippon, 2017).

As discussed earlier, bad firms only raise financing in booms, and they default. As more bad firms obtain credit in more opaque countries, there are more defaults in periphery economies. This implies larger dispersion of productivity in

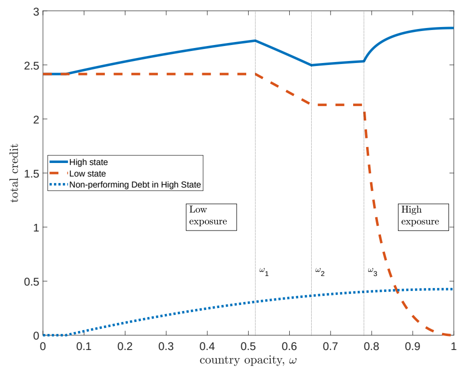
[18]The trade-off of between investment and saving towards the cost of liquidity in adverse states is akin to the investment and risk-management trade-off explored by Rampini and Viswanathan (2013). See also Rampini et al. (2020) for evidence on the relevance of this channel.



(a) Cross-country Firm Scale of Operation



(b) Cross-country output



(c) Cross-country credit and non-performing debt

Figure 2. : Total scale of operation, $\lambda I(\omega, g) + (1-\lambda)I(\omega, b)$, output, $Y(\omega, \theta)$, credit, $C(\omega, \theta)$ and non-performing debt, $(1-\lambda)\phi\xi i(\omega, b, H)$ in each country. Output and credit are separately plotted for high state (blue, solid) and low state (red, dashed). Parameters are $\lambda = 0.85$, $\xi = 5.9$, $\phi = 0.7$, $\pi_H = 0.75$, $w(s) = 5(1-s)$.

booms between the core and the periphery, with $\rho_g > \rho_b$. Furthermore, let *non-performing debt* denote the amount of credit for which the borrowers default and do not pay the lenders back. A similar argument implies that within each country, more non-performing debt is initiated in booms compared to busts. Figure 2c also depicts the cross-section of non-performing debt issued during booms, and the following corollary summarizes these results.

COROLLARY 3 (Testable Predictions II: Real Economy):

(i) In booms, productivity dispersion among firms obtaining credit is larger in the periphery than in the core, leading to capital misallocation.

(ii) *More non-performing debt is issued in booms than in busts. Furthermore, more non-performing debt is issued in the periphery.*

These predictions are consistent with the observed increase in the misallocation of capital during the pre-crisis years in the south of Europe (Reis, 2013; Gopinath et al., 2017).

Credit Market: Price and Return on Debt, and Portfolio Compositions

The most straightforward implication of the model concerning the credit market is that while they are integrated in booms, they become fragmented in recessions, and yields in periphery economies spike especially relative to the core countries. In relation to the European sovereign debt crisis, this fragmentation was observed not only in the market of sovereign bonds, but also on financial and non-financial corporate debt (Battistini et al., 2014; Farhi and Tirole, 2016; Gilchrist and Mojon, 2018), and bank credit (Darracq Paries et al., 2014).

The European debt crisis provides a good example: as shown in Gilchrist and Mojon (2018), non-financial firms active in the corporate bond market who were treated almost as equals before the Greek crisis, suddenly started facing very different market conditions depending on their country of origin in 2010. Whether an investment grade firm was French or Italian did not seem to matter before or even during the crisis in 2008–2009. By 2011, Italian firms were paying a much higher interest rate for credit than French firms. We connect this figure with the interest rate schedule on Figure 1. The shift from the high to the low aggregate state in our model corresponds to the fragmentation of the corporate bond market around 2010. This is the first prediction of the model about the cross-section of bonds.

COROLLARY 4 (Testable Predictions I: Credit Market):
Credit markets are integrated in booms and fragmented in busts. The nominal yields for comparable firms are close to equal across countries in booms, while they are higher in the periphery than in the core economies in busts.

A novel testable implication of our information structure is the significant change in the concentration of ownership of the portfolio of securities across states. In booms, a wide range of investors with various skill levels hold the debt of each country. Hence, the concentration of ownership of bonds is low in each country as even low-skilled investors lend to firms in periphery economies. However, during busts, low-skilled investors stop lending to firms in peripheral countries and re-balance their portfolio towards core countries where they can confidently identify the good firms. In contrast, highly skilled investors rebalance toward periphery countries where they can earn high returns. This leads to a high concentration of ownership of credit in high exposure peripheral countries during busts.[19]

Indeed, in context of the eurozone crisis, Ivashina et al. (2015) and Gallagher et al. (2018) find that in 2011 a group of US money market funds stopped lending only to European banks but not to other banks which had similar risk. In particular, Gallagher et al. (2018) find that when these money market funds stopped financing firms in a European country, they did so irrespective of a firm's implied risk of default. Moreover, Ivashina et al. (2015) also find evidence that this process led to a significant disruption in the syndicated loan market.

The final testable implication of the model concerns realized average return across countries, investors, and states. In booms, although the yields are the same across all countries, more bonds issued in peripheral countries default. Thus in booms, the average realized return is higher in core countries compared to periphery, and higher for the high-skilled investors who held fewer bad bonds. Alternatively, in busts, nominal yields are higher for periphery bonds. As such, the average realized return in peripheral countries is higher than the core in busts, and for investors who hold more high return bonds, who are again the higher skilled investors.

We summarize these observations as testable predictions in the following corollary.

COROLLARY 5 (Testable Predictions II: Credit Market):

(i) In each country, ownership of debt is more concentrated in busts relative to booms. Moreover during busts, ownership of debt is more concentrated in the periphery than in the core.

(ii) In busts, unskilled investors shed assets in peripheral countries and rebalance towards core countries with low yields, while skilled investors rebalance toward peripheral countries with high yields.

(iii) Average realized return on bonds issued in booms is higher in core countries versus peripheral countries, while it is the opposite for bonds issued in busts.

Let us emphasize that we generate these facts in a model where countries are exante identical in their production fundamentals. As such, the aggregate shock would be neutral without investors being heterogeneously informed about firms in different countries. While we do not doubt that fundamental differences across European countries also contributed to their differential performance, we abstract away from this channel in order to emphasize the role of the informational frictions affecting credit supply in global cycles.

[19]Battistini et al. (2014) document a pattern consistent with our prediction on concentration of ownership in the context of sovereign lending in the eurozone. However, we are not aware of any test of these predictions on corporate credit.

Corporate Credit, Sovereign Bonds and Safe Asset Determination

Sovereign bonds and corporate credit do not have separate roles in our framework since we have chosen not to explicitly model governments as decision makers. Nevertheless, it is reasonable to use the spread on the corporate bond portfolio of a given country in our model as a prediction for the corresponding sovereign spread, as sovereign bond and average corporate spreads move in tandem in the data. For instance, the average correlation between the non-financial corporate spreads and the sovereign bond spreads between 1999 and 2017 in European countries was 0.92 (ranging from 0.88 in Italy to 0.95 in Germany) as estimated by Gilchrist and Mojon (2018).

With this caveat in mind, our model has implications on the set of countries where safe assets, public or private, can be issued. We follow the definition of He et al. (2016) and Maggiori (2017), who define safe assets as those which are traded at a lower yield during bad times, often due to flight-to-quality episodes. We state our model prediction in the following corollary.

COROLLARY 6 (Safe asset determination): *Safe assets are issued only in sufficiently transparent countries. The real outcomes of these countries have a low-exposure to credit fluctuations.*

Note that our framework provides a novel mechanism for safe asset determination. He et al. (2016) emphasizes the role of coordination of investors. Farhi and Maggiori (2017) focuses on the issuer's limited commitment not to default on the asset, or devaluate the underlying currency. There is also a strand of literature (Caballero et al., 2008; Caballero and Farhi, 2013) where a given country is capable of issuing safe assets and/or certain investors demand safe assets by assumption, and only the quantity is determined in equilibrium. In contrast, in our paper, the set of issuers and the set of buyers are endogenously determined through a critical level of opacity, ω_1. As ω_1 responds to parameter changes, so does the marginal country's ability to issue safe assets.[20]

In the next section we study the type of shocks which might lead some countries to move across exposure groups.

B. Changes in the Anatomy of the Global Economy

A unique feature of our model is the endogenous determination of the exposure groups, i.e. the core and the periphery. This allows us to study effect of global

[20]It is useful to recognize that in the existing literature the distinction between the concepts of "reserve currency" and "safe asset" is not always clear. A reserve currency is traditionally defined as an asset which serves three roles simultaneously: it is an international store of value, a unit of account, and a medium of exchange. Because of the second and third role, almost by definition, there can be only very few reserve currencies in the world. While reserve currencies usually qualify, our definition of safe assets can also include a large set of other securities, potentially ranging from sovereign bonds and currencies of some small developed countries (e.g. Swedish sovereign bond, Swiss Franc), to highly rated commercial papers, or certain asset-backed securities.

shocks on the composition and relative size of core and periphery, that is, how the global economy is structured.

We first explain how the skill distribution of capital supply interacts with demand for capital from different countries and determines the exposure groups in our model. We then use two experiments to illustrate the most important insights of this mechanism. The first scenario explores the effect of a permanent credit demand shock, and the second one examines an increase in the supply of capital by low-skilled global investors. Since the capital demand and supply interact through different channels in each experiment, the outcomes have similarities and differences. Counterintuitively, both shocks lead to a contraction in the set of core countries. However, while the credit demand shock leads to a suppressed boom and an amplified bust across all countries, the supply shock culminates in a suppressed bust in more core countries accompanied with an amplified boom in more peripheral ones.

Figure 3 illustrates how the demand and supply of capital across skill groups determine the exposure groups. Let $\hat{s}(\omega) = \omega$ denote the *marginal* investor for opacity ω, defined as the lowest-skill investor who recognizes the (pledgeability) type of the firm with opacity ω. The solid diagonal line is $w(\omega)$, the supply of capital by the marginal investor $\hat{s}(\omega) = \omega$.

Recall that the credit demand of a good firm $j = (\omega, g)$ in $\theta = L$, conditional on the interest rate $r(\omega, \tau; L)$ that the firm faces, is independent of the firm opacity ω. The red dash-dotted horizontal line (higher) corresponds to credit demand of any good firm facing zero interest rate, $d_h = \phi\lambda\xi\sigma(\omega, g; L|r_L(\omega) = 0)$. Thus ω_1 is the highest opacity level for which the wealth of the corresponding marginal investor, s_{ω_1}, is sufficient to cover d_h. When $\theta = L$, good firms in the least opaque countries, $\omega \leq \omega_1$, enjoy zero interest rate and have the maximum credit demand. Therefore, countries with $\omega = [0, \omega_1]$ constitute the low exposure group, alternatively the *core*. Note that the total supply of capital in this market, area ACDE, is larger than the total demand from firms serviced in this market, area BCDE. That is, investors with low skill "queue up" to finance the transparent good firms which they can identify confidently.

Next, consider the interest rate $\bar{r}$, above which good firms prefer to abandon production after a liquidity shock and instead invest at a higher scale and continue only when they are not hit by the shock. The blue dashed horizontal line (lower) corresponds to credit demand of any good firm facing the maximum interest rate $\bar{r}$, $d_l = \phi\lambda\frac{\xi}{1+\bar{r}}\sigma(\omega, g; L|r_L(\omega) = \bar{r})$. Then, ω_2 is the highest opacity level such that the wealth of the corresponding marginal investor is sufficient to cover credit demand d_l.

It also follows that every firm in the $[\omega_2, 1]$ region faces the same interest rate $\bar{r}$. Since the price of credit is constant in this region but the supply is lower for more opaque firms, the quantity of credit instead adjusts. Recall that each investor active in a market distributes her endowment pro-rata across all the good firms that she can identify. As such, threshold ω_3 is the opacity level for

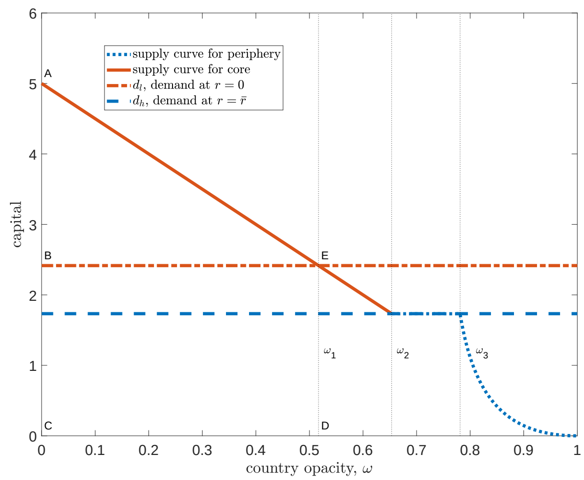


Figure 3. : Supply and demand for investors' funds, and the determination of thresholds $\omega_1, \omega_2, \omega_3$.

which the corresponding total supply of capital allocated to good firms with this level of opacity is exactly sufficient to cover their demand. Firms in more opaque countries, $\omega > \omega_3$, get rationed for credit and their investment takes a larger hit when $\theta = L$. Thus, countries in $[\omega_3, 1]$ constitute the high exposure region, or the *periphery*.

As Figure 3 illustrates, for a fixed supply curve of capital $w(\cdot)$, ω_1 is determined using demand curve for international capital at zero interest rate, d_1. Changes in parameters that lead to an upward shift of this demand curve implies a lower ω_1, that is, a shrinking group of core countries. Intuitively, larger capital demand by good firms implies that the marginal investor who absorbs the demand from core countries has to be richer. Since $w(s)$ is downward sloping, this implies that investor $s_{\omega_1} = \omega_1$ is less skilled, which in turn implies that ω_1 has to correspond to a less opaque country. In the following section we argue the global pandemic of 2020-21 might lead to shrinkage of the set of core countries through this mechanism.[21]

Note that on top of the direct effects described above, parameter shifts also induce an indirect, general equilibrium effect through the adjustment of r_H. Section IV.B illustrates how at the beginning of the 2000-2010 decade, the global saving glut might have pushed some countries out of the core through this indirect effect.

[21] For the interested reader, Online Appendix E.1 summarizes our analytical results on the effect of parameters $\phi, \lambda, \pi_\theta, \xi$ on the size of high and low exposure groups.

Increase in Credit Demand: The Case of More Distressed Firms

In this section, we consider the effect of a permanent shift in credit demand by increasing the fraction of distressed firms, ϕ, in both aggregate states and every country. Figure 4a illustrates how the structure of the global economy changes as a result.

The higher probability that a firm is hit by a liquidity shock implies that each firm chooses a smaller scale and does more precautionary saving to cover their interim liquidity needs. While lower scale of investment reduces the maintenance cost of the firm conditional on being hit by a liquidity shock, the aggregate credit demand still increases in each state and country as it is more likely to be hit by a liquidity shock.

Higher aggregate liquidity demand in each country pushes certain countries out of the core as the threshold ω_1 gets smaller, as explained above. This is illustrated on panel (a) of Figure 4. Furthermore, there is a suppressed boom and a more pronounced bust in virtually all countries, as illustrated by panel (c). The suppressed boom is mainly due to the smaller investment by good firms. For non-periphery countries, $\omega \in [0, \omega_3]$, the same effect is responsible for the larger bust in the low state.

Interestingly, there is heterogeneity in how much the bust worsens within the peripheral countries, $\omega \in [\omega_3, 1]$. Recall that in the periphery, short supply of skilled capital leads to credit rationing, which is more extreme for more opaque countries in this region. After the credit demand shock, the higher demand of relatively more transparent countries in the core absorbs even more of the available skilled capital. This leaves the most opaque countries even more squeezed, leading to a more dramatic collapse in output.

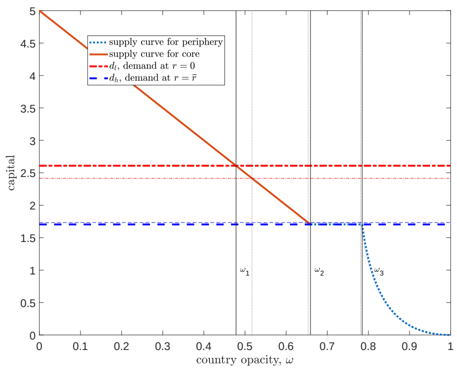
To sum up, our model predicts that a global increase in firm credit demand pushes some countries out of the core. Furthermore, growth rates will be suppressed across the board, and the credit crunch will be even more severe in the most opaque countries. Arguably, this represents a first order consequence of the global pandemic of 2020-21.

Excess Savings: The Case of the Global Saving Glut

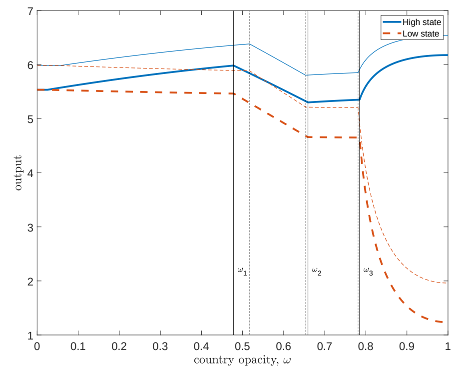
In this section, we examine the effect of an increase in the supply of capital by low-skilled global investors. We interpret this exercise as a representation of the excess global savings phenomena described by Caballero et al. (2017). In particular, we consider the effect of substituting $w(s)$ with a $\tilde{w}(s)$ such that

$$\tilde{w}(s) = \begin{cases} w(s) + k & \text{if } s \leq \bar{s} \\ w(s) & \text{if } s > \bar{s} \end{cases}$$

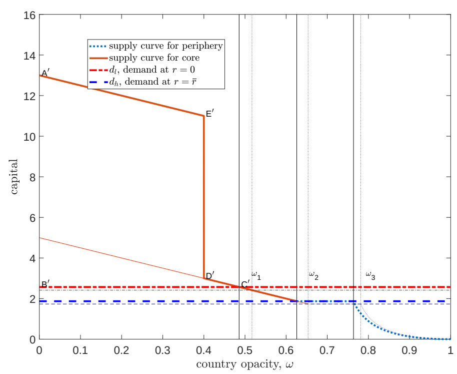
for some positive constant k. Consider the endogenous boundary of the core region, ω_1, associated with $w(s)$. In order to emphasize the role of the indirect



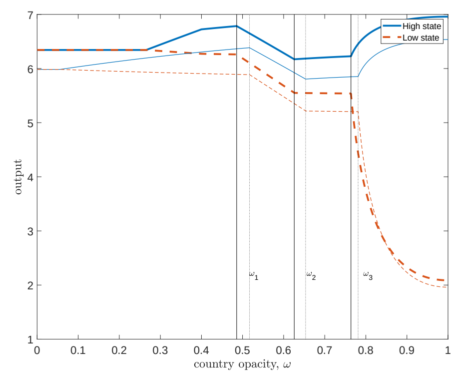
(a) The effect of more distressed firms on cross-country credit supply and demand



(b) The effect of more distressed firms on cross-country output



(c) The effect of more low-skill capital on cross-country credit supply and demand



(d) The effect of more low-skill capital on cross-country output

Figure 4. : The effect of increase in low-skill wealth on output and investment. Parameters are as in the baseline, except that $\phi = 0.8$ on panels (a) and (b), and $w(s) = 5(1 - s) + \mathbb{1}_{0.6>s} \times 8$ on panels (c) and (d).

general equilibrium forces, we choose $\bar{s} < \hat{s}(\omega_1) = \omega_1$. This choice implies that the increase in the supply of global capital does not have a direct effect on demand and supply around thresholds ω_1 and ω_3. As such, all the results are driven by the change in the boom interest rate, r_H.

Panel 4c illustrates this exercise. The key observation is that both thresholds ω_1 and ω_3 move to the left: the core shrinks and the periphery expands. The reason is that increase in the supply of capital decreases the common interest rate in the boom, r_H. This improvement in funding conditions during the boom allows firms to invest more, which increases their liquidity demand for low and high expertise capital alike, both in the boom and the bust. The upward shift in the demand curve when $\theta = L$ implies that both thresholds ω_1 and ω_3 shift to the left. Moreover, this increase in the supply of capital affects the real economic outcomes across countries in both states. Panel 4d illustrates this by showing the total output after the increase in credit supply.

As mentioned above, the higher availability of the global supply of capital increases the investment of both good and bad firms during booms. Good firms simply increase investment because of lower interest rate r_H, which leads to a bigger boom everywhere. The boom is further amplified in the more opaque countries as their bad firms' access to capital increases disproportionately, since the additional capital is channeled towards low-skilled investors who mistakenly lend to them. The more pronounced boom fueled by bad investment implies a larger collapse and a larger volume of non-performing debt in the peripheral countries during the bust.

