Macro Shocks Cause Equilibrium Price Dispersion*

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Abstract

Price dispersion is shown to arise when demand is stochastic, *ex-ante* identical competitive firms set price prior to the realization of uncertainty and *ex-ante* identical buyers cannot switch sellers if rationed.

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1 Introduction

Price dispersion, common in seemingly competitive markets (e.g. Hong and Shum, 2006, for textbook sellers), is normally attributed to information frictions. For example, multiple price equilibria are possible in models in which buyers search a fixed number of sellers (Burdett and Judd, 1983), or with sequential search when search costs are heterogeneous (e.g. Albrecht and Axell, 1984).

This paper explores an alternative explanation involving macro shocks and fixed capacity. Competitive sellers of a perishable good must set their prices before demand is realized. Buyers are identical *ex ante,* observe all prices but can only visit one seller and must infer service probabilities. Restaurants are the exemplar. Models with these features are provided by, amongst others, Carlton (1978) and Deneckere and Peck (1995). Although it has been recognized that a single-price equilibrium may not be the only possibility, price dispersion has not been explicitly analyzed. Replacing the unit-demand assumption of Deneckere and Peck (1995) by smoothly downward sloping demand, it is shown that when the number of sellers is at intermediate levels, the equilibrium involves multiple prices. Within this interval, the entry of sellers lowers the number setting the high price. When the number of sellers is low, there is a single-price equilibrium with rationing in the high-demand state and market clearing when demand is low. With many sellers, there is a single price with unused capacity when demand is low but market clearing in the high demand. When the number of sellers is intermediate, high and low price sellers coexist, and rationing and waste are both present in the market. In this zone, entry of sellers lowers the number setting the low price.[1]

In an alternative formulation, Prescott (1975) and Dana (1999) assume buyers can costlessly visit all sellers. Buyers first queue for the lowest priced sellers where they are served at random. Any unserved buyers switch to the second-cheapest sellers and so on. Crucially, a buyer that deviates by first visiting a seller pricing above the lowest gets no service priority for arriving before others. If capacity is limited, equilibrium prices are strictly dispersed with high-price sellers attracting no buyers in low-demand states. In our model, unserved buyers cannot go elsewhere. A high-price is therefore compensated by a greater chance of buying, and all sellers earn revenue even in low-demand states.[2]

[1] A different model of price increasing entry is Rosenthal (1980).

[2] Eden (1990, 2018) provides an alternative formulation where consumers arrive in exogeneously determined waves, sellers commit capacity for each wave before the uncertainty is resolved, but do not need to set price *ex ante.* The solution is essentially the same.

2 The Model

Consider an economy where N sellers, each with capacity of measure 1, supply a homogeneous, perishable good to potential consumers of measure M. There are two macro states, high (H), which occurs with probability $P(H) = x$, and low (L), which occurs with $P(L) = 1 - x$. In the high state, all M consumers are active, each with linear demand, $1 - p$. In the low state, a fraction $1 - \phi$ of consumers randomly drop out of the market (are inactive), whilst the rest are unaffected.[3]

The timing is: 1) sellers simultaneously post prices; 2) consumers observe prices and their own valuation; 3) consumers can visit at most one seller and if there is excess demand, it is random which are served but those chosen can buy as much as they want at the posted price. Individual sellers are sufficiently small they have negligible impact on the rest of the market.[4] All agents are risk neutral.

As Carlton (1978) notes, perfectly competitive firms may be utility takers, but this does not make them price takers. A seller charging more than its rivals loses customers (as well as reducing the demand of the remaining buyers), diminishing queues and making stockouts less likely. This creates a more attractive offer mitigating the decline in sales. A downward sloping demand curve does not though guarantee that marginal revenue is monotonic in price. We show that there is an interval for N in which this cannot be the case, which provides the basis for price dispersion.

To evaluate price offers active buyers must infer the macro state. The unconditional probability of a consumer being active is $x + \phi(1 - x)$. From Bayes' rule, the probability of the high state, conditional on the buyer being active, is

$$P(H \mid active) = \tfrac{x}{x+(1-x)\phi} \equiv \pi.$$

Our main objective is to show that there is an interval for N in which the equilibrium involves sellers dividing between two prices. The characteristics of the proposed equilibrium are that at the low price demand equals capacity in the low state but there is rationing in the high state, whilst at the high price demand equals capacity in the high state but there is unused capacity in the low state. These properties require

$$\phi(1 - p_1)M_1 = N_1; \tag{1}$$

$$(1 - p_2)(M - M_1) = N - N_1. \tag{2}$$

[3]de Meza and Reito (2020) term this a drastic shock whereas depressing willingness to pay but not to zero is a non drastic shock. Their focus is on welfare and price dispersion is not covered.

