Investment, Capital Stock, and Replacement Cost of Assets when Economic Depreciation is Non-Geometric

Dmitry Livdan
Haas School of Business,
University of California, Berkeley
livdan@haas.berkeley.edu

Alexander Nezlobin*
London School of Economics
a.nezlobin@lse.ac.uk

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Abstract

This paper extends the Q-theory of investment to capital goods with arbitrary efficiency profiles. When efficiency is non-geometric, the firm's capital stock and the replacement cost of its assets are fundamentally different aggregates of the firm's investment history. If capital goods have constant efficiency over a finite useful life, then simple proxies are readily available for both the replacement cost of assets in place and capital stock. Under this assumption, we decompose the total investment rate along two dimensions: into its net and replacement components, and into its cash and non-cash components. We then show that these components exhibit significantly different economic determinants and behavior.

1 Introduction

Traditional Q-theoretic models of investment rely on the assumption that the economic depreciation of capital goods is either geometric (in discrete-time) or exponential (in continuous time); see, e.g., Hayashi (1982). This assumption is analytically convenient since it leads to a homogeneous capital stock whose future economic efficiency is independent of its current vintage composition. While geometric efficiency may be descriptive for some assets, its general applicability has long been challenged on both empirical and theoretical grounds (see, e.g., Feldstein and Rothschild 1974, Ramey and Shapiro 2001). Moreover, firm-level data on capital goods, such as property, plant, and equipment (PP&E), is prepared in practice almost exclusively under the assumption that the capital goods' efficiency is constant over a finite useful life. The latter assumption also underlies one of the most commonly used empirical procedures for estimating Tobin's Q suggested by Lewellen and Badrinath (1997). However, from a theoretical perspective, it is not clear whether and how the insights from models with geometric efficiency carry over to other settings. In this paper, we fill this gap by extending the Q-theory of investment to capital goods with arbitrary efficiency profiles.

In traditional Q-theory, a firm's current productive capacity, i.e., its capital stock, is the only aggregate of its investment history that is relevant for future investment decisions. This is an artifact of the geometric depreciation assumption: since the productive capacity of capital goods of all vintages declines at the same rate, the aggregate current capacity of assets-in-place fully determines how much capacity these assets will provide in all future periods. To see why this is not the case generally, consider a firm investing in capital goods with constant efficiency over a finite useful life. Such efficiency pattern is often referred to in the economics literature as the *one-hoss shay*.[1] While the same productive capacity today can be generated by either old or new capital goods, the firm will need to replace its existing assets sooner if they are old. Consequently, all future investment decisions of such a firm depend not only on its current capital stock but also on the vintage composition of its assets-in-place.

Our model of capital goods is based on Rogerson (2008), who provided a simple closed-form expression for the user cost of capital for assets with arbitrary efficiency profiles. Consistent with much of the earlier literature, the value of the firm in our model is equal to the sum of the replacement cost of its assets-in-place and the present value of the expected future economic profits (see, e.g., Lindenberg and Ross, 1981; Salinger, 1984; Abel and Eberly, 2011; and Nezlobin, 2012). However, under non-geometric efficiency, the replacement cost of

[1]The term originates from a 19-th century poem by Oliver Wendell Holmes that tells a story of a wonderful one-hoss shay that "ran a hundred years to a day" and went to pieces all at once in the Great Lisbon earthquake.

the firm's assets-in-place is not equal to the value of its capital stock – they are two different aggregates of the firm's investment history. We formally define the firm's capital stock as the amount that the firm would have to pay today to replicate it current productive capacity with *new* capital goods. By way of contrast, the replacement cost of assets-in-place reflects the age composition of the firm's capital goods: it is equal to the value of their current and future productive capacity in a hypothetical competitive rental market for capital goods. While capital stock is the appropriate measure of the firm's current scale of operation, the replacement cost of assets-in-place effectively measures the prepaid future capital costs.

Our model generates particularly crisp empirical predictions when the efficiency of capital goods follows the one-hoss shay pattern. In this special case, we show that both the firm's effective capital stock and the replacement cost of its assets can be readily measured from the information reported in its financial statements. Furthermore, it turns out that the ratio of these two quantities, which we label RC/K, serves as one of the main determinants of the future investment rates. To estimate firm-level *total* investment, we adopt a variant of the measure developed in Lewellen and Badrinath (1997), which relies on the information reported in two consecutive balance sheets. The main advantage of this measure over the widely used capital expenditures (CapEx) numbers reported in firms' cash flow statements is that it accounts for capital goods acquired with non-cash transactions, such as capital leases and business combinations. Armed with a total investment measure, we then decompose it along two alternative dimensions: into its cash and non-cash components, and the net and replacement investment components. While the former decomposition holds for arbitrary efficiency patterns, the latter one relies on the one-hoss shay efficiency assumption. This assumption once again allows us to construct simple proxies for both net and replacement investment that rely only on two periods of data and do not require a perpetual inventory procedure.

Outside the case of geometric efficiency, both net and replacement investment rates contribute to the variation in the firm's total investment rate. The firm's replacement investment is the amount the firm needs to spend today to maintain its current capital stock for another period. As such, replacement investment does not depend on the current conditions in the firm's output product markets and is determined solely by the vintage composition of its assets-in-place. The firm's net investment is the amount that the firm spends on capital goods in excess of the replacement investment, i.e., to grow its capital stock. In our model, the firm's net investment rate is determined solely by the expected growth in demand for its output and is unaffected by the vintage composition of its assets-in-place.

We show that in the one-hoss shay setting, the net and replacement investment rates have fundamentally different economic behavior and determinants. For instance, since net investment is primarily driven by growth in the firm's product markets, it should be positively associated with Q and RC/K. On the other hand, the replacement investment rate is shown to be negatively associated with the realized past growth rates and RC/K. This effect obtains because faster growing firms have, on average, newer assets and a higher RC/K ratio. Under geometric efficiency, this has no effect on the future replacement investment rates; yet under the one-hoss shay assumption, newer capital goods lead to a lower future replacement rate.

To validate our empirical measures of net and replacement investment, we first study their explanatory power for future sales growth. As expected, the variation in the future sales growth is primarily explained by the variation in the net investment rate, and the relation between the two retains economic and statistical significance after controlling for a host of other explanatory variables. This finding supports the notion that our proxy for the net investment rate indeed captures the growth-related component of total investment. Next, we find that the sensitivity of total investment to Q and cash flow is almost entirely due to the net investment component. In most specifications, the replacement investment rate does not have economically significant relations with these two variables. In addition, consistent with our theory, the net investment rate is positively associated with RC/K.

The main determinants of replacement investment are the two variables that we label vintage capital proxies: RC/K and the estimated reciprocal of the useful life of capital goods, T^{-1}. In agreement with our theoretical predictions, replacement investment is positively associated with T^{-1} and negatively with RC/K. While the latter variable exhibits strong explanatory power for both future net and replacement investment rates, it is less significant in regressions of the total investment rate since the relations with the individual components are of opposite signs. Still, however, in most of our multivariate analyses, vintage capital proxies have higher economic and statistical significance than cash flow and comparable statistical significance to that of Q and lagged investment rates.

Our second decomposition of total investment – into its cash and non-cash components – demonstrates the importance of non-cash investment, which is largely overlooked in the existing literature. For instance, we find that total investment is almost twice as sensitive to Q as its cash component. In fact, in all specifications, the net investment component alone is more sensitive to Q and cash flow than the cash component. The sensitivity of total investment to cash flow is highly dependent on the estimation procedure. When we use plain OLS estimates, the sensitivity of total investment to cash flow exceed that of cash investment by about two thirds. However, when we use Erickson et al. (2014) estimates corrected for errors-in-variables problem in Q, the effect of cash flow on total investment disappears completely. While cash investment is still positively associated with lagged cash flow, the sensitivity of non-cash investment to cash flow is negative, and the two effects offset each other. A possible interpretation for this finding is that cash constrained firms do not necessarily reduce their total investment but simply acquire a larger share of new capital goods under non-cash arrangements such as capital leases.

Our paper is related to several strands of literature in economics and finance. First, multiple papers have analyzed investment problems with capital goods with non-geometric economic efficiency. Particularly closely related are the studies by Rogerson (2008), which serves as a basis for our model, and Lewellen and Badrinath (1997), from which we derive our measure of total investment. Models with vintage capital effects have also been considered in, for instance, Benhabib and Rustichini (1991), Sakellaris (1997), Sakellaris and Wilson (2004), and Rampini (2019). To the best of our knowledge, our paper is the first one to emphasize the difference between the replacement cost of assets-in-place and capital stock arising from relaxing the assumption of geometric efficiency, as well as to suggest simple empirical proxies for these quantities.

Our study provides new insights on the structure of total investment by decomposing it along two dimensions: into the net and replacement, and cash and non-cash components. The former decomposition has been studied at least since Jorgenson (1967), yet no empirical measures have been suggested for the net and replacement investment components outside of the geometric efficiency case. Our model allows us to construct such measures and verify that their empirical behavior is consistent with the analytical predictions. Our second investment decomposition – into the cash and non-cash components – sheds more light on the structure of non-cash investment expenditures. The importance of non-cash investment has been recognized in, e.g., Eisfeldt and Rampini (2009), but there appears to be no widely accepted measure for non-cash investment that can be used in broad samples of firms.

The rest of the paper is organized as follows. Our theoretical model is set up in Section 2. Section 3 presents the main theoretical results. Section 4 focuses on data selection and empirical variable construction. Section 4 reports empirical findings. Section 5 concludes.

2 Model Setup

2.1 Production Technology

Consider a firm that uses a single type of capital goods to produce a single non-storable output good. Capital goods have a useful life of T periods and their efficiency declines with age. Specifically, a unit of capital good purchased in period t comes online in period $t + 1$

and allows the firm to produce x_τ units of the output good in period $t+\tau$, where

$$1 = x_1 \geq ... \geq x_T.$$

The vector $\boldsymbol{x} = (x_1, \ldots, x_T)$ will be referred to as the *efficiency pattern* of the firm's assets. For notational convenience, let $x_{T+1} \equiv 0$. The firm's effective capital stock in period t, i.e., its aggregate production capacity, can then be written as:

$$K_t = \sum_{\tau=1}^{T} x_\tau \cdot I_{t-\tau}, \tag{1}$$

where $I_{t-\tau}$ is the firm's gross investment in period $t-\tau$.[2] Let $\Theta_{t-1} \equiv (I_{t-1}, ..., I_{t-T})$ denote the firm's relevant investment history in period t. We normalize the purchase price of new capital goods to unity, so that the direct cost of investment in period t is measured by I_t.

There are two efficiency patterns commonly considered in the earlier literature: *geometric* economic depreciation and *one-hoss shay* efficiency. In the geometric depreciation scenario, assets are infinitely lived, $T = \infty$, and the amount of investment surviving to date τ of its life is declining exponentially in τ:

$$x_\tau = (1-\delta)^{\tau-1} \tag{2}$$

for some $0 \leq \delta \leq 1$. An important property of this pattern is that the rate by which the productive capacity of a unit of capital good decreases over a given period is independent of the age of that unit. Under this assumption the firm's capital stock becomes homogeneous, i.e., the vintage composition of the firm's current stock is irrelevant for future investment choices. The efficiency pattern in (2) has been observed to be descriptive for some types of capital goods but not for others. For instance, solar PV installations appear to comport well to this assumption, see, e.g. Reichelstein and Yorston (2013); however, Ramey and Shapiro (2001) strongly reject the geometric efficiency model in their analysis of equipment-level data from the aerospace industry. Hulten et al. (1989) consider a broad class of efficiency patterns and find that geometric efficiency is reasonably descriptive for their data on machine tools and construction equipment.

While the assumption of geometric efficiency is prevalent in the academic literature due

[2] Our model of vintage capital builds on Rogerson (2008). Similar to that paper, we assume that the firm purchases only new capital goods. For models with investment in used capital goods, see, e.g., Eisfeldt and Rampini (2007) and Jovanovic and Yatsenko (2012). For a more general, but arguably less analytically tractable, model of vintage capital, see, e.g., Benhabib and Rustichini (1991). Several papers examined, empirically and analytically, the behavior of Tobin's Q in models with vintage capital (e.g., Lewellen and Badrinath 1997, McNichols et al. 2014, Nezlobin et al. 2016).

to its analytical convenience, it is rarely used in practice. Instead, managers often view capital goods as having an approximately constant productive capacity over a finite useful life. This assumption is usually invoked to justify the widespread use of the straight-line depreciation rule in financial reporting. In the academic literature this efficiency pattern is usually referred to as one-hoss shay (see, e.g., Fisher and McGowan 1983, Laffont and Tirole 2000, and Rogerson 2011). In our notation, one-hoss shay productivity is given by:

$$1 = x_1 = ... = x_T,$$

for some finite T. In the investment literature, this pattern underlies the popular empirical procedure for estimating Tobin's Q suggested by Lewellen and Badrinath (1997). We discuss the relation between the assumptions in Lewellen and Badrinath (1997) and one-hoss shay efficiency in greater detail below. The one-hoss shay assumption has also been applied by McNichols et al. (2014) in developing an alternative methodology for Tobin's Q estimation. Since this assumption appears to be widely used by managers in practice and, therefore, is already largely incorporated in the reported financial statements, we adopt it as a natural foundation for our empirical analysis later in this paper. Moreover, as we show below, it also leads to simple empirical proxies for many economic quantities of interest.

Let $K_{t,t+j}$ denote the capacity provided in period $t+j$ by assets already in place at the beginning of period t . Formally, it can be expressed as:

$$K_{t,t+j} = \sum_{\tau=1}^{T-j} x_{j+\tau} \cdot I_{t-\tau}$$

for $j \geq 0$. Note that $K_t = K_{t,t}$. If depreciation is geometric, then we have:

$$K_{t,t+j} = (1-\delta)^j K_t,$$

i.e., the current capacity of the firm's assets in place determines how much capacity those assets will provide in each future period. Consequently, in this case, we obtain the usual law of motion for the firm's capital stock:

$$K_{t+1} = (1-\delta)K_t + I_t.$$

However, in general, K_t does not uniquely determine $K_{t,t+j}$, which depends on the full vintage composition of the capital stock at date t, Θ_t. Then, the firm's capital stock in period $t+1$ can be written as:

$$K_{t+1} = K_{t,t+1} + I_t. \tag{3}$$

Assume that capital is the only input required for production. Suppose further that the inverse demand function for the firm's output good takes the following form:

$$P\left(\hat{Z}_t, K_t\right) = \left(\frac{K_t}{\hat{Z}_t}\right)^{\alpha-1},$$

where $0 < \alpha < 1$, and $\hat{Z}_t$ is a stochastic demand shift parameter. In this specification, $1-\alpha$ is the the price elasticity of demand, with $\alpha \to 1$ corresponding to the case of perfect competition and $\alpha \to 0$ to the case of high monopoly power. The firm's revenue in period t, $R\left(\hat{Z}_t, K_t\right)$, can then be written as:

$$R\left(\hat{Z}_t, K_t\right) = \hat{Z}_t^{1-\alpha} K_t^{\alpha},$$

and the firm's net cash flow is equal to $R\left(\hat{Z}_t, K_t\right) - I_t$. In Appendix B, we discuss how our results carry over to a model with a competitive product market and convex capital adjustment costs.

Each period, the firm faces two types of demand shocks: permanent and transitory. Let Z_t denote the permanent component of the demand shift parameter. We assume that it evolves according to:

$$Z_{t+1} = \mu_{t+1} \cdot Z_t, \tag{4}$$

where the (gross) growth rates μ_{t+1} are independently drawn from some time-invariant distribution with a bounded support on $[\mu_{min}, \mu_{max}]$ and mean $\bar{\mu}$.

The actual, realized, demand shift parameter is different from its permanent component by an additional multiplicative transitory error term:

$$\hat{Z}_{t+1} = \epsilon_{t+1} \cdot Z_{t+1},$$

where ϵ_t are distributed identically and independently of each other and of $\{\mu_s\}_{s=1}^{\infty}$, with a mean of one. Recall that investments "come online" with a lag of one period, i.e., when the firm decides on the investment level I_t it effectively chooses its capital stock for period $t+1$, K_{t+1}. To simplify the exposition, we assume that the firm and the equity market observe μ_{t+1}, ϵ_{t+1}, and, therefore, $\hat{Z}_{t+1}$ just before the choice of I_t is made. It is straightforward to extend our results to a setting where investment I_t has to be made before μ_{t+1} and ϵ_{t+1} are observed. In Appendix B, we also consider an extension of our model in which the permanent component of demand follows a regime-switching process as in Eberly et al. (2008), Abel and Eberly (2011), and Eberly et al. (2012). This extension is important because these papers demonstrate that the assumption of regime-switching demand can significantly improve the empirical plausibility of the model. Figure 1 illustrates the evolution of the demand shift parameter, $\hat{Z}_t$, in our main model.

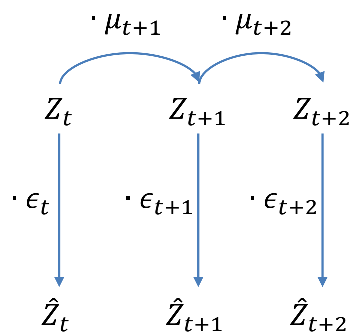

Figure 1: Time Evolution of the Stochastic Demand Shift Parameter
The permanent component of the demand shift parameter evolves according to: $Z_{t+1} = \mu_{t+1} \cdot Z_t$. The actual value of the demand shift parameter in period $t+1$ is given by: $\hat{Z}_{t+1} = \epsilon_{t+1} \cdot Z_{t+1}$.

The firm is all-equity financed, and all cash flows are disbursed to (or supplied by) the firm's shareholders immediately. Let $r > 0$ be the firm's discount rate and $\gamma \equiv \frac{1}{1+r}$ be the corresponding discount factor. To ensure that the firm's value is always finite, we impose that $1 + r > \mu_{max}$. The firm makes its investments so as to maximize the expected present value of its future cash flows. As in Abel and Eberly (2011), we focus on a setting where the firm's investments are fully reversible, i.e., I_t can be less than zero. In periods where I_t is negative, the firm sells a capacity stream which is equivalent to $|I_t|$ units of new capital goods for the price of $|I_t|$ units of new capital goods. Equivalently, the firm can be assumed to be able to sell its used capital goods at their perfectly competitive price, which is formally defined below.

2.2 User Cost of Capital and Replacement Cost of Assets

To characterize the optimal investment policy, it is helpful to employ the notion of the user *cost of capital,* i.e., a hypothetical perfectly competitive rental rate per unit of capital stock (see, e.g., Jorgenson 1963 and Arrow 1964). The user cost of capital will be denoted by c. It is well-known that in the geometric depreciation scenario, c is equal to $r + \delta$.

