Construct Validity in Accruals Quality Research*

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Abstract

A large body of empirical research in accounting investigates the causes and consequences of accruals quality, reaching numerous influential conclusions. Yet little work has been done to systematically evaluate the validity of the underlying measures of accruals quality. We evaluate these measures using three criteria: (i) Is the measure unaffected by the underlying economic determinants of accruals? (ii) Does the measure consistently reflect errors in accruals? and (iii) Does the measure facilitate tests with sufficient power to detect plausible variation in accrual errors? Using a combination of theoretical modelling and numerical simulations, we show that all measures fail at least one of these criteria. Our evaluation provides new interpretations of existing research and guides the choice of measures and the interpretation of results in future research.

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ABSTRACT

A large body of empirical research in accounting investigates the causes and consequences of accruals quality, reaching numerous influential conclusions. Yet little work has been done to systematically evaluate the validity of the underlying measures of accruals quality. We evaluate these measures using three criteria: (i) Is the measure unaffected by the underlying economic determinants of accruals? (ii) Does the measure consistently reflect errors in accruals? and (iii) Does the measure facilitate tests with sufficient power to detect plausible variation in accrual errors? Using a combination of theoretical modelling and numerical simulations, we show that all measures fail at least one of these criteria. Our evaluation provides new interpretations of existing research and guides the choice of measures and the interpretation of results in future research.

Keywords: accruals quality; construct validity; power; specification.

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Data Availability: Data are available from the public sources identified in the paper.

I. INTRODUCTION

A large body of accounting research analyzes the determinants and consequences of accruals quality [see Dechow, Ge, and Schrand (2010) for a review]. This research reaches numerous influential conclusions. Determinants of high accruals quality include big N auditors (Becker, DeFond, Jiambalvo, and Subramanyam 1998), US exchange listing (Lang, Raedy, and Yetman 2003), investor protection (Leuz, Nanda, and Wysocki 2003), being domiciled in the US (Lang, Raedy, and Wilson 2006), voluntary adoption of IAS (Barth, Landsman, and Lang 2008), but not mandatory adoption of IFRS (Ahmed, Neel, and Wang 2013). Consequences of higher accruals quality include a lower cost of capital (Bhattacharya, Daouk, and Welker 2003; Francis, LaFond, Olsson, and Schipper 2004, 2005), higher trading volume (Bhattacharya et al. 2003) and greater investment efficiency (Biddle and Hilary 2006). While this research employs a variety of different measures of accruals quality, there has been little attempt to validate these measures. Moreover, the ability of particular measures to separate accruals quality from economic performance has been called into question (e.g., Schipper and Vincent 2003; Butler, Leone, and Willenborg 2004; Hribar and Nichols 2007; Wysocki 2009; Dechow et al. 2010; Liu and Wysocki 2017; Nikolaev 2018).

In this paper, we provide a comprehensive evaluation of construct validity for popular measures of accruals quality. We evaluate construct validity using three criteria. First, we examine the extent to which each measure is dependent on the underlying economic properties of accruals. This criterion is often referred to as discriminant validity. If a measure is affected by variation in economic accruals, then its ability to discriminate between errors in accruals and economic accruals is compromised. Second, we examine whether each measure consistently reflects errors in accruals. This criterion is often referred to as convergent validity. A perfect measure of accruals quality would allow systematic identification of all types of errors in accruals. Convergent validity concerns how the actual measures converge to such a perfect measure. Third, we examine the ability of each measure to correctly detect differences in accruals quality at statistically significant levels. This criterion is often referred to as test power.

Our evaluation employs a combination of theoretical modelling and numerical simulations. We start by constructing a parameterized model of economic earnings and net operating assets. Our model builds on and extends those in Dechow and Dichev (2002), Richardson, Sloan, Soliman, and Tuna (2005), Barth, Clinch, and Israeli (2016) and Nikolaev (2018). For a measure to separate the quality of accruals from the underlying economic performance, it must be invariant to the parameters of the economic earnings process. We evaluate the discriminant validity of different accruals quality measures by checking if they always indicate that accruals quality is perfect when the firm consistently reports its economic earnings and financial position.

To assess convergent validity, we augment our model by adding errors to accruals. Accruals quality is then decreasing in the standard deviation of the added errors. A key challenge in modelling these errors is that their relation with the parameters of the economic earnings process in practice is unknown. We address this challenge by considering four types of errors that have been widely considered in the existing literature: (i) 'white noise errors' that are unrelated to the economic accruals, (ii) 'omitted accrual errors' that reflect the omission of a subset of economic accruals; (iii) 'scaling errors' that are proportional to the underlying economic accruals; and (iv) 'smoothing errors' that are negatively correlated with the underlying shocks to economic earnings. An ideal measure of accruals quality should move in a consistent direction as the magnitude of any of the four types of errors increases. Finally, to assess test power, we examine the values and significance levels of test statistics rejecting the null hypothesis of no difference in accruals quality across simulated samples with plausible levels of variation in accruals quality.

Our analysis relies heavily on numerical simulations of the model. The advantage of our model is that it is sufficiently general to allow for a wide range of plausible properties for both the underlying economic accruals and for accrual errors. But this generality comes at a cost. The derivation of closed form solutions for many of the accrual quality measures becomes intractable and the comparative statics become ambiguous. We therefore calibrate our model using a range of plausible parameters and use numerical simulations to evaluate each measure's discriminant validity, convergent validity and test power.

Our analysis reveals that the construct validity of existing measures of accruals quality is poor. All of the measures have a systematic relation with at least one underlying economic determinant of accruals. Thus, the measures lack discriminant validity and may incorrectly attribute differences in the underlying economic determinants of accruals to differences in accrual quality. Second, none of the measures moves in a consistent direction in the presence of the four types of accrual errors that we consider. Thus, the measures lack convergent validity and may fail to correctly identify differences in accruals quality. Finally, we document wide variation in the power of the different measures to detect plausible differences in accruals quality.

Given that existing measures of accruals quality have poor construct validity, we consider two additional measures that are inspired by our model: (i) the covariance between earnings and past accruals, and (ii) the relative ability of past earnings versus past cash flows to predict current cash flows. While both measures perform relatively well, neither provides a panacea. The former measure ranks highly on discriminant validity but has issues with convergent validity, while the latter measure ranks highly on convergent validity but ranks poorly on discriminant validity and test power. We thus recommend using these measures in combination with existing measures, and we provide more specific recommendations below.

Our analysis has important implications for research on accruals quality. First our analysis highlights that some popular measures of accrual quality have an ambiguous relation with the magnitude of accrual errors. This is because the structure of the underlying errors is typically unknown, and some measures are differentially impacted by different accrual error types. For example, measures of earnings smoothness are decreasing in smoothing errors, but increasing in other types of accrual errors. Consequently, we recommend caution in accepting the conclusions of prior research relying on measures of earnings smoothness. Impactful studies that are subject to caution in this respect include Bhattacharya et al. (2003), Lang et al. (2003), Leuz et al. (2003), Biddle and Hilary (2006), Lang et al. (2006) and Barth et al. (2008).

Second, our analysis provides a comprehensive reference for selecting measures of accruals quality and interpreting results in empirical tests of accruals quality. Given that no single measure achieves either perfect discriminant validity or perfect convergent validity, we recommend independently confirming results across multiple measures that combine to achieve discriminant and convergent validity. Based on our analysis, three measures that we recommend in this respect are *Persistence*, *Accrual Reversal,* and *Relative Information Content*.

Our paper is related to a large body of previous research. Most closely related are other studies that analyze the impact of errors in accruals on measures of accruals quality (e.g., Dechow and Dichev 2002; Richardson et al. 2005; Barth et al. 2016; and Nikolaev 2018). We discuss the relation between our analysis and this prior research in Section 3. Our study is also related to prior analytical research on the determinants of earnings quality (e.g., Marinovic 2013; Ewert and Wagenhofer 2015; Ewert and Wagenhofer 2016). In particular, Ewert and Wagenhofer (2015) evaluate the theoretical properties of several measures of earnings quality in a two period rational expectations model. An important difference between their analysis and ours lies in the definition of 'quality'. Ewert and Wagenhofer (2015) define 'earnings quality' as the amount of information that accounting earnings convey about the terminal value, whereas we define 'accruals quality' as the amount of noise in the reported accounting numbers *relative* to the ideal economic earnings and net operating assets.[1] This difference in focus can lead to different results for the same quality measure. For instance, while Ewert and Wagenhofer (2015) find that *Persistence* is closely aligned with their notion of earnings quality, we show that it generally lacks discriminant validity for our notion of accruals quality since it is affected by the persistence of economic earnings even in the absence of noise in accounting numbers. Furthermore, we show that *Persistence* can incorrectly indicate higher accruals quality in the presence of smoothing errors that do not convey any useful information about firm value.

Finally, our paper is related to prior research evaluating measures of earnings or accruals quality using various empirical proxies for 'quality' (e.g., Richardson, Sloan, Soliman, and Tuna 2006; Hribar, Kravet and Wilson 2014; Perotti and Wagenhofer 2014; Peterson, Schmardebeck and Wilks 2015; Bloomfield, Gerakos, and Kovrijnykh 2018; Du, Huddart, Xue, and Zhang 2020). The proxies used in this research include, for instance, accounting and auditing enforcement releases, restatements, internal control weaknesses, SEC comment letters, audit fees, textual analysis and stock return volatility. The limitation of this approach is that the proxies themselves are also imperfect measures of quality. Thus, a high association between a measure and one of these proxies could be due to misspecification that is common to both the measure and the proxy.

[1] Our criterion of discriminant validity requires that an accruals quality measure must be independent of the underlying parameters of economic earnings. In contrast, according to the definition in Ewert and Wagenhofer (2015), even firms consistently reporting their true economic earnings can have different levels of earnings quality due to the differences in, for instance, their operating risk. The definition employed in Ewert and Wagenhofer (2015) is common in the existing theoretical literature (see, e.g., Ewert and Wagenhofer, 2011, and references therein) but is different from the one that we adopt in this paper.

The remainder of our paper proceeds as follows. Section II describes the measures of accruals quality considered in this paper. Section III presents our model of accruals quality and section IV derives the theoretical properties of the measures of accruals quality. Section V summarizes the simulation results and section VI concludes.

II. MEASURES OF ACCRUALS QUALITY

This section briefly summarizes the seven measures of accruals quality considered in the paper. We begin by describing five measures of accruals quality that are popular in the existing literature and routinely relied upon to make inferences regarding accruals quality. We then summarize two additional measures of accruals quality that are inspired by our model. The measures are summarized in Table 1 and their empirical definitions are provided in the Appendix 2.

The extant literature employs a large number of proxies for accruals quality. The most common proxies are summarized in Dechow et al. (2010). Our analysis focuses on their first four categories of proxies: (i) earnings persistence, (ii) the variability of accruals, (iii) the variability of accruals residuals and (iv) earnings smoothness. These four categories are all based on the properties of earnings, accruals and/or cash flows. Other categories considered by Dechow et al. include timely loss recognition, meeting benchmarks, ERCs and other indirect indicators of accruals quality. As discussed by Dechow et al., an important limitation of these other measures is that they are not based directly on accruals and may reflect factors other than errors in accruals. For example, timely loss recognition also reflects curtailments (Lawrence, Sun, and Sloan 2017), meeting benchmarks reflects managerial effort (Dechow, Richardson, and Tuna 2003) and ERCs reflect asset-pricing parameters (Collins and Kothari 1989).