[4]This property can be satisfied by replicating the economy. That is, take the number of sellers and buyers as rN, rM and let r tend to infinity (see de Meza and Reito, 2020).

In (1) and (2), N_1 and $N - N_1$ are the numbers of firms at the two prices, and M_1 and $M - M_1$ the measure of high-state buyers at the respective prices. According to (1), at p_1 there is market clearing in the low state and by (2), at p_2 there is market clearing in the high state.

A competitive equilibrium with these prices must satisfy the following: *i*) expected consumer surplus and expected profit must be equalized at the two prices; *ii*) a deviant has no incentive to choose an intermediate price; *iii*) deviating to a price above p_2 is unprofitable as is deviating to a price below p_1.

Equalization of consumer surplus at p_1 and p_2 is achieved if

$$\tfrac{N_1}{(1-p_1)M_1}\pi\sigma(p_1) + (1-\pi)\sigma(p_1) = \tfrac{N-N_1}{(1-p_2)(M-M_1)}\pi\sigma(p_2) + (1-\pi)\sigma(p_2), \tag{3}$$

where the expected surplus of an unrationed consumer buying at price p is $\sigma(p) = (1-p)(1+p)/2 - p(1-p)$. Equation (3) allows for rationing in the high state.

Profit equalization with possibility of waste in the low involves

$$xp_1 + (1-x)p_1\phi(1-p_1)\tfrac{M_1}{N_1} = xp_2 + (1-x)p_2\phi(1-p_2)\tfrac{M-M_1}{N-N_1}. \tag{4}$$

Solving the system of equations (1) to (4) (by means of the software Mathematica–Wolfram Research), the equilibrium values of the endogeneous variables are

$$p_1^e = \tfrac{x+\phi(1-x)-(1-x)\alpha}{1+\phi(1-x)};$$
$$p_2^e = \tfrac{x}{x+\phi(1-x)+(1-x)\alpha};$$
$$M_1^e = \tfrac{\phi(1-x)[M+\alpha(M-N)]-\alpha N}{\phi(1-\phi)(1-x)[\phi(1-x)+1]};$$
$$N_1^e = \tfrac{\phi^3(1-x)^2(M-N)-\alpha N-\phi^2(1-x)[N-x(M-N)]+\phi\{(1-x)(1+2\alpha)M-[x+\alpha(1-x)]N\}}{(1-\phi)[\phi(1-x)+1]^2},$$

where $\alpha = \sqrt{\phi[\phi(1-x)+x]}$. The solution is in the economically meaningful range if $N_1^e \in (0, N)$ and $M_1^e \in (0, M)$. Note that $N_1^e < N$ when

$$N > \tfrac{\phi(1-x)(1+\alpha)M}{\phi(1-x)+1} \equiv N_L,$$

and $N_1^e > 0$ when

$$N < \tfrac{\phi(1-x)(1+\alpha)M}{[\phi(1-x)+1]\alpha} \equiv N_H.$$

According to these conditions, $N_L < N_H$. A similar procedure for $M_1^e \in (0, N)$ yields the same interval for N. Hence the proposed equilibrium exists for an interval of N. From the expressions for p_1^e and p_2^e, $p_1^e > 0$,[5] $p_2^e < 1$, and

$$p_2^e - p_1^e = \tfrac{x\{1-[\phi(1-x)+x]\}}{x+\phi(1-x)+(1-x)\alpha} > 0.$$

[5]As $x = 0$, when $p_1^e = 0$, and the derivative of p_1^e with respect to x is

$$\tfrac{\phi^2(1-x)[\phi(1-x)+x+2]+3\phi x+2\sqrt{\phi[\phi(1-x)+x]}-\phi}{2[1+\phi(1-x)]^2\sqrt{\phi[\phi(1-x)+x]}} > 0,$$

Within the two-price interval, the equilibrium price solutions are invariant to N. This requires that the number of buyers choosing each seller does not vary with increases in N. As there are more buyers per low-price seller, an increase in N must be accompanied by a decrease in N_1^e and therefore an increase in average market price.