The generalization of the concept of the user cost of capital to the setting with arbitrary efficiency patterns is due to Rogerson (2008). Following the approach of that paper, consider a hypothetical perfectly competitive rental market for capital goods. In this market, a provider of rental services can buy one unit of the capital good (at a cost of one dollar) and then rent out its capacity in future periods. In period τ of the asset's useful life, the asset will generate $x_\tau \cdot c$ in rental income. Then, the net present value of the rental firm's investment project will be:

$$-1 + \gamma x_1 c + ... + \gamma^T x_T c.$$

Since the rental market is assumed to be perfectly competitive, the quantity above must equal zero, i.e.,

$$1 = \sum_{\tau=1}^{T} \gamma^\tau x_\tau c. \tag{5}$$

Then, it follows that the user cost of capital is given by:

$$c = \frac{1}{\sum_{\tau=1}^{T} \gamma^\tau x_\tau}. \tag{6}$$

It is straightforward to verify that when the economic depreciation of assets is geometric and given by (2), the right-hand side of (6) reduces to $r + \delta$. On the other hand, for asset with one-hoss shay efficiency, the user cost of capital is given by:

$$c = \frac{1}{\sum_{\tau=1}^{T} \gamma^\tau} = \frac{r}{1 - \gamma^T}. \tag{7}$$

We will refer to cK_t as the *current cost* of the capital stock in period t. Note that the current cost of capital can be expressed as a function of past investment cash outflows:

$$cK_t = c \sum_{\tau=1}^{T} x_\tau I_{t-\tau}, \tag{8}$$

Furthermore, let π_t be the firm's *economic profit* in period t defined as the difference between its operating cash flow and the current cost of the capital stock in that period:

$$\pi_t \equiv R\left(\hat{Z}_t, K_t\right) - cK_t.$$

In our model, an important difference arises between the firm's effective capital stock in a given period and the replacement cost of assets-in-place at the beginning of that period. To formally define the latter quantity, consider a unit of asset purchased in period $t - \tau$ from the perspective of date t. This asset will provide the following stream of capacity in the remaining $T - \tau$ periods of its useful life: $\{x_{\tau+1}, ..., x_T\}$. If we again consider a hypothetical rental market for capital goods, the value of such a stream in that market would be:

$$v_\tau \equiv \left(\gamma x_{\tau+1} + ... + \gamma^{T-\tau} x_T\right) c. \tag{9}$$

So defined v_τ can also be interpreted as the perfectly competitive price for a unit of capital good of age τ. Note that $v_0 = 1$, i.e., the replacement cost of an asset just acquired is equal to its price, and $v_T = 0$. The total replacement cost of assets in place at date t (just after the new investment I_t is made) is equal to:

$$RC_t \equiv \sum_{\tau=0}^{T-1} v_\tau I_{t-\tau}. \tag{10}$$

It can be verified that in the scenario with geometric depreciation,

$$v_\tau = (1-\delta)^\tau .$$

It follows that the replacement cost of assets under the assumption of geometric depreciation is

$$RC_t = \sum_{\tau=0}^{\infty} (1-\delta)^\tau I_{t-\tau} = K_{t+1},$$

i.e., the replacement cost of assets in place at the beginning of period $t+1$ is simply equal to that period's capital stock.

However, for arbitrary efficiency profiles, the replacement cost of assets in place and the effective capital stock will be two different linear aggregates of the relevant investment history. Specifically, according to (8), the weights on past investments in the calculation of K_{t+1} are proportional to the *current* efficiency of those investments. On the other hand, equations (9-10) show that the weight on $I_{t-\tau}$ in the expression for RC_t is proportional to the present value of all capacity levels that a unit of capital good of that vintage is yet to generate in the future periods.

Of key importance for our empirical analysis will be the case of one-hoss shay efficiency, in which the difference between the replacement cost of assets-in-place, RC_t, and capital stock, K_{t+1}, is particularly pronounced. For assets with one-hoss shay efficiency, the capital stock in period $t+1$ is given by

$$K_{t+1} = \sum_{\tau=0}^{T-1} I_{t-\tau}. \tag{11}$$

It is noteworthy that, at least under the assumptions imposed so far, K_{t+1} is simply equal to the firm's *gross* investment up to time t. This equality holds as long as the price of new capital goods is constant over time and the firm's accounting system correctly estimates the useful life of capital goods, T.

Now consider the replacement cost of assets-in-place in the case of one-hoss shay efficiency. Applying the expression for c in (7), v_τ in equation (9) can be calculated as:

$$v_\tau \equiv c\left(\gamma + ... + \gamma^{T-\tau}\right) = \frac{1-\gamma^{T-\tau}}{1-\gamma^T}. \tag{12}$$

Therefore, the total replacement cost of assets-in-place at date t is equal to:

$$RC_t = \sum_{\tau=0}^{T-1} \frac{1-\gamma^{T-\tau}}{1-\gamma^T} I_{t-\tau}. \tag{13}$$

Comparing equations (11) and (13), one can see that while both RC_t and K_{t+1} are determined by the same investments, the two quantities are not equal to each other.

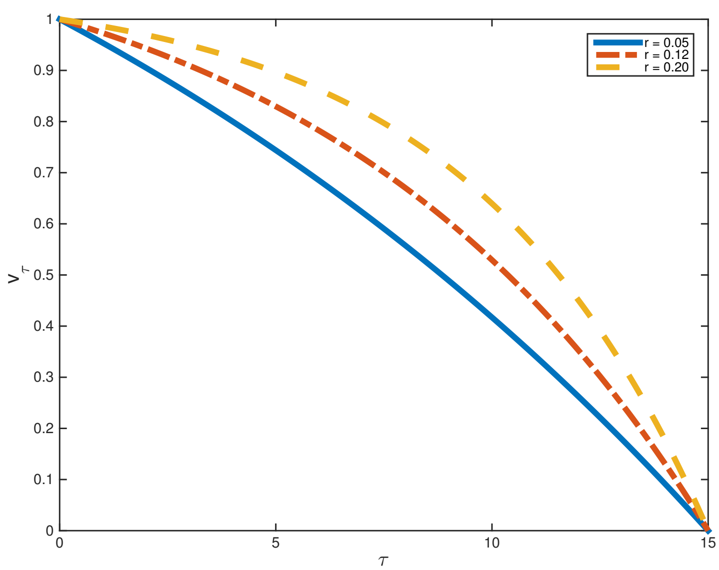

Figure 2: One-Hoss Shay Efficiency - Replacement Cost per Unit Investment
The useful life of assets is 15 years. The discount rates are 5% (solid line), 12% (dash-dotted line), and 20% (dashed line).

Figure 2 illustrates how the replacement cost of one unit of the capital good declines with its age for assets with one-hoss shay efficiency, i.e., the behavior of v_τ in (12) as a function of τ. Note that while the current capacity of such assets, x_τ, is constant over their useful life and vanishes instantaneously at T, their replacement cost declines to zero gradually. Figure 2 shows that for relatively low values of the discount rate, r, the pattern of decline is almost linear. In this case, RC_t would be close to the *net* book value of capital goods calculated under the straight-line depreciation rule.[3] Therefore, when efficiency is one-hoss shay and r is low, both RC_t and K_{t+1} can be measured as simple aggregates of the firm's investment history – the gross and net book value of the capital goods, respectively. For assets such as property, plant, and equipment, both gross and net book values are directly provided by firms in the annual reports.

3 Model Analysis

3.1 Optimal Investment Policy and Firm Valuation

We now jointly characterize the firm's optimal investment policy and its equity value on the optimal investment path. Let $V\left(\hat{Z}_t, Z_{t+1}, \epsilon_{t+1}, \Theta_{t-1}\right)$ denote the firm's cum-dividend value of equity at date t. In our model, the firm's cum-dividend value at date t depends on: (i) the current demand shift parameter $\hat{Z}_t$ because it determines the operating cash flow at date t, $R\left(\hat{Z}_t, K_t\right)$; (ii) next period's demand shift parameter $\hat{Z}_{t+1} = \epsilon_{t+1} Z_{t+1}$ since the firm makes its investment decision I_t after observing $\hat{Z}_{t+1}$; (iii) the permanent component of next period's demand, Z_{t+1}, because as will become clear later, this parameter affects the expected value of the firm's economic profits after period $t+1$; (iv) investment history Θ_{t-1} since it determines K_t and also affects future investments.

The function $V\left(\hat{Z}_t, Z_{t+1}, \epsilon_{t+1}, \Theta_{t-1}\right)$ must satisfy the following Bellman equation:

$$V\left(\hat{Z}_t, Z_{t+1}, \epsilon_{t+1}, \Theta_{t-1}\right) = \underbrace{\hat{Z}_t^{1-\alpha} K_t^{\alpha}}_{R\left(\hat{Z}_t, K_t\right)} + \max_{I_t} \left\{ \gamma \mathbb{E}_t \left[V\left(\hat{Z}_{t+1}, Z_{t+2}, \epsilon_{t+2}, \Theta_t\right)\right] - I_t \right\}. \tag{14}$$

In the proof of Proposition 1, we show that the optimal investment policy is to choose I_t so as to maximize the firm's economic profit in the following period:

$$I_t^* = \arg\max_{I} \underbrace{\hat{Z}_{t+1}^{1-\alpha} K_{t+1}^{\alpha} - cK_{t+1}}_{\pi_{t+1}}. \tag{15}$$

[3] Applying L'Hôpital's rule to (12) yields $v_\tau = 1 - \frac{\tau}{T}$ when $r \to 0$. The assumption of straight-line economic depreciation is used as a starting point in constructing an empirical measure of Tobin's Q in Lewellen and Badrinath (1997). While the two assumptions, one-hoss shay efficiency and straight-line economic depreciation, are consistent for low values of r, Figure 2 suggests that they diverge from each other as r increases. In fact, it can be verified that the assumption of straight-line economic depreciation translates into the following *linear* efficiency pattern (see, e.g., Rajan and Reichelstein 2009):

$$x_\tau = 1 - \frac{r}{1+rT}(\tau - 1).$$

The first-order condition for the optimal level of capital stock in period $t+1$ then is:

$$\alpha \hat{Z}_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} = c,$$

from which it follows that the optimal capital stock is the given by

$$K_{t+1}^{*} = M^{\frac{1}{\alpha}} \hat{Z}_{t+1}, \tag{16}$$

where the constant M is defined as $M \equiv (\alpha/c)^{\frac{\alpha}{1-\alpha}}$. Furthermore, it is straightforward to check that the maximized economic profit in period $t+1$ is:

$$\begin{aligned} \pi_{t+1}^{*} &= \frac{1-\alpha}{\alpha} c K_{t+1}^{*} \\ &= (1-\alpha) M \hat{Z}_{t+1}. \end{aligned} \tag{17}$$

Let $V^{ex}(Z_{t+1}, \epsilon_{t+1}, \Theta_t)$ denote the firm's ex-dividend value at date t:

$$V^{ex}(Z_{t+1}, \epsilon_{t+1}, \Theta_t) \equiv V\left(\hat{Z}_t, Z_{t+1}, \epsilon_{t+1}, \Theta_{t-1}\right) - R\left(\hat{Z}_t, K_t\right) + I_t.$$

Noe that since $V^{ex}(Z_{t+1}, \epsilon_{t+1}, \Theta_t)$ depends on I_t, we write the last argument of this function as Θ_t, not Θ_{t-1}. Proposition 1 characterizes the firm's equity value on the optimal investment path.

Proposition 1 *On the optimal investment path,*

$$K_{t+1}^{*} = M^{\frac{1}{\alpha}} \hat{Z}_{t+1} \tag{18}$$

and

$$I_t^{*} = K_{t+1}^{*} - K_{t,t+1}. \tag{19}$$

The firm's ex-dividend equity value at date t is given by:

$$V^{ex}(Z_{t+1}, \epsilon_{t+1}, \Theta_t) = RC_t + \gamma \underbrace{(1-\alpha) M \hat{Z}_{t+1}}_{\pi_{t+1}^{*}} + \nu(\bar{\mu}) Z_{t+1}, \tag{20}$$

where

$$\nu(\bar{\mu}) \equiv \frac{\gamma(1-\alpha) M \bar{\mu}}{1+r-\bar{\mu}}. \tag{21}$$

Equation (20) in Proposition 1 shows that the firm's equity value is comprised of the following three components: the replacement cost of assets-in-place, RC_t, the present value of next period's economic profit, $\gamma\pi^*_{t+1}$, and the present value of all future expected economic profits, $Z_{t+1}\nu(\bar{\mu})$. If the firm could rent its capacity on an as-needed basis at the cost of c per unit per period of time, then its value would be simply given by the sum of the last two terms of equation (20). The replacement cost of assets-in-place, RC_t, can be viewed as the present value of future rental costs that the firm has effectively "prepaid" by investing in long-lived assets in the past. The two shaded areas in Figure 3 illustrate the two components of the firm value: the darker shaded (orange) area represents the future economic profits whereas the lighter shaded (blue) area corresponds to the replacement cost of assets.

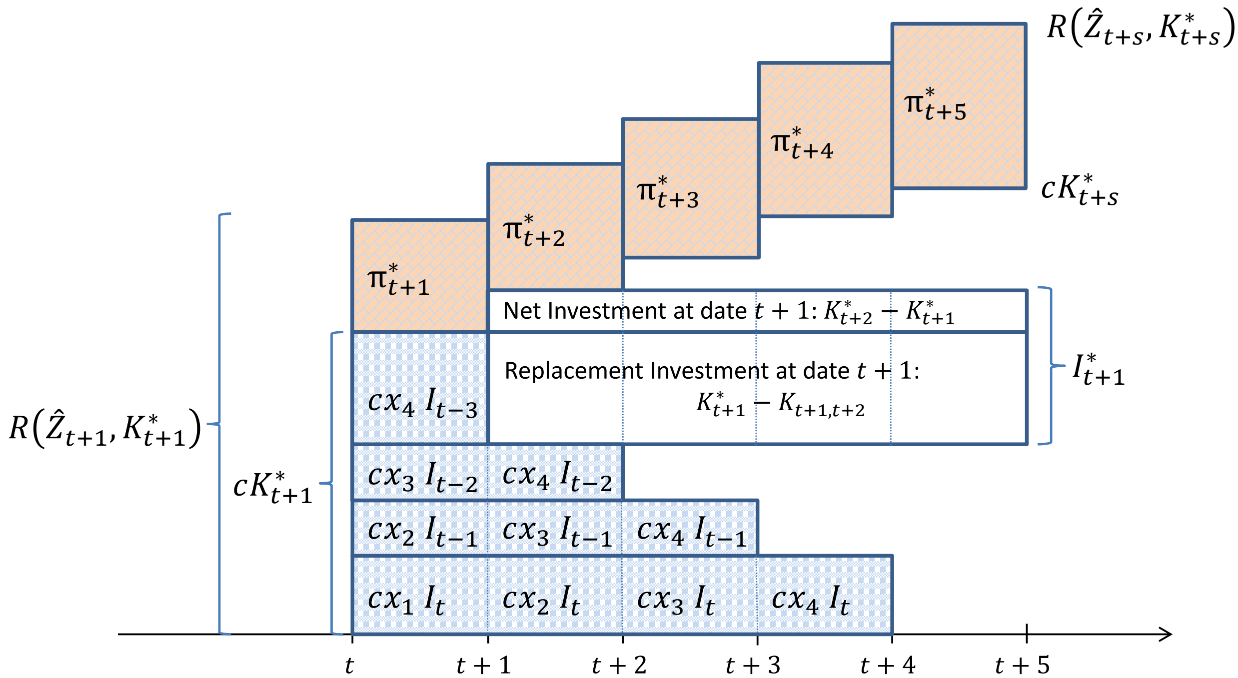

Figure 3: Optimal Investment and Valuation with Vintage Capital

Assets have a useful life of four periods. In period $t+1$, the firm's capital stock, K^*_{t+1}, consists of four vintages corresponding to investments I_{t-3} through I_t, depicted along the vertical axis. The firm generates revenues of $R(\hat{Z}_{t+1}, K^*_{t+1})$ and has a current cost of capital of cK^*_{t+1}, leaving it with optimal economic profits of π^*_{t+1}. At the end of period $t+1$, the oldest vintage, I_{t-3}, is fully retired, and the firm experiences a positive demand shock. The firm's total investment at date $t+1$ is decomposed into its replacement component, $K^*_{t+1} - K_{t+1,t+2}$, and net investment, $K^*_{t+2} - K^*_{t+1}$.

Recall that in the case of geometric efficiency, $RC_t = K^*_{t+1}$ in all periods. It then follows from equations (18) and (20) that

$$V^{ex}(Z_{t+1}, \epsilon_{t+1}, \Theta_t) = \hat{Z}_{t+1} \underbrace{\left(\text{constant} + \frac{\nu(\bar{\mu})}{\epsilon_{t+1}}\right)}_{\text{Tobin's } Q}.$$

The firm's ex-dividend value on the optimal investment path then turns out to be independent of its investment history Θ_t and the state variable effectively reduces to the vector $(Z_{t+1}, \epsilon_{t+1})$. For arbitrary efficiency profiles, however, RC_t and K^*_{t+1} will generally be two different linear aggregates of the firm's investment history up to date t. Accordingly, the firm's value will depend not only on Z_{t+1} and ϵ_{t+1} but also on RC_t. Furthermore, the firm's optimal investment policy will also be path-dependent: it follows from equations (18-19) that I^*_t is not determined solely by $\hat{Z}_{t+1}$ but also by past investments $I^*_{t-\tau}$ entering $K_{t,t+1}$. Accordingly, the problem of forecasting next period's investment becomes more complex because at each point in time one needs to take into account the full vintage composition of assets-in-place. In the next section, we decompose the firm's investment rate into its net and replacement components and examine more closely the relations between these components and several explanatory variables.

3.2 Net Investment Rate

Let i^*_{t+1} be the firm's investment rate in period $t+1$:

$$i^*_{t+1} \equiv \frac{I^*_{t+1}}{K^*_{t+1}}.$$

We now turn to the main question of the paper: how is investment rate i^*_{t+1} related to the observable information at time t? As we demonstrate below, the total investment rate in our model consists of two components both of which vary over time but for different economic reasons. Specifically, we decompose i^*_{t+1} into its *net* component, i^n_{t+1},

$$i^n_{t+1} \equiv \frac{K^*_{t+2} - K^*_{t+1}}{K^*_{t+1}},$$

and *replacement* component i^r_{t+1},

$$i^r_{t+1} \equiv \frac{K^*_{t+1} - K_{t+1,t+2}}{K^*_{t+1}}.$$

Under these definitions, $i^*_{t+1} = i^n_{t+1} + i^r_{t+1}$.