Within each of the four categories, we identify at least one representative measure of accruals quality. In the earnings persistence category, we identify two popular measures with sufficiently different properties: (i) earnings persistence and (ii) the differential persistence of the cash flow and accrual components of earnings. This leads to the following five measures.[2]

Measure 1: Persistence. Earnings persistence is the estimated coefficient from a regression of current period earnings on last period earnings. A higher coefficient signifies higher accruals quality (see Dechow et al. 2010). This measure reflects the idea that low quality accruals should add transitory errors to earnings that will reduce the persistence of earnings. The obvious shortcoming of this measure is that observed earnings persistence also reflects the persistence of underlying economic earnings.

Measure 2: Differential Persistence. The differential persistence of the cash flow and accrual components of earnings is the difference between the estimated coefficients from a regression of current period earnings on last period cash flows and last period accruals. A relatively higher coefficient on cash flows signifies lower accrual quality (see Sloan 1996). The intuition underlying this measure is that the coefficients on both cash flows and accruals reflect the persistence of underlying economic earnings, while the coefficient on accruals is additionally biased downward by the presence of any transitory errors in accruals. The difference between the coefficient on cash flows and the coefficient on accruals provides an estimate of the amount of transitory errors in accruals (see Richardson et al. 2005).

Measure 3: Standard Deviation of Accruals. Prior research has used the volatility of accruals to measure accruals quality, with higher volatility interpreted as lower quality (e.g., Bartov, Gul, and

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[2] We also evaluated two additional measures that are not presented in this paper. The first is an alternative measure of unsmoothness, defined as the correlation between changes in cash flows and changes in accruals (e.g., Leuz et al. 2003). The associated results are very similar to those for *Unsmoothness*. The second is the estimated coefficient on net operating assets from a regression of current period cash flows on last period net operating assets and current period earnings (Bloomfield et al. 2018). The performance of this measure is relatively poor on all three of the criteria that we consider. These results are available from the authors upon request. Finally, we note that the list of measures evaluated in this paper is by no means exhaustive. Additional accruals quality measures have been suggested in, for instance, Hribar and Nichols (2007), Wysocki (2009), Gerakos and Kovrijnykh (2013), Nikolaev (2018), Bloomfield et al. (2018) and Du et al. (2020).

Tsui 2000; Bergstresser and Philippon 2006). We use the standard deviation of accruals to represent such measures with a higher standard deviation signifying lower accruals quality. The idea behind this measure is that more errors in accruals will lead to a higher standard deviation of accruals. The main shortcoming of this measure is that the standard deviation of accruals should also be higher for firms with greater volatility in their economic working capital (Dechow 1994; Butler et al. 2004; Hribar and Nichols 2007). As such, the measure does not discriminate between legitimate accruals and errors in accruals.

Measure 4: Standard Deviation of Accruals Residuals. Dechow and Dichev (2002) introduce a measure of accruals quality based on the standard deviation of the residuals from a regression of current accruals on current, lead and lagged cash flows. The fitted component of accruals is intended to capture economic accruals. Higher residual standard deviation therefore signifies higher accrual errors and lower accrual quality. Subsequent research questions the ability of this measure to effectively extract the economic component of accruals (e.g., Hribar and Nichols 2007; Wysocki 2009; Liu and Wysocki 2017; Nikolaev 2018).

Measure 5: Unsmoothness. Earnings smoothness is typically measured as the ratio of the standard deviation of earnings to the standard deviation of cash flows (e.g., Leuz et al. 2003). In this paper, we call this ratio *Unsmoothness*, since larger values reflect less smoothness. There is disagreement in the literature regarding the interpretation of this measure (see Dechow et al. 2010). Dechow (1994) popularized the idea that the role of legitimate economic accruals is to smooth the impact of transitory shocks to economic working capital. Consistent with this idea, Dechow (1994) and Francis et al. (2004) interpret a lower ratio as a more desirable attribute of earnings. Yet following Leuz et al. (2003), another line of literature argues that a lower ratio is indicative of lower accruals quality. This line of literature argues that managers introduce ‘smoothing’ errors in accruals to conceal economic performance from outsiders.

Measure 6: Accrual Reversal. This is the first of our two additional measures and is estimated as the covariance between current period earnings and last period accruals. The intuition behind this measure is that any errors in last period accruals will reverse in the current period. Thus, the errors will be positively related to last period accruals and negatively related with current period earnings. This will manifest as a negative covariance between last period accruals and current period earnings. Similar intuition is discussed in prior research including Defond and Park (2001), Baber, Kang, and Li (2011), Dechow, Hutton, Kim, and Sloan (2012) and Allen, Larson, and Sloan (2013). This measure of accruals quality is also implied by simple models of accruals, such as the simple model that we consider in the next section, where it achieves perfect discriminant validity.

Measure 7: Relative Information Content. The second of our two additional measures and is based on a ratio employing the adjusted R-squared (R^2_{adj}) from two regressions. The numerator is ($1 - R^2_{adj}$) from a regression of current cash flows on last period earnings and last period net operating assets, capturing the proportion of the variation in future cash flows that cannot be forecast using accrual accounting. The denominator is ($1 - R^2_{adj}$) from a regression of current cash flows on last period cash flows, capturing the proportion of the variation in future cash flows that cannot be forecast using cash accounting. The ratio should achieve its global minimum in the absence of errors in accruals. It is closely related to previous measures based on the forecasting ability of accounting information. For example, Mikhail, Walther, and Willis (2003) measure earnings quality as the adjusted R-squared from a regression of current cash flows on last period earnings. The key innovation in our new measure is that the denominator attempts to control for the inherent predictability of underlying economic earnings. This measure is also related to various notions of *earnings* quality studied in the theoretical literature (see Ewert and Wagenhofer 2011).

III. MODEL DESCRIPTION

Accounting and Economic Earnings and Net Operating Assets

Assume that there is a filtration, $\{\mathcal{F}_t\}$, that determines the information sets that become available at each date t, $\mathcal{I}_t$. The filtration and the corresponding information sets are exogenous to our model, and our definitions of economic earnings and net operating assets below are relative to filtration $\{\mathcal{F}_t\}$.[3] We assume that the cash flow process $\{CF_t\}$ is adapted to $\{\mathcal{F}_t\}$, i.e., the firm's cash flow in period t is in $\mathcal{I}_t$.

We will call two processes, $\{E_t\}$ and $\{O_t\}$, the firm's economic earnings and economic net operating assets (NOA) relative to filtration $\{\mathcal{F}_t\}$ if they satisfy the following three conditions.

(i) First, both processes are adapted, i.e., both E_t and O_t are in $\mathcal{I}_t$.

(ii) Second, processes $\{E_t\}$, $\{O_t\}$, and $\{CF_t\}$ satisfy the usual clean surplus relation at each date t:

$$CF_t = E_t + O_{t-1} - O_t. \tag{1}$$

(iii) Finally, vector (E_t, O_t) must preserve all value relevant information in $\mathcal{I}_t$: for any $\tau > 0$, the distribution of $(E_{t+\tau}, O_{t+\tau})$ conditional on $\mathcal{I}_t$ must depend only on E_t and O_t.

[3] For instance, information sets $\{\mathcal{I}_t\}$ can reflect all information that is available to managers at date t, all information that is available at date t except for the information that must be ignored by accountants in preparing date-t financial statements (such as the expected profitability of not yet started projects), or all information available at date t plus the eventual cash outcomes of the transactions recognized by accountants during period t (e.g., the actual future cash collections of accounts receivable outstanding at date t). An earnings process satisfying the definition of economic earnings relative to one filtration will not necessarily satisfy the definition of economic earnings relative to a different filtration.

Conditions (ii) and (iii) together imply that vector (E_t, O_t) accumulates all the information in $\mathcal{I}_t$ that is useful for predicting future cash flows, economic earnings, and net operating assets. This concept of economic accruals is consistent with the FASB Conceptual Framework, which states that accrual accounting is important because information about an entity's economic resources and claims and changes in those claims during a period provides a better basis for assessing future performance than information solely about realized cash flows (FASB 2018, OB17).

Assume that processes $\{E_t\}$ and $\{O_t\}$ satisfying conditions (i)-(iii) exist and evolve according to the following VAR(1) model:[4]

$$E_{t+1} = \omega_E E_t + \omega_{EO} O_t + \epsilon_{t+1}, \tag{2}$$

$$O_{t+1} = \omega_O O_t + \omega_{OE} E_t + \eta_{t+1}, \tag{3}$$

where each vector of innovations $(\epsilon_{t+1}, \eta_{t+1})$ is independently drawn from a time-invariant bivariate normal distribution with mean zero and covariance matrix Σ given by

$$\Sigma \equiv \begin{pmatrix} \sigma_\epsilon^2 & \sigma_{\epsilon\eta} \\ \sigma_{\epsilon\eta} & \sigma_\eta^2 \end{pmatrix}. \tag{4}$$

The model in equations (2)-(4) is a standard bivariate VAR(1) process, which is characterized by seven parameters. The first four parameters are the four autoregressive coefficients, ω_E, ω_O, ω_{EO}, and ω_{OE}. The remaining three parameters come from the covariance matrix Σ, in which σ_ϵ^2 is the variance of innovations in the earnings process, σ_η^2 is the variance of innovations in the NOA time series, and $\sigma_{\epsilon\eta}$ is the covariance between contemporaneous innovations in earnings and NOA. In the theoretical model, we assume that the unconditional means of E_t and O_t are equal to zero. This assumption does not entail a loss of generality since unconditional means do not affect any

[4] It is straightforward to confirm that if cash flows can be written as $CF_t = E_t + O_{t-1} - O_t$ for some VAR(1) process $\{(E_t, O_t)\}$, then $\{E_t\}$ and $\{O_t\}$ automatically satisfy conditions (i)-(iii) in our definition of economic earnings and NOA if $\{\mathcal{F}_t\}$ is the natural filtration of process $\{(E_t, O_t)\}$.

of the accrual quality measures that we consider in this paper if such measures are estimated at the firm level. When estimating the seven parameters of the model using a panel of firms, we allow for firm-level fixed effects in regressions corresponding to equations (2)-(3), thus eliminating any effects that the unconditional means might have on the estimated parameters of the VAR(1) model.

Some of the assumptions of our model can be traced back to earlier studies of accruals quality. For instance, it is common to model the time series of economic earnings as an AR(1) process, which is a special case of the VAR(1) model with $\omega_{EO} = 0$; see, e.g., Sloan (1996), Richardson et al. (2005), Gerakos and Kovrijnykh (2013), Barth et al. (2016), Bloomfield et al. (2018), Nikolaev (2018) and Lewellen and Resutek (2019). However, the VAR(1) model in equations (2)-(4) is more general than the AR(1) models employed in most earlier studies. Owing to its larger number of parameters, the general VAR(1) model has the ability to capture additional plausible economic effects. First, the beginning balance of net operating assets, O_t, can have positive or negative predictive power for next period's economic earnings, E_{t+1}, and this relation is captured by ω_{EO}. For instance, a high balance of deferred revenue at date t (which reduces O_t since deferred revenue is a liability) can predict a higher level of sales and hence earnings in the next period, E_{t+1}. On the other hand, a high beginning balance of accounts receivable or inventory, which increases O_t, can also indicate growing demand for the firm's products and a higher value of E_{t+1}.

Second, the general VAR(1) model also captures the idea that economic earnings in period $t + 1$ can have explanatory power for the *ending* balance of net operating assets, O_{t+1}. The direction of this relation is primarily determined by $\sigma_{\epsilon\eta}$. To see this, note that by equation (2), E_{t+1} is informative about ϵ_{t+1}, which, depending on the sign of $\sigma_{\epsilon\eta}$, can be positively or negatively correlated with η_{t+1}. Next, note that through equation (3), O_{t+1} is positively associated with η_{t+1}. In practice, it seems likely that shocks to economic earnings in period $t + 1$ may also affect the ending balances of operating asset accounts, such as inventory and accounts receivable. The parameters ω_{OE} and ω_O also allow for a relation between O_{t+1} and the date-t conditional expectation of E_{t+1}. This is because the conditional expectation of E_{t+1} is determined solely by E_t and O_t by equation (2), and these two variables also enter the right-hand side of equation (3).

