Green Product Innovation in Industrial Networks: A Theoretical Model

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Abstract

Previous studies have modeled green technological change as innovations in the *process* of production (e.g., abatement technologies or energy sources). But greening the economy also requires changing *products*. The automotive industry, for example, needs to massively deploy alternative-fuel vehicles. Product manufacturing occurs within supply-chain networks, and developing new products typically requires complementary investments by suppliers. We study the incentives for green product innovation in industrial networks and how policies can affect them. We follow the industrial organization theory of product differentiation, and model green product innovations as upgrades in product quality where inputs from suppliers are essential for upgrading quality. We show that suppliers can be innovation bottlenecks and render policy instruments less effective. We provide an explicit mechanism for the role of institutions that help actors coordinate on the long-term direction of innovation. We discuss how our results help organize several findings from case studies in the automotive industry.

JEL Classification: Q55, Q58, L52, O31
Keywords: green products, innovation, production networks, buyer-supplier relationships, supply chains

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1 Introduction

Any significant progress in reconciling economic activities with environmental goals requires technological change (Aghion et al. 2009a; Barrett 2009). Most of the literature on induced green technological change has modeled environmental innovations as changes in the *process* through which goods are produced (as opposed to changes in the *products* themselves). For example, the innovation is often represented as a decrease in the cost of abatement technologies (e.g. Goulder et al. 2000; Jaffe et al. 2003) or as an improvement in clean intermediate inputs making dirty technologies less attractive (e.g. Acemoglu et al. 2012; Fischer et al. 2008; Nordhaus 2010)[1].

Greening the economy indeed requires cleaner sources of energies or technologies to decrease pollution. But the *goods* produced have to change as well. For example, the automotive industry needs to deploy large fleets of electric or hydrogen-based vehicles (Deep Decarbonization Pathways Project 2015). In the construction sector, new types of designs such as zero-energy buildings have to be broadly adopted (Olsthoorn et al. 2019). The scholarship has so far placed less attention on the drivers and barriers of green technological change when the change required is a new type of product. Doing so, however, requires modeling inter-firms relationships because, nowadays, the design or manufacturing of most products typically occur within supply-chain networks (Timmer et al. 2014).

Indeed, vertically integrated firms are much rarer than a few decades ago. In the 1990s, firms outsourced manufacturing of many components to refocus on core competencies (Feenstra 1998). Among other objectives, they sought to lower labor costs and gain economies of scale. As a result, "mega-suppliers" emerged: that is, upstream firms that supply many, if not most, of final competing producers (Jacobides et al. 2016). For example, in the car manufacturing sector, companies such as Denso or Bosch now supply most final producers.

[1]For reviews of both the empirical and theoretical literature on induced green innovation, see Popp (2010) and Popp et al. (2010).

Similarly, in the construction industry, a few firms dominate the upstream market for materials. In this paper, we focus on how the structure of supplier-buyer networks may impact incentives to innovate, and how policies may affect those incentives. Ambitious product innovations typically require changing several components of a product. This in turn means that several firms in the supply-chain must engage in risky investments concurrently. Understanding the incentives in supply-chain networks is therefore essential to learn about how to foster green product innovations. This is particularly pertinent for green product innovation because we need qualitative leaps in the material efficiency of products. This in turn calls for the redesign of whole systems rather than marginal changes to individual components of a product (Geels et al. 2018).

We model green product innovations as upgrades in product quality[2]. To do so, we use and extend a Bertrand duopoly game with vertical product differentiation (Anderson et al. 1992; Shaked et al. 1982; Tirole 1988). We start with a baseline of vertically integrated duopolists and then focus on the case when one upstream firm supplies several competing final producers. An important implication of sharing a supplier is that innovation activities of all the players are now interdependent. Indeed, shared suppliers control the level of innovation for inputs that are complements to final producers. We find that these shared suppliers face weaker incentives to innovate, thwarting competition for quality between the final producers and acting as innovation bottlenecks. The reason is that shared suppliers have a monopoly choice over the level of innovative effort in the network. Yet, they do not control prices since the producers still compete on prices. Consequently, suppliers are not sufficiently rewarded if they choose to be technologically ambitious, in contrast to vertically integrated producers who control both quality and price. We also study policies that can modify these incentives. We find that price-based policies (e.g., taxes) are less effective than subsidies or procurement to foster a switch to greener products.

[2]The focus of our paper is on the supply side, so we model green preferences solely as tastes for quality.

The key features of our model are as follows. Firms invest in innovative quality upgrades which increase the environmental quality of their products. Their ambition, however, is bounded by an exogenously defined innovation research frontier. Importantly, we model suppliers as essential actors to the innovation project: their investments are complementary to that of the final producer. This assumption reflects that most ambitious projects require changes to more than one component. Mistakes or underinvestments in any of the critical components can jeopardize the project as a whole (Fixson et al. 2008; Kremer 1993). Policies enter our model in the following way: 1) a carbon tax increases the marginal value for consumers of a product's environmental quality, and therefore increases the appeal of green products to consumers; 2) subsidies lower the fixed cost of the innovation; and 3) procurement policies increase the market size for a given innovation.

Finally, in an extension, we introduce uncertainty in the direction of technological change and show that institutions that can coordinate expectations about technological direction will increase players' effort towards green products. Think for example of electric cars vs. hydrogen cars. Both are low-emission vehicles, but they require different components. In our model, we introduce a probability that final producers choose different directions. If they do, shared suppliers lose economies of scope. We find that shared suppliers are then even more reluctant to innovate. In this context, a policy (or institution) that can help coordinate the direction of innovation is found to be complementary to the economic instruments considered (tax, procurement or subsidy).

These results help understand the conditions that enable green product innovation in a mature industry with established players and relationships. First, for all producers to switch to more environmental products, there needs to be a target level of effort (the exogenous research frontier or a technological standard) and a complementary set of economic incentives (tax and market size relative to fixed costs) of the right magnitude given this target. Otherwise, producers differentiate and environmental products remain niche rather than mainstream. Second, industrial structure matters. Innovation is facilitated by vertical integration or exclusive relationships with suppliers. In more complex networks, shared suppliers are key players. In the absence of sufficient incentives, they become roadblocks to innovation. Institutions that can coordinate players then become important. In the discussion section of this paper, we consider each of these implications in more detail. We illustrate them with findings from case studies of the automotive industry and case studies of institutions that have buttressed innovation in previous waves of industrial policy during the 20th century.

Our paper draws on the fields of industrial organization and management to contribute to scholarship on the economics of induced environmental innovation. Our contribution is two-fold. First, we contribute to a small but important literature that draws on the theory of product differentiation to study the incentives for green *product* innovation (André et al. 2009; Conrad 2005; Cremer et al. 1999; Eriksson 2004). The closest to our paper is that of André et al. 2009, who revisit the Porter Hypothesis in light of a duopoly model of vertical product differentiation. Their main result is similar to our baseline model. They show that a regulatory standard changes the Nash equilibrium of the innovation game in a way that can be beneficial to firms. Indeed, consumers will pay higher prices for higher quality products. Without the standard, firms have an incentive to differentiate to relax price competition.

