Wolf Pack Activism*

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Abstract

Blockholder monitoring is central to corporate governance, but blockholders large enough to exercise significant unilateral influence are rare. Mechanisms that enable moderately-sized blockholders to exert collective influence are therefore important. Existing theory suggests that engagement by moderately-sized blockholders is unlikely, especially when the blocks are held by delegated asset managers who have limited skin in the game. We present a model in which multiple delegated blockholders engage target management in parallel, i.e., "wolf pack activism." Delegation reduces skin in the game, which decreases incentives for engagement. However, it also induces competition over investor capital (i.e, competition for flow). We show that this increases engagement incentives and helps ameliorate the problem of insufficient engagement, though it can also foster excess engagement. Under competition for flow the total amount of capital seeking skilled activist managers is relevant to engagement incentives, which helps to predict when and where wolf packs arise. Flow incentives are particularly valuable in incentivizing engagement by packs with smaller members.

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1 Introduction

Economists have long recognized the key role of blockholder engagement in ameliorating problems arising from the separation of ownership and control. In particular, Shleifer and Vishny (1986) argue that ownership of a large block by a single shareholder enhances firm value, and more so the larger is the block. However, while blockholding is widely prevalent in the U.S., most blockholders are not large enough to exert significant unilateral influence in the face of recalcitrant management. Holderness (2009) documents that 96% of U.S. firms have at least one blockholder with 5% ownership. Yet, La Porta, Lopes de Silanes, and Shleifer (1999) document that 80% of the largest U.S. firms lack any single blockholder with a stake of at least 20%, a level that they argue generates effective control. Using data on a broader sample from Dlugosz et al (2006), we find that fewer than 15% of U.S. firms have a 20% outside blockholder. Mechanisms that enable non-controlling blockholders to exert collective influence are therefore key to effective monitoring.

In this paper, we theoretically examine how parallel engagement by non-controlling blockholders may arise. This question is interesting because, while the existing literature on multiple blockholders (Winton, 1993, and Edmans and Manso, 2011) suggests that blockholders with moderate stakes are unlikely to engage in costly interventions, the past two decades have witnessed a significant amount of simultaneous costly engagement by holders of such blocks. Indeed, legal scholars allege that institutional investors such as activist hedge funds engage via so-called "wolf packs,"[1] in which multiple funds with small to moderate stakes (who do not act as a formal group) each

[1]See, for example, Briggs (2006), Nathan (2009), Coffee and Palia (2015). Interestingly, though hedge funds are involved in a majority of debt restructurings in the US (Jiang et al 2012), we are not aware of any evidence of parallel engagements amongst such creditors.

engage in costly efforts to change firm policies.[2] This process has been described as "conscious parallelism" (Tevlin, 2016).

Block sizes in wolf packs range from around 8% to 9% (e.g., the 2013-2014 Sotheby's wolf pack involving Third Point, Trian, and Marcato) all the way down to around 1% (e.g., the 2005 Deutsche Borse wolf pack involving, among others, eight activist funds with stakes of 1% to 2%).[3] Furthermore, such campaigns typically involve significant costs for each activist. In addition to research costs and expenses incurred in persuading management and other shareholders, significant costs arise from legal and compliance risk due to SEC rules concerning communication between shareholders, and/or potential lawsuits relating to undeclared 13D group formation.[4] Importantly, legal risks and associated costs attach to each participant in a wolf pack, regardless of their size.

It is noteworthy that activist wolf packs involve institutional investors who manage *delegated* blocks. Standard agency theory would seem to imply that, relative to blockholders who invest their own capital (as in the existing theoretical models), holders of delegated blocks like hedge funds should engage less because delegation by definition results in lower "skin in the game." What could explain the willingness of delegated

[2]A starting point of our analysis is that the actions of the different investors is *formally* uncoordinated. This is consistent with legal constraints in the activism process: U.S. disclosure rules require investors to file together as a group when their activities are formally coordinated, which risks triggering poison pills and thus restricts total group holdings.

[3]See Third Point LLC vs Ruprecht C.A. No. 9469-VCP (Del. Ch. May. 2, 2014) and Becht et al (2017) for details. We are grateful to Julian Franks for providing us with further detail on the Deutsche Borse case.

[4]Legal risk derives from two sources. First, the Security and Exchange Act (SEA) of 1933 defines "proxy solicitation" expansively as any "communication to security holders under circumstances reasonably calculated to result in the procurement, withholding, or revocation of a proxy." Black (1990) notes that this effectively covers any form of communication between shareholders, all of which are subject to stringent anti-fraud provisions under SEA 14a-9. Second, SEA Section 13D requires any person *or groups of persons* that owns 5% or more of a company's shares to file Form 13D declaring their intentions. This fosters an incentive for target managers to sue on the grounds of an undeclared group, even if there is no evidence of explicit coordination.

funds to take these types of costly actions that is not in the standard agency model? We show that the desire to attract delegated investor capital, i.e., "competition for flow," can help ameliorate the problem of insufficient engagement.

When a blockholder invests only her own capital, her incentive to engage with management is limited to the impact on the value of that capital, even though successful engagement positively affects all shareholders. In other words, engagement is a public good and is therefore underprovided relative to the social optimum. A possible solution to such a public goods problem would be to subsidize those who undertake socially valuable but costly actions at the expense of those who don't. We show that competition for flow creates an *endogenous* set of transfers across agents that (imperfectly) achieves this goal. By undertaking a costly action—activism—to advertise her skill, a fund can attract capital at the expense of other, non-engaging, funds. Flow thus endogenously compensates those who undertake costly engagement. Such endogenous, decentralized flows do not, however, perfectly replicate the social optimum. There may still be some underprovision, while the possibility of attracting flow can also over-incentivize funds, leading to socially wasteful excess engagement.

In our model there is a target firm that is partially owned by activist funds that differ in their skill at engaging target management. Funds own blocks via a combination of proprietary (i.e., the fund's own) and delegated capital. Delegated capital is provided by investors who wish to reinvest with skilled funds, which provide a higher continuation return. Investors observe the engagement actions of all activist funds and the outcome of engagement, and (rationally) reallocate their capital to those funds that they view as most likely to be skilled. Funds, in turn, receive fees for any capital that is reinvested with them. As a result, funds are incentivized to compete for reallocatable investor capital, i.e., flow. This setup is consistent with empirical evidence about hedge funds provided by Lim, et al. (2016), who show that indirect incentives deriving from future fund flows are larger in magnitude than direct incentives coming from incentive fees and returns from proprietary investment. Furthermore, Boyson (2008) finds that performance persistence is more prevalent among smaller and younger hedge funds, exactly those that Lim, et al. (2016) find are the most flow motivated.

As discussed above, if blocks are proprietary engagement is under-provided relative to the social optimum. As delegated blockholders, funds differ in two key ways: (1) they have less "skin in the game" and (2) they have an incentive to attract flow due to their fee structure. The reduction in skin in the game exacerbates the underprovision problem. Competition for flow, on the other hand, gives funds the incentive to engage in order to advertise their skill. This delivers a unique feature of competition for flow as a solution to under-engagement: with competition for flow, the *total* size of activist capital becomes relevant to engagement incentives. For a given number of engaging funds, the larger is the total pool of activist capital, the greater is the amount that each fund can attract by advertising their skill. At an applied level, if we associate the growth of activist capital in the model with the size of the activist industry, our model thus predicts that wolf packs would be most common when the industry has grown and matured.

Another unique feature of competition for flow is that its incentivizing effect is most pertinent for smaller blocks. This is because, while a smaller block size directly discourages engagement in the proprietary case, for a given amount of total activist capital the relative per-blockholder gains from potential flow increase as blocks get smaller. At an applied level, this makes our model well suited to the analysis of parallel engagements involving smaller blocks.

The total size of activist capital clearly plays a central role in our analysis. We intentionally take the conservative approach that all available flow *nets out* across funds, i.e., there is no capital flowing in from outside the model. As a result, the impact of flow is self-limiting: the larger the number of funds advertising their skill by engagement, the lower is the flow reward per fund, ceteris paribus. While this limits the effect of competition for flow, we show that the incentivizing effect of competition for flow can still sometimes overcome the negative effects of reduced skin in the game.

However, the engagement incentives stemming from competition for flow are not costless. Unlike price appreciation from successful engagement, capital inflows are excludable benefits for funds. As a result, we show that there are equilibria where funds engage even when they know they are not pivotal: i.e., there is excess engagement. Overall, we find that the positive welfare impact of increased engagement is likely to dominate at higher engagement costs (such as those relevant for hedge fund activism), while the negative impact of excess engagement is likely to dominate at lower costs.

In recent years the empirical literature in finance has taken an active interest in the wolf pack phenomenon, beginning with Becht at al (2017). They provide an overview of global hedge fund activism between 2000 and 2010 and document that as many as a fifth of such events involve multiple activists intervening in parallel. They find that wolf packs are associated with a greater probability of successful engagement and with higher announcement returns when stakes are disclosed. Both of these findings are broadly consistent with our model: for any given required level of engagement, the probability of success is (weakly) increasing in the amount of activist capital present; furthermore, as long as there is ex ante uncertainty about the required level of engagement, the increased presence of activists will lead to larger increases in anticipated firm value. More recently, Artiga Gonzalez and Caluzzo (2019) show that wolf packs are more common in larger firms. This is also consistent with our model: a particular dollar value of capital held by a hedge fund will translate into a smaller block size for a larger firm; thus, for a given level of required engagement, success will require a larger number of engaging blocks.[5]

The papers in this nascent literature identify wolf packs using regulatory discosures by multiple activists per target firm, which thus limits their scope (as Becht et al, 2017 recognize) to block sizes that cross the relevant reporting thresholds (5% in the US). It is therefore likely that wolf packs are under-counted in the data, because anecdotal evidence (e.g. the Deutche Borse example discussed above) suggests that they can involve players that never cross the relevant reporting thresholds. In this context, it is salient that our mechanism suggests that the flow motivation may be strongest for exactly such relatively small participants. Thus, as the empirical literature finds new ways (e.g., starting from 13F holdings data) to study the presence of smaller blocks in parallel engagements, our results will be increasingly relevant. Furthermore, our analysis also suggests that those hedge funds that have the strongest incentives to gain reputation but cannot acquire larger stakes (i.e., smaller and younger funds) will be most affected by the flow incentives that we identify.

1.1 Related theoretical literature

At a broad theoretical level our analysis is related to the large literature on blockholder monitoring (surveyed by Edmans and Holderness, 2017). Papers in this literature tend to focus either on blockholders who exercise "voice" by directly intervening in the firm's activities, or those who use informed trading, also called "exit," to improve

[5] Other recent contributions include Wong (2020), who considers pre-filing trade, and He and Li (2020) who show that social ties matter in wolf pack engagements.

stock price efficiency and encourage correct actions by managers.[6] Our paper belongs in the former strand of the literature, and is most closely linked to papers that focus on multiple blockholders. As noted above, these include Winton (1993) and Edmans and Manso (2011), both of which show that intervention via voice is less likely when blocks are smaller. We show that competition for flow among delegated blockholders can ameliorate the underprovision of intervention, and especially so for smaller blocks. Some papers within the multiple blockholders literature also examine how trading opportunities in a non-transparent market may be of key importance in facilitating or incentivizing governance. For example, Edmans and Manso (2011) show that having multiple blockholders can increase the effectiveness of governance via exit due to competition among blockholders. In a similar vein, Cornelli and Li (2002) and Noe (2002) show that small blockholders' ability to generate trading profits in a non-transparent financial market can encourage engagement or tendering. In contrast to these papers, trading plays no role in our model.[7] Further, none of these papers consider the effect of delegation on engagement incentives.