Both effects discussed above – the relations between O_t and E_{t+1}, and between E_{t+1} and O_{t+1} – have been studied in the earlier literature.[5] In addition to these effects, our model allows for the persistence of economic NOA as captured by parameter ω_O. This effect can be present in the model even when the economic earnings and NOA time series are independent of each other, i.e., when $\omega_{OE} = \omega_{EO} = \sigma_{\epsilon\eta} = 0$. This distinguishes our model from earlier studies such as Barth et al. (2016), in which the persistence of economic NOA arises due to linear relations between economic earnings and the beginning and ending balances of economic NOA. Parameter ω_O is important from an empirical perspective because, as we discuss below, the estimated persistence of economic NOA is relatively high in empirical data, yet the relation between economic NOA and economic earnings is relatively weak. There are also economic reasons to expect a positive persistence in economic NOA. For instance, if customers buy a firm's goods or services on a subscription basis, then a product promotion in one year is likely to affect deferred revenue balances over multiple periods as subscribers responding to the initial promotion extend their subscriptions.

In our empirical estimation of the general VAR(1) model in equations (2)-(4), all seven parameters exhibit significant variation across industry and size portfolios. In the population of firms, the parameters representing the persistence of economic earnings and NOA (ω_E and ω_O) are relatively high, but the relation between the earnings and NOA time series (captured by

[5] Both effects are present in the model of Barth et al. (2016), which appears to be a special case of the VAR(1) model with an additional constraint on ω_O. Bloomfield et al. (2018) study only the second effect (the explanatory power of economic earnings for the ending balance of net operating assets), while Lewellen and Resutek (2019) focus on the first effect.

parameters ω_{OE}, ω_{EO}, and $\sigma_{\epsilon\eta}$) is, on average, weak. Therefore, to simplify analytics, we also consider a special case of our model in which $\omega_{OE} = \omega_{EO} = \sigma_{\epsilon\eta} = 0$. Under these assumptions, the processes of economic earnings and NOA reduce to two independent AR(1) time series:

$$E_{t+1} = \omega_E E_t + \epsilon_{t+1}, \tag{5}$$

$$O_{t+1} = \omega_O O_t + \eta_{t+1}, \tag{6}$$

We will refer to this special case as the *simple* model, with four parameters – ω_E, ω_O, σ_ϵ^2, and σ_η^2.

We assume that all parameters of our model are such that the resulting processes are stationary. In particular, in the simple model, a sufficient condition for stationarity that we impose is that $0 \leq \omega_O, \omega_E < 1$. The assumption of stationarity is common in the literature but is not innocuous since, in practice, both economic NOA and earnings exhibit growth over time. Empirically, however, accrual models like ours are usually estimated using time series of scaled variables, for which the assumption of stationarity is sufficiently descriptive. One potential issue with this approach is that for scaled variables, the clean surplus relation in (1) should be expected to hold only approximately. We follow the earlier literature on accruals quality in assuming that the clean surplus relation is reasonably descriptive even for scaled variables.

Accrual Measurement Errors

In practice, accounting measurements are imperfect. Let O_t' denote the reported value of net operating assets at date t, E_t' be the reported accounting earnings in period t, and err_t be the NOA measurement error so that:

$$O_t' = O_t + err_t. \tag{7}$$

Cash flow, accounting earnings, and accounting NOA processes satisfy the clean surplus relation in each period, and therefore it follows that

$$E_t' = E_t + err_t - err_{t-1}. \tag{8}$$

Some of the measures of accruals quality that we consider in this paper, such as *Differential Persistence* and *SD of Accruals Residuals*, decompose accounting earnings into its cash and accrual components. Let A_t denote the accrual component of economic earnings, E_t, and A_t' denote the accrual component of the reported earnings, E_t'. Then, by the clean surplus relation,

$$A_t \equiv E_t - CF_t = O_t - O_{t-1}, \tag{9}$$

and, similarly,

$$A_t' \equiv E_t' - CF_t = O_t' - O_{t-1}'. \tag{10}$$

We assume that the time-series of measurement errors $\{err_t\}$ is stationary, and that processes $\{err_t\}$, $\{E_t\}$, and $\{O_t\}$ are jointly normal. These assumptions exclude several intuitive types of accrual measurement errors, such as random walks or errors that result from the application of conditionally conservative asset valuation rules. Yet, as we demonstrate below, the class of errors that we consider is still sufficiently rich to include many of the error types discussed in the earlier literature. Each new error, err_t, can be correlated with the current or past values of the vector of innovations $(\epsilon_{t-\tau}, \eta_{t-\tau})$ for $\tau \geq 0$ but must be independent of the future values of this vector. In fact, any correlation between err_t and $(\epsilon_{t+\tau}, \eta_{t+\tau})$ for $\tau > 0$ would violate condition (iii) of our definition of economic earnings and NOA. The concept of *accruals quality* (AQ) in our model is captured by the unconditional standard deviation of err_t, denoted by σ_{err}. When σ_{err}=0, accruals are of perfect quality; as $\sigma_{err} \to \infty$, accruals quality becomes very low.

The definition of accrual measurement errors introduced above is rather general since we allow for arbitrary covariances between err_t and the contemporaneous and lagged innovations in the economic earnings and NOA processes. However, the reaction of existing AQ measures to measurement errors generally depends on these covariances. Therefore, to explicitly characterize the behavior of different AQ measures in σ_{err}, we consider four types of measurement errors that are commonly discussed in the literature.

Type 1: White Noise Errors. The first type of errors that we consider are independent of the economic earnings and NOA processes. Formally, assume that each err_t is independently drawn from a normal distribution with mean zero and variance σ_{err}^2. This type of error is frequently considered in the theoretical literature on disclosure as well as in the empirical literature on accruals quality, e.g., Dechow and Dichev (2002), Barth et al. (2016), Bloomfield et al. (2018) and Nikolaev (2018). It is also the default option in cases where the researcher does not have strong priors concerning systematic relations between the errors and the underlying economic parameters. Since err_t are assumed to be independent over time, each err_t affects accounting earnings in the origination period, t, and the reversal period, $t+1$. Arguably, this type of error more closely describes estimation errors related to working capital rather than to long term operating assets. For such white noise errors, it will be convenient to express σ_{err} as a share of σ_η, so we will write $\sigma_{err} = \delta\sigma_\eta$ for some $\delta \geq 0$. Note that accruals quality is decreasing in δ.

Type 2: Accrual Omission Errors. Our second type of accrual errors reflects situations where accountants fail to record a component of perfect economic accruals (e.g., Penman and Zhang 2002).[6] Recall that by equation (3), the economic NOA at date $t+1$ is given by:

$$O_{t+1} = \omega_O O_t + \omega_{OE} E_t + \eta_{t+1}.$$

Assume that at date $t+1$ accountants know O_t and E_t with certainty. Assume further that the accountant does not fully recognize the new shock to the economic NOA, η_{t+1}, either because she does not have sufficient information to make the perfect accrual, or because accounting rules disallow it (e.g., the requirement to immediately expense of all investments in research). We model such omission errors by decomposing η_{t+1} into two orthogonal components. The first component, η_{t+1}^i, is included in the accounting NOA, and the second component, η_{t+1}^e, is excluded:

[6] We thank Valeri Nikolaev for helpful discussions regarding this type of accrual error.

$$O_{t+1} = \omega_O O_t + \omega_{OE} E_t + \underbrace{\eta_{t+1}^i + \eta_{t+1}^e}_{\eta_{t+1}}. \tag{11}$$

Then, the reported NOA at date $t+1$ is given by

$$O'_{t+1} = \omega_O O_t + \omega_{OE} E_t + \eta_{t+1}^i = O_{t+1} - \eta_{t+1}^e, \tag{12}$$

and the accruals measurement error is $err_{t+1} \equiv -\eta_{t+1}^e$.

To ensure that the total variance of η_{t+1} and the covariance between η_{t+1} and ϵ_{t+1} are unchanged from our original VAR(1) model, the variables ϵ_{t+1}, η_{t+1}^i, and η_{t+1}^e have the following covariance matrix for some $0 \leq \delta \leq 1$:

$$\begin{pmatrix} \sigma_\epsilon^2 & (1-\delta^2)\sigma_{\epsilon\eta} & \delta^2\sigma_{\epsilon\eta} \\ (1-\delta^2)\sigma_{\epsilon\eta} & (1-\delta^2)\sigma_\eta^2 & 0 \\ \delta^2\sigma_{\epsilon\eta} & 0 & \delta^2\sigma_\eta^2 \end{pmatrix} \tag{13}$$

Comparing this matrix (13) to matrix Σ in equation (4), it is straightforward to see that the variance of $\eta_{t+1}^i + \eta_{t+1}^e$ is equal to σ_η^2 and the covariance between $\eta_{t+1}^i + \eta_{t+1}^e$ and ϵ_{t+1} is equal to $\sigma_{\epsilon\eta}$. Parameter δ regulates the amount of accrual measurement error since $\sigma_{err} = \text{StdDev}[-\eta_t^e] = \delta\sigma_\eta$. Therefore, the model of economic NOA in equations (11) and (13) is equivalent to that in (3) and (4). When $\delta = 0$, the accountant makes the perfect full accrual, and accounting NOA is equal to the economic NOA at each date. When $\delta = 1$, the accountant excludes the full amount of the shock to economic NOA.

In contrast to the white noise errors, which are independent of η_t, omission errors have a negative covariance with η_t, the contemporaneous shock to the economic NOA. Moreover, the absolute value of this covariance is equal to the variance of the error itself:

$$\text{Cov}[err_t, \eta_t] = \text{Cov}\left[-\eta_t^e, \eta_t^i + \eta_t^e\right] = -\delta^2\sigma_\eta^2 = -\sigma_{err}^2.$$

Nikolaev (2018) shows that errors with this property arise naturally when accountants estimate the perfect accruals in an unbiased fashion but based on imperfect information sets. Intuitively, when the actual realization of the economic accrual, η_t, is high, accountants who are unbiased but only have access to imperfect information will have underestimated the accrual. Then, their realized error, err_t, will be a relatively large negative number, Conversely, when the actual realization of the economic accrual is surprisingly low, the error of unbiased accountants with imperfect information is going to be positive. The same argument applies when accountants have access to informative signals but are required to ignore them by accounting rules, such as the requirement to expense research costs regardless of their potential benefits.

Type 3: NOA Scaling Errors. For the third type of accrual errors that we consider, the reported NOA understates or overstates the economic NOA by a constant factor at each date:

$$O_t' = (1+\delta)O_t \tag{14}$$

for some $-1 \leq \delta \leq 1$. Accrual measurement errors are therefore given by:[7]

$$err_t = O_t' - O_t = \delta O_t.$$

The magnitude of accrual errors is increasing in $|\delta|$, since $\sigma_{err} = |\delta| \, \mathrm{StdDev}[O_t]$. When $\delta = -1$, accountants do not recognize any operating assets and implement cash accounting. Since most firms operate with positive NOA, the range $-1 \leq \delta < 0$ represents conservative accounting and causes NOA to be understated. At $\delta = 0$, accounting NOA always coincides with the economic NOA, and accruals quality is perfect. The range where $0 < \delta \leq 1$ corresponds to aggressive accounting, and when $\delta = 1$, the reported NOA is overstated by a factor of two at each date.

