Pricing with Bargain Hunting Consumers

Matthew Gentry[*]and Martin Pesendorfer[†]

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Abstract

A single-product retailer faces bargain hunting consumers whose willingness to pay incorporates sensations of gain and loss driven by differences between the observed price and prices they rationally expect in the spirit of Koszegi and Rabin [2006]. We examine the Bayesian Nash equilibrium (non-commitment) pricing solution in which (i) the retailer maximizes profit given consumers' beliefs and (ii) consumers' beliefs are consistent with the retailer's choice. We show two novel results: First, a pure-strategy, uniform-price, equilibrium does not exist when consumers are bargain hunters who value gains more than losses. Second, in this case there exists a mixed strategy equilibrium and all mixed strategy equilibria involve the same retailer profit. The equilibrium retailer profit is (weakly) lower than in the absence of reference effects.

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[*]Department of Economics, Florida State University. Email: mgentry@fsu.edu.

[†]Department of Economics, London School of Economics and Political Science. Email: m.pesendorfer@lse.ac.uk.

1 Introduction

"It's not what I spent, it's what I saved." In the US 83 percent of consumers are bargain shoppers and 23 percent sometimes buy things they don't need just because they're on sale, according to a recent Consumer Reports National Research Center study.[1] Evidence in other countries is similar. 84 percent of the British population are self-confessed bargain hunters with one in four adults admitting to losing their temper with more than one in ten admitting to having fought with other shoppers.[2] When bargain hunting becomes compulsive psychiatrist have attached the term compulsive buying behavior (CBB). Koran et al. [2006] estimate the prevalence of CBB in the US population to be 5.8%. Black [2007] reports that patients with compulsive buying disorder, anecdotally, often report buying a product based on its attractiveness or because it was a bargain.

This paper examines retail pricing decisions with bargain hunting consumers. We consider a linear demand framework augmented with an expectation-based reference-effects similar to Koszegi and Rabin [2006, 2007, 2009]. Consumers' willingness to pay is affected by differences between the observed price and prices they rationally expect. Motivated by empirical evidence, see Mazumdar et al. [2005] and Gentry and Pesendorfer [2018], we study the novel case in which consumers willingness to pay responds to the magnitude of the bargain. We say that consumers are bargain hunters when willingness to pay responds more strongly to perceived gains than to perceived losses: as may be true, for instance, in contexts such as retail shopping, where savings from "sales" are saliently highlighted but losses on non-sales are not, and where consumers are known to respond strongly (and perhaps excessively) to perceived bargains. We analyze the equilibrium pricing decisions of a single-product retailer (monopolist) facing consumers with this willingness to pay function. Consumers make rational purchase decisions given their beliefs. Beliefs are required to be consistent with the monopolist's pricing decision. We analyze the Bayesian Nash equilibrium

[1] https://www.consumerreports.org/cro/news/2014/04/america-s-bargain-hunting-habits/index.htm

[2] The Independent on 22 November 2017, article entitled "Average Brit spends nine months of their life bargain hunting."

solution. We contrast our main findings with an alternative setting in which the monopolist can commit to a distributional pricing policy *ex ante*.

A number of papers study pricing decisions when consumers are loss averse in the *expectations-based* paradigm due to Koszegi and Rabin, see Heidhues and Koszegi [2018] for a survey. Loss aversion can reduce or eliminate price variation.[3] Heidhues and Koszegi [2008], who consider oligopolistic competition in the Salop circular model with loss averse consumers, show that the same price is a Bayesian Nash equilibrium for a range of marginal costs giving rise to focal equilibria. The same price equilibrium emerges as loss aversion introduces a concave curvature into the demand curve, with a kink at the point where loss aversion kicks in initially as pointed out by Sibly [2002]. The resulting demand curve is reminiscent of the kinked demand literature, see Hall and Hitch [1939], Sweezy [1939], and Maskin and Tirole [1988] for a game theoretic analysis.

In contrast to the loss aversion literature, this paper provides novel results with expectation-based price reference effects when consumers are bargain-hunters: First, a uniform price policy does not constitute an equilibrium. The usual law of one price fails to hold. The main reason for this novel result is that bargain-hunting consumers assign incremental value to prices slightly below the expected uniform price introducing a local convexity in demand and in profit. For any uniform price equilibrium candidate, there is a profitable deviation by lowering (or increasing) the price slightly.

Second, we show that an equilibrium in mixed strategies exists and that all mixed strategy equilibria involve the same monopoly profit and aggregate reference beliefs. These mixed strategy equilibria have the feature that equilibrium profit is (weakly) lower than monopoly profit in the absence of reference effects. Interestingly, there are multiple individual consumer-level beliefs that generate the uniquely determined aggregate reference beliefs.

[3]Heidhues and Koszegi [2014] and Rosato [2016] study monopolistic markets under loss aversion, in the product and price dimension, and when the seller can commit to stochastic pricing policy ex ante. They show that a mixed pricing distribution, that has high and low prices, is an optimal commitment solution because low prices induce expectations of purchase. Yet mixed pricing is not optimal, even under commitment, when reference effects arise in the price dimension only as we shall illustrate.

We provide four examples of individual reference expectations. The mixed pricing strategy equilibrium accords well with the related empirically well-documented evidence on randomized price promotions in retailing, see Pesendorfer [2002] and Seim and Sinkinson [2016], as well as the HiLo pricing phenomenon described in Ho et al. [1998], see Fassnacht and Husseini [2013] for a recent survey.

Third, we contrast this mixed strategy pricing equilibrium with the alternative commitment solution arising when the monopolist can commit to a pricing distribution *ex ante.* We show that this commitment solution also involves a distribution of prices, but whose support is shifted to the right relative to the non-commitment pricing equilibrium. Monopoly profit is increased and exceeds the canonical monopoly profit absent reference effects.

Building on these results, we analyze deadweight loss of monopoly in the presence of reference effects, defined as the usual Harberger triangle between demand and marginal costs. We distinguish long-run deadweight loss, in which consumers' reference point is the competitive price, from short-run deadweight loss, in which the reference point is the distribution of equilibrium prices. We show that if consumers are pure bargain hunters who like gains but do not dislike losses, then short-run deadweight loss exceeds canonical monopoly deadweight loss, which in turns exceeds long-run deadweight loss.

We re-consider the case of loss averse consumers and show that, as expected, uniform prices do emerge in equilibrium. In fact, there are multiple uniform price equilibria. Equilibrium prices are always weakly lower than the canonical monopoly price, with strict inequality if consumers strictly like bargains. With loss aversion the commitment solution involves a uniform price which equals the canonical monopoly price without reference effects, which again is (weakly) higher than in the Bayesian Nash equilibrium.

Retail shopping consumers are known to react strongly (and perhaps excessively) to perceived price gains, as exemplified by 'sales' periods, which is well documented in consumer surveys. There is a body of psychological studies exploring the related compulsive buying behavior, see Black [2007] for a survey. Armstrong and Chen [2020] study the implications of discount pricing which is that consumers are more likely to buy if they perceive the price as a bargain relative to the earlier price.

There is an extensive marketing literature arguing that reference effects are important in retail markets. This literature includes both behavioral and quantitative branches; see, e.g., Mazumdar et al. [2005] for a detailed review of both branches.[4] A series of quantitative studies within the internal-reference-price paradigm has found evidence consistent with both bargain-hunting and loss-averse behavior. Bargain-hunting is found more important than loss-aversion for coffee, peanut butter, liquid detergent, and tissue products in Krishnamurthi et al. [1992] and Briesch et al. [1997]. Gal and Rucker [2018] review the empirical evidence on loss aversion more broadly and conclude that the evidence does not support that losses, on balance, tend to be any more impactful than gains. Mukherjee et al. [2017] find that the monetary value of the item in question matters when measuring bargain hunting and loss aversion, with potential losses never looming larger than gains for low monetary values in their experiments. Gentry and Pesendorfer [2018] analyze household ketchup purchases within a dynamic framework incorporating expectations-based reference effects, finding evidence of bargain hunting in this context. Insofar as they represent over-weighting of gains relative to losses, phenomena such as loss-leader pricing and excessive search may also be at least tangentially consistent with bargain-hunting behavior.

Section 2 describes our model and introduces the equilibrium concept. Section 3 starts with a characterization of the Bayesian Nash equilibrium played between the monopolist and consumers when consumers are bargain hunting and value gains more than losses. We then contrast the Bayesian Nash equilibrium outcome with the commitment solution in which

[4] Broadly, work within marketing has conceptualized reference effects in terms of *internal reference prices:* inner reference points, formed on the basis of past experience, against which consumers judge prospective purchase prices. Prices below this reference point are perceived as gains, while prices above are perceived as losses. The internal reference price literature has shown that the nature in which the internal reference effect is formed plays an important role in shaping firm pricing behavior, pushing firms toward focal, mixed, or cyclical pricing. For example, Kopalle et al. [1996] consider a multi-period setting in which the internal reference price is defined as the most recent purchase price. They show that cyclical pricing can emerge when consumers experience gains or losses vis-a-vis the most recent purchase price. In contrast to this internal reference price literature, we follow the expectations-based paradigm of Koszegi and Rabin [2006, 2007, 2009], in which the reference expectations are rational and part of the equilibrium.

the monopolist can commit to a pricing distribution ex ante. Finally, we illustrate that the dead-weight loss associated with the Bayesian Nash equilibrium tends to be lower than that of the canonical monopoly solution without reference effects. Section 4 examines the Bayesian Nash equilibrium played between the monopolist and consumers when consumers are loss averse. Section 5 concludes. All technical proofs are given in the appendix.

2 Framework

This section introduces our demand model with bargain hunting consumers, describes the monopoly problem, and defines the equilibrium concepts. We then provide an equilibrium existence result.

2.1 Linear demand with bargain hunting consumers

Our model considers a linear demand function for one product augmented with reference dependent price effects. This linear demand function can be derived from a stochastic consumer level utility model in which the willingness to pay consists of a random term augmented with reference effects. To see this derivation, assume that consumer $i \in N$ has indirect utility of purchasing one unit at price p given by

$$\alpha_i - bp + r(p, F_i),$$

where consumer i's type α_i is an i.i.d. error drawn from a uniform distribution on $[-a, a]$, b is the slope coefficient for price p, and $r(p, F_i)$ is a reference dependent utility component to be specified shortly, evaluated by comparing the purchase price p to the reference distribution F_i. Following Sakovics [2011] we can interpret the reference effect $r(p, F_i)$ as influencing a consumer's willingness to pay. The indirect utility of not buying is normalized to 0. The total mass of consumers equals $2a$.

Following Koszegi and Rabin [2006], we specify $r(p, F_i)$ as follows:

$$r(p, F_i) = \int \rho(R - p)\, dF_i(R), \tag{1}$$

where $\rho(\cdot)$ is a continuous, weakly increasing *gain-loss function*. Intuitively, if consumer i expects a single price R with certainty, then $r(p, F_i)$ simply equals the willingness to pay gain or loss $\rho(R - p)$ associated with a deterministic monetary gain or loss of size $R - p$. A 'bargain' price p, lower than R, increases the willingness to pay, while a 'rip-off' price p, above R, reduces the willingness to pay. With multiple prices in the support of F_i, $r(p, F_i)$ equals the gain-loss in willingness to pay $\rho(R - p)$ induced by the actual price p relative to each potential price R in the support of F_i, averaged with respect to reference measure dF_i.

We assume that the idiosyncratic taste shock α_i does not affect the reference expectation F_i. This arises when the reference expectation F_i is formed before realizations of the individual taste shock α_i are observed. The expected consumer i demand can be calculated by taking expectation over the taste shock. It equals $D(p, F_i) = \frac{a - bp + \int \rho(R-p)\, dF_i(R)}{2a}$ at interior demand prices p. The expected demand at price p equals zero when consumer i does not buy at the high taste shock, $a - bp + r(p, F_i) < 0$, and equals one when consumer i buys at the low taste shock, $-a - bp + r(p, F_i) > 0$. When demand at price p is interior for all i, we obtain aggregate expected demand by summing over all consumers $i \in N$ as $\frac{2a}{N} \sum_{i \in N} \frac{a - bp + \int \rho(R-p)\, dF_i(R)}{2a}$. As the individual reference effect $r(p, F_i)$ is linear in F_i, we may rewrite aggregate expected demand as a linear demand curve augmented with an aggregate reference effect:

$$D(p, F) = a - bp + \int \rho(R - p)\, dF(R),$$

where $F = E_i[F_i]$ denotes the average of the reference distributions F_i across individuals. The aggregate reference distribution F is the distribution obtained by first sampling an individual i at random from the population, then sampling R at random from F_i. Observe that, so long as demand at price p is interior for all i, aggregate demand $D(p, F)$ is completely characterized by the aggregate reference distribution F. In what follows, we therefore frame our analysis primarily in terms of the aggregate reference distribution F, and only occasionally involve the individual reference distributions F_i.

