Pacioli's Lens: Through a Glass, Darkly

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ABSTRACT

It has long been argued that double-entry bookkeeping ('DEB') was important for enabling capitalism's development in the West and heralded the beginning of 'modern accounting'. However, these claims remain contested so it is important to understand the history of DEB's emergence about 700 years ago and its underlying rationale. Sangster (2018a) [Pacioli's Lens: God, Humanism, Euclid, and the Rhetoric of Double Entry. *The Accounting Review*, 93(2): 299-314] argues that, in the first printed manual on DEB in 1494, Pacioli presented a novel 'axiomatic' approach to explaining DEB that requires a corresponding 'paradigmatic shift' in our appreciation of his contribution. This paper challenges Sangster's interpretation of Pacioli's mathematical contribution and calls for deeper understanding of the historical development of DEB in the West by comparison with accounting developments in the East.

Keywords: Pacioli; double-entry bookkeeping (DEB); algebraic axioms; comparative international accounting history.

JEL Classifications: B11; C00; M40; P52; Z1

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I. INTRODUCTION: MOTIVATION, METHOD AND CONTRIBUTION

It is well known that Luca Pacioli, a friar and university teacher, published the first printed exposition of double-entry bookkeeping ('DEB') in 1494, within his compendium on mathematics *Summa de Arithmetica, geometria, proportioni e proportionalità* (von Gebsattel 1994). Here he set out the method primarily as it had developed in Venice,[1] while innovating by publishing in the vernacular rather than in Latin and by utilising Arabic numerals.

It has been repeatedly argued that DEB was fundamental for enabling capitalism's development in the West as it heralded the beginning of the 'modern accounting' that is embedded as such a powerful institution in the functioning and regulation of modern organizations worldwide and in particular of international stockmarkets (e.g. Soll 2014). However, its properties and role have also long been contested (e.g. Yamey 1949) and it has recently been argued that it constrains the possibilities available to accounting standard setters such as FASB and IASB for innovating financial reporting rules under uncertainty (Barker, Lennard, Penman, and Texeira 2020).

Examples of accounts kept in DEB from around Italy (including in particular Florence and Genoa) have been traced back as far as about another 200 years before Pacioli's treatise (de Roover 1955, 1956; Goldthwaite 2015). Today, DEB still underlies the FASB's and IASB's conceptual frameworks that adopt a 'balance sheet approach' to the determination of enterprise income (c.f. Bromwich, Macve, and Sunder 2010) and it is the hallmark of the accounting and auditing profession around the world.

[1] However he also confusingly presents a Tuscan approach to balancing and closing the ledger (Yamey, 1994a).

As the claims for DEB's economic importance remain contested (e.g. Macve 1996; Yuan, Macve, and Ma 2017) it is important to explore the history of how this particular book-keeping and accounting practice became institutionalized. Such exploration requires thorough examination of historical archives of extant account books from the Renaissance onwards (e.g. Goldthwaite 2015; Matringe 2016), together with exploration of accounting manuals and the development of their approaches to teaching DEB (e.g. Yamey 1994a), which were supplemented from the 19th century onwards, first in the UK and US, by the emergence of accounting as an organized profession and then as an academic field of study.

Sangster (2018a) goes very significantly beyond well-known evaluations of Pacioli's contribution (e.g. Yamey 1994a) and argues (in the Abstract) that 'Pacioli reveals a simplicity in the then-unrecognized axiomatic foundation of double entry that has been largely overlooked. The findings represent a paradigm shift in how we perceive Pacioli, his treatise, and double entry.'[2]

The bulk of Sangster's paper sets out a view of Pacioli's life, motivation, and approach in publishing his treatise that is in most respects broadly consistent with previous work (e.g. Macve 1996) while refining some of the minor historical details of Pacioli's life and work (c.f. Vollmers 2015).[3] And it has indeed been recognised that Pacioli's treatment of algebra in the *Summa* was innovative in generalising from individual problems and their numerical solutions to providing 'keys' that showed how to approach a class of similar problems (Heeffer 2012). But, while innovative, these 'keys' are arrived at from repeated numerical examples that establish a generalizable pattern (as given in the example in Figure 1 on p.307 of Sangster

[2] The argument is reiterated in Sangster (2018b).

[3] Sangster (2021) now collates the available evidence about Pacioli's life, teaching roles and writings but also repeats (albeit with less hyperbole: but c.f. p132) the main claims of Sangster (2018a) (e.g. p.128; pp.147-8). However, while emphasising Pacioli's devotion to the Church, it sheds no light on why he was threatened with excommunication in 1491 (p.141) or on his disputes with his convent in San Sepolcro in 1509 and 1511 (p.145).