To establish that the proposed configuration is an equilibrium, the consequences of deviation must be examined. It will first be shown that profit is lower for a seller choosing a price between any two others (including the equilibrium values) involving rationing at p_1 in the high state and unsold capacity in the low state at p_2, and which generate equal profit and surplus. For this demonstration, only (3) and (4) are required, not the market clearing conditions (1) and (2). The distribution of sellers and buyers that satisfy (3) and (4) are:

$$M_1 = \frac{(1-p_1)[\phi(1-x)p_2(2-p_1-p_2)M - x(1-p_2)N]}{\phi(1-x)(p_2-p_1)(2-p_1-p_2)}; \tag{5}$$

$$N_1 = \frac{p_1[\phi(1-x)p_2(2-p_1-p_2)M - x(1-p_2)N]}{x(p_2-p_1)}. \tag{6}$$

Consider a seller deviating to a price p_d between p_1 and p_2. Write the measure of buyers attracted to the deviant, which equalizes expected surplus with that at p_1 and p_2, as $m(p_d, p_1, p_2)$. Using (5) and (6), this satisfies (at either of the two prices, here p_1)

$$\frac{1}{(1-p_d)m(p_d,p_1,p_2)}\pi\sigma(p_d) + (1-\pi)\sigma(p_d) = \frac{N_1}{(1-p_1)M_1}\pi\sigma(p_1) + (1-\pi)\sigma(p_1). \tag{7}$$

This requires $M_1/N_1 > m(p_d, p_1, p_2) > (M - M_1)/(N - N_1)$.

The deviant's expected return (details in the Appendix) is

$$\rho(p_d, p_1, p_2) = xp_d + (1-x)p_d\phi(1-p_d)m(p_d, p_1, p_2) = \frac{x(1-p_1p_2)p_d}{2p_d - p_d^2 - p_1p_2}, \tag{8}$$

which has a unique turning point at $\sqrt{p_1p_2}$, but it is a minimum. So, there is no profitable deviation to a price between p_1 and p_2. It is easily checked that, evaluated at p_1, $d\rho(p_d, p_1, p_2)/dp_d < 0$ whilst, at p_2, $d\rho(p_d, p_1, p_2)/dp_d > 0$.

That only leaves to check whether a deviant has an incentive to choose a price below p_1^e or above p_2^e. As p_1^e clears the market in the low state, deviation to an even lower price results in rationing in both states. That is, at p_1^e, sales equal capacity in both states, and a price cut will definitely lose revenue. Similarly, as p_2^e clears the market in the high state, consumers are unrationed and a deviation to a higher

the signing following since $\sqrt{\phi[\phi(1-x)+x]} > \phi$.

price is unprofitable as all sales will be lost. Figure 1 illustrates the returns to a deviant. In the numerical example, the parameters are $M = 10000$, $N = 2000$, $x = 0.5$, $\phi = 0.25$, yielding solutions $p_1^e = 0.38$, $p_2^e = 0.61$, $N_L = 1550$, $N_H = 3922$, $M_1^e = 8104$ and $N_1^e = 1256$.

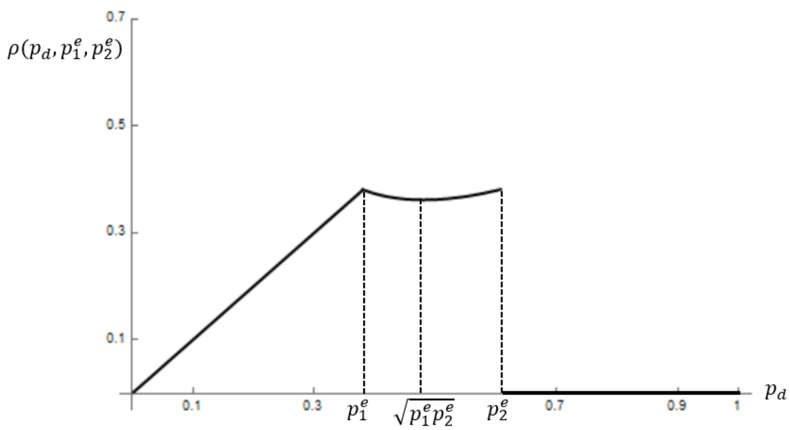

Figure 1. Deviant's profit function.