Outside the multiple blockholder literature, Dasgupta and Piacentino (2015) consider the role of delegation in governance but, unlike us, focus on exit as a governance mechanism. They show that the threat of exit is weakened when the blockholder is a flow-motivated fund manager. In that paper, the reduced skin in the game and competition for flow arising from delegation work in the same direction: both reduce the incentives to govern via exit. In contrast, the two forces oppose each other in our model. While reduced skin in the game reduces the incentives to pay engagement costs,

[6] A few papers (e.g., Maug, 1998; Kahn and Winton, 1998; Faure-Grimaud and Gromb, 2004) allow blockholders to choose between exerting voice and exiting.

[7] Another important but less related contribution to this literature is Zwiebel (1995), which models the sharing of private control benefits as part of a coalitional bargaining game, and derives the equilibrium number and size of blockholders who try to optimally capture these benefits.

thus exacerbating the underprovision of engagement, competition for flow engenders an endogenous transfer mechanism that alleviates such underprovision.

Our paper is also related to the literature on free riding in takeovers, starting with Grossman and Hart (1980). Many follow up papers have proposed possible solutions to the free rider problem, the most relevant of which is Bagnoli and Lipman (1988), who, like us, study a model with discretely sized shareholders who can be pivotal in equilibrium. While some of the equilibria in our model have similarities to those in Bagnoli and Lipman (1988), our focus on costly engagement (as opposed to costless tendering) as well as delegation takes our paper in a very different direction.

2 The Model

Consider a publicly traded firm that is amenable to shareholder activism, in that value can be created by inducing a change in management's policies. Such a change can be induced only if investors who own shares successfully engage with management. All players are risk neutral and there is no discounting.

Ownership. The firm has a unit continuum of shares outstanding. Some of these shares are held by *activist funds* in blocks of size $b \in (0, 1)$ each. Each fund owns one block and holds cash $c > 0$. The remaining shares are owned by passive shareholders who neither engage nor invest in active funds. The ownership structure is common knowledge at the beginning of the game ($t = 0$).

Shareholder engagement. The firm is characterized by η, a random variable that measures the degree of difficulty in implementing changes in strategy. Two natural sources of such difficulty—which may vary across firms—are the willingness of current management to resist any proposed changes in strategy and the difficulty in convincing passive shareholders to vote with activists in a proxy contest. We assume that η is distributed Uniformly on $(b, U]$ for some $U > 1$, and is publicly revealed at $t = 1$.

After observing η, each fund must choose (simultaneously) at $t = 1$ whether to engage target management ($a = E$) or not ($a = NE$). We assume that engaging the target is costly. This could arise from research costs and potential legal risks, as discussed in the introduction.

Engagement succeeds if the measure of shares that engage is at least η. Given success, the firm's value at the end of the game ($t = 2$) will be $P_2 = P_h$; otherwise it will be $P_2 = P_l$, which we normalize to zero.[8] This "threshold" characterization is meant to capture the idea that, for any given level of η, there is some level of pressure from shareholders that will induce a change in strategy.[9] Our interest is in collective engagement across multiple blockholders: this is captured in our assumption that $\eta > b$.

Skill. Activist funds differ in their skill ($\theta \in \{S, U\}$) at engaging management. Funds discover their engagement skills at $t = 1$. N funds are skilled ($\theta = S$), and can engage management at cost c, i.e., by spending their available cash. M funds are unskilled ($\theta = U$), and face an infinite cost for engaging management. We show in Online Appendix B that the model's qualitative results are similar if unskilled funds have the same cost of engagement as skilled funds, but are unable to observe η.

We denote by α the total share ownership of skilled funds and by β the total share ownership of unskilled funds, so that $\alpha = Nb$ and $\beta = Mb$. Thus $\alpha + \beta$ represents total activist capital. At $t = 2$, after the engagement concludes, activist funds reinvest their capital elsewhere. Skilled funds have a gross $t = 2$ reinvestment rate of return $R_S > 1$. Unskilled funds have a reinvestment rate of $R_U = 1$. These should be understood to

[8]We discuss the implications of this normalization in section 5.3.

[9]Bebchuk et al (2019) document that many activist campaigns result in settlements rather than in proxy fights. Accordingly, η does *not* necessarily correspond to a particular voting threshold.

represent the value of payoffs from future engagements.

Skin in the game. Funds hold blocks using a combination of delegated and proprietary capital. In particular, each block of size b is made up of: (i) a fraction $\phi \in (0,1)$ from the fund's proprietary capital, i.e., its "skin in the game"; and (ii) a fraction $1-\phi$ from a continuum of (randomly matched) identical investors who give capital to the fund to manage, i.e., delegated capital. For example, for a hedge fund, ϕ represents general partner investment while $1-\phi$ represents limited partner investment.

Competition for flow. Funds are evaluated by their investors at $t=2$, after the outcome of engagement is determined. Investors in each fund observe η and the actions of all funds.[10] They then update their beliefs about the skill of all funds, and all delegated capital is reallocated to the set of funds that share the highest posterior probability of being skilled. The reallocated capital is spread evenly among all such funds. We thus limit reallocatable capital to the amount of delegated activist ownership of the target firm. In other words, we exclude from reallocation funds' skin in the game and their initial cash holdings (in effect, funds never invest their proprietary capital in other funds) as well as capital held by passive investors. Since we show later that capital reallocation provides a potential solution to the collective engagement problem, limiting the size of such reallocatable capital works against us.[11]

Fund fees. At $t=2$, after capital is reallocated, each fund earns a proportional assets under management (AUM) fee, $w \in (0,1)$, on delegated capital. We define f_2^i, $i \in \{1, ..., M+N\}$, as the $t=2$ value of delegated capital allocated to fund i, so that fees earned at $t=2$ are wf_2^i. The fund then reinvests all available capital—including its

[10] Since our investors are best interpreted as sophisticated limited partners in activist hedge funds, it is plausible that they have access to similar information about their target firms as their fund managers. Furthermore, our results would be unchanged if investors did not observe η.

[11] Allowing for inflows of external capital or the reallocation of skin in the game, initial cash holdings, or passive capital would bias the model in favor of competition for flow delivering higher engagement.

fees and cash c if it did not engage—at its reinvestment rate of return R_θ. The fund's final payoff is therefore $(\phi b P_2 + w f_2^i + c I_{NE}) R_\theta$, where I_{NE} is an indicator function that is 1 if the fund does not engage, i.e., if $a = NE$.

For parsimony, we have abstracted from the fact that many funds, particularly hedge funds, have more complex compensation structures that include, for example, carried interest on delegated capital. Since the AUM fee parameter w only enters the model as a $t = 2$ multiplier on the value of delegated capital, it can be interpreted as including the expected value of carried interest as well as up front AUM fees.[12]

Socially optimal engagement

Engagement is a public good. In the subsequent analysis we analyze when engagement is underprovided (or excessively provided) in equilibrium. In order to benchmark this analysis, we first characterize the unconstrained socially optimal level of engagement.

For a given η, denote by K_η the *smallest* integer such that $K_\eta b \geq \eta$ (if it exists), i.e., $K_\eta \equiv argmin_{x \in \mathbb{N}} x$ satisfying $xb \geq \eta$. Since $K_\eta > N$ for any $\eta > \alpha$, engagement cannot succeed and should not occur for such η. At any given $\eta \leq \alpha$, engagement is socially optimal if the cost of having exactly K_η skilled funds engage is less than the benefits associated with successful engagement. Engagement by greater than K_η funds is never optimal as this generates costs with no social benefits. In our equilibrium analysis starting in Section 3, we refer to engagement by strictly more than K_η funds in any equilibrium as *excess engagement*.

[12] For example, in our model if delegated capital with $t = 2$ value Z is allocated to a given fund following the engagement game, they might earn a percentage AUM fee of ς on that value plus some percentage ϱ of the reinvestment gain, which is $(R_\theta - 1)Z$. Then, we can think of w being set such that $wZ = \varsigma Z + \varrho(R_\theta - 1)Z$, i.e, $w = \varsigma + \varrho(R_\theta - 1)$. As an illustration, a traditional hedge fund with a "two and twenty" fee structure offering an expected gross return of $R_\theta = 1.2$ would then have a w equal to $.02 + .2(1.2 - 1) = .06$.

The social payoffs from exactly K_η funds engaging at a given η include (i) the reinvestment value of activist capital given successful engagement and potential reallocations, (ii) the post-engagement value of capital held by passive shareholders, and (iii) the investment of any cash not spent on engagement.

Consider (i) first. Let $\Upsilon \in [\alpha, \alpha + (1-\phi)\,\beta]$ be the proportion of total firm value held by skilled funds at $t = 2$ following any reallocation of capital. While social welfare would always be improved by reallocating as much capital as possible to skilled funds, we treat Υ as a parameter taking values over the indicated range in order to respect the constraints on reallocation imposed in different parts of the equilibrium analysis below. The lower bound of Υ reflects the fact that skilled funds already control an α proportion of the firm, and social welfare cannot be improved by reallocating such capital to unskilled funds. The upper bound of Υ reflects the fact that—as assumed in the model—skin in the game cannot be reallocated; thus at most a $(1-\phi)\,\beta$ proportion of total firm value can be reallocated to skilled funds. The post-engagement value of activist capital given success is therefore $P_h\,(\Upsilon R_S + (\alpha + \beta - \Upsilon))$ for the relevant Υ.

As for (ii), since passive shareholders do not invest in activist funds, the post-engagement value of capital held by such investors given success is $P_h\,(1 - \alpha - \beta)$. Finally, with respect to (iii), since the initial cash can only be reinvested at the rate of return for the fund in question, the investment value of residual cash for a non-engaging fund is cR_θ.

The total welfare enjoyed by all shareholders if K_η skilled funds engage is therefore $P_h(\Upsilon R_S+(1-\Upsilon))+(N-K_\eta)cR_S+Mc$. Alternatively, social welfare with no engagement equals $Mc + NcR_S$. This proves the following result.