Our main analysis follows Heidhues and Koszegi [2008, 2014] and Spiegler [2012] in specifying the reference function $\rho(x)$ as piecewise linear, with a potential kink at zero to accommodate the fact that consumers may weigh gains and losses differently:

$$\rho(x) = \begin{cases} \delta^+ x & \text{if } x \geq 0; \\ \delta^- x & \text{if } x < 0, \end{cases}$$

where the non-negative parameters δ^+ and δ^- describe the changes in willingness to pay that consumers associate with perceived gains and perceived losses, respectively. We say that consumers are bargain hunting if they weigh gains more heavily than losses, $\delta^+ > \delta^-$, and loss averse if they weigh losses more heavily than gains, $\delta^+ < \delta^-$.

Let $\underline{p}$ and $\bar{p}$, respectively, denote the infimum and supremum prices in the support of the reference distribution F. Substituting for $r(p, F)$ from (1), plugging in our assumed piecewise linear form for $\rho(x)$, and simplifying, we obtain our primary expression for the demand function $D(p, F)$:

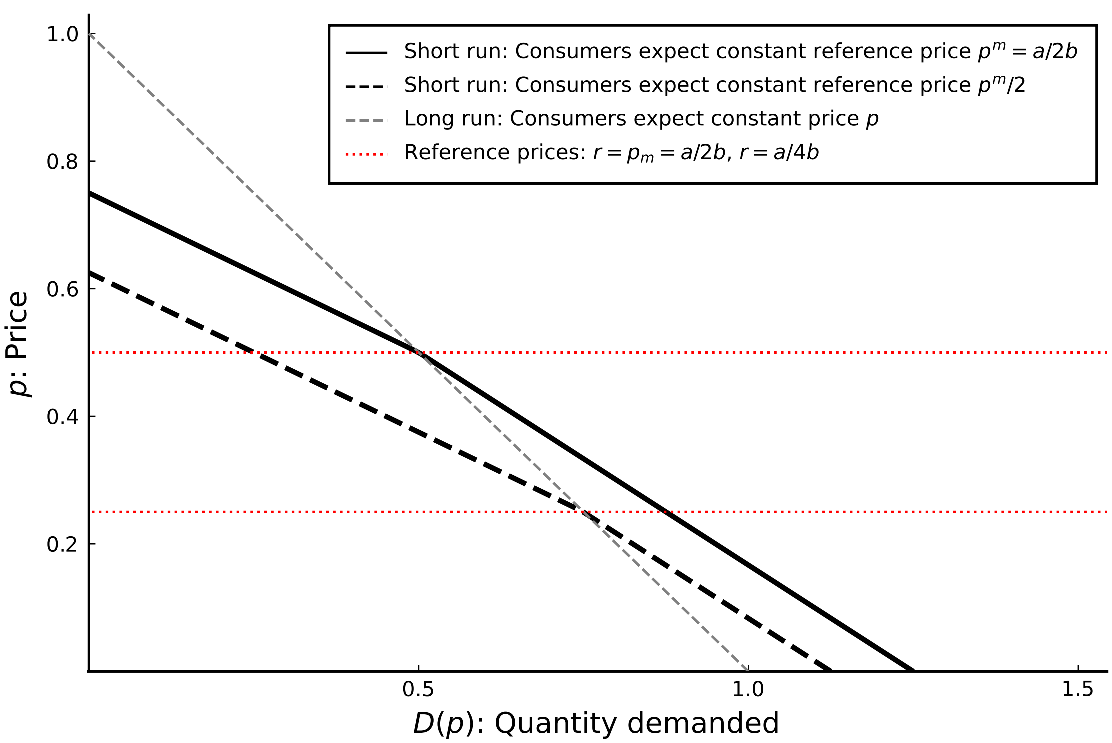
$$D(p, F) = a - bp + \delta^+ \int_p^{\bar{p}} (R - p)\, dF(R) + \delta^- \int_{\underline{p}}^{p} (R - p)\, dF(R). \tag{2}$$

Figure 1 illustrates the interaction between direct price effects, reference expectations F, and demand in two special cases of this model. In Panel (a), we consider a case in which consumers are pure bargain hunters, characterized by parameters $a = b = \delta^+ = 1$ and $\delta^- = 0$. We plot the demand curve implied by a continuous reference distribution F defined on $[0.35, 0.5]$, which in this case is also the equilibrium reference distribution characterized in Proposition 3 below. For comparison, we also plot two benchmark demand curves based on alternative reference expectations. The first of these arises when consumers expect the

Figure 1: Demand with reference-dependent willingness to pay

(a) Bargain hunting consumers: $a = 1$, $b = 1$, $\delta^+ = 1$, $\delta^- = 0$

(b) Loss averse consumers: $a = 1$, $b = 1$, $\delta^+ = 0.5$, $\delta^- = 1$



canonical monopoly price $p^m = 0.5$ with certainty. The second is the long run demand curve that would arise after consumer beliefs update to reflect the deviation price p, which corresponds to the canonical linear demand curve $a - bp$.

Meanwhile, Panel (b) of Figure 1 illustrates demand when consumers are loss averse, here illustrated at parameters $a = b = \delta^- = 1$ and $\delta^+ = 0.5$. We plot three demand curves in this figure, induced by three sets of reference beliefs: the first arising when consumers expect the canonical monopoly price $p^m = \frac{a}{2b}$ with certainty, the second arising when consumers instead expect the price $R = \frac{p^m}{2} = \frac{a}{4b}$ with certainty, and the third the canonical linear demand curve $a - bp$, which as above also reflects the long-run demand curve after reference beliefs F update to reflect the monopolist's deviation price p.

Inspecting Figure 1 illustrates three important features of this demand model. First, if consumers expect a constant reference price R, both bargain hunting and loss aversion induce kinks in short-run demand at the reference price R: outward if consumers are net bargain-hunting, inward when consumers are net loss averse. In contrast, if the reference distribution F is continuous, then the corresponding demand curve transitions smoothly from a slope of $-b - \delta^+$ at prices $p \leq \underline{p}$ to a slope of $-b - \delta^+$ at prices $p \geq \bar{p}$. Second, shifting the reference point—here from $R = \frac{a}{2b}$ to $R = \frac{a}{4b}$—shifts consumers' entire schedule of willingness to pay. Third, given any set of reference expectations, the short-run demand curve is substantially more price-elastic than the long-run demand curve. As we will see in Sections 3 and 4.2, these effects have important implications for pricing, particularly in the absence of commitment.

2.2 The monopolist's pricing problem

For simplicity we assume the monopolist's marginal cost c equals average cost and is zero. Thus, taking aggregate reference expectations F as given, monopoly profit at price p is given by the product of price and demand,

$$\pi(p, F) = p \cdot D(p, F)$$

The monopolist's problem is to select prices that maximize profit when taking consumers' reference expectations into account. The monopolist adopts a pricing policy which may in general involve a distribution of prices G defined on the support $S \subseteq \mathbb{R}$.

Consumers have beliefs about the monopolist's pricing policy G and form their individual reference expectations F_i based on these beliefs. We require consumers' beliefs about prices to be consistent with the monopolist's actual pricing policy G. Specifically, we require that for each consumer i, the individual reference distribution F_i is absolutely continuous with respect to the monopolist's pricing policy G:

$$F_i \ll G, \tag{3}$$

This formulation is consistent with a *personal equilibrium* as defined by Koszegi and Rabin [2006], in which F_i is derived from consumers' rational forecasts of F_i given G, accounting for the possibility that consumers weigh prices differently than the monopolist. For example, if consumer i weighs prices with respect to their ex ante probability of purchasing at those prices, then

$$F_i(p; G) = \int_{\underline{p}}^{p} \left[D(p, F_i) / \int_{\underline{p}}^{\bar{p}} D(x, F_i)\, dG(x) \right] dG(p) \quad \forall p \in S \quad ,$$

where the function $D(p, F_i)/\int_{\underline{p}}^{\bar{p}} D(x, F_i)\, dG(x)$ describes the purchase weights, which is the specific notion of reference used by consumer i. Alternatively, if consumer i weighs prices in the same way as the monopolist, then $F_i = G$. The former specification assumes, paralleling Koszegi and Rabin [2006] and Heidhues and Koszegi [2008, 2014], that consumers form price beliefs using prices they rationally expect to pay, while the latter assumes, paralleling Spiegler

[2012], that consumers form price beliefs using prices they rationally expect to see. These are the two leading special cases we consider below. Our analysis also applies to any other model of reference beliefs satisfying the consistency requirement (3).

2.3 Solution concepts

In solving for the monopolist's price distribution G, we consider two solution concepts: one with commitment and the other without. The no commitment solution requires all prices in the support S of G to be optimal given aggregate reference expectations F, so that there is no incentive for the monopolist to deviate from the pricing plan G. We can view this as a Bayesian Nash equilibrium between the monopolist and consumers:

Definition 1. A Bayesian Nash Equilibrium (BNE) is a price distribution G for the monopolist, together with a reference distribution F_i for each consumer, such that (i) for every consumer i, $F_i \ll G$, and (ii) G maximizes the monopolist's profits taking the aggregate reference distribution $F = E_i[F_i]$ as given.

An alternative solution concept is the commitment solution, in which the monopolist can commit to a pricing policy ex ante. In this case, the monopolist chooses a price distribution G^c to maximize expected profits, accounting for the effect that G^c has on consumer reference expectations:

Definition 2. A commitment solution is a price distribution G^c for the monopolist, together with an induced reference distribution $F_i^c(G^c)$ for each consumer, such that (i) for each consumer i, $F_i^c \ll G^c$, and (ii) letting $F^c(G^c) = E_i[F_i^c(G^c)]$ denote the aggregate reference distribution, the price distribution G^c maximizes the monopolist's expected profit:

$$G^c = \arg\max_G \int \pi(p, F^c(G))\, dG(p).$$

Note that in this case the monopolist may set prices p' which are not optimal, in the sense that $p' \notin \arg\max \pi(p, F^c)$. The monopolist therefore requires a commitment device to ensure that all prices in the support of G^c are indeed chosen.

Existing work on monopoly pricing under reference dependence has focused mainly on commitment solutions: see, e.g., Spiegler [2012] and Heidhues and Koszegi [2014] among others. In contrast, we focus mainly on Bayesian Nash equilibrium, referencing the commitment solution primarily for comparison. In retail markets, where all consumers face identical prices, it not clear what formal commitment devices might exist, although informal considerations such as consumer learning could generate a flavor of commitment as noted by Spiegler [2012]. In other settings, explicit commitment may be more reasonable. For example, category pricing for opera tickets allows the monopolist to commit to an allocation of seats at ex ante specified category prices. To the extent that seat quality differences are small, this ex ante published pricing plan is a commitment device.

2.4 Existence

Before turning to our main analysis, which specializes the reference utility function $\rho(\cdot)$ to be piecewise linear, we establish a general result on existence of both the BNE and commitment solutions in our demand context. We say that a BNE (respectively commitment solution) exists if there exists a solution satisfying Definition 1 (respectively Definition 2) when the monopolist is restricted to prices contained in any bounded interval in $\mathbb{R}$. Adapting standard arguments due to Glicksberg [1952], we then obtain the following result, proved formally in the appendix:

Proposition 1. *Let the support of G be contained in the interval $\left[\underline{p}, \overline{p}\right]$, and suppose that, for each individual i, the reference distribution F_i is continuous (under the weak* topology) in the pricing policy G. Then both the BNE and commitment solutions exist.*

3 Equilibrium with bargain hunting consumers

This section analyses the Bayesian Nash equilibrium of the pricing game between the monopolist and consumers when consumers are bargain hunters who value gains more than losses: i.e. cases with $\delta^+ > \delta^- \geq 0$ if $\rho(x)$ is piecewise linear. We begin by showing that, in this case, no pure strategy equilibrium exists. Hence the monopolist will not adopt a uniform-price policy facing bargain hunting consumers. We then characterize mixed strategy equilibria of the pricing game, showing that all equilibria involve the same aggregate reference beliefs $F = E_i[F_i]$. Finally, we contrast this mixed strategy equilibrium with the commitment solution described in Definition 2 and examine welfare effects.