Viewing environmental innovations as complementary investments across firms is our second contribution. Here, we bridge the literature on the economics of innovation and (green) industrial policy (Aghion et al. 2009b; Alic et al. 2003; Ansell 2000; Mazzucato 2011; Rodrik 2014). The latter often emphasizes public institutions to coordinate actors and catalyze technological change. But, typically, precise mechanisms about the market failures are omitted. Here we provide theoretical support for one mechanism. It recognizes that shared suppliers are critical players in coordinating around a shift to superior technology. But they can also be biased in favor of the status-quo product. Modeling innovations as projects requiring complementary investments also relate to the organization literature that has focused on supply-chain management (e.g., Argyres et al. 2010; Cachon 2003; Jacobides et al. 2005; Sturgeon et al. 2008). Finally, our paper also links to the scholarship bridging IO and the economics of innovation. Much of this literature examines the relationship between the boundaries of the firm, market structure and innovation with an important focus on contracting issues (e.g., Aghion et al. 1994; Teece 1986 or more recently Gilson et al. 2009 and Chen et al. 2011)[3].

Section 2 presents our baseline model of green product innovation. It includes the case of integrated producers and the case of parallel supply chains (cases a and b on Figure 1). Section 3 covers the model with shared suppliers (case c on Figure 1), and Section 4 the extension on technological uncertainty and coordination failures. We discuss our results and implications in Section 5.

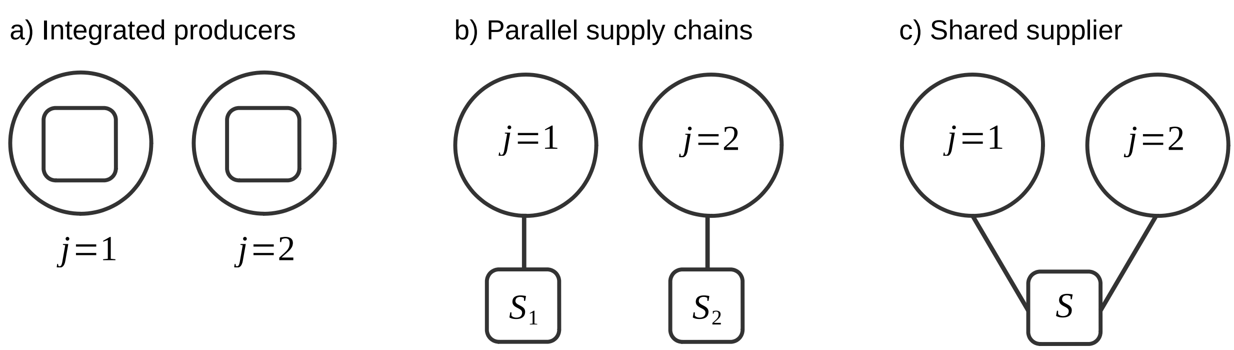


Figure 1: The three configurations studied in this paper.

2 A Baseline Model of Green Product Innovation

We start by presenting a simple model of duopolistic competition, in which producers compete in environmental quality and prices. This allows us to characterize the conditions under which both producers would switch to producing a higher quality product. This first model serves as a baseline for the rest of the analysis. We then introduce suppliers who produce inputs that are perfect complements to that of the producers.

[3]For a review of this literature, see Cohen 2010 and Teece 2010.

2.1 Model

We first assume that suppliers and producers are vertically integrated. The model consists of two producers, indexed by subscripts 1 and 2. To keep the model simple, we study the case of symmetric producers, and use 'producer 1' to refer to any of the two producers without loss of generality.

We model demand for industries where products are differentiated and where consumers choose only one of the competing products. Examples include cars, appliances, construction services. We use a multinomial logit model to represent discrete-choice demand (Anderson et al. 1992) derived from a random utility model of consumer behavior (McFadden 1973). Consumers' idiosyncratic preferences are assumed to be distributed i.i.d. type I extreme value, which gives an intuitive expression for the demand function. In a market with two producers, each producing a good, the aggregate demand for product 1 takes the following form:

$$q_1(p_1, p_2, a_1, a_2) = M \frac{e^{\frac{a_1 - p_1}{\mu}}}{e^{U_0} + e^{\frac{a_1 - p_1}{\mu}} + e^{\frac{a_2 - p_2}{\mu}}}, \tag{1}$$

where a_1 stands for the environmental quality of product 1. In other words, a_1 is the marginal value (in utility terms) that consumers associate to environmental properties. This may be affected by environmental norms, health benefits or a tax on negative environmental externalities[4]. p_1 stands for the price of product 1; M the number of consumers (the size of the market); U_0 the utility derived from the outside option, which we normalize to 0; and μ the scale parameter of the i.i.d. type 1 extreme values distribution of consumers' preferences.

Firms can increase the environmental quality of their product by innovating. We model the innovation effort that producer 1 chooses as a variable $z_1 \in [0; Z]$, where Z stands for the technological frontier. If z_1 is zero, the firm chooses not to innovate. Higher innovation

[4]We write the utility that consumer i derives from product i as: $U_i(a_j, z) = a_j + \epsilon_{ij} + z$, where z stands for the outside option and ϵ_{ij} the idiosyncratic preference of consumer i for good j unrelated to environmental properties.

efforts lead to higher environmental quality a_1: $a_1(z_1) = \beta z_1$, where β is a positive constant. However, innovating requires paying for investments: fixed costs are incurred upfront and increase with innovation efforts: $R_p z_1$, where R_p is a constant. The variable cost for producer 1 is $c_p * q_1$ where c_p is a positive constant. We denote s the share of revenue accruing to each producer[5]. For vertically integrated producers, producers capture all of the revenues and s is simply 1. But this parameter will be useful in Section 2.4 and 3 where we split revenues between a producer and its supplier. Putting all this together, the profit function for producer 1 is:

$$\pi_{p_1} = (sp_1 - c_p) \times q_1(p_1, p_2, a_1, a_2) - R_p z_1 \tag{2}$$

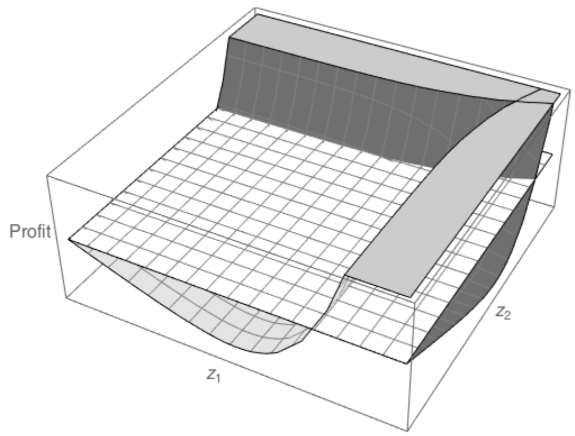
2.2 Nash Equilibria

The sequence of the game is as follows: 1) producers choose their innovation effort and incur fixed costs, and 2) producers choose prices. We solve for the subgame perfect Nash equilibrium by backward induction. In the last stage of the game, producers choose prices given qualities $a_1 = \beta z_1$ and $a_2 = \beta z_2$. Such games have been studied extensively in industrial organization, and we know the equilibrium exists and is unique (Anderson et al. 1992). The equilibrium prices are defined implicitly by $p_1^* = c_p + \frac{\mu}{1 - \frac{q_1^*}{M}}$, with the property that $\frac{dp_1^*}{dz_1} > 0$ (see Appendix). Importantly, as the environmental quality of product 1 increases relative to product 2, its market share increases, and consequently, producer 1 charges a higher price, above marginal cost.