Proposition 1. *The social optimum for a given* Υ *is characterized as follows: (1) for every* $\eta \leq \alpha$*, if* $c \leq \frac{P_h(\Upsilon R_S+(1-\Upsilon))}{K_\eta R_S}$*, exactly* K_η *skilled funds engage and engagement* *succeeds, while if* $c > \frac{P_h(\Upsilon R_S+(1-\Upsilon))}{K_\eta R_S}$ *there is no engagement, (2) for every* $\eta > \alpha$*, there is no engagement.*

3 Engagement in equilibrium

As discussed above, activism is a public good which may be underprovided in a decentralized equilibrium. Public goods provision problems are usually characterized by multiplicity of equilibria. As a result, the full equilibrium set of our game is complex. Since our interest is in examining when engagement across multiple blockholders may succeed in equilibrium, we characterize equilibria with "maximal success" for a given set of parameters, i.e., equilibria with the highest overall probability of successful engagement.[13] Such equilibria maximize the expected terminal value of the firm.

Since only skilled funds can engage, our game simplifies into a sequence of state-contingent subgames, one for each η, at which the skilled funds choose their actions. Maximal success equilibria are, therefore, equivalent to the following:

Definition 1. A maximal success equilibrium is one in which for each η, the probability of successful engagement is maximal.

While our interest is in characterizing maximal success equilibria, we begin with the observation that there is always a free-riding equilibrium with no engagement. All proofs are in Online Appendix A.

Lemma 1. *There exists an equilibrium with no engagement.*

If no activist engages, no individual activist can unilaterally change the engagement outcome (because $\eta > b$). Furthermore, since $P_l = 0$ attracting additional delegated

[13]Our interest in analyzing the maximal scope for successful engagement also explains why we do not instead focus on symmetric mixed strategy equilibria, which generate a lower success probability.

capital is worthless. As a result, there are no benefits to engagement, and costly engagement never occurs.

3.1 Proprietary blocks

We start our characterization of maximal success equilibria by considering a proprietary blocks benchmark that corresponds to most of the existing literature on blockholder governance.[14] In this benchmark, $\phi = 1$ so that funds manage only their own money, and competition for flow and delegation fees are irrelevant. We begin with a definition.

Definition 2. For a given η, a pure strategy profile in which K_η skilled funds engage is a K_η *profile.*

We next show that K_η profiles form the basis for successful engagement within a maximal success equilibrium.

Proposition 2. *In the proprietary blocks game, a maximal success equilibrium is characterized as follows: (1) for every $\eta \leq \alpha$, if $c \leq bP_h$ then a K_η profile is played and engagement succeeds, while if $c > bP_h$ there is no engagement, (2) for every $\eta > \alpha$, there is no engagement.*

When $\eta > \alpha$, engagement involving only skilled funds cannot succeed, so the only scope for success is when $\eta \leq \alpha$. For such η, the result shows that the maximal success equilibrium involves engagement by exactly the number of blockholders required for success. Since each fund cares only about the impact of engagement on the value of her individual block, she will engage if and only if she believes she is pivotal with respect to

[14] Our comparison below between the proprietary and delegated environments holds the ownership structure, including individual block sizes, constant. In reality, ownership structure may be endogenous to the nature of blockholders.

engagement success, and moreover will be willing to forego the benefits of saving c to invest at $t = 2$ only if her individual net payoff from engagement conditional on being pivotal, $bP_h R_S$, is sufficiently large. The relevant condition for engagement is therefore $cR_S \leq bP_h R_S$, i.e., $c \leq bP_h$ as stated in the result. When the cost of engagement is higher, the only possible equilibrium outcome is free-riding.[15]

We now compare this equilibrium to the social optimum derived above, specifying $\Upsilon = \alpha$ to reflect the restrictions of the model with proprietary blocks, i.e., the absence of any capital reallocation. The social optimum has two key features: (1) a constraint on the number of engaging activists to K_η whenever engagement occurs (to avoid excessive deadweight costs) and (2) a range of costs over which such engagement is optimal. The decentralized equilibrium in Proposition 2 delivers one of these characteristics: since funds will not engage unless they believe they are pivotal, there is never engagement by more than K_η funds. However, because each activist cares only about the value of her individual stake, her incentive to engage is strictly lower than society would prefer. In particular, as we have shown above, society desires sufficient skilled engagement to achieve success at every $\eta \leq \alpha$ whenever $c \leq \frac{P_h(\Upsilon R_S + (1-\Upsilon))}{K_\eta R_S}$. Given $\Upsilon = \alpha$ and $K_\eta \leq N = \frac{\alpha}{b}$, it is immediate that $\frac{P_h(\Upsilon R_S + (1-\Upsilon))}{K_\eta R_S} > bP_h$.[16] Thus, we state without proof:

Corollary 1. *In the maximal success equilibrium of the proprietary blocks game:*

1. *Successful engagement arises for a strictly smaller range of parameters in equilibrium than in the social optimum, i.e., engagement is underprovided.*

2. *There is no excess engagement.*

[15]Note that for each $\eta \leq \alpha$ there are $\binom{N}{K_\eta}$ different K_η profiles that deliver identical engagement outcomes for that η. We do not distinguish across such subgame equilibria.

[16]The latter inequality would clearly also hold for any $\Upsilon > \alpha$.

3.2 Delegation without competition for flow

Delegated blockholders differ in two key ways from proprietary blockholders in our model: (1) they have less "skin in the game" ($\phi < 1$) since some of their capital comes from outside investors, and (2) they have an incentive to attract additional outside capital due to their fees, i.e., they compete for flow. To isolate the effects of these two factors on engagement incentives, we start by considering a version of the model in which blocks are owned by funds using a combination of proprietary and delegated capital but there are no flows, so only the first difference is operable. This allows us to illustrate the "standard" intuition that delegation reduces incentives since funds who have less skin in the game face lower incentives to take privately costly actions that benefit all investors.

Proposition 3. *In the delegated game without competition for flow, a maximal success equilibrium is characterized as follows: (1) for every $\eta \leq \alpha$, if $c \leq \phi b P_h + w(1-\phi) b P_h$ then a K_η profile is played and engagement succeeds, while if $c > \phi b P_h + w(1-\phi) b P_h$ there is no engagement, (2) for every $\eta > \alpha$, there is no engagement.*

Intuitively, the first term on the right hand side of the upper boundary on c, $\phi b P_h$, is similar to the analogous condition in Proposition 2 and reflects incentives coming from pivotality. This incentive is lower than in the proprietary case because $\phi < 1$, reflecting lower incentives to engage with less skin in the game. The second term on the right hand side reflects incentives coming from fees on locked-up delegated capital. These fees are maximized when the firm's value is maximized. Despite this, given $w < 1$ the right hand side of the existence condition in Proposition 3 is clearly less than the right hand side of the existence condition in Proposition 2. Intuitively, while fees on delegated capital offset the reduction in returns on proprietary capital, the offsetting is not complete because the fee rate on delegated capital, w, is less than 1. Thus, the scope for successful equilibrium engagement is reduced compared to the proprietary blocks game, and more so the lower are ϕ and w. However, at any given η no more than K_η funds engage as in the proprietary case. Thus:

Corollary 2. *In the maximal success equilibrium of the delegated game without competition for flow:*

1. *Successful engagement arises for a strictly smaller range of parameters in equilibrium than in the proprietary blocks game, i.e., the underprovision problem is exacerbated.*

2. *There is no excess engagement.*

3.3 Delegation with competition for flow

In this section we provide our main results on equilibria with competition for flow. First, we lay out some further definitions.

Definition 3. For a given η and any $L_\eta \in \{K_\eta + 1, ..., N-1\}$, a pure strategy profile in which L_η skilled funds engage is an L_η *profile.*

Definition 4. For a given η a pure strategy profile in which N skilled funds engage is an N *profile.*

Definition 5. Let

$$\hat{c}(x) \equiv w\left((1-\phi)\, bP_h + \frac{1}{x}\,(M+N-x)\, b(1-\phi)P_h\right).$$

Intuitively, $\hat{c}(x)$ represents the fees that each engaging fund earns as a result of capital flows in a situation where exactly x funds engage and those funds are assigned a higher posterior belief by investors than the $M+N-x$ non-engaging funds. The total reallocatable capital in each fund is $(1-\phi)b$. Each engaging fund retains their own reallocatable capital and receives inflows worth $1/x$ times the total capital reallocated from non-engaging funds. Such fees are then reinvestable at rate R_S for a total flow reward of $\hat{c}(x)R_S$. Note that $\hat{c}(x)$ is strictly decreasing in x.

Given the complexity of the full model, we split our analysis of maximal engagement equilibria into two results. First, we derive the existence of relevant subgame equilibria at each η.

Lemma 2. *In the delegated game with competition for flow, for any $\eta \leq \alpha$:*

1. *When $\eta \leq (N-1)b$,*
   (a) *if $c \in (\hat{c}(K_\eta+1), \phi b P_h + \hat{c}(K_\eta)]$ a subgame equilibrium exists in which a K_η profile is played, or*
   (b) *if $c \in (\hat{c}(L_\eta+1), \hat{c}(L_\eta)]$ a subgame equilibrium exists in which an L_η profile is played, for $L_\eta \in \{K_\eta+1, ..., N-1\}$, or*
   (c) *if $c < \hat{c}(N)$ a subgame equilibrium exists in which an N profile is played.*
2. *When $\eta \in ((N-1)b, \alpha]$, if $c \leq \phi b P_h + \hat{c}(N)$ a subgame equilibrium exists in which an N profile is played.*

It is noteworthy that the ranges of c over which the equilibria with different strategy profiles (i.e., K_η, L_η and N profiles) exist are non-overlapping, i.e., only one such profile can be played at a given c. We next show that the subgame equilibria derived above are part of a maximal success equilibrium.

Proposition 4. *In the delegated game with competition for flow, a maximal success equilibrium is characterized as follows: (1) for every $\eta \leq \alpha$, if $c \leq \phi b P_h + \hat{c}(K_\eta)$ a K_η, L_η, or N profile is played and engagement succeeds, while if $c > \phi b P_h + \hat{c}(K_\eta)$ there is no engagement, (2) for every $\eta > \alpha$ there is no engagement.*

This result is similar to Propositions 2 and 3 in that successful engagement only occurs if $\eta \leq \alpha$, but note that the condition on c in part (1) of the proposition depends on K_η, and therefore on η, whereas in Propositions 2 and 3 the condition was independent of η. Thus, in the benchmark games engagement is "all or nothing" for all η's below α: either there is successful engagement for all such η, or for none of them. In the delegated game with competition for flow, however, there can be different engagement outcomes at different η's depending on the exact value of c.

It is important to note that our characterization of maximal success equilibria is not limited to pure strategy equilibria, even though those are the only equilibria that appear in our results. For $c \leq \phi b P_h + \hat{c}(K_\eta)$, the identified pure strategy equilibria deliver success for certain, which no mixed equilibrium could do, so must constitute the maximal success subgames. Furthermore, in the proofs we show that mixed strategy equilibria with positive probability of engagement cannot arise at $c's$ above the upper limits in the proposition, so there is no way to achieve success with positive probability at those $c's$.