3.1 No pure strategy equilibrium with bargain hunting consumers

We first show that a pure strategy equilibrium does not exist when consumers are strictly bargain hunting. Thus, a single (uniform) monopoly price is not optimal. This result in fact obtains whenever, locally around 0, consumers values gains more than losses, in the sense that the gain-loss function $\rho(x)$ has an upward kink at zero. Given our piecewise linear form for $\rho(x)$, this is of course equivalent to the condition that $\delta^+ > \delta^-$. But we state the result more generally as follows:

Proposition 2. *If* $\left.\frac{\partial \rho^+(x)}{\partial x}\right|_{x=0} > \left.\frac{\partial \rho^-(x)}{\partial x}\right|_{x=0}$*, then there does not exist a pure strategy Bayesian Nash equilibrium.*

The idea of the proof is to show that it cannot be optimal for the monopolist to set a uniform price p^*. Optimality requires that the profit is necessarily both non-decreasing as prices approach p^* from the left, i.e. $lim_{p \nearrow p^*} \partial\pi(p, F)/\partial p \geq 0$, and non-increasing as prices approach p^* from the right, i.e. $lim_{p' \searrow p^*} \partial\pi(p, F)/\partial p \leq 0$. But if the monopolist sets a uniform price p^*, then by absolute continuity of F_i in G consumers must expect this price with certainty. Hence the left- and right-hand side derivatives above simplify to

$$lim_{p\nearrow p^*}\partial\pi(p,F)/\partial p = a - 2bp^* - \left.\frac{\partial\rho^+(x)}{\partial x}\right|_{x=0} \cdot p^* \geq 0,$$
$$lim_{p'\searrow p^*}\partial\pi(p',F)/\partial p = a - 2bp^* - \left.\frac{\partial\rho^-(x)}{\partial x}\right|_{x=0} \cdot p^* \leq 0.$$

These inequalities can hold simultaneously only if $\left.\frac{\partial\rho^+(x)}{\partial x}\right|_{x=0} \leq \left.\frac{\partial\rho^-(x)}{\partial x}\right|_{x=0}$, contracting the hypothesis of Proposition 2. Hence, when consumers value gains more than losses in a neighborhood of zero, no pure strategy Nash equilibrium exists.[5]

Intuitively, given any candidate p^* for a uniform-price equilibrium, local bargain hunting behavior by consumers will induce a local convexity in the monopolist's profit function at p^*. Hence the monopolist will always be able to gain either by setting a price slightly below p^*, or by setting a price slightly above. We illustrate this fact for the canonical monopoly price $p^* = p^m = \frac{a}{2b}$ in Figure 2 discussed below.

3.2 Mixed strategy Bayesian Nash equilibrium with bargain hunting consumers

We now focus on the case of piecewise linear gain-loss utility $\rho(x)$, and characterize the set of mixed-strategy Bayesian Nash equilibria when consumers are bargain hunters who value gains more than losses: $\delta^+ > \delta^- \geq 0$. We show a uniqueness result in the sense that the all Bayesian Nash equilibria involve the same aggregate reference beliefs $F = E_i[F_i]$. Before stating our formal result, we give an intuitive derivation of the equilibrium. A complete proof is given in the Appendix.

For purposes of this derivation, it will frequently be more convenient to work with the following alternative expression for the demand function $D(p,F)$, which we refer to as the

[5]The statement in Proposition 2 is based on the first order condition of profit maximization. A slightly stronger statement can be obtained by additionally appealing to the second order condition. Specifically, if the first order condition holds with equality, $\left.\frac{\partial\rho^+(x)}{\partial x}\right|_{x=0} = \left.\frac{\partial\rho^-(x)}{\partial x}\right|_{x=0}$, then the second order condition requires that the second derivative of the gain-loss function evaluated at zero cannot be 'too' convex.

"bargain hunting" representation of demand. Let p_e denote the expected price under the aggregate reference distribution F: $p_e \equiv \int_{\underline{p}}^{\bar{p}} R\, dF(R)$. We may then rewrite (2) as

$$\begin{aligned} D(p, F) &= \big(a + \delta^- p_e\big) - (b + \delta^-) \cdot p + (\delta^+ - \delta^-) \cdot \int_{\min\{p,\bar{p}\}}^{\bar{p}} (R - p)\, dF(R) \\ &\equiv a' - b'p + \Delta \int_{\min\{p,\bar{p}\}}^{\bar{p}} (R - p)\, dF(R), \end{aligned} \tag{4}$$

where the first equation follows from adding and subtracting $\delta^- \int_p^{\bar{p}} (R - p)\, dF(R)$ on the right-hand side of (2), and we define $a' \equiv a + \delta^- p_e$, $b' \equiv b + \delta^-$, and $\Delta \equiv \delta^+ - \delta^-$.

For a tuple $\big((F_i)_{i \in N}\,, G\big)$ to be a mixed-strategy equilibrium, all prices in the support S of the pricing strategy G must yield equal payoffs to the monopolist. In particular, every price $p \in S$ must yield the same profit as the supremum price $\bar{p} \in S$: $\pi(p; F) = \pi(\bar{p}, F), \forall p \in S$. Dividing both sides of this expression by p, we obtain the equivalent indifference condition

$$\phi(p, F) = \frac{\pi(p, F) - \pi(\bar{p}, F)}{p} = D(p, F) - \frac{\bar{p} D(\bar{p}, F)}{p} = 0, \qquad \forall p \in S. \tag{5}$$

Since $\phi(p, F)$ is constant over the support S, we must have $\partial\phi(p, F)/\partial p = 0$ for all $p \in S$. In view of the bargain-hunting demand representation (4), this in turn implies the following necessary condition for a mixed-strategy equilibrium:

$$\frac{\partial\phi(p, F)}{\partial p} = -b' - \Delta\,[1 - F(p)] + \frac{\bar{p}(a' - b'\bar{p})}{p^2} = 0 \qquad \forall p \in S. \tag{6}$$

Solving for $F(p)$ in (6), we conclude that, in any Bayesian Nash equilibrium, the aggregate reference distribution $F(p)$ must satisfy the necessary relationship:

$$F(p) = 1 + \frac{b'}{\Delta} - \frac{1}{\Delta} \frac{\bar{p}(a' - b'\bar{p})}{p^2}. \tag{7}$$

To completely characterize $F(p)$, we still need to solve for two endogenously determined unknowns: the supremum $\bar{p}$ of the price support S, and the expected price $p_e = \int R\, dF(R)$ under the reference distribution F. We do so in the technical appendix containing all our proofs.

It only remains to specify the pricing policy G and consumer level reference reference distributions F_i. We impose the technical condition $\delta^+ - \delta^- \leq 2b$ to ensure that demand of individual consumers is strictly between zero and one for any personal priors, and that personal beliefs F_i have the same support as aggregate beliefs F. This condition is verified in the technical appendix. Whereas the aggregate reference distribution F is uniquely determined by 7 under any model of reference formation satisfying our consistency requirement, G and F_i will depend on the assumed personal equilibrium notion, that is, the way in which consumers form reference beliefs. We defer further details to Section 3.3, which presents equilibrium pricing policies under four models of reference formation, including two with heterogeneous reference expectations, as well as discussing how to derive equilibrium policies more generally.

We are now ready to state our proposition.

Proposition 3. *Suppose that consumers are strictly bargain hunting, $\delta^+ > \delta^- \geq 0$ and $\delta^+ - \delta^- \leq 2b$. Then there exists a mixed strategy Bayesian Nash equilibrium. Furthermore, in any Bayesian Nash equilibrium the aggregate reference distribution F is described by the cumulative distribution function $F(p) = 1 - \left(\frac{b+\delta^-}{\Delta}\right)\left(\frac{\bar{p}^2}{p^2} - 1\right)$ defined on the support $S = [\underline{p}, \bar{p}]$, with supremum price $\bar{p} = \frac{a+\delta^- p_e}{2b+2\delta^-}$, infimum price $\underline{p} = \frac{a+\delta^+ p_e}{2b+2\delta^+} = \frac{\sqrt{b+\delta^-}}{\sqrt{b+\delta^+}}\bar{p}$, and expected price $p_e = a\frac{\sqrt{b+\delta^+}-\sqrt{b+\delta^-}}{\delta^+\sqrt{b+\delta^-}-\delta^-\sqrt{b+\delta^+}}$.*

The characterization of the unique aggregate reference distribution yields the following qualitative implications summarized in Corollary 1. π^m denotes the canonical monopoly profit without reference effects, and $\Pi^*(G, F)$ denotes the expected Bayesian Nash equilibrium profit.

Corollary 1. *Suppose that consumers are strictly bargain hunting, $\delta^+ > \delta^- \geq 0$ and $\delta^+ - \delta^- \leq 2b$. The following properties hold in any Bayesian Nash equilibrium:*

(i) $\Pi^*(G, F) = \frac{(a+\delta^- p_e)^2}{4b+4\delta^-}$;

(ii) $\Pi^*(G,F) \le \pi^m$ *for all* (δ^-, δ^+)*, with* $\Pi^*(G,F) < \pi^m$ *if* $\delta^- > 0$*;*

(iii) If gain sensitivity δ^+ *or loss sensitivity* δ^- *individually increase (ceteris paribus), then* F *decrease in the sense of first-order stochastic dominance.*

First, in any equilibrium, the monopolist earns identical profits which are equal to the profit level the monopolist would have earned facing the simple linear demand curve $a' - b'p$ without reference effects: $\Pi^*(G,F) = \frac{(a')^2}{4b'} = \frac{(a+\delta^- p_e)^2}{4b+4\delta^-}$. This follows because, as seen above, the monopolist's profit at every $p \in S$ must equal its profit at the supremum price $\bar{p}$, at which the consumer experiences loss but not gain effects.

Constant monopoly profits irrespective of the notion of personal beliefs imply that neither the aggregate reference distribution F, the demand function $D(p,F)$, the profit function $\pi(p,F)$, nor the monopolist's expected profit $\Pi^*(G,F) = \int \pi(p,F)\, dG(p)$ depend on the notion of personal beliefs. That is, they do not depend on whether reference beliefs are equal to prices observed or prices paid or any other notion of consistent beliefs. This invariance may initially appear counterintuitive, but in fact it follows directly from price optimality, once one recognizes that F, rather than G, ultimately determines the monopolist's deviation payoffs. The monopoly price distribution G does vary with the notion of personal beliefs, and is chosen such that the specific notion of personal beliefs imply the 'optimal' reference distribution F when aggregated. We shall provide further details on the relationship between G, F_i and F in Section 3.3.

Second, without commitment, the monopolist cannot exploit bargain-hunting behavior to increase profit: letting $\pi^m \equiv \frac{a^2}{4b}$ denote monopoly profit against the canonical linear demand curve $a - bp$ without reference effects, we have $\Pi^*(G,F) \le \pi^m$ for all (δ^-, δ^+), with $\Pi^*(G,F) < \pi^m$ if $\delta^- > 0$. Intuitively, bargain-hunting behavior induces the firm to set prices below $p^m = \frac{a}{2b}$ with positive probability. But the consumer then perceives p^m as a loss relative to other prices in the support of F, increasing the effective elasticity of demand at p^m and thereby pushing prices and profits below their canonical monopoly levels.

Third, if either gain sensitivity δ^+ or loss sensitivity δ^- individually increase, then F

decreases in the sense of first-order stochastic dominance. In either case, if relative bargain hunting $\Delta = \delta^+ - \delta^-$ increases, then the relative price spread $\frac{\bar{p}}{\underline{p}}$ increases. In the limit, as $\Delta \to 0$, both F and G converge to mass points at a single limit price, which if $\delta^- = 0$ is the usual monopoly price solution $p^m = \frac{a}{2b}$. With loss aversion ($\delta^- > 0$) it can be any of the prices characterized in Proposition 5 below.

Finally, reducing the slope parameter of demand b results in more dispersed prices, in terms of both the price distribution function G and the reference price distribution function F. In the limit as demand becomes perfectly elastic, $b \to \infty$, the support S shrinks to a singleton, and the price distributions G and F converge to a single mass point at $p^m = 0$.

Illustration 1: Equilibrium with pure bargain hunting We first illustrate Proposition 3 for the special case of *pure bargain hunting* consumers: $\delta^+ > 0$, $\delta^- = 0$. In this case, both the supremum price and the monopolist's profit are the same as the monopoly solution against the canonical linear demand model $a - bp$ without reference effects: $\bar{P} = p^m = \frac{a}{2b}$, $\Pi^*(G, F) = \frac{a^2}{4b}$. For simplicity, we set $a = 1$, $b = 1$, and $\delta^+ = 1$, with $\delta^- = 0$ as noted above.