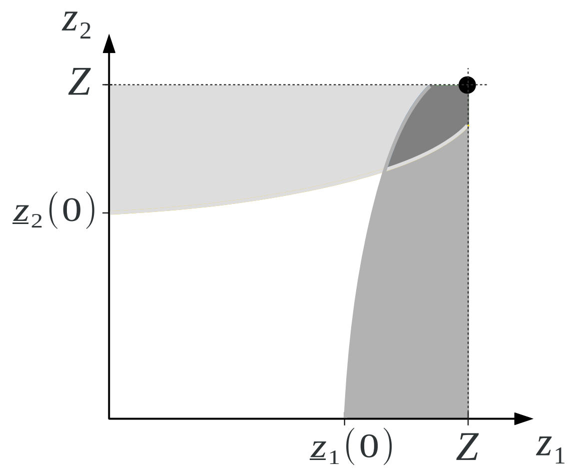
We now turn to the first stage of the game, the choice of innovation efforts. For the rest of the paper, we will assume that $R_p < \beta s M$ to rule out scenarios in which innovating is too expensive relative to the size of the market. Otherwise neither of the producers would want to innovate, even if it allowed them to capture the entire market.

We first examine how an increase in effort z_1 (given the competitor's effort, z_2) affects

[5]Since we focus on symmetric producers, costs and revenue shares are the same across producers: $c_{p_1} = c_{p_2} = c_p$, $R_{p_1} = R_{p_2} = R_p$ and $s_1 = s_2 = s$.



(a) Example of profit surfaces as a function of z_1 and z_2. The surfaces correspond to Π_{p_1} and Π_{p_2}. The horizontal plane represents profits when z_1 or z_2 is 0. $\Pi_{p_1}(z_1, z_2)$ initially decreases and then increases with z_1. It becomes positive above some threshold value $\underline{z}_1(z_2)$ (which depends on z_2).



(b) Positive profits in the (z_1, z_2) plane. The darker grey area corresponds to where Π_{p_1} is positive, that is to the right of the threshold function $\underline{z}_1(z_2)$. The lighter grey area corresponds to where Π_{p_2} is positive, that is above the threshold function $\underline{z}_2(z_1)$. The "overlapping" region corresponds to the area where both profits are positive. In this example, the frontier Z (and the dot at (Z, Z)) represents a Nash equilibrium.

Figure 2: Producers' profits as a function of their innovation effort.

profits in stage 1 which we denote Π_{p_1}[6]. Remark 1 states that profits increase with z_1 and become positive beyond a threshold $\underline{z}_1(z_2)$. This threshold depends on z_2, the innovation effort of the competitor.

Remark 1. *Given* z_2, $\exists\ \underline{z}_1(z_2)$ *s.t.:* $\forall z_1 \geq \underline{z}_1(z_2),\ \Pi_{p_1}(z_1, z_2) \geq \Pi_{p_1}(0, z_2)$ *and* $\frac{d\Pi_{p_1}(z_1,z_2)}{dz_1} > 0$.

All proofs are given in the Appendix. Figure 2 illustrates Remark 1 for specific parameter values. It shows the function $\underline{z}_1(z_2)$, delineating the region where profits are positive, i.e. where innovating is more profitable than not innovating. Below the threshold, the marginal benefit of innovating is lower than its marginal cost because the increase in market share is insufficient relative to the upfront costs.

[6]Profits in stage 1 are a function of z_1 and z_2: $\Pi_{p_1}(z_1, z_2) = \pi_{p_1}(p_1^*(z_1, z_2), p_2^*(z_1, z_2), z_1, z_2)$

We want to identify equilibria where both producers innovate. On Figure 2 for example, both producers can maximize profits by innovating at the research frontier $z^* = Z$. Graphically, the existence of such equilibrium relies on the two profit surfaces "overlapping". Mathematically, the overlapping region corresponds to the set of points (z_1, z_2) where both profit surfaces are positive. Given the definition of $\underline{z}_1(z_2)$, we can write this as: $\{(z_1, z_2) : z_1 > \underline{z}_1(z_2) \text{ and } z_2 > \underline{z}_2(z_1)\}$. Such region may or may not exist; it will depend on the parameters.

We first formally presents the Nash equilibria in Result 1 below and then use Figure 3 to illustrate and provide an intuition. Result 1 establishes that an overlapping area exists only if the ratio of the market size over the fixed cost (M/R_p) is larger than $g(\beta, \mu, c_p)$, an implicit function of β (the marginal value of environmental quality), μ (the parameter affecting the dispersion of idiosyncratic tastes), and c_p (the marginal cost of production). In short, three situations can occur: 1) both producers innovate at the frontier (Z, Z) (we call this joint innovation); 2) only one producer innovates at the frontier $(0, Z)$ or $(Z, 0)$ (we call this anti-coordination); 3) neither producers innovate $(0, 0)$.

Result 1. *The values of β, μ, c_p, and Z determine whether we have a unique equilibrium with joint innovation, two equilibria (anti-coordination) or no innovation. The table below specifies all possible cases. $g(\beta, \mu, c_p, s)$ denotes a function of β, μ, c_p and s.*

	Parameter Values	*Equilibrium* (z_1^*, z_2^*)
If $M/R_p > g(\beta, \mu, c_p, s)$	$Z < \underline{z}_1(0)$	$(0, 0)$
	$\underline{z}_1(0) \leq Z < \zeta^L$	$(0, Z)$ $(Z, 0)$
	$\zeta^L \leq Z \leq \zeta^U$	(Z, Z)
	$\zeta^U < Z$	$(0, Z)$ $(Z, 0)$
If $M/R_p < g(\beta, \mu, c_p, s)$	$Z < \underline{z}_1(0)$	$(0, 0)$
	$\underline{z}_1(0) \leq Z$	$(0, Z)$

On Figures 3a, 3b and 3c, an overlapping region exists and is bounded. Within this region, firms both innovate and make a profit: we call this "joint innovation". Since producers are symmetric, if one innovate, the other does as well and with the same effort z^*. On the figures, the segment $[\zeta^L, \zeta^U]$ defines the range of possible symmetric outcomes[7]. If a frontier Z exists and is in $[\zeta^L, \zeta^U]$ (as on Figure 3a), then there is a unique Nash equilibrium with joint innovation at $(z_1^* = Z, z_2^* = Z)$ because producers out-compete each other by increasing their effort z until they reach the technological frontier Z.