The expansion of the periphery and the exacerbation of the bust in these countries is a pure general equilibrium effect, a negative spillover from the boom to the bust. It is through the more elaborate Holmström and Tirole (1998) technology that the model is able to capture this crucial spillover effect and address the global saving glut.

This discussion also sheds new light on the problem of scarce safe assets described in Caballero et al. (2017). It is argued that during the last few decades, the supply of safe assets has not been able to keep up with the increasing demand, which has led to excessively low interest rates on these assets. Under our interpretation of safe assets, introduced in Section IV.A, an increase in capital of low-skilled investors represents increasing demand for safe assets. In line with Caballero et al. (2017), this increasing demand results in a lower interest rate, r_H, during booms. Similar to the previous literature, our model predicts an increase in the supply of safe assets by each core country, but not nearly enough to offset the increase in demand. This is apparent by the increase in the area of the ADE triangle in Figure 3 to the A'B'C'D'E' area in Figure 4c, which corresponds to the increase in idle capital.

V. Conclusion

We argue that in the presence of heterogeneous information frictions between international investors and firms, financial liberalization impacts countries heterogeneously, even if these countries have similar fundamentals. Our main premise is that identifying good lending opportunities requires skill, and investor skill is particularly important in certain countries, which we refer to as opaque. As such, countries are subject to varying degrees of information frictions vis-a-vis investors.

We show that countries subject to the most severe frictions become highly exposed to aggregate shocks to investors' information, which leads to countercyclical credit flows and output. Countries that are less exposed to these information frictions experience a much smaller drop in output in a bust, and benefit from low spreads and large capital-inflows. We further illustrate that the key to both the existence of the credit cycle and the heterogeneous exposure spectrum across countries is the scarcity of skilled investor capital.

Our framework provides a wealth of predictions about credit spreads, investment, safe asset supply, concentration of debt ownership, and the return on debt during the boom-bust cycle, both in the time series and in the cross-section. These implications provide a useful guidance for future empirical work. Moreover, we plan to examine the normative implications of our framework and the consequences of different policy interventions on the structure of the global equilibrium.

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Heterogeneous Global Booms and Busts

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Online Appendix

The appendix is organized as follows. Appendix A introduces the required formalization to solve for the credit market equilibrium, with endogenous and exogenous prudence.

Appendix B solves for the general form of credit market equilibrium, at $t = 1$, in a global equilibrium with exogenous prudence. Appendix C uses the credit market equilibrium in Appendix B to construct the equilibrium in real market, at $t = 0$, specialized to a simple global equilibrium. To be more precise, Appendices B and C separately address the two sub-problems that a firm solves. First, the firm chooses initial and maintained investment levels, $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_\theta$ at $t = 0$, solved in Appendix C in a simple global equilibrium. Second, the firm chooses how to raise the required liquidity on the international markets at $t = 1$, solved in Appendix B in a global equilibrium.

Finally, Appendix D provides the proofs for the results in the text, and Appendix E discusses extensions.

Appendices A and B build heavily on Kurlat (2016).

A International Credit Market Formalization

There are many markets at $t = 1$, indexed by m, open simultaneously, where firms can demand credit. M denotes the set of all markets. Each market in aggregate state θ is defined by two features. The first feature is the market interest rate, $\tilde{r}(m; \theta)$, paid by firms to international investor in exchange for bonds. If in market m only firms from a single opacity ω are serviced, we use $r_\theta(\omega) = \tilde{r}(m; \theta)$ to denote the interest rate associated with that market m.

The second is a clearing algorithm. A clearing algorithm is a rule that determines which bonds are traded first, as a function of demand and supply in a market. Since investors have different information sets, different clearing algorithms result in different allocations and we need to specify what algorithm will be used. We will expand on clearing algorithms in Section A.3.

A.1 Firm Problem

We start with two definitions.

Definition A.2 [**Maximum Market Demand**] *There is a maximum amount of credit each firm j can demand in each market m, denoted by $\bar{\sigma}_\theta$. We require $\bar{\sigma}_\theta \geq \max_\omega \ell(\omega, g; \theta)$.*

We need to impose an exogenous upper bound on how much demand for bond issuance firm can submit, in order to prevent firms from submitting excess demand $\hat{y}$ at $t = 1$ to undo the rationing. To keep the analysis as simple as possible, we set the maximum in each state such that the best good firms are not restricted and the maximum repayment promised by any bad firm is consistent with the highest repayment promised by good firms, $\bar{\sigma}_\theta = \max_\omega \ell(\omega, g; \theta)$, $\theta = H, L$. In particular, in the proof of Propositions 1, 2, and Lemma 1, we have $\bar{\sigma}_H \equiv \frac{1}{r_H}$ and $\bar{\sigma}_L \equiv \frac{1}{\underline{r}}$. For the rest of the paper, $\bar{\sigma}_\theta \equiv \xi \frac{I(0,g)}{1+r(0,g;\theta)}$.

Definition A.3 [**Rationing Function**] *A rationing function η assigns a measure $\eta(., \omega, \tau; \theta)$ on M to each bond issued by firm $j = (\omega, \tau)$.*

Let $M_0 \subset M$ denote a set of markets. Then $\eta(M_0, \omega, \tau; \theta)$ is the number of bonds firm (ω, τ) issues if he submits one unit of credit demand to each market $m \in M_0$ in aggregate state θ. The firm receives one unit per bond issued, and $r(\omega, \tau; \theta)$ denotes the average interest rate firm $j = (\omega, \tau)$ has to pay back if aggregate state is θ.

Firm Optimization in International Markets. The firm participates in the international markets in each state θ if he is hit by the liquidity shock, to raise liquidity required to maintain investment. We closely map the problem of the firm in the international market to the seller problem of Kurlat (2016). In order to do so, we introduce the following auxiliary variable, $\hat{y}$.

Definition A.4 [**Credit Capacity**] *$\hat{y}(\omega, \tau; \theta, r_H, r_L)$ is the maximum number of bonds that the firm $j = (\omega, \tau)$ can issue when aggregate state is θ, and the firm faces interest rate $r_{\theta'}$ in state $\theta' \in \{H, L\}$. By definition, $\hat{y}(\omega, \tau; \theta, r_H, r_L) \leq \bar{\sigma}_\theta$.*

We define the firm's problem on the international credit market as an independent problem, which takes one state variable, credit capacity $\hat{y}(\omega, \tau; \theta, r_H, r_L)$. When $\beta = 0$, the credit capacity of firm j is $\hat{y}(\omega, \tau; \theta, r_H, r_L) \equiv \frac{1}{r(\omega,\tau;\theta)}$, as explained in the text. When $\beta > 0$, we will relate $\hat{y}(\omega, \tau; \theta, r_H, r_L)$ to the firm's pledgeability constraint and the technological constraint $\bar{\sigma}_\theta$, in Section C. Here, we assume $\hat{y}(\omega, \tau; \theta, r_H, r_L)$ is continuous and weakly decreasing in $r_{\theta'}$, $\forall \theta'$. Later, in Section C, we verify that in equilibrium, $\hat{y}(\omega, \tau; \theta, r_H, r_L)$ is weakly decreasing in r_H and r_L.

Finally, we show that $y(\omega,\tau;\theta,r_H,r_L)$ in problem (A.1) below maps to $\ell(\omega,\tau;\theta)$ defined in Equation (5).

$$V_{\omega,\tau}\left(\hat{y}\left(.;\theta,r_H,r_L\right)\right) \equiv \max_{\{\sigma(m,\omega,\tau;\theta)\}_m} \quad (1+r(\omega,\tau;\theta))\left(\frac{\rho_\tau}{\xi}-\mathbb{1}_{\tau=g}\right) y(\omega,\tau;\theta,r_H,r_L) \quad \text{(A.1)}$$

s.t.

$$\begin{aligned}
y(\omega,\tau;\theta,r_H,r_L) &= \int_M \sigma(m,\omega,\tau;\theta)d\eta(m,\omega,\tau;\theta) \\
y(\omega,\tau;\theta,r_H,r_L) &\leq \hat{y}(\omega,\tau;\theta,r_H,r_L) \qquad \text{(A.2)} \\
0 \leq \sigma(m,\omega,\tau;\theta) &\leq \bar{\sigma}_\theta \\
r(\omega,\tau;\theta) &= \frac{\int_M \tilde{r}(m;\theta)\sigma(m,\omega,\tau;\theta)d\eta(m,\omega,\tau;\theta)}{\int_M \sigma(m,\omega,\tau;\theta)d\eta(m,\omega,\tau;\theta)}
\end{aligned}$$

To any unit of bonds that the firm issues to international investors, he adds $r(\omega,\tau;\theta)$ units of what he has saved using the bankers. He then injects this as the required liquidity to maintain investment. Thus by issuing $y(\omega,\tau;\theta,r_H,r_L)$ bonds, the firm continues at scale $\frac{1+r(\omega,\tau;\theta)}{\xi}y(\omega,\tau;\theta,r_H,r_L)$, which pays off ρ_τ at date $t=2$. Good firms then have to pay back $1+r(\omega,\tau;\theta)$ per unit bond issued, which leads to the objective (A.1).

Similar to Kurlat (2016), the choice of $\sigma(m,\omega,\tau;\theta)$ for any single market m such that $\eta(m,\omega,\tau;\theta)=0$ has no effect on the funding obtained by the firm. Formally, this implies that program (A.1) has multiple solutions. We follow Kurlat (2016) and assume that when this is the case, the solution has to be robust to small positive perturbations of $\eta(m,\omega,\tau;\theta)$, meaning that the firm must attempt to issue bonds in all the markets where if he could he would want to, and must not attempt to issue bonds in any market where if he could he would not want to.

Definition A.5 [Robust Program] *A solution to program* (12) *is robust if for each* θ *and every* (m_0,ω_0,τ_0) *such that* $\eta(m_0,\omega_0,\tau_0;\theta)=0$ *there exists a sequence of strictly positive real numbers* $\{z_n\}_{n=1}^{\infty}$ *and sequences of credit demand* σ^n *and total credit allocation* y^n*, such that defining*

$$\eta^n(M_0,\omega_0,\tau_0;\theta) = \eta(M_0,\omega_0,\tau_0;\theta) + z_n\mathbb{I}(m_0\in M_0)\mathbb{I}\left(j=j_0\right),$$

(i) $\forall\,\theta$*, given* $\hat{y}(\omega,\tau;\theta,r_H,r_L)$*,* $\{\sigma^n,y^n\}$ *solve program*

$$\max_{\sigma,y} \quad (1+r(\omega,\tau;\theta))\left(\frac{\rho_\tau}{\xi} - \mathbb{1}_{\tau=g}\right) y(\omega,\tau;\theta,r_H,r_L) \tag{A.3}$$

$$\begin{aligned}
&s.t. \\
&\quad y(\omega,\tau;\theta,r_H,r_L) = \int_M \sigma(m,\omega,\tau;\theta) d\eta^n(m,\omega,\tau;\theta) \\
&\quad y(\omega,\tau;\theta,r_H,r_L) \leq \hat{y}(\omega,\tau;\theta,r_H,r_L) \\
&\quad 0 \leq \sigma(m,\omega,\tau;\theta) \leq \bar{\sigma}_\theta
\end{aligned}$$

(ii) $z_n \to 0$

(iii) $\sigma^n \to \sigma,\ y^n \to y;\ \forall(\omega,\tau), m$

Cross-sectional differences across $\sigma(m,\omega,\tau;\theta)$, at the same market m_0, do not reveal the identity of the firm $j = (\omega,\tau)$. This is so because wee assume that $\sigma(m,\omega,\tau;\theta)$ is divisible, and firms submit unit by unit. The investment that can potentially serve as collateral, if $\tau = g$, is verified and "marked", to avoid double promising.

A.2 International Investor Problem

$\int_0^s w(s')ds'$ denotes the mass of investors with skill not higher than s. Each investors is endowed with one unit of wealth. We refer to $w(s)$ as investor skill distribution and require it to be a non-negative function on $s \in [0,1]$ and continuous almost everywhere. International investors consume at dates $t = 1, 2$ and participate in international markets at $t = 1$. Since they are active after realization of aggregate shock θ, we will suppress the dependence of their decisions on θ.

Definition A.6 [Acceptance Rule] *An acceptance rule is a function* $\chi : [0,1] \times \{g,b\} \times [0,1] \times \{0,1\} \to \{0,1\}$.

Definition A.7 [Feasibility] *An acceptance rule* χ *is feasible for investor* s *if it is measurable with respect to his information set, i.e. if*

$$\chi(\omega,\tau;s,\iota) = \chi(\omega',\tau';s,\iota) \quad \textit{whenever} \quad x(\omega,\tau,s,\iota) = x(\omega',\tau',s,\iota).$$

Let X denote the set of all possible acceptance rules, and X_s the set of acceptance rules that are feasible for investor s.

Definition A.8 [Allocation Function] *An allocation function* A *assigns a measure* $A(.;\chi,m,\theta)$ *on* $[0,1]$ *to each acceptance rule-market pair* $(\chi,m) \in X \times M$. $a(.;\chi,m,\theta)$ *denotes the density of the allocation function.*

Consider an investor demanding to buy one unit in market m and imposing acceptance rule χ. If $I_0 \subseteq [0,1] \times \{g,b\}$, then $A(I_0; \chi, m, \theta)$ represents the amount of bonds of firms $j = (\omega, \tau) \in I_0$ she will obtain, i.e. the fraction of her one unit that goes to financing firms j.

In each aggregate state θ, investor s chooses the test ι (or endowed with one), the market she participates in, m, how many bonds he intends to finance δ, and a feasible acceptance rule χ to maximize

$$\max_{\iota,m,\chi,\delta} c_1 + c_2$$

s.t.

$$\chi \in X_s \tag{A.4}$$

$$\delta \int_{(\omega,\tau)} dA(\omega, \tau; \chi, m, \theta) \leq 1 \tag{A.5}$$

$$c_1 = 1 - \delta \int_{(\omega,\tau)} dA(\omega, \tau; \chi, m; \theta) - \delta\kappa \tag{A.6}$$

$$c_2 = \big(1 + \tilde{r}(m;\theta)\big)\delta \int_{\omega} dA(\omega, g; \chi, m, \theta) \tag{A.7}$$

Constraint (A.4) restrict the investor to using feasible rules. Constraint (A.5) says that each investor can only provide credit from her own wealth. Constraint (A.6) says the investor consumes her leftover endowment at $t = 1$ net of cost of sampling, while (A.7) says that at $t = 2$ she is paid back by good firms and consumes. Substitute the consumption into investor utility function and simplify to get the objective function (8) in the text.

A.3 Clearing Algorithm

A clearing algorithm is a total order on X, which determines which acceptance rule is executed first. We will use the clearing algorithms proposed by Kurlat (2016), adjusted to our framework. In order to adopt these algorithms to our settings, let $T = \{g, b\}$.

Definition A.9 [LRF Clearing Algorithm] *ζ is a less-restrictive-first (LRF) algorithm if it orders nested acceptance rules according to $\chi(.;s,.) <_\zeta \chi(.;s',.)$ if $\chi(.;s',.)$ is nested in $\chi(.;s,.)$; i.e. the less restrictive acceptance rule first.*

Thus, acceptance rules of the form $\chi(\omega, \tau; s, \iota) = \mathbb{1}\,(\tau \in T' \subset T \,||\, (\tau \in T - T' \;\&\; \omega \geq s))$ are ordered according to $\chi(.;s,.) <_\zeta \chi(.;s',.)$ if $s < s'$, when ζ is an LRF clearing algorithm. Given the signal structure of investors when $\theta = H$, the less restrictive acceptance rule is also the less accurate.

Definition A.10 [NMR Clearing Algorithm] *ζ is a nonselective-then-more-restrictive-first (NMR) algorithm if it orders nested acceptance rules according to $\chi(.;s,.)$ first if it imposes no restriction, and among acceptance rules with restrictions, the more restrictive acceptance rule first; i.e. $\chi(.;s,.) <_\zeta \chi(.;s',.)$ if $\chi(.;s,.)$ is nested in $\chi(.;s',.)$.*

Thus acceptance rules of the form $\chi(\omega,\tau;s,\iota) = \mathbb{1}\,(\tau \in T' \subseteq T \;\&\; \omega \leq s)$, if ζ is an NMR clearing algorithm, are ordered according to

(i) $\chi(\omega,T;s,\iota) <_\zeta \chi(\omega,T' \subset T;s,\iota)$;

(ii) $\chi(\omega,T';s,\iota) <_\zeta \chi(\omega,T';s',\iota)$ if $s < s'$, for all $s,s' < 1$;

Given the signal structure of investors when $\theta = L$, the more restrictive acceptance rule is the less accurate.

Let $S(m,\omega,\tau;\theta)$ denote the total measure of bonds (ω,τ) offered in market m, which is the total bonds that firms of type (ω,τ) who are hit by a liquidity shock supply in market m when the aggregate state is θ.

$$S(m,\omega,\tau;\theta) = \phi\sigma(m,\omega,\tau;\theta). \qquad \forall\ \theta$$

Furthermore, let $S^s(m,\omega,\tau;\theta)$ denote the residual supply that is faced by an investor with skill s. The allocation function A and rationing function η are determined in the identical manner as Appendix B in Kurlat (2016). Note that in the proofs, we will suppress the dependence of $S(.)$ and $S^s(.)$ on θ as it is clear from the context.

Kurlat (2016) proves that in the presence of markets with different clearing algorithms, there exist an equilibrium where investors self-select into markets using LRF algorithm when the information structure is akin to ours in $\theta = H$, and markets using NMR algorithm when the information structure is that of $\theta = L$. For simplicity, we will directly assume that the clearing algorithm is LRF when $\theta = H$ and NMR when $\theta = L$. These algorithms guarantee that each investor receives a representative sample of the overall supply of bonds he is willing to accept, in the market where he participates.

B Global Equilibrium. Construction of Equilibrium in Credit Market (t = 1)

At $t = 1$, we take firm credit capacity $\hat{y}(\omega,\tau;\theta,r_H,r_L)$, i.e. the maximum level of liquidity that it can raise in the credit market, satisfying the properties of Definition A.4, as given. Given $\hat{y}\,(\omega,\tau,\theta,r_H,r_L)$, the equilibrium in international mar-

kets is such that firms maximize problem (A.1)**, international investors maximize problem** (8)**, and active markets clear, under certain parametric restrictions.**

We construct a more general version of the equilibrium compared to the one used in the main text.

We solve for the credit market equilibrium state-by state. We start with a lemma which simplifies the set of relevant strategies of firms in the credit market.

Lemma B.1 *Every solution to robust program* (A.3) *satisfies*

$$\begin{cases} \sigma(m,\omega,\tau;\theta) \geq \hat{y}\left(\omega,\tau;\theta,r_H(\omega),r_L(\omega)\right) & \text{if } \tilde{r}(m;\theta) < r^R(\omega,\tau;\theta) \\ \sigma(m,\omega,\tau;\theta) = 0 & \text{if } \tilde{r}(m;\theta) > r^R(\omega,\tau;\theta) \end{cases}$$

for some reservation interest rate, $r^R(\omega,\tau;\theta)$*.*

Furthermore, if $\tilde{r}(m;\theta) < r^R(\omega,\tau;\theta)$, $\frac{d\sigma(m,\omega,\tau;\theta)}{d\tilde{r}(m;\theta)} \leq 0$.

Proof. We start with the first part of the proposition. For simplicity, let j denote the firm (ω,τ), $\hat{y}(\omega,\tau) \equiv \hat{y}\left(\omega,\tau;\theta,r_H,r_L(\omega)\right)$, $\sigma(m,j) \equiv \sigma(m,\omega,\tau;\theta)$, and $\eta(m,j) \equiv \eta(m,\omega,\tau;\theta)$. Also, we suppress the dependence of interest rate on prudence shock $\theta = H, L$ and write $\tilde{r}(m)$. Each individual firm is small and takes the prices as given, and does not affect the schedule of prices either.

Assume the contrary. This implies that there are two markets, m and m' with $\tilde{r}(m') < \tilde{r}(m)$ such that, for some j, the firm chooses $\sigma(m,j) > 0$ and $\sigma(m',j) < \hat{y}(\omega,\tau)$. There are four possible cases:

(i) $\eta(m;i) > 0$ and $\eta(m',j) > 0$. Then the firm can increase his utility by choosing demand $\tilde{\sigma}$ with $\tilde{\sigma}(m',j) = \sigma(i,m') + \epsilon$ and $\tilde{\sigma}(m,j) = \sigma(m',j) - \epsilon\frac{\eta(m',j)}{\eta(m,j)})$ for some positive ϵ.