Figure 2 compares the monopolist's equilibrium profit function $\pi(p, F)$ to two non-equilibrium profit functions: that arising when consumers expect the canonical monopoly price $p^m = \frac{a}{2b}$ with certainty, and that arising when demand takes the canonical linear form $a - bp$ without reference effects. Inspecting this figure, it clear that setting a constant monopoly price $p = p^m$ is not an equilibrium: if bargain-hunting consumers expect the constant reference price $p^m = \frac{a}{2b}$, then the monopolist will always prefer to exploit this expectation by deviating to a price $p < p^m$. In contrast, when consumers have reference expectations described by the equilibrium reference distribution F, the firm is indifferent between all prices in the support $[\underline{p}, \bar{p}]$. This confirms that the pair (G, F) is in fact an equilibrium. The demand functions inducing these profit functions have already been illustrated in Panel (a) of Figure 4 above.

Figure 2: Equilibrium profit $\pi(p, F)$ arising when consumers have equilibrium reference expectations F (solid line), versus the monopolist's profits when consumers have non-equilibrium reference expectations (dashed lines), in a setting with pure bargain-hunting consumers: $a = 1$, $b = 1$, $\delta^{+} = 1.0$, $\delta^{-} = 0$.

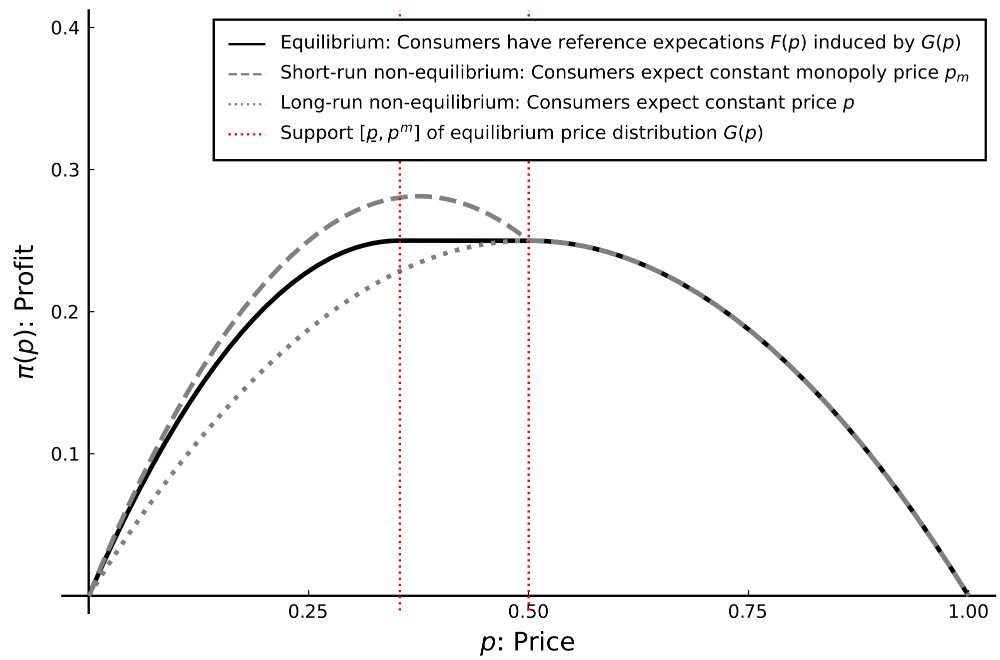


Figure 3: Equilibrium profit $\pi(p, F)$ arising when consumers have equilibrium reference expectations F (solid line), versus the monopolist's profits when consumers have non-equilibrium reference expectations (dashed lines), in a setting with mixed bargain-hunting consumers: $a = 1$, $b = 1$, $\delta^{+} = 1.0$, $\delta^{-} = 0.5$.

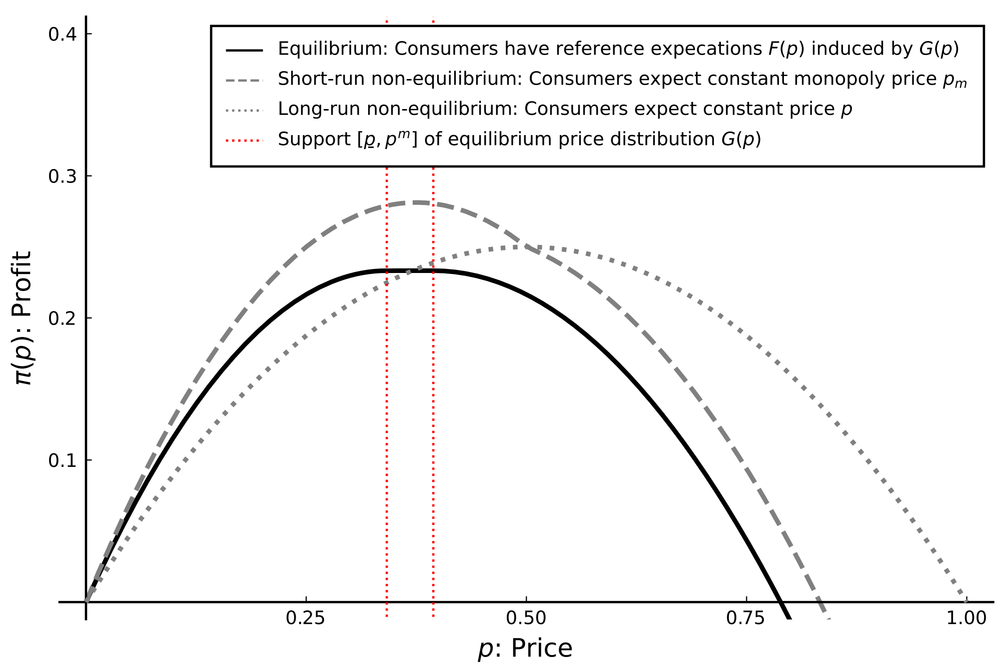


Illustration 2: Equilibrium with mixed bargain hunting Figure 3 illustrates the equilibrium profit functions arising under *mixed bargain hunting*: in this case at parameters, $a = 1$, $b = 1$, $\delta^+ = 1$, $\delta^- = 0.5$. As expected, both prices and profits are lower than in the pure bargain hunting case. This is particularly evident at the upper support $\bar{p}$, which is more than 20 percent lower at $\delta^+ = 1$, $\delta^- = 0.5$ ($\bar{p} < 0.4$) than at $\delta^+ = 1$, $\delta^- = 0$ ($\bar{p} = p^m = 0.5$). Intuitively, this arises because when $\delta^- > 0$, setting prices below p^m pushes the consumer into the loss domain at $p = p^m$. This causes the firm to face an effectively higher elasticity which renders it unprofitable to set $p = p^m$.

3.3 Individual reference expectations and the monopolist's pricing policy

This section completes the characterization of equilibrium by deriving the monopolist's equilibrium price distribution G under several specific models of reference formation. We reiterate that, while the equilibrium aggregate reference distribution F is unique, both the individual reference distributions F_i and the price distribution G will depend on the specific notion of reference beliefs considered. We give four examples of possible personal equilibrium notions, then comment on the general construction.

Example 1. All consumers have reference beliefs formed based on the relative frequency of the prices they see, that is $F_i = G$ for all i. Then $F_i = F$ and $G = F$.

Example 2. All consumers have reference beliefs which reflect the prices that consumers rationally expect to pay. Then $F_i = F$ and $F(p) = \int_{\underline{p}}^{p} \left[D(p, F) / \int_{\underline{p}}^{\bar{p}} D(p, F) \right] dG(p) \;\; \forall p \in S$. We can find G through the inverse of the Radon Nikodym derivative:

$$dG(p) = \left[D(p, F) / \int_{\underline{p}}^{\bar{p}} D(p, F) \right]^{-1} \cdot dF(p).$$

Example 3. Half of consumers have reference beliefs formed based on the relative frequency of the prices they see, that is $F_i = F_1 = G$ for all $i = 1, .., N/2$, while the remaining consumers have reference beliefs that prices are uniformly distributed $F_i = F_2 = \frac{p}{\bar{p}-\underline{p}}$ for all $i = N/2+1, ..., N$. Then the reference pdf satisfies $f(p) = \frac{1}{2}g(p) + \frac{1}{2}\frac{1}{\bar{p}-\underline{p}}$, and we obtain $g(p) = 2f(p) - \frac{1}{\bar{p}-\underline{p}}$.

Example 4. Half of consumers have reference beliefs formed based on the relative frequency of the prices they see, that is $F_i = F_1 = G$ for all $i = 1, .., N/2$, while the remaining consumers have reference beliefs based on $F_i = F_2 = G^2$ for all $i = N/2+1, ..., N$, where G^2 denotes the cdf of the maximum of two price draws. Then $\frac{1}{2}F_1 + \frac{1}{2}F_2 = \frac{1}{2}G + \frac{1}{2}G^2 = F$, and we obtain $G(p) = -\frac{1}{2} + \sqrt{\frac{1}{4} + 2F(p)}$.

All four examples are personal equilibria in which consumers have beliefs F_i about prices consistent with the monopolist's pricing distribution G and the individual beliefs induce the aggregate reference distribution F, that is $\frac{1}{N}\sum F_i = F$. In each case, F is the unique choice of aggregate reference distribution of the monopolist so that every price in S is optimal for the monopolist. What was the principle that allowed us to find the price distribution G induced from F? Consistency of personal beliefs requires absolute continuity with respect to G, that is that there exists a Lebesgue integrable function λ_i such that $F_i(p) = \int_{\underline{p}}^{p} \lambda_i dG$. The function $\lambda_i(p)$ measures the personal weight consumer i assigns to purchase price p in addition to the monopolist's weight $g(p)$. Let $\lambda = \frac{1}{N}\sum \lambda_i$, then the average consumer beliefs are given by: $F(p) = \frac{1}{N}\sum F_i = \frac{1}{N}\sum \int_{\underline{p}}^{p} \lambda_i dG = \int_{\underline{p}}^{p} \left(\frac{1}{N}\sum \lambda_i\right) dG = \int_{\underline{p}}^{p} \lambda dG$. Looking at the outer equality, G is obtained through the inverse of the Radon Nikodym derivative: $dG(p) = [\lambda(p)]^{-1} \cdot dF(p)$.

4 Reference Beliefs Equal to the Price Frequency

In this section only, we focus on the consistency condition (3), in which the consumer's reference distribution F_i is equal to the posted price distribution G for all i and $F = G$. In this special case, the Bayesian Nash equilibrium with bargain hunting consumers is fully described in Proposition 3. The analysis in Section 3 demonstrates that, absent a commitment device, the monopolist is rendered at least weakly, and often strictly, worse off by reference effects. This section characterises the commitment solution in this case and compares it to the Bayesian Nash equilibrium characterised in Proposition 3. We also characterise the Bayesian Nash equilibrium under loss aversion.

4.1 Commitment Solution

What if the monopolist can commit to a price distribution ex ante, as considered by, e.g., Spiegler [2012], Heidhues and Koszegi [2014], and Rosato [2016] among others? In this case, the monopolist will commit to the distribution G^c which maximizes expected monopoly profits

$$\Pi\left(G^c, F^c\left(G^c\right)\right) = \int \pi(p, F^c\left(G^c\right))\, dG^c(p).$$

Let p_c denote the expected price under the aggregate reference distribution F^c: $p_c \equiv \int_S R\, dF^c(R)$. Under this assumption, the following Proposition fully characterizes the monopolist's commitment solution against bargain-hunting consumers:

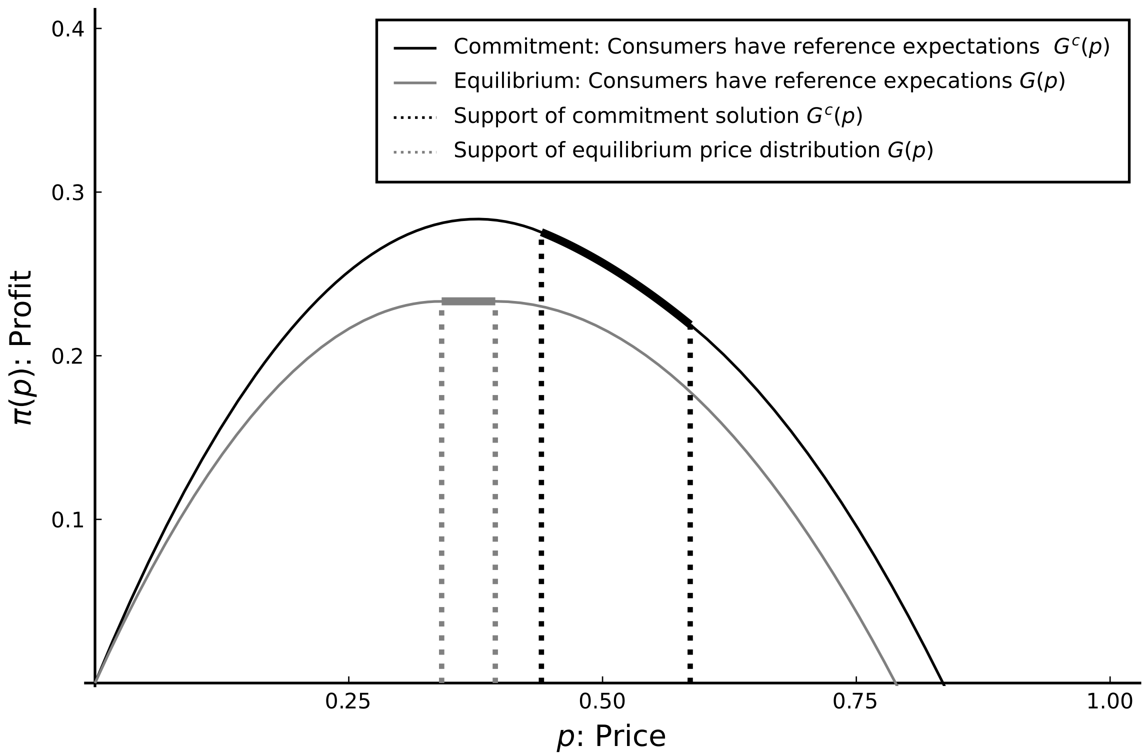
Proposition 4. *If* $\delta^+ > \delta^- \geq 0$*, then the optimal pricing policy is* $G^c(p) = \frac{\delta^+ + b}{\delta^+ - \delta^-} - \frac{a + \left(\delta^+ + \delta^-\right) p_c}{2(\delta^+ - \delta^-)p}$ *with price support* $\left[\frac{a+\left(\delta^+ + \delta^-\right)p_c}{2(b+\delta^+)}, \frac{a+\left(\delta^+ + \delta^-\right)p_c}{2(b+\delta^-)}\right]$*, where* $p_c = \frac{a\left[ln(b+\delta^+) - ln(b+\delta^-)\right]}{2(\delta^+ - \delta^-) - (\delta^+ + \delta^-)[ln(b+\delta^+) - ln(b+\delta^-)]}$. *Furthermore, if* $\delta^+ > \delta^- = 0$, the equilibrium profit $\pi(p, F^c)$ *is decreasing in* p *over the price support.*

The proof idea can be illustrated by considering the first order condition with respect to f^c evaluated at a point x:

$$\left\{x\left[a - bx\right] + \delta^+ x \int_x^l (y - x)\, f^c(y) dy + \delta^- x \int_0^x (y - x)\, f^c(y) dy\right\}$$
$$+ \delta^+ \int_0^x y\,(x - y)\, f^c(y) dy + \delta^- \int_x^l y\,(x - y)\, f^c(y) dy = 0 \qquad for\ all\ x \in S. \quad (8)$$

Notice that the term in braces $\{\,\cdot\,\}$ is the profit evaluated at price x. The first term in the second line is increasing in x. Thus, when $\delta^- = 0$, profit is decreasing in price x over the

Figure 4: Profit $\pi(p, G^c)$ arising under the commitment solution $G^c(p)$ (black) compared to profit $\pi(p, G)$ arising under the Bayesian Nash Equilibrium solution G (gray), in a setting with mixed bargain-hunting consumers: $a = 1$, $b = 1$, $\delta^+ = 1.0$, $\delta^- = 0.5$. In this example the canonical monopoly price is $p^m = 0.5$ and canonical monopoly profit is $\pi^m = 0.25$. Under the Bayesian Nash equilibrium, all prices in the support are optimal given consumer beliefs, and the monopolist earns profit $\pi = 0.233 < \pi^m$. Under the commitment solution, the monopolist's optimal deviation price is outside the price support, and the monopolist earns profit $\pi^c \approx 0.253 > \pi^m$.



price support, which establishes the second statement in the Proposition. Furthermore, as the first order condition (8) must hold for all prices in the support, we can take derivatives on both sides with respect to x, which yields:

$$a - 2bx + \left(\delta^+ + \delta^-\right)\left(\delta^+ - \delta^-\right) - \delta^+ 2x[1 - F^c(x)] - \delta^- 2x F^c(x) = 0$$

The optimal pricing policy $F^c = G^c$ is obtained from this equation, as are the boundary points in the price support.

When consumers are bargain hunters, both the Bayesian Nash equilibrium (G, F) and the commitment solution (G^c, F^c) involve mixed-strategy pricing by the monopolist. As illustrated in Figure 4, however, these solutions qualitatively differ in at least three important respects. First, whereas without commitment every price in the support of the equilibrium price distribution must be individually optimal, with commitment almost all prices charged by the monopolist are individually sub-optimal. Indeed, the Figure illustrates a situation in which all prices are suboptimal. The reason is that inclusion of the high-profit price in the support would generates losses, proportional to the deviation mass, on all the prices above. Second, whereas the equilibrium price support is always at least weakly below the canonical monopoly price $p^m = \frac{a}{2b}$, with commitment the monopolist charges prices above p^m with positive probability. Third, whereas without commitment reference effects at least weakly, and usually strictly, reduce equilibrium profit, with commitment the monopolist is able to exploit bargain hunting behavior to strictly increase profit.

In other words, the qualitative effects of bargain hunting behavior depend critically on whether or not the monopolist can commit ex ante to individually sub-optimal prices. If such commitment is feasible, then the monopolist will exploit bargain hunting consumers to increase prices and profits. If not, then the monopolist's *temptation* to exploit bargain hunting behavior will reduce prices and profits.

4.2 Loss-averse consumers

What if instead consumers are loss averse in the price dimension, in the sense that $\delta^+ \leq \delta^-$? Proposition 5 shows that when consumers are more concerned about losses than gains, uniform pricing is optimal. However, when $\delta^- > 0$, the equilibrium price is not unique. Rather, there exist a continuum of uniform-price equilibria:

Proposition 5. *If $\delta^+ - \delta^- \leq 0$, then any uniform price in the interval $\left[\frac{a}{2b+\delta^-}, \frac{a}{2b+\delta^+}\right]$ is a Bayesian Nash equilibrium.*

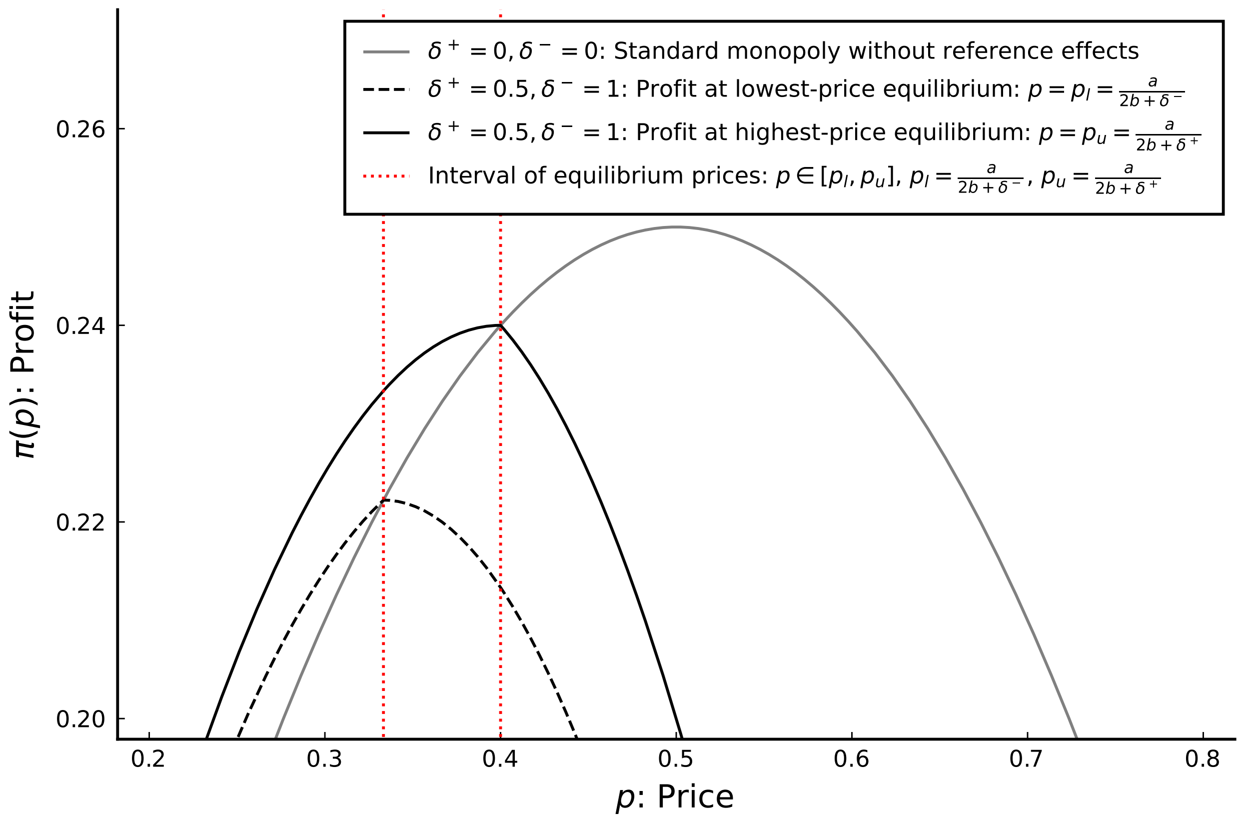
The proof idea is simple. Under loss aversion, the profit function is strictly concave for any reference distribution F, implying that any equilibrium must involve a uniform price.

Furthermore, for any candidate uniform price p, loss aversion implies that the derivative of the profit function is greater for prices approaching from the left than for those approaching from the right: $\lim_{p' \nearrow p} \partial\pi(p,F)/\partial p \geq \lim_{p' \searrow p} \partial\pi(p,F)/\partial p$. In particular, for any $p \in \left[\frac{a}{2b+\delta^-}, \frac{a}{2b+\delta^+}\right]$, one can show that both $\lim_{p' \nearrow p} \partial\pi(p,F)/\partial p \geq 0$ and $\lim_{p' \searrow p} \partial\pi(p,F)/\partial p \leq 0$. Hence, as shown by Heidhues and Koszegi [2008] in a related context, any uniform price in the non-empty interval $\left[\frac{a}{2b+\delta^-}, \frac{a}{2b+\delta^+}\right]$ constitutes a Bayesian Nash equilibrium.

What if we instead consider the commitment solution? When $\delta^- \geq \delta^+$, the monopolist can do no better than commit to the uniform price $p^m = \frac{a}{2b}$; this follows intuitively from the fact that demand is concave under loss aversion, or more formally from the proof of Proposition 4 as shown in Appendix A. As above, when $\delta^+ > 0$, this will not be a Bayesian Nash equilibrium, since if consumers expect $p = p^m$ with certainty, then the monopolist would optimally set $p^* = \frac{a+\delta^+ p^m}{2b+2\delta^+}$, which is strictly less than p^m when $\delta^+ > 0$. But given access to a credible commitment device, the monopolist's optimal policy is to set $p^m = \frac{a}{2b}$ with certainty.

Figure 5 illustrates Proposition 5 for the parameters $a = 1$, $b = 1$, $\delta^- = 1$, and $\delta^+ = 0.5$. Specifically, this figure depicts the monopolist's profit functions at the high-price and low-price Nash equilibria, i.e. those involving uniform prices $p = \frac{a}{2b+\delta^+}$ and $p = \frac{a}{2b+\delta^-}$ respectively. For comparison, the canonical monopoly profit function arising in the absence of reference effects is also depicted. At each equilibrium price, the profit function looks like a tip of a triangle, with an inward kink at this price. The maximum price $p = \frac{a}{2b+\delta^+}$ is the largest price such that the right-hand derivative $\lim_{p' \searrow p} \partial\pi(p,F)/\partial p$ is nonpositive, whereas the minimum price $p = \frac{a}{2b+\delta^-}$ is the smallest price such that the left-hand derivative $\lim_{p' \nearrow p} \partial\pi(p,F)/\partial p$ is nonnegative. Equilibrium profits range from $\frac{2}{9} = 0.2\dot{2}$ to $\frac{6}{25} = 0.24$ and are lower than the canonical monopoly profit of $\frac{1}{4} = 0.25$.

Figure 5: Equilibrium profit with loss-averse consumers



5 Welfare effects of the Bayesian Nash Equilibrium

This section compares the welfare of the Bayesian Nash equilibrium outcome to the benchmark of the perfectly competitive solution in which price $p_{comp} = 0$. Our main result is that if consumers are pure bargain hunters who like gains but do not dislike losses, then short-run deadweight loss exceeds canonical monopoly deadweight loss, which in turns exceeds long-run deadweight loss.