2018a), rather than exhibiting a 'paradigm shift' of being derived from first principles (or 'axioms') as Sangster claims.

Moreover Sangster then further argues that Pacioli also adopted this approach based on 'keys' in his section of the *Summa* on DEB (usually referred to as the *de Scripturis*—e.g. Yamey 1994a) and that his explanation of DEB was correspondingly 'axiomatic'. But it will be argued here that the illustrations in Sangster's Figures 1 and 2 do not bear this out.[4]

The contribution of this comment is therefore to reestablish a realistic understanding of Pacioli's place both in the development of mathematics and in the understanding of the logic of DEB in order to clarify the significance of the Renaissance Italian developments in the evolution of 'modern accounting'. This will better 'provide us with a clear view of what occurred in the past and why, to identify the values of that time, contrast them with our values today and consider if there are lessons to be learned from the historical account' (Sangster 2018a, 311).

The method I adopt in challenging Sangster's central claims is a close reading of his arguments, and in particular its illustration in his Figure 2 (reproduced here as Appendix 2), alongside his cited sources. Making comparisons with accepted understandings of the nature of 'axioms', 'postulates' and 'theorems', and using simple logic together with reviewing relevant literature on Pacioli and on the history of mathematics, I consider first the nature and role of axioms (Section II); then the nature and role of 'keys' in Pacioli's algebra, noting their absence in his treatise on DEB (Section III); followed by an outline discussion of the historical significance of his treatise and of DEB in world history (Section IV). Section V concludes and indicates where further research will be valuable.

[4] Stone (2021) strangely does not comment directly on Sangster (2018a) but he argues (consistent with the suggestion in Macve, 1996 about his *implicit* understanding) that Pacioli 'recognised that the Venetian method of bookkeeping had an algebraic underpinning, although, when writing his treatise, he converted this algebra into rules' (p.31). So he quotes (p. 34) from Heeffer that 'there is no direct relationship between algebra and bookkeeping'.

II. THE ROLE OF 'AXIOMS'

Sangster (2018a, 306) adopts a generally accepted definition of axioms as 'prerequisites that are neither provable nor in need of proof', [5] while postulates are 'a thing suggested or assumed as true as the basis for reasoning, discussion, or belief'. Given his set of five axioms and five postulates, Euclid (around 300 BC) was able to deduce all the theorems of his geometry as a logically consistent system.[6]

Euclid's axioms (or 'common notions'), on which he built his geometry, can be expressed as:

1. Things are equal to one another if those things are equal to the same thing.
2. The wholes are equal if equals are added to equals.
3. The remainders are equal if equals are subtracted from equals.
4. Things are equal to one another if they coincide with one another.
5. The whole is greater than the part

Clearly these axioms are 'self-evident' truths applicable to *any field* of scientific knowledge and, as Appendix 1 here shows, they are sufficient to show how to derive all the rules of DEB from the 'Balance Sheet Equation'. While 'axioms' and 'postulates' are often regarded as interchangeable concepts, the stricter distinction is that postulates are generally related to *particular fields* of knowledge (so Euclid's postulates for geometry relate to what can be said about lines, angles and circles). The Balance Sheet Equation itself might therefore be regarded as the fundamental postulate of DEB, so that, together with the axioms, that is sufficient to generate all the rules of the system.[7]

[5] In correspondence (July 19, 2020) Sangster has suggested that he would now prefer 'A self-evident principle or one that is accepted as true without proof as the basis for argument' based on https://www.thefreedictionary.com/Axioms . But the essence of both definitions as generally understood is that mathematical axioms are not just assumed to be true but are self-evidently true and therefore do not require proof.

[6] https://www.sfu.ca/~swartz/euclid.htm (accessed July 29, 2020). I do not discuss more modern theoretical developments in mathematics and in the philosophy of mathematics.

[7] I classify it as a postulate not an axiom as, like Euclid's fifth postulate (https://mathworld.wolfram.com/EuclidsPostulates.html), it is contestable, as shown by the economic controversies over whether 'capital' is adequately represented by the total of 'net assets' (e.g. Bromwich et al. 2010). And modern developments in financial instruments have created

However, while an axiomatic approach allows exposition of DEB through the 'Balance Sheet Equation' (as in Appendix 1 here), this approach was not developed until the 19th century (Gentili and Giacomello 2017). Nevertheless, it has been argued (e.g. Macve 1996) that Pacioli had an *implicit* understanding of this logic as he begins his treatise by showing how to prepare an opening 'inventory' (i.e. effectively a balance sheet of (net) assets and the equivalent amount of owner's equity capital); then explains how transactions are to be both 'debited and 'credited' so that the total of the debits is always equal to the total of the credits; and concludes by explaining how to prepare a closing profit and loss account and balance sheet from the resulting balances in the ledger. While there are several technical problems with his exposition (Yamey 1994a) the underlying logic does seem clear.