(ii) $\eta(m,j) > 0$ and $\eta(m',j) = 0$. Consider a sequence such that $\eta^n(m',j) > 0$. By the argument in part 1, for any n the solution to robust firm problem must have either $\sigma^n(m,j) = 0$ or $\sigma^n(m',j) \geq \hat{y}(\omega,\tau)$ (or both). Therefore either the condition that $\sigma^n(m,j) \to \sigma(m,j)$ or the condition that $\sigma^n(m',j) \to \sigma(j,m')$ in a robust solution is violated.

(iii) $\eta(m,j) = 0$ and $\eta(m',j) > 0$. Consider a sequence such that $\eta^n(m',j) > 0$. By the argument in part 1, for any n the solution to robust firm problem must have either $\sigma^n(m,j) = 0$ or $\sigma^n(m',j) \geq \hat{y}(\omega,\tau)$ (or both). Therefore either the condition that $\sigma^n(m,j) \to \sigma(m,j)$ or the condition that $\sigma^n(m',j) \to \sigma(m',j)$ in a robust solution is violated.

(iv) $\eta(m, j) = \eta(m', j) = 0$. Consider a sequence such that $\eta^n(m', j) > 0$ and suppose that there is a sequence of solutions to robust firm problem which satisfies $\sigma^n(m', j) \to \sigma(m', j) < \hat{y}(\omega, \tau)$. This implies that for any sequence such that $\eta^n(m, j) > 0$ and for any n, the solution to robust firm problem must have $\sigma^n(m, j) = 0$. Therefore the condition that $\sigma^n(m, j) \to \sigma(m, j)$ in a robust solution is violated.

■

Lemma B.1 implies that firms use a threshold strategy across markets with different interest rates: They submit demand to all the markets with prevailing interest rate lower than a threshold $r^R(\omega, \tau; \theta)$. The threshold interest rate depends on both the firm and the aggregate state.

To save on notation we often suppress the dependence on r_H, and r_L, and sometimes the dependence on θ, unless useful to clarify the context. Finally, we suppress the argument ι of $\chi(\omega, \tau; s, \iota)$, as it is implied by θ in each subsection.

B.1 $\theta =$ H: Bold International Investors

Equilibrium description. The equilibrium consists of a a single active market, m_H, pair (r_H, s_H), firm and investor optimization, an allocation function, and a rationing function. Market m_H is the market defined by interest rate r_H and an LRF algorithm. The equilibrium is described as follows.

(i) $(\mathbf{r_H}, \mathbf{s_H})$ is the solution to the pair of equations

$$r = \frac{(1-\lambda)\int_s^1 \hat{y}(\omega, b; H)d\omega}{\lambda \int_0^1 \hat{y}(\omega, g; H)d\omega} \tag{B.8}$$

$$\phi = \int_s^1 \frac{1}{(1-\lambda)\int_{s'}^1 \hat{y}(\omega, b; H)d\omega + \lambda \int_0^1 \hat{y}(\omega, g; H)d\omega} w(s')ds' \tag{B.9}$$

(ii) **Firm optimization**

- Good firm

$$\sigma(m, \omega, g; H) = \begin{cases} \min\{\bar{\sigma}_H, \hat{y}(\omega, g; H)\} = \hat{y}(\omega, g; H) & \text{if } \tilde{r}(m) = r_H \\ \bar{\sigma}_H & \text{if } \tilde{r}(m) < r_H \\ 0 & \text{otherwise} \end{cases}$$

where the first line in $\sigma(.)$ follows from Definition A.2 along with construction of $\hat{y}(\omega, g; H)$.

- Bad firm

$$\sigma(m,\omega,b;H) = \min\left\{\bar{\sigma}_H, \hat{y}(\omega,b;H)\right\} = \hat{y}(\omega,b;H) \qquad \forall\, m$$

(iii) **International investor optimization**

- $s < s_H$

$$\begin{aligned} \delta_s &= 0 \\ m_s &= m_H \\ \chi(\omega,\tau;s) &= \mathbb{I}\left(\tau = g \;||\; (\tau = b \;\&\; \omega \geq s)\right) \end{aligned}$$

- $s \geq s_H$

$$\begin{aligned} \delta_s &= 1 \\ m_s &= m_H \\ \chi(\omega,\tau;s) &= \mathbb{I}\left(\tau = g \;||\; (\tau = b \;\&\; \omega \geq s)\right) \end{aligned}$$

(iv) **Allocation function**

- For market m_H and $\chi(\omega,\tau;s) = \mathbb{1}\left(\tau = g \;||\; (\tau = b \;\&\; \omega \geq s)\right)$ for some $s \in [0,1]$

$$a(\omega,\tau;\chi,m_H) = \frac{\left(\mathbb{I}(\tau = g) + \mathbb{I}(\tau = b \;\&\; \omega \geq s)\right)\sigma(m_H,\omega,\tau;H)}{\lambda\int_0^1 \sigma(m_H,\omega',g;H)d\omega' + (1-\lambda)\int_s^1 \sigma(m_H,\omega',b;H)d\omega'} \tag{B.10}$$

- For market m_H and any other acceptance rule

$$a(\omega,\tau;\chi,m_H) = \tag{B.11}$$
$$\begin{cases} \frac{\chi(\omega,\tau;s)\sigma(m_H,\omega,\tau;H)[1-\eta(m_H,\omega,\tau;H)]}{\sum_{\tau'}\int_{\omega'}\chi(\omega',b;s')\sigma(m_H,\omega',b;H)[1-\eta(m_H,\omega',b;H)]d\omega'} & \\ \quad \text{if } \chi(\omega,\tau;s) \notin X_0 \;\&\; \sum_{\tau'}\int_{\omega'}\chi(\omega',b;s')\sigma(m_H,\omega',b)[1-\eta(m_H,\omega',b)]d\omega' > 0 & \\ \frac{\chi(\omega,\tau;s)\sigma(m_H,\omega,\tau;H)[1-\eta(m_H,\omega,\tau;H)]}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)\sigma(m_H,\omega',\tau';H)[1-\eta(m_H,\omega',\tau';H)]} & \\ \quad \text{if } \chi(\omega,\tau;s) \notin X_0 \;\&\; \sum_{\tau'}\int_{\omega'}\chi(\omega',b;s')\sigma(m_H,b,\tau')[1-\eta(m_H,\omega',b)]d\omega' = 0, & \\ \quad \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',b';s')\sigma(m_H,\omega',b)[1-\eta(m_H,\omega',b)] > 0 & \\ 0 & \\ \quad \text{otherwise} & \end{cases}$$

where $\eta(m_H,\omega,\tau;H)$ is defined below.

- For any other market

$$a(\omega,\tau;\chi,m) = \tag{B.12}$$
$$\begin{cases} \frac{\chi(\omega,\tau;s)S(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega'} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega' > 0 \\ \frac{\chi(\omega,\tau;s)S(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega' = 0, \\ & \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau') > 0 \\ 0 & \text{otherwise} \end{cases}$$

where

$$S(m,\omega,\tau) = \begin{cases} \phi\sigma(m,\omega,\tau;H) & \text{if } \begin{cases} \tau = b \\ \text{or} \\ \tau = g \;\&\; \tilde{r}(m) \in (0, r_H] \\ \text{or} \\ \tilde{r}(m) \leq 0 \end{cases} \\ 0 & \text{if } \tau = g,\ \tilde{r}(m) > r_H \end{cases}$$

(v) **Rationing function**

$$\eta(M_0,\omega,\tau;H) = \tag{B.13}$$
$$\begin{cases} 1 & \text{if } m_H \in M_0 \text{ and } \tau = g \\ \int_{s_H}^{\omega} \frac{1}{\phi(1-\lambda)\int_s^1 \hat{y}(\omega',b;H)d\omega' + \phi\lambda\int_0^1 \hat{y}(\omega',g;H)d\omega'} w(s)ds & \text{if } m_H \in M_0 \text{ and } \tau = b \text{ and } \omega \geq s_H \\ 0 & \text{otherwise} \end{cases}$$

Proof.

(i) $(\mathbf{r_H}, \mathbf{s_H})$. There is a single market m_H, with $\tilde{r}(m_H) = r_H$, where all trades take place. In this market, firms try to issue as many bonds as they can. Total supply is therefore $\lambda\int_0^1 \hat{y}(\omega',g;H)d\omega'$ good bonds and $(1-\lambda)\int_0^1 \hat{y}(\omega',b;H)d\omega'$ bad bonds. Supply decisions in markets $m \neq m_H$ have no effect on firm utility since $\eta(m,\omega,\tau;H) = 0$, so they are determined in equilibrium by the robustness requirement.

Buying from markets with interest rate other than r_H is not optimal for investors. At interest rates above r_H, the supply includes only bad firms, so investors prefer to stay away, whereas at interest rate below r_H, the supply of bonds is exactly the same as at interest rate r_H but the interest rate is lower. This does not settle the question of whether an investor chooses to buy at all. Investor optimization below then shows that investor with $s = s_H$ faces terms of trade of $\upsilon(s_H) = 1$ in market m_H, and is indifferent between buying and not buying. This results in Equation (B.8).

All investors with $s > s_H$ thus spend all of their wealth buying in market m_H and those with $s < s_H$ choose not to buy at all. The fraction of bonds by firm $j = (\omega, \tau)$ that can be issued in market m_H is given by the ratio of the total allocation of that bond across investors, to the supply of that bond. Noticing that only firms hit by liquidity shock issue bonds, and adding across investors and imposing that all good bonds are issued results in (B.9).

Proposition 7 uses an appropriate monotonic transformation of Equation (B.8) along with Lemma D.1 to show that a solution to the pair of Equations (B.8) and (B.9) such that $r_H \geq 0$ and $0 \leq s_H \leq 1$, constitutes an equilibrium.

(ii) **Firm optimization.** Taking the equilibrium market structure, rationing function and allocation function as given, $y(\omega, \tau) = \sigma(m_H, \omega, \tau)\eta(m_H, \omega, \tau)$. Since $\frac{\rho_\tau}{\xi} - \mathbb{1}_{\tau=g} > 0$, firm j's optimal choice of $\sigma(m_H, \omega, \tau)$ is determined by the corresponding constraints. For a good firm, $\eta(m_H, \omega, g) = 1$ from rationing function (B.13), which implies $y(\omega, g) = \sigma(m_H, \omega, g)$. As such, condition (A.2) is the binding constraint which in turn implies $y(\omega, g) = \sigma(m_H, \omega, g) = \hat{y}(\omega, g)$, when $\theta = H$.

For a bad firm $\eta(m_H, \omega, b) = \int_{s_H}^{\omega} \frac{1}{(1-\lambda)\int_s^1 \hat{y}(\omega', b; H)d\omega' + \lambda \int_0^1 \hat{y}(\omega', g; H)d\omega'} \frac{w(s)}{\phi} ds$ from rationing function (B.13). From equation (B.9), $\eta(m_H, 1, b) = 1$, so $\eta(m_H, \omega, b) < 1$, $\forall\ s_H \leq \omega < 1$, thus $y(\omega, b) \leq \sigma(m_H, \omega, b)$. Since $\sigma(m_H, \omega, b) = \hat{y}(\omega, b) = \bar{\sigma}_H$, constraint (A.2) is satisfied, which in turn implies $y(\omega, b) = \eta(m_H, \omega, b)\hat{y}(\omega, b)$.

Put together, the rationing function (B.13) implies that in market m^H, all good firms will be able to issue as many bonds as they demand to issue. A bad firm with opacity ω will be able to sell a fraction $\eta(m_H, \omega, b) < 1$ of bonds they demand to issue. No other bond can be issued. Thus

$$y(\omega, \tau) = \begin{cases} \hat{y}(\omega, \tau) & \text{if } \tau = g \\ \eta(m_H, \omega, \tau)\hat{y}(\omega, \tau) = \eta_H(\omega)\hat{y}(\omega, \tau) & \text{if } \tau = b \end{cases} \tag{B.14}$$

Off equilibrium, in all cheaper markets (lower interest rate), all good firms submit $\bar{\sigma}_H$. In all more expensive markets, they submit zero demand. All bad firms submit the maximum that they can submit, $\bar{\sigma}_H$, on every other market. These decisions satisfy the robust program (A.3). Note that the equilibrium $\sigma(m, \omega, \tau)$ satisfy the form of Lemma B.1.

(iii) **International investor optimization.** Choosing any feasible acceptance rule other than $\chi(\omega, \tau; s) = \mathbb{1}(\tau = g \,||\, (\tau = b \,\&\, \omega \geq s)) = 1 - \mathbb{1}[x(\omega, \tau, s, 1) = b]$ by investor s using test $\iota = 1$ in market m_H would, according to (B.10) and (B.11), result in a lower fraction of good assets, so choosing $\chi(\omega, \tau; s) = \mathbb{1}(\tau = g \,||\, (\tau = b \,\&\, \omega \geq s))$ is optimal.

Let $\upsilon(m, \chi)$ denote the terms of trade that an investor obtains in market m with acceptance rule χ,

$$\upsilon(m, \chi) = \begin{cases} \frac{(1+\tilde{r}(m)) \int_\omega \mathbb{1}[\tau=g]\sigma(m,\omega,g;\theta)dA(\omega,g;\chi,m,\theta)}{\int_{\omega,\tau} \sigma(m,\omega,\tau;\theta)dA(\omega,\tau;\chi,m,\theta)} & \text{if } A(\{g,b\},[0,1];\chi,m) > 0 \\ 0 & \text{otherwise} \end{cases}$$

which is her expected repayment per unit of bond she finances, i.e. the principal and interest rate she receives at $t = 2$. Let

$$\upsilon^{max}(s) \equiv \max_{m \in M, \chi \in X_s} \upsilon(m, \chi)$$

be the best term of trade that investor s can achieve, and let $M^{max}(s)$ be the set of markets where investor s can obtain terms of trade υ^{max} with a feasible acceptance rule.

Necessary and sufficient condition for investor optimization are that investors for whom $\upsilon^{max} < 1$ choose not to finance any bonds, investors for whom $\upsilon^{max} > 1$ spend their entire endowment in a market $m \in M^{max}(s)$, and investors for whom $\upsilon^{max} = 1$ choose a market $m \in M^{max}(s)$. Using Equation (B.10), an investor s who uses acceptance rule $\chi(\omega, \tau; s) = \mathbb{1}(\tau = g \,||\, (\tau = b \,\&\, \omega \geq s))$ in market m obtains terms of trade

$$\upsilon(m, \chi) = \begin{cases} \frac{(1+\tilde{r}(m))\lambda \int_0^1 \hat{y}(\omega,g;H)d\omega}{(1-\lambda)\int_s^1 \hat{y}(\omega,b;H)d\omega + \lambda \int_0^1 \hat{y}(\omega,g;H)d\omega} & \tilde{r}(m) \leq r_H \\ 0 & \text{otherwise} \end{cases}$$

Thus for all investors

$$\upsilon^{max}(s) = \frac{(1 + r_H)\lambda \int_0^1 \hat{y}(\omega, g; H)d\omega}{(1 - \lambda) \int_s^1 \hat{y}(\omega, b; H)d\omega + \lambda \int_0^1 \hat{y}(\omega, g; H)d\omega}$$

and the maximum is attained in any market where the interest rate is r_H, including m_H. Rewrite

$$\upsilon^{max}(s) = (1 + r_H)J(s)$$

$$J(s) = \frac{\lambda \int_0^1 \hat{y}(\omega, g; H)d\omega}{(1 - \lambda) \int_s^1 \hat{y}(\omega, b; H)d\omega + \lambda \int_0^1 \hat{y}(\omega, g; H)d\omega},$$

Note that from Equation (B.8), $J(s_H) = \frac{1}{1+r_H}$, so $\upsilon^{max}(s_H) = 1$. Moreover, $J'(s) > 0$. This implies that investors $s < s_H$ have $\upsilon^{max}(s) < 1$, so not financing any bonds is optimal for them. Investors of types $s \geq s_H$ have $\upsilon^{max}(s) \geq 1$, so financing bonds such that they spend their entire wealth in market m_H at $t = 2$ is optimal for them.

(iv) **Allocation function.** In all markets except m_H (off equilibrium path), there are no investors, so for any clearing algorithm the residual set of bonds any investor faces is just the original set of bonds demanded by firm on that market. In this case, (B.12) follows from Appendix A, Equation (39) and Appendix B, Equation (65) in Kurlat (2016).

For market m_H, the LRF algorithm implies that an investor who imposes $\chi(\omega, \tau; s) = \mathbb{1}\left(\tau = g \,||\, (\tau = b \;\&\; \omega \geq s)\right)$ faces a residual firm credit demand of acceptable bonds that is proportional to the original firm credit demand (credit demand $\equiv$ bond supply). Therefore, the measure of assets she will obtain is the same as if she traded first. Therefore (B.10) follows from Appendix A, Equation (37) and Appendix B, Equation (65) in Kurlat (2016).

For market m_H and rules that are not of the form $\chi(\omega, \tau; s) = \mathbb{1}\left(\tau = g \,||\, (\tau = b \;\&\; \omega \geq s)\right)$, (off equilibrium path), their trades clear after all other investors, so the bond financing demand they face only includes bonds of bad firms. Therefor (B.11) follows from Appendix A, Equation (38) and Appendix B, Equation (65) in Kurlat (2016)..

(v) **Rationing function.** Equation (B.13) follows from (B.9) using Appendix B of Kurlat (2016), Equation (67). It is the fraction of bonds that the firm is able to issue, out of the total bonds he offers (i.e. a number between zero and one).

■

B.2 $\theta = \mathbf{L}$: Cautious International Investors

Equilibrium description. A global equilibrium consists of an interest rate schedule $0 \leq r_L(\omega) \leq \bar{r}(r_H)$, cut-offs $\{\omega_k\}_{k=1..K} \in [0, 1]$, $K \geq 3$, firm and investor optimization, an allocation function, and a rationing function. Any global equilibrium has at least 3 thresholds, $\omega_1 < \omega_2 < \omega_3$. However, in general K can exceed 3 in a global equilibrium, with $\omega_1 < \omega_{k'} < \omega_2$ for $k' > 3$. A simple global equilibrium is a global equilibrium where there is no nonselective region, and thus $K = 3$.

For any $\omega \in [0, 1]$, let $m(\omega)$ denote the market where the price is $r_L(\omega)$, where $r_L(\omega)$ is found by the procedure described in the proof below, and the clearing algorithm is NMR.

Because of bunching, $m(\omega)$ could mean the same market for different ω. For any $\Omega_0 \subseteq [0,1]$, let the set of markets $M(\Omega_0)$ be $M(\Omega_0) = \{m(\omega) : \omega \in \Omega_0\}$. The set of active markets is $M([0,1])$. A global equilibrium is described as follows.

(i) **Premium schedule $\mathbf{0} \leq \mathbf{r_L}(\omega) \leq \bar{\mathbf{r}}(\mathbf{r_H})$** such that the interest rate falls into one of the cash-in-the-market, bunching, bunching-with-scarcity, or nonselective regions as described below.

(ii) **Firm optimization**

- Good firm

$$\sigma(m,\omega,g;L) = \begin{cases} \min\{\bar{\sigma}_L, \hat{y}(\omega,g;L)\} = \hat{y}(\omega,g;L) & \text{if } \tilde{r}(m) = r_L(\omega), \omega < \omega_2 \\ \hat{y}(\omega_2,g;L) & \text{if } \tilde{r}(m) = r_L(\omega), \omega \geq \omega_2 \\ \bar{\sigma}_L & \text{if } \tilde{r}(m) < r_L(\omega) \\ 0 & \text{otherwise} \end{cases}$$

$$y(\omega,g;L) = \int_{M([0,\omega])} \sigma(m,\omega,g;L)\,d\eta(m,\omega,g;L)$$

where the first line in σ follows from Definition A.2 along with construction of $\hat{y}(\omega,g;L)$.

- Bad firm

$$\sigma(m,\omega,b;L) = \begin{cases} \min\{\bar{\sigma}_L, \hat{y}(\omega,b;L)\} = \hat{y}(\omega,b;L) & m \in \text{nonselective region} \\ \bar{\sigma}_L & \text{otherwise} \end{cases}$$

$$y(\omega,b;L) = \int_{M([0,1])} \sigma(m,\omega,b;L)\,d\eta(m,0,b;L)$$

The rationing functions $\eta(m,\omega,\tau;L)$ are defined in Equations (B.20) and (B.21).