The concept of welfare is not obvious in our setting. Should expectation-based reference effects be taken into account, or just the "standard" consumption preferences? How should individual level reference effects be aggregated? We consider welfare comparisons based on the aggregate demand function only. We do so because the aggregate beliefs are uniquely determined as shown in Proposition 3, while there is multiplicity of possible personal beliefs. Of course, our analysis also applies to the case in which personal reference beliefs equal aggregate reference beliefs.

When assessing the Bayesian Nash equilibrium outcome we can distinguish two welfare scenarios depending on whether consumers' expectations have adjusted or not: *short-run* and *long-run*. The *short-run* analysis assumes that consumers maintain the equilibrium expectations of the Bayesian Nash equilibrium. The *long-run* analysis allows consumers to adjust their expectations to the perfectly competitive equilibrium and calculate any deadweight losses by using the new expectations. Additionally, we contrast our welfare findings to the outcome emerging from the canonical monopoly solution in the absence of reference effects in which the monopoly price equals $p_m = \frac{a}{2b}$.

5.1 Welfare Effects under Bargain Hunting

We begin our analysis with the case of bargain hunting consumers. Let the Bayesian Nash equilibrium aggregate price beliefs, which are fully characterised in Proposition 3, be denoted by F . In the *short-run* consumers remain with these beliefs and the *short-run* demand curve

D_s is given by equation (2),

$$D_s(p,F) = D(p,F).$$

In the *long-run* consumers adjust their beliefs to the perfectly competitive equilibrium in which F_0 has all the mass at the competitive price $p = 0$. The *long-run* demand curve D_l is given by,

$$D_l(p,F_0) = a - bp - \delta^- p$$

The *long-run* dead-weight-loss calculation at a price p involves the usual triangle under the linear demand curve and equals $DWL_l(p) = \frac{p^2}{2}(b+\delta^-)$. It reflects that consumers perceive the inflated price of the Bayesian Nash equilibrium as a loss relative to the competitive price in the long run. The *short-run* dead-weight-loss does not involve a simple triangle as the demand function is non-linear over the support of equilibrium prices. To overcome this difficulty, we approximate the *short-run* dead-weight loss DWL_s by using an upper and a lower bound. The bounds are obtained by using the usual triangle under the linear demand curve but evaluated at the lower and upper end point of the support of equilibrium prices. Our lower bound $\underline{DWL_s}$ is given by the dead-weight loss at the low equilibrium price $\underline{p}$, and satisfies:

$$\underline{DWL_s} = \frac{\underline{p}^2}{2}\left(b+\delta^+\right) = DWL_s(\underline{p}) \leq DWL_s$$

The upper bound is based on the observation that the *short-run* demand curve D_s is convex. To see the convexity observe that D_s has a positive second derivative on the interval $\left[\underline{p},\bar{p}\right]$, $\frac{\partial^2 D_s(p,F)}{(\partial p)^2} = (\delta^+ - \delta^-)\,dF(p) > 0$, and is linear for prices below $\underline{p}$. It is thus convex. Due to convexity, an upper bound on the dead-weight loss $\overline{DWL_s}$ is given by the *short-run* dead-weight loss triangle at $\bar{p}$ with consumers' expectations centered at price $\bar{p}$. To see that it is indeed an upper bound observe the following::

$$DWL_s \leq DWL_s(p^*) = \left[\left(\delta^+ - \delta^-\right)ER + \left(b+\delta^-\right)\bar{p}\right]\frac{\bar{p}}{2}$$

$$\leq \frac{(b+\delta^+)}{2}\bar{p}^2 = \overline{DWL_s}$$

Notice that both the upper and lower bound have the interpretation that the competitive equilibrium price is perceived as a gain relative to a price in the Bayesian Nash equilibrium price support. For the lower bound we use the lower end point in the price support, while for the upper bound we use the upper end-point plus expectations centered at the upper end point. The following theorem uses these bounds to make a comparison between the *short-run* and *long-run* dead-weight-loss as well as with respect to the canonical monopoly dead-weight-loss in absence of reference effects, $DWL_m = \frac{a^2}{8b}$.

Theorem 1. Suppose $\delta^+ - \delta^- \geq 0$ *and* $\delta^+ - \delta^- \leq 2b$. The following properties hold for the Bayesian Nash equilibrium:

(i) $DWL_l \leq DWL_s$.

(ii) *If* $\delta^- = 0$*, then* $DWL_l \leq DWL_m \leq DWL_s$.

(iii) If $\delta^- \geq \delta^{-*}$, then $DWL_s \leq DWL_m$, where

$$\delta^{-*} = \left[\frac{1}{2}(\frac{b}{\sqrt{b+\delta^+}} - \sqrt{b}) - \sqrt{\frac{1}{4}(\frac{b}{\sqrt{b+\delta^+}} - \sqrt{b})^2 + \sqrt{b\,(b+\delta^+)}}\right]^2 - b.$$

Part (i) of Theorem 1 establishes that with bargain hunting consumers the perceived *long-run* losses at the high Bayesian Nash equilibrium price are less than equal to the perceived *short-run* gains of moving to the perfectly competitive equilibrium. The reason is that the upper bound on the *long-run* dead-weight-loss coincides with the lower bound on the *short-run* dead-weight loss.

Parts (ii) and (iii) compare the equilibrium outcome under bargain hunting to the canonical outcome in the absence of reference expectations. Part (ii) says that when the loss aversion parameter δ^- is zero, then the canonical monopoly dead weight loss falls in between the *long-run* and *short-run* Bayesian Nash equilibrium dead-weight loss. This finding emerges because when $\delta^- = 0$ the *short-run* dead-weight loss upper bound reduces to the canonical monopoly dead-weight loss DWL_m.

Part (iii) says that for sufficiently large loss aversion parameters the *short-run* Bayesian Nash equilibrium dead-weight-loss will be smaller than the canonical monopoly dead-weight loss. Part (iii) uses the bounds to establish that for any bargain hunting parameter there exists a range of loss aversion parameters so that the Bayesian Nash equilibrium dead-weight loss is smaller than the canonical monopoly dead-weight loss. Thus, combining parts one and three, for any bargain hunting parameter δ^+ there exists a range of loss aversion parameters so that welfare losses in the equilibrium with bargain hunting consumers are smaller, both in the *short-run* and in the *long-run*, than the canonical monopoly dead-weight loss in the absence of reference effects.

5.2 Welfare Effects under Loss Aversion

What if instead consumers are loss averse in the price dimension, in the sense that $\delta^+ \leq \delta^-$? Proposition 5 shows that the Bayesian Nash equilibrium beliefs F_R have all the mass at $p = R$. Thus, the dead-weight-loss calculations under loss aversion involve the usual triangle under a linear demand curve. In the *short-run* consumers remain with Bayesian Nash equilibrium beliefs F_R and the *short-run* demand curve D_s is given by,

$$D_s(p, F_R) = a - bp + \delta^+ (R - p) \cdot 1_{\{p<R\}} + \delta^- (R - p) \cdot 1_{\{p>R\}}$$

In the *long-run* consumers adjust their beliefs to the perfectly competitive equilibrium in which F_0 has all the mass at the competitive price $p_{comp} = 0$. The *long-run* demand curve D_l is given by,

$$D_l(p, F_0) = a - bp - \delta^- p$$

The *short-run* dead-weight-loss of the Bayesian Nash equilibrium reflects that consumers perceive the *short-run* reduced price of the perfectly competitive outcome as gain and equals $DWL_s(R) = \frac{R^2}{2}(b + \delta^+)$. The *long-run* dead-weight-loss of the Bayesian Nash equilibrium equals $DWL_l(R) = \frac{R^2}{2}(b+\delta^-)$ and reflects that consumers perceive the inflated price of the Bayesian Nash equilibrium as a loss in the *long run*.

The following theorem states our comparison between the dead-weight-loss *short-run* and *long run* as well as with respect to the canonical monopoly dead-weight-loss in absence of reference effects, DWL_m.

Theorem 2. Suppose $\delta^+ - \delta^- < 0$. The following properties hold for any Bayesian Nash equilibrium with reference price R:

(i) $DWL_l(R) > DWL_s(R)$.

(ii) $DWL_s(R) < DWL_m$.

(iii) If $\delta^- < b + 2\delta^+ + \frac{(\delta^+)^2}{2b}$, *then* $DWL_l(R) < DWL_m$.

The first part states that *long-run* DWL considerations outweigh *short-run* DWL when consumers are loss averse. The reason is that the perceived losses due to the inflated price of the Bayesian Nash equilibrium exceed the perceived gains of moving to the perfectly competitive equilibrium as $\delta^+ < \delta^-$. The second and third part compare the equilibrium outcome under loss aversion to the canonical outcome in the absence of reference expectations. There are two opposing effects: On the one hand equilibrium prices with loss averse consumers are lower than the canonical monopoly price in the absence of reference effects which lowers the DWL. On the other hand reference effects shift the demand curve adversely, which is absent in the canonical monopoly model, and results in an increase in DWL. In the *short-run* the perfectly competitive price is perceived as a gain, while in the *long-run* moving to the inflated price of the Bayesian Nash equilibrium creates a loss. Part (ii) of Theorem 2 states that the first effect always dominates the second in the *short-run*. The reduction in equilibrium price relative to the monopoly price due to loss averse consumers, $R \leq \frac{a}{2b+\delta^+} \leq \frac{a}{2b} = p_m$, outweighs the *short-run* perceived gains of moving to the competitive outcome. Part (iii) of Theorem 2 states the the first effect dominates the second also in the *long-run* provided the loss parameter δ^- is not too large. In that case, the welfare loss perceived by loss averse consumers stemming from the inflated price R in the Bayesian Nash equilibrium is outweighed by the price reduction relative to the canonical monopoly price, $R < p_m$.

6 Conclusion

This paper explores retail pricing decisions with bargain hunting consumers whose willingness to pay is elevated when prices are unexpectedly low. This bargain hunting notion is the flip side of loss aversion which has attracted a lot of attention in the literature. We show that bargain hunting introduces a local convexity in demand, so that the monopolist finds it beneficial to deviate from any uniform price. While there is no pure strategy pricing equilibrium, we characterise the mixed strategy equilibrium in which the monopolist randomizes over a connected support of prices.

Retail markets exhibit a lot of variation in prices. To the extent that loss aversion predicts price inertia, in which prices do not change even if marginal costs or demand parameters change, it does not explain the observed pricing patterns. Of course, an alternative interpretation of the multiple pricing equilibria under loss aversion is that they may also explain dispersed prices. Bargain hunting behavior has stronger implication implying that prices may vary even if cost or demand parameters do not change. Some empirical evidence in the marketing literature and in economics suggests that consumers may in fact be in part motivated by bargain hunting in retail markets. A future research agenda is to explore to what extent reference price effects may explain price variation in retail markets.

Our analysis has focused on the simplest case with (i) one firm and (ii) a demand curve which is linear absent reference effects. Natural extensions to consider in future work include competition between firm, say in a Cournot model, or bargain hunting in alternative models of demand such as with differentiated products.

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Appendix: Technical proofs

Proof of Proposition (1). Let $\mathcal{F}$ denote the space of probability measures with support contained in $[\underline{p}, \overline{p}]$. $\mathcal{F}$ is compact (in the weak* topology) and convex. By assumption, $G \in \mathcal{F}$, and from belief consistency, $F_i \in \mathcal{F}$ for all i and thus $F \in \mathcal{F}$. Absolute continuity requires that there exists a Lebesgue integrable function λ_i such that $F_i(p) = \int_{\underline{p}}^{p} \lambda_i dG$ for all i. Let $\lambda = \frac{1}{N} \sum \lambda_i$, which by construction is also a Lebesgue integrable function, then as F is equal to the average belief, we obtain: $F(p) = \frac{1}{N} \sum F_i = \frac{1}{N} \sum \int_{\underline{p}}^{p} \lambda_i dG = \int_{\underline{p}}^{p} \left(\frac{1}{N} \sum \lambda_i\right) dG = \int_{\underline{p}}^{p} \lambda dG \equiv C^*(G)$. The mapping $C^* : \mathcal{F} \to \mathcal{F}$ denotes the average beliefs consistent with G. By assumption, each F_i is continuous in the weak* topology, which implies that the average mapping $C^*(G)$ is continuous.

Next consider the monopolist's problem. For given aggregate reference expectations $F \in \mathcal{F}$, let $G^*(F) = \arg\max_{G \in \mathcal{G}} \Pi(G, F) \equiv \int \pi(p, F)\, dG(p)$ denote the monopolist's set of profit-maximizing price distributions. $\Pi(G, F)$ is a continuous function from $\mathcal{F} \times \mathcal{F} \to \mathbb{R}$. By Berge's Maximum Theorem, $G^*(F)$ is non-empty, compact-valued, and upper hemicontinuous. Linearity of $\Pi(G, F)$ in G implies that for any $G_1, G_2 \in G^*(F)$, we also have $\alpha G_1 + (1 - \alpha) G_2 \in G^*(F)$ for all $\alpha \in [0, 1]$. Hence, $G^*(F)$ is convex-valued.