What do Sangster's (2018a) arguments add to this picture of Pacioli's achievement? His Figure 2 (at p.309—reproduced here as Appendix 2) claims to show how Pacioli's bookkeeping instructions are derived from underlying 'axioms, postulates and theorems'.[8] This is claimed to be pedagogically effective but, if so, why in explaining to his readers his 'axiomatic system' does Pacioli only reach 'Axiom 1' in Chapter 9? And then wait until Chapter 12 to reach 'Axiom 4' and 'Axiom 5'? Euclid had recognised that the whole edifice of his geometry could be built logically on his axiomatic foundations and he demonstrated it through his exposition, stating the 'common notions' (axioms and postulates) *at the start* and building up from there.[9]

In order to operationalise the DEB system in a given situation, or when regulators set a common accounting standard to apply to all situations to facilitate

ongoing dilemmas as to how to fully distinguish debt from equity within the Balance Sheet equation (e.g. IASB 2018).

[8] A theorem is 'A general proposition not self-evident but proved by a chain of reasoning; a truth established by means of accepted truths.' https://www.lexico.com/definition/theorem (accessed. 12.16.20). A well-known example is Pythagoras's Theorem about the properties of a right-angled triangle.

[9] It is also not clear why Sangster does not present the 'Axioms' sequentially: 'Axiom 2' in his Figure 2 (see Appendix 2 here) is presented after 'Axiom 3' and 'Axiom 4'.

comparability (e.g. Macve 2014), not only is choice needed of the appropriate common measurement unit to be the *numéraire* (such as $ or £ or 'constant purchasing power units')—labelled by Sangster as 'Axiom 5'—but a choice must also be made of how the recognised assets and liabilities are to be 'valued' (such as at historical cost; replacement cost; fair value; discounted present value; or even 'target price' as recommended by Pacioli (von Gebsattel 1994: 54)). These of course are the areas of controversy that are the primary focus of ongoing debates over FASB and IASB 'financial accounting standards' (e.g. Macve 2015).

By contrast the 'axioms, postulates and theorems' proposed in his Figure 2 by Sangster (2018a) (see Appendix 2 here) are a bewildering mixture of the intuitively obvious (his 'Axiom 5'), some definitions, and several plausible but not necessary propositions. It is also unclear how it has been decided which are 'axioms', which are 'postulates', and which are 'theorems'. Taking them in sequence (although as noted, Pacioli does not set them out or summarize them as a sequence, rather they are scattered throughout the *de Scripturis* without any particular emphasis, so that their logical function is obscured), 'Axiom 1' sounds plausible for business transactions but it is not clear how it covers gifts received or given.[10] And the opening inventory of assets, with the entry of which in the accounts Pacioli begins his treatise, is the result of unknown previous transactions not of current transactions.

It is not clear what 'Axiom 2' (which is given twice) adds to the structure.[11] 'Axiom 3' and 'Postulates 1 and 3'[12] (or more generally 'Theorem 2' and 'Theorem 4') are simply a naming convention (as the convention could as well have been 'an item on the left' and 'an item on the right' or indeed 'called dog' and 'called cat' or

[10] In the 'Balance Sheet equation approach' (Appendix 1 here) one would be able to record a cash gift received as 'Dr Cash; Cr. (Equity) Capital (or Cr. Profit and Loss or Other Comprehensive Income). C.f. https://advisory.kpmg.us/articles/2019/government-grants-ifrs-compared-to-us-gaap.html (accessed 12.16. 2020).

[11] In correspondence (July 19 2020) Sangster agrees that 'Axiom 2' is redundant but maintains that it had pedagogical value.