(iii) **International investor optimization**

Recall that $\hat{s}(\omega) = \omega$ is the lowest-skill investor who recognizes the type of a firm with opacity ω. Furthermore, let ω_1 denote the highest-ω opacity whose firm face a zero interest rate when investors are cautious, $r_L(\omega) = 0$. Define s_N by

$$\int_{s_N}^{\hat{s}(\omega_1)} w(s)\,ds = \phi\lambda \int_0^{\omega_1} \hat{y}(\omega,g;L)\,d\omega.$$

We have assumed that there is sufficient wealth by all investors to cover the aggregate credit demand by all firms, which implies that $s_N \geq 0$. The above equation implies that the aggregate wealth of investors in the interval $[s_N, \hat{s}(\omega_1)]$ is just sufficient to finance all the bonds offered by good firms with opacity $\omega < \omega_1$ at interest rate 0, and each of these investors can identify some good bond in this interval. Furthermore, any investor $s \leq s(\omega_1)$ breaks even, and is indifferent between market participation or not. We focus on an equilibrium where investors with $s \in [s_N, \hat{s}(\omega_1)]$ finance all the bonds offered by good firm $\omega < \omega_1$ at interest rate 0, while investors with $s < s_N$ either buy nonselectively or do not buy at all.

Let $\varepsilon(\omega)$ denote the unfinanced fraction of bonds that are offered in market $m(\omega)$, i.e., those whose supply is not fully absorbed in markets $M([0, \omega))$.

Define the function $\tilde{s}(\omega)$ as the solution to the following differential equation

$$\tilde{s}'(\omega) = -\frac{1}{w\left(\tilde{s}(\omega)\right)}\phi\left[\lambda\int_{\omega}^{1}\hat{y}(\omega', g; L)d\omega' + (1-\lambda)\int_{0}^{1}\hat{y}(\omega', b; L)d\omega'\right]\varepsilon'(\omega) \tag{B.15}$$

with boundary condition $\tilde{s}(1) = s_N$. Finally, let $s_0 = \tilde{s}(0)$ and define $\tilde{\omega}(s)$ for $s \in [s_0, s_N]$ by

$$\tilde{\omega}(s) = \min\left\{\omega : \tilde{s}(\omega) = s\right\}$$

(a) for $s \geq s_N$

$$\begin{aligned}\delta_s &= 1\\ m_s &= m(s)\\ \chi(\omega, \tau; s) &= \mathbb{I}(\tau = g \,\&\, \omega \leq s)\end{aligned}$$

(b) $s \in [s_0, s_N)$

$$\begin{aligned}\delta_s &= 1\\ m_s &= m(\tilde{\omega}(s))\\ \chi(\omega, \tau; s) &= 1\end{aligned}$$

(c) $s < s_0$

$$\begin{aligned}\delta_s &= 0\\ m_s &= m(1)\\ \chi(\omega, \tau; s) &= 1\end{aligned}$$

Investors $s \geq s_N$ spend their entire endowment financing bonds in market $m(s)$, i.e. the market for the most opaque firms for which they can observe a good signal, and they use the selective acceptance rule $\mathbb{I}(\tau = g \; \& \; \omega \leq s)$, which only accepts good assets. Some of these investor are in cash-in-the-market region, some in bunching, and some in bunching-with-scarcity. Investors $s \in [s_0, s_N)$ are nonselective. The function $\tilde{\omega}(s)$ assigns each one to a market: in market $m(\omega)$, nonselective investors bring down the fraction of unfinanced bonds by $\varepsilon'(\omega)$, which requires financing $\varepsilon'(\omega)\phi\lambda \int_\omega^1 \hat{y}(\omega', g; L)d\omega'$ good firms and $\varepsilon'(\omega)\phi\,(1-\lambda)\int_0^1 \hat{y}(\omega', b; L)d\omega'$ bad firms. If investor $\tilde{s}(\omega)$ is the nonselective investor that buys in market $m(\omega)$ then the total nonselective wealth available in that market is $-w\left(\tilde{s}(\omega)\right)\tilde{s}'(\omega)$, so market clearing implies (B.15). Inverting this function results in investor s choosing market $m\left(\tilde{\omega}(s)\right)$. Investors $s < s_0$ don't finance (buy) anything. Since they are indifferent between buying and not buying, many other patterns of demand among nonselective investors are possible.

(iv) **Allocation function**

- For markets $m(\omega) \in M([0,1])$ where ω falls in either a cash-in-the-market or a nonselective region

$$a(\omega, \tau; \chi, m) = \tag{B.16}$$
$$\begin{cases} \dfrac{\chi(\omega,\tau;s)S(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega'} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega' > 0 \\ \dfrac{\chi(\omega,\tau;s)S(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')} & \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega' = 0, \\ & \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S(m,\omega',\tau')d\omega' > 0 \\ 0 & \text{otherwise} \end{cases}$$

where

$$S(m,\omega,\tau) = \begin{cases} \phi\sigma(m,\omega,\tau;L) & \text{if } \begin{cases} \tau = b \\ \text{or} \\ \tau = g \; \& \; \tilde{r}(m) \in (0, r_L(\omega)] \\ \text{or} \\ \tilde{r}(m) \leq 0 \end{cases} \\ 0 & \text{if } \tau = g,\ \tilde{r}(m) > r_L(\omega) \end{cases}$$

- For market $m(\omega)$ where ω falls in $[\omega^z, \omega^{z+1}]$ for some z which is either a bunching or bunching-with-scarcity region; and $\chi = \mathbb{I}\left(\tau = g \ \& \ \omega \leq s\right)$.

$$a(\omega,\tau;\chi,m) = \tag{B.17}$$
$$\begin{cases} \frac{\chi(\omega,\tau;s)S^s(m,\omega,\tau)}{\sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S^s(m,\omega',\tau')d\omega'} \\ \qquad \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S^s(m,\omega',\tau')d\omega' > 0 \\ \frac{\chi(\omega,\tau;s)S^s(m,\omega,\tau)}{\sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S^s(m,\omega',\tau')} \\ \qquad \text{if } \sum_{\tau'}\int_{\omega'}\chi(\omega',\tau';s)S^s(m,\omega',\tau')d\omega' = 0, \\ \qquad \text{but } \sum_{\tau'}\sum_{\omega'}\chi(\omega',\tau';s)S^s(m,\omega',\tau')d\omega' > 0 \\ 0 \\ \qquad \text{otherwise} \end{cases}$$

where $S^s(m,\omega,\tau)$ is the solution to differential equation

$$\frac{dS^s(m,\omega,\tau)}{ds} = \begin{cases} -w(s)\frac{S^s(m,\omega,\tau)\mathbb{I}[\omega^z \leq \omega \leq s]}{\sum_{\tau'}\int_{\omega^z}^{s} S^s(m,\omega',\tau')d\omega'} & \text{if } \tau = g \text{ and } s \in [\omega^z, \omega^{z+1}] \\ 0 & \text{otherwise} \end{cases} \tag{B.18}$$

with boundary condition

$$S^0(m,\omega,\tau) = \begin{cases} \phi\sigma(m,\omega,\tau;L) & \text{if } \tau = b \text{ or } (\tau = g \text{ and } \omega \in [0,\omega^{z+1}]) \\ 0 & \text{otherwise} \end{cases} \tag{B.19}$$

Note that if $m(\omega)$ is in a bunching-with-scarcity region, $\sigma(m,\omega,g;L) = \sigma(m,\omega^z,g;L)$, $\forall\ \omega$. Furthermore, note that $s = 0$ is the least skilled investor, who imposes the most restrictive acceptance rule, which is cleared first via NMR algorithm.

Except for bunching and bunching-with-scarcity markets, the clearing algorithm implies that all investors draw bonds from a sample that is proportional to the original supply. This results in (B.16). In bunching markets, investor s imposes acceptance rule of the form $\chi_s(\omega,\tau;s) = \mathbb{I}\left(\tau = g \ \& \ \omega \leq s\right)$; therefore when he buys his bond portfolio, the supply of bonds from good firms in opacity ω falls in proportion to his wealth, $w(s)$, times the ratio between the supply of bonds by good firms with opacity ω and all the other bonds acceptable by investor s. This results in differential Equation (B.18) which characterizes how the supply for bonds fall as the clearing algorithm progresses.

(v) **Rationing function**

- Firm (ω,τ), $\omega \leq \omega_3$

$$\eta(M([0,l]),\omega,\tau;L) = \begin{cases} 1 & \omega \leq l \text{ and } \tau = g \\ 1-\varepsilon(l) & \text{otherwise} \end{cases} \tag{B.20}$$

- Firm (ω,τ), $\omega > \omega_3$

$$\eta(M([0,l]),\omega,\tau;L) = \begin{cases} \int_{\omega}^{1} \frac{1}{R_D(\omega_3,\omega_2,\bar{r}(r_H),1)+(s-\omega_3)\phi\lambda\hat{y}(\omega_2,g;L)} w(s)ds & \omega \leq l \text{ and } \tau = g \\ 1-\varepsilon(l) & \text{otherwise} \end{cases} \tag{B.21}$$

where $R_D(.)$ and $\tilde{\omega}$ are defined in Equations (B.26) and (B.27), respectively. For good firms with opacity $\omega \leq \omega_3$ and $\omega > \omega_3$, the rationing function is separately defined. It says that if $\omega \leq \omega_3$, good firms with opacity $\omega \leq l$ are fully financed on the markets with interest rate $r(m) \in [0, r_L(l)]$. Alternatively, good firms with $\omega > \omega_3$ are rationed. They can only raise up to the maximum specified in Equation (B.21), which is the same as $\eta_L(\omega) = \eta(m(\omega),\omega,g;L)$ defined in Equation (C.9), and can be achieved at the market with (maximum) interest rate $\bar{r}(r_H)$.

Every other firm, including good firms with opacity $\omega > l$, who offers a bond at markets with interest rate $r(m) \in [0, r_L(l)]$, $(r_L(l) < \bar{r}(r_H))$ will be able to issue a fraction $1-\varepsilon(l)$, so that the unfinanced fraction of these bonds at market l is $\varepsilon(l)$. If the issuer is a good firm with $\omega < \omega_3$, the $\varepsilon(\omega)$ fraction can be issued in market $m(\omega)$.

Before moving to the proof, we present the following lemma which we will use in what follows.

Lemma B.2 *In any equilibrium $r_L(\omega)$ is non-decreasing in ω everywhere.*

Proof.

Assume the contrary. Then when $\theta = L$, there exists bonds offered by good firms with opacity ω, ω' with $\omega' < \omega$ such that $r_L(\omega') > r_L(\omega)$.For this to be consistent with firm optimization, it must be that

$$\eta(M_0,\omega',g;L) < \eta(M_0,\omega,g;L) = 1,$$

where the inequality follows from firm optimization, and the equality from definition of $r_L(\omega)$ and M_0, where $M_0 = \{m : \tilde{r}(m) \leq r_L(\omega)\}$. But investor optimization and the signal structure when $\theta = L$ requires that investors only use rules of the form $\chi(\omega,\tau;s) =$

$\mathbb{1}(\tau = g \;\&\; \omega \leq s)$. This implies that for any $M_0 \subset M$,

$$\eta(M_0, \omega', g; L) \geq \eta(M_0, \omega, g; L),$$

a contradiction. ■

Proof. Lemma B.1 expresses the decision of each firm in terms of a reservation interest rate $r^R(\omega, \tau; L)$ when $\theta = L$. Here we show the following statements: all bad firms are identified under equilibrium acceptance rule, so $r^R(\omega, b; L) = 0$. $r^R(\omega, g; L)$ is different for good firms of different ω, unlike $\theta = H$. Finding $r^R(\omega, g; L)$ is equivalent to finding the highest interest rate at which bonds of a good firm with opacity ω trades.

Moreover, unlike $\theta = H$, a firm (ω, g) might be able to sell some bonds at interest rate below $r^R(\omega, g; L)$, so the equilibrium must characterize $r^R(\omega, g; L)$ and any other prices at which bonds of firm (ω, g) are sold.

(i) **Premium schedule $\mathbf{0} \leq \mathbf{r_L}(\omega) \leq \bar{\mathbf{r}}(\mathbf{r_H})$.** Let $r_L(\omega) = r^R(\omega, g; L)$ denote the highest interest rate at which bonds of a good firm with opacity ω trades. $r_L(\omega)$ falls into four possible classes: a "cash-in-the-market" interest rate, a"bunching" interest rate, a"bunching-with-scarcity" interest rate, or a "nonselective" interest rate.

Cash-in-the-market. The cash-in-the-market interest rate $r^C(\omega)$ for the bonds issued by the good firm of opacity ω is determined by equating demand and supply in the corresponding market. The total amount of liquidity demanded by firm $j = (\omega, g)$ at interest rate $r^C(\omega)$ should be equal to total wealth of investor $\hat{s}(\omega) = \omega$ which is the financier in that market.

$$\varepsilon(\omega)\phi\lambda\hat{y}(\omega, g; L, r_H, r^C(\omega)) = w(\omega) \tag{B.22}$$

As long as $r^C(\omega)$ is a strictly increasing function and in the correct range, the equilibrium would be a cash-in-the-market pricing equilibrium. Each good firm of opacity ω demands bonds in all markets where $r(m) \leq r^C(\omega)$, and no market with a higher interest rate, while bad firms demand maximum bonds on every (active) market. Given the prudence shocks, each investor imposes $\chi(\omega, \tau; s) = \mathbb{I}(\tau = g \;\&\; \omega \leq s)$, i.e. she finances bond in the most profitable (highest interest rate) market for which he observes $x(g; \omega, s, 0) = g$. Now consider a market with $r = r^C(\omega)$. Firms with opacity $\omega' \geq \omega$ demand credit in that market, but no firm with opacity $\omega' < \omega$ demand in this market because they have been able to issue all the bonds that they want at lower interest rate. Investor $s = \omega$ is able to recognize good assets in this markets, but investors

$s < \omega$ are not. Moreover, if $r^C(\omega)$ is strictly increasing, this is the highest interest rate where $s = \omega$ can detect good firms, so he will spend his entire wealth financing bonds demanded on this market. Then Equation (B.22) implies all the bonds demanded by firm $j = (\omega, g)$ are financed at this market, and there will be non of them for sale at interest rate higher than $r^C(\omega)$.

Bunching. If $r^C(\omega)$ turns out to be downward sloping in any range, the logic of cash-in-the-market pricing breaks down because it implies the good firm with a lower opacity is paying a higher interest rate to issue bonds, $\omega < \omega'$ and $r^C(\omega) > r^C(\omega')$. The investor who is financing the firm from higher opacity, ω', can also identify the firm from a lower opacity, ω, so he is better off financing the more transparent firm and collect a higher interest rate $r^C(\omega) > r^C(\omega')$, so there will be no financier for the more opaque firm ω'; a contradiction. In this region, there will be "bunching" of all the firms $[\omega, \omega']$ at a single price, i.e. an ironing procedure that restores a weakly monotone function. The clearing algorithm is such that the lower s investor picks the bonds that she finances first in a bunching market.

Since $w(.)$ function is decreasing in s, and $\hat{y}(.)$ function is decreasing in r^C, for low enough ω, and appropriate set of parameters, Equation (B.22) requires $r^C(\omega) < 0$. Let $\hat{\omega} = \max\ \omega$ such that $r^C(\omega) \leq 0$, then as long as $r^C(\omega)$ is increasing, or ironed as explained above, $\forall\ \omega'$ s.t. $\hat{\omega} > \omega' \geq 0$, $r^C(\omega') < 0$. Thus the requirement that there is a zero lower bound on the interest rate (no negative interest rate), implies there is a range of transparencies at the bottom, $\omega \leq \hat{\omega}$, whose good firms face zero interest rate in issuing bonds. Investors with $s \leq \hat{\omega}$ have idle wealth that is not financing any bonds, as there is not enough credit demand from good firms that they can recognize. In order for $\hat{\omega} > 0$ it must be that the least skilled investor has sufficient wealth to cover all the demand of the most transparent good firm, i.e.,

$$w(0) > \phi\lambda\hat{y}(0, g; L), \tag{B.23}$$

where we have used that $r_L(0) = 0$. Later in proof of Proposition 7 we make the appropriate parametric assumption to ensure this condition holds.

Nonselective pricing. Consider a market m with interest rate $r = \tilde{r}(m)$, where good firms with opacity ω submit credit demand in that market. That implies all the good firms with opacity $\omega' > \omega$ also submit demand in market m, as well as all the bad firms with any level of opacity. An investor can choose to impose $\chi_s(\omega, \tau; s) = 1$ in market m and buy a representative sample of the pool.

The terms of trade that he will get is

$$\begin{aligned} \upsilon^N(r) &= \frac{(1+r)\lambda[g \text{ supply at interest rate r in FN}]}{((1-\lambda)\,[b \text{ supply at interest rate r in FN}] + \lambda[g \text{ supply at interest rate r in FN}])} \\ &= \frac{(1+r)\lambda \int_\omega^1 \hat{y}(\omega, g; L)d\omega'}{(1-\lambda)\int_0^1 \hat{y}(\omega, b; L)d\omega' + \lambda \int_\omega^1 \hat{y}(\omega, g; L)d\omega'} \end{aligned}$$

As long as $\omega_1 > 0$, there are (low expertise) international investors who finance bonds issued by good firms with opacity $\omega < \omega_1$. The interest rate for these bonds is zero, so these investors make zero profits and are indifferent between financing and not financing bonds. Alternatively, if they trade nonselectively at a market at interest rate r, they can get the above terms of trade. As a result, if $\upsilon^N(r) > 1$ these investors are better off trading at interest rate r nonselectively, which in turn implies no good bond from opacity ω can be offered at a interest rate above $r^{NS}(\omega)$. With some algebra, one can show that $\upsilon_N(r) \leq 1$ is equivalent to $r \leq r^{NS}(\omega)$ where

$$r^{NS}(\omega) \equiv \frac{(1-\lambda)\int_0^1 \hat{y}(\omega, b; L)d\omega'}{\lambda \int_\omega^1 \hat{y}(\omega', g; L)d\omega'}. \tag{B.24}$$

When this upper bound interest rate is operative, bonds are finances in markets where both selective and nonselective buyers are active. In the markets where the interest rate is $r^{NS}(\omega)$, nonselective buyers will buy just enough assets (distributed pro-rate among the assets offered) such that the interest rate $r^C(\omega)$ is pushed down such that marginal investor $s = \omega$ can charge exactly interest rate $r^{NS}(\omega)$:

$$\varepsilon(\omega) = \frac{w(\omega)}{\phi\lambda\hat{y}(\omega, g; L, r^{NS}(\omega))} \tag{B.25}$$

In other words, if international investors with skill $s = \omega$ are poor, that requires a high interest rate to push the demand of firms (ω, g) down so that Equation (B.25) is satisfied. At this high interest rate, investors financing low ω good firms will enter this market and be nonselective financiers. This takes some bonds off of the market, which in turn implies a lower interest rate.

Bunching-with-scarcity. If there is a maximum interest rate $\bar{r}(r_H)$ that firms are willing to pay to get bonds from investors, and if the wealth of smart investors, in the sense precisely defined below, is in short supply, then there will be a bunching region where some good firms will be rationed.

At any interest rate $r > \bar{r}(r_H)$, good firms have zero demand for bonds, and (with linear objective function) at $r = \bar{r}(r_H)$ they are indifferent between all levels of bond issued. So if the interest rate hits $\bar{r}(r_H)$ in any market, it cannot increase any further than that.