Combining the properties established so far, we have that (C^*, G^*) is a non-empty, convex-valued, and upper-hemicontinuous correspondence from the compact, convex set $\mathcal{F} \times \mathcal{F}$ to itself. By the Kakutani-Glicksberg-Fan FPT, there exists a fixed point, which is a BNE. To show existence of a commitment solution, note from above that $F^c = C^*(G^c)$, with $C^*(\cdot)$ a continuous function from $\mathcal{F} \to \mathcal{F}$. Furthermore, the monopolist's profit function $\Pi(G^c, F^c) : \mathcal{F} \times \mathcal{F} \to \mathbb{R}$ is continuous. Hence the composition $\Pi(G^c, C^*(G^c))$ is a continuous function from the compact domain $\mathcal{F}$ to $\mathbb{R}$. By the Berge maximum theorem, the set of maximizers G^{c*} to $\Pi(G^c, C^*(G^c))$ is nonempty, and a commitment solution exists.

□

Proof of Proposition (2). Consider any uniform price equilibrium candidate p. A necessary condition for optimality is that marginal profit is non-decreasing as prices approach p from the left, $lim_{p' \nearrow p} \partial\pi(p, F)/\partial p \geq 0$, and non-increasing as prices approach p from the right, $lim_{p' \searrow p} \partial\pi(p, F)/\partial p \leq 0$. The left- and right-hand-side derivatives are readily calculated. We obtain $lim_{p' \uparrow p} \partial\pi(p, F)/\partial p = a - 2bp - \left.\frac{\partial\rho^+(x)}{\partial x}\right|_{x=0} \cdot p \geq 0$ and $lim_{p' \downarrow p} \partial\pi(p, F)/\partial p = a - 2bp - \left.\frac{\partial\rho^-(x)}{\partial x}\right|_{x=0} \leq 0$. The inequalities imply that $\left.\frac{\partial\rho^+(x)}{\partial x}\right|_{x=0} \leq \left.\frac{\partial\rho^-(x)}{\partial x}\right|_{x=0}$. This condition is violated when small gains are valued more than small losses. Thus, there is no uniform price equilibrium.

□

Proof of Proposition (3). The proof proceeds in steps. Section 3 shows that any mixed bargain hunting demand function can be represented by a pure bargain hunting demand function in which the loss aversion parameter is zero by making the appropriate changes in the intercept and slope.

We shall first establish a number of properties of the equilibrium using the pure bargain hunting representation. We shall return to the mixed bargain hunting case at the end.

Let $\bar{p}$ be the high price in the support S, $\bar{p} = sup\, S$. Observe that all prices in the support of the mixed strategy must yield equal payoffs. The equal payoff implications, can be expressed in terms of the high price as $\forall p \in S : \pi(p) = \pi(\bar{p})$. Thus,

$$p\left(a - bp + \delta^{+}\int_{p}^{\bar{p}}(R-p)dF(R)\right) = \bar{p}\,[a - b\bar{p}] \qquad \forall p \in S.$$

Notice that the equal profit condition implies that the first derivative with respect to p vanishes:

$$a - 2bp + \delta^{+}\int_{p}^{\bar{p}}(R-p)\,dF(R) - p\delta^{+}\int_{p}^{\bar{p}}dF(R) = 0 \qquad \forall p \in S$$

The high price in the support is the canonical monopoly price, $\bar{p} = p^m$. Proof by contradiction. Otherwise, as there are no reference effects at the high price, the firm could increase profit by charging the canonical monopoly price. Notice, the profit must thus equal $\pi(p^m)$.

There cannot be a mass point in the support S. Let $\hat{S} \subseteq S$ be the set of mass points. Consider the high price $\widehat{p} = sup\ \hat{S}$ at which there is mass $\delta_G > 0$. This mass point in G induces a mass of δ_F in the reference price distribution as demand $D(\widehat{p})$ at price $\widehat{p} \epsilon S$ is strictly positive. The monopoly profit equals $\Pi(\widehat{p}, F) = \widehat{p}\,(a - b\widehat{p}) + \delta^{+}\int_{\widehat{p}}^{\bar{p}}(R - \widehat{p})\,dF(R)$. Now consider the price $\widetilde{p} = \widehat{p} - \varepsilon$, for some $\varepsilon > 0$ small. We shall show that the monopoly profit at price $\widetilde{p}$ exceeds the monopoly profit at $\widehat{p}$. To see this, consider the following inequalities for profit $\pi(\widetilde{p})$:

$$\begin{aligned}
\pi(\widetilde{p}) &= \widetilde{p}\,(a - b\widetilde{p}) + \delta^{+}\widetilde{p}\int_{\widetilde{p}}^{\bar{p}}(R - \widetilde{p})\,dF(R) \\
&\geq \widetilde{p}\,(a - b\widetilde{p}) + \delta^{+}\widetilde{p}\int_{\widehat{p}}^{\bar{p}}(R - \widehat{p} + \varepsilon)\,dF(R) + \delta^{+}\widetilde{p}\varepsilon\delta_F \\
&= \widehat{p}\left(a - b\widehat{p} + \delta^{+}\int_{\widehat{p}}^{\bar{p}}(R - \widehat{p})\,dF(R)\right) + \varepsilon\left(2b\widehat{p} - a + \delta^{+}\widehat{p}\int_{\widehat{p}}^{\bar{p}}dF(R) - \delta^{+}\int_{\widehat{p}}^{\bar{p}}(R - \widehat{p})\,dF(R)\right) \\
&+ \varepsilon\delta^{+}\left(\widehat{p}\delta_F + \varepsilon\left[b - \delta_F - \int_{\widehat{p}}^{\bar{p}}dF(R)\right]\right).
\end{aligned}$$

Optimality of price $\widehat{p}$ implies from above that the first derivative of the monopoly profit evaluated at the price $\widehat{p}$ vanishes, that is $a - 2b\widehat{p} + \delta^{+}\int_{\widehat{p}}^{\bar{p}}(R - \widehat{p})\,dF(R) + \widehat{p}\delta^{+}\left[-\int_{\widehat{p}}^{\bar{p}}dF(R)\right] = 0$. Using this expression, we conclude from the outer inequality that:

$$\pi(\tilde{p}) \geq \pi(\widehat{p}) + \varepsilon\delta^{+}\left[\widehat{p}\delta_F - \varepsilon\left(b - \delta_F - \int_{\widehat{p}}^{\bar{p}}dF(R)\right)\right].$$

For ε small, the right hand side expression in square brackets is positive, which contradicts the optimality of price $\widehat{p}$. This establishes that there cannot be a mass point in the support of S.

The support S is connected. Proof by contradiction. Suppose the points $p, p' \in S$ but the interval (p, p') is not contained in S. Let $p' = p + d$. We shall show that both prices cannot be optimal. No mass in the interval (p, p') implies that $\int_{p}^{p'} f(r)\,dF(R) = 0$ for any real valued function f. Observing that there is no mass in the interval (p, p'), the optimality condition evaluated at p'

yields:

$$a - 2bp - 2d + \delta^+ \int_{p'}^{\bar{p}} (R - p) dF(R) + \delta^+ \int_{p'}^{\bar{p}} (-d) dF(R) - (p + d)\, \delta^+ \int_{p'}^{\bar{p}} dF(R) = 0.$$

Substituting the optimality condition evaluated at price p yields:

$$-2d + \delta^+ \int_{p'}^{\bar{p}} (-d)\, dF(R) - d\delta^+ \int_{p'}^{\bar{p}} dF(R) = 0,$$

a contradiction which establishes that the support is S is connected.

We have shown so far that the optimal price policy consists of a reference price distribution $F(p)$ induced by $G(p)$ with non-atomistic interval support $S = [\underline{p}, p^m]$, where prices $p \in [\underline{p}, p^m]$ must satisfy the constant profit condition

$$\delta^+ \int_p^{p^m} (R - p) dF(R) - \frac{p^m \left[a - bp^m\right]}{p} + (a - bp) = 0 \qquad \forall p \in S.$$

As this equality holds for all $p \in S$, the derivative with respect to p must vanish, which gives

$$-\delta^+ \left[1 - F(p)\right] - b + \frac{p^m \left[a - bp^m\right]}{p^2} = 0 \qquad \forall p \in S.$$

Re-arranging gives the desired equation for $F(p)$. The lower endpoint in the support satisfies $F(\underline{p}) = 0$, which implies

$$\underline{p} = \sqrt{\frac{p^m \left[a - bp^m\right]}{b + \delta^+}}.$$

This last argument completes the proof for the pure bargain hunting case in which $\delta^- = 0$.

Finally, we shall consider the mixed bargain hunting case. Recall that the intercept now is $a' = a + \delta^- p_e$, where $p_e = \int_{\underline{p}}^{\overline{p}} R\, dF(R)$ and the slope coefficient equals $b' = b + \delta^-$. To complete the equilibrium characterization under mixed bargain hunting we need to obtain the expression for the expected reference price p_e. Using the formula for the lower end point of the support, we obtain $\underline{p} = \sqrt{\frac{b+\delta^-}{b+\delta^+}} p^m$ with $p^m = \frac{a+\delta^- p_e}{2b+2\delta^-}$.

An alternative expression of the lower bound based on the equal profit condition. Recall that the profit function is

$$\pi(p) = p \left[a - bp + \delta^+ \int_p^{\overline{p}} (R - p) dF(R) + \delta^- \int_{\underline{p}}^{p} (R - p) dF(R) \right].$$

In a mixed strategy all prices yield equal profit, which implies that the derivative of profit with respect to p vanishes for prices in the support S:

$$\begin{aligned} \frac{\partial \pi(p)}{\partial p} &= a - bp + \delta^+ \int_p^{\overline{p}} (R - p)\, dF(R) + \delta^- \int_{\underline{p}}^{p} (R - p)\, dF(R) \\ &\quad + p \left[-b - \delta^+ \left[F(\overline{p}) - F(p)\right] - \delta^- \left[F(p) - F(\underline{p})\right] \right] = 0 \qquad \forall p \in S \end{aligned}$$

$$\frac{\partial \pi(p)}{\partial p} = a - 2bp - 2p\delta^{+}\left[F(\overline{p}) - F(p)\right] - 2p\delta^{-}\left[F(p) - F(\underline{p})\right]$$
$$+ \delta^{+}\int_{p}^{\overline{p}} R\,dF(R) + \delta^{-}\int_{\underline{p}}^{p} R dF(R) = 0 \qquad \forall p{\in}S.$$

Evaluated at $\underline{p}$, this condition gives

$$a + \delta^{+}p_e - 2\left(b + \delta^{+}\right)\underline{p} = 0$$

or equivalently

$$\underline{p} = \frac{a + \delta^{+}p_e}{2b + 2\delta^{+}}.$$

Combining this with our earlier expressions $\underline{p} = \sqrt{\frac{b+\delta^{-}}{b+\delta^{+}}}p^m$ and $p^m = \frac{a+\delta^{-}p_e}{2b+2\delta^{-}}$, we have

$$\sqrt{\frac{b+\delta^{-}}{b+\delta^{+}}}\frac{a+\delta^{-}p_e}{2b+2\delta^{-}} = \frac{a+\delta^{+}p_e}{2b+2\delta^{+}}$$

Rearranging yields

$$p_e = a\frac{\sqrt{b+\delta^{+}} - \sqrt{b+\delta^{-}}}{\delta^{+}\sqrt{b+\delta^{-}} - \delta^{-}\sqrt{b+\delta^{+}}}.$$

Observe that the ratio is between zero and one.

To conclude, we verify that the condition $\delta^{+} - \delta^{-} \leq 2b$ is sufficient to guarantee that equilibrium demand is nonzero for each consumer at all possible prices $[\underline{p}, \bar{p}]$. Toward this end, observe that demand is interior at any price $p \in \left[\underline{p}, \bar{p}\right]$ for all consumers irrespective of their personal priors if it is so at the worst case gain seeking and worst case loss aversion priors. The worst case gain seeking prior is $Pr(p = \bar{p}) = 1$. To see this, observe that it gives the maximal gain at low prices. A consumer will buy at a price of zero at the low taste shock if $-a + \delta^{+}\bar{p} \geq 0$. Thus, the complementary condition $\bar{p} < \frac{a}{\delta^{+}}$ ensures that expected demand is less than one for all consumers irrespective of their priors. The worst case loss aversion prior is $Pr(p = 0) = 1$. To see this, observe that it has a maximal loss of $\delta^{-}\bar{p}$. It has zero demand at the high taste shock if $a - b\bar{p} - \delta^{-}\bar{p} \leq 0$. The complementary condition $\bar{p} < \frac{a}{b+\delta^{-}}$ ensures that expected demand is positive for all consumers. Now, equilibrium prices satisfy $\bar{p} = \frac{a+\delta^{-}p_e}{2b+2\delta^{-}} \leq \frac{a+\delta^{-}\bar{p}}{2b+2\delta^{-}} \leq \frac{a}{2b+\delta^{-}}$. Thus, the second condition is satisfied, and we only need to check the first condition, $\bar{p} < \frac{a}{\delta^{+}}$, which indeed holds as $\frac{a}{2b+\delta^{-}} \leq \frac{a}{\delta^{+}}$, or $\delta^{+} - \delta^{-} \leq 2b$.