[12] The explanation given in Sangster's (2018a) fn.32 for why 'Postulate 2' is not needed is based on the assumption that the owner and 'the business' are separate entities. But Pacioli's opening inventory includes both business and personal items as was common in most economies until much later (e.g. Macve 1985; Goldthwaite 2015).

have adopted any other arbitrary labels as long as they are to be consistently applied).[13]

'Axiom 4' appears to be derived from 'Axiom 3' and from 'Theorem 1' (which follows from 'Axiom 1' and 'Axiom 2' and is also applied twice), rather than entailing 'Theorem 1'. It is not clear how 'Theorem 1' is to be applied to (economically) 'unequal exchanges', such as 'fire sales' and the medieval and Islamic practices of disguising interest within the contracted prices through 'dry exchange' contracts to circumvent the religious proscriptions on usury—e.g. Lopez and Raymond 2001, 163).[14]

'Axiom 5' is simply a fundamental rule of standardising the measurement unit if arithmetic calculations are to be operable.

'Theorems 2, 3, 4 and 5' merely confirm internally consistent application of the elements so far derived, but just 2 and 4 would be sufficent.

Where does this leave us? With a basic structure where transactions have two elements which are equal and opposite (so-called 'Axiom 1' and 'Theorem 1'); together with a naming convention for which element shall consistently be labelled 'debit' and which 'credit'. And arguably 'all transactions involve two elements: an item exchanged and an equal form of settlement' (so-called 'Axiom 1') is simply what the word 'transaction' normally *means* in business (but subject to the reservations stated above).[15]

As is well known from the history of DEB, authors of manuals, teachers, and students have all always had difficulty in seeing clearly how to apply Pacioli's structure when faced with individual transactions of any complexity (e.g. Yamey 1994a). Sangster's (2018a) attempt to present his structure as purportedly 'axiomatic'

[13] The choice of 'debit' and 'credit' as the labels of course reflects the historical evolution of Italian bookkeeping wherein debtors were originally the main recorded assets and creditors the main recorded liabilities (c.f. Goldthwaite 2015; Sangster 2016).

[14] Under current US and international accounting standards where 'negative goodwill' arises on a takeover/merger transaction when the acquisition price is less than the fair value of the net assets acquired the difference is taken as a gain in the income statement (e.g. IASB 2008).

[15] Even in barter transactions (which Pacioli deals with in Chapter 20) once the 'deal' is done the two parties are accepting the equivalence of the items exchanged.

adds no clarity. By contrast, the logic of the 'Balance Sheet Equation' (Appendix 1 here) enables the student and practitioner to *deduce* how to keep accounts by DEB.

A further crucial limitation of the so-called 'axiomatic' approach as it is described by Sangster (2018a, Figure 2) is that it only deals with *transactions* (e.g. sales, purchases, current expenses). By contrast the 'Balance Sheet Equation' enables one to deduce the correct bookkeeping treatment of *any* item one wishes to record, including accruals such as provisions for depreciation, for bad and doubtful debts, for pensions, for deferred taxes, and for all the other (often controversial) items that constitute the major problems in modern financial accounting. Sangster argues (p.310) that 'It should be emphasized that Pacioli taught the fundamentals of double entry—how to record transactions. This did not include making entries for adjustments. At that time, when calculation of profit and wealth were the exception, this was sufficient for most, if not all, of his intended audience'.[16]

But, even if it is not his primary objective, Pacioli does explain that his system will enable one to track one's profits and one's wealth (e.g. in Chapters 20, 23, 25, 27, 34 (von Gebsattel 1994; see also Macve 1996). Moreover examples of such adjustments are found in contemporary Italian DEB accounts of that time (e.g. de Roover 1955, 1956; Yamey 2012; Goldthwaite 2015; Kuter, Gurskaya, and Aleinikov 2020).[17] Indeed they can be found in non-DEB accounts from earlier times and from elsewhere in the world (e.g. Macve 2002; Yuan and Macve 2019) so the claimed 'axiomatic' structure of Pacioli's DEB would be deficient if it cannot cope with these.

[16] The axiomatic approach given here in Appendix 1 also covers all the other kinds of entries that might be made within DEB including correction of mis-postings, transfers of account balances in 'closing the ledger', and consolidation of the accounts of branches / subsidiaries. It is stretching Sangster's 'Axiom 1' to regard these as 'transactions'. The explanation for Pacioli's restricted scope is normally taken to be that Venetian trade was primarily long-distance voyages to the Far East so accounts were only settled and closed once the voyages were completed on return to Italy and so 'accruals' (Sangster's 'adjustments') were not needed. However, Goldthwaite (2015, 627) comments that Pacioli's exposition reflects 'little of the complexity of the Venetian practice he wrote about, in the light of which some of his methods are downright "crude"'.