Proposition 1 *When $N \in (N_L, N_H)$, there is a two-price equilibrium. In this interval, prices are invariant to N, but the proportion of sales at the low price is decreasing in N.*

Outside this interval it is shown in the Appendix that there is a single price equilibrium. When $N \leq N_L$ all firms charge the price that clears the market in the low state, and when $N \geq N_H$ all firms charge the market clearing price in the high state. Figure 2 plots the relationship between N and price(s) for the earlier parameter set.

3 Summary

If all sellers set the price that clears the market in the low state, rationing arises in the high state. A seller setting a higher price loses some customers, but those remaining are compensated by a lower probability of rationing. As more sellers switch to the high price, rationing at the low price decreases, enhancing its appeal and leading to the emergence of a multiple-price equilibrium. Augmenting supply then decreases the number of low-price sellers, although this does not harm consumers.

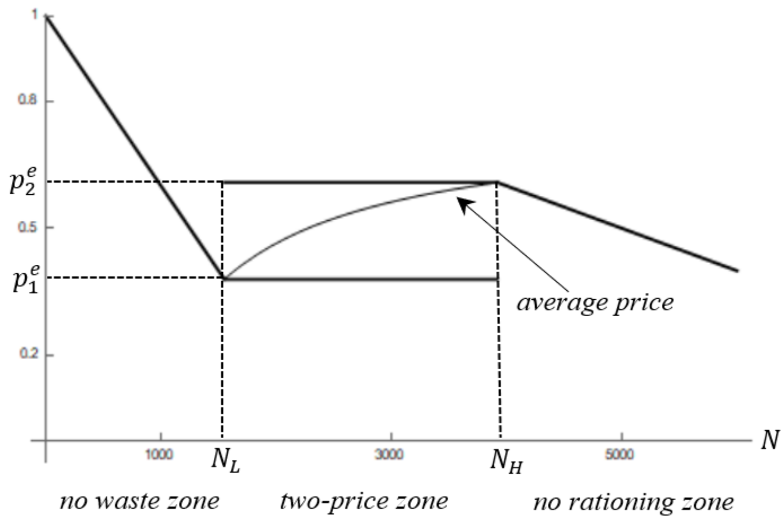

Figure 2. Relationship between price and N.

Appendix

Two-price equilibrium

Propose an equilibrium with two distinct prices, $p_1 < p_2$, both of which attract buyers. Using M_1 and N_1 from (5) and (6), a deviant charging p_d (an intermediate price) must satisfy the equal surplus condition (at one of the two prices, here p_1),

$$\tfrac{1}{(1-p_d)m(p_d,p_1,p_2)}\pi\sigma(p_d) + (1-\pi)\sigma(p_d) = \tfrac{N_1}{(1-p_1)M_1}\pi\sigma(p_1) + (1-\pi)\sigma(p_1),$$

yielding

$$m(p_d,p_1,p_2) = \tfrac{x(1-p_d)}{\phi(1-x)(2p_d-p_d^2-p_1p_2)}.$$

The revenue of the deviant is

$$\rho(p_d,p_1,p_2) = xp_d + \phi(1-x)p_d(1-p_d)m(p_d,p_1,p_2) = \tfrac{x(1-p_1p_2)p_d}{2p_d-p_d^2-p_1p_2}.$$

The revenue of a non-deviant at p_1 is

$$\rho(p_1,p_1,p_2) = xp_1 + \phi(1-x)p_1(1-p_1)\tfrac{M_1}{N_1} = \tfrac{x(1-p_1p_2)}{2-p_1p_2},$$

so

$$\rho(p_1,p_1,p_2) - \rho(p_d,p_1,p_2) = \tfrac{x(p_1-p_d)(p_d-p_2)(1-p_1p_2)}{(2-p_1-p_2)(2p_d-p_d^2-p_1p_2)},$$

which is always positive if $p_1 < p_d < p_2$ (for a deviant from p_2, the condition is the same as above).