$\square$

Proof of Proposition (4). The monopoly profit under commitment becomes

$$\Pi = \int_{l}^{u} x\left[a - bx + \delta^{+}\int_{x}^{u}(y - x)\,f(y)dy + \delta^{-}\int_{l}^{x}(y - x)\,f(y)dy\right]f(x)dx$$

For any price x consider the choice of the weight $w = f(x)$. The optimal weight must satisfy the necessary first order condition, $\frac{\partial \Pi}{\partial w} = 0$, which is

$$\phi(x) = \left\{ x\left[a - bx\right] + \delta^{+} x \int_{x}^{l} (y - x)\, f(y)dy + \delta^{-} x \int_{0}^{x} (y - x)\, f(y)dy \right\}$$

$$+\delta^{+} \int_{0}^{x} y\,(x - y)\, f(y)dy + \delta^{-} \int_{x}^{l} y\,(x - y)\, f(y)dy = 0 \qquad for\ all\ x$$

Notice, that the second order condition, , $\frac{\partial^2 \Pi}{\partial^2 w} \leq 0$, is satisfied. Also, notice that the term in $\{\}$ brackets is the profit evaluated at price x. The terms in the second line are increasing in x. Thus, profit is decreasing in price x over the price support, establishing the second part of the Proposition when consumers are (weakly) bargain hunting.

The first order condition of optimal weights is a necessary condition that applies to all prices in the support, that is $\phi(x) = 0$ for all x. Taking the derivative with respect to x yields:

$$\frac{\partial \phi(x)}{\partial x} = a - 2bx + \left(\delta^{+} + \delta^{-}\right) p_c - \delta^{+} 2x[1 - F(x)] - \delta^{-} 2xF(x) = 0$$

Expressing the cdf F yields,

$$F(x) = \frac{\delta^{+} + b}{\left(\delta^{+} - \delta^{-}\right)} - \frac{a + \left(\delta^{+} + \delta^{-}\right) p_c}{2\left(\delta^{+} - \delta^{-}\right) x}$$

with pdf,

$$f(x) = \frac{a + \left(\delta^{+} + \delta^{-}\right) p_c}{2\left(\delta^{+} - \delta^{-}\right) x^2}.$$

The boundary conditions require $F(\underline{p}) = 0$ and $F(\overline{p}) = 1$. We find the boundary points as:

$$\underline{p} = \frac{a + \left(\delta^{+} + \delta^{-}\right) p_c}{2(b + \delta^{+})}$$

$$\overline{p} = \frac{a + \left(\delta^{+} + \delta^{-}\right) p_c}{2\left(b + \delta^{-}\right)}.$$

Using the pdf, we can solve for the expectation p_c of the price distribution G as

$$p_c = \int_{\underline{p}}^{\overline{p}} x \cdot \frac{a + \left(\delta^{+} + \delta^{-}\right) p_c}{2\left(\delta^{+} - \delta^{-}\right) x^2} dx$$

$$= \frac{a + \left(\delta^{+} + \delta^{-}\right) p_c}{2\left(\delta^{+} - \delta^{-}\right)}\ ln(x)|_{\underline{p}}^{\overline{p}}$$

$$= \frac{a + \left(\delta^{+} + \delta^{-}\right) p_c}{2\left(\delta^{+} - \delta^{-}\right)} \left[ln(b + \delta^{+}) - ln(b + \delta^{-})\right]$$

$$= \frac{a\left[ln(b + \delta^{+}) - ln(b + \delta^{-})\right]}{2\left(\delta^{+} - \delta^{-}\right) - \left(\delta^{+} + \delta^{-}\right)\left[ln(b + \delta^{+}) - ln(b + \delta^{-})\right]}.$$

This completes the proof for the bargain-hunting case.

Observe that when $\delta^+ < \delta^-$, the boundary conditions above are violated: $\delta^+ < \delta^-$ implies $\underline{p} > \bar{p}$. Thus, the optimal pricing rule consists of a single price. With a single price policy we are back to the canonical monopoly problem absent of reference effects. The optimal price is thus the canonical monopoly price.

$\square$

Proof of Proposition (5). Consider any optimal uniform price candidate p. Due to the reference effects there will be a kink in demand at that price p. At prices not equal to p, the demand and profit functions are differentiable. A necessary condition for optimality is that marginal profit is non-decreasing as prices approach p from the left, $lim_{p' \nearrow p} \partial\pi(p)/\partial p \geq 0$, and non-increasing as prices approach p from the right, $lim_{p' \searrow p} \partial\pi(p)/\partial p \leq 0$. The left- and right-hand-side derivatives are readily calculated. We obtain $lim_{p' \uparrow p} \partial\pi(p)/\partial p = a - 2bp - \delta^+ p \geq 0$ and $lim_{p' \downarrow p} \partial\pi(p)/\partial p = a - 2bp - \delta^+ p \leq 0$. The inequalities imply that equilibrium prices must be in the interval $\left[\frac{a}{2b+\delta^-}, \frac{a}{2b+\delta^+}\right]$. This interval is non-empty when losses are valued more than gains, $\delta^+ - \delta^- \leq 0$. Moreover, the profit function is globally concave under loss aversion. Thus, any price in the interval described is a Bayesian Nash equilibrium.

$\square$

Proof of Theorem 1. The proof idea is to consider bounds on the *short-run* dead-weight loss DWL_s. The bounds are obtained by using the usual triangle under the linear demand curve but evaluated at the lower and upper end point of the support of equilibrium prices. Our lower bound $\underline{DWL_s}$ is given by the dead-weight loss at the low equilibrium price $\underline{p}$, and satisfies:

$$DWL_s \geq DWL_s(\underline{p}) = \frac{\underline{p}^2}{2}\left(b + \delta^+\right) = \underline{DWL_s}$$

The upper bound is based on the observation that the *short-run* demand curve D_s is convex. To see the convexity observe that D_s has a positive second derivative on the interval $\left[\underline{p}, \bar{p}\right]$, $\frac{\partial^2 D_s(p,F)}{(\partial p)^2} = \left(\delta^+ - \delta^-\right) dF(p) > 0$, and is linear for prices below $\underline{p}$. It is thus convex. Due to convexity, an upper bound on the dead-weight loss $\overline{DWL_s}$ is given by the *short-run* dead-weight loss triangle at $\bar{p}$ with consumers' expectations centered at price $\bar{p}$. To see that it is indeed an upper bound observe the following::

$$\begin{aligned} DWL_s \leq DWL_s(\bar{p}) &= \left[\left(\delta^+ - \delta^-\right) p_e + \left(b + \delta^-\right) \bar{p}\right] \frac{\bar{p}}{2} \\ &\leq \frac{\left(b + \delta^+\right)}{2} \bar{p}^2 = \overline{DWL_s} \end{aligned}$$

Both the upper and lower bound have the interpretation that the competitive equilibrium price is perceived as a gain relative to a price in the Bayesian Nash equilibrium price support. For the lower bound we use the lower end point in the price support, while for the upper bound we use the upper end-point plus expectations centered at the upper end point.

An upper bound on the *long-run* dead-weight loss is given by $DWL_l(\bar{p})$,

$$DWL_l(\bar{p}) = \frac{(b+\delta^-)}{2}\bar{p}^2 = \frac{(b+\delta^-)}{2}\left(\frac{a+\delta^- p_e}{2b+2\delta^-}\right)^2 = \frac{a^2}{8(b+\delta^-)}\left[\frac{\sqrt{b+\delta^-}(\delta^+-\delta^-)}{\delta^+\sqrt{b+\delta^-}-\delta^-\sqrt{b+\delta^+}}\right]^2$$

A lower bound on the *short-run* dead-weight-loss is given by $\underline{DWL_s}$:

$$\underline{DWL_s} = \frac{\underline{p}^2}{2}\left(b+\delta^+\right) = \frac{a^2}{8}\left[\frac{\delta^+-\delta^-}{\delta^+\sqrt{b+\delta^-}-\delta^-\sqrt{b+\delta^+}}\right]^2$$

To complete the proof of part (i) observe that the lower bound on the *short-run* dead-weight loss equals the upper bound on the *long-run* dead-weight loss, $\underline{DWL_s} = DWL_l(\bar{p})$. This establishes part (i).

With $\delta^- = 0$ the lower bound on the *short-run* dead-weight-loss equals the canonical monopoly dead-weight loss DWL_m. From part (i) we already know that the lower bound on the *short-run* dead-weight loss equals the upper bound on the *long-run* dead-weight loss.

The upper bounds on the *long-run* dead-weight loss equals

$$\overline{DWL_s} = \frac{(b+\delta^+)}{2}\bar{p}^2 = \frac{(b+\delta^+)}{8(b+\delta^-)^2}\left(a+\delta^- a\frac{\sqrt{b+\delta^+}-\sqrt{b+\delta^-}}{\delta^+\sqrt{b+\delta^-}-\delta^-\sqrt{b+\delta^+}}\right)^2$$

$$= \frac{a^2(b+\delta^+)}{8(b+\delta^-)}\left(\frac{(\delta^+-\delta^-)}{\delta^+\sqrt{b+\delta^-}-\delta^-\sqrt{b+\delta^+}}\right)^2.$$

The dead-weight loss of the canonical monopoly solution in the absence of reference effects equals $DWL_m = \frac{a^2}{8b}$. We need to establish a condition under which $\overline{DWL_s} \leq DWL_m$. Using the substitutions $x = \sqrt{b+\delta^+}$ and $y = \sqrt{b+\delta^-}$, and taking the square root, we obtain the following quadratic inequality:

$$\frac{ax}{\sqrt{8}y}\left(\frac{x^2-y^2}{(x^2-b)\,y-(y^2-b)\,x}\right) = \frac{ax}{\sqrt{8}y}\left(\frac{x+y}{xy+b}\right) \leq \frac{a}{\sqrt{8b}}.$$

Rearranging yields:

$$y^2 + \left(\frac{b}{x}-\sqrt{b}\right)y - \sqrt{b}x \geq 0$$

with boundary solution

$$y = -\frac{1}{2}\left(\frac{b}{x} - \sqrt{b}\right) + \sqrt{\frac{1}{4}\left(\frac{b}{x} - \sqrt{b}\right)^2 + \sqrt{b}x}.$$

Thus, the cut-off point where the bound equals the canonical monopoly dead-weight loss is

$$\delta^{-*} = \left[\frac{1}{2}\left(\frac{b}{\sqrt{b+\delta^+}} - \sqrt{b}\right) - \sqrt{\frac{1}{4}\left(\frac{b}{\sqrt{b+\delta^+}} - \sqrt{b}\right)^2 + \sqrt{b}\sqrt{b+\delta^+}}\right]^2 - b.$$

For $\delta^- \in [\delta^{-*}, \delta^+]$, the inequality $\overline{DWL_s} \leq DWL_m$ holds. This completes the proof.

$\square$

Proof of Theorem2. By assumption, $\delta^+ \leq \delta^-$, and property (i) follows immediately. To see property (ii), observe that $DWL_s(R) = \frac{R^2}{2}\left(b + \delta^+\right)$ is maximal when R equals the upper endpoint of the support, $\overline{R} = \frac{a}{2b+\delta^+}$. Now, $DWL_s(\overline{R}) = \left(\frac{a}{2b+\delta^+}\right)^2 \cdot \frac{b+\delta^+}{2} < \frac{a^2}{4b} = DWL_m$, which established property (ii). To see property (iii), observe that $DWL_l(R) = \frac{R^2}{2}\left(b + \delta^-\right)$ is maximal when R equals the upper endpoint of the support, $\overline{R} = \frac{a}{2b+\delta^+}$. Now, $DWL_l(\overline{R}) = \left(\frac{a}{2b+\delta^+}\right)^2 \cdot \frac{b+\delta^-}{2} < \frac{a^2}{4b} = DWL_m$, which holds provided $\delta^- < b + 2\delta^+ + \frac{\left(\delta^+\right)^2}{2b}$, establishing property (iii).

$\square$