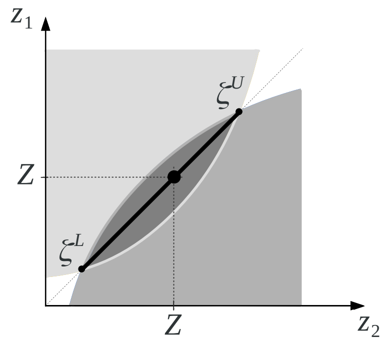
To understand why, consider a point z on the segment $[\zeta^L, \zeta^U]$. By definition, we have $z \geq \underline{z}_1(z)$. Using Remark 1, we know producer 1 can increase its profit by increasing z. It is therefore not an equilibrium, and producers will keep increasing z until they reach Z. Are the outcomes on the segment always Nash equilibria? The answer depends on the research frontier Z. If Z is larger than ζ^U, as on Figure 3b, we obtain an anti-coordination game with two equilibria at $(0, Z)$ and $(Z, 0)$: if producer 1 innovates at Z, producer 2's best response is not to innovate (and vice-versa)[8].

To see why, suppose producer 1 chooses the most ambitious, yet feasible, innovation effort: $z_1 = Z$. If producer 2 were to choose $z_2 = Z$ as well, both producers would have negative profits. Indeed, on Figure 3b, the point (Z, Z) is not in the region of positive profits for either producer. However, if producer 2 chooses a low enough z_2, the point (Z, z_2) moves downwards and reaches the region where producer 1's profits are positive. Producer 2, however, does not profit from innovating, and hence chooses $z_2 = 0$. Since producers are symmetric, the same reasoning applies to producer 2. We, therefore, get two equilibria where producers anti-coordinate: $(Z, 0)$ and $(0, Z)$.

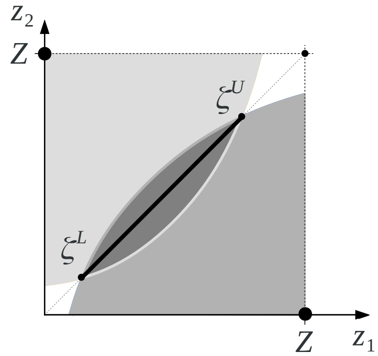
The intuition here is that the market is not big enough to allow two producers to recoup

[7] ζ^L and ζ^U are defined as the solutions to the equation $\underline{z}_1(z) = \underline{z}_2(z)$.

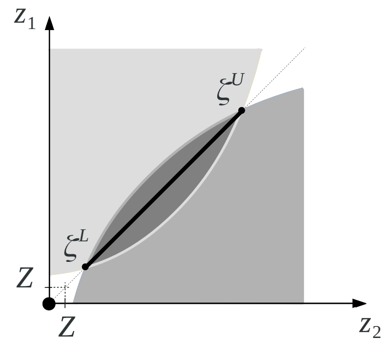
[8] We obtain a similar anti-coordination outcome if Z is smaller than ζ^L, yet larger than $\underline{z}_1(0)$; we don't illustrate this case on the figures for simplicity.



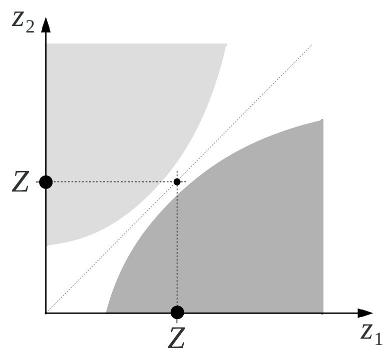
(a) A region exists where joint innovation is profitable. The Nash Equilibrium is at (Z, Z).



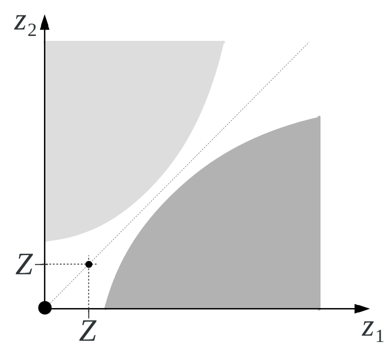
(b) A region exists where joint innovation is profitable, but Z is "too" high and leading to an anti-coordination equilibria at $(Z, 0)$ and $(0, Z)$.



(c) A region exists where joint innovation is profitable, but Z is too low leading to an equilibrium with no innovation.



(d) There is no region where joint innovation is profitable which leads to an anti-coordination equilibria at $(Z, 0)$ and $(0, Z)$.



(e) There is no region where joint innovation is profitable and Z is low leading to an equilibrium with no innovation.

Figure 3: Illustration of best responses functions and Nash equilibria.

their large initial investments[9]. They compete in prices while offerring the same environmental quality, and therefore do not enjoy monopolistic markups from offering the highest environmental quality. Result 1 and our discussion below highlights that this is a common prediction from models of vertical differentiation.

Finally, Figure 3c represents the case where Z is smaller than $\underline{z}_1(0)$. Then, there exists no z smaller or equal to the technological frontier that would allow any producer to make positive profits. The Nash equilibrium is therefore $(0, 0)$. Figures 3d and 3e illustrate situations where the profit surfaces do not overlap. That is, there exists no point (z_1, z_2) where both producers have positive profits when innovating. If Z is larger than $\underline{z}_1(0)$, as on Figure 3d, we have an anti-coordination outcome similar to Figure 3b. But if Z is smaller, as on Figure 3e, we have a unique equilibrium at $(0, 0)$.

Result 1 is useful to establish how two integrated producers innovate. We investigate how adding parallel and shared suppliers affects their best responses in the next sections. Before moving on, it is worth reflecting on the implications. Although models of vertical differentiation are often used to model the effect of industry structure on R&D, such models are rarely used to study green innovation. This is despite the fact that introducing green products is often akin to introducing higher environmental quality products. Details of the present model may differ from typical models of vertical differentiation, but Result 1 is broadly consistent with findings in the literature (Shaked et al. 1982; Tirole 1988). Indeed, the prediction is that firms will differentiate to relax price competition. Here, this is what we find, unless there is an exogenous frontier within the region of joint profitable innovation. This is important because it indicates that in many cases, we should not expect the whole market to switch to green products, even with strong tax incentives. Empirically, this is what we see in many sectors: green products remain niche products that are considered higher-end and are, consequently, sold at a higher price.

[9]Recall that fixed costs increase with z.

2.3 Policy-Induced Innovation

In many cases, policy-makers wish to induce innovation. This could be, for example, to foster climate change mitigation. In this model, this is equivalent to increasing the environmental quality a of a product, and a can be thought as the carbon intensity of the product. Policies shape several parameters of our model that determine whether innovation is profitable. First, there are parameters that influence demand. The size of the market, M, can be changed by procurement policies, while the marginal value of environmental quality a to the consumer's utility, β, can be changed by a carbon tax (which increases the attractiveness of low-carbon products for consumers). Second, policies influence cost parameters. For example, subsidies can lower the upfront cost R_p.