To see why this is true, it is useful to compare incentives to engage between the K_η subgame equilibrium and a potential mixed equilibrium. Such incentives may differ in two possible ways. First, in mixed equilibria, funds are pivotal only with positive probability, while in K_η subgame equilibria each engaging fund is pivotal for sure. Since pivotality increases incentives to engage, mixed equilibria must feature lower incentives to engage on the basis of pure monetary rewards. Second, in contrast to K_η subgame equilibria, the number of engaging funds in any mixed equilibrium can be either higher than K_η (in which case the flow rewards are smaller than in a K_η subgame equilibrium), or lower than K_η (in which case engagement will fail and capital is worthless). In either case, the flow incentives to engage are lower in the mixed equilibrium than in the K_η equilibrium. Thus, for any c for which the K_η subgame equilibrium cannot exist, mixed equilibria also cannot exist.

We can also state several comparative statics.

Proposition 5. *The following statements hold in the the maximal success equilibrium identified in Proposition 4:*

1. For any $\eta \leq \alpha$, the highest c for which engagement occurs in equilibrium is weakly decreasing in η.

2. For any given c, there exists a threshold level of η, $\bar{\eta} \leq \alpha$, such that in the equilibrium engagement occurs iff $\eta \leq \bar{\eta}$.

3. For any given c, the number of funds engaging in equilibrium is weakly increasing in η as long as η is below the threshold for engagement to occur.

The first result is straightforward: for larger η a greater number of funds must engage to achieve success, which means smaller flow rewards to each engaging fund and therefore a smaller range of c for which success can be achieved. The second result is essentially a corollary to the first: since it is harder to sustain engagement at higher η, for a given c there will exist some maximum η at which success can be achieved. The third result reflects the fact that, for a given c, as η increases the number of funds required for success (K_η) increases. Thus, as long as a K_η subgame equilibrium can still be supported, the number of equilibrium engagers will weakly increase.

4 The Role of Competition for Flow

The key difference between the full model analyzed in Section 3.3 and the benchmarks analyzed in Section 3.1 and 3.2 is that funds compete for flow. We now analyze the manner in which competition for flow affects incentives to engage, and subsequently highlight two key implications of such incentives and analyze their welfare implications.

We first show that with competition for flow the aggregate amount of activist capital, $\alpha + \beta$, has a key effect on engagement incentives.

Proposition 6. *A change in $\alpha + \beta$ holding b constant affects a fund's incentive to engage only in the delegated game with competition for flow. In that game, increasing $\alpha + \beta$ increases the fund's incentive to engage.*

In the proprietary blocks game, a given fund's incentive to engage is limited to the impact on her own capital; she is indifferent to the effect on other shareholders. The amount of total activist capital is therefore irrelevant to her incentives, and engagement is underprovided. As one may suspect on the basis of standard agency theory, this problem is exacerbated in the delegated game without competition for flow since the fund's effective block size is reduced from b to $b\,(\phi + w(1-\phi)) < b$. However, in the delegated game with competition for flow, a fund's incentive to engage depends on how much capital is available in the economy for her to attract if she can advertise her skill. Thus competition for flow fundamentally alters the incentives for engagement: When funds compete for flow, the *total* size of activist capital is relevant to their incentives.

There are two interrelated ways in which changes in activist capital affects engagement incentives. A change in the availability of unskilled activist capital, β, affects engagement incentives only along the *intensive* margin: for each $\eta \leq \alpha$ at which engagement occurs, an increase in β causes the upper limit of c (as identified in Proposition 4) to rise ($\hat{c}(K_\eta)$ is increasing in β), since the amount of capital to attract from "unskilled" funds expands. An increase in the availability of skilled capital, α, on the other hand affects both the intensive and extensive margins: apart from a symmetric increase in the range of c over which engagement occurs (as in the case of an increase in β), it also increases the range of η over which success can be achieved, i.e., there are some η for which engagement may now occur but for which engagement was infeasible before.

At a deeper level, the reason that the total amount of activist capital affects incentives is that, unlike price appreciation, flow rewards from successful engagement are excludable benefits: funds that expend the cost of engagement advertise their skill and thus gain capital flows that do not accrue to non-engaging funds. We show that such endogenously generated excludable benefits have a dual effect. On the one hand, they can ameliorate the underprovision of engagement identified in Sections 3.1 and 3.2. On the other hand, they can lead to excess engagement, as funds engage even when their engagement is unnecessary in order to attract flow. We examine each of these in turn.

4.1 Competition for flow ameliorates underengagement

The potential to attract flow by advertising skill incentivizes engagement. We can see this by comparing engagement in the full model with that in our two benchmarks.

Proposition 7. *In the maximal success equilibrium of the delegated game with competition for flow:*

1. *For each $\eta \leq \alpha$, if $c \in (\phi b P_h + w(1-\phi) b P_h, \phi b P_h + \hat{c}(K_\eta)]$ engagement succeeds where success is impossible in the delegated game without competition for flow. Furthermore, this range is always non-empty.*

2. *For each $\eta \leq \alpha$, if $c \in (bP_h, \phi bP_h + \hat{c}(K_\eta)]$ engagement succeeds where success is impossible in the proprietary blocks game. Furthermore, if $M + N > \frac{2}{w}$, this range is non-empty for some η.*

Part (1) of Proposition 7 compares the delegated games with and without competition for flow. The potential for attracting flow enhances rewards for engagement, raising incentives to engage in the game with competition for flow. This results in successful engagement over a larger parameter space. The comparison in Part (2), with respect to the proprietary game, is more subtle. Here, the increased incentive to engage due to competition for flow must compete with the lowered incentive to engage due to reduced skin in the game (relative to the proprietary game). Nevertheless, we show that under certain conditions, there is a range of c over which the positive effect of competition for flow dominates the negative effect of reduced skin in the game for some η. This occurs when the number of activist blocks is sufficiently large. Fixing the pool of activist capital, $\alpha + \beta$, the smaller is block size, b, the more likely it is that this condition will be satisfied. This is because while a smaller block size directly discourages engagement in the proprietary case, the relative per-blockholder gains from potential flow increase as blocks get smaller.

The aggregation across the incentive effects of competition for flow and reduced skin in the game is fundamentally a quantitative issue, and our model is not ideally suited to such quantitative comparisons. For example, in our model we have taken the conservative view that only delegated capital invested in the single firm at issue can be reallocated based on the engagement outcome. In reality, capital may flow to skilled funds from investments outside this particular firm. Thus, it is conceivable that the quantitative effect of competition for flow is in reality greater than what is captured by the model.

4.2 Competition for flow leads to excess engagement

Inspection of Lemma 2 indicates that for any $\eta \leq (N-1)b$, if $c < \hat{c}(K_\eta + 1)$ a subgame equilibrium exists in which $K_\eta + 1$ or more funds engage in equilibrium. Such engagement is non-pivotal or *excess* engagement, which never occured in the previous benchmark models (see Corollaries 1 and 2), but now occurs in equilibrium anywhere an L_η or N profile is played. Thus, we can state:

Corollary 3. *Excess engagement arises in maximal success equilibria if and only if funds compete for flow. For any $\eta \leq (N-1)b$, excess engagement occurs in the maximal success equilibrium of the delegated game with competition for flow if $c < \hat{c}(K_\eta + 1)$.*

This is a result of the fact that capital inflows are an excludable benefit, and thus funds may wish to engage in equilibrium even if their engagement is not essential for success. This also explains why there are lower as well as upper bounds on the range of permissible costs in parts (a) and (b) of Lemma 2: the lower bounds ensure that only the prescribed number of funds engage.

We now provide further insight into the ranges of c and η over which excess engagement arises in equilibrium. In Figure 1 we map the different maximal success subgame equilibria identified in Lemma 2 for varying levels of η and c, focusing on the range of η for which K_η is between $N-3$ and N. For purposes of the figure we define $\pi \equiv \phi b P_h$. Each vertical line corresponds to a break-point where K_η increases in steps as η increases to the right. The empty regions above the upper red lines within each vertical band correspond to the part of the parameter space where there is no engagement because c is too high. For $c's$ below the red line, an equilibrium with successful engagement exists. For the highest range of $c's$ within each band below the red line, the equilibrium involves a K_η profile being played, i.e., only the required number of

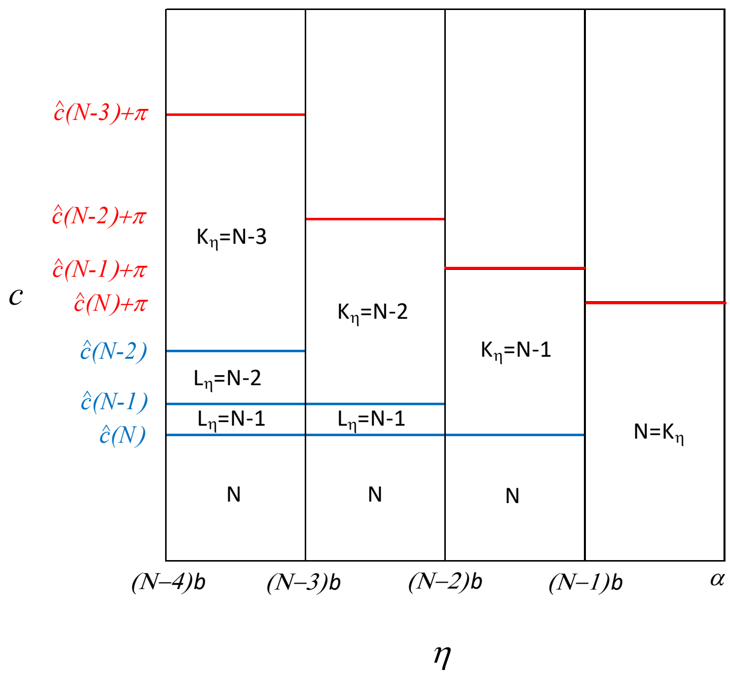

Figure 1: The figure illustrates the number of engaging skilled funds in the maximal success equilibrium for different values of η and c.

skilled funds engage and they are all pivotal. For ranges of c below the blue lines, maximal success equilibria involve engagement by more than the required number of skilled funds (i.e., involve an L_η or N profile being played), and thus feature excess engagement.

4.3 Competition for flow can be beneficial or harmful

Delegation with competition for flow can ameliorate the underprovision of engagement in equilibrium and allow for valuable reallocation of capital, but at the expense of fostering the possibility of excess engagement. Thus, deriving general results about the overall welfare effect is complex. However, we are able to identify two ranges of engagement costs over which the welfare effect of delegation with competition for flow is unambiguous.

Proposition 8. *In maximal success equilibria:*

1. *For each $\eta \leq \alpha$, whenever $c \in (max\left\{bP_h, \hat{c}(K_\eta+1)\right\}, \phi bP_h + \hat{c}(K_\eta)]$ delegation with competition for flow increases social welfare relative to the proprietary blocks game.*

2. *For each $\eta \leq (N-1)b$, whenever $c \leq min\left\{bP_h, \hat{c}(K_\eta+1)\right\}$ delegation with competition for flow decreases social welfare relative to the proprietary blocks game as long as R_S is sufficiently low.*

Over the range of costs in part (1), engagement does not arise in the proprietary blocks game (because $c > bP_h$) but does arise in the delegated game with competition for flow (because $c \leq \phi bP_h + \hat{c}(K_\eta)$). Furthermore, the number of engaging funds in the delegated game with competition for flow is K_η, i.e., there is no excess engagement (because $c > \hat{c}(K_\eta+1)$). Finally, we show in the proof that $\phi bP_h + \hat{c}(K_\eta)$ is always lower than the relevant socially optimal engagement cost threshold, and hence engagement at such c's is efficient. Thus, delegation with competition for flow is unambiguously beneficial in this cost range.