Let $\bar{m}$ denote the market with interest rate $\bar{r}(r_H)$, $\tilde{r}(\bar{m}) = \bar{r}(r_H)$, and let $\bar{\omega}$ denote the lowest opacity level whose good firms demand credit on market $\bar{m}$. Firms $(\bar{\omega}, g)$ submit $\sigma(\bar{m}, \bar{\omega}, g; L) = \hat{y}(\bar{\omega}, g; L)$ on market $\bar{m}$ and by definition their demand is exactly fully satisfied at interest rate $\bar{r}(r_H)$. Good firms with opacity $\omega > \bar{\omega}$ also demand credit on this market. Since these firms are indifferent about how many bonds they raise on market $\bar{m}$ (given the linearity of $t = 0$ objective function), we assume that all of them submit the maximum that they can, $\hat{y}(\bar{\omega}, g; L)$: $\forall\ \omega > \bar{\omega},\ \sigma(\bar{m}, \bar{\omega}, g; L) = \hat{y}(\bar{\omega}, g; L)$; and how many bond they raise is determined by rationing explained next.[20]

Bad firms with any opacity level also demand credit on market $\bar{m}$, but none is able to issue any bonds in this market. Thus the demand submitted on market $\bar{m}$ is given by

$$\sigma(\bar{m}, \omega, \tau; L) = \begin{cases} \hat{y}(\omega, b; L) & \text{if } \tau = b \\ \hat{y}(\bar{\omega}, g; L) & \text{if } \tau = g \text{ and } \omega > \bar{\omega} \\ 0 & \text{otherwise} \end{cases}$$

As such, if

$$(1 - \bar{\omega}) \times \phi\lambda\hat{y}(\bar{\omega}, g; L) > \int_{\bar{\omega}}^{1} w(s)ds,$$

then the wealth of investors who are able to recognize good firms with opacity $\omega \in (\bar{\omega}, 1)$ is collectively in short supply. As such, some of the good firm demand is rationed at maximum interest rate $\bar{r}(r_H)$. We next determine the subset of good firms whose credit demand is fully satisfied at interest rate $\bar{r}(r_H)$, i.e. those who are not rationed. In order to do so, introduce the following function.

[20] This is slightly stronger than what we actually need to simplify the equilibrium derivation. What we need is that when $\theta = L$, on the market where interest rate is $\bar{r}(r_H)$, no good firm submits more than the credit capacity of the lowest-opacity good firm in that market. The latter firm is $j = (\omega_2, g)$, and even absent this assumption, $\hat{y}(\omega, g; L) = \hat{y}(\omega_2, g; L)$ for $\omega_3 \geq \omega > \omega_2$. So what we need is $\hat{y}(\omega, g; L) = \hat{y}(\omega_3, g; L)$ for $\omega > \omega_3$, weaker than what specified here.

$\mathbf{R_D}(\omega', \omega, \mathbf{r}, \varepsilon)$. For $\omega' > \omega$, and interest rate r, let

$$R_D(\omega', \omega, r, \varepsilon) \equiv \varepsilon\phi\lambda \int_{\omega}^{\omega'} \hat{y}(z, g; L, r)dz - \int_{\omega}^{\omega'} w(s)ds. \tag{B.26}$$

where x is a parameter.

$R_D(\omega', \omega, r, \varepsilon)$ measures the excess residual bonds offered, ε, by good firms with opacity in the interval (ω, ω'), at interest rate r, which is not met by the cumulative wealth of the investors who are able to identify some good firm in this interval but no good firms with opacity $\omega'' > \omega'$, i.e. $\omega \leq s \leq \omega'$.

For $\omega = \bar{\omega}$ and $\varepsilon(\bar{\omega}) = 1$, we have

$$R_D(\omega', \bar{\omega}, \bar{r}(r_H), 1) = \phi\lambda(\omega' - \bar{\omega})\hat{y}(\bar{\omega}, g; L, \bar{r}(r_H)) - \int_{\bar{\omega}}^{\omega'} w(s)ds.$$

Recall that in markets where there is bunching, the clearing algorithm used lets lower-s investors, who impose more restrictive acceptance rules, trade before higher-s investors.

Moreover, note that $R_D(\omega', \bar{\omega}, \bar{r}(r_H), 1) > 0$, $\forall\, \omega' > \bar{\omega}$. The reason is the following. By the logic of cash-in-the-market pricing, $\bar{r}(r_H)$ is the interest rate at which demand of good firms of opacity $\bar{\omega}$ is exactly absorbed by wealth of the marginal investor $s = \bar{\omega}$. Consider a good firm with opacity ω' right above $\bar{\omega}$. Let $\tilde{r}'$ denote the hypothetical interest rate which clears the market for such good firm $\omega' > \bar{\omega}$, if this firm was still in a cash-in-the-market pricing. Again, using the logic of cash-in-the-market pricing, and the downward sloping skill distribution of investors, it must be that $\tilde{r}' > \bar{r}(r_H)$ as $\omega' > \bar{\omega}$. However, since $\bar{r}(r_H)$ is the maximum interest rate any good firm accept, good firm $\omega' > \bar{\omega}$ faces a lower interest rate compared to what would clear his demand using only the wealth of his marginal investors, $s = \omega'$. Let $\tilde{\omega} \in (\bar{\omega}, 1)$ be the highest opacity where the demand of good firms is fully absorbed by all the investors active in market $\bar{m}$.

$$R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1) = \int_{\tilde{\omega}}^{1} \frac{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1)}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1) + (s - \tilde{\omega})\, \phi\lambda\hat{y}(\bar{\omega}, g; L, \bar{r}(r_H))} w(s)ds$$

which implies $\tilde{\omega}$ is the solution to

$$1 = \int_{\tilde{\omega}}^{1} \frac{1}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1) + (s - \tilde{\omega})\, \phi\lambda\hat{y}(\bar{\omega}, g; L, \bar{r}(r_H))} w(s)ds \tag{B.27}$$

In Proposition 7 we argue that under our assumptions, $\tilde{\omega} < 1$.

For a good firm from any opacity $\omega > \tilde{\omega}$, none of his offered bonds can be bought by investors of expertise $s < \tilde{\omega}$, since those investors cannot identify him as good. Thus he can only sell what can be absorbed by the residual wealth of the subset of investors $s > \tilde{\omega}$ who can identify him, $s > \omega > \tilde{\omega}$.

For $s > \tilde{\omega}$, let

$$\zeta(s) = \frac{(s - \tilde{\omega})\,\phi\lambda\hat{y}(\bar{\omega}, g; L, \bar{r}(r_H))}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1) + (s - \tilde{\omega})\,\phi\lambda\hat{y}(\bar{\omega}, g; L, \bar{r}(r_H))}$$

$\zeta(s)$ captures how much of the portfolio held by investor $s > \tilde{\omega}$ are bonds issued "collectively" by good firms with opacity $\omega > \tilde{\omega}$ that s can identify. The measure of those good firms is $(s - \tilde{\omega})\,\phi\lambda$. Thus for an individual firm of opacity $\omega > \tilde{\omega}$, aggregating over holdings of his bonds, by all the investors $s > \omega$, we find how much $j = (\omega, g)$ can issue.

Let $\eta_L(\omega) = \eta(\bar{m}, \omega, g; L)$ denote the rationing function in this market. The above argument implies

$$\begin{aligned}
\eta(\bar{m}, \omega, g; L) = \eta_L(\omega) &= \frac{1}{\hat{y}(\bar{\omega}, g; L; \bar{r}(r_H))} \int_{\omega}^{1} \frac{1}{(s - \tilde{\omega})\,\phi\lambda} \zeta(s) w(s) ds \\
&= \int_{\omega}^{1} \frac{1}{R_D(\tilde{\omega}, \bar{\omega}, \bar{r}(r_H), 1) + (s - \tilde{\omega})\,\phi\lambda\hat{y}(\bar{\omega}, g; L; \bar{r}(r_H))} w(s) ds
\end{aligned}$$

This is the rationing function stated in equation (B.21) in part (v) of the equilibrium, for $\bar{\omega} = \omega_2$ and $\tilde{\omega} = \omega_3$. For good firms with opacity $\tilde{\omega} \geq \omega > \bar{\omega}$, $\eta_L(\omega) = 1$.

Interest Rate Regimes. Next, we determine when $r_L(\omega)$ falls into each of the four possible classes of "cash-in-the-market", "bunching", "bunching-with-scarcity", or "nonselective" interest rate." In order to do so, introduce the following function.

$\mathbf{E}(\omega, \mathbf{r}, \varepsilon)$. Define

$$E(\omega, r, \varepsilon) \equiv \max_{\omega' \in [\omega, 1]} \int_{\omega}^{\omega'} w(s) ds - \varepsilon\phi\left((1 - \lambda) \int_{\omega}^{\omega'} \hat{y}(z, b; L; r) dz + \lambda \int_{\omega}^{\omega'} \hat{y}(z, g; L; r) dz \right)$$

For a bond issued by good firm of opacity ω, interest rate r and remaining firm demand for bonds issuance ε, $E(\omega, r, \varepsilon)$ measures the maximum over $\omega' > \omega$ of the difference between the endowment of all investors with skill $s \in [\omega, \omega']$, and how much is needed to finance ε units of all the bonds in $[\omega, \omega']$ which firms offer, if they all face interest rate r. A bond interest rate can only be determined by cash-in-the-market if $E(\omega, r^C(\omega), \varepsilon(\omega)) = 0$. A strictly positive value would mean that there exists a range of investors $s \in [\omega, \omega']$ for some $\omega' > \omega$, all of whom can identify some bond in the range $[\omega, \omega']$ as a good bond (but not any bonds offered by firms with opacity higher than ω') and whose collective endowment exceeds what is necessary to finance all the bonds demanded by firms in $[\omega, \omega']$ facing a interest rate $r^C(\omega)$. Since these investors will want to spend their entire endowment financing bonds, it must be that some bond in the range $[\omega, \omega']$ must face a interest rate lower than $r^C(\omega)$. This is because firms' credit demand is downward sloping, hence a lower interest rate would push the firm credit demand up and bring firm credit demand closer to investor credit supply. But then monotonicity implies that the interest rate faced by a good firm with opacity ω must be lower than $r^C(\omega)$, a contradiction.

Next, suppose one knows that $\breve{\omega}$ is the lower limit of one type of region. In a similar manner to (Kurlat, 2016), the following procedure finds the higher end of that region, the type of region immediately above and the prices within the region.

1. For a cash-in-the-market region, the higher end is

$$\inf\{\omega > \breve{\omega} : r^{NS}(\omega) < r^C(\omega) \text{ or } E(\omega, r^C(\omega), \varepsilon(\omega); r_H) > 0 \text{ or } r^C(\omega) > \bar{r}(r_H)\} \tag{B.28}$$

   and the region to the right is a nonselective region (first condition) or a bunching region (second condition), and bunching-with-scarcity (third condition), respectively. Within the region, $r_L(\omega) = r^C(\omega)$ and $\varepsilon(\omega) = \varepsilon(\breve{\omega})$.

2. For a bunching region, the higher end is

$$\min\{\omega > \breve{\omega} : E(\omega, r^C(\bar{\omega}), \varepsilon(\bar{\omega}); r_H) = 0\} \tag{B.29}$$

   and the region to the right is a cash-in-the-market region. Within the region, $r_L(\omega) = r_L(\breve{\omega})$ and $r(\omega) = r(\breve{\omega})$.

3. For a nonselective region, the higher end is

$$\inf\left\{\omega > \breve{\omega} : \frac{w(\omega)}{\phi\lambda\hat{y}(\omega, g; L, r^{NS}(\omega))} > r(\omega') \text{ for some } \omega' \in (\breve{\omega}, \omega) \right.$$
$$\left. \text{or } E(\omega, r^C(\breve{\omega}), \varepsilon(\breve{\omega}); r_H) > 0 \text{ or } r^{NS}(\omega) \geq \bar{r}(r_H) \right\} \quad \text{(B.30)}$$

and the region to the right is a cash-in-the-market region (former condition), bunching region (second condition), and bunching-with-scarcity (third condition), respectively. Within the region, $r_L(\omega) = r^{NS}(\omega)$ and $\varepsilon(\omega) = \frac{w(\hat{s}(\omega))}{\phi\lambda\hat{y}(\omega,g;L,r^{NS}(\omega))}$.

4. For a bunching-with-scarcity-region, the higher end is 1. Within the region, $r_L(\omega) = \bar{r}(r_H)$ and $\varepsilon(\omega) = \varepsilon(\breve{\omega})$.

We have assume that the aggregate wealth of investors is sufficient to finance all good bonds. Thus the first region to the left is a bunching region with $\breve{\omega} = 0$, $r_L(0) = 0$, and $\varepsilon(0) = \phi\lambda\bar{\sigma}_L$. This region ends at $\omega = \omega_1$, and the region to the right is always a cash-in-the-market region. There after, if either one of the sets defined by (B.28), (B.29), (B.30) is empty, that region extends up. Since there is a maximum interest rate $\bar{r}(r_L)$ that investors are willing to accept, $r_L(\omega) \leq \bar{r}(r_L)$, firms from very opaque countries all bunch at the same interest rate. Furthermore, as we assume that wealth of skilled investors is in short supply, this is a bunching-with-scarcity region. Thus at $\omega = \omega_2$ bunching-with-scarcity region starts, with $r_L(\omega_2) = \bar{r}(r_H)$, and extends all the way to 1. Note that $\omega_2 = \bar{\omega}$ in the proof above. $\omega_3 \in (\omega_2, 1)$ is the degree of opacity at which scarcity starts, i.e. good firms that are more opaque than this level are rationed even at the maximum interest rate. Note that $\omega_3 = \tilde{\omega}$ in the proof above.

(ii) **Firm optimization.** Bond issuance decisions follow the reservation interest rate strategy. A good firm (ω, g) raises total liquidity equal to all the bonds they are able to sell on all $M([0, \omega])$ markets. A bad firm (ω, b) tries to sell in all markets $M([0, 1])$. Since in equilibrium all bad assets sell at the same ratio, $\eta(m, \omega, b; L) = \eta(m, 0, b; L)$, $\forall\, m, \forall\, \omega \in [0, 1]$.

Let ω_2 denote the lowest opacity firm in the bunching-with-scarcity region, i.e. the lowest ω firm who face the interest rate $\bar{r}(r_H)$

$$w(\omega_2) = \phi\lambda\hat{y}(\omega_2, g; L),$$

and let ω_3 denote the index of the lowest ω opacity whose good firms do face rationing in the bunching-with-scarcity region, defined as the solution to Equation (B.27).

For any good firm with opacity $\omega < \omega_2$, since $r(\omega) > 0$, the rationing function (B.20) implies that in order to issue all of the firm bonds, the reservation interest rate should be $r_L(\omega)$. Good firm with opacity $\omega > \omega_2$ are indifferent between raising any number of bonds, so the issuance decision is optimal. For any bad firm, the rationing function implies that reservation interest rate is $\bar{r}(r_H)$. Therefor credit issuance decisions are optimal for all firms and the total number of bonds they issue follows directly.

(iii) **International investor optimization.** For $s \in [s_N, 1]$, each investor chooses the highest interest rate market on which there is an opacity level ω such that $x(g; \omega, s, 0) = g$ and $S(m, \omega, g) > 0$. Since $\tilde{r}(m) \geq 0$, this is optimal. For $s \in [s_0, s_N)$, investors only place weight on markets where nonselective pricing prevails. Equation (B.24) implies they are indifferent between financing bonds and staying out, since the highest interest rate market where there is a ω such that $x(g; \omega, s, 0) = g$ and $S(m, \omega, g) > 0$ has $\tilde{r}(m) = 0$, there is no other market in which they would strictly prefer to finance. For $s < s_0$, the same logic implies that no financing is optimal.

(iv) **Allocation function.** For any market $m(\omega)$ where ω falls in either a cash-in-the-market or a nonselective range, the NMR algorithm implies that all the investors face a residual supply proportional to the original supply, so Equation (B.16) follows from Appendix B of Kurlat (2016), Equation (65).

For markets $m(\omega)$ where ω falls in a bunching or bunching-with-scarcity region as described above and χ is of the form $\chi(\omega, \tau; s) = \mathbb{I}(\tau = g \ \& \ \omega < s)$, then the differential Equations (B.18) follows from Appendix B of Kurlat (2016), Equation (66), along with Equation (D.19). Then (B.17) follows from applying the NMR algorithm.

(v) **Rationing function.** Follows from applying Equation (67) in Appendix B of Kurlat (2016).

■

C Simple Global Equilibrium. Construction of Equilibrium in Real Investment (t = 0)

In this appendix we restrict attention to a simple global equilibria in the financial market at $t = 1$. A simple global equilibrium is a global equilibrium where there is no nonselective pricing region.

We first connect $\hat{y}(\omega, \tau; \theta, r_H, r_L)$ to date zero variables, and then show that it satisfies the required conditions in equilibrium. At $t = 0$, investors are not active. Firms anticipate the date $t = 1$ continuation value and choose the initial and maintained investment levels, $I(\omega, \tau), \{i(\omega, \tau; \theta)\}_\theta$, to maximize their expected utility as defined in program (12).

We start by constructing $\hat{y}(\omega, \tau; \theta)$, i.e. the maximum liquidity that a firm can raise on the international markets. Maintaining $i(\omega, \tau; \theta)$ units allows a good firm to issue up to $\ell(\omega, \tau; \theta) = \frac{1}{1+r(\omega,\tau;\theta)} \xi i(\omega, \tau; \theta)$ bonds, with unit face value each, without violating the pledgeability constraint.

Bad firms value each unit of continued investment more than good firms since investors cannot seize anything from their output. Moreover, (1) they do not need any liquidity if $\theta = L$, since they cannot continue if hit by a liquidity shock, and (2) they face the same financing condition as good firms if $\theta = H$ but can only partially continue. It follows that bad firms save less liquidity, and they have enough collateral (initial scale) to issue up to $\bar{\sigma}_\theta$. See section "Firm problem given the optimal choice of issuance" in C.2 for more detail.

Putting this together we have[21]

$$\hat{y}(\omega, \tau; \theta) = \begin{cases} \ell(\omega, \tau; \theta) & \tau = g;\ \theta = H \text{ or } (\theta = L \text{ and } \omega \leq \omega_2) \\ \ell(\omega_2, \tau; \theta) & \tau = g;\ \theta = L \text{ and } \omega > \omega_2 \\ \bar{\sigma}_\theta & \tau = b;\ \forall\, \theta, \forall\, \omega \end{cases} \tag{C.1}$$

Remark. Recall that in Section B we assumed $\hat{y}(\omega, \tau; \theta, r_H, r_L)$ is decreasing in the (common) interest rate when $\theta = H$, and in the firm specific r_L. With the above mapping, we need to verify that the equilibrium $\ell(\omega, \tau; \theta)$ is in fact downward sloping in r_H, r_L, which we will do in this section.

We impose the following parameter restrictions going forward, to focus on a simple global equilibrium. They are also a sufficient condition for existence of a simple global equilibrium, as shown in the proof of Proposition 7.

Assumption C.1

(i) $\xi \geq \frac{1}{1-\phi}$

(ii) $\frac{1-\lambda}{\lambda} \leq \frac{(\rho_g-\xi)}{\rho_g\xi(1-\phi)+(\rho_g-\xi)(\phi\pi_L\xi-1)}$

(iii) $w(0) \geq \phi\lambda\xi$ *and* $\lim_{s\to 1} w(s) = 0$.

(iv) $\min\left\{\frac{(\rho_g-\xi)(1+(1-\lambda)\phi\xi\pi_H)}{(\rho_g(1-\phi)+\phi(\rho_g-\xi)\pi_H)\xi}, \frac{\xi\phi\lambda-w(\omega)}{\xi\phi(\lambda+w(\omega)\pi_L)}\right\} \leq \frac{1-\lambda}{1-\lambda\omega} \qquad \forall\omega$

[21] We will show that when $\theta = L$, good firms with opacity $\omega > \omega_2$ are indifferent in the scale at which they continue. Thus the above $\hat{y}$ is an equilibrium. We pick this tie-breaking rule because it simplifies the exposition. For more detail see Section B.2, Bunching-with-scarcity.

Condition (i) ensures that when there is liquidity risk, $\phi > 0$, without access to credit markets firms prefer to invest all of their initial endowment and do not use any part of it to manage liquidity risk. It also implies that without access to credit markets firms do want to invest (rather than consume right away), which requires $\rho_\tau > \frac{1}{1-\phi}$ and follows since $\forall\ \tau,\ \rho_\tau > \xi$. Condition (ii) ensures that the common interest rate is not prohibitively high when $\theta = H$, so that firms use international markets and part of their own endowment to manage liquidity risk, as opposed to investing all of their initial endowment. Condition (iii) ensures two properties of the investor skill distribution function. First, low-expertise investors have sufficient wealth so that some bonds are issued at zero interest rate. Second, expert capital is in short supply. Condition (iv) ensures that when investors are cautious, there is no equilibrium interest rate for which some investors are willing to buy up all the offered securities independent of their signal, given the investor skill distribution function $w(s)$ in the next section.

Proof of Proposition 7 provides a formal description of what each condition guarantees.