Figure 3 depicts the decomposition of investment i^*_{t+1} into its net and replacement components. Such decomposition is often considered even in models with geometric efficiency, yet in such models, the replacement investment rate is always constant and equal to δ. In our setting, the replacement investment rate varies over time and is generally path-dependent. The main determinants of i^r_{t+1} are the firm's T-period investment history and the efficiency profile of its capital goods. This component of the firm's investment rate captures the share of the capital stock in period $t+1$ going offline in that period. In contrast, the firm's net investment rate reflects the expected growth in demand for the firm's output and is independent of the current vintage composition of assets in place.

In this section, we focus on the net investment rate. It follows from Proposition 1 that this investment component has a particularly straightforward relation with future sales growth. Recall that K^*_{t+1} is proportional to $\hat{Z}_{t+1}$. Since revenue in period $t+1$ is equal to $\hat{Z}_{t+1}^{1-\alpha}\left(K^*_{t+1}\right)^{\alpha}$, it is also proportional to K^*_{t+1}. Therefore, on the optimal investment path, the net investment rate in period t is equal to the sales growth rate in period $t+1$, i.e., the net investment rate leads the sales growth rate by one period. The two rates are exactly equal in our model because we assume that the firm's manager perfectly foresees demand one period ahead. Without such perfect foresight, the net investment rate would be an imperfect predictor of future demand growth. In the empirical part of our paper, we use the predictive ability of the net investment rate for future sales growth to validate our decomposition of total investment into its net and replacement components: a decomposition is valid if it is the net investment component that predicts future sales growth.

Let us now turn to the question of predicting future net investment. Note that i^n_{t+1} can be expressed as:

$$i^n_{t+1} \equiv \frac{K^*_{t+2} - K^*_{t+1}}{K^*_{t+1}} = \frac{\hat{Z}_{t+2}}{\hat{Z}_{t+1}} - 1 = \mu_{t+2}\frac{\epsilon_{t+2}}{\epsilon_{t+1}} - 1. \tag{22}$$

Therefore, the conditional expectation at date t of i^n_{t+1} can be calculated as:

$$\mathbb{E}_t\left[i^n_{t+1}\right] = \frac{1}{\epsilon_{t+1}}\mathbb{E}_t\left[\mu_{t+2}\right] - 1 = \frac{\bar{\mu}}{\epsilon_{t+1}} - 1. \tag{23}$$

The expected value of the net investment rate is increasing in $\bar{\mu}$ and decreasing in ϵ_{t+1}. Intuitively, the firm's expected net investment rate deviates from the long-term growth rate in demand due to the transitory demand shocks. For instance, a firm can experience a period of higher-than-expected demand growth (high ϵ_{t+1}). Then, in future periods, its net investment rate should be expected to be lower since the firm has just built up extra capacity to accommodate short-term growth. While in our main model $\bar{\mu}$ is assumed to be a firm-level constant, in Appendix B we present a variant of our model in which demand follows a regime-switching process. In this extended model, the firm-level variation in $\mathbb{E}_t\left[i^n_{t+1}\right]$ arises not only due to the transitory shocks to demand but also due to the shocks to the growth rate of the permanent demand component.

One of the traditional arguments linking Tobin's Q to the expected investment rate is based on the observation that both quantities reflect the underlying growth rate in demand.

With non-geometric efficiency, there are at least two possible definitions of Tobin's Q consistent with the original idea of this measure: the market value of equity divided by the replacement cost of assets in place and the market value of equity divided by the current capital stock. We proceed with the second definition:

$$Q_t \equiv \frac{V^{ex}\left(Z_t, \mu_{t+1}, \epsilon_{t+1}, \Theta_{t+1}\right)}{K^*_{t+1}}. \tag{24}$$

Applying Proposition 1, we then arrive at the following decomposition of Tobin's Q for non-geometric efficiency patterns:

$$Q_t = \frac{RC_t}{K^*_{t+1}} + \underbrace{\gamma\left(1-\alpha\right)M + \frac{\nu\left(\bar{\mu}\right)}{\epsilon_{t+1}}}_{\text{Adjusted}\, Q}. \tag{25}$$

The first term in the right-hand side above (RC_t/K^*_{t+1}) is always equal to one in models with geometric efficiency. In general, this term is time-varying and path-dependent; we will analyze its behavior in greater detail below. It will be convenient to refer to the sum of the last two terms in (25) as the *Adjusted Q*. Note that the second term, $\gamma\left(1-\alpha\right)M$, is a time-invariant constant for a given firm. In cross-section, however, this term will reflect variations in profitability across firms since M determines the economic (monopoly) profit generated by a firm per unit of capital stock. The last term in (25) increases in $\bar{\mu}$ and decreases in the current transitory shock to demand, ϵ_{t+1}, which is directionally consistent with the relations between these variables and the expected net investment rate (see equation 23). Therefore, the traditional argument linking Q to the firm's investment rate carries over to our model with non-geometric efficiency with the qualification that it now describes the relation between Adjusted Q and the expected *net* investment rate.

Another explanatory variable commonly considered in the investment literature is the firm's scaled cash flow from operations. Operating cash flows are typically scaled in investment regressions by the same variable that is used in the denominator of Tobin's Q. As discussed above, in our model, there are two natural generalizations for this scaling variable: RC_t and K^*_{t+1}. For consistency with the definition of Tobin's Q, we scale operating cash flows by K^*_{t+1}. Using once again Proposition 1, we obtain the following expression for the scaled operating cash flow on the optimal investment path:

$$\frac{R\left(\hat{Z}_t, K^*_t\right)}{K^*_{t+1}} = \frac{M\hat{Z}_t}{M^{\frac{1}{\alpha}}\hat{Z}_{t+1}} = M^{-\frac{1-\alpha}{\alpha}}\frac{\epsilon_t}{\epsilon_{t+1}\mu_{t+1}}. \tag{26}$$

Recalling that the expected future net investment rate is proportional to $\bar{\mu}/\epsilon_{t+1}$, the expression above suggests two relations between the normalized cash flow and $\mathbb{E}_t\left[i^n_{t+1}\right]$. First, note that ϵ_{t+1} affects both variables in the same direction. A high value of scaled cash flow in period t may be due to relatively low growth in total capacity from period t to $t+1$. If this low growth is due to a transitory unfavorable demand shock (low ϵ_{t+1}), then future net investment rate should be expected to be higher since it will be calculated relative to a temporarily depressed capacity level K^*_{t+1}. On the other hand, the long-term expected growth rate $\bar{\mu}$ has opposite effects on scaled cash flow, which decreases in μ, and the expected net investment rate, which increases in μ. Scaled cash flow decreases in μ since growth in capital stock leads revenue growth in our model. The denominator of period-t scaled cash flow is usually the firm's period-$t+1$ capital stock. This denominator already captures the growth in demand between periods t and $t+1$,whereas the numerator is unaffected by this growth.

To summarize, for a given $\bar{\mu}$, our model predicts a positive relation between cash flow and expected net investment. However, this relation should be weaker, possibly turning negative, if $\bar{\mu}$ is not controlled for. We note that this cash flow effect arises in our model because of its discrete nature and the presence of transitory demand shocks. Abel and Eberly (2011) demonstrate that cash flow effect can arise in continuous-time models with reversible investment due to fluctuations in the firm's user cost of capital. In our model the user cost of capital, c, is constant and the effect is largely driven by the assumption of discrete time.

When economic depreciation is geometric, $RC_t = K^*_{t+1}$, and Tobin's Q exceeds Adjusted Q by one. However, as discussed above, the equality between the replacement cost of assets and the effective capital stock is an artifact of the geometric depreciation assumption. In general, the relation between Tobin's Q and future net investment will also depend on the relation between the ratio of RC_t to K^*_{t+1} and $\mathbb{E}_t\left[i^n_{t+1}\right]$. Recall that the ratio of RC_t to K^*_{t+1} is a function of the firm's relevant investment history and the efficiency pattern of the firm's capital goods:

$$\frac{RC_t}{K^*_{t+1}} = \frac{\sum_{\tau=0}^{T-1} v_\tau I^*_{t-\tau}}{\sum_{\tau=0}^{T-1} x_{\tau+1} I^*_{t-\tau}}. \tag{27}$$

Even in the case of one-hoss shay efficiency and constant growth in demand, the ratio above can follow different processes depending on the long-term (i.e., prior to year $t-T$) history of the firm.

Consider, for instance, the case of zero growth in output. With one-hoss shay assets, a firm can maintain constant capacity if it makes the same investment every period. Then, using equation (13), the firm's ratio of RC_t to K^*_{t+1} will be constant over time and equal to:

$$\frac{RC_t}{K^*_{t+1}} = \frac{1}{T}\sum_{\tau=0}^{T-1}\frac{1-\gamma^{T-\tau}}{1-\gamma^T} = \frac{1}{1-\gamma^T} - \frac{1}{rT}.$$

The firm can also maintain constant capacity by making a T times larger investment every T-th period and zero investments in all other periods. Then, its RC_t/K^*_{t+1} ratio will fluctuate over time from the value of 1 (right after an investment period) to $\frac{1-\gamma}{1-\gamma^T}$ right before a new investment period. Yet under our assumption, the firm's net investment rate is equal to zero in all periods under both scenarios. This example demonstrates that general claims about the relation between the RC_t/K^*_{t+1} ratio and the firm's future net investment rate are unlikely to hold for all investment paths.

To proceed further, we make two additional simplifying assumptions. First, we restrict attention to the case of one-hoss shay capital goods since this efficiency profile serves as the foundation for our empirical analysis and is also often invoked to justify the use of straight-line depreciation in practice. Second, we abstract from considering the effects of the replacement components of past investments on the RC_t/K^*_{t+1} ratio, i.e., we will consider a "new" firm that did not have any significant capital stock more than T periods ago. This assumption is descriptive of firms whose most recent T vintages are significantly larger than the older ones. Formally, consider a firm that starts operations at date 0 and builds up its capital stock for T periods. Proposition 2 characterizes how the time-τ conditional expectation of the ratio of RC_T/K^*_{T+1} depends on $\mu_{\tau+1}$.

Proposition 2 *Assume that capital goods have one-hoss shay efficiency. Then, for each* $0 \leq \tau \leq T$, $\mathbb{E}_\tau\left[\frac{RC_T}{K^*_{T+1}}\right]$ *increases in* $\mu_{\tau+1}$.

Proposition 2 demonstrates that, at least if one ignores the replacement components of the past T investments, the ratio of RC_t to K^*_{t+1} increases in the past T realized growth rates of demand. To the extent that each μ_τ is informative about the expected demand growth rate $\bar{\mu}$, Proposition 2 suggests a positive relation between RC_t/K^*_{t+1} and future net investment. The intuition behind this result is as follows. Firms that have experienced faster growth in recent periods tend to have newer assets. On a per unit of current capacity basis, a newer capital good has a higher replacement cost: the ratios v_i/x_{i+1} decrease in i. Specifically, in the one-hoss shay scenario, $x_{i+1} = 1$ for all i, and $\{v_i\}_0^{T-1}$ are declining in i to zero as depicted in Figure 2. Therefore, for firms with newer assets, the ratio of RC_t to K^*_{t+1} in equation (27) moves in the direction of v_i/x_{i+1} corresponding to lower values of i, i.e., is lower than what it would have been for older capital goods. The ratio of RC_t to K^*_{t+1} thus captures the "newness" of the firm's capital stock, which is irrelevant for future investment decisions if and only if capital goods have geometric efficiency.

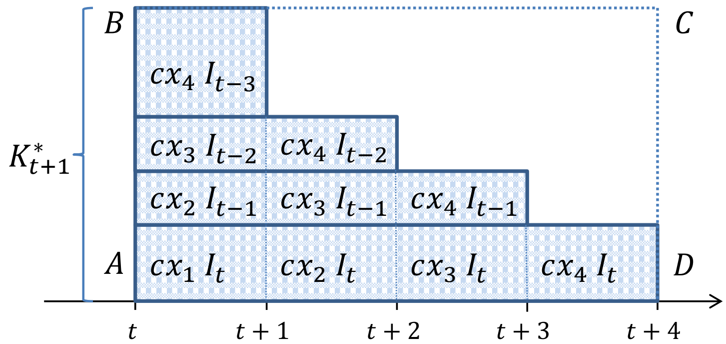

Figure 4: Ratio of RC_t to K^*_{t+1}

Assets have a useful life of four periods. In period $t+1$, the firm's capital stock, K^*_{t+1}, consists of four vintages corresponding to investments I_{t-3} through I_t, depicted along the vertical axis. At the end of period $t+1$, the oldest vintage, I_{t-3}. The shaded area represents RC_t, and the area of the rectangle $ABCD$ represents K^*_{t+1}.

In Figure 4, we reproduce the structure of capital goods from Figure 3 to illustrate the behavior of RC_t/K^*_{t+1}. The shaded area in Figure 4 can be interpreted as RC_t: according to equations (9) and (10), the replacement cost of capital goods in place at date t is equal to the present value of the future (hypothetical) rental payments that would be needed to replicate the productive capacity of these capital goods. The area of the rectangle ABCD can be interpreted as the firm's current capital stock, K^*_{t+1}, since it is equal to the replacement cost of brand new capital goods with the current capacity equal to K^*_{t+1}. For a firm that has experienced high growth in the past, the newer vintages of assets (depicted in Figure 4 at the bottom of the stack) are relatively larger, leading to a higher value of RC_t/K^*_{t+1}. If, for instance, the firm's capacity has gone from 0 to K^*_{t+1} in period t, all of the firm's capital goods are brand new and thus $RC_t/K^*_{t+1} = 1$. The special case of geometric efficiency studied extensively in the earlier literature is degenerate in the sense that $RC_t/K^*_{t+1} = 1$ regardless of the firm's investment history.

3.3 Replacement Investment Rate

Now consider the firm's replacement investment rate in period $t+1$, given by:

$$i^r_{t+1} \equiv \frac{K^*_{t+1} - K_{t+1,t+2}}{K^*_{t+1}} = 1 - \frac{x_2 I_t + \ldots + x_T I_{t-T+2}}{x_1 I_t + \ldots + x_T I_{t-T+1}}.$$

In the geometric scenario, i^r_{t+1} is constant and equal to δ. In general, the equation above demonstrates that the firm's replacement investment rate depends on its entire investment history as well as on the efficiency profile of its capital goods. Even when demand for the firm's output follows a relatively simple process, the dynamic behavior of i^r_{t+1} can be quite complicated.

Consider, for instance, two firms facing a completely stationary product market, with all $\mu_t = \epsilon_t = 1$,and using one-hoss shay assets. One firm makes a constant investment each T-th period and zero investment in periods not divisible by T. The second firm makes a constant (T times smaller) investment in each and every period. Both firms maintain constant capacity. The replacement investment rate of the first firm is equal to zero in all periods not divisible by T and equal to one in each investment period. The replacement investment rate of the second firm is always equal to $1/T$.

Note that the firm's replacement investment rate in period $t+1$ is generally not random given the information available at date t. For example, under the assumption of one-hoss shay efficiency:

$$i^r_{t+1} = \frac{I_{t-T+1}}{K^*_{t+1}},$$

i.e., it can be estimated by dividing the investment made exactly T periods ago by the current capital stock. The equation above would produce correct estimates of i^r_{t+1} for both firms described in the previous paragraph. In practice, however, this approach is not useful for at least two reasons. First, for large values of T, information about I_{t-T+1} may not be available due to data limitations or special circumstances, such as mergers and acquisitions, which we discuss in greater detail in the next section. Second, this approach relies heavily on the precision of the employed estimate of T since it is used to pinpoint the investment being replaced. In the next section, we describe a procedure for measuring replacement investment that requires only information from the firm's contemporaneous financial statements; in the current section our goal is to describe explanatory variables for i^r_{t+1} that can be constructed from the firm's one-period-lagged (i.e., period-t) financial statements.

At least two variables emerge as natural candidates for explaining the firm's replacement investment rate. First, consider the case of one-hoss shay productivity and a stationary market for the firm's output. As the two-firm example above illustrates, *any* T-periodic investment process leads to a constant amount of productive capital stock over arbitrary many periods. Consequently, any T-periodic replacement investment rate process is consistent with the assumptions of one-hoss shay efficiency and stationary product market. Yet all of such processes will share one feature in common: the firm's *average* replacement investment rate over any T subsequent periods will be equal to $1/T$. The assumption of one-hoss shay efficiency will be imposed throughout the empirical part of the paper; the assumption of a stationary product market should not be expected to hold in general but is likely to be descriptive of firms operating in low-growth environments. We expect T^{-1} to be positively associated with the firm's replacement investment.

Our second explanatory variable for i^r_{t+1} is the ratio RC_t/K^*_{t+1} introduced in the previous section. In contrast with the net investment rate, we expect RC_t/K^*_{t+1} to be *negatively* associated with i^r_{t+1}. The intuition for this prediction can be gleaned from Figure 4. If the firm has been growing its capacity at a high rate, then its more recent investments (depicted in Figure 4 at the bottom of the stack) will be relatively large. Thus only a small portion of its current capacity is generated by its oldest surviving investment I_{t-3}, which determines the replacement investment rate in period $t+1$. This intuition also holds up in the two-firm example discussed above. The firm making constant investments in all periods has a constant RC_t/K^*_{t+1} ratio as well as a constant replacement investment rate. The RC_t/K^*_{t+1} ratio of the firm that invests every T-th period will vary from the value one at the end of each investment period to the value of v_{T-1} at the end of periods preceding investment. Note that the lowest value of this ratio predicts the highest value of the replacement investment rate.

There is, in fact, a more formal theoretical argument that suggests a negative relation between RC_t/K^*_{t+1} and the firm's replacement investment for one-hoss shay capital goods. To state it, let us consider the following long-term measure of replacement investment. Let $PVRI_{t+1}$ be the present value of the replacement investments in periods $t+1$ to $t+T$. Since in period $t+\tau$, the firm replaces its investment made in period $t+\tau-T$, $PVRI_{t+1}$ is given by:

$$PVRI_{t+1} = \gamma I_{t-T+1} + \gamma^2 I_{t-T+2} + ... + \gamma^T I_t.$$

It turns out that $PVRI_{t+1}$ can be expressed as the following linear combination of K^*_{t+1} and RC_t.