Over the range of costs in part (2), engagement arises in both the proprietary blocks game (because $c \leq bP_h$) and delegated game with competition for flow (because $c \leq \hat{c}(K_\eta + 1)$). However, there is excess engagement in the delegated game: for $\eta \leq (N-1)b$, we have $K_\eta < N$, so the L_η or N equilibrium that arises with $c \leq \hat{c}(K_\eta+1)$ represents excess engagement. This has a negative welfare effect. However, there is an offsetting positive effect due to the reallocation of capital from unskilled to skilled funds that is not possible in the proprietary blocks game. For sufficiently low values of R_S, the negative effect is guaranteed to outweigh the positive effect, resulting in an overall welfare loss.

Overall, this result implies that delegation is more likely to be beneficial for high cost forms of engagement. As discussed in the introduction, these are the situations most likely to be relevant for hedge fund activism.

5 Discussion

In this section we discuss some of our modeling assumptions and compare our model's implications to other possible explanations of wolf packs.

5.1 Other models of wolf packs

It is clear that there may be alternative models of wolf pack formation. For example, one may conjecture that wolf packs arise as a result of private benefits available to each wolf pack member or because of tacit collusion across multiple activist campaigns over time by groups of hedge funds. However, our mechanism generates unique implications that may help separate our findings from these and other alternative explanations. In our model, funds have the ability to attract capital from other activist funds by advertising their skill. They do so by selectively engaging only when their information indicates that doing so is appropriate. This renders the total size of activist capital relevant to their engagement incentives. The larger is the pool of total activist capital, the higher is the proportionate amount that each fund can attract by advertising their skill. This distinguishes our predictions from a number of alternative explanations of wolf pack formation. For example, it may be natural to associate the growth of reallocatable capital in the model with the size of the total activist industry (e.g., if our target firm is "representative"). If so, our model predicts that wolf packs would be seen mostly when the industry has grown and matured, so that a significant pool of reallocatable activist capital is available to successful activists. This stands in contrast with both tacit collusion or private benefit models of wolf pack activism. It is arguable that tacit collusion across multiple targets is harder to sustain in an industry with a larger number of activists. Similarly, it is unclear that private benefits scale with the size of the activist industry. Finally, our framework shows that it is precisely when blocks are relatively small that the positive effect of competition for flow is most likely to overcome the negative effect of reduced skin in the game. It is not obvious that other theories of wolf pack formation, including tacit collusion and private benefits, generate such an implication.

5.2 Single blockholder engagements and hidden wolf packs

As discussed above, for data-driven reasons the empirical literature has focused on blockholders who cross the relevant reporting thresholds (5% ownership in the US), and thus may undercount wolf packs involving smaller blockholders. In other words, activist campaigns identified in the empirical literature as single-blockholder engagements may involve "hidden" wolf packs.[17] However, it is noteworthy that our model of flow motivation could, in principle, apply to single blockholder engagements as well. Formally, this would be captured in our model by setting the lower bound of η below b, so that a single blockholder sometimes has effective control. In practice this could mean that either management in these cases sees the benefits of the activist's agenda

[17] Hidden wolf packs could also arise if a single activist hedge fund receives implicit support from other types of institutional investors, including, e.g., flow motivated mutual funds. An earlier version of this paper (Brav et al. 2019) explored such a model. Evidence for such support across different types of institutional investors can be found in Kedia et al. (2021).

and is easily persuaded to capitulate, or non-activist shareholders representing a significant proportional ownership stake can be persuaded by the activist to provide passive voting support. Our analysis in the bulk of this paper could therefore be viewed as capturing situations in which neither of these things are true, i.e., where management is not interested in yielding and/or there are not sufficient persuadable shareholders to allow a single activist blockholder to prevail. Such difficult targets may emerge when the activism industry is large or mature and easy targets are in short supply. Interestingly, even in such cases where a single activist *may* prevail, our characterization of excess engagement suggests that one may still see wolfpacks.

5.3 Non-zero P_l

Throughout the paper we have maintained the assumption that $P_l = 0$. However, this is not required for our qualitative results. If $P_l > 0$, it is straightforward to show for both the proprietary game and the delegated game without competition for flow that engagement incentives are dampened for all players (because the difference in firm value based on the engagement outcome is smaller), and that the social optimum involves less engagement. Thus, the maximal success equilibria in these games are qualitatively unchanged. In the delegated game with competition for flow, a sufficiently high P_l may affect incentives in perverse ways because skilled funds may wish to engage even when they know success is impossible in order to protect their delegated capital. A sufficient condition to rule out such effects is $P_l < \frac{c}{wb(1-\phi)\frac{1}{2}(M+N)}$. When this condition is satisfied, the maximal success equilibria in the delegated game with reallocation will be qualitatively similar to those analyzed in Proposition 4, with the proviso that we can no longer definitively rule out mixed strategy equilibria with some probability of success at higher $c's$.

6 Conclusion

The possibility of collective engagement by non-controlling blockholders has important implications for corporate governance. We show that parallel engagement by institutional blockholders can play a powerful role in activist campaigns, thus providing a lens through which to view activist wolf packs, a tactic that has generated significant attention. In doing so we analyze the key role of delegation in determining the level of engagment across non-controlling blockholders. Our analysis highlights two key differences between delegated and non-delegated blockholders: reduced skin in the game, and competition for investor flow. We show that while reduced skin in the game weakens engagement incentives, competition for flow fosters an endogenous set of transfers across funds that strengthens incentives to engage, though it can also foster incentives to engage excessively. As Franklin Allen emphasized in his AFA Presidential Address (Allen, 2001), the incentives faced by institutional money managers can have a significant impact on financial markets. Our study suggests that these incentives can have even wider-ranging implications, for example by affecting the nature of shareholder activism.

Our results shed light on existing empirical results regarding wolfpacks, and its new testable predictions should enable empirical researchers to better study the mechanics and implications of collective shareholder engagement. Future work could also examine the role that explicit collusion or intentional information leakage might play in either substituting for or complementing the mechanism we model.

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Online Appendix for "Wolf Pack Activism"

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Online Appendix A: Omitted proofs

Additional notation. In the proofs below, wherever relevant, we index the N skilled activist funds by $i \in \{1, 2, ...N\}$. Investors cannot observe the indices of individual funds, because otherwise fund's types would become publicly known. We denote strategies by $\sigma_i : \eta \to \{E, NE\}$.

Proof of Lemma 1: In equilibrium, $P_2 = 0$. Since $\eta \in (b, 1]$, unilateral engagement by any deviator cannot change P_2, and thus the payoff to engagement is 0 while the payoff to not engaging is cR_S.■

Proof of Proposition 2: First, we show that, for every $\eta \le \alpha$, if $c \le bP_h$ a subgame equilibrium exists in which a K_η profile is played. For each such η, let us specify a strategy profile in which $\sigma_i(\eta) = E$ if and only if $i \in \varepsilon_{K_\eta}$ where ε_{K_η} is a subset of $\{1, 2, ...N\}$ with cardinality K_η. Given such a strategy profile, for each $i \in \varepsilon_{K_\eta}$, the payoff to engaging is bP_hR_S, while the payoff to not engaging is cR_S, since engagement

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fails if less than K_η funds engage. For each $i \notin \varepsilon_{K_\eta}$, the payoff to engaging is bP_hR_S, while the payoff to not engaging is $bP_hR_S + cR_S$, since engagement succeeds even without the fund's own engagement since K_η funds are already engaging in equilibrium. Hence, for $c \leq bP_h$, it is a best response for all $i \in \varepsilon_{K_\eta}$ to engage, and for all $i \notin \varepsilon_{K_\eta}$ not to engage.

Next, note that these subgame equilibria achieve successful engagement for sure for all $\eta \leq \alpha$, and so characterize a maximal success equilibrium for $c \leq bP_h$ at those η. Holding $\eta \leq \alpha$, suppose instead that $c > bP_h$. Now, for any $i \in \{1, ..., N\}$, the maximal payoff to engaging would be bP_hR_S, whereas the minimal payoff to not engaging would be cR_S. Therefore it is a dominant strategy not to engage. Hence, for all $\eta > \alpha$, we have $P_2 = 0$, regardless of $\sigma_i(\eta)$ for $i = 1, ..., N$. Therefore it is a dominant strategy for each skilled activist to not engage. ■

Proof of Proposition 3: First, we show that, for every $\eta \leq \alpha$, if $c \leq \phi bP_h + w(1-\phi)bP_h$ a subgame equilibrium exists in which a K_η profile is played. For each such η, let us specify a strategy profile in which $\sigma_i(\eta) = E$ if and only if $i \in \varepsilon_{K_\eta}$ where ε_{K_η} is a subset of $\{1, 2, ...N\}$ with cardinality K_η. Given such a strategy profile, for each $i \in \varepsilon_{K_\eta}$, the payoff to engaging is $\phi bP_hR_S + wR_S(1-\phi)bP_h$, while the payoff to not engaging is cR_S, since engagement fails if less than K_η funds engage, rendering the value of both proprietary and delegated capital zero. For each $i \notin \varepsilon_{K_\eta}$, the payoff to engaging is $\phi bP_hR_S + wR_S(1-\phi)bP_h$, while the payoff to not engaging is $\phi bP_hR_S + wR_S(1-\phi)bP_h + cR_S$, since engagement succeeds even without the fund's own engagement since K_η funds are already engaging in equilibrium. Hence, for $c \leq \phi bP_h + w(1-\phi)bP_h$, it is a best response for all $i \in \varepsilon_{K_\eta}$ to engage, and for all $i \notin \varepsilon_{K_\eta}$ not to engage.

Next, note that these subgame equilibria achieve successful engagement for sure for all $\eta \leq \alpha$, and so characterize a maximal success equilibrium for $c \leq \phi bP_h + w(1-\phi)bP_h$

at those η. Holding $\eta \leq \alpha$, suppose instead that $c > \phi b P_h + w(1-\phi)bP_h$. Now, for any $i \in \{1, ..., N\}$, the maximal payoff to engaging would be $\phi b R_S P_h + w R_S(1-\phi)bP_h < 0$, whereas the minimal payoff to not engaging would be cR_S. Therefore it is a dominant strategy not to engage. Hence, for all $\eta > \alpha$, we have $P_2 = 0$, regardless of $\sigma_i(\eta)$ for $i = 1, ..., N$. Therefore it is a dominant strategy for each skilled fund to not engage. ■

Proof of Lemma 2: We begin by specifying off equilibrium beliefs that are used to support the equilibria that we construct. Investors who evaluate funds base their inferences on each fund's enagement and the publicly observed value of η. When they observe an off-equilibrium amount of engagement at a given η, we assume that that all engagers are assigned a posterior of 1, while all non-engagers are assigned a posterior strictly less than 1.