[17] Nobes (1982: 309) gives an illustration from the Sienese Gallerani's London branch books of 1305-8 of writing off a horse that had gone lame and was given away. Venetians would have had to write off voyages that failed to return (unless they were insured), providing the storyline for Shakespeare's *Merchant of Venice*.

It should also be noted that although Sangster (2018a) argues that Pacioli was only concerned to explain how DEB worked for transactions, in fact 'Axioms 3, 4, and 5' and 'Postulate 1' are introduced in Chapters 11 and 12 where Pacioli is explaining how the opening inventory of assets (and the owner's corresponding total capital) should be entered via the journal into the ledger.

If 'axioms and postulates' are the foundation of Pacioli's analysis of DEB one might also have expected to find them collected together in the Summary (Chapter 36). So-called 'Axioms 3, 4, and 5' do appear or are implied there but are mixed in without particular emphasis with the other prescriptions given as guidance on how to keep the books properly.[18]

III THE ROLE OF 'KEYS': ARE THEY DEPLOYED IN THE *DE SCRIPTURIS*?

Sangster (2018a, Figure 1) illustrates how Pacioli deployed 'keys' (*chiave*) in his exposition of algebra in the *Summa*. That example is taken from Heeffer (2012, 39). However, Heeffer comments:

> These "general principles" are presented without any argumentation except for numerical examples as a test. Pacioli continues to solve Antonio's problem by again applying a general rule for the division of a number into two parts proportional to the mean term. This rule is also extracted from previous algebraic practice. We can be certain that Pacioli mined Antonio's treatise for general principles such as this, because they are used nowhere else than for solving the problems taken from Antonio. The only sense we can give to the definition of a general principle, which is used only one time, is precisely rhetorical. Pacioli has chosen to present some typical derivations as general rules which are later applied to solve problems in a clear and concise way. Even with the body of evidence against him, we should be careful in accusing Pacioli of plagiarism. At best, we observe here an appropriation of problems and methods. The restructuring of material and the shift in rhetoric is in itself an important aspect in the development of 16th-century textbooks on algebra. Pacioli raised the testimonies of algebraic problem solving from the abbacus masters to the next level of scientific discourse, the textbook. When writing the *Summa*, Pacioli had already almost twenty years of experience in teaching mathematics at several universities. His restructuring of abbacus problem solving methods is undoubtedly inspired by this teaching experience.

[18] For discussion of the differences between Pacioli's approach in Chapter 36 and that in the preceding chapters see Yamey 1994a.

So, as argued above, Pacioli does not rely on axiomatic algebra in formulating these so-called general principles. And although he uses them in the *Summa*, when one turns to the *de Scripturis* one does not find any instances where a similar exposition is applied to developing illustrative examples of DEB: indeed a major criticism of Pacioli has always been that he did not include a complete worked example and it was left to later authors of manuals to provide them (e.g. Yamey 1994a). Sangster (2018a) however argues that examples were unnecessary given the clear 'axiomatic' structure of Pacioli's exposition.[19]

I therefore conclude that Sangster's argument fails in the following respects:

1. Pacioli's exposition of algebra in the *Summa* through 'keys' is not axiomatic (see Section II above).
2. Pacioli does not actually use similar 'keys' in the *de Scripturis* to explain DEB.
3. So even if the 'keys' in the *Summa* were axiomatic, Pacioli does not explain the rules for DEB in the *de Scripturis* by deriving them from axioms.

So a more realistic description of Pacioli's propositions that Sangster (2018a) labels 'axioms, postulates and theorems' in his Figure 2 is that they are simply general rules that the student of DEB must learn to follow. But how to apply them in individual cases is not made intuitively more obvious than is following the rules that later manuals and instructors developed in trying to further explain DEB but which Sangster (2018a, 309-310) criticizes as pedagogically confusing. As noted above, while it has been argued (e.g. Macve 1996) that Pacioli did understand the underlying logic of the Balance Sheet Equation (as in Appendix 1 here), that was not clearly expounded until the 19th century (Gentili and Giacomello 2017).

[19] Pacioli does give numerical examples of journal entries in Chapters 12, 15, 16, 18, 20, 21, 22, 24, 28, 34, and the untitled appendix. But of these only Chapters 12 and 18 are argued in Sangster (2018a) Figure 2 (see Appendix 2 here) to contain 'axioms' and 'postulates' and only Chapters 18 and 20 relate to trading transactions.