Observation 1 *Assume capital goods have one-hoss shay efficiency, then:*

$$PVRI_{t+1} = K^*_{t+1} - \frac{r}{c} RC_t.$$

According to Observation 1, if instead of calculating instantaneous replacement investment we focus on the present value of the future T replacement investments, then the ratio of $PVRI_{t+1}$ to K^*_{t+1} will be decreasing in RC_t/K^*_{t+1}:

$$\frac{PVRI_{t+1}}{K^*_{t+1}} = 1 - \frac{r}{c} \frac{RC_t}{K^*_{t+1}}.$$

To summarize, we expect the ratio of RC_t to K^*_{t+1} to be positively associated with the firm's net investment rate and negatively associated with its replacement investment rate.

A natural question to ask in this situation is whether the firm's total investment is increasing in RC_t/K^*_{t+1} together with its net investment rate. As one might expect, in general, there may be different possibilities. However we can, in fact, demonstrate that RC_t/K^*_{t+1} is positively associated with the total investment rate at least for natural "constant growth" investment paths. To see this, consider a firm that grows its investments in one-hoss shay capital goods by factor $\bar{\mu}$ in every period, so that

$$I_{t+1} = \bar{\mu} I_t \tag{28}$$

for all t.

This firm's capital stock grows by factor $\bar{\mu}$ each period, and its net investment rate is always equal to $\bar{\mu}-1$. To calculate its replacement investment rate, note that in period $t+1$, it replaces investment I_{t-T+1}, so

$$i^r_{t+1} = \frac{I_{t-T+1}}{I_{t-T+1} + ... + I_t} = \frac{1}{1 + \bar{\mu} + ... + \bar{\mu}^{T-1}} = \frac{\bar{\mu} - 1}{\bar{\mu}^T - 1}. \tag{29}$$

For future reference, observe that while the net investment rate increases in $\bar{\mu}$, the replacement investment rate in the equation above declines in $\bar{\mu}$. The firm's total investment rate is given by:

$$\underbrace{\frac{\bar{\mu} - 1}{\bar{\mu}^T - 1}}_{i^r_{t+1}} + \underbrace{\bar{\mu} - 1}_{i^n_{t+1}} = \frac{\bar{\mu}^T (\bar{\mu} - 1)}{\bar{\mu}^T - 1}.$$

It can be verified that the expression above increases in $\bar{\mu}$ for both growing and shrinking firms, $\bar{\mu} \in (0, \infty)$.

Finally, we can also calculate the RC_t to K^*_{t+1} ratio for this scenario. After some algebra, this ratio reduces to:

$$\frac{RC_t}{K^*_{t+1}} = \frac{\bar{\mu}^T (1 - \gamma\bar{\mu}) + \gamma (\bar{\mu} - 1) (\gamma\bar{\mu})^T + \gamma - 1}{(\bar{\mu}^T - 1) (1 - \gamma^T) (1 - \gamma\bar{\mu})}. \tag{30}$$

In Appendix A, we show that the ratio above increases in $\bar{\mu}$. Hence this ratio is positively associated with the demand growth rate, which is consistent with the behavior of the net investment rate and opposite to that of the replacement investment rate.

Observation 2 *Assume capital goods have one-hoss shay efficiency, and the firm's investment growth rate, $\bar{\mu}$, has been constant over the last T periods. Then, RC_t/K^*_{t+1} is increasing in $\bar{\mu}$.*

To summarize, we expect the replacement investment rate to be increasing in T^{-1} and declining in RC_t/K^*_{t+1}. As illustrated by the above discussion, we further expect the positive relation between the net investment rate and RC_t/K^*_{t+1} to dominate the negative relation between replacement investment and RC_t/K^*_{t+1}, leading to an overall positive relation between the total investment rate and RC_t/K^*_{t+1}.

4 Data and Variable Construction

4.1 Data Selection

Our sample consists of firms in the Compustat North America annual files from 1971 to 2017. We start our sample in 1971 because several of our variables require information from the statement of cash flows, which is available in Compustat from 1971. We employ a screening procedure similar to that in Hennessy et al. (2007), Erickson and Whited (2012), and Peters and Taylor (2017). First, we remove firms with SIC codes between 4900 and 4999 (regulated utilities), between 6000 and 6999 (financial services), or greater than 9000 (public services). The excluded sectors are arguably subject to significant accounting and economic conditions that are outside the scope of our model, such as extensive regulatory and government oversight.

We drop firm-year observations in which any of the required data values are missing and apply the following four additional screens. First, we require that a firm's net Property, Plant, and Equipment (Compustat item PPENT) measured in real 2012 dollars be not less than 5 million. To calculate the real net PP&E, we use the Gross Private Domestic Investment price deflator from NIPA Table 1.1.9 (item 7). Second, we drop firm-years in which the absolute value of pre-tax writedowns (item WDP) exceeds 10% of the beginning-of-period gross PP&E (item PPEGT). Significant write-downs can indicate that the firm's capital stock is impaired or obsolete, yet they can also be related to other accounts on the firm's balance sheet such as inventory, accounts receivable, or goodwill. When write-downs are very large relative to the firm's gross PP&E, they are particularly likely to be at least partially driven by these items. We also drop observations in which our estimate of the useful life of the firm's capital goods (presented below) is less than zero. Finally, we only keep firms that have at least five years of usable data. All variables that are defined as ratios are winsorized at the 0.1% level. Our ultimate sample consists of 124,728 firm-years with 8,255 unique firms.

4.2 Capital Stock and Replacement Cost of Assets

In this section, we describe the main empirical measures employed in our study. In defining these measures, we seek to make them easily constructable from the firms' most recent financial statements and not reliant on the availability of long investment histories. Such measures are useful for two reasons. First, sufficiently long investment histories are only available for a relatively small share of firm-years. Second, even when such histories are available, the performance of perpetual inventory algorithms for estimating the replacement cost of assets has been questioned in the earlier literature (see, e.g., Erickson and Whited 2006).

According to our model, there are two important economic quantities that describe the composition of the firm's capital goods: its current capital stock, K_{t+1}, and the replacement cost of its assets in place, RC_t. The importance of the capital stock comes from the fact that it serves as a natural deflator in the calculation of Tobin's Q and other variables that need to be scaled by some measure of size of the firm's productive capacity. Under the assumption of one-hoss shay productivity that we impose throughout this section, K_{t+1} is simply equal to the sum of all investment expenditures that survive up to date t:

$$K_{t+1} = I_t + ... + I_{t-T+1}.$$

Therefore, a straightforward measure for K_{t+1} in our model is the firm's *gross* PP&E (item PPEGT) at date t. While the use of PPEGT as a scaling variable is relatively common in the investment literature (see, e.g., Fazzari et al. 1988, Erickson and Whited 2012, Peters and Taylor 2017, and Lin et al. 2018), our model appears to provide the first theoretical justification for this measure.

Now let us turn to the replacement cost of assets in place, given by equation (13) in the one-hoss shay setting. Even under all the simplifying assumptions imposed so far, precise estimation of this quantity requires measures of r, T, and the latest T investments. Note, however, that according to Figure 2, the *net* book value of a capital good, calculated under the straight-line depreciation rule, approximates its replacement cost reasonably well, in particular for low values of r.[4] We will therefore use the net book value of PP&E (item PPENT) as our proxy for RC_t. Accordingly, our measure of RC_t/K_{t+1} is the ratio of net-to-gross PP&E at date t.

[4]See McNichols et al. (2009) for numerical estimates of this bias. When $r = 0$, RC_t is in fact equal to PPENT at date t.

4.3 Cash and Non-Cash Investment

The most prevalent measure of a firm's total investment used in the earlier literature is the firm's capital expenditures as reported in its statement of cash flows (item CAPX). Another commonly used measure is the firm's net investment cash flow, which is also reported in the cash flow statement (item IVNCF). Both of these measures, however, exclude significant components of investment in capital stock and only capture the cash component of the firm's investment.

Firms often acquire capital goods without immediately paying their full value in cash. Particularly common examples of such transactions include leasing, purchases of capital goods in exchange for a firm's stock or liability, and capacity expansion through mergers and acquisitions. Consider, for instance, capital leases. Both accounting standard-setters and practitioners have long recognized that such transactions are essentially equivalent to a combination of a debt issuance (for the present value of future lease payments) and an asset acquisition (for the same amount). Property, plant, and equipment acquired on capital leases have lone long been included in firms' PP&E accounts. In fact, under the new accounting standards for leases (IFRS 16 and ASC 842), the capitalization of lease obligations and the corresponding right-of-use assets is extended to most leases with a duration of more than 12 months. Yet the acquisition of capital goods with leases is never reflected in the firm's capital expenditures or its investment cash flow. This is because the interest component of lease payments is usually included in the firm's cash flow from operations, and the principal repayment component is a part of the cash flow from financing activities.[5] We expect this discrepancy between the balance sheet PP&E accounts and capital expenditures reported in cash flow statements to become even more pronounced under the new accounting standards for leases. For example, ASC 842 is expected to add \$2 trillion to balance sheets of S&P 500 firms (see, e.g., "Transforming The Balance Sheet: Navigating New Lease Standards For Success," *Forbes*, May 1, 2019).

A more comprehensive approach to measuring total investment was suggested by Lewellen and Badrinath (1998) and involves comparing the PP&E values in the firm's two successive balance sheets. Both at the beginning and at the end of each period, the firm's balance sheet reflects all PP&E that is recognized by accountants as of the current measurement date, regardless of how the equipment was procured and paid for. The firm's total investment in

[5]Somewhat surprisingly, Compustat documentation states that item CAPX includes "expenditures for capital leases." Firms are not required to disclose the present value of new lease obligations that they had entered in during the latest accounting period. Some firms provide this information voluntarily, but at least in cases that we were able to identify, those amounts were not included in item CAPX. We elaborate on this point below in our discussion of Amazon disclosures.

period $t+1$ can then be measured according to the following relation:

$$\text{Total Investment}_{t+1} = PPENT_{t+1} - PPENT_t + WDP_{t+1} + DPC_{t+1}, \tag{31}$$

where $PPENT_t$ is the net book value of PP&E at date t, WDP_{t+1} is the pre-tax write-down in period $t+1$, and DPC_{t+1} is the firm's depreciation expense in period $t+1$ as reported on its cash flow statement.[6] Intuitively, the firm's investment in period $t+1$ should explain the change in the PP&E balance from the beginning to the end of the accounting period. We know, however, that in the absence of new investment, the balance of PP&E would have been reduced by the depreciation expense and write-down of period $t+1$. Therefore, we define the firm's total investment as the change in PP&E unexplained by depreciation and write-downs.

To illustrate the more comprehensive nature of the investment measure in (31), consider the financial disclosures of Amazon.com, Inc. for financial year 2014. The amount of capital expenditures reported in its statement of cash flows is $4,893 million, and this is exactly the value reported in Compustat item CAPX. At the same time, Amazon recognized $4,746 million in depreciation expense in 2014, and its net PP&E increased by $6,018 million during the year. Clearly, this increase in net PP&E cannot be explained by CAPX alone. Our measure of total investment for Amazon in 2014 is equal to $10,764 million. Conveniently for our purposes, Amazon also voluntarily discloses its own measure of free cash flow used by the management and reconciles this measure with the numbers reported in its cash flow statement. This reconciliation makes it clear that in addition to $4,893 million in CAPX, Amazon also acquired $4,008 million in PP&E under capital leases in 2014.[7]

The importance of leasing and non-cash asset acquisitions has been long recognized by both academics and practitioners. For instance, Eisfeldt and Rampini (2009) show that leasing is comparable in importance to long-term debt for large firms and is perhaps the most important source of external finance for small firms. They also find it surprising that "given its quantitative importance, leasing has been essentially ignored in the theoretical

[6]Note that it is important to take the value of the firm's depreciation expense from its cash flow statement rather than its income statement. The reason for this is that in the income statement, the depreciation expense related to the manufacturing equipment must be reported as a part of the cost of goods sold (COGS) and not as a separate expense below the gross margin line. If an income statement of a manufacturing firm includes a depreciation expense outside of COGS, then such expense is related to the administrative facilities of the firm and not its manufacturing property or equipment. The depreciation expense reported on the cash flow statement includes in most cases both the manufacturing and administrative components.

[7]See subsection "Non-GAAP Financial Measures" in Item 7 (Management Discussion and Analysis) of Amazon's 2014 10-K report. The report states that in the calculation of the free cash flow measure, "property and equipment acquired under capital leases is reflected as if these assets had been acquired with cash." In Supplemental Cash Flow Information, Amazon further discloses that it acquired $920 million of PP&E under build-to-suit leases.

and empirical literature on investment in both finance and macroeconomics." Both IASB and FASB have long tried to achieve an equivalence in presentation between long-term leases and levered asset acquisitions; they have recently passed standards (IFRS 16 and ASC 842, respectively) that call for capitalization of not only capital but also operating leases. As a consequence of these standards, the investment measure in equation (31) will become even more comprehensive in the future.

To relate our results to the earlier literature that has largely focused on CAPX, we break down total investment into its cash and non-cash components. We define Cash Investment in period $t + 1$ as:

$$\text{Cash Investment}_{t+1} = CAPX_{t+1} - SPPE_{t+1}, \tag{32}$$

where $SPPE_{t+1}$ denotes cash proceeds from sales of PP&E as reported on the firm's cash flow statement (item SPPE). Then, Non-Cash Investment in period $t + 1$ can be measured as:

$$\text{Non-Cash Investment}_{t+1} = \text{Total Investment}_{t+1} - \text{Cash Investment}_{t+1}. \tag{33}$$

We acknowledge that these measures are imperfect for at least two reasons. First, some part of what we label "Non-Cash Investment" may actually be paid for in cash. This can happen if a firm obtains new capital stock as a part of a cash acquisition of another company. The newly acquired PP&E will be reflected in the ending balance of the combined PP&E account, but the corresponding share of the cash expenditure will not be included in CAPX. Second, when PP&E is sold, an issue arises with the realized gains or losses on such sales. The firm's total investment should perhaps be unaffected by such gains or losses. A gain, for instance, might indicate that a firm's capital good was more valuable than expected (which can be treated as a positive investment), but, since this capital good is now sold, the firm's disinvestment increases by the same amount. If a firm makes significant gains on sold assets, however, our measure of Cash Investment will be biased downwards and Non-Cash Investment upwards.[8]

4.4 Net and Replacement Investment

Our model suggests another economically meaningful decomposition of the firm's total investment: into its net and replacement components. Since we measure K_{t+1} as gross PP&E

[8]A potential solution here would be to add gains on sales of PP&E to our definition of Cash Investment. An issue with this approach is that the corresponding Compustat item (SPPIV) includes not only gains/losses on sale of PP&E but also gains/losses on sale of other long- and short-term investments (such as minority interests in other companies).

at date t, we have a correspondingly simple expression for Net Investment in period $t+1$:

$$\text{Net Investment}_{t+1} = PPEGT_{t+1} - PPEGT_t. \tag{34}$$

Consequently, Replacement Investment in period $t+1$ is estimated as:

$$\text{Replacement Investment}_{t+1} = \text{Total Investment}_{t+1} - \text{Net Investment}_{t+1}. \tag{35}$$

Following the exposition of our model, we scale all measures of investment in period $t+1$ by our measure of capital stock in that period. For instance, our empirical measure of the total investment rate in period $t+1$ is given by:

$$i_{t+1} \equiv \frac{\text{Total Investment}_{t+1}}{PPEGT_t},$$

where Total Investment_{t+1} is measured according to (31). As in the theory part of the paper, i^n_{t+1} and i^r_{t+1} will denote our measures of the net and replacement investment rates, respectively. Let i^c_{t+1} and i^{nc}_{t+1} denote the cash and non-cash investment rates. All these rates are scaled by our proxy for K^*_{t+1}, i.e., $PPEGT_t$.

While our empirical measures for net and replacement investment require little data and are easy to construct, they have several important limitations. For instance, our result equating the firm's capital stock to its gross PP&E relies on the assumption that the price of new capital goods stays constant over time. In inflationary environments, gross PP&E will understate the firm's capital stock, which generally needs to be adjusted for inflation. On the other hand, new vintages of capital goods can become more productive over time due to the technological progress. In that case, the firm's gross PP&E can overstate its capital stock since older asset vintages owned by the firm are not as valuable as an equivalent dollar amount of capital goods of the newest vintage.

The accounting system introduces further biases in our measure of capital stock. First, while as discussed above, PP&E accounts in our sample include assets on capital leases, they still do not include assets on operating leases. This leads to a downward bias in our measure of K_{t+1}. Conversely, accountants often underestimate the useful life of capital goods, in which case some capital that is still providing useful capacity may not be reflected in the PP&E account. The overall effect of these biases on net investment is even more ambiguous since it is measured as the *change* in gross PP&E. Conceivably, these biases can also affect the breakdown between net and replacement investment in non-trivial ways. For instance, consider a firm that invests \$$I$ in a plant, uses it for 25 years, and, in year 25, replaces it with a new one. Assume further that accountants estimate the useful life of the firm's plants to be only 20 years. Then, our measure of replacement investment will spike from zero to I in year 20, i.e., five years ahead of the actual replacement. The net investment rate will drop to $-I$ in year 20 year, thus offsetting the spike in replacement investment, and then jump to I when the actual replacement takes place. Economically, however, the firm's net investment remains zero in all periods in this example and replacement investment equals I every 25th year. Despite of these potential biases, we show below that our measures of both net and replacement investment have empirical properties consistent with the theoretical predictions.

4.5 Other Variables

In our model, the useful life of capital goods, T, was assumed to be constant over time. In the empirical section, we measure T for each firm-year as the rounded ratio of the average of the beginning and ending balances of gross PP&E to the depreciation expense for that year:

$$T_t \equiv \left\| \frac{PPEGT_t + PPEGT_{t-1}}{2 \cdot DPC_t} \right\|. \tag{36}$$

The measure in (36) is often used by financial analysts to estimate the average useful life of a firm's PP&E. It is justified by the observation that firms overwhelmingly use the straight-line depreciation rule to account for their fixed assets, and under this rule, the ratio of gross PP&E to the depreciation expense should be roughly equal to the useful life assumed by the accountants.

While equation (36) provides a reasonable estimate of T in most cases, there are also situations in which this estimate is significantly biased upwards. For example, for a firm building its PP&E but not yet using it, the amount of depreciation recognized in a construction year can be close to zero, while the gross PP&E balance already reflects the amount of investment incurred to date. To mitigate the impact of very high estimates of T, we winsorize our measure at 25 years. In addition, we provide robustness checks based on the subsample of firms for which the estimated value of T is strictly below 25 years. Figure 5 graphs the distribution of our estimates of T for all firm-years in the sample, as well as for firms in the Manufacturing and HiTech industries. The mean (median) estimated value of T for all firm years is 12.6 (12) years, whereas the corresponding numbers for the HiTech industry are 8.4 (8) years, and for the Manufacturing industry – 14.7 (14) years. Approximately 5.7% of firm-years in our sample have an estimated value of T equal to 25 years.