C.1 No Nonselective Region. Investor Skill Distribution

Consider $r^C(\omega)$ and $r^{NS}(\omega)$ defined in Equation (B.22) and (B.24), respectively. First, note that we have assumed the investor skill distribution function is monotonically decreasing, $w'(s) < 0$, so $r^C(\omega)$ does not become non-monotone. As such, bunching region can only emerge below some threshold, $0 \leq \omega < \underline{\omega}$, and bunching-with-scarcity only above some threshold, $\bar{\omega} < \omega \leq 1$.

In what follows, we derive a parametric assumption to ensure that nonselective region does not emerge. Nonselective interest rate schedule is an upper bound on the prevailing interest rate in each market. Thus a sufficient condition for this upper bound to never be active, i.e. for the nonselective pricing region not to emerge, is to have $r^C(\omega) \leq r^{NS}(\omega)$ for markets where $0 < \tilde{r}(m) < \bar{r}(r_H)$.

$$r^C(\omega) = \ell^{-1}\left(\frac{w(\omega)}{\phi\lambda}\right) \leq \frac{(1-\lambda)\,\bar{\sigma}_L}{\lambda\int_\omega^1 \ell(\omega, g; L, r_H, r^C(\omega))d\omega'},$$

where $\ell^{-1}(.)$ denotes the inverse of function $\ell(\omega, g; L; \{r_H, r^C(\omega)\})$ with respect to $r^C(\omega)$, and $\{r_H, r^C(\omega)\}$ indicates the dependence of demand function on (H, L) interest rate explicitly.[22] Note that $\hat{y}(\omega', g; L) = \bar{\sigma}_L$ $(\forall\ \omega')$ minimizes the right hand side on the above equation, which

[22] We have also used that $\hat{y}(\omega, g; L; \{r_H, r^C(\omega)\}) = \ell(\omega, g; L; \{r_H, r^C(\omega)\})$ for $r^C(\omega) < \bar{r}(r_H)$, and that no nonselective region in equilibrium implies $\varepsilon(\omega) = 1\ \forall\omega$.

yields the following sufficient condition

$$r^C(\omega) = \ell^{-1}\left(\frac{w(\omega)}{\phi\lambda}\right) \leq \frac{(1-\lambda)}{\lambda(1-\omega)}. \tag{C.2}$$

This implies that under Assumption C.1, there is no nonselective region when $\theta = L$, $\omega_1 > 0$ and $\omega_3 < 1$. The equilibrium pricing regions are thus characterized by three thresholds $\omega_1 < \omega_2 < \omega_3$ such that starting from the most transparent firms, $\omega = 0$:

(i) Good firms with opacity $0 \leq \omega \leq \omega_1$ are in bunching region and face zero interest rate.

(ii) Good firms with opacity $\omega_1 < \omega \leq \omega_2$ are in cash-in-the-market pricing region.

(iii) Good firms with opacity $\omega_2 < \omega \leq \omega_3$ are in bunching-with-scarcity market $\bar{m}$ at interest rate $\bar{r}(r_H)$, defined in (C.4), and $\eta(\bar{m}, \omega, g; L) = 1$.

(iv) Good firms with opacity $\omega_3 < \omega \leq 1$ are in bunching-with-scarcity market $\bar{m}$ at interest rate $\bar{r}(r_H)$, defined in (C.4), and $\eta(\bar{m}, \omega, g; L) < 1$.

(v) No bad firm issues any bonds in any market.

In this equilibrium

$$y(\omega, \tau; L) = \begin{cases} \hat{y}(\omega, \tau; L) & \text{if } \tau = g \text{ and } \omega \leq \omega_3 \\ \eta(m(\bar{r}), \omega, \tau; L)\hat{y}(\omega, \tau; L) = \eta_L(\omega)\hat{y}(\omega, \tau; L) & \text{if } \tau = g \text{ and } \omega > \omega_3 \\ 0 & \text{if } \tau = b \end{cases} \tag{C.3}$$

C.2 Firm Optimal Decision

Consider the firm problem (12). Each firm j takes his optimal behavior at $t = 1$ as given, which along with $t = 1$ prices in different prudence shocks, the allocation function and the rationing function fully describes firm j continuation payoff. Firm j then chooses his business plan to maximize his expected utility given this continuation payoff.

Derivation of firm optimal choice of bond issuance, Equations (2) **and** (13)**.** A firm hit by liquidity shock has three possible options, at $t = 0$, in how to manage a liquidity shock in each aggregate state at $t = 1$. First, the firm can choose not to insure against the liquidity risk and abandon investment if a liquidity shock happen. This would lead to the highest scale of operation, $I(\omega, \tau)$. Second, the firm can choose to save enough out of his own endowment, through the banker, such that he has sufficient liquidity at $t = 1$ and does not need to raise any extra financing on the international markets. This option leads to the lowest scale of operation. Third, the firm can choose to save a lower amount from his initial endowment and borrow the rest from international investors. This leads to an intermediate level of scale of operation.

From the linearity of the firm problem, the firm chooses the same option for all units of investment. Moreover, Assumption C.1.(i) implies the first option dominates the second. Assumption C.1.(iii) implies that borrowing on the international markets is sufficiently cheap that the third option dominates the first one, which in turn leads to firm's optimal liquidity choice, Equation (2).

Alternatively, a good firm who is not hit by a liquidity shock is indifferent between issuing bonds or not if $r(\omega, \tau; \theta) = 0$, and otherwise prefers not to issue. Thus these firms do not participate in the international markets. It follows that, if a bad firm not hit by a liquidity shock tries to issue bonds, his type is revealed and he does not succeed in raising funding, and he will not participate either. As such, only firms hit by liquidity shock attempt to raise funding from international investors at $t = 1$, which in turn implies the exante budget constraint (13).

Firm problem given the optimal choice of issuance. Since problem (12) is linear, Equations (4)-(11) determine the optimal firm choices, $i(\omega, \tau; \theta)\ \forall\ \theta$ whenever they are non-zero. Plugging these solutions into (13) determines $I(\omega, \tau)$.

The rest of the argument follows from a parallel logic to (Holmström and Tirole, 1998, 2011).

Good firms. Consider a good firm $j = (\omega, g)$. First, conjecture that good firms continue at full scale in high state, $i(\omega, g; H) = I(\omega, g)$. Next, let $0 \leq x \leq 1$ denote the fraction of the initial scale that firm $j = (\omega, g)$ chooses to continue when $\theta = L$. Formally $x \equiv \frac{i(\omega,g;L)}{I(\omega,g)}$.

Use the $t = 2$ interest rate along with Equation (13) to get $I(\omega, \tau)$. Substitute $I(\omega, \tau)$ and x in the objective function (12). The objective function of the good firm then boils down to

$$\Pi(x) = \frac{\phi(\rho_g - \xi)(\pi_H + \pi_L x) + (1 - \phi)\rho_g}{1 + \phi\xi(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r_L(\omega)}{1+r_L(\omega)} x)} - 1.$$

The optimal investment is the continuation scale x such that $\Pi'(x) = 0$, where

$$\Pi'(x) = \frac{\pi_L \phi \Big(\rho_g - \xi - \pi_H \phi \xi^2 \big(\frac{r_H}{1+r_H} - \frac{r_L(\omega)}{1+r_L(\omega)}\big) - \rho_g \xi \big(\frac{r_L(\omega)}{1+r_L(\omega)} (1 - \pi_L \phi) - \pi_H \frac{r_H}{1+r_H} \phi\big)\Big)}{\Big(1 + \phi \xi \big(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r_L(\omega)}{1+r_L(\omega)} x\big)\Big)^2}.$$

The numerator of $\Pi'(x)$ is independent of x. As such, if the numerator is strictly positive (negative), the firm chooses $x^* = 1(x^* = 0)$. If the numerator is zero, good firm j is indifferent between any level of continuation when he receives a liquidity shock in $\theta = L$.

This implies that iff

$$r_L(\omega) < \bar{r}(r_H) \equiv \frac{(\rho_g - \xi)\left(1 + \phi \xi \pi \frac{r_H}{1+r_H}\right)}{\rho_g (1 - \phi) \xi + (\rho_g - \xi)\left(\frac{\phi \pi \xi}{1+r_H} - 1\right)}, \tag{C.4}$$

a good firm $j = (\omega, g)$ hit by a liquidity shock continues at full scale when $\theta = L$. If $r_L(\omega) > \bar{r}(r_H)$ he does not maintain any investment if he receives a liquidity shock. If $r_L(\omega) = \bar{r}(r_H)$ the good firm might be rationed when issuing bonds on the international markets and continue at lower scale.

Next, we need to make sure that our conjecture for continuation at full scale in high state, $i(\omega, g; H) = I(\omega, g)$, is correct. For this conjecture to hold, it must be that $r_H < \bar{r}_H$ such that every good firm j prefers to submit liquidity demand to international markets when $\theta = H$. Using Assumption C.1.(i), the alternative is to set $i(\omega, \tau; H) = 0$, do not do any liquidity risk management and abandon production if hit by a liquidity shock in state $\theta = H$, and instead increase $I(\omega, \tau)$. Since firms with opacity $\omega = 0$ are those who face zero interest rate in $\theta = L$ such deviation is most profitable for them. Thus it is sufficient to ensure that they do not want to deviate. Thus $\bar{r}_H$ solves

$$\rho_g(1 - \phi) + (\rho_\tau - \xi)\phi \pi_L = \frac{\rho_g(1 - \phi) + (\rho_g - \xi)\phi}{1 + \phi \pi_H \xi \frac{\bar{r}_H}{1+\bar{r}_H}}.$$

It follows that if

$$r_H < \bar{r}_H \equiv \frac{(\rho_g - \xi)}{\rho_g \xi (1 - \phi) + (\rho_g - \xi)(\phi \pi_L \xi - 1)}, \tag{C.5}$$

all good firms prefer to do liquidity management using a combination of own saving and international markets.

Finally, consider the most transparent good firm, $j_{0,g} = (0, g)$. When $\theta = L$, this firm faces zero interest rate and thus does not need to hold any precautionary liquidity against this state. When $\theta = H$, every good firm, including $j_{0,g}$, prefers to do liquidity management against the liquidity shock and save $\pi_H \phi \frac{r_H}{1+r_H}$ per unit of scale, as long as $r_H < \bar{r}_H$. Putting the two states together, $j_{0,g}$ faces the lowest possible interest rate in both states of the world, and has the highest investment level among all good firms, $I(0,g)$. As explained at the end of this section, we have chosen $\bar{\sigma}_\theta \equiv \xi \frac{I(0,g)}{1+r(0,g;\theta)}$.

Bad firms. Consider any bad firm. Assumption C.1.(i) implies that firms either do liquidity management using international markets, or do not do any liquidity management. When $\theta = L$ a bad firms hit by a liquidity shock is not able to raise any international financing, so he has to fully liquidate his initial investment. Thus bad firms do not save any liquidity against $\theta = L$ aggregate state. Next, consider the most opaque bad firm, $j_{1,b} = (1,b)$. When $\theta = H$, $\eta_H(1) = 1$, thus $j_{1,b}$ is not rationed, and is treated as a good firm. Thus he needs to save $\pi_H \phi \frac{r_H}{1+r_H}$, per unit of scale, to be able to continue at full scale. It follows that $j_{1,b}$ saves the same amount of liquidity as $j_{0,g}$, and thus chooses the same level of investment to maintain.

Every other bad firm, $\omega < 1$ is rationed when $\theta = H$, thus they hold lower liquidity, compared to $j_{1,b}$, against this state of the world. This in turn implies they choose a larger scale of operation: $I(\omega,b) > I(1,b)$, $\forall\, \omega < 1$ by (13). Furthermore, bad firms face the same interest rate r_H as good firms when $\theta = H$, and moreover they do not pay back, so if good firms participate in the international markets when $\theta = H$, it is optimal for bad firms to do so as well. Because these firms are rationed when $\theta = H$ they choose maximal credit demand $\bar{\sigma}_\theta$. It follows that $\hat{y}(\omega, b; \theta, r_H, r_L)$ as defined in Equation (C.1) is optimal.

Firm investment at $\mathbf{t = 0}$. Next we characterize the scale of operation of the firm at $t = 0$. For $\tau = g$ firms, substitute the optimal continuation decisions (16)-(17), as well as the interest rates into Equation (13) to get the optimal investment decision

$$I(\omega,g) = \begin{cases} \frac{1}{1+\phi\xi\left(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r_L(\omega)}{1+r_L(\omega)}\right)} & \text{if } \omega < \omega_2 \\ \frac{1}{1+\phi\xi\left(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{\bar{r}(r_H)}{1+\bar{r}(r_H)}\right)} & \text{if } \omega_2 \leq \omega < \omega_3 \\ \frac{1+(1-\eta_L(\omega))\frac{\phi\xi\pi_L \frac{\bar{r}(r_H)}{1+\bar{r}(r_H)}}{1+\phi\xi\pi_H \frac{r_H}{1+r_H}}}{1+\phi\xi\left(\pi_H \frac{r_H}{1+r_H} + \pi_L \frac{r_L(\omega)}{1+r_L(\omega)}\right)} & \text{if } \omega \geq \omega_3 \end{cases} \tag{C.6}$$

where $\eta_L(\omega) = \eta\left(m(\bar{r}(r_H)), \omega, g; L\right)$, ω_1 is defined by (D.28).

Alternatively, for $\tau = b$ firms,

$$I(\omega,b) = 1 - r_H \xi \phi \pi_H \eta_H(\omega) \bar{\sigma}_H \tag{C.7}$$

where $\eta_H(\omega) = \eta(m_H, \omega, b; H)$.

Next we verify that for good firms who do payback the international investors, the liquidity a firm raises at $t = 1$ on the international market, $y(\omega, \tau; \theta)$ in problem (A.1), is equal to its liquidity need, $\ell(\omega, \tau; \theta)$ associated with optimal investment decision (C.6). This is immediate from comparing Equations (B.14), (C.3) and (C.1).

Consider $D(\cdot)$ and $\bar{D}(\cdot)$, the individual and aggregate expenditure on maintenance, as defined by (D.18) and (D.19) in Section D, respectively. It follows that firm j's realized issuance of bonds on the international market, $\ell(\omega, \tau; \theta)$, is given by:

(i) Good firm $j = (\tau, g)$

$$\ell\left(\omega, g; \theta\right) = \frac{\xi I(\omega, \tau)}{1 + r(\omega, g; \theta)} \eta\left(m_{j,\theta}, \omega, g; \theta\right) \tag{C.8}$$

where $\eta\left(m, \omega, g; \theta\right)$ is given by

$$\eta\left(m, \omega, g; \theta\right) = \begin{cases} \int_\omega^1 \frac{1+\bar{r}(r_H)}{\phi\lambda D(\frac{\bar{r}(r_H)}{1+\bar{r}(r_H)}; \frac{r_H}{1+r_H})(s-\omega_2) - \int_{\omega_2}^{\omega_3} w(s)ds} w(s)ds, & \tilde{r}(m) = \bar{r}(r_H) \ \& \ \omega > \omega_3 \ \& \ \theta = L \\ 1 & \left(\tilde{r}(m) < \bar{r}(r_H) \text{ or } \left(\tilde{r}(m) = \bar{r}(r_H) \ \& \ \omega \leq \omega_3\right)\right) \\ & \& \ \theta = L \\ & \text{or} \\ & \tilde{r}(m) = r_H \ \& \ \theta = H \\ 0 & \text{otherwise} \end{cases} \tag{C.9}$$

In each aggregate state, each good firm $j = (\omega, g)$ issues bonds in a single market, $m_{j,\theta}$. $m_{j,\theta}$ is given by $r(m) = r_H$ if $\theta = H$, and is defined in Section C.1 if $\theta = L$.

(ii) Bad firm $j = (\tau, b)$

$$\ell\left(\omega, \tau; \theta\right) = \eta(m_{j,\theta}, \omega, \tau; \theta)\bar{\sigma}_\theta \tag{C.10}$$

where $\eta\left(m, \omega, b; \theta\right)$ is given by

$$\eta(m, \omega, b; \theta) = \begin{cases} \int_{s_H}^\omega \frac{1+r_H}{(1-\lambda)(1-s)D(0; \frac{r_H}{1+r_H}) + \lambda\bar{D}(\frac{r_H}{1+r_H})} \frac{w(s)}{\phi} ds & \tilde{r}(m) = r_H \ \& \ \omega \geq s_H \ \& \ \theta = H \\ 0 & \text{otherwise} \end{cases} \tag{C.11}$$

Bad firms $j = (\omega, b)$ issue bonds on a single market when $\theta = H$ and do not issue any bonds if $\theta = L$. Thus $m_{j,\theta}$ is given by $r(m) = r_H$ if $\theta = H$, and $m_{j,\theta} = \emptyset$ if $\theta = L$.

To complete the proof we need to verify that there is a fixed point to the joint $t = 0, 1$ problem, i.e. date $t = 0$ optimal outcomes do constitute an equilibrium in the international markets at $t = 1$. We do this in Proposition 7.

D Proofs

Proof of Propositions 1, 2, and Lemma 1 . We prove the results in Section 3 backward. However, in the main text the results are stated forward. As such, we will present the 3 proofs jointly and point to the corresponding result accordingly. Furthermore, the credit market is as described in Appendix A. Furthermore, with the same argument as in Appendix B, a bold investor extends loans to all firms who do not produce conclusive evidence of being bad, while a cautious investor only extends loans to firms who produce conclusive evidence for being good.

We proceed in two steps. First we conjecture investors' choice of test (information choice) and acceptance rule in each state, describe the equilibrium for the conjectured test and acceptance rule, and prove that it is an equilibrium. We then step back and prove that investors' optimal choice of test is consistent with the conjectured choice and the proposed equilibrium in each state.

High Aggregate Shock ($\theta = \lambda_{\mathbf{H}}$.) We conjecture that all investors are bold in this state and accept applications with $x(\omega, \tau) = g$ only. In equilibrium, a fraction of unskilled and all investors with $s > s_0$ advertise r_H. All good firms demand $\bar{\sigma}_H \equiv \frac{1}{r_H}$ credit at all interest rates not higher than r_H and all bad firms demand $\bar{\sigma}_H$ credit at all interest rates. Good firms' demand is fulfilled at r_H while bad firms are allocated

$$\ell(\omega, b; \lambda_H) = \begin{cases} 0 & \text{if } \omega \in [0, s_0] \\ \frac{1}{r_H} - \frac{w(s_1)}{\lambda_H + (1-\lambda_H)(1-s_1)} - \frac{1}{\lambda_H} w(1) & \text{if } \omega \in [s_0, s_1] \\ \frac{1}{r_H} - \frac{1}{\lambda_H} w(1) & \text{if } \omega \in [s_1, 1] \end{cases} \tag{D.1}$$

at r_H. r_H is defined using the condition that unskilled investors break even. It is the solution to

$$\frac{\lambda_H}{\lambda_H + (1 - \lambda_H)(1 - s_0)} (1 + r_H) - \kappa \left(\frac{1}{\lambda_H + (1 - \lambda_H)(1 - s_0)} \right) - 1 = 0, \tag{D.2}$$

i.e. the zero profit condition of an unskilled bold investor when all firms submit the same credit demand at r_H, and Assumption A.9 implies that she is the first investor who samples the pool. $\lambda_H + (1-\lambda_H)(1-s_0)$ is the fraction of applications which do not provide conclusive evidence that they are bad in the bold test, when used by an investor with skill s_0, and the investor lends to them. Therefore, the investor has to test $\frac{1}{\lambda_H+(1-\lambda_H)(1-s_0)}$ applications to be able to lend out her 1 unit. Out of these projects, λ_H fraction are actually good and pay back, generating the total revenue from lending. The second term is the cost of these tests. Therefore, the unskilled group is indifferent whether to lend at r_H or stay out.

$$\frac{\lambda_H w(s_0)}{\lambda_H + (1-\lambda_H)(1-s_0)} > \frac{\lambda_H}{r_H} - w_1 - \frac{\lambda_H w(s_1)}{\lambda_H + (1-\lambda_H)(1-s_1)} > 0 \tag{D.3}$$

is sufficient to ensure that there exists an $s_H < s_0$ such that

$$\left(1 - \frac{s_H}{s_0}\right) \frac{\lambda_H w(s_0)}{\lambda_H + (1-\lambda_H)(1-s_0)} + \frac{\lambda_H w(s_1)}{\lambda_H + (1-\lambda_H)(1-s_1)} + w(1) = \frac{\lambda_H}{r_H} \tag{D.4}$$

In turn, (D.4) ensures that at interest rate r_H, the total credit supplied by a fraction $\left(1 - \frac{s_H}{s_0}\right)$ of unskilled investors along with every other investors is exactly sufficient to satisfy the credit demand of all good firms.