NOA understatement errors are often employed in accounting theory to model unconditional conservatism, see, e.g., McNichols, Rajan, and Reichelstein (2014). Note that errors of this type are similar to accrual omission errors discussed above in that, in both cases, accountants fail to make an economic accrual. The main difference is that for the accrual omission errors, the omitted

[7] Note that for this type of errors, err_t is correlated with the contemporaneous and all past values of η_t.

accrual is not correlated with the accrual that is actually made ($\text{Cov}\left[\eta_t^e, \eta_t^i\right] = 0$). In contrast, for NOA understatement errors with $\delta < 0$, the omitted accrual is perfectly correlated with the accrual that is made by the accountants. Similarly, NOA overstatement errors (with $\delta > 0$) resemble the white noise errors because in both cases, accountants make an extra accrual that should not have been made. For white noise errors, this extra accrual is uncorrelated with the economic accrual, whereas for the NOA overstatement errors, the extra accrual is perfectly correlated with the economic accrual. In practical situations, the correlation between missing (extra) accruals and accounting (economic) accruals is unlikely to be precisely one or zero, but understanding these two polar cases should be useful for making predictions about the intermediate scenarios.

It might appear that, in the presence of scaling errors, accounting earnings and NOA are still consistent with our definition of perfect economic earnings and NOA since, when δ is a known constant, the information contained in $\{E_t', O_t', O_{t-1}'\}$ is exactly the same as in $\{E_t, O_t, O_{t-1}\}$. Note, however, that to avoid defining economic earnings and NOA only up to an accruals scaling factor, we require in condition (iii) that all value-relevant information at date t must be summarized in $\{E_t \;, O_t \;\}$, without a reference to O_{t-1}. It is easy to see that in our VAR(1) model with scaling errors, vector $\{E_t', O_t'\}$ generally carries less information about future cash flows than $\{E_t \;, O_t \;\}$, and therefore accounting earnings and NOA with errors of this type will not satisfy the definition of economic earnings and NOA.

Type 4: Smoothing. Leuz et al. (2003, p. 509) popularize an earnings management scenario in which managers generate accrual errors to "conceal changes in their firm's economic performance." While Leuz et al. do not formally model this phenomenon, Gerakos and Kovrijnykh (2013) study a setting in which managers manipulate earnings in an effort to conceal innovations in economic performance. In their model, the firm's economic earnings follow an

AR(1) process, and in each period, the manager introduces an error into the ending balance of NOA which offsets a δ-share of the new shock to economic earnings:

$$err_t = -\delta\epsilon_t, \tag{15}$$

for some $0 \leq \delta \leq 1$. We follow the approach of Gerakos and Kovrijnykh (2013) in modeling smoothing errors according to equation (15).

Gerakos and Kovrijnykh (2013) do not explicitly derive an expression for the unconditional variance of accounting earnings in the presence of smoothing accruals. They interpret parameter δ as the degree of smoothing, with $\delta = 1$ corresponding to the most aggressive smoothing scenario. In our terminology, the magnitude of accrual measurement errors is increasing in δ since $\sigma_{err}^2 = \delta^2\sigma_\epsilon^2$. The unconditional variance of accounting earnings, however, is generally not monotonic in δ. For example, in our simple AR(1) model, which is most similar to the setting in Gerakos and Kovrijnykh (2013), the unconditional volatility of accounting earnings is quadratic in δ. It decreases for small values of δ, achieves, achieves its minimum value at $\delta = \frac{1}{2}(1 - \omega_E)$, and increases thereafter. Importantly, at $\delta = 1$, i.e., when the error introduced by the manager exactly offsets the new shock to economic earnings, the unconditional variance of accounting earnings is, in fact, *greater* than the variance of economic earnings. In other words, when the magnitude of smoothing errors is sufficiently large, accounting earnings become more volatile (i.e., less smooth) than economic earnings without error. This is a consequence of the reversal of accrual errors in the subsequent period.

IV. MODEL ANALYSIS

An ideal empirical measure of accruals quality should (i) be independent of the economic parameters of the model, (ii) be monotonic in σ_{err} in a consistent direction for accrual errors of all types, and (iii) facilitate tests with sufficient power to detect accrual errors of plausible magnitude.

The last criterion is addressed solely through simulations. In this Section, we focus on the first and second criteria, which we refer to as discriminant and convergent validity, respectively. None of the measures that we consider have perfect discriminant or convergent validity, and it may be impossible to construct a measure with these properties without further restrictions on model parameters. Our goal is to evaluate the extent to which each measure has compromised validity.

Discriminant Validity

Discriminant validity is the first of our three criteria for evaluating measures of accruals quality. A well-specified measure should not vary as a function of the underlying economic parameters that drive legitimate variation in accruals. Absent discriminant validity, a difference in the average value of an accruals quality measure between two samples of firms cannot be unequivocally attributed to differences in accruals quality, since it may also reflect differences in the underlying economic parameters between the two samples. Failure to control for the parameters of the underlying economic earnings and NOA processes, which generally are not directly observable, can introduce a correlated omitted variable bias in tests of accruals quality.
Perfect discriminant validity would require that an accrual quality measure is independent of the parameters of economic earnings and NOA processes for any value of σ_{err}. We focus on a less demanding notion of discriminant validity that requires independence of model parameters only in the absence of accrual measurement errors, i.e., when $\sigma_{err} = 0$. We will say that a measure achieves discriminant validity if under the economic accounting, it takes on a fixed value regardless of the model parameters.

All of the theoretical results in this paper focus on asymptotic values of accrual quality measures that obtain when such measures are calculated from very long time series of data. In practice, time series of financial numbers available for each firm are relatively short, and therefore empirical accruals quality measures can be subject to small sample biases. We ignore such biases in our theoretical analysis and address them in our simulations. Proposition 1 summarizes theoretical results on the discriminant validity of different accruals quality measures. Supporting proofs are provided in Appendix 1.

Proposition 1.

(1) *In the general model, none of the measures achieve discriminant validity.*

(2) *In the simple model, only Differential Persistence and Accrual Reversal achieve discriminant validity. The relations between the other AQ measures and model parameters are shown in Table 2 panel A.*

Proposition 1 indicates that no measures achieve discriminant validity in the general model, while only *Differential Persistence* and *Accrual Reversal* achieve discriminant validity in the simple model. However, as we demonstrate in the proof of Proposition 1, both of these measures vary in some of the model parameters when the economic NOA has predictive power for next period's economic earnings, $\omega_{EO} \neq 0$. Similar results for *Differential Persistence* are also documented by Lewellen and Resutek (2019), and also hold for our new *Accrual Reversal* measure.

Convergent Validity

We next evaluate the convergent validity of the accruals quality measures. Holding the economic parameters fixed, an ideal accruals quality measure should be monotonically increasing (or decreasing) in σ_{err} for all error types. Without this property, a higher (lower) value of the measure cannot be unequivocally interpreted as indicating higher (lower) accruals quality. The quality of full convergent validity turns out to be very demanding. As we show below, none of the measures we consider in this paper achieves it, even in the simple model and with only four error types. Therefore, we also consider a weaker notion of *partial* convergent validity that only requires that, holding the economic parameters fixed, the measure must achieve its minimum (or maximum) value under economic accounting. Measures with partial convergent validity never identify perfect economic accounting to be of lower quality. However, such measures may still incorrectly rank two accounting treatments with errors. Proposition 2 summarizes our theoretical results on convergent validity with supporting proofs provided in Appendix 1.

Proposition 2.

(1) *In the general model, none of the measures achieve convergent validity and only Relative Information Content achieves partial convergent validity.*

(2) *In the simple model, none of the measures achieve convergent validity and only Relative Information Content achieves partial convergent validity. The relations between the measures of accruals quality and the magnitude of the errors in accruals (σ_{err}) are shown in Table 2 panel B.*

Proposition 2 shows that convergent validity is a demanding criterion that is not met by any of the measures, despite the fact that we consider only four error types. Table 1 provides the closed-form expressions for the different AQ measures under the four error types in the simple model. Depending on the parameter values, each measure can be non-monotonic in the magnitude of errors of at least one of the four types. Importantly, as discussed in greater detail in the next section, all commonly used measures of accruals quality react in opposite directions to some types of errors. Our new *Relative Information Content* measure is constructed to achieve a notion of *partial* convergent validity. While holding the economic parameters fixed, *Relative Information Content* achieves its minimum value under economic accounting.[8] However, this result is of limited utility

[8] This is because the denominator of *Relative Information Content* is determined by the cash flow process alone, and the numerator is inversely related to the predictive power of accounting earnings and NOA for one-period-ahead cash

to an empiricist without access to a perfect accounting control group because *Relative Information Content* is not always monotonically increasing in σ_{err}. For instance, for some parameter values, it initially increases but then decreases in the magnitude of omission errors.

V. NUMERICAL SIMULATIONS

In order to corroborate and extend the insights of our model, we use numerical simulations. Up to this point, most of our theoretical predictions are based on the simple AR(1) model, a limitation convenient for analytical tractability. The first advantage of simulations is that they can clarify the ambiguous relations in Propositions 1 and 2 at realistic parameter values in the actual data (e.g., those marked with a '?' in Table 2). Second, numerical simulations can be used to document the relations in the general model where the economic earnings and NOA processes are linked. Third, simulations can characterize the economic magnitude and statistical significance of the underlying relations. In particular, while Propositions 1 and 2 identify some relations to be monotonic, they do not calibrate the economic magnitude and statistical significance of these relations. In particular, the simulations allow us to calibrate the extent to which the comparative statics relating accrual errors to the accrual quality measures in Proposition 2 translate into powerful tests for differences in accruals quality. Finally, the simulations incorporate a realistic time series length of 15 years per firm. In contrast, the theoretical results in this paper used asymptotic values of accrual quality measures that obtain when such measures are calculated from long time series of data. It is well-known that, for instance, small sample OLS estimates of autoregressive parameters are generally biased; see, e.g., Phillips (1977). We describe details of the simulations in the Online Appendix and summarize the key results below.

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flows. Since vector (E_t, O_t) is predictive sufficient for CF_{t+1}, *Relative Information Content* always achieves its minimum value under economic accounting.

Simulating Relations with Economic Parameters to Evaluate Discriminant Validity

Table 3 summarizes the relations observed in the simulations between the seven measures of accruals quality and the seven economic parameters. The table reports arrows to signify the directions of any observed significant relations and asterisks to signify the statistical significance based on the average *p* value, using one-tailed tests and evaluated at the limit of the range having the smallest average *p* value. In these tests, a significant relation indicates that the economic parameter represents a potentially important omitted variable in tests for differences in AQ. Thus, a significant relation reveals that a measure has compromised discriminant validity. The first major takeaway from Table 3 is that every one of the 7 measures has a significant relation with at least one of the underlying economic parameters. For instance, our simulation results for the *SD of Accruals Residuals* measure corroborate findings in the earlier literature demonstrating that this measure is related to economic parameters; see, e.g., Wysocki (2009) for theoretical results and Nikolaev (2018) for simulations. Importantly, our results show that such misspecifications also pervade other measures of accruals quality, particularly the popular *Unsmoothness* measure. Moreover, once we drop the assumption that the economic earnings and economic NOA processes are unrelated, measures relying on this assumption, such as *Differential Persistence* and *Accrual Reversal*, are also misspecified. The second major takeaway is that all of the relations that are predicted for the simple model in Proposition 1 are observed in the simulations for the general model in Table 3. Finally, all of the measures except for *Persistence* are also related to at least one of the additional three parameters in the general model.