IV. THE HISTORICAL SIGNIFICANCE OF PACIOLI'S *DE SCRIPTURIS*

I would argue that Pacioli's exposition of DEB is a reasonably useful mercantile appendage to the *Summa* rather than a derivation from its mathematics and in this respect is therefore similar to the section on weights and measures, currencies etc. ('Tariffs') that he also included (Yamey 1994a).[20]

This does not diminish its significance as the first printed exposition of DEB, along with the pathbreaking accessibility of the *Summa* given its use of the vernacular rather than Latin and Arabic rather than Roman numerals. But it suggests that the lens through which to appreciate it is one that scans its location within the longer history of European and then North American accounting development; and one that contrasts this history with that of other cultures that developed significant mercantile economies. Here I give only a tentative outline and further research is clearly needed.

What is important about 'double-entry bookkeeping' (DEB)?

The most important intellectual advance historically was the development of *monetization* of all assets so that stocks of 'apples and pears' and the flows of changes in their quantities could be accounted for not only as physical quantities (as in Ancient Greece and Rome—Macve 2002) but could be included in the accounts at monetary amounts in calculating a monetary total of net assets and income. And once that had been achieved there was potentially no limit to what could be included. So now there can be arguments over how far 'intangible' assets should be included

[20] Pacioli plagiarised his *Summa, Distinctio 9, Tractatus 12* on *Tariffa* by copying it from *Questo e el libro cho tracta di mercanti…* originating from 1481 in Florence. The author is unknown but was possibly Giorgio Chiarini. A copy of the original is held in the library of the Institute of Chartered Accountants in England and Wales in London UK. (Pacioli's *de Scripturis*, on DEB, is *Tractatus 11* in that *Distinctio* and Yamey 1994a argues that this was not plagiarised from another manuscript.)

alongside traditional physical assets, and even whether the 'present value of expected future residual earnings' should be included too, so that the balance sheet total represents the 'value of the firm' while maintaining the Balance Sheet Equation (see Appendix 1, c.f. Horton, Macve and Serafeim 2011).

Accounting has built on this key advance in various ways to enable changes in net assets to be tracked and these recording techniques have often been developed (as in China and Japan)[21] without any formal theoretical underpinning. What does DEB add?

I have argued above that Pacioli's exposition was not based on an explicit axiomatic foundation, as claimed in Sangster (2018a). But it conveyed the essential structure of DEB. While there are several subsidiary features of techniques of writing and processing the 'books' that may vary within medieval Western bookkeeping systems (e.g. Sangster 2016), the essence of the commercial accounting by DEB that captivated Weber and Sombart is that the rigid requirements for *completely* doubled entries (labelled 'debit' and 'credit' in the West) produce a 'closed system' (e.g. Mattessich 2000, Introduction, p.13), that provides as periodic precipitates a Profit and Loss Account (income statement) and a Balance Sheet of assets and liabilities (see also Goldthwaite 2015, 618-9).

These financial statements are widely interpreted as summarising a business's progress and the state of its capital (cf. Macve 1996).[22] In this view of 'What is the importance of DEB?', although the presence of integrated and cross-referenced real, personal, and nominal accounts may be the most significant medieval advances in Western bookkeeping technique, nevertheless, as the nominal accounts (e.g. for sales, purchases, expenses) are conceptually only temporary subdivisions showing the sources of changes in equity, they do not add to the underlying logic of the 'closed system' where Equity = Assets – Liabilities and where Income (allowing for

[21] See e.g. Yuan and Macve, 2019; Ogura 1982; Zhang 2018, 2019. See also Previts, Walton, and Wolnizer 2011.

[22] For modern economic analyses of 'capital' and 'income' see Bromwich et al. 2010.

dividends to owners etc.) represents changes in Equity (as Mattessich 2000, Introduction p.13 observes)—see Appendix 1 here for analysis of the structural relationships.

However Sangster (2016) imposes further processing conditions for recognising DEB. In the supposed evolution from single-entry to DEB he recognises an intermediate stage of what he labels 'dual entry', where there are the functional reciprocal or 'doubled' entries needed for credit transactions and their settlement but the *location* of the corresponding account in the books is not identified. In his view it is the *cross-referencing* of each side of the entry *to identify the location of the corresponding account* that he regards as crucial for there to be fully developed DEB.[23] His emphasis is on the *form* rather than the *functio*n.

Such efficient and accurate processing requires full pagination of the DEB books for cross referencing (nowadays built into the logic of computer systems). Sangster (2016) argues that before this there was often only 'dual entry' within banks' accounts but not full 'double-entry' (DEB).