Suppose $\eta \leq (N-1)b$. For each such η, let us first specify a strategy profile in which $\sigma_i(\eta) = E$ if and only if $i \in \varepsilon_{K_\eta}$ where ε_{K_η} is a subset of $\{1, 2, ...N\}$ with cardinality K_η. Given such a strategy profile, for each $i \in \varepsilon_{K_\eta}$, the payoff to engaging is

$$\phi b R_S P_h + w R_S \left((1-\phi)\, bP_h + \frac{1}{K_\eta}(M + N - K_\eta)b(1-\phi)P_h \right),$$

where the payoffs follow from the facts that: (i) if that fund engages engagement succeeds (each fund is pivotal); (ii) engagement reveals the fund to be skilled (because only skilled funds engage in equilibrium), and thus the fund retains its own delegated capital and further gains a $1/K_\eta$-th share of the capital of the $M + N - K_\eta$ funds that do not engage. In contrast, the payoff to not engaging is cR_S, because engagement fails if less than K_η funds engage so any remaining proprietary or delegated capital is worthless. Thus, for each $i \in \varepsilon_{K_\eta}$, engagement is a best response if:

$$c \leq \phi b P_h + w \left((1-\phi)\, b P_h + \frac{1}{K_\eta}(M+N-K_\eta) b(1-\phi) P_h \right) = \phi b P_h + \hat{c}(K_\eta).$$

For each $i \notin \varepsilon_{K_\eta}$, the equilibrium payoff to not engaging is $\phi b P_h R_S + c R_S$, because (as above) all delegated capital is transferred to funds that have $i \in \varepsilon_{K_\eta}$. If they deviate to engaging, their off equilibrium payoff (given the beliefs specified above) is

$$\phi b P_h R_S + w R_S \left((1-\phi) b P_h + \frac{1}{K_\eta + 1}(M+N-K_\eta - 1) b(1-\phi) P_h \right),$$

because with more than K_η engagers engagement succeeds, and all $K_\eta + 1$ engagers are considered skilled and capture delegated capital from the $M + N - (K_\eta + 1)$ non-engagers. Thus, any fund $i \notin \varepsilon_{K_\eta}$, will choose not to engage if

$$c > w \left((1-\phi) b P_h + \frac{1}{K_\eta + 1}(M+N-K_\eta - 1) b(1-\phi) P_h \right) = \hat{c}(K_\eta + 1).$$

Thus, a K_η profile is a subgame equilibrium if $c \in (\hat{c}(K_\eta + 1), \phi b P_h + \hat{c}(K_\eta)]$.

Next, consider $c \leq \hat{c}(K_\eta + 1)$. For any $L_\eta \in \{K_\eta + 1, ..., N-1\}$, let us next specify a strategy profile in which $\sigma_i(\eta) = E$ if and only if $i \in \varepsilon_{L_\eta}$ where ε_{L_η} is a subset of $\{1, 2, ...N\}$ with cardinality L_η. Given such a strategy profile, for each $i \in \varepsilon_{L_\eta}$, the payoff to engaging is

$$\phi b R_S P_h + w R_S \left((1-\phi)\, b P_h + \frac{1}{L_\eta}(M+N-L_\eta) b(1-\phi) P_h \right),$$

where the payoffs follow from the facts that: (i) if that fund engages engagement succeeds (more than K_η funds are engaging); (ii) engagement reveals the fund to be skilled (because only skilled funds engage in equilibrium), and thus the fund retains its own delegated capital and further gains a $1/L_\eta$-th share of the capital of the $M+N-L_\eta$ funds that do not engage. In contrast, the payoff to not engaging is $\phi b R_S P_h + cR_S$, because engagement still succeeds without that fund's participation (no individual fund is pivotal for $L_\eta > K_\eta$) but non-engagement leads to an outflow of all delegated capital, consistent with our assumed off-equilibrium beliefs. Thus, for each $i \in \varepsilon_{L_\eta}$, engagement is a best response if:

$$c \leq w\left((1-\phi)\, bP_h + \frac{1}{L_\eta}(M+N-L_\eta)b(1-\phi)P_h\right) = \hat{c}(L_\eta).$$

For each $i \notin \varepsilon_{L_\eta}$, the equilibrium payoff to not engaging is $\phi b P_h R_S + cR_S$, because (as above) all delegated capital is transferred to funds that have $i \in \varepsilon_{L_\eta}$. If they deviate to engaging, their off equilibrium payoff (given the beliefs specified above) is

$$\phi b P_h R_S + wR_S\left((1-\phi)bP_h + \frac{1}{L_\eta+1}(M+N-L_\eta-1)b(1-\phi)P_h\right),$$

because with more than L_η engagers engagement succeeds, and all $L_\eta+1$ engagers are considered skilled and capture delegated capital from the $M+N-(L_\eta+1)$ non-engagers. Thus, any fund $i \notin \varepsilon_{K_\eta}$, will choose not to engage if

$$c > w\left((1-\phi)bP_h + \frac{1}{L_\eta+1}(M+N-L_\eta-1)b(1-\phi)P_h\right) = \hat{c}(L_\eta+1).$$

Thus, a L_η profile is a subgame equilibrium if $c \in (\hat{c}(L_\eta+1), \hat{c}(L_\eta)]$.

Since $\hat{c}(L_\eta)$ is strictly decreasing in L_η, the ranges of c for which the L_η subgame equilibria exist do not overlap for different L_η. So, now consider the case where $c \leq \hat{c}(N)$. Now let us next specify a strategy profile in which $\sigma_i(\eta) = E$ if and only if

$i \in \{1, 2, ...N\}$. Given such a strategy profile, for each i, the payoff to engaging is

$$\phi b R_S P_h + w R_S \left((1-\phi)\, b P_h + \frac{1}{N} M b (1-\phi) P_h \right),$$

where the payoffs follow from the facts that: (i) if that fund engages engagement succeeds (each fund is pivotal); (ii) engagement reveals the fund to be skilled (because only skilled funds engage in equilibrium), and thus the fund retains its own delegated capital and further gains a $1/N$-th share of the capital of the M funds that do not engage. In contrast, the payoff to not engaging is $\phi b R_S P_h + c R_S$, because engagement still succeeds without that fund's participation (no individual fund is pivotal for $N > K_\eta$) but non-engagement leads to an outflow of all delegated capital, consistent with our off-equilibrium beliefs. Thus, for each i, engagement is a best response if:

$$c \leq w \left((1-\phi)\, b P_h + \frac{1}{N} M b (1-\phi) P_h \right) = \hat{c}(N).$$

Finally, for $\eta \in ((N-1)b, \alpha]$ we have $K_\eta = N$. Thus, by repetition of the argument for K_η subgame equilibria above, we have that if $c \leq \phi b P_h + \hat{c}(N)$ a subgame equilibrium exists in which an N profile is played. Note that we do not need to compute a lower bound on c because all skilled funds engage in this subgame equilibrium.■

Proof of Proposition 4: For $c \leq \phi b P_h + \hat{c}(K_\eta)$ the specified equilibrium strategies achieve successful engagement for sure for all $\eta \leq \alpha$, and so we have characterized a maximal success equilibrium for such parameters. For $\eta > \alpha$, engagement can never succeed, so any reallocatable capital is worthless and it is a dominant strategy not to engage. Thus, to prove the result we need to show that for $\eta \leq \alpha$ and $c > \phi b P_h + \hat{c}(K_\eta)$ skilled funds never engage. Holding $\eta \leq \alpha$, suppose that $c > \phi b P_h + \hat{c}(K_\eta)$. We first show that for such c, there is never any pure strategy subgame equilibrium with successful engagement. Successful engagement requires that, at each η, at least K_η funds engage. Thus, the only possible pure strategy subgame equilibria are ones in which, for $\eta \leq (N-1)b$, K_η, L_η,or N profiles are played at each η while for $\eta \in ((N-1)b, \alpha]$ an N profile is played at each η. To demonstrate the non-existence of each such pure strategy subgame equilibrium, we shall show that the upper bound on c was not only sufficient, but also necessary, to ensure that funds due to engage in equilibrium do not deviate to non-engagement. This may, in principle, depend on off equilibrium beliefs.

In a K_η subgame equilibrium, the deviation of any fund to non-engagement leads to failure, in which case all capital, proprietary and delegated, is worthless and thus the deviation payoff is unaffected by flows driven by off equilibrium investor beliefs. For L_η or N subgame equilibria, the incentive to deviate to non-engagement for such agents relies on off-equilibrium inferences about such agents.

We shall show that the off-equilibrium beliefs used to construct the equilibria in Lemma 2 already imposed maximal penalties for deviation to non-engagement. In other words, any other off equilibrium belief would strictly reduce incentives to engage. Hence, if engaging funds would wish to deviate under the off equilibrium beliefs of Lemma 2, they would certainly wish to do so for any other off equilibrium beliefs.

The off equilibrium beliefs that were used to support the subgame equilibria in Lemma 2 are as follows. Investors who evaluate funds base their inferences on each fund's enagement and the publicly observed value of η. When they observe an off-equilibrium amount of engagement at a given η, we assume that that all engagers are assigned a posterior of 1, while all non-engagers are assigned a posterior strictly less than 1.

These beliefs imply that in any L_η or N subgame equilibrium, if a fund deviates to non-engagement then investors observe a strictly smaller number of engagers than specified and the deviating fund is assigned a posterior strictly smaller than 1, meaning that it loses all delegated capital to funds that do engage. This is clearly the maximal punishment that can be imposed on the deviating fund subject to engagement being successful.

Consider $\eta \leq (N-1)b$. Suppose that a K_η profile is played in equilibrium. In other words, for each such η, let us first specify a strategy profile in which $\sigma_i(\eta) = E$ if and only if $i \in \varepsilon_{K_\eta}$ where ε_{K_η} is a subset of $\{1, 2, ...N\}$ with cardinality K_η. For each $i \in \varepsilon_{K_\eta}$, the payoff to engaging is $\phi b R_S P_h + R_S \hat{c}(K_\eta)$,while the payoff to not engaging is cR_S. Thus, for $c > \phi b P_h + \hat{c}(K_\eta)$, player i will deviate for any $i \in \varepsilon_{K_\eta}$. So the K_η profile cannot be played in equilibrium.

Now, considering L_η profiles instead, it is clear from the proof of Lemma 2 that for any player due to engage in such a subgame equilibrium, it is necessary to have $c < \hat{c}(L_\eta) < \hat{c}(K_\eta) < \phi b P_h + \hat{c}(K_\eta)$. Hence, L_η profiles cannot be played in equilibrium. Similarly, for N equilibria for each skilled fund, it is necessary to have $c < \hat{c}(N) < \hat{c}(K_\eta) < \phi b P_h + \hat{c}(K_\eta)$. Hence, N profiles cannot be played in equilibrium. The argument for N profiles for $\eta \in ((N-1)b, \alpha]$ is identical to the proof for K_η profiles, since for $\eta \in ((N-1)b, \alpha]$, $K_\eta = N$.