As we know from Result 1, for both producers to innovate at equilibrium, Z must lie in the region of joint innovation. A larger region means that there is a greater range of technological frontiers Z that will induce all producers to switch to greener products. A larger region, therefore, makes such a switch more likely. The size of the region depends on all the policy parameters: β, M and R_p.

Figure 4 illustrates how the policy parameters affect the size of the region and, accordingly, which are most likely to induce an equilibrium where all producers switch to the greener product. It shows the value of ζ^U in the $(\frac{M}{R_p}, \beta)$ plane[10]. In Figure 4, we can also see the region where $M/R_p < g(\beta, \mu, c_p)$, i.e. ζ^U does not exist and therefore there is no joint innovation Nash equilibrium (i.e., no overlapping area as on Figures 3d and 3e). When ζ^U exists, we see that it increases with M/R_p; in fact, it reaches a maximum equal to $\frac{M\mu}{R_p}$. Since β and $\frac{M}{R_p}$ are related through the function g, the figure can also be interpreted as showing the minimum β given $\frac{M}{R_p}$ (or the minimum $\frac{M}{R_p}$ given β) needed to induce both producers to innovate[11].

[10]Since the function $g(\beta, \mu, c_p)$ cannot be solved in closed form, it is here computed numerically.
[11]Recall that a higher value for ζ^U allows for a higher joint innovation equilibrium (Z, Z).

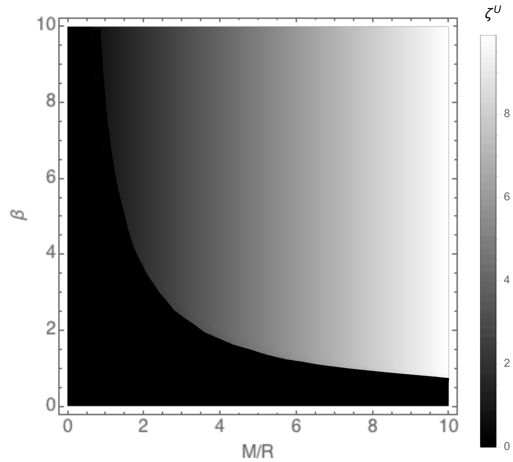


Figure 4: Effect of key parameters on innovation: the $(M/R, \beta)$ plane is divided by $M/R = g(\beta, \mu, c_p)$ into a region which does not support symmetric equilibria with positive innovation (black region), and a region where the range of possible symmetric equilibria (Z, Z) varies from a low level to a maximum level of ambition equal to $\frac{M\mu}{R_p}$.

Figure 4 indicates that, if the goal is to ensure that both firms have an incentive to innovate, increasing $\frac{M}{R_p}$ and increasing β are substitutable policies. Indeed, the minimum value of $M/\underline{R}_p$ such that both producers innovate decreases with β. Conversely, the minimum value $\underline{\beta}$ such that both producers innovate decreases with $\frac{M}{R_p}$. However, the two policies are not equivalent. Setting a high β while being at a low value for $\frac{M}{R_p}$ may support a equilibrium where both producers innovate, but for low innovation efforts. Setting instead a high value of $\frac{M}{R_p}$ and a low minimum β will support a greater and higher range of innovation efforts.

2.4 Adding suppliers

As a stepping stone to our main model, in which the duopolists will buy from a shared supplier, we consider here the case where they each buy from separate suppliers. Results are very similar to the baseline model of integrated producers. The main differences is that now both suppliers and buyers have to weigh the costs and benefits of innovating, which leads to more complicated conditions for characterising the equilibria where all players innovate. There is now the possibility that suppliers and buyers fail to both decide to innovate, and

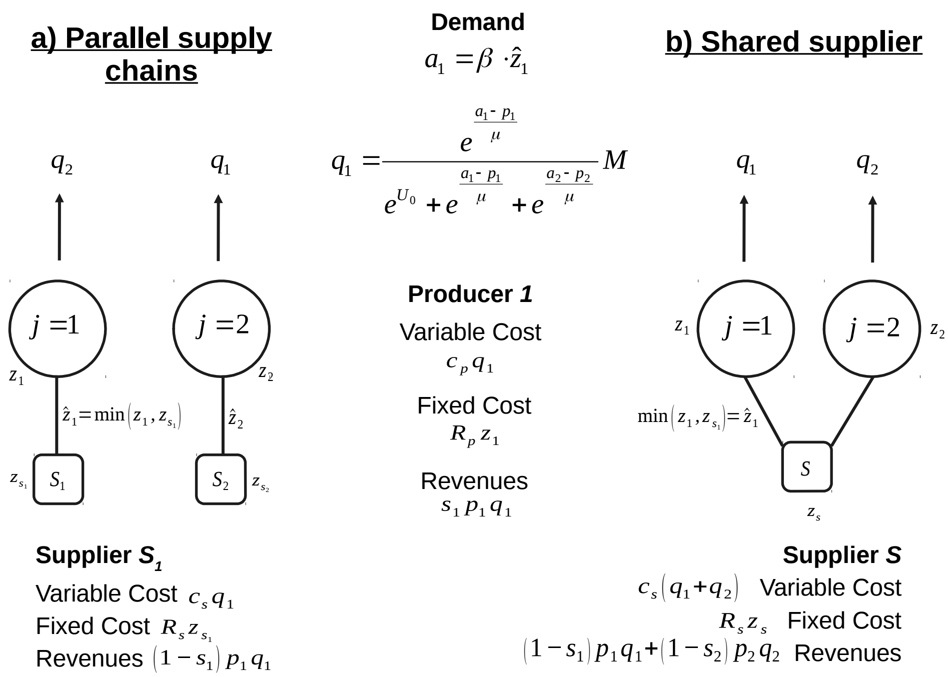


Figure 5: Summary diagram of the producer-supplier models

Note: The "parallel supply chains" model (left-hand side) is described in Section 2.4. The "shared supplier" model is described in Section 3. In both cases, the producers are symmetric. q_1 and a_1 refers to the quantity and environmental quality of the good produced by producer 1; p_1 the price; and the demand function is derived from a multinomial logit model. z_1 refers to the innovation effort chosen by producer 1. When the supplier is shared, z_s denotes the innovation effort chosen by the supplier. When supply chains are parallel, we use the subscripts s_1 for variables relating to the supplier of producer 1. $\hat{z}_1$ corresponds to the *effective* innovation effort for the producer 1 and its supplier. s refers to the share of revenues accruing to producer j.

instead settle for the status quo of no innovation.

Figure 5 summarizes the key elements of the model. The model still consists of two symmetric producers, but we now add a supplier to each of the producers[12]: producer 1 manufactures a good using inputs from its supplier, whose choices, revenues and costs are indexed by the subscript s_1. A key difference with the model in Section 2 is that we now have producers *and* suppliers choosing independently their innovation effort, z. Here, we assume that if producer 1 and supplier s_1 choose different efforts, only the lowest effort can be implemented. In other words, efforts are perfect complements within each supply-chain[13], and we think of innovation as requiring common effort from the actors producing different components of the technology. We denote $\hat{z}$ the *effective* innovation effort and define as: $\hat{z}_1 = \min\{z_1, z_{s_1}\}$[14].