It is clear that indigenous Chinese accounting, as found in surviving examples from the 16th century onwards (for example in the records analysed in Yuan et al. 2017 and Yuan and Macve 2019), like other Eastern systems using traditional Chinese characters, did not have the indexed and page-numbered books that Pacioli recommends and that are needed to satisfy Sangster's criterion—but this processing technique is not essential to what Mattessich (2000) correctly identifies as the 'logic'

[23] The medieval evolution of such indexing and ordering of books, beginning in European universities with the Bible, is traced in Hoskin and Macve (1986). Sangster (2016) ascribes its adoption in commercial account books to Florentine legal requirements but without explaining *why* courts would have demanded such full DEB books in evidence. Yamey (1994b, 258, fn.34) comments 'I am not aware of any specific evidence that courts of law or other judicial authorities gave greater probative weight to a merchant's account books when they were kept by double entry rather than in any other way'. Yamey (2012), looking at various jurisdictions, develops the argument further that well-kept books (whether or not by DEB) and the character of the merchant were what weighed with the courts. Goldthwaite (2015) describes the Florentine obsession with carefully keeping DEB account books for both personal and business affairs that went beyond any obvious economic benefits.

of DEB; and I would argue for Eastern accounting having the essential contents and functional features of DEB *if* it could be shown that there was indeed integration across the three levels of the books (i.e. day books / classified 'ledger' accounts / financial statements of profit and loss and net assets), as has been claimed by other authors (e.g. Aiken and Lu 1998). This would be sufficient to achieve Mattessich's 'closed system' and provide the functional feature of DEB that has generally been regarded as the most significant, i.e. its ability to systematically track 'profit' and 'capital' (albeit that it is not the only method by which this can be done, e.g. Macve 1985).

So there is no reason to believe that the lack of 'full' (i.e. paginated, indexed and cross-referenced) DEB outside the West would inhibit the development of more complex businesses, given that major Western companies such as the Dutch East India Company (Robertson and Funnell 2012), and the Fugger and Rothschild banks, did not use DEB (e.g. Yamey 1978, 1982); and even during the Industrial Revolution companies were successful without DEB (King 2010; Lemarchand 1994; cf. Soll 2014). The initial development of DEB in the West can be seen as primarily reflecting the developments in textual ordering and reordering within the academic world of the new European universities rather than a response to 'business needs' (Hoskin and Macve 1986; c.f. Frandsen and Hoskin 2018).

Looking ahead, the 19th century embedding of DEB in the US was part of the 'managerial revolution' in administrative coordination of huge M-form businesses (Chandler 1977; Hoskin and Macve 2000). What became the early-20th century 'Weber-Sombart' thesis has created the 'myth' of DEB's crucial *economic* role in Western capitalism (cf. Yamey 1949; Yuan et al. 2017) (and hence its use being banned in Chairman Mao's PRC). [24]

[24] Along with other Western technologies, DEB was then reimported into the PRC under Deng Xiao Ping's 'reform and open' policy that started in 1978 (e.g. Ezzamel and Xiao 2015; Macve 2020).

V. CONCLUDING REMARKS

Sangster (2018a) attributes to Pacioli in 1494 derivation from axioms of both his algebra and his exposition of DEB.[25] Although, as Sangster argues, it is indeed an advance on the *abbaco* teachers' schoolroom methods of instruction (Heeffer 2012; cf. Goldthwaite 2015), Pacioli's exposition of algebra in his *Summa* still only gives generalised principles (rules or 'keys') that are demonstrated by numerical examples, but without formal proof. It is only some 150 years later in the middle of the 17th century that 'algebra starts from some simple facts that can be formulated as axioms….[w]ith specific reference to Euclid's *Elements*' (Heefer 2012, 38-39; 47). Pacioli's exposition of DEB similarly only states some generalised rules that need to be memorised, but they are not presented or summarised in a logical order (c.f. Sangster 2018a, 309, Figure 2 (see Appendix 2 here)) and do not even exhibit the structure of the algebraic 'keys' he developed in his *Summa*. So we do not need any 'paradigmatic shift' in our understanding of Pacioli.

Sangster concludes (2018a, 311): '[Pacioli] developed a sound, straightforward, and reliable principles-based (*sic*) method of instruction in double entry… Are these revelations not what historical research should seek to do, to provide us with a clear view of what occurred in the past and why, to identify the values of that time, contrast them with our values today and consider if there are lessons to be learned from the historical account?'. I have argued here that Sangster's 'lens' still only enables us to 'see through a glass, darkly'[26] and historical understanding can only be achieved by considering the context of the emergence, dissemination and institutionalization of DEB in the West and by putting it in comparative perspective with developments in the East, where further research can now be most valuably

[25] Pacioli translated Euclid from Greek into Latin in 1509.
[26] *1 Corinthians 13:12*.

focussed (Dobie and McCollum-Oldroyd 2020). Rather than further poring over Pacioli, the research importance of further collaborative study of China and its accounting history, alongside that of other Eastern economies, is clear in order to illuminate and inform the mainstream of ‘comparative international historical accounting research’ (Carnegie and Napier, 2012).