To complete the proof, we now need to show that for $c > \phi b P_h + \hat{c}(K_\eta)$, there are no equilibria where skilled funds mix. Suppose skilled funds engage with some index-dependent probability σ_i. Since only skilled funds mix, engagement is a positive signal, regardless of success. For any skilled fund the payoff from engagement is as follows

$$\Pi^{mixed}_{engage} \equiv \phi b E(P_2 | engage) R_S$$

$$+wR_S(1-\phi)\sum_{j=1}^{N}Pr(j\text{ total skilled funds engage})\left(1+\frac{1}{j}(M+N-j)\right)P_2\left(j\right)b$$

where $P_2(j)$ is the terminal price of the target firm when j funds engage. The payoff to not engaging is:

$$\Pi^{mixed}_{not-engage}\equiv\phi bE(P_2|not-engage)R_S+wR_S(1-\phi)Pr(0\text{ skilled funds engage})P_2\left(0\right)b+cR_S,$$

because a non-engaging fund can retain its delegated capital only if no other fund engages in equilibrium. Comparing these payoffs to those of a K_η subgame equilibrium, we note that $\Pi^{mixed}_{engage}<\phi bR_SP_h+R_S\hat{c}(K_\eta)$ and $\Pi^{mixed}_{not-engage}\geq cR_S$. To see that $\Pi^{mixed}_{engage}<\phi bR_SP_h+R_S\hat{c}(K_\eta)$, note that $E(P_2|engage)\leq P_h$ and $P_2\left(j\right)=0$ for all $j<K_\eta$. Thus, for $c>\phi bP_h+\hat{c}(K_\eta)$, such a mixed equilibrium cannot exist.■

Proof of Proposition 5:

Proof of (1). The highest c for which engagement occurs at any given η is $\phi bP_h+\hat{c}(K_\eta)$. ϕbP_h is independent of η, while $\hat{c}(K_\eta)$ is decreasing in K_η, which in turn is weakly increasing in η.

Proof of (2). Given (1), one of the following statements must be true: Either $c>\phi bP_h+\hat{c}(2)$ in which case engagement never occurs for any $\eta\leq\alpha$; or $c\leq\phi bP_h+\hat{c}(N)$, in which case engagement occurs for all $\eta\leq\alpha$; or $c\in(\phi bP_h+\hat{c}(N),\phi bP_h+\hat{c}(2)]$, in which case engagement occurs for some η's, for which $c\leq\phi bP_h+\hat{c}(K_\eta)$, but not for higher η's for which $c>\phi bP_h+\hat{c}(K_\eta)$.

Proof of (3). Engagement never fails in equilibrium, and thus in equilibrium, the minimal engagement for each η involves K_η funds, where K_η is weakly increasing in η. Further, if for any η, if $c\in(\hat{c}(L_\eta+1),\hat{c}(L_\eta)]$ for some $L_\eta\in\{K_\eta+1,...,N-1\}$, Lemma 2 immediately implies that an L_η strategy constitutes a subgame equilibrium for any $\eta' > \eta$ such that $K_{\eta'} < L_\eta$.■

Proof of Proposition 6: The engagement conditions for a blockholder in the proprietary case and the delegated case without competition for flow are, respectively, $c \leq bP_h$ and $c \leq \phi bP_h + w(1-\phi)bP_h$. Since b is held constant, neither of these equations depends on α or β. Now note that the engagement conditions for all of the equilibria in the delegated case with competition for flow depend on $\hat{c}(\cdot)$, which is defined for a given argument x as $\hat{c}(x) \equiv w\left((1-\phi)\, bP_h + \frac{1}{x}(M+N-x)b(1-\phi)P_h\right)$, which can be expressed as $\hat{c}(x) \equiv wb\left((1-\phi)\, P_h + \frac{1}{x}(\frac{\alpha+\beta}{b} - x)(1-\phi)P_h\right)$. Thus, the range of c for which any of the equilibria identified in Lemma 2 exists is affected by α and β holding b constant, and the upper boundary of the existence range for each equilibrium is increasing in $\alpha + \beta$.■

Proof of Proposition 7: Proof of part (1): In the delegated game without competition for flow, for each $\eta \leq \alpha$, whenever $c \leq \phi bP_h + w(1-\phi)bP_h \equiv \bar{c}_{NR}$ then a K_η profile is played and engagement succeeds. In the delegated game with competition for flow, for each $\eta \leq \alpha$, whenever $c \leq \phi bP_h + \hat{c}(K_\eta)$ a K_η, L_η, or N profile is played and engagement succeeds. This proves the first statement. Since $\hat{c}(K_\eta) = w\left((1-\phi)\, bP_h + \frac{1}{K_\eta}(M+N-K_\eta)b(1-\phi)P_h\right)$, we can rewrite $c \leq \phi bP_h + \hat{c}(K_\eta)$ as $c \leq \bar{c}_{NR} + w\left(\frac{1}{K_\eta}(M+N-K_\eta)b(1-\phi)P_h\right) > \bar{c}_{NR}$. This proves the second statement.

Proof of part (2): Engagement occurs in the proprietary game iff $\eta < \alpha$ and $c \leq bP_h$, while in the delegated game with competition for flow it occurs for a given η if $c \leq \phi bP_h + \hat{c}(K_\eta)$. This proves the first statement. Since

$$\hat{c}(K_\eta) = w\left((1-\phi)\, bP_h + \frac{1}{K_\eta}\,(M+N-K_\eta)\, b(1-\phi)P_h\right),$$

the RHS of the condition for the delegated game with competition for flow is decreasing in K_η, so the condition is easiest to satisfy close to the lower end of the support of η, where $K_\eta = 2$. Thus, to show that there are some η for which the range is non-empty, it suffices to have

$$(1-\phi)bP_h < w\left((1-\phi)\,bP_h + \frac{1}{2}\,(M+N-2)\,b(1-\phi)P_h\right)$$

or, simplifying, $\frac{2}{w} < M + N$.■

Proof of Proposition 8: Proof of part (1): There is successful engagement with exactly K_η engagers in the maximal success equilibrium of the delegated game with competition for flow in this cost range according to Lemma 2 and Proposition 4. There is no engagement in the proprietary game in this cost range according to Proposition 2. Thus, it suffices to prove that engagement in the delegated game with competition for flow is efficient over this entire range. To do so we show that $\phi bP_h + \hat{c}(K_\eta) < \frac{P_h(1+(\alpha+(1-\phi)\beta)(R_S-1))}{K_\eta R_S}$, where the RHS is the maximum c for which engagement is socially beneficial for a given η when $\Upsilon = \alpha + (1-\phi)\beta$, which is the appropriate benchmark for the delegated game with competititon for flow since all reallocatable capital is invested in skilled funds following successful engagement. Expanding and simplifying the LHS gives $\phi bP_h + wb(1-\phi)P_h\left(\frac{M+N}{K_\eta}\right)$. This is clearly increasing in w, so if the inequality holds at $w = 1$ it will always hold. We thus set $w = 1$ and suppose, by way of contradiction, that

$$\phi bP_h + b(1-\phi)P_h\left(\frac{M+N}{K_\eta}\right) > \frac{P_h\,(1+(\alpha+(1-\phi)\beta)\,(R_S-1))}{K_\eta R_S}$$

$$\Rightarrow -\phi\alpha > \frac{1-(\alpha+(1-\phi)\beta)}{R_S} - \phi bK_\eta.$$

Noting that $bK_\eta < \alpha$, this can hold only if

$$0 > \frac{1-(\alpha+(1-\phi)\beta)}{R_S},$$

which is a contradiction.

Proof of part (2): Welfare in the maximal success equilibrium of the proprietary blocks game for this range of c and η is

$$P_h(1+\alpha(R_S-1))+(N-K_\eta)cR_S+Mc$$

since exactly $K_\eta < N$ skilled funds engage. In the maximal success equilibrium of the delegated game with competition for flow an L_η equilibrium (or N equilibrium) is played, resulting in welfare of

$$P_h(1+(\alpha+(1-\phi)\beta)(R_S-1))+(N-L_\eta)cR_S+Mc$$

(where L_η would be replaced by N for an N equilibrium). Letting $R_S \to 1$ provides the result since $K_\eta < N$, and wherever an L_η equilibrium exists in the delegated game with competition for flow, $L_\eta > K_\eta$ holds. ■

Online Appendix B: Asymmetric information model

In our baseline analysis we assumed that unskilled funds faced a prohibitively high cost of engagement. An alternative formulation which may also be relevant for real world applications is one in which skilled and unskilled firms face the same engagement costs, but are differentially informed. In this extension, we show that for empirically relevant parameters, the qualitative results of such a formulation would be identical to our baseline case.

The model remains broadly unchanged, with the following difference: both skilled and unskilled funds can engage at cost c, but η is no longer publicly revealed at $t = 1$. Instead, skilled funds enjoy an informational advantage by observing η privately at $t = 1$, while unskilled funds do not. The parameter η is publicly observed at $t = 2$, and used by fund investors along with the engagement outcome and the actions of all funds to evaluate funds' skill as before.

Given the presence of asymmetric information, we need to lay out some additional notation and definitions. For notational convenience, we assign indices $i = 1, ..., N$ to skilled activists and indices $i = N + 1, ..., N + M$ to unskilled activists. The $t = 1$ information set of fund i is $\mathcal{I}_i$. For skilled funds, $\mathcal{I}_i = \eta$, while for unskilled funds $\mathcal{I}_i = \emptyset$. Strategies profiles take the form $\sigma_i : \mathcal{I}_i \to \Delta\{E, NE\}$. Fund investors observe the action choices by all funds $\{a_i\}$, the outcome of the engagement (P_2), and η and form beliefs about each fund $\gamma(a_i, P_2, \eta) = Pr(\theta_i = S|a_i, P_2, \eta)$ for $i = 1, ..., M + N$. Note that fund investors cannot observe a fund's index itself – otherwise inferences would be trivial. As a result the posterior function $\gamma(\cdot)$ is not indexed by i. In other words, inferences about any two funds that take the same action are identical in equilibrium. A perfect Bayesian equilibrium is characterized by:

1. State contingent strategies $\sigma_i(\eta)$ for $i = 1, ...N$, for each η,

2. State uncontigent strategies $\sigma_i(\emptyset)$ for $i = N + 1, ..., N + M$, and

3. Investor beliefs $\gamma(a_i, P_2, \eta)$ for all a_i, P_2, and η.

such that:

1. For each $i \in \{1, ..., N\}$ and each η, $\sigma_i(\eta)$ is a best response to $\sigma_j(\eta)$ for $j \in \{1, ..., N\} \backslash \{i\}$ and $\sigma_j(\emptyset)$ for $i \in \{N+1, ..., N+M\}$, given investor beliefs (which determine capital reallocation as described above).