We assume incomplete contracts between suppliers and producers. Revenues are therefore shared ex-post, after investment decisions and costs are incurred. As said before, s is the share of revenues accruing to a producer; we keep the shares as exogeneous parameters[15]. Contrary to the previous section, we now have $s < 1$ since some of the revenue flow to the supplier. The producers and supplier independently choose their effort and share the revenues from their joint production.

As shown on Figure 5, the variable cost of the supplier is $c_s q_1$, and its fixed cost $R_s z_{s_1}$. The suppliers' profits equation is therefore very similar to Equation 2:

$$\pi_{s_1} = (s_1 p_1 - c_s) \times q_1(p_1, p_2, a_1, a_2) - R_s z_{s_1} \tag{3}$$

[12]We assume that the two suppliers are symmetric.

[13]By supply chain, we mean a producer and the supplier. Since we have two producers in the model, there are also two supply chains.

[14]The environmental quality of product 1 is now a function of $\hat{z}_1$: $a_1(\hat{z}_1) = \beta \hat{z}_1$.

[15]Note here that we do not model how players' choices affect their options outside of the relationship. Hence, our model does not include the effect of hold-up, since this is not the focus of our analysis. Here, the share s merely rescales revenues relative to costs. It doesn't necessarily restrict the range of conditions favorable to innovation. It restricts the range only if s poorly reflects the distribution of costs (for example if $s = .5$ when in fact the producer's costs are twice as high as the supplier's).

The sequence of the game is as follows: 1) all players choose their innovation effort (z_1, z_2, z_{s_1} and z_{s_2}) and incur the respective fixed cost; 2) producers choose the price of their product; 3) revenues are divided between producers and suppliers according to the shares s. Since we define $\hat{z}_1$ as the lowest innovation effort chosen between the producer and the supplier, we can derive the Nash Equilibria by solving for the best response $\hat{z}_1(\hat{z}_2)$ of each type of player and then considering which player has the lowest.

Solving for the best responses is very similar to Section 2 where we showed that $M/R_p = g(\beta, \mu, c, s)$ was a critical condition. We now have the same condition for the supplier, $M/R_s = g(\beta, \mu, c_s, 1-s)$, and the key thresholds are now $\zeta^L = max(\zeta_p^L, \zeta_S^L)$, $\zeta^U = min(\zeta_p^U, \zeta_S^U)$ and $\underline{z} = max(\underline{z}_1(0), \underline{z}_s(0))$. Result 2 below formally establishes these conditions.

Result 2. *The Nash Equilibria* $(z_1^*, z_{s_1}^*, z_2^*, z_{s_2}^*)$ *of the two parallel supply-chains innovation game are specified below, where* $g(.)$ *is an implicit function of* β, μ*, the variable costs* (c_p *or* c_s) *and the revenue share* (s *or* $1-s$)*:*

	Parameter Values	*Equilibrium* $(z_1^*, z_{s_1}^*, z_2^*, z_{s_2}^*)$
	$Z < \underline{z}$	$(0,0,0,0)$
If $M/R_p > g(\beta, \mu, c_p, s)$ *and*	$\underline{z} \leq Z < \zeta^L$	$(0,0,Z,Z)$ $(Z,Z,0,0)$ $(0,0,0,0)$
$M/R_s > g(\beta, \mu, c_s, 1-s)$	$\zeta^L \leq Z \leq \zeta^U$	(Z,Z,Z,Z) $(Z,Z,0,0)$ $(0,0,Z,Z)$ $(0,0,0,0)$
	$\zeta^U < Z$	$(0,0,Z,Z)$ $(Z,Z,0,0)$ $(0,0,0,0)$
If $M/R_p < g(\beta, \mu, c_p, s)$ *or*	$Z < \underline{z}$	$(0,0,0,0)$
$M/R_s < g(\beta, \mu, c_s, 1-s)$	$\underline{z} \leq Z$	$(0,0,Z,Z)$ $(Z,Z,0,0)$ $(0,0,0,0)$

Result 2 is a corrolary of Result 1. Each actor is bound by similar conditions as in Result 1. The main addition is to check that these conditions are fulfilled for both supplier and producer. Importantly, there are additional equilibria due to the possibility that suppliers and producers decide to remain at the status quo instead of innovating. Now that we have seen how the model generalizes when adding suppliers, we are ready to analyze the incentives and strategic frictions that arise when suppliers are shared.

3 Green Product Innovation with Shared Suppliers

3.1 Model

We now consider the case where two symmetric producers source inputs from one common supplier. As mentioned in Section 1, and examined in more detail in Section 5, shared suppliers are very common in globalized industrial networks. Our goal, here, is to investigate how shared suppliers affect the Nash equilibria of the innovation game modeled in the previous section. An important implication of sharing a supplier is that the three players are interdependent. The supplier's decision constrains the choices of both downstream competitors. In that sense, the supplier holds monopoly power in deciding the innovation effort. Yet, the producers still compete in prices. This erodes the markups that can be obtained from increasing quality. As a result, the shared supplier's incentives to innovate are lower than if there was a vertically integrated monopolist setting both quality and price, and lower than the cases considered earlier of parallel supply chains.

As shown on Figure 5, the main difference is that we now refer to the common supplier with the subscript s. The *effective* innovation effort in producer 1's supply chain, therefore, is $\hat{z}_1 = \min\{z_1, z_s\}$. The supplier's first-stage profit function can be written as follows:

$$\Pi_s = \Big((1-s)p_1^*(\hat{z}_1, \hat{z}_2) - c_s\Big)q_1^*(\hat{z}_1, \hat{z}_2) + \Big((1-s_2)p_2^*(\hat{z}_1, \hat{z}_2) - c_s\Big)q_2^*(\hat{z}_1, \hat{z}_2) - R_s z_s \tag{4}$$

3.2 Nash Equilibria

As in Section 2.4, we can consider separately what each player would do when they are not constrained by the decision of the other players. Then, the player choosing the lowest optimal effort will de facto "impose" its equilibrium. Let's assume the supplier could choose the effective effort $\hat{z}$ (i.e. $z_s = \hat{z}_1 = \hat{z}_2$). We then obtain the following expression for the supplier's first-stage profits[16]:

$$\Pi_s(z_s) = 2\Big((1-s)p_1^*(z_s) - c_s\Big)q_1^*(z_s) - R_s z_s \tag{5}$$

Remark 2. *The supplier's profit function,* Π_s*, reaches a maximum for some value* z_s^{max}*.*

It is useful to contrast Remark 2 with Remark 1. When we studied the case of two integrated producers, we found that, above a threshold, the profits of the two *competing* producers increased monotonically with z (i.e. they never reached a maximum). With a shared supplier, a critical difference is that the supplier's profits reach a maximum. The reason for this is that the supplier here controls the innovation effort of both downstream producers who then control prices. This thwarts competition over z: instead of competing with another innovating actor, the supplier only competes with the outside option. So eventually, it obtains a high market share. At the same time, prices are kept in check by competition between the producers. From the supplier's viewpoint, the benefits from innovating therefore display decreasing marginal returns, which leads to an interior solution to the supplier's profit maximization problem.