APPENDIX 1

Structure of DEB

The Balance Sheet Equation and the Articulation of Financial Statements

Resources	=	Claims		
Assets	=	Claims by Outsiders ('Liabilities')	+	Claims by Owners ('Equity capital')
A	=	L	+	E
A - L[27]	=			E
Δ (A - L)[28]	=			ΔE
Δ(A - L)	=			Δ[E1 + E2]
Δ(A - L)	=			Δ[E1a + E1b] + ΔE2

- ΔE1 = 'Comprehensive income'
- ΔE1 = ΔE1a + ΔE1b
- ΔE1a = 'net income' (US) / 'net profit' (UK)
- ΔE1a = 'income - expenditure' or 'revenue - costs' (as in 'income statement' / 'Profit & Loss Account')
- ΔE1b = 'other comprehensive income'
- ΔE2 = transactions with owners (e.g. new share issues; dividends)

If ΔE2 is typically dividends it can equivalently be written as -D (and any new capital contributions are 'negative dividends'). The 'clean surplus equation' (Feltham and Ohlson 1995) is then:

$$BV_t = BV_{t-1} + Y_t - D_t$$

where:
BV_t is ending book value (i.e. A-L) and BV_{t-1} is opening book value for the period ended at time t,
Y_t is the period's 'clean surplus' income (i.e. ΔE1),
and D_t is the period's dividends (i.e. ΔE2).[29]

The 'clean surplus equation' in its simple algebra therefore also reflects Euclid's fundamental axioms (in particular that if a=b, then a+c = b+c).[30]

Note however that Hicks argued that this bookkeeping approach to *measuring capital and income* (which still underpins the Conceptual Frameworks of modern accounting standard setting bodies—e.g. Macve 1997—as well as modern financial analysis—e.g. Penman 2010) does not necessarily supply the most relevant information for business and investment decisions (Bromwich et al., 2010).

[27] 'A' and 'L' can be subdivided into 'accounts' for any number of component assets and liabilities (or groups of these) as required. If these are presented so that increases in assets are entered on their left side, and increases in liabilities and equity on their right side, they form the familiar pattern of 'T' accounts.

[28] Δ to be read as 'change(s) in'.

[29] Relatedly, the 'value of the firm' at time $_t$ can be expressed as (BV_t + 'the present value of expected future residual earnings') (e.g. Feltham and Ohlson 1995).

[30] Equivalently, a-c = b-c; a+c-c = b; and a = b+c-c (the possible variant entries within DEB).

APPENDIX 2
Figure 2 from Sangster (2018a) p.309

FIGURE 2: Double entry using axioms

- ❒ **All transactions involve two elements: an item exchanged and a form of settlement {Axiom 1}** *(Chapter 9)*
- ❒ **One element is debit and the other is credit {Axiom 3}** *(Chapter 11)*
- ❒ **Debit = credit {Axiom 4}** *(Chapter 12)*

    ⇒ The amount of the form of settlement = the amount of the item exchanged [Theorem 1]

- ❒ The amount of the form of settlement = the amount of the item exchanged [Theorem 1]
- ❒ **All forms of settlement can substitute for each other {Axiom 2}** *(Chapter 9)*
- ❒ Cash received is a debit <Postulate 1> *(Chapter 12)*
- ❒ Cash is a form of settlement *(Chapter 9)*

    ⇒ If a form of settlement is received, it is a debit [Theorem 2]

    ⇒ And the item exchanged is a credit [Theorem 3]

- ❒ The amount of the form of settlement = the amount of the item exchanged [Theorem 1]
- ❒ **All forms of settlement can substitute for each other {Axiom 2}** *(Chapter 9)*
- ❒ Cash given is a credit <Postulate 3> *(Chapter 18)*
- ❒ Cash is a form of settlement *(Chapter 9)*

    ⇒ If a form of settlement is given, it is a credit [Theorem 4]

    ⇒ And the item exchanged is a debit [Theorem 5]

- ❒ **The entries in the money column are to be in one currency only {Axiom 5}** *(Chapter 12)*

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