Following earlier literature, we define the market value of a firm as the sum of the book value of its long-term debt, DLTT, the book value of debt in current liabilities, DLC, and the

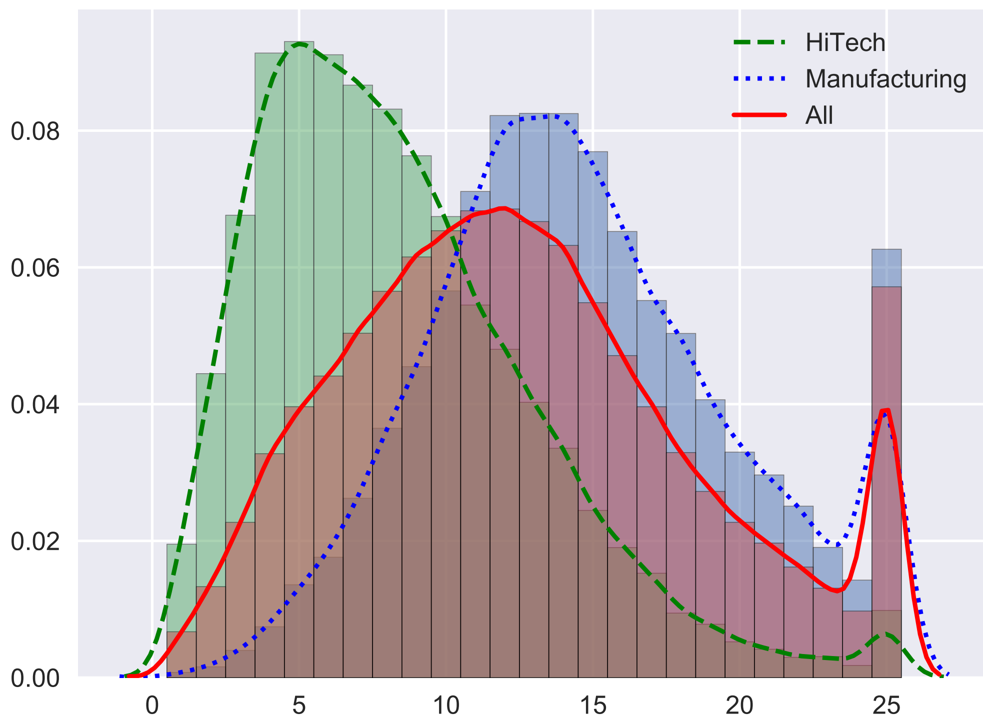

Figure 5: Distribution of Useful Life Estimates

The useful life of capital goods, T, is estimated as the minimum between the value of equation (36) and 25 years. The figure shows the probability distribution of estimated T for all firms in our sample (solid line), and firms in the HiTech (dashed line) and Manufacturing (dotted line) industries. Each bar shows the percentage of the sample with the corresponding T.

product of the annual closing price of equity, PRCC_F, and the number of common shares outstanding, CSHO. Tobin's Q is then calculated as the ratio of the firm's market value to its capital stock:

$$Q_t = \frac{DLTT_t + DLC_t + PRCC\_F_t \cdot CSHO_t}{PPEGT_t}. \tag{37}$$

Cash flow is measured as the sum of the following two items from the firm's cash statement: income before extraordinary items (IBC) and depreciation expense (DPC). As with other variables, we scale cash flow by PPEGT.

5 Empirical Analysis

5.1 Descriptive Statistics

Table 1 reports summary statistics for our main investment variables. The first row of Panel A reports statistics for the total investment rate; this rate is then decomposed into the net and replacement components in rows two and three, and, alternatively, into cash and non-cash components in rows four and five. Prior literature has largely focused on the cash component of investment, i^c_{t+1}. While Table 1 indicates that i^c_{t+1} is a large component of i_{t+1}, the mean non-cash investment rate still accounts for approximately one quarter of the average total investment rate. The average net investment rate is about twice the magnitude of the replacement investment rate. Therefore, the net investment rate appears to be more volatile than any other investment components regardless of the total variance decomposition. In terms of volatility, i^r_{t+1} is the least volatile investment component, whereas i^n_{t+1} is the most volatile. Notably, the replacement investment rate also has the lowest percentages of variance attributable to within-firm, 64.5%, and within-industry variation, 96.1%. For the net investment rate, both of these percentages are the highest at 91.3% and 99.2%, respectively.

Cash investment rate is the most persistent component: its AR(1) coefficient is more than four times that of non-cash investment when estimated using either Han and Phillips (2010) or Arellano-Bond (1991) procedure. Consistent with conventional wisdom, Arellano-Bond (1991) procedure leads to lower persistence parameters in most cases, which is arguably due to the weak instruments problem. There are at least two reasons for the low persistence of non-cash investment. First, firms often resort to non-cash investment when the amount to be invested is particularly large, and such large investments are relatively transitory. Second, our measure of non-cash investment includes capital stock acquired through mergers and acquisitions, i.e., events that also have relatively low persistence. Column "Large i_{t+1}" presents some additional evidence on these observations. In this column, we report the mean

Table 1: Summary Statistics: Investment Variables

The data is obtained from the Annual Compustat Files from 1971 to 2017. Total investment rate, i_{t+1}, is the ratio of Total Investment$_{t+1}$ defined in equation (31) to PPEGT at date t. Net investment rate, i^n_{t+1}, is the ratio of Net Investment$_{t+1}$ (equation 34) to PPEGT at date t. Replacement investment rate, i^r_{t+1}, is the ratio of Replacement Investment$_{t+1}$ (equation 35) to PPEGT at date t. Cash investment rate, i^c_{t+1}, is the ratio of Cash Investment$_{t+1}$ (equation 32) to PPEGT at date t. Non-Cash investment rate, i^{nc}_{t+1}, is the ratio of Non-Cash Investment$_{t+1}$ (equation 33) to PPEGT at date t. All investment rates are winsorized at the 0.1% level. In Panel A, column "Large i_{t+1}" presents the mean values of different investment rate in the subsample of firm-years for which i_{t+1} exceeds three times its unconditional mean of 0.198. Columns "Within-Variation %" report the share of the total variance that remains unexplained after accounting for the variation in the firm-level and industry-level means. Industry definitions are based on the Fama-French 10 industry portfolios. Column titled "AR(1)" report Han and Phillips (2010) dynamic panel estimates of the first-order autoregressive coefficients (subcolumn "HP") and the corresponding Arellano-Bond (1991) estimates (subcolumn "AB"). Panel B reports correlations among different investment measures. Red-colored elements in the lower triangle report Pearson correlations calculated from the full sample. Blue-colored elements in the upper triangle reported firm-level time-series correlations averaged across firms. The sample consists of 124,728 firm-years with 8,255 unique firms for all columns except "Large i_{t+1}". The sample size for the latter column is 7,533 firm-years with 3,786 unique firms. Superscripts ‡, †, and * in the upper triangle of Panel B indicate whether the average correlation coefficient is statistically different from zero at the 1%, 5%, and 10% levels, respectively.

Panel A: Summary Statistics

	Mean	Std. Dev.	25%	50%	75%	Large i_{t+1}	Within-Variation % Firm	Within-Variation % Industry	AR(1) HP	AR(1) AB
i_{t+1}	0.198	0.339	0.060	0.124	0.236	1.131	0.844	0.980	0.272	0.224
i^n_{t+1}	0.134	0.329	0.020	0.081	0.181	0.937	0.913	0.992	0.219	0.194
i^r_{t+1}	0.063	0.135	0.012	0.033	0.076	0.185	0.645	0.961	0.154	0.173
i^c_{t+1}	0.147	0.193	0.056	0.101	0.176	0.565	0.818	0.978	0.546	0.367
i^{nc}_{t+1}	0.050	0.239	-0.003	0.007	0.049	0.556	0.851	0.990	0.119	0.085

Panel B: Correlation Matrix

	i_{t+1}	i^n_{t+1}	i^r_{t+1}	i^c_{t+1}	i^{nc}_{t+1}
i_{t+1}	1	0.91‡	-0.09	0.75‡	0.65*
i^n_{t+1}	0.91	1	-0.37	0.72‡	0.58
i^r_{t+1}	0.25	-0.17	1	-0.09	-0.05
i^c_{t+1}	0.71	0.71	0.04	1	0.17
i^{nc}_{t+1}	0.81	0.68	0.32	0.18	1

values of the different investment rates in the subsample with unusually large total investment rate, defined as at least three times its unconditional mean. According to this column, such investment spikes are primarily driven by jumps in the net investment rate (which explain, on average, 86% of the total spike) and by increases in the non-cash investment rate. While the non-cash investment rate is ordinarily responsible for 25% of total investment, increases in this rate explain 54% of the extreme investment spikes.

Panel B of Table 1 documents correlations among different investment components. The bottom triangle presents Pearson correlations in the full sample, while the upper triangle reports firm-level time-series correlations averaged across firms. One notable finding present in both triangles is the negative correlation between i^n_{t+1} and i^r_{t+1}. According to our model, one can expect this correlation to be negative since the net and replacement investment rates have opposite relations with firm growth. When a firm is growing, its net investment is high, whereas the replacement investment rate is low since an investment made T periods ago contributes a relatively smaller share of the current capital stock. Several correlations have opposite signs in the upper and lower triangles of Panel B. For instance, the correlation between total and replacement investment rates is positive when estimates in the whole sample, 0.25, but it is, on average, negative when estimated at the firm-level, -0.09. We note, however, that due to a significant variation in firm-level estimates of correlation coefficients, all negative estimates in the upper triangle of Panel B are not statistically different from zero.

Table 2 presents summary statistics for other variables used in our empirical analysis. The main variable that we use to validate our measure of capital stock is future sales growth - the ratio of sales in period $t+1$ to sales revenue in period t minus one. According to the model, this variable should be directly related to the growth in capital stock since revenues in period $t+1$ are proportional to K^*_{t+1} on the optimal investment path. Table 2 shows that the mean value of the sales growth rate is 14.4%, and 92% of its variance is due to the within-firm component. These numbers are comparable to the ones reported in Panel A of Table 1 for the net investment rate. However, sales growth appears to be significantly more volatile than i^n_{t+1}. This indicates that our assumption that managers have a perfect foresight of the transitory shocks to demand (ϵ_{t+1}) is not descriptive. If this assumption is relaxed to accommodate transitory shocks that are not perfectly observable at investment time, then it can be verified that demand growth will in fact be more volatile than net investment. Importantly, the mean sales growth rate suggests that CAPX is indeed an incomplete measure of total investment since to sustain a sales growth rate of 14.4% with an investment rate of 14.7%, the replacement investment rate would have to be only 0.3%, which is an improbably low long-run value.

Lastly, we note that the estimates of the average sales growth rate and the net investment rate are relatively high in our sample because we winsorize all ratios only at the 0.1% level. Such small level of winsorization preserves some of the extreme growth rates: for instance, the maximum sales growth in our sample is 1311%, and the maximum net investment rate is 488%. Excluding the top 1% of observations for each one of these variables leads to the means of 10.7% for the sales growth rate and 11.2% for the net investment rate. In calibrating our model below, we consider 10.7% as an alternative value for the sales growth rate.

Table 2: Summary Statistics: Explanatory Variables

The data is obtained from the Annual Compustat Files from 1971 to 2017. SalesGrowth_{t+1} is the ratio of revenues in period $t+1$ (Compustat item SALE) to revenues in period t minus one. Q_t is measured according to equation (37). CashFlow_t is the sum of Compustat items DPC and IBC scaled by PPEGT. Our measure for RC_t/K_{t+1} is the ratio of PPENT to PPEGT at date t. Useful life T is defined in equation (36), and T^{-1} is the inverse of this variable. Age is the number of years between the current one (Compustat item FYEAR) and the year of the IPO as reported in Compustat item IPODATE. Size is the natural logarithm of the market value of the firm as defined in the numerator of our Tobin's Q measure in equation (37). All variables that are defined as ratios (i.e., all variables other than Size and Age) are winsorized at the 0.1% level. Column "Within-Var(%)" reports the percentage of the total variance that remains unexplained after accounting for the variation in the firm-level unconditional means. For variables other than Age, the sample consists of 124,728 firm-years with 8,255 unique firms. Using Age reduces the sample size to 42,301 firm-years with 3,364 unique firms.

Variable	Mean	SD	25%	50%	75%	Within-Var
SalesGrowth_{t+1}	0.144	0.555	-0.012	0.086	0.204	0.920
Q_t	4.454	9.462	1.044	1.904	4.126	0.473
CashFlow_t	0.187	0.488	0.081	0.156	0.278	0.605
RC_t/K_{t+1}	0.569	0.164	0.455	0.568	0.682	0.463
T	12.580	5.891	8.000	12.000	16.000	0.280
T^{-1}	0.112	0.105	0.062	0.083	0.125	0.356
Age	8.887	9.283	3.000	7.000	13.000	0.492
Size	5.875	2.155	4.233	5.712	7.324	0.198

5.2 Determinants of Future Sales Growth

One of the main implications of our model is that a firm's gross PP&E at date t can be used as a proxy for the firm's capital stock in period $t+1$. If a measure is a good proxy for the firm's capital stock, then, according to model, its growth rate should be directly related to the future sales growth. Therefore, studying the determinants of future sales growth is a natural way to validate our model-implied measures of capital stock and net investment. Table 3 reports regression results of sales growth in period $t+1$ onto different sets of dependent variables, all of which are measured as of date t to avoid a potential look-ahead bias.

Our model suggests three variables that should be associated with a higher future sales growth – the firm's net investment rate, Q (by Proposition 1), and RC_t/K^*_{t+1} (by Proposition 2). Columns (1), (3) and (4) of Table 3 show that all of these variables are indeed positively associated with SalesGrowth_{t+1}, and the lagged net investment rate has the best explanatory power for future sales growth in specifications both with (Panel A) and without (Panel B) firm fixed effects. Columns (2) and (5) demonstrate that our measure of the net investment rate captures almost entirely the growth-related component of the total investment rate: i^r_t does not have a significant explanatory power for SalesGrowth_{t+1}. In columns (6)-(10), we explore whether the coefficient on the net investment rate retains its economic and statistical significance once we control for additional variables that can explain SalesGrowth_{t+1}. There is only a mild decline in this coefficient as more explanatory variables are included, and it remains statistically significant in all specifications.

Table 3: Future Sales Growth

The dependent variable is SalesGrowth_{t+1}. See captions of Tables 1 and 2 for data and variable definitions. Standard errors used to construct the t-statistics (reported in parentheses) are two-way clustered by year and industry (four-digit SIC code). All regressions include year fixed effects; Panel A (B) reports results with (without) firm-level fixed effects. Superscripts ‡, †, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Panel A: With Firm FE										
i_t^n	$0.104^{\ddagger}$				$0.104^{\ddagger}$	$0.100^{\ddagger}$	$0.094^{\ddagger}$	$0.094^{\ddagger}$	$0.090^{\ddagger}$	$0.091^{\ddagger}$
	(8.882)				(8.937)	(8.823)	(8.085)	(8.108)	(5.953)	(6.029)
i_t^r		0.058			0.024					
		(1.221)			(0.513)					
Q_t			$0.009^{\ddagger}$			$0.008^{\ddagger}$	$0.007^{\ddagger}$	$0.009^{\ddagger}$	$0.009^{\ddagger}$	$0.009^{\ddagger}$
			(8.108)			(7.846)	(7.521)	(8.893)	(9.049)	(9.501)
$\frac{RC_t}{K_{t+1}}$				$0.516^{\ddagger}$			$0.297^{\ddagger}$	$0.305^{\ddagger}$	$0.297^{\ddagger}$	$0.323^{\ddagger}$
				(9.53)			(5.982)	(6.164)	(6.545)	(6.826)
$\frac{CF_t}{K_{t+1}}$								$-0.090^{\ddagger}$	$-0.089^{\ddagger}$	$-0.088^{\ddagger}$
								(-3.688)	(-3.686)	(-3.677)
i_t^c									0.015	0.015
									(0.613)	(0.616)
T^{-1}										$-0.477^{\ddagger}$
										(-5.896)
Adj. R^2	0.131	0.100	0.111	0.108	0.131	0.139	0.142	0.146	0.146	0.148
Adj. Within-R^2	0.035	0.000	0.013	0.012	0.035	0.045	0.050	0.054	0.054	0.057
Panel B: Without Firm FE										
i_t^n	$0.127^{\ddagger}$				$0.127^{\ddagger}$	$0.121^{\ddagger}$	$0.109^{\ddagger}$	$0.108^{\ddagger}$	$0.092^{\ddagger}$	$0.092^{\ddagger}$
	(11.425)				(11.503)	(11.026)	(9.888)	(9.747)	(6.366)	(6.367)
i_t^r		0.053			0.012					
		(1.128)			(0.300)					
Q_t			$0.008^{\ddagger}$			$0.007^{\ddagger}$	$0.007^{\ddagger}$	$0.008^{\ddagger}$	$0.008^{\ddagger}$	$0.009^{\ddagger}$
			(7.600)			(7.145)	(7.493)	(9.803)	(9.264)	(9.830)
$\frac{RC_t}{K_{t+1}}$				$0.528^{\ddagger}$			$0.374^{\ddagger}$	$0.367^{\ddagger}$	$0.349^{\ddagger}$	$0.341^{\ddagger}$
				(8.945)			(7.136)	(7.124)	(7.536)	(7.603)
$\frac{CF_t}{K_{t+1}}$								$-0.117^{\ddagger}$	$-0.116^{\ddagger}$	$-0.114^{\ddagger}$
								(-4.778)	(-4.724)	(-4.719)
i_t^c									0.056^{*}	$0.059^{\dagger}$
									(1.913)	(2.006)
T^{-1}										$-0.142^{\ddagger}$
										(-2.990)
Adj. R^2	0.063	0.012	0.029	0.034	0.063	0.075	0.086	0.095	0.096	0.097
Adj. Within-R^2	0.051	0.000	0.018	0.023	0.051	0.064	0.075	0.084	0.085	0.086