We now restrict the parameter values to focus our attention on how the industrial structure considered here changes the possible equilibria, instead of enumerating all possible cases. Specifically, we restrict $R_s = 2R_p$ and $c_s = 2c$ and $s = 1 - s = \frac{1}{2}$. Together, this implies that the cost structure of the supplier is equivalent to that of each producer. Hence, the differences in behavior that we highlight below do not arise from differences in costs, but from differences in the structural position in the network and the incentives inherent to that position.

Remark 3. *If* $M/R_p > g(\beta, \mu, c_p, s)$*,* $\exists\ z_s^{max} > 0$ *s.t.* $\zeta^L < z_s^{max} < \zeta^U$*.*

[16] Since producers are symmetric, we have $q_1^*(z_s) = q_2^*(z_s)$, and $p_1^*(z_s) = p_2^*(z_s)$.

Remark 3 implies that the shared supplier *restricts* the range of innovation efforts that can be sustained as a joint innovation equilibrium. The Nash equilibrium in this case is that all players innovate with effort equal to $min(Z, z_s^{max})$. The shared supplier will innovate only up to its preferred effort, even if it is below the frontier, and the downstream producers, to remain compatible with their supplier's products, must do the same. On the contrary, as seen previously, players in parallel supply chains would all innovate up to the frontier (as long as $Z \in [\zeta^L, \zeta^U]$).

Further restricting the parameters to the case where $M/R_p > g(\beta, \mu, c_p, s)$, the Nash equilibria of the 3-player innovation game are given in Result 3.

Result 3. *The Nash Equilibria* (z_1, z_2, z_s) *of the 3-player innovation game are:*

	Parameter Values	*Equilibrium* (z_1^*, z_2^*, z_s^*)
	$Z < \underline{z}_1(0)$	$(0, 0, 0)$
If $M/R_p > g(\beta, \mu, c_p, s)$	$\underline{z}_1(0) \leq Z < \zeta^L$	$(0, 0, 0)$ $(0, Z, Z)$ $(Z, 0, Z)$
	$\zeta^L \leq Z \leq \zeta^U$	$(0, 0, 0)$ $(\min\{z_s^{max}, Z\}, \min\{z_s^{max}, Z\}, \min\{z_s^{max}, Z\})$

3.3 Policy-Induced Innovation

We saw in Section 2.2 that the policy parameters affect the existence and size of the region of joint innovation. Here, we consider the effect of these policy parameters on z_s^{max}, the chosen level of effort in a joint innovation equilibrium. Although in our model the three parameters R_S, β and M can all induce a shift from no innovation to some innovation, they are not equally effective in encouraging higher innovation efforts:

Remark 4. *It can be shown that:*

$$\frac{dz_s^{max}}{dR_S} = 0 \qquad \frac{dz_s^{max}}{dM} > 0 \qquad \frac{dz_s^{max}}{d\beta} < 0$$

Remark 4 indicates that changing upfront costs has no effect on the optimal innovation effort since it merely shifts the profit function. However, increasing the market size raises innovation efforts, and increasing β lowers it. As we saw in Result 1 and Figure 4, β must exceed a minimum level to satisfy $M/R > g(\beta, \mu, c, s)$. However, beyond this minimum, it does little to increase ambition. Here we see that with a shared supplier, raising it further is even counter-productive. The reason is that a higher β means that a quality increase can beat the utility from the outside option at a lower value of z_s, and so the market is "seized" for lower innovation efforts. In contrast, raising M increases the range of efforts that are sustained as a joint innovation equilibrium. We interpret this as a note of caution regarding the reliance on carbon taxes to induce innovation in contexts in which green innovation takes the form of vertical product differentiation, at least within the scope conditions of this model.

In Section 5, we use case studies to illustrate the frictions modeled above. We also discuss why such industrial structures arise despite the fact that they impose constraints on the actions of some players. One important limitation of our model is that it is static. In a dynamic setting, we would have to consider the possibility of new entrants ready to offer higher investments. Yet, such entries may be fairly rare in industries with high capital costs. From the point of view of a policy-maker attempting to induce incumbents to upgrade their technologies, the arrival of competing entrants cannot be assumed.

4 Extension: Technological Uncertainty and Coordination Failures

We have so far considered innovation as being of a single type and assumed players had a shared understanding about what it is. However, this is rarely the case. When opportunities for radical technological change first arise, there is usually an initial period of "ferment", in which multiple technological concepts coexist. This creates considerable uncertainty as to the direction that technological change will take (Abernathy et al. 1978; Anderson et al. 1990). For example, in the case of the switch to alternative fuel vehicles, Sierzchula et al.

(2012) documents seven different technologies potentially replacing the internal combustion engine, all at various stages of prototyping and production by different players.

What is the impact of such technological ambiguity on players in mature, highly interdependent supply chains? We think that the configuration we studied in the last section is characteristic of supply chains producing established designs and organized to maximize economies of scale and scope. Yet such a configuration is not necessarily well suited to embrace radical technological change. This is not only because there is a chance of landing on the no-innovation equilibrium due to miscoordination with suppliers (especially when there are many of them), but also because of the risk that players will innovate in different directions. This would destroy the current organization of production and the efficiency gains that it allows.

In what follows, we model the scale and scope efficiencies that may justify the existence of mega-suppliers. We then introduce the possibility that actors mis-coordinate on the direction of innovation and thereby lose these efficiencies.

4.1 Model

We consider again the shared supplier configuration of Figure 5. The key modification is that the variable cost of the supplier now takes the form of a CES function so to capture the existence of scale and scope economies:

$$C_s(q_1, q_2|\rho, k) = \left(q_1^\rho + q_2^\rho\right)^{\frac{k}{\rho}}, \tag{6}$$

where k $\in [0; 1]$ governs the returns to scale, and $\rho > 0$ governs the extent to which the inputs produced by the supplier are substitutable in the cost function[17]. The variable cost and fixed cost for the producers are unchanged, and so is the sequence of the game.

[17] $\rho > 1$ corresponds to economies of scope from producing multiple products, $\rho = 1$ corresponds to purely substitutable products, and $\rho < 1$ corresponds to diseconomies of scope from producing multiple products.

We introduce the possibility that producers innovate in *different technological directions*. In other words, whatever the innovation effort chosen by the producers and the supplier (for example, it could be e.g. $z_1 = z_2 = z_s = Z$), there is a probability, θ, that the producers innovate in different *directions* imposing on the supplier the need for producing very different inputs. This is what we call *miscoordination*. For example, the two producers could decide to launch radically innovative cars, but producer 1 chooses to invest in plug-in electric cars, while producer 2 in hydrogen cars. For the supplier, such directions are not compatible because they require different inputs. To formalize this, we assume that the realization of θ impacts the supplier's economies of scope through the parameter ρ:

$$\rho = \begin{cases} \rho_0 & \text{if Coordination } (Prob = 1 - \theta) \\ \rho_0 - \sigma \hat{z} & \text{if Miscoordination } (Prob = \theta) \end{cases} \tag{7}$$

If producers coordinate, the supplier continues to benefit from economies of scope ($\rho_0 > 1$). If instead they miscoordinate, the supplier loses economies of scope, and the loss increases with the level of effort because higher effort implies larger departures from the status quo technology.