2. For each $i \in \{N+1, ..., N+M\}$, $\sigma_i(\emptyset)$ is a best response to $\sigma_j(\eta)$ for $j \in \{1, ..., N\}$ and $\sigma_j(\emptyset)$ for $i \in \{N+1, ..., N+M\} \setminus \{i\}$, given investor beliefs (which determine capital reallocation as described above).

3. Investor beliefs $\gamma(a_i, P_2, \eta)$ for each a_i, P_2, and η are computed according to Bayes rule along the equilibrium path and arbitrary otherwise.

We can now state:

Proposition 9. *For*

$$c > \phi b Pr[\eta \leq \alpha + \beta] P_h + Pr[\eta \leq \alpha + \beta] w P_h \left((1-\phi)b + \frac{1}{2}(M+N-2)b(1-\phi) \right)$$

unskilled funds never engage in equilibrium.

Proof: Consider a generic (possibly mixed) equilibrium in which some skilled funds engage with some probability at some η's and some unskilled funds engage unconditionally with some probability. Consider any arbitrary unskilled fund i. Conditional on engaging, fund i will receive a cash flow payoff of $\phi b E(P_2 | a_i = E)$ on its proprietary capital. Further, if (i) engagement succeeds conditional on engagement by fund i and (ii) if, upon engaging, fund i attains the highest posterior in the cross section of funds, then it will retain its own delegated capital and receive further inflows, which we denote $f(a_i = E | \eta, a_{-i})$. Denote event (i) by Λ and event (ii) i.e., $\gamma(a_i = E, a_{-i}, P_h, \eta) = max_{j \in 1,..,i-1,i+1,..,M+N\}} \gamma(a_j, a_i = E, a_{-j}, P_h, \eta)$, by Θ. If event Λ does not occur, flows are worthless, while if event Θ does not occur, all delegated capital flows to funds other than fund i. Thus, fund i's expected payoff from engaging is as follows:

$$\phi b Pr\left(\Lambda\right) P_h + Pr[\Lambda, \Theta] w \left((1-\phi) b P_h + E\left(f(a_i = E|\eta, a_{-i})|\Lambda, \Theta\right)\right)$$

Now observe that:

1. $Pr\left(\Lambda\right) \leq Pr[\eta \leq \alpha + \beta]$,

2. $Pr[\Lambda, \Theta] \leq Pr[\Lambda] \leq Pr[\eta \leq \alpha + \beta]$, and

3. $f(a_i = E|\eta, a_{-i}) \leq \frac{1}{2}(M + N - 2) b (1 - \phi) P_h$, because the maximal flow that fund i could gain on the basis of engagement is the total delegated activist capital and – since success can only be obtained with at least two funds engaging – such flows must be shared with at least one other fund.

Thus, the engagement payoff is bounded above by

$$\phi b Pr[\eta \leq \alpha + \beta] P_h + Pr[\eta \leq \alpha + \beta] w P_h \left((1-\phi) b + \frac{1}{2}(M + N - 2) b (1 - \phi)\right)$$

The minimal payoff to not engaging is c. Thus, if

$$c > \phi b Pr[\eta \leq \alpha + \beta] P_h + Pr[\eta \leq \alpha + \beta] w P_h \left((1-\phi) b + \frac{1}{2}(M + N - 2) b (1 - \phi)\right),$$

fund i will not engage with positive probability in equilibrium.■

The intuition for this result is as follows. Unskilled funds are uninformed, and hence cannot judge precisely when engagement is likely to earn them returns by way of either gains on their proprietary capital or capital infows. Proposition 9 establishes a sufficient condition, whereby even if—whenever engagement can in principle be successful (i.e., whenever $\eta \leq \alpha + \beta$)—unskilled funds were to be guaranteed the full return on their proprietary capital (i.e., $\phi b P_h$) and if in each such instance they received the highest capital inflow consistent with successful engagement (i.e., $P_h\left((1-\phi)b + \frac{1}{2}(M+N-2)b(1-\phi)\right)$) they would find the cost c too high. Clearly, the lower in the ex ante probability that engagement can be successful, i.e., the lower is $Pr[\eta \leq \alpha + \beta]$, the harder it is incentivize unskilled funds to engage.

As long as the condition in Proposition 9 holds, all our analysis in Sections 3 and 4 go through unchanged. Further, note that if $Pr[\eta \leq \alpha+\beta]$ is low enough then the lower bound on c identified in Proposition 9 will be strictly lower than $\phi b P_h + w(1-\phi)bP_h$, which is the lowest upper bound on c required to induce skilled funds to engage in Section 3. Then there is a non-empty range of parameters supporting all our analysis in Sections 3 and 4.

High engagements costs are empirically relevant because hedge fund activism is widely recognized to be costly to activists. For example, Gantchev (2013) estimates that the *average* costs of activist campaigns by hedge funds range from $2 million to $11 million depending on the difficulty of the campaign and whether it ultimately culminates in a proxy contest. Nevertheless, we provide a brief characterization of the low-cost case next. When c is low the set of potential equilibria is particularly complex. We characterize maximal success equilibria that have successful engagement for all $\eta \leq \alpha + \beta$, with a focus on the effect of competition for flow on the ability to achieve this level of success. Such equilibria are not achievable without engagement by unskilled funds. We have the following result.

Proposition 10.

(i) Without reallocation, if $c \leq \frac{\alpha+b}{U-b} b P_h(\phi + w(1-\phi))$ *there exists a maximal success equilibrium in which engagement succeeds for all* $\eta \leq \alpha + \beta$*, all unskilled funds engage*

unconditionally, and skilled funds engage iff they are pivotal.

(ii) With reallocation, if $c < min[\phi b \frac{b}{U-b} P_h + \frac{\alpha+\beta-b}{U-b} wb(1-\phi)P_h, w\left((1-\phi)bP_h\right)]$*, there exists a maximal success equilibrium in which engagement succeeds for all* $\eta \leq \alpha+\beta$*, all unskilled funds engage unconditionally, and all skilled funds engage when* $\eta \leq \alpha + \beta$.

Proof: (i) Without reallocation, no fund will ever engage if they believe there is zero chance they are pivotal. Thus, the only feasible equilibrium with success at all $\eta \leq \alpha + \beta$ will be one in which (1) all unskilled always engage unconditionally, (2) skilled funds only engage when they think they could be pivotal, and therefore (3) only the number of skilled funds actually needed at a given η will engage in equilibrium. In other words, equilibrium behavior of skilled funds is as follows: for $\eta \leq \beta$, the unskilled by themselves are sufficient for success, so no skilled funds engage; for $\eta \in (\beta, \alpha + \beta]$, the unskilled funds alone are not sufficient for success, so $K_\eta - M$ skilled funds engage; and for $\eta > \alpha + \beta$, success is impossible so no skilled funds engage. Now consider whether the required behavior can be supported in equilibrium.

First consider unskilled funds. They are supposed to engage unconditionally. For the range $\eta \in (\beta - b, \alpha + \beta]$, every engaging fund is pivotal, and for the range $\eta \leq \alpha + \beta$ engagement succeeds. Unskilled funds' engagement payoff will be

$$\phi b Pr[\eta \leq \alpha + \beta]P_h + wb(1-\phi)Pr[\eta \leq \alpha + \beta]P_h$$

while their non engagement payoff will be

$$\phi b Pr[\eta \leq \beta - b]P_h + wb(1-\phi)Pr[\eta \leq \beta - b]P_h + c$$

Thus, the unskilled will engage if $c < Pr[\eta \in (\beta - b, \alpha + \beta]bP_h(\phi + w(1-\phi))$.

Now consider skilled funds. Any skilled fund not expected to engage is happy not to engage since they cannot change the succes outcome by doing so. Any skilled fund expected to engage faces the same incentive as in Proposition 3 (since they are always pivotal when expected to engage), i.e., those who are supposed to engage in equilibrium will do so if $c \leq \phi b P_h + w(1-\phi)bP_h$. This condition is clearly easier to satisfy than the above condition for the unskilled, so the condition for existence of this equilibrium is $c < Pr[\eta \in (\beta - b, \alpha + \beta] b P_h(\phi + w(1-\phi)) = \frac{\alpha+b}{U-b} b P_h(\phi + w(1-\phi))$.

(ii) We assume the off-equilibrium belief that any fund that does not engage when $\eta \leq \alpha+\beta$ is unskilled. In the proposed equilibrium, unskilled funds all engage unconditionally while skilled funds are expected to behave as follows: for $\eta \leq \alpha + \beta$, all skilled funds engage, while for $\eta > \alpha + \beta$, no skilled funds engage. Now consider whether the required behavior can be supported in equilibrium.

First consider unskilled funds. For the range $\eta \in (\alpha + \beta - b, \alpha + \beta]$, every fund is pivotal, and for the range $\eta \leq \alpha + \beta$ engagement succeeds. An unskilled fund's engagement payoff is

$$\phi b Pr[\eta \leq \alpha + \beta] P_h + Pr[\eta < \alpha + \beta] w \left((1-\phi) b P_h\right),$$

because when every fund engages, there is no update about any fund's skill and there is no reallocation. Their non engagement payoff given our assumed off equilibrium belief is

$$\phi b Pr[\eta \leq \alpha + \beta - b] P_h + c,$$

so they will engage if $c < \phi b Pr[\eta \in (\alpha+\beta-b, \alpha+\beta] P_h + Pr[\eta < \alpha+\beta] w \left((1-\phi) b P_h\right)$.

Now consider the skilled funds. When they are expected to engage but they are not pivotal (i.e., for $\eta \in (b, \alpha + \beta - b)$), their engagement payoff is

$$\phi b R_S P_h + w\left((1-\phi) b P_h\right) R_S,$$

while their non engagement payoff under our assumed off equilibrium belief is

$$\phi b R_S P_h + c R_S,$$

so they will engage if $c < w\left((1-\phi) b P_h\right)$. Note that when they are expected to engage but are pivotal, i.e., $\eta \in (\alpha + \beta - b, \alpha + \beta]$, they will be even more likely to engage. Thus, this equilibrium will exist if (using the uniform distribution)

$$c < min[\phi b \frac{b}{U-b} P_h + \frac{\alpha + \beta - b}{U-b} w b (1-\phi) P_h, w\left((1-\phi) b P_h\right)]. \blacksquare$$

Without competition for flow, there is a single equilibrium type that delivers this level of success: a pure strategy equilibrium in which all unskilled engage unconditionally, while skilled players engage only when pivotal. With competition, we have constructed a pooling equilibrium where all unskilled players engage unconditionally while all skilled players engage whenever there is success. This pooling equilibrium is supported by the off equilibrium belief that any fund failing to engage when $\eta \leq \alpha + \beta$ is unskilled. Thus, the high level of engagement in this equilibrium is supported by the fear of *losing* existing delegated capital rather than the hope of *gaining* delegated capital from others.

Note that it is possible, as in the baseline analysis, for the existence range for the game with competition for flow to be greater than the range for the game without. This is more likely to be true when ϕ is small and β is large. A small ϕ means that there is little incentive coming from being pivotal, so flow incentives are relatively more important for those cases. A large β expands the range over which engagement succeeds, which increases the incentive for each fund to preserve its capital.