Table 4: Model Calibration

Columns (1), (2), (3), and (5) present calibration result for the constant-growth steady state version of the model described by equation (28): $I_{t+1} = \bar{\mu} I_t$ for all t. Parameters α, T, $\bar{\mu}$,and r are primitives of the model. User cost c is calculated using equation (7). The model-implied value of RC_t/K^*_{t+1} and i^r_{t+1} are calculated using equations (30) and (29), respectively. Tobin's Q is calculated using the following variant of equation (20) obtained under the additional assumption that $\hat{Z}_{t+1} = Z_{t+1}$, i.e., in the absence of transitory demand shocks:

$$Q_t = \frac{V^{ex}(Z_{t+1}, \Theta_t)}{K^*_{t+1}} = \frac{RC_t}{K^*_{t+1}} + \frac{(1-\alpha)c}{\alpha(1+r-\bar{\mu})}.$$

In the constant-growth steady state described above, $i^n_{t+1} \equiv \bar{\mu} - 1$. Color-coded column (4) presents estimates of the model parameters and variables for the whole sample. The sensitivities presented in this column are estimated from univariate OLS regressions of one-period-ahead sales growth on RC_t/K_{t+1}, Q_t, i^n_{t+1}, and i^r_{t+1}, with year fixed effects but without firm fixed effects. Superscripts ‡, †, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	(1)	(2)	(3)	(4) Data
		Calibration parameters		
α	0.70	0.70	0.70	-
T	12	12	12	12.58
$\bar{\mu} - 1$	0.144	0.107	0.107	0.144
r	0.165	0.125	$\rightarrow 0$	-
c	0.196	0.165	0.083	-
		Calibrated variables		
RC_t/K^*_{t+1}	0.793	0.742	0.640	0.569
Q_t	4.801	4.676	-	4.454
i^r_{t+1}	0.036	0.045	0.045	0.063
i^n_{t+1}	0.144	0.107	0.107	0.134
		Calibrated sensitivities		
$\partial\bar{\mu}/\partial\left(RC_t/K^*_{t+1}\right)$	1.640	1.391	1.200	$0.528^\ddagger$
$\partial\bar{\mu}/\partial Q_t$	0.005	0.005	-	$0.008^\ddagger$
$\partial\bar{\mu}/\partial i^n_{t+1}$	1	1	1	$0.618^\ddagger$
$\partial\bar{\mu}/\partial i^r_{t+1}$	-4.544	-3.695	-3.695	-0.143

Columns (8)-(10) show that scaled cash flow is negatively associated with the future sales growth. This finding can be consistent with our model but only in situations when the permanent component of growth is not fully controlled for. We note, however, that our theoretical results relating to cash flow should be interpreted with caution since, for instance, we do not model the difference between revenues and operating cash flow. In columns (9) and (10), we add two more variables to the specification: cash investment rate (Column 9) and T^{-1} (Column 10). While there appears to be a statistically negative relation between T^{-1} and future sales growth, neither of the two new variables contributes significantly to the explanatory power of the empirical model. In an unreported result, we confirm that the growth in, for instance, net PP&E does not predict future sales growth after controlling for i^n_t.

To provide a quantitative link between our model and the empirical evidence reported in Tables 1-3, we perform a simple calibration exercise based on the special case of the model presented in Section 3.3 (equation 28) with a constant (steady-state) investment growth. While this special case is arguably quite restrictive since, for example, it implies that all investment rates are constant at the firm level, this exercise can still shed light on the expected magnitude of the main variables and sensitivities in our model. Table 4 presents the model calibration results.

The main parameters to be calibrated are α (capital elasticity of revenue), T (useful life of capital goods), $\bar{\mu} - 1$ (sales growth rate), and r (cost of capital). Following Abel and Eberly (2011), we set $\alpha = 0.7$. As discussed above, the average sales growth rate is relatively high in our sample due to our high winsorization threshold. In column (1), we report results in which the sales growth rate is matched to the one estimated from the full sample; in columns (2) and (3), we set the sales growth rate equal to the mean of the bottom 99%-quantile. We set T equal to the sample median of 12 years. Lastly, recall that our model requires that $1 + r > \bar{\mu}$. Therefore, in columns (1) and (2), we set r equal to 16.5% and 12.5%, respectively. While these values are higher than what CAPM-like models typically imply, they are close to the range of values considered in Table 1 of Abel and Eberly (2011), 8%-14%, and broadly consistent with the hurdle rates used by CEO's internally as reported in Poterba and Summers (1995). Finally, recall that the net-to-gross PP&E ratio under the straight-line depreciation rule corresponds to RC_t/K^*_{t+1} only when r is close to zero. Accordingly, we consider this case in column (3), even though it does not allow us to calculate Q since it violates the assumption that $1 + r > \bar{\mu}$. For comparison, column (4) reports the estimated values from the full sample.

The calibrated values of Q and i^n_{t+1} appear to be close to their counterparts estimated in the data. The calibrated values of RC_t/K^*_{t+1} are somewhat higher and the calibrated values of the replacement investment rate are lower than the ones observed in the data. This suggests that the constant-growth special case is limited in its ability to describe the actual investment processes: the average composition of capital goods in the sample is older than the one that would be implied by maintaining the investment rate equal to the average sales growth for a long period of time. In terms of sensitivities, one notable result is that the model implies a low coefficient on Q even in the absence of financing frictions. This result is consistent with Abel and Eberly (2011). The model predicts correct signs for the remaining three sensitivities that we consider in Table 4 and matches the order of magnitude for the sensitivities of $\bar{\mu}$ to RC_t/K^*_{t+1} and i^n_{t+1}. The absolute value of the estimated sensitivity of $\bar{\mu}$ to i^r_{t+1} is, however, statistically not different from zero and is significantly below its predicted value. This is again indicative of only limited descriptive ability of the constant investment-growth assumption since for the replacement investment rate to be highly sensitive to $\bar{\mu}$ that growth rate would need to be maintained for T periods.

5.3 Determinants of Investment Components

Table 5 presents results of multivariate regressions for different components of the total investment rate. Panel A shows that among the investment rate components studied in columns (2-4), it is the net investment rate that is most sensitive to Q. The net investment rate is almost entirely responsible for the sensitivity of i_{t+1} to Q. The coefficient on Q_t in the regression of the cash investment rate in column (4) is close in magnitude to the values reported in the earlier literature, yet it only amounts to about 60% of the total sensitivity of i_{t+1} to Q_t. This result suggests that the focus on cash investment, which is prevalent in the earlier literature, can lead to significant underestimation of the investment-Q relation. In the OLS specification of Panel A, both cash and non-cash investment rates are positively associated with lagged cash flow. As we discussed in connection with equation (26), our model implies that the net investment rate should be positively associated with lagged cash flow even in the absence of financing frictions. Column (2) of Table 5 indeed shows a strong positive relation between i^n_{t+1} and CF_t/K_{t+1}. In unreported univariate tests, we confirm that the relation between i^n_{t+1} and CF_t/K_{t+1} becomes weaker without firm fixed effects, consistent with our discussion immediately following equation (26).

Panel A also provides strong support for our analytical predictions regarding the relations between investment rates and two vintage capital proxies, RC_t/K_{t+1} and T^{-1}. RC_t/K_{t+1} is positively associated with the future net investment rate and negatively with the replacement investment rate. Both of these effects are significant; for instance, the t-statistic of RC_t/K_{t+1} in the regression for future net investment, 11.613, is almost as high as that of Q, 13.003, and significantly higher than that of cash flow, 5.264. Since the relations between RC_t/K_{t+1} and net and replacement investment rates are of opposite signs, the overall effect on the total investment rate is less pronounced, but it is comparable to that of cash flow. T^{-1} has a significant positive relation with the total investment rate but only through the replacement investment channel. Comparing the net/replacement and cash/non-cash decompositions of the total investment, we can see that the non-cash rate leans more toward the replacement component (albeit it is more sensitive to Q and cash flow), whereas the cash component behaves more similar to the net component.

The estimates reported in Panel A are still subject to the potential problem of measurement errors in Tobin's Q identified in the earlier literature (see, e.g., Erickson and Whited 2000, Erickson and Whited 2012, Erickson et al. 2014). Panel B reports estimates corrected

Table 5: Multivariate Regressions for Investment Components

The dependent variables are different components of the investment rate in period $t+1$. See captions of Tables 1, 2, and 3 for more details on data and variable definitions. Panel A includes firm- and year fixed effects; the standard errors are two-way clustered by year and industry (four-digit SIC code). Panel B reports estimates corrected for the measurement error in Q_t using the methodology from Erickson et al. (2017). The highest cumulant order used is 5. In Panel A, the pairs of coefficients in columns (2) and (3) and in columns (4) and (5) sometimes do not exactly add up to the corresponding coefficient in column (1) due to the winsorization of dependent variables. The econometric model applied in Panel B is non-linear, therefore the sums of coefficients in columns (2) and (3) and in columns (4) and (5) are generally different from the corresponding coefficient in column (1).

	(1)	(2)	(3)	(4)	(5)
	i_{t+1}	i^n_{t+1}	i^r_{t+1}	i^c_{t+1}	i^{nc}_{t+1}
Panel A: Multivariate OLS with Firm and Year FE					
Q_t	$0.014^{\ddagger}$	$0.012^{\ddagger}$	$0.002^{\dagger}$	$0.008^{\ddagger}$	$0.006^{\ddagger}$
	(11.710)	(13.003)	(2.390)	(15.945)	(5.094)
CF_t/K_{t+1}	$0.060^{\ddagger}$	$0.066^{\ddagger}$	-0.004	$0.036^{\ddagger}$	$0.022^{\dagger}$
	(4.833)	(5.264)	(-0.874)	(5.005)	(2.568)
RC_t/K_{t+1}	$0.173^{\ddagger}$	$0.336^{\ddagger}$	$-0.161^{\ddagger}$	$0.234^{\ddagger}$	$-0.059^{\ddagger}$
	(7.063)	(11.613)	(-20.041)	(11.211)	(-4.583)
T^{-1}	$0.566^{\ddagger}$	0.022	$0.521^{\ddagger}$	$0.039^{\ddagger}$	$0.525^{\ddagger}$
	(11.649)	(0.837)	(13.924)	(2.987)	(11.846)
Adj. R^2	0.296	0.208	0.446	0.349	0.207
Adj. Within-R^2	0.143	0.103	0.129	0.163	0.063
Panel B: Using Erickson et al. (2017) EIV model					
Q_t	$0.030^{\ddagger}$	$0.024^{\ddagger}$	$0.010^{\ddagger}$	$0.015^{\ddagger}$	$0.023^{\ddagger}$
	(35.73)	(19.81)	(32.32)	(28.31)	(39.94)
CF_t/K_{t+1}	0.004	$0.024^{\dagger}$	$-0.036^{\ddagger}$	$0.015^{\ddagger}$	$-0.041^{\dagger}$
	(0.46)	(2.44)	(-9.01)	(3.05)	(-5.00)
RC_t/K_{t+1}	$0.221^{\ddagger}$	$0.413^{\ddagger}$	$-0.214^{\ddagger}$	$0.297^{\ddagger}$	$-0.113^{\ddagger}$
	(14.72)	(28.02)	(-34.63)	(34.56)	(-9.36)
T^{-1}	$0.277^{\ddagger}$	$-0.211^{\ddagger}$	$0.393^{\ddagger}$	$-0.091^{\ddagger}$	$0.248^{\ddagger}$
	(5.64)	(-4.99)	(15.91)	(-4.40)	(5.32)
ρ^2	0.237	0.171	0.185	0.248	0.139

for this error-in-variables (EIV) problem using the high-order cumulants and moments approach developed in Erickson et al. (2014). Consistent with the results from these earlier studies, the coefficients on Q_t increase significantly (approximately doubling) in all regressions. This reflects a correction for the attenuation bias in OLS stemming from the EIV problem. In contrast, the cash flow effect on total investment declines. The regression coefficient on cash flow is no longer statistically and economically significant for i_{t+1}. An interesting observation in Panel B is that the cash investment rate is still positively associated with cash flow, yet the relation between non-cash investment and cash flow switches sign. A possible explanation for this is that in the absence of internally generated funds, firms do not reduce the total amount of investment but simply switch to non-cash options, such as leases.

The magnitudes of coefficients on RC_t/K_{t+1} increase relative to Panel A. The signs are still consistent with our model predictions: RC_t/K_{t+1} is positively associated with net investment and negatively with replacement investment. Judging by z-statistics, RC_t/K_{t+1} has the highest explanatory power for the net, replacement, and cash investment rates. However, since the effects on net and replacement investment have opposite signs, the coefficient on RC_t/K_{t+1} for i_{t+1} ends up with a lower z-statistic than that on Q_t. The coefficients on T^{-1} are now statistically significant for all investment components, yet as expected, and consistent with Panel A, this variable has by far the highest explanatory power for the replacement investment rate.

Following the earlier literature, in Table 6, we investigate lagged investment effects (see, e.g., Eberly et al. 2012). Since for each firm, we only have a relatively short time series of data, fixed effect estimators in regressions with lagged dependent variables are subject to Nickell (1981) bias and are inconsistent as the number of firms gets large while the number of periods remains small. In Panel A, we present Han and Phillips (2010) estimates that are consistent in large-N, small-T asymptotics. As in Table 1, the cash investment rate is the most persistent component of investment. Regression coefficients on Q_t and cash flow for all components remain similar to those reported in Panel A of Table 5. The RC_t/K_{t+1} ratio is again positively associated with net investment and negatively associated with replacement investment, but notably it is now negatively associated with total investment.

The econometric model in Panel A of Table 6 treats all variables other than the lagged rate as strictly exogenous in all specifications. However, this is unlikely to be the case for our vintage capital proxies. For instance, RC_t/K_{t+1} is a direct function of past investments, and, as such, it is related to structural errors incorporated in them. Likewise, our estimate of T^{-1} is fully determined by past investments and the depreciation expenses associated with them. Therefore, this variable is also likely to be related to the past values of the

Table 6: Lagged Investment Effect

The dependent variables are different components of the investment rate in period $t+1$. See captions of Tables 1, 2, and 3 for details on data and variable definitions. Variable LaggedRate represents the lagged (period-t) value of the dependent variable in each regression. Both panels include year fixed effects. Panel A presents Han and Phillips (2010) estimates of the following econometric model:

$$y_{t+1} = a_1 + a_2 y_t + a_3 \left(x_t - a_2 x_{t-1}\right) + \epsilon_{t+1},$$

where y_t is the investment rate corresponding to the given column, a_1 is a firm-level constant, a_2 is the persistence parameter, x_t is the vector of firm-level explanatory variables (Q_t, CF_t/K_{t+1}, RC_t/K_{t+1}, and T^{-1}), and a_3 is the vector of coefficients on explanatory variables. The first row in Panel A reports a_2, and the subsequent rows report the individual components of a_3. For this estimation procedure, the resulting sample consists of 8,255 unique firms and 111,971 firm-years. Arellano and Bond (1991) estimates in Panel B are calculated under the assumptions that Q_t and CF_t/K_{t+1} are exogenous, and RC_t/K_{t+1} and T^{-1} are predetermined. In Panel B, z-statistics are calculated based on robust standard errors. The sample size in Panel B is 8,244 unique firms and 103,802 firm-years.

	(1) i_{t+1}	(2) i^n_{t+1}	(3) i^r_{t+1}	(4) i^c_{t+1}	(5) i^{nc}_{t+1}
Panel A: Han and Phillips (2010) estimates					
LaggedRate	0.227‡	0.184‡	0.095‡	0.513‡	0.102‡
	(14.03)	(13.71)	(6.75)	(30.10)	(6.85)
Q_t	0.013‡	0.011‡	0.002‡	0.007‡	0.006†
	(74.93)	(59.39)	(28.20)	(74.33)	(42.33)
CF_t/K_{t+1}	0.062‡	0.065‡	-0.003‡	0.034‡	0.027‡
	(24.35)	(24.47)	(-3.17)	(26.63)	(13.89)
RC_t/K_{t+1}	-0.108‡	0.161‡	-0.188‡	-0.166‡	-0.117‡
	(-10.56)	(15.56)	(-55.95)	(-24.96)	(-16.58)
T^{-1}	0.413‡	0.021	0.469‡	0.049‡	0.448‡
	(25.70)	(1.27)	(84.98)	(5.43)	(38.66)
Panel B: Arellano and Bond (1991) estimates					
LaggedRate	0.091‡	0.073‡	0.111‡	0.250‡	0.034‡
	(9.54)	(9.30)	(9.13)	(27.29)	(3.52)
Q_t	0.015‡	0.014‡	0.001‡	0.007‡	0.007‡
	(14.91)	(14.75)	(3.83)	(15.41)	(10.49)
CF_t/K_{t+1}	0.053‡	0.058‡	-0.005	0.033‡	0.021‡
	(5.96)	(6.13)	(-1.53)	(9.63)	(3.13)
RC_t/K_{t+1}	0.584‡	0.696‡	-0.074‡	0.223‡	0.224‡
	(14.95)	(17.50)	(-4.81)	(12.06)	(7.21)
T^{-1}	0.229‡	-0.139*	0.395‡	-0.065*	0.373‡
	(3.75)	(-2.45)	(13.66)	(-2.47)	(7.39)

structural error term. We address this problem in Panel B of Table 6 using Arellano and Bond (1991) dynamic panel estimation procedure and treating Q_t and cash flow as exogenous, and RC_t/K_{t+1} and T^{-1} as predetermined. Comparing Panels A and B of Table 6, we note that Arellano and Bond (1991) estimates of the persistence of investment rate components are consistently lower than the corresponding Han and Phillips (2010) estimates. This is in agreement with the conventional wisdom that Arellano and Bond (1991) estimates suffer from the weak instruments problem. The most significant consequence of treating RC_t/K_{t+1} and T^{-1} as predetermined is the drastic increase in the coefficient on RC_t/K_{t+1} in the regression of the net invested rate reported in column (2), from 0.161 in Panel A to 0.696 in Panel B. When treated as predetermined, RC_t/K_{t+1} is strongly positively associated with the total investment rate even after controlling for the lagged total investment rate.

5.4 Determinants of Vintage Capital Proxies

Having characterized the performance of our two vintage capital proxies, RC_t/K_{t+1} and T^{-1}, we now turn to analyzing their determinants. Recall that RC_t/K_{t+1} captures the "newness" of assets in our model. In Table 7, we present the results of multivariate regressions of RC_t/K_{t+1} and T^{-1} on four variables: Q_t, firm size, firm age, and the lagged net investment rate. Since RC_t/K_{t+1} is expected to be higher for newer and growing firms, we expect this ratio to be positively associated with Q and i_t^n, and negatively associated with firm age. Note that RC_t/K_{t+1} reflects growth in investment over the firm's full relevant (T-period) history. Since large firms are the ones that have undergone more aggressive expansion, one can expect RC_t/K_{t+1} to be positively associated with size. Our model does not make predictions regarding the determinants of T^{-1}, so in Table 7 we use the same explanatory variables for T^{-1} as for RC_t/K_{t+1}.