The results in Table 3 indicate that no measure has perfect discriminant validity. The results nevertheless provide guidance for researchers in selecting measures appropriate for their research setting. First, if the researcher has priors that certain economic parameters are similar between the treatment and control samples, then Table 3 allows for the selection of an appropriate measure. For example, if the researcher has priors that the persistence of economic earnings is similar across the treatment and control samples, then *Persistence* should have good discriminant validity, since it is unrelated to the other economic parameters. Second, Table 3 identifies which combination of measures can be used to achieve discriminant validity. For example, *Persistence* is positively related to ω_E but has no significant relation with the other economic parameters, while *Accrual Reversal* is unrelated to ω_E. Thus, if consistent evidence of a difference in AQ is uncovered using both *Persistence* and *Accrual Reversal*, then discriminant validity is achieved. Note, however, that many of the other measures are related to the same underlying economic parameters. For example, *Differential Persistence*, *SD of Accruals*, *SD of Accruals Residuals, Unsmoothness* and *Relative Information Content* are all related to the variance of the innovation in working capital (σ_η^2). Thus, using these measures in combination will not achieve discriminant validity.

Simulating Relations with Induced Errors to Evaluate Convergent Validity and Test Power

Table 4 summarizes the relations observed in the simulations between the seven measures of accruals quality and the four accrual error types. Arrows signify how the direction of any significant relation between each AQ measure and each error type evolves as the standard deviation of the error is increased from zero toward the limit of the range. *, ** and *** indicate whether the associated average *p* value is significant at the 10%, 5% and 1% significance levels, using one-tailed tests. Remember that in these tests, a significant relation indicates that the AQ measure successfully detects errors in accruals. But to achieve perfect convergent validity, a measure must not only be related to each type of accrual error, but also have a consistent relation both within and across the four types of errors. Table 4 shows that some measures have a significant but non-monotonic relation with accrual errors, while other measures have significant, monotonic relations with each type of error, but the sign of these relations differs across error types. Under such conditions, it is unclear whether a significant result signifies higher accruals quality or lower accruals quality. Thus, these measures lack convergent validity.

The first major takeaway from Table 4 is that all of the predictions of Proposition 2 are borne out with one exception. *Relative Information Content* has no significant relation with NOA scaling errors and thus has low test power for this type of error. The second major takeaway from Table 4 is that none of the seven measures achieves perfect convergent validity. The measures that get the closest to achieving convergent validity are *Persistence* and our two new measures, *Accrual Reversal* and *Relative Information Content*. *Persistence* is strictly decreasing in all types of measurement error with the exception of smoothing errors. For smoothing errors, the simulations show that *Persistence* initially increases up to a smoothing coefficient of around $\delta = 0.3$ and then decreases. Consequently, a higher measure of persistence can be due to less error of the first three types or more error of the fourth type. Thus, a higher measure of persistence does not unambiguously identify higher accruals quality.

The simulations indicate that the non-monotonic relation between *Persistence* and smoothing errors is mirrored in *SD of Accruals Residuals* and *Unsmoothness*. These measures are initially decreasing in smoothing errors up to a smoothing coefficient of around $\delta = 0.3$ and then start increasing. These relations tell us that smoothing only ‘works’, in that it only lowers the volatility of reported earnings and associated measures of smoothness, if the smoothing parameter is not too high. Presumably, managers engaging in opportunistic earnings smoothing will strive to keep the smoothing coefficient below the point at which it stops working. In this case, the only relevant range for evaluating the measures is to the left of $\delta = 0.3$. But note that in this case, many of the measures move in a different direction for smoothing errors than they do for other error types. For example, *SD of Accruals Residuals* and *Unsmoothness* are increasing in white noise errors but locally decreasing in smoothing errors.

The conflicting results for smoothing errors versus many other types of errors explain the inconsistent interpretation of results in the existing literature. Some research interprets <u>higher</u> earnings volatility, accruals residuals volatility and unsmoothness measures as evidence of higher accruals quality (e.g., Leuz et al, 2003; Barth et al., 2008), while other research interprets <u>lower</u> values of these same measures as evidence of higher accruals quality (e.g., Dechow and Dichev, 2002; Francis et al., 2004). The key to reconciling these conflicting findings is in understanding that the research imposes different priors regarding the structure of the underlying errors in accruals. The former research assumes the structure of smoothing errors, while the latter research assumes the structure of white noise errors. But since the structure of the underlying errors is unknown, the conclusions of this research remain open to question.

While the results in Table 4 indicate that no measure has perfect convergent validity, they do offer guidance for researchers in selecting measures appropriate for their research setting. First, if the researcher has priors concerning the structure of the accrual errors, then Table 4 allows for the selection and interpretation of a suitable measure. For example, if the researcher has priors that the accrual errors are white noise, then testing for lower *Persistence* in the treatment versus control samples provides a simple and powerful test. Second, Table 4 identifies which combination of measures can be used to achieve convergent validity. For example, *Persistence* has a strong and negative relation to all types of errors except smoothing errors while *Relative Information Content* has a strong positive relation with all errors except for NOA scaling errors. Thus, observing both lower *Persistence* and higher *Relative Information Content* is consistent with higher errors. Note, however, that many of the other measures fail to move in a consistent direction for different types of errors and so we caution against their use in the absence of strong priors about error type.

Implications for Selecting and Interpreting Measures of Accruals Quality

Tables 3 and 4 provide a comprehensive reference for selecting appropriate measures of accruals quality. Given that no single measure achieves either discriminant or convergent validity, we recommend that researchers corroborate results across select combinations of measures. If the researcher has priors regarding either similarities in economic parameters between the treatment and control samples or the types of errors that are present in the treatment sample, then Tables 3 and 4 guide the selection of suitable measures. Absent such priors, the researcher should employ multiple measures that combine to achieve discriminant and convergent validity. Based on the results in Tables 3 and 4, we recommend using the combination of *Persistence*, *Accrual Reversal* and *Relative Information Content*. For example, if a researcher finds evidence of lower *Persistence*, lower *Accrual Reversal,* and higher *Relative Information Content* in the treatment sample relative to the control sample, the researcher can conclude that earnings quality is lower in the treatment sample. The shortcoming of such an approach, however, is that it sacrifices test power (i.e., more type II errors) in order to improve test specification (i.e., fewer type I errors).[9]

The results summarized in Tables 3 and 4 also provide guidance for interpreting the results in new research or reinterpreting the results in existing research. For example, a large and prominent body of research relies heavily on measures of earnings smoothing to measure earnings quality (e.g., Lang et al. 2003; Leuz et al. 2003; Lang et al. 2006; Barth et al. 2008). This research generally finds that the treatment samples have smoother earnings, concluding that their earnings quality is <u>lower</u> due to higher smoothing errors. However, the results in Table 4 show that their earnings

[9] In our simulations, *Accrual Reversal* is unable to detect errors due to omitted accruals, while *Relative Information Content* is unable to detect NOA scaling errors and exhibits low power in detecting some other error types.

quality could also be interpreted to be higher due to lower white noise errors, lower omission errors or lower NOA scaling errors.

VI. CONCLUSIONS

We conduct a comprehensive analysis of construct validity and statistical power for seven measures of accruals quality. We provide several new insights. First, our analysis indicates that all seven measures lack both discriminant validity and convergent validity. Importantly, some popular measures that have been heavily relied upon to reach important conclusions in previous research appear to have poor construct validity. These measures include *SD of Accruals Residuals* and *Unsmoothness*. Second, we find that some less popular measures, including two measures that we propose in this paper, have somewhat better construct validity. These measures include *Persistence*, *Differential Persistence*, *Accrual Reversal* and *Relative Information Content*.

Another important contribution of our analysis is to highlight that some measures have an ambiguous relation with the magnitude of accrual errors. This is because the structure of the underlying errors is typically unknown, and some measures are differentially impacted by different types of errors. For example, *Unsmoothness* is increasing in white noise errors and (locally) decreasing in smoothing errors. This explains why different researchers have attached opposite interpretations to the same results. Consequently, we recommend caution in accepting the conclusions of prior research relying on such measures.

Finally, we provide a comprehensive reference for selecting measures and interpreting results in empirical tests of accruals quality. Given that no single measure achieves perfect discriminant or convergent validity, we recommend independently confirming results across multiple measures that combine to achieve discriminant and convergent validity. Three measures that we recommend in this respect are *Persistence*, *Accrual Reversal* and *Relative Information Content*.

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APPENDIX 1
Proof of Propositions 1 and 2

Step 1. Deriving analytical expressions for AQ measures in the simple model

For both Propositions 1 and 2, we need to derive closed-form expressions for all accruals quality measures in the four-parameter special case of the model. Moreover, these expressions need to be derived for errors of all four types. Most of these expressions are presented in Table 1; the most cumbersome ones are omitted for brevity and are available from the authors upon request. Here, we describe the process that can be followed to derive these expressions.

To fix ideas, let us focus on the white noise errors. The first step in deriving closed-form expressions for AQ measures is to calculate the covariance matrix of the following five variables: E'_{t+1}, E'_t, O'_{t+1}, O'_t, O'_{t-1}. This five-by-five matrix is presented in expression (18), which can be found at the end of this appendix. To illustrate how this matrix is prepared, consider, for instance, the element in the second row of the first column:

$$\mathrm{Cov}[E'_{t+1}, E'_t] = \frac{\omega_E \sigma_\epsilon^2}{1-\omega_E^2} - \sigma_{err}^2.$$

To prove this expression, note that

$$\begin{aligned}\mathrm{Cov}[E'_{t+1}, E'_t] &= \mathrm{Cov}[E_{t+1} + err_{t+1} - err_t, E_t + err_t - err_{t-1}] \\ &= \mathrm{Cov}[E_{t+1}, E_t] - \mathrm{Var}[err_t] = \mathrm{Cov}[\omega_E E_t + \epsilon_{t+1}, E_t] - \sigma_{err}^2 \\ &= \omega_E \,\mathrm{Var}[E_t] - \sigma_{err}^2.\end{aligned}$$

It remains to show that

$$\mathrm{Var}[E_t] = \frac{\sigma_\epsilon^2}{1-\omega_E^2}.$$

The latter expression follows from the following two facts about the AR(1) process for earnings:

$$\mathrm{Var}[E_t] = \mathrm{Var}[E_{t+1}]\,,$$

and

$$\mathrm{Var}[E_{t+1}] = \mathrm{Var}[\omega_E E_t + \epsilon_{t+1}] = \omega_E^2\,\mathrm{Var}[E_t] + \sigma_\epsilon^2.$$

The covariance matrices of $(E'_{t+1}, E'_t, O'_{t+1}, O'_t, O'_{t-1})$ for type 1, 2, 3, and 4 errors are presented in expressions (18), (19), (20), and (21), respectively. These matrices are sufficient to derive expressions for all AQ measures except for *SD of Accruals Residuals*. The derivation of the latter measure additionally requires the following observation that can be verified with straightforward algebra:

$$\mathrm{Cov}[CF_{t+1}, CF_{t-1}] = \frac{\omega_E^2 \sigma_\epsilon^2}{1-\omega_E^2} - \frac{\omega_O(1-\omega_O)\sigma_\eta^2}{1+\omega_O} \qquad (16)$$

To illustrate how one can derive closed-form expressions in Table 1, consider first the *Persistence* measure. Let M_1 denote the five-by-five covariance matrix for type 1 errors in (18). Since *Persistence* is the coefficient from the regression of E'_{t+1} onto E'_t, it is well-known that when it is computed from a sufficiently long time series, it converges to:

$$\frac{\mathrm{Cov}[E'_{t+1}, E'_t]}{\mathrm{Var}[E'_t]} = \frac{M_1[2,1]}{M_1[2,2]},$$

where $M_1[i,j]$ denotes the element in row i of column j in matrix M_1. To calculate *SD of*

Accruals, note that

$$(\text{SD of Accruals})^2 = \mathrm{Var}[O_t - O_{t-1}].$$

Then, one can use matrix M_1 to calculate $\mathrm{Var}[O_t - O_{t-1}]$ as follows:

$$\mathrm{Var}[O_t - O_{t-1}] = (0,0,0,1,-1)M_1(0,0,0,1,-1)^t,$$

where t denotes the matrix transposition operator.