The allocation of credit to bad firms is given by market clearing conditions. In particular, we have

$$\left(1 - \frac{s_H}{s_0}\right) \frac{(1-\lambda_H)(1-s_1)w(s_0)}{\lambda_H + (1-\lambda_H)(1-s_0)} + \frac{(1-\lambda_H)(1-s_1)w(s_1)}{\lambda_H + (1-\lambda_H)(1-s_1)} = (1-s_1)(1-\lambda_H)\ell(\omega, b; \lambda_H)$$
$$\left(1 - \frac{s_H}{s_0}\right) \frac{(1-\lambda_H)(s_1-s_0)w(s_0)}{\lambda_H + (1-\lambda_H)(1-s_0)} = (s_1-s_0)(1-\lambda_H)\ell(\omega, b; \lambda_H)$$

for $\omega \in [s_1, 1]$ and $\omega \in [s_0, s_1]$ respectively, which, together with Equation (D.4), implies Equation (D.1).

Any investor with $s > s_0$ strictly prefers to enter at r_H as they make strictly fewer bad loans. These investors do not advertise a higher rate as good firms do not demand credit at higher rates. All entrants lend out all of their capital, thus none of them advertise a lower rate either. All good firms can borrow up to their credit capacity, therefore they do not demand credit at a higher rate. As all bad investors are rationed, by our robustness requirement and the fact that they do not intend to pay back, they demand maximum credit at higher interest rates as well, but do not raise any credit in those markets.

Low Aggregate Shock ($\theta = \lambda_{\mathbf{L}}$.) We conjecture that all investors are cautious in this state and accept applications with $x(\omega, \tau) = g$ only. In equilibrium, unskilled advertise the rate $\underline{r}$, moderately skilled advertise $\hat{r}$, and the most skilled advertise $\bar{r}$. Bad firms are not allocated any credit, hence by our robustness requirement, all bad firms demand maximum credit $\bar{\sigma}_L \equiv \frac{1}{\underline{r}}$ at every interest rate. Let $r_L(\omega)$ denote the interest rate at which good firms from country ω can raise financing.

Assume the following two conditions hold:

$$w(0) \geq s_0 \lambda_L \frac{1}{\underline{r}}, \tag{D.5}$$

$$w(1) < (1 - s_1)\,\lambda_L \frac{1}{\bar{r}}, \tag{D.6}$$

Then, $r_L(\omega)$ is given by

$$r_L(\omega) = \begin{cases} \underline{r} & \text{if } \omega \in [0, s_0] \\ \hat{r} = \max\{\min\{\frac{\lambda_L(s_1 - s_0)}{w(s_1)}, \bar{r}\}, \underline{r}\} & \text{if } \omega \in [s_0, s_1] \\ \bar{r} & \text{if } \omega \in [s_1, 1]\,, \end{cases}$$

If in addition, we have

$$\frac{1}{\bar{r}} \leq \frac{w\,(s_1)}{(s_1 - s_0)\,\lambda_L} \leq \frac{1}{\underline{r}} \tag{D.7}$$

then the interest rate schedule corresponds to the following credit allocation for good firms on the international market:

$$\ell(\omega, g; \lambda_L) = \begin{cases} \frac{1}{\underline{r}} & \text{if } \omega \in [0, s_0] \\ \frac{1}{\hat{r}} & \text{if } \omega \in [s_0, s_1] \\ \frac{w(1)}{(1-s_1)\lambda_L} & \text{if } \omega \in [s_1, 1]\ . \end{cases}$$

Demand of the good firms from least opaque countries, $\omega \in [0, s_1]$, is fully satisfied at the market with minimum interest rate, $r_L(\omega) = \underline{r}$, i.e. $\ell(\omega, g; \lambda_L) = \sigma(m(r_L(\omega)), \omega, g; \theta) = \bar{\sigma}_L$. $\underline{r}$ is the solution to the zero profit condition of unskilled cautious investors when all type of firms submit the same credit demand at the given interest rate,

$$(1 + \underline{r}) - \kappa \left(\frac{1}{\lambda_L s_0} \right) - 1 = 0. \tag{D.8}$$

For these investors, only fraction $\lambda_L s_0$ of the sampled firms provide conclusive evidence that they are good, which implies the second term is the cost. However, all the passed applications are from good firms, hence the revenue is the first term. Therefore, unskilled investors are indifferent whether to advertise interest rate $\underline{r}$ or stay inactive. The fraction of unskilled entrants is consistent with market clearing if condition (D.5) is satisfied. The acceptance rule is rationalized by the same argument as in state $\theta = \lambda_H$.

Moderately skilled investors offer a higher rate $\hat{r}$, hence they make a positive rent. $\hat{r}$ is determined by the market clearing condition of these investors, through cash-in-the-market pricing,

$$w\left(s_1\right) = \left(s_1 - s_0\right) \lambda_L \frac{1}{\hat{r}}.$$

Thus, if condition (D.7) is satisfied, then $\hat{r} = \frac{(s_1 - s_0)\lambda_L}{w(s_1)}$, and otherwise $\hat{r}$ takes the boundary values of $\underline{r}$ and $\bar{r}$, correspondingly. The completes proof of Lemma 1.

Note that if an unskilled or moderately skilled investor were to advertise a higher rate, she would not receive any applications she could pass her their skill level.

As long as skilled capital is in short supply, i.e. condition (D.6) is satisfied, skilled investors advertise the maximal interest rate any good firm is willing to accept $\bar{r}$.

Firms from the most opaque countries $\omega \in [s_1, 1]$ who cannot raise capital at any other advertised interest rate demand the maximum $\bar{\sigma}_L$ both at and under $r_L(\omega) = \bar{r}$. They can only raise financing in the market with highest interest rate $\bar{r}$ tho since only in that market they are recognized as good firms. Furthermore since skilled capital is in short supply, good firms are rationed at this rate. Thus, $\ell(\omega, g; \lambda_L)$ for this group is given by the market clearing condition

$$w(1) = (1 - s_1)\lambda_L \ell(\omega, g; \lambda_L).$$

Assumption 2.(ii) ensures that Equations (D.3), and (D.5)-(D.6) hold. This completes the proof of Proposition 2.

Optimal Choice of Test. Now we show that in each aggregate state, each investor prefers to choose the conjectured test and acceptance rule. Note that the skilled group's choice is immaterial as they observe τ of all firms regardless of their choice of test.

Acceptance rule. For the unskilled investors, recall that the acceptance rule has to be measurable with respect to the signal. That is, for any test, investors have three choices.(1) they can accept applications generating $x(\omega, \tau) = g$ only as conjectured, (2) they can reject all applications regardless of the signal, (3) they can accept all applications regardless of the signal. (2) is dominated by choosing to be inactive, while (3) is dominated by choosing the bold test and following (1). The latter is so, because a bold test rejects only bad firms, which is surely better than accepting all for any given pool. Therefore, our conjectured acceptance rule has to be optimal for any choice of test.

Test. First we show that the optimal test is either $\iota = 0$ or $\iota = 1$. Consider an investor with skill s, running an ι-test and advertising an interest rate r understanding that the the corresponding market m, $\gamma_\tau(m,s)$ fraction of τ the applications are submitted by $\omega > s$ firms. Thus, lending out her one unit of capital at market m leads to the profit

$$\frac{\lambda\left(\gamma_g+\iota\left(1-\gamma_g\right)\right)(1+r)-\kappa}{\lambda\gamma_g+\iota\left(\left(1-\gamma_g\right)\lambda+(1-\lambda)\left(1-\gamma_b\right)\right)}-1$$

where we have omitted arguments of $\gamma_\tau(m,s)$ for brevity. This expression is consistent with the zero profit conditions (D.2) and (D.8).

The first order condition with respect to the optimal test is

$$\frac{\partial\left(\frac{\lambda(\gamma_g+\iota(1-\gamma_g))(1+r)-\kappa}{\lambda\gamma_g+\iota((1-\gamma_g)\lambda+(1-\lambda)(1-\gamma_b))}-1\right)}{\partial\iota}=\frac{\kappa\left(\left(1-\gamma_b\right)(1-\lambda)+\lambda\left(1-\gamma_g\right)\right)-\lambda\gamma_g(1-\lambda)\left(1-\gamma_b\right)(1+r)}{\left(\lambda\gamma_g+\iota\left(\left(1-\gamma_g\right)\lambda+(1-\lambda)\left(1-\gamma_b\right)\right)\right)^2}. \tag{D.9}$$

Observe that the sign is independent of ι, implying a corner solution. Therefore, when (D.9) is positive the investor chooses the bold test, and when it is negative she chooses the cautious test.

Now we show the optimal choice when $\theta=\lambda_H$ is the bold test for both the moderately skilled and unskilled investor groups. For that (D.9) must be positive for r_H. As all firms submit demand at that rate, $\gamma_g=\gamma_b=s$. Therefore, we need

$$\kappa\left((1-s)(1-\lambda)+\lambda(1-s)\right)-\lambda s(1-\lambda)(1-s)\left(1+\frac{(\kappa+(1-\lambda)(1-s))}{\lambda}\right)>0$$

or:

$$(1-s)(1-s(1-\lambda))(-s+\kappa+s\lambda)>0.$$

Thus

$$\frac{\kappa}{s_0}>\frac{\kappa}{s_1}>1-\lambda_H \tag{D.10}$$

ensures that both unskilled and moderately skilled strictly prefers to enter as bold.

Finally, we show that $\theta=\lambda_L$ implies that the optimal choice is a cautious test for the moderately skilled and unskilled investors. An unskilled investor, in the market with $\underline{r}$ where all firms submit and so $\gamma_g=\gamma_b=s_0$, we need

$$\begin{aligned}&\kappa\left((1-s_0)(1-\lambda_L)+\lambda_L(1-s_0)\right)-\lambda_L s_0(1-\lambda_L)(1-s_0)\left(1+\frac{\kappa}{\lambda_L s_0}\right)\\&=\lambda_L(1-s_0)(\kappa-s_0+\lambda_L s_0)<0\end{aligned}$$

or

$$\frac{\kappa}{s_0} < 1 - \lambda_L. \tag{D.11}$$

For moderately skilled investors, (D.9) is decreasing in r. Therefore, it is sufficient to show that they prefer to enter as cautious at interest rate $\min \hat{r} = \underline{r}$. Importantly, in the candidate equilibrium $\gamma_g = (s_1 - s_0), \gamma_b = s_1$ as only $\omega \in [s_0, s_1]$ good firms and all bad firms participate in the market with interest rate $\hat{r}$. Therefore, it is sufficient if

$$\begin{aligned} &\kappa\left((1-s_1)(1-\lambda_L)+\lambda_L\left(1-(s_1-s_0)\right)\right) \\ &-\lambda_L(s_1-s_0)(1-\lambda_L)(1-s_1)\left(1+\frac{(\kappa+(1-\lambda_L)(1-s_1))}{\lambda_L}\right)<0 \end{aligned}$$

or

$$\frac{\kappa\left(1-(s_1-s_0)\right)}{(1-s_1)(s_1-s_0)} < 1 - \lambda_L. \tag{D.12}$$

Equations (D.10)-(D.12) follow from Assumptions 2.(i) and 2.(iv). This completes the proof of Proposition 1. ■

Proof of Proposition 3.

The firm problem at date $t = 1$ is defined in (A.1).

(i) The general form of equilibrium for $\theta = H$ is characterized in Section B.1. (r_H, s_H) are given by Equations (B.8) and (B.9), respectively, using $\hat{y}$ defined in (C.1).

(ii) The general form of equilibrium for $\theta = L$ is characterized in Section B.2. The form in (14) is then derived by specializing the investor skill distribution function in Section C.1, which also uses $\hat{y}$ defined in (C.1) as well as Assumption C.1.

■

Proof of Proposition 4.

The firm problem at date $t = 1$ is defined in (A.1).

(i) The general form of equilibrium for $\theta = H$ is characterized in Section B.1. Section C.2 shows that the optimal continuation decision is determined by the constraint. It follows that the equilibrium amount that the firm raises is given by $y(\omega, \tau; H, r_H, r_L)$ in program (A.1), using $\hat{y}$ defined in (C.1) and Equations (2) and (13) with the optimal $i(\omega, \tau; H)$.

(ii) The general form of equilibrium for $\theta = L$ is characterized in Section B.2, and specialized in Section C.1 by specializing the investor skill distribution function under Assumption C.1. Section C.2 shows that the optimal continuation decision is determined by the constraint. It follows that the equilibrium amount that the firm raises is given by $y(\omega, \tau; L, r_H, r_L)$ in program (A.1), using $\hat{y}$ defined in (C.1) and Equations (2) and (13) with the optimal $i(\omega, \tau; L)$ and $\alpha = \xi$.

■

Proof of Proposition 5.

The derivation of optimal firm scale of operation, as well as the optimal continuation decision, is provided in Section C.2. (15) follows from (C.6), where the rationing function is defined in (C.9). ■

Proof of Proposition 6.

The derivation of optimal firm exante investment, as well as the optimal continuation decision, is provided in Section C.2. (18) follows from (C.7), where the rationing function is defined in (C.11). ■

Lemma D.1 *Assume $G(x)$ and $H(x, z)$ are continuous in x. Equation* (D.13) *has a fixed point $x \in [0, 1]$,*

$$F(x) = \frac{(1-\lambda)\int_{s_H(x)}^{1} H(x,z)dz}{(1-\lambda)\int_{s_H(x)}^{1} H(x,z)dz + \lambda G(x)}; \tag{D.13}$$

where $s_H(x)$ is the solution to

$$\int_{s_H(x)}^{1} \frac{1}{(1-\lambda)\int_{s}^{1} H(x,z)dz + \lambda G(x)} w(s)ds = \phi(1-x), \tag{D.14}$$

if Equation (D.14) *has a solution, and $s_H(x) = 0$ otherwise.*

Proof of Lemma D.1.

First note that if Equation (D.14) has a solution in $s_H(x)$, it will be $s_H(x) \in [0, 1]$. The reason is that $w(s) = 0$ for $s > 1$ and $s < 0$, so moving s_H outside the $[0, 1]$ interval does not change the left hand side of Equation (D.14).

Case 1 [Equation (D.14) **holds with equality, $s_H \in [0, 1]$].** Consider the case where s_H is interior. Consider the self-map on $F : [0, 1] \mapsto [0, 1]$. We use Brouwer's fixed-point theorem to prove existence of a fixed point. $[0,1]$ is a compact convex set. We need to show is that $F(x)$ is a continuous function, and maps $[0,1]$ to itself, which is immediate since the ratio in $F(x)$ is positive and (weakly) smaller than one.

Next we move to proving continuity. $G(x)$ is continuous in x. $H(x,z)$ is also continuous in x, and so is $\int H(x,z)dz$. Thus if a solution $s_H(x)$ to Equation (D.14) exists, it is also continuous.

This implies that if a solution to Equation (D.14) exists, then everything on the right hand side of Equation (D.13) is continuous, so $F(x)$ is a continuous map from $[0,1]$ to $[0,1]$, which implies by Brouwer's theorem a fix point exists.

Case 2 [Equation (D.14) **only holds with inequality, thus** $s_H = 0$**].** Then Equation (D.13) becomes one equation in one unknown in x, which with the same argument as the previous case has a fixed point. ∎

Proof of Proposition 7. Let

$$\underline{\lambda} \equiv \frac{\rho_g \xi(1-\phi) + (\rho_g - \xi)(\phi \pi_L \xi - 1)}{\rho_g \xi(1-\phi) + (\rho_g - \xi)\phi \pi_L \xi} \text{ and}$$

$$\Lambda(\omega) \equiv \min\left\{ \frac{(\rho_g - \xi)\left(1 + (1-\lambda)\,\phi \xi \pi_H\right)}{\left(\rho_g\,(1-\phi) + \phi\,(\rho_g - \xi)\,\pi_H\right)\xi}, \frac{\xi \phi \lambda - w(\omega)}{\xi \phi(\lambda + w(\omega)\pi_L)} \right\}.$$

Then, the sufficient conditions of Proposition 7 are equivalent with Assumption C.1 (i)-(iv). Here, we proceed in steps. First, we show that the existence of a simple global equilibrium can be mapped to a fixed point problem. After describing the problem we explain the mapping between Equations (D.15)-(D.30) to the solution developed in Sections C and B. Then, we use Lemma D.1 to prove the existence of equilibrium. Second, we explain the role of Assumption C.1 (i)-(iv).

Equilibrium Existence as a Fixed Point Problem

Equations (D.15)-(D.30) spell out how the equilibrium objects $r_H, r_L(\omega), s_H, \omega_1, \omega_2, \omega_3, \eta_L(\omega)$ and $\eta_H(\omega)$ are constructed.

$$F(x) = \frac{(1-\lambda)\,(1-s_H(x))\,D\,(0;x)}{(1-\lambda)\,(1-s_H(x))\,D(0;x) + \lambda \bar{D}(x)} \tag{D.15}$$

where $s_H(x)$ solves

$$\int_{s_H(x)}^{1} \frac{1}{(1-\lambda)\,(1-s)D(0;x) + \lambda \bar{D}(x)} w(s)ds = (1-x)\phi, \tag{D.16}$$

if Equation (D.16) has a positive solution, and $s_H(x) = 0$ otherwise.

Moreover

$$\bar{y}(x) = \frac{(\rho_g - \xi)(1 + \phi\xi\pi_H x)}{(\rho_g(1-\phi) + \phi(\rho_g - \xi)\pi_H)\xi} \tag{D.17}$$

$$D(y;x) = \frac{\xi}{1 + \phi\xi(\pi_H x + \pi_L y)} \tag{D.18}$$

$$\begin{aligned}\bar{D}(x) =& \omega_1(x) D(0;x) + \int_{\omega_1(x)}^{\omega_2(x)} D(y^C(\omega);x) d\omega \\ &+ \left(1 - \omega_2(x) + \frac{\phi\xi\pi_L\bar{y}(x)}{1+\phi\xi\pi_H x}\int_{\omega_3(x)}^{1}(1-\eta_L(\omega))d\omega\right) D(\bar{y}(x);x).\end{aligned} \tag{D.19}$$

where

$$y^C(\omega;x) \equiv \frac{\xi\phi\lambda - w(\omega)(1+\phi\xi\pi_H x)}{\xi\phi(\lambda + w(\omega)\pi_L)} \qquad \omega \in [\omega_1(x), \omega_2(x)]. \tag{D.20}$$

The rationing functions are given as follows

$$\eta_L(\omega) = \min\left(1, \int_{\omega}^{1} \frac{1}{\phi\lambda(1-\bar{y}(x))D(\bar{y}(x);x)(s-\omega_2(x)) - \int_{\omega_2(x)}^{\omega_3(x)} w(s)ds} w(s)ds\right) \tag{D.21}$$

$$\eta_H(\omega) = \min\left(1, \int_{s_H(x)}^{\omega} \frac{1}{(1-\lambda)(1-s)D(0;x) + \lambda\bar{D}(x)}\frac{w(s)}{\phi(1-x)} ds\right) \tag{D.22}$$

and $\omega_1(x), \omega_2(x), \omega_3(x)$ are defined as follows.

Let $\hat{\omega}_1(x)$ and $\hat{\omega}_2(x)$ be the solution to the following two equations, respectively:

$$w(\omega_2) - \phi\lambda(1-\bar{y}(x))D(\bar{y}(x);x) = 0, \tag{D.23}$$

$$w(\omega_1) - \phi\lambda D(0;x) = 0. \tag{D.24}$$

Then

$$\omega_2(x) = \min\{\max\{\hat{\omega}_2(x), 0\}, 1\}, \tag{D.25}$$

$$\omega_1(x) = \min\{\max\{\hat{\omega}_3(x), 0\}, 1\}. \tag{D.26}$$

Moreover, let $\hat{\omega}_3$ be the solution to

$$1 = \int_{\omega_3}^{1} \frac{1}{\phi\lambda(1-\bar{y}(x))D(\bar{y}(x);x)(s-\omega_2(x))) - \int_{\omega_2(x)}^{\omega_3(x)} w(s)ds} w(s)ds \tag{D.27}$$

$$\omega_3(x) = \min\{\max\{\hat{\omega}_3(x), 0\}, 1\}. \tag{D.28}$$

Finally, given the fixed point x^*, $\bar{r}(x^*) = \frac{\bar{y}(x^*)}{1-\bar{y}(x^*)}$, and interest rates r_H and $r_L(\omega)$ are given by

$$r_H = \frac{x^*}{1-x^*} \tag{D.29}$$

$$\hat{r}(\omega) = \frac{y^C(\omega; x^*)}{1-y^C(\omega; x^*)}. \tag{D.30}$$

and (14).