Table 7 shows that when only year fixed effects are included in specification (1), Age and the lagged net investment rate have the strongest association with RC_t/K_{t+1}. The signs of these relations are as expected: RC_t/K_{t+1} declines with Age and increases with the lagged net investment. Q_t does not appear to be related to RC_t/K_{t+1} after controlling for other variables. This is primarily due to the fact that our multivariate specifications include i_t^n,which captures the impact of current growth. In unreported univariate results, we find that Q_t is positively associated with RC_t/K_{t+1} in empirical models both with and without firm fixed effects. Once firm fixed effects are included in specification (2), Age and Size become the two most important determinants of RC_t/K_{t+1}. It is noteworthy that Size appears to play a more important role at the firm level than in the cross-section. It is not a priori clear whether larger or smaller firms should have higher RC_t/K_{t+1} ratios. Yet for a

Table 7: Determinants of RC_t/K_{t+1} and T^{-1}

See captions of Tables 1, 2, and 3 for variable definitions and presentation details. In Panel A, for all variables other than Age, the sample consists of 124,728 firm-years with 8,255 unique firms. For Age and all regressions in Panel B, the sample size is 42,301 firm-years with 3,364 unique firms.

	(1) RC_t/K_{t+1}	(2) RC_t/K_{t+1}	(3) T^{-1}	(4) T^{-1}
Q_t	-0.001‡	-0.000	0.004‡	0.001‡
	(-3.636)	(-0.204)	(8.513)	(4.721)
Size	0.014‡	0.044‡	-0.001	0.006‡
	(5.677)	(17.857)	(-0.810)	(3.202)
Age	-0.006‡	-0.012‡	-0.001‡	-0.001‡
	(-10.202)	(-28.484)	(-4.451)	(-4.217)
i_t^n	0.031‡	0.016‡	0.004†	0.004‡
	(8.977)	(7.101)	(1.972)	(2.626)
Year FE	Y	Y	Y	Y
Firm FE	N	Y	N	Y
Adj. R^2	0.668	0.730	0.506	0.664
Adj. Within-R^2	0.109	0.304	0.167	0.032

given firm, both RC_t/K_{t+1} and Size capture the cumulative effect of growth over the past several years. Therefore, one should expect these two variables to be positively correlated at the firm-level. Table 7 confirms this intuition. Multivariate results for RC_t/K_{t+1} show that the coefficients on Size and Age increase significantly when firm fixed effects are included, while the coefficient on i_t^n declines.

Columns (3) and (4) of Table 7 show that Q_t and Age have the strongest association with T^{-1}. The explanatory power of Q_t is most evident in the cross-section and is subsumed significantly by firm-level fixed effects. In contrast, the explanatory power of firm size, again, increases when firm fixed effects are included. We note, however, that the adjusted within-R^2 in specification (4) is only 3.2%, suggesting that these relations are of limited economic significance.

5.5 Industry Analysis

In addition to the determinants discussed above, one might expect there to be significant differences in investment variables across industries. In Table 8, we report summary statistics for our investment, sales growth, vintage capital variables, and Q_t in Fama-French 10 industry portfolios. The main focus of our discussion will be on Panel A, which reports mean values for each variable by industry as well as industry rankings per each variable's mean.

Several industries stand out in a casual inspection of Panel A of Table 7. HiTech industry

Table 8: Summary Statistics by Industry

This table presents summary statistics for investment, sales growth, and vintage capital variables by industry. For industry definitions, we use Fama-French 10 industry portfolios excluding utilities. All variables are defined in Tables 1 and 2. Panel A (Panel B) presents mean values (standard deviations) for each variable for each industry. Values in parentheses represent rankings of industries by means (Panel A) or standard deviations (Panel B) of each variable.

Industry	T^{-1}	RC_t/K_{t+1}	Q_t	SalesGrowth_{t+1}	i_t	i_t^n	i_t^r	i_t^c	i_t^{nc}
Panel A: Mean (Rank)									
Non-durables	0.092 (7)	0.558 (6)	3.296 (6)	0.093 (9)	0.150 (8)	0.098 (8)	0.053 (5)	0.119 (8)	0.031 (8)
Durables	0.094 (6)	0.547 (7)	2.846 (7)	0.101 (7)	0.159 (7)	0.110 (7)	0.050 (7)	0.125 (7)	0.034 (6)
Manufacturing	0.080 (8)	0.540 (8)	2.464 (8)	0.097 (8)	0.137 (9)	0.095 (9)	0.042 (8)	0.108 (9)	0.029 (9)
Energy	0.080 (9)	0.611 (2)	1.404 (9)	0.239 (2)	0.207 (5)	0.181 (1)	0.025 (9)	0.174 (2)	0.032 (7)
HiTech	0.179 (1)	0.499 (9)	8.883 (2)	0.155 (4)	0.275 (1)	0.158 (4)	0.114 (1)	0.190 (1)	0.085 (2)
Telecom	0.152 (2)	0.579 (5)	3.671 (4)	0.187 (3)	0.266 (2)	0.166 (2)	0.099 (2)	0.159 (4)	0.106 (1)
Shops	0.099 (5)	0.606 (3)	3.525 (5)	0.124 (6)	0.193 (6)	0.141 (5)	0.052 (6)	0.158 (5)	0.035 (5)
Healthcare	0.129 (3)	0.596 (4)	8.908 (1)	0.259 (1)	0.234 (3)	0.166 (3)	0.068 (3)	0.166 (3)	0.068 (3)
Other	0.110 (4)	0.629 (1)	4.007 (3)	0.153 (5)	0.207 (4)	0.139 (6)	0.068 (4)	0.149 (6)	0.058 (4)
Panel B: Standard Deviation (Rank)									
Non-durables	0.069 (6)	0.142 (7)	5.760 (6)	0.327 (8)	0.238 (7)	0.255 (7)	0.097 (7)	0.136 (7)	0.170 (8)
Durables	0.056 (7)	0.137 (9)	3.712 (7)	0.255 (9)	0.222 (9)	0.242 (9)	0.084 (8)	0.131 (8)	0.162 (9)
Manufacturing	0.048 (9)	0.141 (8)	3.531 (8)	0.339 (7)	0.237 (8)	0.251 (8)	0.080 (9)	0.123 (9)	0.179 (7)
Energy	0.053 (8)	0.184 (1)	1.996 (9)	0.923 (2)	0.410 (2)	0.425 (2)	0.118 (5)	0.245 (1)	0.283 (3)
HiTech	0.156 (2)	0.164 (4)	16.330 (1)	0.469 (5)	0.409 (3)	0.347 (4)	0.192 (2)	0.241 (2)	0.274 (4)
Telecom	0.168 (1)	0.175 (3)	6.807 (4)	0.623 (4)	0.479 (1)	0.438 (1)	0.227 (1)	0.224 (3)	0.356 (1)
Shops	0.071 (5)	0.149 (6)	5.913 (5)	0.389 (6)	0.294 (6)	0.306 (6)	0.098 (6)	0.184 (6)	0.194 (6)
Healthcare	0.099 (4)	0.163 (5)	13.527 (2)	0.986 (1)	0.344 (5)	0.339 (5)	0.133 (4)	0.194 (5)	0.252 (5)
Other	0.114 (3)	0.179 (2)	8.909 (3)	0.638 (3)	0.394 (4)	0.389 (3)	0.143 (3)	0.217 (4)	0.287 (2)

has the highest total and cash investment rates yet its average sales growth is only ranked fourth. This difference is explained by the fact that the HiTech industry employs the shortest-lived capital goods as indicated by the highest average value of T^{-1} and the highest average replacement investment rate. The net investment rate for the HiTech industry is ranked fourth, which is consistent with its sales growth ranking. Tobin's Q is the second highest for HiTech, closely following that of Healthcare, perhaps because the physical capital stock represents only a small share of the economic capital for this industry. By way of contrast, Energy industry utilizes the longest-lived capital goods (i.e., has lowest mean of T^{-1}) and, accordingly, has the lowest average replacement investment rate. However, Energy ranks first both in average sales growth and net investment rate. As a consequence, its total investment rate is ranked fifth, just below the middle of the pack. Also, being a capital intensive industry, Energy exhibits the lowest Q. Finally, Manufacturing ranks last or next-to-last on all components of the investment rate, the sales growth rate, and Tobin's Q. This industry also has the second-longest useful life of capital goods. Panel B further reveals that all of the variables have a fairly low volatility in the Manufacturing sector.

Given the differences between Manufacturing and HiTech industries in terms of sales growth, investment rates and vintage capital proxies, it is worthwhile to compare the relations between investment and its determinants between these two industries. Because of its capital intensity, the Manufacturing industry has played a central role in the earlier investment research. For HiTech industry, it is generally understood that PP&E is a significantly incomplete measure of long-lived assets since it does not include intangibles.[9] The differential importance of PP&E in these two industries is also transparent in our data: while the mean (median) Q_t is equal to 2.46 (1.53) for Manufacturing industry, it is equal to 8.88 (3.79) for HiTech. Still, the reported PP&E for technology firms generally includes significant components of the internally developed IT infrastructure costs and, under more recent standards, certain components of the infrastructure costs incurred under cloud computing arrangements.

Table 9 reports investment regression results for HiTech and Manufacturing industries obtained using the EIV methodology detailed in Erickson et al. (2017). In Table 10, we present model calibration results for these two industries that are based on the same special case of constant investment growth explored in Table 4. In this calibration exercise, we keep the cost of capital, r, and revenue elasticity of capital, α,at their values employed in Table 4 (16.5% and 0.7, respectively) and only change the values of T and $\bar{\mu}$. Conceivably, the

[9]Peters and Taylor (2017) construct a new proxy for Q that takes into account investment in intangibles and show that the Q-theory holds more closely when intangibles are included in the definition of capital stock and investment.

Table 9: Investment Regressions for Manufacturing and HiTech Industries

See caption of Table 5 for details on data and variable definitions. Both panels report estimates corrected for the measurement error in Q_t using the methodology from Erickson et al. (2017), with the highest cumulant order used set equal to 5. Panels A and B report the results for the Manufacturing (27,832 firm-years) and HiTech industries (20,241), respectively. Industry classification is based on the Fama-French 10-industry portfolios. The econometric model is non-linear, therefore the sums of coefficients in columns (2) and (3) and in columns (4) and (5) are generally different from the corresponding coefficient in column (1).

	(1) i_{t+1}	(2) i^n_{t+1}	(3) i^r_{t+1}	(4) i^c_{t+1}	(5) i^{nc}_{t+1}
Panel A: Manufacturing Industry					
Q_t	$0.054^{\ddagger}$	$0.055^{\ddagger}$	$0.002^{\dagger}$	$0.035^{\ddagger}$	0.003^{*}
	(6.59)	(6.81)	(5.50)	(9.91)	(1.94)
CF_t/K_{t+1}	-0.010	0.005	$-0.028^{\ddagger}$	-0.018	$0.068^{\ddagger}$
	(-0.28)	(0.14)	(-4.65)	(-1.04)	(4.42)
RC_t/K_{t+1}	$0.100^{\ddagger}$	$0.229^{\ddagger}$	$-0.138^{\ddagger}$	$0.173^{\ddagger}$	$-0.038^{\dagger}$
	(3.35)	(7.49)	(-18.06)	(10.76)	(-2.58)
T^{-1}	0.196	$-0.341^{\dagger}$	$0.518^{\ddagger}$	$-0.240^{\ddagger}$	$0.556^{\ddagger}$
	(1.17)	(-2.29)	(7.47)	(-3.22)	(5.07)
ρ^2	0.181	0.165	0.078	0.288	0.027
Panel B: HiTech Industry					
Q_t	$0.013^{\ddagger}$	$0.008^{\ddagger}$	$0.011^{\ddagger}$	$0.007^{\ddagger}$	$0.024^{\ddagger}$
	(11.27)	(12.25)	(32.03)	(17.51)	(45.54)
CF_t/K_{t+1}	$0.054^{\ddagger}$	$0.075^{\ddagger}$	$-0.050^{\ddagger}$	$0.037^{\ddagger}$	$-0.090^{\ddagger}$
	(3.74)	(6.97)	(-7.47)	(5.47)	(-6.67)
RC_t/K_{t+1}	$0.355^{\ddagger}$	$0.598^{\ddagger}$	$-0.345^{\ddagger}$	$0.440^{\ddagger}$	$-0.373^{\ddagger}$
	(11.46)	(21.11)	(-18.93)	(22.75)	(-10.77)
T^{-1}	$0.504^{\ddagger}$	0.033	$0.330^{\ddagger}$	0.013	$0.175^{\ddagger}$
	(10.57)	(0.97)	(9.40)	(0.55)	$(2.93)^{\ddagger}$
ρ^2	0.293	0.229	0.278	0.298	0.394

parameters that we hold fixed are, in fact, different for these two industries, but our goal in Table 10 is solely to illustrate the qualitative impact of the parameters that we empirically estimate in this paper.

Two observations stand out as noteworthy. First, investment is much less sensitive to Q in the HiTech industry. The estimates in the blue columns of Table 10 show that future sales growth is also less sensitive to Q in the HiTech industry.[10] The model also predicts that when $r - \bar{\mu}$ is low (and thus Q_t is high), Q_t is relatively more sensitive to $\bar{\mu}$, and therefore growth is less sensitive to Q_t. Calibrated sensitivities in columns (1) and (3) of Table 10 confirm this observation. In addition, it is likely that the omission of intangible investment, which is important for HiTech firms, from our measure of capital stock further contributes to the low sensitivity of investment rates to Q_t.

The second interesting observation in Table 9 is that both vintage capital proxies are significantly more important in explaining investment in the HiTech industry. For instance, the average values of RC_t/K_{t+1} are close between the two industries, yet i^n_{t+1} is significantly more sensitive to RC_t/K_{t+1} for HiTech firms. Calibration results in Table 10 confirm that this is to be expected given the significantly shorter useful life of capital goods in the HiTech sector. Note that while the average values of RC_t/K_{t+1} are approximately the same in columns (1) and (3), the sensitivity of future sales growth to RC_t/K_{t+1} in column (1) is significantly greater than that in column (3). The main force driving this calibration result is the significantly shorter assumed useful life of capital goods in column (1), which is matched to the median estimated value of T in the HiTech industry. Consistent with results for the net investment rate, columns (2) and (4) of Table 10 show that the future sales growth is also more sensitive to RC_t/K_{t+1} for HiTech firms.

6 Conclusion

In this paper, we extend the Q-theory of investment to capital goods with non-geometric efficiency. When efficiency is non-geometric, a firm's replacement cost of assets-in-place and its current capital stock are two different linear aggregates of its investment history. For capital goods with one-hoss shay efficiency, we construct simple proxies for the net and replacement investment rates, capital stock, and the replacement cost of assets in place. We further decompose the total investment rate into its cash and non-cash components. Our findings demonstrate that the investment components we propose have substantially different economic behavior. For instance, the net investment rate has the best explanatory

[10]Recall that in the special case of constant growth considered in Tables 4 and 10, the sales growth rate is equal to the net investment rate in all periods.

Table 10: Model Calibration for HiTech and Manufacturing Industries

See caption of Table 4 for details on model calibration. Color-coded columns (2) and (2) present estimates of the model parameters and variables for the HiTech and Manufacturing industries, respectively. The sensitivities presented in these columns are estimated from univariate OLS regressions of one-period-ahead sales growth on RC_t/K_{t+1}, Q_t, i^n_{t+1}, and i^r_{t+1}, with year fixed effects but without firm fixed effects. Superscripts ‡, †, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	(1)	(2) HiTech	(3)	(4) Mfg
Calibration parameters				
α	0.70	-	0.70	-
T	8	8.355	14	14.660
$\bar{\mu}-1$	0.155	0.155	0.097	0.097
r	0.165	-	0.165	-
c	0.234	-	0.187	-
Calibrated variables				
RC_t/K^*_{t+1}	0.742	0.499	0.787	0.540
Q_t	10.768	8.883	1.966	2.464
i^r_{t+1}	0.072	0.114	0.037	0.042
i^n_{t+1}	0.155	0.158	0.097	0.095
Calibrated sensitivities				
$\partial\bar{\mu}/\partial\left(RC_t/K^*_{t+1}\right)$	2.176	$0.603^{\ddagger}$	1.303	$0.341^{\ddagger}$
$\partial\bar{\mu}/\partial Q_t$	0.001	$0.007^{\ddagger}$	0.055	$0.012^{\ddagger}$
$\partial\bar{\mu}/\partial i^n_{t+1}$	1	$0.589^{\ddagger}$	1	$0.533^{\ddagger}$
$\partial\bar{\mu}/\partial i^r_{t+1}$	-3.808	0.147	-3.770	$-0.626^{\ddagger}$

power for future sales growth and has the strongest association with Q. By way of contrast, replacement investment is mostly determined by the vintage capital proxies that capture the age profile of capital goods.

Our analysis in this paper has relied on several simplifying assumptions. For example, we have assumed that the firm does not face financing constraints. Conceivably, the effects of such constraints on the firm's decisions can depend on the vintage structure of the firm's assets-in-place. For instance, in the model of Hennessy et al. (2007), the cost of equity financing decreases in the firm size (which is captured in our model by its capital stock), and the amount of credit available to the firm is bounded from above by the liquidation value of firm's capital goods (which in our model, would be determined by the replacement cost of assets in place). While these two quantities are exactly the same in models with geometric efficiency, they are quite different when efficiency is non-geometric. It is therefore natural to expect an interplay between the effects of financing constraints and asset vintage composition. The first steps in this area have been made in Rampini (2019), which characterized the relation between asset durability and financing employing the assumption of one-hoss shay efficiency over a two-period useful life.

While in the theoretical part of paper we presented our main results for arbitrary efficiency patterns, the empirical analysis has relied exclusively on the one-hoss shay assumption. In our setting, the main advantage of this assumption is that it is largely consistent with the assumptions made by managers in preparing financial statements and thus enables the construction of simple proxies for net and replacement investment that can be readily calculated for a broad sample of firms. Clearly, however, not all types of capital goods are well described by the one-hoss shay efficiency. Our model can provide guidance on the measurement of capital stock, replacement cost of assets, as well as net and replacement investment rates for capital goods with efficiency patterns other than one-hoss shay. Given availability of equipment-level data, future studies can use our model to tailor their measures of these quantities to the efficiency patterns identified in the data.