Derivations of expressions for some measures require additional algebra. Consider, for instance, *Differential Persistence*. Since this measure is based on coefficients from the regression of E'_{t+1} onto CF_t and accruals $A'_t \equiv O'_t - O'_{t-1}$, we first construct the covariance matrix of these three variables:

$$\begin{array}{c} E'_{t+1} \\ A'_t \\ CF_t \end{array} \begin{pmatrix} \mathrm{Var}[E'_{t+1}] & \mathrm{Cov}[E'_{t+1}, A'_t] & \mathrm{Cov}[E'_{t+1}, CF_t] \\ \mathrm{Cov}[E'_{t+1}, A'_t] & \mathrm{Var}[A'_t] & \mathrm{Cov}[A'_t, CF_t] \\ \mathrm{Cov}[E'_{t+1}, CF_t] & \mathrm{Cov}[A'_t, CF_t] & \mathrm{Var}[CF_t] \end{pmatrix}. \tag{17}$$

All of the elements in this matrix can be derived from M_1. For example:

$$\mathrm{Cov}[E'_{t+1}, A'_t] = (1,0,0,0,0)M_1(0,0,0,1,-1)^t, \text{ and}$$
$$\mathrm{Cov}[A'_t, CF_t] = (0,0,0,1,-1)M_1(0,1,0,-1,1)^t.$$

Since all variables are jointly normally distributed with unconditional means of zero, the conditional expectation of E'_{t+1} takes the following form:

$$\mathbb{E}[E'_{t+1} | A'_t, CF_t] = \alpha_1 A'_t + \alpha_2 CF_t.$$

When *Differential Persistence* is calculated from sufficiently long time series, its value will converge to $\alpha_2 - \alpha_1$. Applying a standard property of multivariate normal distributions, we can calculate vector $\begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix}$ using submatrices of matrix (17) as follows:

$$\begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix} = \left(\mathrm{Cov}[E'_{t+1}, A'_t] \quad \mathrm{Cov}[E'_{t+1}, CF_t]\right) \begin{pmatrix} \mathrm{Var}[A'_t] & \mathrm{Cov}[A'_t, CF_t] \\ \mathrm{Cov}[A'_t, CF_t] & \mathrm{Var}[CF_t] \end{pmatrix}^{-1}.$$

All variances and covariances in the expression above can be calculated using matrix M_1.

Finally, it is instructive to discuss the derivation for *SD of Accruals Residuals* measure. Let B denote the the covariance matrix of $(A'_t, CF_{t-1}, CF_t, CF_{t+1})$. As above, all elements of this matrix except for Cov $[CF_{t-1}, CF_{t+1}]$ can be derived using M_1. For instance, since $CF_t = E'_t - O'_t + O'_{t-1}$ and $CF_{t+1} = E'_{t+1} - O'_{t+1} + O'_t$, we obtain:

$$\mathrm{Cov}[CF_t, CF_{t+1}] = (0,1,0,-1,1)M_1(1,0,-1,1)^t.$$

Since all processes are stationary, the covariance above is also equal to $\mathrm{Cov}[CF_{t-1}, CF_t]$. The expression for $\mathrm{Cov}[CF_{t+1}, CF_{t-1}]$ is provided in equation (16) above. Now let us calculate $\mathrm{Var}[A'_t | CF_{t-1}, CF_t, CF_{t+1}]$. When estimated from long time series, *SD of Accruals Residuals* will be approaching $\sqrt{\mathrm{Var}[A'_t | CF_{t-1}, CF_t, CF_{t+1}]}$. To estimate this quantity, matrix B can be decomposed into four blocks, $B_1, \ldots, B_4$, located as follows:

$$\begin{pmatrix} B_1 & B_2 \\ B_3 & B_4 \end{pmatrix},$$

with dimensions of 1×1, 1×3, 3×1, and 3×3, respectively. Then, the variance of residuals from the regression of A'_t on CF_{t-1}, CF_t, and CF_{t+1} will approach:

$$\mathrm{Var}[A'_t | CF_{t-1}, CF_t, CF_{t+1}] = B_1 - B_2 B_4^{-1} B_3.$$

Analytical expressions for AQ measures under type 2, 3, and 4 errors are obtained as above but using matrices in (19), (20), and (21) in place of M_1. Analytical expressions for AQ measures in the absence of errors can be obtained from the expressions for type 1 (noise) errors by setting $\sigma_{err} = 0$.

Step 2. Proving directional results in Propositions 1 and 2

Directional results in Propositions 1 and 2 can be verified by differentiating the expressions in Table 1 with respect to the model parameters (Proposition 1) or the magnitude of accruals error measured by σ_{err} (for white noise errors in Proposition 2) or δ (for omission, scaling and smoothing errors in Proposition 2) and confirming that the derivatives have the predicted signs. Whenever the derivatives can be signed unequivocally in the region $0 \leq \omega_O, \omega_E < 1$, Propositions 1 and 2 indicate this with an up or down arrow. For smoothing errors, the derivatives of two measures (*Persistence* and *Unsmoothness*) with respect to δ are predicted to switch signs at some critical level δ^*. These critical levels are given by the following expressions:

For *Unsmoothness*, $\delta^* = \frac{1}{2}(1 - \omega_E)$.

For *Persistence*, δ^* is a root of the following equation:

$$-(1 + \omega_E - \omega_E^2) + 2(1 + 2\omega_E)\delta^* + 2\omega_E^2(1 - \omega_E^2)\delta^{*2} = 0.$$

The equation above has two roots, one of which is negative and another one which is between zero and one. The critical value δ^* for the *Persistence* measure corresponds to the second root.

Step 3. *Differential Persistence* and *Accrual Reversal* achieve discriminant validity in the four-parameter but not seven-parameter model.

Discriminant validity of *Differential Persistence* and *Accrual Reversal* in the four-parameter model follows from the fact that in the absence of accrual measurement errors, both measures are equal to zero regardless of other parameter values. To confirm that the two measures do not achieve discriminant validity in the general model, consider a five-parameter setting in which ω_{EO} is different from zero but $\omega_{OE} = \sigma_{\epsilon\eta} = 0$. Using the approach discussed above, it can be verified that:

$$\text{Differential Persistence} = -\frac{\omega_{EO}(1-\omega_O)(1-\omega_E\omega_O)^2\sigma_\epsilon^2 + \omega_{EO}^3(1-\omega_E^2\omega_O)\sigma_\eta^2}{2(1-\omega_O)(1-\omega_E\omega_O)^2\sigma_\epsilon^2 + \omega_{EO}^2\big(1+\omega_E^2(1-2\omega_O)\big)\sigma_\eta^2}.$$

This expression depends on all five parameters. It is monotonically decreasing in ω_{EO}, positive when $\omega_{EO} < 0$, and negative otherwise. Similarly, in the five-parameter model,

$$\text{Accrual Reversal} = \frac{(1-\omega_E-\omega_E\omega_O)\omega_{EO}\sigma_\eta^2}{(1+\omega_O)(1-\omega_E\omega_O)}.$$

The expression above depends on four out of five non-zero parameters of the model.

Step 4. *Relative Information Content* achieves its global minimum value when no errors are present

To prove that *Relative Information Content* achieves its global minimum value when no errors are present, note that the joint normality assumptions imposed on the processes of economic earnings, NOA and accruals measurement errors imply that when this measure is estimated from sufficiently long time series, it converges to:

$$\frac{\text{Var}[CF_{t+1}|E_t', O_t']}{\text{Var}[CF_{t+1}|CF_t]}.$$

The denominator of the ratio above does not depend on the presence of accrual measurement errors. Therefore, it remains to show that when accruals are imperfect. $\text{Var}[CF_{t+1}|E_t', O_t'] \geq$

$\mathrm{Var}[CF_{t+1}|E_t, O_t]$. By joint normality of all processes, we know that $\mathrm{Var}[CF_{t+1}|E_t', O_t'] \geq \mathrm{Var}[CF_{t+1}|E_t', O_t', E_t, O_t]$. Finally, by property (iii) of economic earnings, $\mathrm{Var}[CF_{t+1}|E_t', O_t', E_t, O_t] = \mathrm{Var}[CF_{t+1}|E_t, O_t]$, which concludes the proof of the global minimum property of *Relative Information Content*.

To prove that *Relative Information Content* does not achieve convergent validity, consider omission errors in the four-parameter model with the following parameters: $\sigma_\epsilon^2 = 0.25$, $\sigma_\eta^2 = 1$, $\omega_O = 0.5$, $\omega_E = 0$. It can be verified that for these values of parameters, *Relative Information Content* is increasing up to the value of δ approximately equal to 0.857, but then decreases up to $\delta = 1$.

Covariance Matrix of $(\boldsymbol{E'_{t+1}}, \boldsymbol{E'_t}, \boldsymbol{O'_{t+1}}, \boldsymbol{O'_t}, \boldsymbol{O'_{t-1}})$ in the Presence of White Noise Errors

$$
\begin{array}{cccccc}
 & E'_{t+1} & E'_t & O'_{t+1} & O'_t & O'_{t-1} \\
E'_{t+1} & \dfrac{\sigma_\epsilon^2}{1-\omega_E^2} + 2\sigma_{err}^2 & . & . & . & . \\
E'_t & \dfrac{\omega_E \sigma_\epsilon^2}{1-\omega_E^2} - \sigma_{err}^2 & \dfrac{\sigma_\epsilon^2}{1-\omega_E^2} + 2\sigma_{err}^2 & . & . & . \\
O'_{t+1} & \sigma_{err}^2 & 0 & \dfrac{\sigma_\eta^2}{1-\omega_O^2} + \sigma_{err}^2 & . & . \\
O'_t & -\sigma_{err}^2 & \sigma_{err}^2 & \dfrac{\omega_O \sigma_\eta^2}{1-\omega_O^2} & \dfrac{\sigma_\eta^2}{1-\omega_O^2} + \sigma_{err}^2 & . \\
O'_{t-1} & 0 & -\sigma_{err}^2 & \dfrac{\omega_O^2 \sigma_\eta^2}{1-\omega_O^2} & \dfrac{\omega_O \sigma_\eta^2}{1-\omega_O^2} & \dfrac{\sigma_\eta^2}{1-\omega_O^2} + \sigma_{err}^2
\end{array}
\tag{18}
$$