To simplify the formulas, the proposition is stated in terms of premia rather than interest rates, using the monotone transformation

$$q = \frac{r}{1+r}. \tag{D.31}$$

Using this transformation, Equation (B.8), which defines the interest rate when $\theta = H$, can be written as

$$q = \frac{(1-\lambda)\int_s^1 \hat{y}(\omega, b; H)d\omega}{(1-\lambda)\int_s^1 \hat{y}(\omega, b; H)d\omega + \lambda \int_0^1 \hat{y}(\omega, g; H)d\omega} \tag{D.32}$$

Let $H(r_H) = \hat{y}(\omega, b; H, r_H)$ and $G(r_H) = \int_0^1 \hat{y}(\omega, g; H, r_H)d\omega$, noting that $H(r_H) = \hat{y}(\omega, g; H)$ depends on q_H through r_H, using Equation (D.31). We have shown in Section C that $\hat{y}$ is continuous in r_H, and r_H is by construction continuous in q_H. As such, existence of a solution to pair of Equations (D.32) and (B.9) such that $q_H \geq 0$ and $0 \leq s_H \leq 1$, implies that there exists a solution to pair of Equations (B.8) and (B.9) such that $r_H \geq 0$ and $0 \leq s_H \leq 1$.
We proceed in two steps. We first explain the mapping between Equations (D.15)-(D.30) to the solution developed in Sections C and B. We will then use Lemma D.1 to prove the existence of equilibrium.

Equation (D.18) writes the general form of expected maintenance cost of a good firm who faces premia x when $\theta = H$, and y when $\theta = L$, or interest rates $\frac{x}{1-x}$ and $\frac{y}{1-y}$, respectively. It uses the equilibrium firm scale of operation, defined by Equation (C.6), and optimal continuation scale. Substitute in Equation (2) (with $\alpha = \xi$) to get firm liquidity demand in international markets: $\ell(\omega, g; \theta) = \frac{D(\frac{r_L(\omega)}{1+r_L(\omega)}; \frac{r_H}{1+r_H})}{1+r(\omega,g;\theta)}$. Using this demand functions, $\hat{y}$ at $t = 1$ is defined in (C.1). It is straight forward to verify that using (C.1) to solve for the firm problem at $t = 1$ (Section B), $y(\omega, \tau; \theta) = \ell(\omega, \tau; \theta)$.

Under the appropriate sufficient conditions on the parameters (see the end of this proposition), firms choose to participate in international markets when $\theta = H$, with the equilibrium described in B.1, and when $\theta = L$ with the equilibrium described in sections B.2 and C.1,. Under this equilibrium structure, Equation (D.19) aggregates the total required maintenance across the pricing regions when $\theta = L$.

Equation (D.17) rewrites the maximum premium $\bar{q}$ in $\theta = L$, defined in Equation (C.4), when the common premium in high state is x. The threshold transparencies ω_1, ω_2 and ω_3 are defined in Equations (D.26), (D.25), and (D.28), respectively, and $\omega_3 = \tilde{\omega}$ and $\omega_2 = \bar{\omega}$ in the $\theta = L$ equilibrium in Section B.2. This leads to the rationing function in Equation (C.9). Equation (D.24) determines the threshold where bunching region ends, at zero interest rate, given liquidity demand function (D.18). Equation (D.23) determines the threshold where bunching-with-scarcity region starts, at premium $\bar{q}$ (interest rate $\bar{r}(r_H)$). Equation (D.27) determines the threshold where rationing starts in bunching-with-scarcity region, given the liquidity demand

Finally, Equations (D.15) and (D.16) jointly determine the pooling premium and marginal investor when $\theta = H$, at the above liquidity demand levels.

The last equilibrium object to determine is demand function for credit $\{\sigma(m, \omega, \tau; \theta)\}_{\theta=H,L}$. It is implied from Lemma B.1 and Equation (C.1), Proposition 4, and Equations (C.6) and (C.7) to relate each firm demand for credit in each state to its the equilibrium scale of operation.

Lastly, we will use Lemma D.1 to prove existence of equilibrium. Let $G(x) = \bar{D}(x)$ and $H(x) = D(0; x)$. As such we need to show both functions are continuous.

$\bar{y}(x)$ is continuous. $D(y; x)$ is continuous in x for any $x, y > 0$ since $1 + \phi\xi(\pi_H x + \pi_L y) > 0$. Thus $D(0; x)$ and $D(\bar{y}(x); x)$ are also continuous.

Now turn to $\omega_1(x)$, $\omega_2(x)$ and $\omega_3(x)$. $w(.)$ and $D(y; x)$ are continuous in x. $D(y, x)$ is constant in ω and $w(.)$ is increasing in ω, so Equations (D.23) and (D.24) have a unique solution in ω, so $\hat{\omega}_1(x)$ and $\hat{\omega}_2(x)$ exist, are unique, and continuous.

Next, $D(0, x)$ is decreasing in x. Moreover

$$D(\bar{y}(x); x) - (1 - \bar{y}(x))D(\bar{y}(x); x) = \bar{y}(x)D(\bar{y}(x); x) = \frac{\rho_H - \xi}{\rho_g - \phi\xi},$$

$$\frac{d\Big((1 - \bar{y}(x))D(\bar{y}(x); x)\Big)}{dx} = \frac{dD(\bar{y}(x); x)}{dx} = \frac{\xi^2 \pi_H \phi(\rho_g(1 - \phi) + \phi\pi_H(\rho_g - \xi))}{(\rho_g - \phi\xi)(1 + \phi\xi\pi_H x)^2} < 0.$$

Thus both $\hat{\omega}_2(x)$ and $\hat{\omega}_1(x)$ are monotonically decreasing in x. Since $\hat{\omega}_2(x)$ and $\hat{\omega}_1(x)$ are continuous, (D.25) and (D.26) imply that $\omega_2(x)$ and $\omega_1(x)$ are also continuous and weakly decreasing in x.

Next consider the right hand side of (D.27). $\omega_2(x)$ is continuous. Moreover, (D.27) is the simplified version of (B.27). We have already shown that $R_D(\omega_1, \omega_2(x), \bar{y}(x), 1) > 0$, and $\hat{y}(\omega_2(x), g; L) = D(\bar{y}(x); x)(1-x) > 0$, thus the denominator is positive. Each term is also continuous in x, which in turn implies the right hand side is continuous in x. Thus $\hat{\omega}_3(x)$ is continuous as well, and using Equation (D.28), $\omega_3(x)$ is also continuous.

Finally, continuity of $\omega_i(x)$ $i = 1, 2, 3$, along with continuity of $w(.)$, $\eta(.)$ and $D(y; x)$ (in x) implies $\bar{D}(x)$ is continuous. So by Lemma D.1, the fixed point exists.

Explanation of Parameter Restrictions

Optimal Firm Decision without Access to International Market [$\xi \geq \frac{1}{1-\phi}$.] Assume the firm does not have access to international investors. So the firm can do one of the two things. The first option is to invest all of his initial endowment. Then the firm continues with a high scale, $I_I = 1$, if not hit by a liquidity shock, and terminate the project if hit. Thus the payoff is $\Pi_I = \rho_\tau(1-\phi)I_1 = \rho_\tau(1-\phi)$. Alternatively, the firm can save enough of his own endowment using bankers to insure against the liquidity shock in either or both aggregate states. Since the aggregate state is only relevant in the interaction with the international investors, if the firm choose to insure against liquidity shock from own endowment, it will be for both aggregate states. The firm investment scale is given by $I_S = \frac{1}{1+\phi\xi}$, and his expected payoff is $\Pi_I = \rho_\tau I_2$. Thus for $\Pi_I > \Pi_S$ we need

$$1 - \phi > \frac{1}{1+\phi\xi} \Rightarrow \phi < \frac{\phi\xi}{1+\phi\xi} \Rightarrow \xi > \frac{1}{1-\phi},$$

which is Assumption C.1.(i). Under this assumptions when firms can access the international credit market, we only need to compare borrowing on the international markets with investing all of their endowment. This is the next parametric restriction that we consider.

Sufficient Condition for Inequality (C.5) [$\underline{\lambda} < \lambda$] Equations (D.30), (D.15), and the monotonicity of (D.15) in s_H implies

$$\frac{r_H}{1+r_H} = \frac{(1-\lambda)(1-s_H(r_H))}{(1-\lambda)(1-s_H(r_H)) + (\lambda)\frac{\bar{D}(\frac{r_H}{1+r_H})}{D(0;\frac{r_H}{1+r_H})}} \leq \frac{(1-\lambda)}{(1-\lambda) + \lambda\frac{\bar{D}(\frac{r_H}{1+r_H})}{D(0;\frac{r_H}{1+r_H})}},$$

which in turn implies

$$r_H \leq \frac{1-\lambda}{\lambda} \frac{D(0; \frac{r_H}{1+r_H})}{\bar{D}(\frac{r_H}{1+r_H})}.$$

So to find an upper bound on r_H, it is sufficient to find an upper bound on $\frac{D(0; \frac{r_H}{1+r_H})}{\bar{D}(\frac{r_H}{1+r_H})}$. Note that $D(0; \frac{r_H}{1+r_H})$ is what the most transparent good firms spending on maintenance. As these firms face the most favourable credit market conditions, $\bar{D}(\frac{r_H}{1+r_H}) \leq D(0; \frac{r_H}{1+r_H})$ holds which in turn implies

$$r_H \leq \frac{(1-\lambda)}{\lambda} \Rightarrow q_H \leq (1-\lambda)$$

In Assumption C.1.(ii) we assume $\frac{(1-\lambda)}{\lambda} \leq \bar{r}_H$, where $\bar{r}_H$ is defined in Equation (C.5). This in turn insures that $r_H \leq \bar{r}_H$. Moreover, one can substitute $(1-\lambda)$ for x in (D.17) to get an upper bound on $\bar{q}$.

Sufficient Condition for Inequality (B.23) $[w(0) \geq \phi\lambda\xi]$. Using (C.1), we can write condition (B.23) as

$$w(0) > \phi\lambda\ell(0, g; L) = \phi\lambda D(0; r_H)$$

Note that in $\hat{\omega} = \omega_3(r_H)$. A sufficient condition for the above inequality to hold is

$$w(0) \geq \phi\lambda\xi, \tag{D.33}$$

which ensure that $\omega_1 > 0$, and constitutes the first part of Assumption C.1.(iii).

Sufficient Condition for a smaller than one solution to Equation (B.27). $[\lim_{s\to 1} w(s) = 0]$ This condition directly ensures insures that $\tilde{\omega}$ that solves Equation (B.27) is smaller than 1, i.e. $1 > \omega_3$. This is the second part of Assumption C.1.(iii).

Sufficient Condition for Inequality (C.2). $[\min\left\{\frac{(\rho_g-\xi)(1+(1-\lambda)\phi\xi\pi_H)}{(\rho_g(1-\phi)+\phi(\rho_g-\xi)\pi_H)\xi}, \frac{\xi\phi\lambda-w(\omega)}{\xi\phi(\lambda+w(\omega)\pi_L)}\right\} \leq \frac{1-\lambda}{1-\lambda\omega}\ \forall\omega]$. The only set of markets we need to consider are those with cash-in-the-market pricing. Let $q^C(\omega) = \frac{r^C(\omega)}{1+r^C(\omega)}$ and $\bar{q}(r_H) = \frac{\bar{r}(r_H)}{1+\bar{r}(r_H)}$. From (C.2)

$$q^C(\omega) \leq \frac{(1-\lambda)}{(1-\lambda)+\lambda(1-\omega)}$$

Start by noting that $\bar{q}(r_H)$ is the maximum $q^C(\omega)$ can achieve, so a sufficient condition for inequality (C.2) is

$$\min\{\bar{q}(r_H), q^C(\omega)\} \leq \frac{(1-\lambda)}{(1-\lambda)+\lambda(1-\omega)}.$$

Next from (D.17)

$$\bar{q}(r_H) = \frac{(\rho_g - \xi)(1+\phi\xi\pi_H q_H)}{(\rho_g(1-\phi)+\phi(\rho_g-\xi)\pi_H)\,\xi} \leq \frac{(\rho_g - \xi)\,(1+(1-\lambda)\,\phi\xi\pi_H)}{(\rho_g\,(1-\phi)+\phi\,(\rho_g-\xi)\,\pi_H)\,\xi}$$

where the inequality used part (ii) to replace q_H with it's maximum, $(1-\lambda)$. Next, from (D.20)

$$q^C(\omega) = \frac{\xi\phi\lambda - w(\omega)(1+\phi\xi\pi_H x)}{\xi\phi(\lambda + w(\omega)\pi_L)} \leq \frac{\xi\phi\lambda - w(\omega)}{\xi\phi(\lambda+w(\omega)\pi_L)}$$

where the inequality just uses $q_H \geq 0$. Substitute both back to get a sufficient condition

$$\min\left\{\frac{(\rho_g - \xi)\,(1+(1-\lambda)\,\phi\xi\pi_H)}{(\rho_g\,(1-\phi)+\phi\,(\rho_g-\xi)\,\pi_H)\,\xi}, \frac{\xi\phi\lambda - w(\omega)}{\xi\phi(\lambda+w(\omega)\pi_L)}\right\} \leq \frac{1-\lambda}{1-\lambda\omega}.$$

which is Assumption C.1.(iv).

■

E Further Results and Extensions

E.1 Determination of Exposure Groups: Analytical Results

As we note in the main text, we can decompose the total effect of our parameters to the relative size of exposure groups into two parts. First, keeping the interest rate in the high state (r_H) fixed, changes in parameters have a direct effect on maintained investment $i(\omega, \tau, L)$. Second, there is an indirect effect through the spill-over across aggregate states. A change in credit demand in the low state affects scale $I(\tau, g)$ and, through the budget constraint (13), affects credit demand in the high state as well. This in turn changes the equilibrium interest rate in the high state, r_H, which then feeds back into the initial and maintained investment in both high and low states.

In the following proposition, we characterize the direct effect. In particular, we show that an increase in the probability or the size of the liquidity shock, ϕ and ξ, in the fraction of good firms, λ, and in the probability of the low aggregate state, π_L, all increase the total credit demand of good firms at zero interest rate, and, consequently, shrink the set of core countries. Similarly, an increase in the size of the liquidity shock, in the fraction of good firms, and in the productivity of good firms increases the total credit demand of good firms at $\bar{r}(r_H)$ interest rate, and, consequently, increases the set of peripheral countries. While we do not have analytical results on the indirect effect, the direct effect dominates in all our numerical simulations.

Proposition E.1 *In a simple global equilibrium, keeping r_H fixed,*

(i) the set of low exposure countries shrinks if there is an increase in ξ, ϕ, λ, or π_L,

$$\frac{\partial \omega_1}{\partial \xi}, \frac{\partial \omega_1}{\partial \phi}, \frac{\partial \omega_1}{\partial \lambda}, \frac{\partial \omega_1}{\partial \pi_L}|_{r_H \text{ fixed}} < 0.$$

(ii) The set of high exposure countries grows if there is an increase in ξ, λ or ρ_g,

$$\frac{\partial \omega_3}{\partial \xi}, \frac{\partial \omega_3}{\partial \lambda}, \frac{\partial \omega_3}{\partial \rho_g}|_{r_H \text{ fixed}} < 0$$

Proof of Proposition E.1. Using Equation (D.24), the size of the low exposure group is determined by

$$w(\omega_1) = \phi\lambda\xi i\left(\omega, g, L\right)|_{\omega\in[0,\omega_1]}. \tag{E.34}$$

The direct effects come from simple differentiation using Equations (15) and (17) and noting that $\eta_H(\omega) = \eta_L(\omega) = 1$ in the low exposure region. The size of the group of high exposure countries is defined implicitly in Equation (D.27). Let $Z_1 = \phi\left(\lambda\right)\frac{\xi}{1+\bar{r}(r_H)} i\left(\tau_j = \omega, g, L\right)|_{\omega\in[\omega_1,\omega_2]}$, the amount an unrationed representative good firm borrow facing the maximum interest rate $\bar{r}$. In the left panel of Figure 3, we plot the supply of capital of a $k \geq \omega_3$ firm, $\eta_L(\omega)Z_1$ as the dashed curve, which, using the definition of ω_2 in (D.23), we can rewrite as

$$Z_1 \int_\omega^1 \frac{1}{Z_1\left(s - w^{-1}\left(Z_1\right)\right) - \int_{w^{-1}(Z_1)}^{\omega_3} w(s)ds} w(s)ds. \tag{E.35}$$

By definition, ω_1 is determined by the point where this curve is equal to the demand Z_1, the dashed line, as this is the least transparent country where firms demand for credit is fully met. While a change in Z_1 moves both curves, using the implicit function theorem, we can verify that $\frac{\partial\omega_3}{\partial(Z_1)} < 0$. The direct effects then come from simple differentiation using equations (15) and (17) and noting that $\eta_H(\omega) = \eta_L(\omega) = 1$ in the region $\omega \in [\omega_1, \omega_2]$. ■

E.2 Partitioned Opacity Groups

In the baseline model, we assume that investors have an uninformative prior about ω, the average opacity of firms in a given country. That is, if an investor does not find conclusive evidence on a firm, the country of origin does not help her do any further inference.

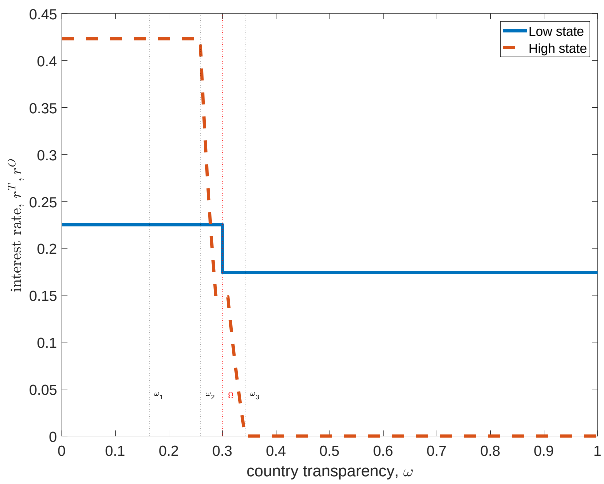

Figure 5: Interest rates for transparent and opaque countries in the high (solid) and low (dashed) aggregate state.

In this section, we weaken this assumption. In particular, suppose that a public signal partitions countries into a transparent and an opaque group. That is, observing the public signal, each investor knows that the opacity, ω of the given country is $\omega > \Omega$ or $\omega < \Omega$, where Ω is an arbitrary cut-off. Intuitively, investors understand that a firm from a southern country in Europe tends to be more opaque than a northern country firm, but they have no information on how firms in different south European countries compare to each other.

Figure 5 illustrates the effect of this treatment on the equilibrium interest rate schedules. Compared to the corresponding figure for the baseline case, the left panel of Figure 1, it is clear that the qualitative difference is small. The main effect of the extra signal is the partial separation in the high aggregate state. With the public signal, investors have an additional choice. They can choose to accept only firms from the transparent group to lend to. For less skilled investors, this implies a portfolio with less bad firms, as their mistakes are concentrated in opaque countries. Therefore, in equilibrium, less skilled investors lend to firms from the transparent group only, albeit at a lower interest rate. On the other hand, more skilled investors lend to firms from the opaque group but for higher interest rate. The marginal investor who is just indifferent between these two choices is determined in equilibrium.[23]

While it is an intuitive assumption that investors have some prior knowledge on the average opacity of firms in different countries, we assume this away in the baseline model

[23]The public signal also introduces a small bunching region around Ω in the low aggregate state interest rate schedule. As we explain in Appendix B, this comes from the requirement that the interest rate schedule has to be weakly monotonically decreasing in ω, and is obtained by an ironing procedure.

because of two main reasons. First, we believe the additional analytical complexity does not justify the additional insight. Second, one of the main focuses of our analysis is how investors endogenously classify countries into low and high exposure groups in equilibrium. As this extension illustrates, a public signal on ω classify countries exogenously, and obscures our analysis.

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The author, Maryam Farboodi, declares that she has no relevant or material financial interests that relate to the research described in this paper.

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