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Appendix A

Proof of Proposition 1.

Recall that the cum-dividend value function must satisfy the following Bellman equation:

$$V\left(\hat{Z}_t, Z_{t+1}, \epsilon_{t+1}, \Theta_{t-1}\right) = \hat{Z}_t^{1-\alpha} K_t^{\alpha} + \max_{I_t} \left\{\gamma \mathbb{E}_t \left[V\left(\hat{Z}_{t+1}, Z_{t+2}, \epsilon_{t+2}, \Theta_t\right)\right] - I_t\right\}. \tag{38}$$

Consider the following candidate solution for $V\left(\hat{Z}_t, Z_{t+1}, \epsilon_{t+1}, \Theta_{t-1}\right)$:

$$V\left(\hat{Z}_t, Z_{t+1}, \epsilon_{t+1}, \Theta_{t-1}\right) = \hat{Z}_t^{1-\alpha} K_t^{\alpha} + \sum_{\tau=1}^{T-1} v_\tau I_{t-\tau} + \gamma\left(1-\alpha\right) M \hat{Z}_{t+1} + \nu\left(\bar{\mu}\right) Z_{t+1}, \tag{39}$$

where $\nu\left(\cdot\right)$ is some function yet to be determined.

Substituting the candidate solution in (39) into the Bellman equation (38), we obtain:

$$\sum_{\tau=1}^{T-1} v_\tau I_{t-\tau} + \gamma\left(1-\alpha\right) M \hat{Z}_{t+1} + \nu\left(\bar{\mu}\right) Z_{t+1} =$$
$$\max_{I_t} \left\{\gamma \mathbb{E}_t \left[\hat{Z}_{t+1}^{1-\alpha} K_{t+1}^{\alpha} + \sum_{\tau=1}^{T-1} v_\tau I_{t-\tau+1} + \gamma\left(1-\alpha\right) M \hat{Z}_{t+2} + \nu\left(\bar{\mu}\right) Z_{t+2}\right] - I_t\right\}. \tag{40}$$

It is straightforward to verify that for v_τ given by (9), the following condition holds:

$$\gamma v_\tau - v_{\tau-1} = -c\gamma x_\tau. \tag{41}$$

Applying (41) and recalling that Z_{t+1} and $\hat{Z}_{t+1}$ are realized at date t, equation (40) can be rewritten as

$$\gamma (1-\alpha) M\hat{Z}_{t+1} + \nu(\bar{\mu}) Z_{t+1} = \\ = \max_{K_{t+1}} \left\{ \gamma \left(\hat{Z}_{t+1}^{1-\alpha} K_{t+1}^{\alpha} - cK_{t+1} \right) + \gamma \mathbb{E}_t \left[\gamma (1-\alpha) M\hat{Z}_{t+2} + \nu(\bar{\mu}) Z_{t+2} \right] \right\}. \quad (42)$$

It follows that K_{t+1}^* is given by (16).

Since

$$\max_{K_{t+1}} \left\{ \hat{Z}_{t+1}^{1-\alpha} K_{t+1}^{\alpha} - cK_{t+1} \right\} = (1-\alpha) M\hat{Z}_{t+1},$$

equation (42) reduces to:

$$\nu(\bar{\mu}) Z_{t+1} = \gamma \mathbb{E}_t \left[\gamma (1-\alpha) M\hat{Z}_{t+2} + \nu(\bar{\mu}) Z_{t+2} \right]. \quad (43)$$

Given that $\mathbb{E}_t \left[\hat{Z}_{t+2} \right] = \mathbb{E}_t [Z_{t+2}] = \bar{\mu} Z_{t+1}$, we have:

$$\nu(\bar{\mu}) = \gamma^2 (1-\alpha) \bar{\mu} M + \gamma \bar{\mu} \nu(\bar{\mu}), \quad (44)$$

which implies

$$\nu(\bar{\mu}) = \frac{\gamma (1-\alpha) \bar{\mu} M}{1 + r - \bar{\mu}}.$$

■

Proof of Proposition 2.

We will show that $\frac{RC_T}{K_{T+1}^*}$ increases in μ_{t+1} for any given trajectory of future uncertainty resolution. This will imply that $\mathbb{E}_t \left[\frac{RC_T}{K_{T+1}^*} \right]$ increases in μ_{t+1}. Observing all realizations of ϵ_s and μ_s up to $s = t+1$, the ratio of the replacement cost of assets at date T to the capital stock in period $T+1$ can be written as:

$$\begin{aligned} \frac{RC_T}{K_{T+1}^*} &= \frac{\upsilon_0 I_T^* + \ldots + \upsilon_{T-1} I_1^*}{K_{T+1}^*} \\ &= \sum_{i=1}^{T} \left(\upsilon_{T-i} \frac{I_i^*}{K_i^*} \frac{K_i^*}{K_{T+1}^*} \right) \\ &= \sum_{i=1}^{T} \left(\upsilon_{T-i} \left(\mu_{i+1} \epsilon_{i+1} - \epsilon_i \right) \epsilon_{T+1}^{-1} \prod_{\tau=i+1}^{T+1} \mu_\tau^{-1} \right), \end{aligned} \quad (45)$$

where the last equality follows from the fact that on the optimal investment path,

$$\frac{K_\tau^*}{K_{\tau-1}^*} = \frac{\mu_\tau \epsilon_\tau}{\epsilon_{\tau-1}},$$

and that $I_i^* = K_{i+1}^* - K_i^*$ for a new firm under the one-hoss shay productivity assumption.

We will now show that RC_T/K_{T+1}^* increases in each μ_{t+1} holding constant all future values of μ_{t+i} and ϵ_{t+i}. Differentiating (45) with respect to μ_{t+1} yields:

$$\begin{aligned}\frac{\partial\left(RC_T/K_{T+1}^*\right)}{\partial\mu_{t+1}} &= \upsilon_{T-t}\mu_{t+1}^{-1}\epsilon_t\epsilon_{T+1}^{-1}\prod_{\tau=t+1}^{T+1}\mu_\tau^{-1} \\ &- \mu_{t+1}^{-1}\sum_{i=1}^{t-1}\left\{\upsilon_{T-t+i}\left(\mu_{t-i+1}\epsilon_{t-i+1}-\epsilon_{t-i}\right)\epsilon_{T+1}^{-1}\prod_{\tau=t-i+1}^{T+1}\mu_\tau^{-1}\right\} \\ &= \frac{1}{\mu_{t+1}K_T}\left\{\upsilon_{T-t}K_t - \sum_{i=1}^{t-1}\upsilon_{T-t+i}I_{t-i}\right\}.\end{aligned}$$

It remains to show that

$$\upsilon_{T-t}K_t \geq \sum_{i=1}^{t-1}\upsilon_{T-t+i}I_{t-i}.$$

Observe that

$$\begin{aligned}\upsilon_{T-t}K_t &\geq \upsilon_{T-t+1}K_t = \upsilon_{T-t+1}I_{t-1} + \upsilon_{T-t+1}K_{t-1} \\ &\geq \upsilon_{T-t+1}I_{t-1} + \upsilon_{T-t+2}K_{t-1} \\ &= \upsilon_{T-t+1}I_{t-1} + \upsilon_{T-t+2}I_{t-2} + \upsilon_{T-t+2}K_{t-2} \\ &\geq ... \geq \sum_{i=1}^{t-1}\upsilon_{T-t+i}I_{t-i},\end{aligned}$$

where the last inequality follows by recalling that $K_0 = 0$. ■

Proof of Observation 1.

Note that

$$\begin{aligned}PVRI_{t+1} &= \gamma I_{t-T+1} + \gamma^2 I_{t-T+2} + ... + \gamma^T I_t \\ &= K_{t+1}^* - \sum_{\tau=0}^{T-1}\left(1-\gamma^{T-\tau}\right)I_{t-\tau} \\ &= K_{t+1}^* - \frac{r}{c}\sum_{\tau=0}^{T-1}\frac{\left(1-\gamma^{T-\tau}\right)}{1-\gamma^T}I_{t-\tau} = K_{t+1}^* - \frac{r}{c}RC_t,\end{aligned}$$

where the last two equalities are obtained using equations (7) and (13). ■

Proof of Observation 2.

When a firm with one-hoss shay capital goods follows a constant-growth investment path, its RC_t/K_{t+1} ratio can be written as:

$$\begin{aligned}\frac{RC_t}{K_{t+1}} &= \frac{I_{t-T+1}\upsilon_{T-1} + ... + I_t\upsilon_o}{I_{t-T+1} + ... + I_t} \\ &= \upsilon_{T-1}\left(\frac{1}{1 + ... + \bar{\lambda}^{T-1}}\right) + ... + \upsilon_o\left(\frac{\bar{\lambda}^{T-1}}{1 + ... + \bar{\lambda}^{T-1}}\right).\end{aligned}$$

Let

$$\kappa_\tau\left(\bar{\lambda}\right) \equiv \frac{\bar{\lambda}^\tau}{1 + ... + \bar{\lambda}^{T-1}}.$$

To prove that $\frac{\partial(RC_t/K_{t+1})}{\partial\lambda} \geq 0$, we need to show that

$$\upsilon_{T-1}\kappa_0^{'}\left(\bar{\lambda}\right) + ... + \upsilon_o\kappa_{T-1}^{'}\left(\bar{\lambda}\right) \geq 0. \tag{46}$$

Note that $\sum_0^{T-1}\kappa_\tau\left(\bar{\lambda}\right) = 1$ by construction, thus implying that $\sum_0^{T-1}\kappa_\tau^{'}\left(\bar{\lambda}\right) = 0$. Note, furthermore, that since $\kappa_\tau\left(\bar{\lambda}\right) = \bar{\lambda}\kappa_{\tau-1}\left(\bar{\lambda}\right)$, we have that

$$\kappa_\tau^{'}\left(\bar{\lambda}\right) = \kappa_{\tau-1}\left(\bar{\lambda}\right) + \bar{\lambda}\kappa_{\tau-1}^{'}\left(\bar{\lambda}\right).$$

The equation implies that if $\kappa_s^{'}\left(\bar{\lambda}\right) \geq 0$ for some s, then it must be that $\kappa_\tau^{'}\left(\bar{\lambda}\right) \geq 0$ for all $\tau > s$. Then, the fact that $\sum_0^{T-1}\kappa_\tau^{'}\left(\bar{\lambda}\right) = 0$ implies that there exists an $s^* > 0$ such that $\kappa_\tau^{'}\left(\bar{\lambda}\right) \geq 0$ if and only if $\tau \geq s^*$.

Now consider the expression in the right-hand side of (46) and note that it can be rewritten as:

$$\underbrace{\upsilon_{T-1}\kappa_0^{'}\left(\bar{\lambda}\right) + ... + \upsilon_{T-s^*}\kappa_{s^*-1}^{'}\left(\bar{\lambda}\right)}_{\geq\left(\kappa_0^{'}\left(\bar{\lambda}\right) + ... + \kappa_{s^*-1}^{'}\left(\bar{\lambda}\right)\right)\upsilon_{T-s^*}} + \underbrace{\upsilon_{T-s^*-1}\kappa_{s^*}^{'}\left(\bar{\lambda}\right) + ... + \upsilon_o\kappa_{T-1}^{'}\left(\bar{\lambda}\right)}_{\geq\left(\kappa_{s^*}^{'}\left(\bar{\lambda}\right) + ... + \kappa_{T-1}^{'}\left(\bar{\lambda}\right)\right)\upsilon_{T-s^*}},$$

where the inequalities in the underbraces follow from the fact that for one-hoss shay capital goods $\upsilon_{T-1} \leq ... \leq \upsilon_0$ and the definition of s^* above. The expression above is then not less than

$$\left(\kappa_0^{'}\left(\bar{\lambda}\right) + ... + \kappa_{s^*-1}^{'}\left(\bar{\lambda}\right)\right)\upsilon_{T-s^*} + \left(\kappa_{s^*}^{'}\left(\bar{\lambda}\right) + ... + \kappa_{T-1}^{'}\left(\bar{\lambda}\right)\right)\upsilon_{T-s^*} = 0.$$

■

Appendix B. Model Extensions

In this Appendix, we discuss two extensions of the main model of the paper. The goal of the first extension is to make the model more empirically plausible by introducing a lagged investment effect documented in the earlier literature (see, e.g., Eberly et al. 2012). To this end, we allow for a regime-switching process for the stochastic demand shift parameter as in Eberly et al. (2008), Abel and Eberly (2011), and Eberly et al. (2012). The goal of the second extension is to demonstrate that the main results from our model carry over to variants of the neoclassical investment model with adjustment costs and perfectly competitive product markets; see, e.g., Hayashi (1982).

In the main model of the paper, the expected growth rate ($\bar{\mu}$) in the permanent component of the demand shift parameter (Z_t) is constant for each firm; see equation (4). Due to this assumption, all variation in the expected net investment rate comes from the transitory demand shocks, as demonstrated in equation (23). Therefore, our main model does not capture the lagged investment effect observed in the data. Earlier literature shows that the lagged investment effect and cash flow effects can arise naturally in models with regime-switching demand growth; see, e.g., Eberly et al. (2008), Abel and Eberly (2011), and Eberly et al. (2012). In particular, the model in Abel and Eberly (2011) is similar to ours along several dimensions (such as investment reversibility, zero capital adjustment costs, and a Cobb-Douglas production function) but relies on the assumption of a homogeneous capital stock.

It is straightforward to extend our model to a regime-switching demand scenario in the spirit of Abel and Eberly (2011). Specifically, assume that in each period

$$Z_{t+1} = \mu_{t+1} \cdot Z_t, \tag{47}$$

where with probability λ, the gross rate μ_{t+1} is drawn from some time-invariant distribution with a finite support in $[\mu_{min}, \mu_{max}]$, and with probability $1-\lambda$, the growth rate remains the same as in the previous period, $\mu_{t+1} = \mu_t$. Let $\mathbb{E}_{\tilde{\mu}}[\cdot]$ denote the expectation operator over the values of μ conditional on the arrival of a new regime, and let $\bar{\mu} \equiv \mathbb{E}_{\tilde{\mu}}[\tilde{\mu}]$, i.e., now $\bar{\mu}$ is the unconditional mean of μ.

It can be verified that in this extended model, the following results hold. First, the optimal investment policy is still linear in $\hat{Z}_{t+1}$: $K^*_{t+1} = M^{\frac{1}{\alpha}}\hat{Z}_{t+1}$. Second, the firm's ex-dividend equity value at date t is given by:

$$V^{ex}(Z_t, \mu_{t+1}, \epsilon_{t+1}, \Theta_{t+1}) = RC_t + \gamma(1-\alpha) M\hat{Z}_{t+1} + Z_{t+1}\nu(\mu_{t+1}), \tag{48}$$

where

$$\nu(\mu_{t+1}) \equiv \frac{(1-\alpha) M\omega}{1 + r - (1-\lambda)\mu_{t+1}} - \gamma(1-\alpha) M \tag{49}$$

and ω is a constant given by

$$\omega \equiv \left\{ \mathbb{E}_{\tilde{\mu}} \left[\frac{1+r-\tilde{\mu}}{1+r-(1-\lambda)\,\tilde{\mu}} \right] \right\}^{-1}. \tag{50}$$

Note that now the last term in the value function depends not only on the current value of Z_{t+1} but also on the current growth regime μ_{t+1}. Finally, the expected value of the future net investment rate is given by:

$$\mathbb{E}_t \left[i^n_{t+1} \right] = \frac{1}{\epsilon_{t+1}} \left\{ (1-\lambda)\,\mu_{t+1} + \lambda \bar{\mu} \right\} - 1.$$

The equation above shows that $\mathbb{E}_t \left[i^n_{t+1} \right]$ depends on the current growth regime, μ_{t+1}, which with some probability is the same as the growth regime of the previous period. Therefore, in this model, lagged investment rate is positively associated with the expected future net investment rate. Since μ_{t+1} also enters the last term of the value function in (48), Q is positively associated with $\mathbb{E}_t \left[i^n_{t+1} \right]$, and both of these variables vary over time for each individual firm.

Let us now turn to the second extension of our model. So far, we have assumed that the firm faces decreasing returns to capital in the product market and does not incur any capital adjustment costs. As discussed in Abel and Eberly (2011), these modeling assumptions are historically more prevalent in the industrial organization literature. In the finance literature, it is common to assume that the firm participates in perfectly competitive product market but its capital adjustment decisions are costly. Our results can be extended to this latter setting, albeit with some additional assumptions on the adjustment cost function.

Specifically, assume that the firms revenue is linear in K_t,

$$R\left(\hat{Z}_t, K_t\right) = \hat{Z}_t K_t,$$

but, in addition to the direct cost of investment (I_t), the firm also incurs a capital adjustment cost of the following form:

$$\phi\left(\frac{K_{t+1}}{K_t}\right) K_t,$$

where $\phi(\cdot)$ is a convex function. The assumption that the adjustment cost function is homogeneous of degree one, as in the expression above, is standard in the literature. It is important for our analysis, however, that the adjustment cost depends only on the firm's *net*, not total, investment rate. In this model, the firm incurs adjustment costs when it changes its scale of operations; replacement investment is subject only to the direct cost. As before, we assume that $\hat{Z}_{t+1}$ is observed just before investment I_t is made.

It is well-known that in the model described above, under the assumption of geometric economic depreciation, the optimal net investment rate is a function of $\hat{Z}_{t+1}$ alone, so that:

$$K_{t+1}^{*} = \xi_K \left(\hat{Z}_{t+1} \right) K_t,$$

where $\xi_K(\cdot)$ is some function that depends on the structure of the adjustment cost. Furthermore, the ex-dividend value function is also linear in K_t due to the homogeneity of the problem:

$$V^{ex} \left(\hat{Z}_{t+1}, K_t \right) = \xi_V \left(\hat{Z}_{t+1} \right) K_t.$$

It can be verified that when the assumption of geometric economic depreciation is relaxed, the optimal net investment rate is still determined solely by Z_{t+1}. Yet the value function now depends on two aggregates of the investment history – RC_t and K_t:

$$V^{ex} \left(\hat{Z}_{t+1}, \Theta_t \right) = RC_t + \tilde{\xi}_V \left(\hat{Z}_{t+1} \right) K_t.$$

This result is consistent with the valuation function presented in our Proposition 1. Tobin's Q can be again written as a sum of two components: one capturing the current state, $\tilde{\xi}_V \left(\hat{Z}_{t+1} \right)$, and another one determined by the firm's investment history, RC_t/K_t. Our results in this paper are primarily driven by assumptions that are imposed on the capital evolution process and are robust to alternative specifications of the revenue and cost functions commonly used in the literature.