Covariance Matrix of $(\boldsymbol{E'_{t+1}}, \boldsymbol{E'_t}, \boldsymbol{O'_{t+1}}, \boldsymbol{O'_t}, \boldsymbol{O'_{t-1}})$ in the Presence of Omission Errors

$$
\begin{array}{cccccc}
 & E'_{t+1} & E'_t & O'_{t+1} & O'_t & O'_{t-1} \\
E'_{t+1} & \dfrac{\sigma_\epsilon^2}{1-\omega_E^2} + 2\delta^2\sigma_\eta^2 & . & . & . & . \\
E'_t & \dfrac{\omega_E \sigma_\epsilon^2}{1-\omega_E^2} - \delta^2\sigma_\eta^2 & \dfrac{\sigma_\epsilon^2}{1-\omega_E^2} + 2\delta^2\sigma_\eta^2 & . & . & . \\
O'_{t+1} & \omega_O \delta^2 \sigma_\eta^2 & -(1-\omega_O)\omega_O \delta^2 \sigma_\eta^2 & \sigma_\eta^2 \left(\dfrac{1}{1-\omega_O^2} - \delta^2\right) & . & . \\
O'_t & 0 & \omega_O \delta^2 \sigma_\eta^2 & \omega_O \sigma_\eta^2 \left(\dfrac{1}{1-\omega_O^2} - \delta^2\right) & \sigma_\eta^2 \left(\dfrac{1}{1-\omega_O^2} - \delta^2\right) & . \\
O'_{t-1} & 0 & 0 & \omega_O^2 \sigma_\eta^2 \left(\dfrac{1}{1-\omega_O^2} - \delta^2\right) & \omega_O \sigma_\eta^2 \left(\dfrac{1}{1-\omega_O^2} - \delta^2\right) & \sigma_\eta^2 \left(\dfrac{1}{1-\omega_O^2} - \delta^2\right)
\end{array}
\tag{19}
$$

Covariance Matrix of $(\boldsymbol{E}'_{t+1}, \boldsymbol{E}'_{t}, \boldsymbol{O}'_{t+1}, \boldsymbol{O}'_{t}, \boldsymbol{O}'_{t-1})$ in the Presence of NOA Scaling Errors

$$
\begin{array}{cccccc}
 & E'_{t+1} & E'_t & O'_{t+1} & O'_t & O'_{t-1} \\
E'_{t+1} & \frac{\sigma_\epsilon^2}{1-\omega_E^2}+\frac{2\delta^2\sigma_\eta^2}{1+\omega_O} & \cdot & \cdot & \cdot & \cdot \\
E'_t & \frac{\omega_E\sigma_\epsilon^2}{1-\omega_E^2}-\frac{\delta^2(1-\omega_O)\sigma_\eta^2}{1+\omega_O} & \frac{\sigma_\epsilon^2}{1-\omega_E^2}+\frac{2\delta^2\sigma_\eta^2}{1+\omega_O} & \cdot & \cdot & \cdot \\
O'_{t+1} & \frac{\delta(1+\delta)\sigma_\eta^2}{1+\omega_O} & \frac{\omega_O\delta(1+\delta)\sigma_\eta^2}{1+\omega_O} & \frac{(1+\delta)^2\sigma_\eta^2}{1-\omega_O^2} & \cdot & \cdot \\
O'_t & -\frac{\delta(1+\delta)\sigma_\eta^2}{1+\omega_O} & \frac{\delta(1+\delta)\sigma_\eta^2}{1+\omega_O} & \frac{(1+\delta)^2\omega_O\sigma_\eta^2}{1-\omega_O^2} & \frac{(1+\delta)^2\sigma_\eta^2}{1-\omega_O^2} & \cdot \\
O'_{t-1} & -\frac{\omega_O\delta(1+\delta)\sigma_\eta^2}{1+\omega_O} & -\frac{\delta(1+\delta)\sigma_\eta^2}{1+\omega_O} & \frac{(1+\delta)^2\omega_O^2\sigma_\eta^2}{1-\omega_O^2} & \frac{(1+\delta)^2\omega_O\sigma_\eta^2}{1-\omega_O^2} & \frac{(1+\delta)^2\sigma_\eta^2}{1-\omega_O^2}
\end{array}
\tag{20}
$$

Covariance Matrix of $(\boldsymbol{E}'_{t+1}, \boldsymbol{E}'_{t}, \boldsymbol{O}'_{t+1}, \boldsymbol{O}'_{t}, \boldsymbol{O}'_{t-1})$ in the Presence of Smoothing Errors

$$
\begin{array}{cccccc}
 & E'_{t+1} & E'_t & O'_{t+1} & O'_t & O'_{t-1} \\
E'_{t+1} & \frac{\sigma_\epsilon^2}{1-\omega_E^2}-2\delta(1-\omega_E-\delta)\sigma_\epsilon^2 & \cdot & \cdot & \cdot & \cdot \\
E'_t & \frac{\omega_E\sigma_\epsilon^2}{1-\omega_E^2}+\delta(1-\delta-\omega_E+\omega_E^2)\sigma_\epsilon^2 & \frac{\sigma_\epsilon^2}{1-\omega_E^2}-2\delta(1-\omega_E-\delta)\sigma_\epsilon^2 & \cdot & \cdot & \cdot \\
O'_{t+1} & -\delta(1-\delta)\sigma_\epsilon^2 & 0 & \frac{\sigma_\eta^2}{1-\omega_O^2}+\delta^2\sigma_\epsilon^2 & \cdot & \cdot \\
O'_t & -\delta(\delta+\omega_E)\sigma_\epsilon^2 & -\delta(1-\delta)\sigma_\epsilon^2 & \frac{\omega_O\sigma_\eta^2}{1-\omega_O^2} & \frac{\sigma_\eta^2}{1-\omega_O^2}+\delta^2\sigma_\epsilon^2 & \cdot \\
O'_{t-1} & -\delta\omega_E^2\sigma_\epsilon^2 & -\delta(\delta+\omega_E)\sigma_\epsilon^2 & \frac{\omega_O^2\sigma_\eta^2}{1-\omega_O^2} & \frac{\omega_O\sigma_\eta^2}{1-\omega_O^2} & \frac{\sigma_\eta^2}{1-\omega_O^2}+\delta^2\sigma_\epsilon^2
\end{array}
\tag{21}
$$

APPENDIX 2
Variable Definitions

Compustat firm-year variables used for model estimation	
$E'_{i,t}$	Operating earnings are calculated as $OIBDP - XINT - TXC$ scaled by $[max(AT_t, LT_t) + max(AT_{t-1}, LT_{t-1})]/2$. $XINT$ and TXC are set to zero if missing. Missing values of $OIBDP$ are filled in with $OIBAP + DP$ where DP is set to zero if missing, then, if still missing, with PI.
$O'_{i,t}$	Net current operating assets are calculated as $(ACT - CHE) - (LCT - DLC)$ scaled by the greater of total assets (AT) or total liabilities (LT). Missing values of CHE are filled in with CH. DLC is set to zero if missing.
$A'_{i,t}$	Working capital accruals are calculated as changes in net current operating assets, $O'_{i,t} - O'_{i,t-1}$.
$CF_{i,t}$	Imputed cash flows from operations are calculated as $E_{i,t} - A_{i,t}$.

Note: $E'_{i,t}$ is winsorized at -1 and 1. $O'_{i,t}$, $A'_{i,t}$ and $CF_{i,t}$ are not winsorized.

Estimated AR(1) model parameters (simple model)	
ω'_E	The persistence of operating earnings is ω'_E from the AR(1) earnings regression, $E'_t = constant + \omega'_E E'_{t-1} + \epsilon_t$.
ω'_O	The persistence of working capital is ω'_O from the AR(1) working capital regression, $O'_t = constant + \omega'_O O'_{t-1} + \eta_t$.
$var[\epsilon'] = \sigma^2_{\epsilon'}$	This is the variance of the earnings innovation from the earnings AR(1) process.
$var[\eta'] = \sigma^2_{\eta'}$	This is the variance of the working capital innovation from the working capital AR(1) process.

Estimated VAR(1) model parameters (general model)	
ω'_E	The conditional persistence of operating earnings is ω'_E from the panel VAR(1) regression, $E'_t = constant + \omega'_e E'_{t-1} + \omega'_{EO} O'_{t-1} + \epsilon_{t+1}$.
ω'_{EO}	The conditional persistence of working capital for future operating earnings is ω'_{EO} from the panel VAR(1) regression above.
ω'_O	The conditional persistence of working capital is ω'_O from the panel VAR(1) regression $O'_t = constant + \omega'_O O'_{t-1} + \omega'_{OE} E'_{t-1} + \eta_t$.
ω'_{OE}	The conditional persistence of operating earnings for future working capital is ω'_{OE} from the panel VAR(1) regression above.
$var[\epsilon'] = \sigma^2_{\epsilon'}$	This is the variance of the earnings innovation from the panel VAR(1).
$var[\eta'] = \sigma^2_{\eta'}$	This is the variance of the working capital innovation from the panel VAR(1).
$cov(\epsilon'\eta') = \sigma_{\epsilon'\eta'}$	This is the covariance of the earnings innovation and the working capital innovation from the panel VAR(1).

Appendix 2 (continued)
Variable Definitions

Accruals quality measures	
Persistence$_i$	The persistence of earnings for firm *i*. It is the estimated β_1 from the regression $E'_t = \beta_0 + \beta_1 \cdot E'_{t-1} + \epsilon_t$.
Differential Persistence$_i$	Differential persistence of accruals and cash flows for future earnings for firm *i*. It is estimated as $\alpha_2 - \alpha_1$ from the regression $E'_t = \alpha_0 + \alpha_1 \cdot A'_{t-1} + \alpha_2 \cdot CF_{t-1} + \varepsilon_t$.
SD of Accruals$_i$	The population standard deviation of working capital accruals A'_t for firm *i*.
SD of Accruals Residuals$_i$	The population standard deviation of working capital accruals residuals firm i. Residuals are estimated for firm i using the regression $A'_t = \gamma_0 + \gamma_1 \cdot CF_t + \gamma_2 \cdot CF_t + \gamma_3 \cdot CF_t + \varepsilon_t$.
Unsmoothness$_i$	The ratio of the standard deviation of earnings (E'_t) to the standard deviation of cash flows (CF_t) for firm i .
Accrual Reversal$_i$	The covariance between current period earnings (E'_t) and lagged working capital accruals (A'_{t-1}) for firm i .
Relative Information Content$_i$	This measure is calculated for firm i as (1 – Adj. R^2) from the first regression divided by (1 – Adj. R^2) from the second regression: $CF_t = c_0 + c_1 \cdot O_{t-1} + c_2 \cdot E_{t-1} + \varepsilon_t$ and $CF_t = d_0 + d_1 \cdot CF_{t-1} + \varepsilon_t$.

TABLE 1
Measures of Accruals Quality

Panel A: Summary of the five extant and two new measures of accruals quality that we consider in this paper.

Name of Quality Measure	Estimation of Measure (assumed direction for lower quality)	Representative Papers Using Measure	Proportion of Papers Using Measure[#]
Persistence	The coefficient on lagged earnings from a regression of earnings on lagged earnings (-)	Dechow and Dichev (2002), Francis et al. (2004), Doyle, Ge, and McVay (2007)	12%
Differential Persistence	Difference between the coefficient on lagged cash flows and the coefficient on lagged accruals from a regression of earnings on lagged cash flows and lagged accruals (+)	Sloan (1996), Richardson et al. (2005)	8%
SD of Accruals	Standard deviation of accruals (+)	Bartov et al. (2000), Bergstresser and Philippon (2006)	12%[##]
SD of Accruals Residuals	Standard deviation of the residuals from a regression of accruals on current, lead and lag cash flows (+)	Francis et al. (2004), Francis et al. (2005), Biddle and Hilary (2006), Doyle et al. (2007)	46%[###]
Unsmoothness	Ratio of standard deviation of earnings to standard deviation of cash flows (+,-)	Leuz et al. (2003), Francis et al. (2004)	19%[####]
Accrual Reversal	Covariance between earnings and lagged accruals (-)	Additional Measure	
Relative Information Content	(1-adjusted R^2) from of a regression of cash flows on lagged earnings and lagged NOA divided by (1-adjusted R^2) from a regression of cash flows on lagged cash flows (+)	Additional Measure	

[#]To compute this proportion, we start with the 500 most highly cited articles published in the "top 6" accounting journals as of September of 2020. We then identify 26 articles containing "accrual(s) quality," "earnings quality," "accounting quality," "financial reporting quality," "quality of accruals," "quality of earnings," "quality of accounting" and "quality of financial reports" in their titles. Finally, we collate and report the proportion of papers using each of the measures (or minor variants thereof, as detailed below). The percentages do not sum to 100% because some papers use multiple measures and some papers do not use any of these measures.
[##]Includes the absolute value of accruals.
[###]Includes variations using the standard deviation and absolute value of residuals from a regression of accruals on variables including current, lead and lag cash flows.
[####]Includes variations using ratios of the standard deviations of residual earnings and residual cash flows and the correlation between (changes in) accruals and (changes in) cash flows.