Equilibrium at p_L if $N \leq N_L$

The market-clearing price in the low state satisfies $\phi(1-p)M = N$, so $p = (\phi M - N)/\phi M \equiv p_L$. A deviant charging $p_d > p_L$, will attract $m(p_d, p_L)$ consumers where

$$\tfrac{1}{(1-p_d)m(p_d,p_L)}\pi\sigma(p_d) + (1-\pi)\sigma(p_d) = \tfrac{N}{(1-p_L)M}\pi\sigma(p_L) + (1-\pi)\sigma(p_L),$$

yielding

$$m(p_d, p_L) = \tfrac{\phi x(1-p_d)M^2}{N^2-\phi^2(1-x)(1-p_d)^2M^2}.$$

A deviation from p_L is not profitable iff $\rho(p_L, p_L) - \rho(p_d, p_L) \geq 0$, or

$$\begin{aligned}&\left[xp_L + \phi(1-x)p_L(1-p_L)\tfrac{M}{N}\right] - \left[xp_d + \phi(1-x)p_d(1-p_d)m(p_d,p_L)\right]\\ &= \tfrac{xp_dN^2}{N^2-\phi^2(1-x)(1-p_d)^2M^2} \geq 0,\end{aligned}$$

which holds when

$$p_L \leq p_d \leq \tfrac{\phi^2(1-x)M^2-N^2}{\phi(1-x)(\phi M-N)M} \equiv \overline{p}.$$

As before, it can never be profitable to deviate to a price above the high-state sell-out price defined by $(1-p_d)m(p_d,p_L) = 1$. Using the expression for $m(p_d,p_L)$, the sell-out price satisfies

$$\pi\sigma(p_d) + (1-\pi)\sigma(p_d) = \tfrac{N}{(1-p_L)M}\pi\sigma(p_L) + (1-\pi)\sigma(p_L),$$

The solution is $p_d = 1 - N/\alpha M \equiv p_H^S$, and $\overline{p} > p_H^S$ when

$$N \leq \tfrac{\phi(1-x)(1+\alpha)M}{\phi(1-x)+1} \equiv N_L.$$

Equilibrium at p_H if $N \geq N_H$

The price that clears the market in the high state satisfies $(1-p)M = N$ so is $p = (M-N)/M \equiv p_H$. A deviant charging $p_d < p_H$, attracts $m(p_d, p_H)$ consumers where

$$\tfrac{1}{(1-p_d)m(p_d,p_H)}\pi\sigma(p_d) + (1-\pi)\sigma(p_d) = \tfrac{N}{(1-p_H)M}\pi\sigma(p_H) + (1-\pi)\sigma(p_H),$$

yielding

$$m(p_d, p_H) = \tfrac{x(1-p_d)M^2}{\phi(1-x)[(1-p_d)M^2+N^2]+xN^2}.$$

A deviation from p_H is not profitable iff $\rho(p_H, p_H) - \rho(p_d, p_H) \geq 0$, or

$$\left[xp_H + \phi(1-x)p_H(1-p_H)\tfrac{M}{N}\right] - \left[xp_d + \phi(1-x)p_d(1-p_d)m(p_d, p_H)\right]$$
$$= \tfrac{[\phi(1-x)+x]xp_dN^2}{(1-x)[2\phi p_dM^2 - \phi M^2 - \phi(p_dM-N)(p_dM-N)]+xN^2} \geq 0,$$

which holds if

$$p_H \geq p_d \geq \tfrac{\phi(1-x)M^2-[\phi(1-x)+x]N^2}{\phi(1-x)(M-N)M} \equiv \underline{p}.$$

It cannot be profit maximising to price below the sell-out price in the low state, at which $\phi(1-p_d)m(p_d, p_H) = 1$. Substituting for $m(p_d, p_H)$, at the sell-out price,

$$\pi\phi\sigma(p_d) + (1-\pi)\sigma(p_d) = \tfrac{N}{(1-p_H)M}\pi\sigma(p_H) + (1-\pi)\sigma(p_H),$$

with solution $p_d = \{\phi[M-(1-x)\beta N]-x\beta N\}/\phi M \equiv p_L^S$, with $\beta = \sqrt{\phi/[\phi(1-x)+x]}$. As $\underline{p} < p_L^S$ when

$$N \geq \tfrac{\phi(1-x)(1+\alpha)M}{[\phi(1-x)+1]\alpha} \equiv N_H,$$

this is the condition for a single equilibrium price of p_H.

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