TABLE 1 (continued)
Measures of Accruals Quality

Panel B: Select expressions for AQ measures using the 'simple model' of earnings embodied in equations (5) and (6). '…' indicates that the expression is omitted for brevity and is available from the authors upon request.

AQ Measure	No Error	White Noise Error	Omission Error
Persistence	ω_E	$\frac{\omega_E\sigma_\epsilon^2-(1-\omega_E^2)\sigma_{err}^2}{\sigma_\epsilon^2+2(1-\omega_E^2)\sigma_{err}^2}$	$\frac{\omega_E\sigma_\epsilon^2-(1-\omega_E^2)\delta^2\sigma_\eta^2}{\sigma_\epsilon^2+2(1-\omega_E^2)\delta^2\sigma_\eta^2}$
Differential Persistence	0	$\frac{\sigma_\epsilon^2\sigma_{err}^2(1+2\omega_E)(1+\omega_O)}{2\sigma_\epsilon^2\left(\sigma_\eta^2+(1+\omega_O)\sigma_{err}^2\right)+4\sigma_\eta^2\sigma_{err}^2(1-\omega_E^2)}$	…
SD of Accruals	$\sqrt{\frac{2\sigma_\eta^2}{1+\omega_O}}$	$\sqrt{2\sigma_{err}^2+\frac{2\sigma_\eta^2}{1+\omega_O}}$	$\sqrt{\frac{2\sigma_\eta^2(1-\delta^2+\omega_O^2\delta^2)}{1+\omega_O}}$
SD of Accruals Residuals	$\sqrt{2\sigma_{err}^2+\mathrm{F}}$, see note (1) below	$\sqrt{2\sigma_{err}^2+\mathrm{F}}$, see note (1)	…
Unsmoothness	$\sqrt{\frac{\sigma_\epsilon^2(1+\omega_O)}{\sigma_\epsilon^2(1+\omega_O)+2\sigma_\eta^2(1-\omega_E^2)}}$	$\sqrt{\frac{(\sigma_\epsilon^2+2\sigma_{err}^2(1-\omega_E^2))(1+\omega_O)}{\sigma_\epsilon^2(1+\omega_O)+2\sigma_\eta^2(1-\omega_E^2)}}$	$\sqrt{\frac{(\sigma_\epsilon^2+2\delta^2\sigma_\eta^2(1-\omega_E^2))(1+\omega_O)}{\sigma_\epsilon^2(1+\omega_O)+2\sigma_\eta^2(1-\omega_E^2)}}$
Accrual Reversal	0	$-\sigma_{err}^2$	0
Relative Info Content	$\frac{\left(\sigma_\epsilon^2+\sigma_\eta^2\right)\left((1+\omega_O)\sigma_\epsilon^2+2(1-\omega_E^2)\sigma_\eta^2\right)}{\left(\sigma_\epsilon^2+(1-\omega_E)\sigma_\eta^2\right)\left((1+\omega_O)\sigma_\epsilon^2+(1+\omega_E)(3-\omega_O)\sigma_\eta^2\right)}$	…	…

TABLE 1 (continued)
Measures of Accruals Quality

AQ Measure	Scaling Error	Smoothing Error
Persistence	$\dfrac{\omega_E(1+\omega_O)\sigma_\epsilon^2-(1-\omega_E^2)(1-\omega_O)\delta^2\sigma_\eta^2}{(1+\omega_O)\sigma_\epsilon^2+2(1-\omega_E^2)\delta^2\sigma_\eta^2}$	$\dfrac{\omega_E+\delta(1-\omega_E^2)(1-\delta-(1-\omega_E)\omega_E)}{1-2\delta(1-\delta-\omega_E)(1-\omega_E^2)}$
Differential Persistence	$\dfrac{\delta(1+2\omega_E-\omega_O)}{2(1+\delta)}$	...
SD of Accruals	$(1+\delta)\sqrt{\dfrac{2\sigma_\eta^2}{1+\omega_O}}$	$\sqrt{2\delta^2\sigma_\epsilon^2+\dfrac{2\sigma_\eta^2}{1+\omega_O}}$
SD of Accruals Residuals	$(1+\delta)\sqrt{\mathrm{F}}$, see note (1)	...
Unsmoothness	$\sqrt{\dfrac{\sigma_\epsilon^2(1+\omega_O)+2\delta^2\sigma_\eta^2(1-\omega_E^2)}{\sigma_\epsilon^2(1+\omega_O)+2\sigma_\eta^2(1-\omega_E^2)}}$	$\sqrt{\dfrac{\sigma_\epsilon^2\left(1-2\delta(1-\delta-\omega_E)(1-\omega_E^2)\right)(1+\omega_O)}{\sigma_\epsilon^2(1+\omega_O)+2\sigma_\eta^2(1-\omega_E^2)}}$
Accrual Reversal	$-\dfrac{(1-\omega_O)\delta(1+\delta)\sigma_\eta^2}{1+\omega_O}$	$-\delta(\delta+\omega_E-\omega_E^2)\sigma_\epsilon^2$
Relative Info Content	…	...

Note:

(1) $\Phi \equiv \dfrac{2\sigma_\epsilon^2\sigma_\eta^2\left(\sigma_\epsilon^2+\sigma_\eta^2\right)}{2\left(1-\omega_E^2\right)\sigma_\eta^4+(1+\omega_O)\sigma_\epsilon^4+\sigma_\epsilon^2\sigma_\eta^2(4+2\omega_E(2+\omega_E)-\omega_O-4\omega_E\omega_O+\omega_O^2)}$.

TABLE 2
Theoretical Relations

Table 2 shows the relations between AQ measures and economic parameters (panel A) and types of errors (panel B) derived in the simple AR(1) model. In panel A, arrows indicate that the AQ measure is either monotonically increasing (↑) or decreasing (↓) in the parameter. '–' indicates that the measure is unrelated to the parameter. '?' denotes a non-monotonic relation. In panel B, arrows indicate that the AQ measure is either monotonically increasing (↑) or decreasing (↓) in the magnitude of accrual errors for all parameter values. If the relation is U-shaped (or inverse U-shaped) for all parameter values, we indicate this as "↓, then ↑" (or "↑, then ↓"). '–' indicates that the measure does not change in the error magnitude. '?' denotes a relation that can be non-monotonic depending on the parameter values.

Panel A: Theoretical relations between AQ measures and economic parameters

	Parameter			
AQ Measure	ω_E	ω_O	σ_ϵ^2	σ_η^2
Persistence	↑	–	–	–
Diff Persistence	–	–	–	–
SD of Accruals	–	↓	–	↑
SD of Accruals Residuals	?	?	↑	↑
Unsmoothness	↑	↑	↑	↓
Accrual Reversal	–	–	–	–
Relative Info Content	↓	↑	?	?

Panel B: Theoretical relations between AQ measures and types of error

	Type of Error			
Measure	White Noise	Omission	Scaling	Smoothing
Persistence	↓	↓	↓	↑, then ↓
Diff Persistence	↑	?	↓ for $\delta < 0$, ↑ for $\delta > 0$	↑
SD of Accruals	↑	↓	↓ for $\delta < 0$, ↑ for $\delta > 0$	↑
SD of Accruals Residuals	↑	?	↓ for $\delta < 0$, ↑ for $\delta > 0$	?
Unsmoothness	↑	↑	↑	↓, then ↑
Accrual Reversal	↓	–	↑ for $-\frac{1}{2} < \delta < 0$, ↓ otherwise	↓
Relative Info Content	↑	?	↑	?

TABLE 3
Relations Between AQ Measures and Economic Parameters

Table 3 summarizes the results of the numerical simulations of the relations between accruals quality measures and the economic model parameters. Arrows signify the direction of any significant relation between each AQ measure and each parameter across the range of parameter values considered. *, ** and *** indicate whether the associated average p value is significant at the 10%, 5% and 1% levels, using one-tailed tests and evaluated at the limit of the range having the smallest average p value.

	ω_E (1)	ω_{EO} (2)	ω_O (3)	ω_{OE} (4)	σ_ϵ^2 (5)	σ_η^2 (6)	$\sigma_{\epsilon\eta}$ (7)
Persistence	$\uparrow^{***}$	.	.	.	.	.	.
Diff Persistence	.	$\downarrow^{***}$	.	.	.	$\uparrow^{*}$	.
SD of Accruals	.	.	$\downarrow^{***}$	.	.	$\uparrow^{***}$	.
SD of Accruals Residuals	$\downarrow^{**}$	$\downarrow^{***}$	.	$\uparrow^{*}$	$\uparrow^{***}$	$\uparrow^{***}$	.
Unsmoothness	.	$\downarrow^{***}$	$\uparrow^{***}$	$\uparrow^{***}$	$\uparrow^{***}$	$\downarrow^{***}$	$\uparrow^{***}$
Accrual Reversal	.	$\uparrow^{*}$	.	.	.	.	$\uparrow^{***}$
Relative Info Content	$\downarrow^{***}$	$\downarrow^{***}$	$\uparrow^{***}$	$\uparrow^{***}$	$\uparrow^{***}$	$\downarrow^{***}$	$\downarrow^{***}$

TABLE 4
Relations Between AQ Measures and Induced Errors

Table 4 summarizes the results of the numerical simulations of the relations between accruals quality measures and the four types of errors in accruals. Arrows signify how the direction of any significant relation between each AQ measure and each error type evolves as the error standard deviation is increased from zero toward the limit of the range. *, ** and *** indicate whether the associated average *p* value is significant at the 10%, 5% and 1% levels, using one-tailed tests.

			NOA scaling error		
	White noise error (1)	Omission error (2)	Understate NOA (3a)	Overstate NOA (3b)	Smoothing error (4)
Persistence	$\downarrow^{***}$	$\downarrow^{***}$	$\downarrow^{***}$	$\downarrow^{***}$	$\uparrow^{***}$ then $\downarrow^{***}$
Diff Persistence	$\uparrow^{***}$	$\downarrow^{***}$	$\downarrow^{***}$	$\uparrow^{***}$	$\uparrow^{***}$
SD of Accruals	$\uparrow^{***}$	$\downarrow^{***}$	$\downarrow^{***}$	$\uparrow^{***}$	$\uparrow^{***}$
SD of Accruals Residuals	$\uparrow^{***}$	$\uparrow^{***}$ then $\downarrow^{***}$	$\downarrow^{***}$	$\uparrow^{***}$	$\downarrow^{***}$ then $\uparrow^{***}$
Unsmoothness	$\uparrow^{***}$	$\uparrow^{***}$	$\uparrow^{***}$	$\uparrow^{***}$	$\downarrow^{***}$ then $\uparrow^{***}$
Accrual Reversal	$\downarrow^{***}$	.	$\uparrow^{*}$ then $\downarrow^{***}$	$\downarrow^{***}$	$\downarrow^{***}$
Relative Info Content	$\uparrow^{***}$	$\uparrow^{***}$	.	.	$\uparrow^{***}$