The sequence of the game is as follows: 1) all players choose the innovation effort z, incurring their respective fixed costs; 2) a move of nature determines whether the producers coordinate or miscoordinate on the direction of innovation, which then affects the marginal costs of production of the shared supplier; 3) producers choose the price of their product; 4) revenues are divided between producers and suppliers according to the share s.

4.2 Effects of Miscoordination on the Nash Equilibria

We find that, as the probability of miscoordination θ increases, the Nash Equilibrium effort decreases. We saw in Result 3 that at equilibrium, if $\zeta^L < Z < \zeta^U$, all players choose the same innovation effort, which is "set" by the shared supplier via $min(Z, z_s^{max})$.

Result 4. *As the probability of miscoordination, θ, increases, z_s^{max} decreases.*

Under miscoordination, the supplier must produce different types of inputs for each of the producers. Higher efforts lead to a greater departure from the status quo technology. Logically, this requires more different inputs from the supplier and increases the loss of economies of scope in the event of miscoordination. Consequently, the supplier decreases its innovation effort. In the model, the parameter σ captures how sensitive the supplier is to the risk of miscoordination: in the supplier's cost function, σ governs how innovation efforts affect the elasticity of substitution between the components supplied to the two producers when miscoordination happens. Figure A.1 in the Appendix illustrates this relationship for specific parameter values. It shows that z_s^{max} decreases slowly with θ; then comes a value of θ above which the supplier chooses not to innovate ($z_s^{max} = 0$).

4.3 Policy Effectiveness under Miscoordination Risk

We now investigate how the probability of miscoordination affects the ability to induce radical innovation via the three policy variables β,M and R_S.

Result 5. *We find that the threshold values $\bar{R}_s$, $\underline{M}$ and $\underline{\beta}$ that can induce innovation by the shared supplier are affected by θ:*

$$d\bar{R}_S/d\theta < 0 \qquad d\underline{M}/d\theta > 0 \qquad d\underline{\beta}/d\theta > 0$$

Result 5 indicates that a higher probability of miscoordination reduces the parameter space under which radical innovation is attractive to the supplier. All other things equal, if θ increases, the upfront cost cannot be as high, the market must be larger, and the marginal utility to the consumer of environmental quality improvements must be higher. In turn, if subsidies are used to induce innovation by reducing the upfront costs borne by the firm, these subsidies need to be higher, while procurement policies to increase the market size need to be more vigorous. Similarly, if the marginal utility to consumers of products being green is raised by a carbon tax, then this tax needs to be higher.

The implication of Result 5 is that policies or institutions that help coordinate firms' expectations about the long-term direction of technological change are complementary to economic incentives such as taxes, subsidies and procurement. We discuss the empirical evidence for this claim from past technological transitions in Section 5.

5 Discussion

The last IPCC report warns that, if the world wants to limit anthropogenic warming to less than 1.5 degrees Celsius, greenhouse gases emissions need to decrease to zero net emissions by mid-century (IPCC 2018). Many argue that revolutionary changes in technology are needed to achieve such objectives (Hoffert et al. 2002). For example, Barrett (2009) contends that the required change looks like a technological "revolution" because it "will require fundamental change, achieved within a relatively short period of time." This "revolution" consists as much in bringing some technologies from paper to proofs of concept (e.g., new carbon capture technologies), as it does pushing advanced technologies through the challenges of mass-scale production and diffusion. In that spirit, Pacala et al. (2004) have claimed that much could be achieved with what is already known (for an update, see Davis et al. (2013)). Similarly, the Deep Decarbonization Pathway Project attempts to demonstrate that, by relying on what we already know, the world can achieve a reduction between 70% and 100% by 2100 (Deep Decarbonization Pathways Project 2015). These pathways require fast and massive scaling-up of production and diffusion of technologies and products that are currently still niche. For example, they project about 134 million electric vehicles in 2030 together with 75 million plug-in hybrid electric vehicles, 31 million hydrogen fuel cell vehicles, 27 million compressed pipeline gas vehicles (ibid.). Much of this change requires that large networks of firms redirect their production towards radically different products.

Challenges for green product innovations in industrial networks Our analysis sheds light on why product innovations can take time to occur in mature industries with established players and relationships. We have identified two critical challenges for green product innovations. The first arises in our baseline model and builds on a well-established finding on vertical differentiation in the industrial organization literature. Namely, firms tend to differentiate the environmental quality of their products to relax price competition, with some producers specializing in high-quality (here green) products and others in low-quality (or dirty) products. We see this in many industries. For example, innovative buildings with high environmental performance represent a tiny share of the construction sector. Green buildings remain cutting-edge products with little spillover to the bulk of the market. The challenge then is to find policies to change incentives so that more firms switch to green products.

Although not our main result, we think this insight has important implications for the question of how to induce green innovations. In particular, it has implications for the debate on the effectiveness of taxes, versus subsidies, versus standards in stimulating environmental innovations (e.g. Amir et al. 2018; Bruneau 2004; Fischer et al. 2008). In our static setting with homogeneous consumers and producers, a joint switch to higher quality green products requires an exogenous research frontier. And, the frontier must be located in a certain range relative to other variables (neither too ambitious given the upfront cost and market size, nor too unambitious). We can interpret the research frontier as a common technological goal, but also as a standard (as in the product differentiation model of André et al. (2009)). Thus, our model can also be understood as a situation where standards are necessary for the widespread deployment of new environmental technologies. Second, we find that a larger market relative to upfront costs allows for more ambitious frontiers. In contrast, a tax that increases the appeal of environmental quality doesn't provide adequate incentives beyond a minimal level. Future work should explore whether these insights are robust in a more general model of green product innovation, and test them empirically.

The second challenge comes from our main result (Result 3). To bring about ambitious innovations, producers need to coordinate with suppliers, so that they make complementary investments. It can, in fact, be difficult to find suppliers with the right incentives. In 2014, the Community Innovation Survey found that 20% of surveyed companies attributed their failure to innovate to the difficulty of finding innovation partners. In our model, the problem is most acute when suppliers are shared between several competing producers. This is for two reasons. First, because a shared supplier controls a complement to the innovative project. It can therefore become an innovation bottleneck, and its incentives to innovate are weaker than other players. Second, the supplier risks losing economies of scale and scope that it enjoys under the status quo technology. A number of case studies of the automotive industry illustrate the complex role of relationships with suppliers in the innovation process (and in particular shared suppliers). Our model helps to organize a number of their findings, so we now turn to reviewing some of these cases.

The Case of the automotive industry The automobile is a complex product for which parts and sub-parts that interact are often produced by different firms (MacDuffie et al. 2010). It is not surprising then that supplier-buyer relationships in this industry have received a high degree of scrutiny (e.g., Dyer (1996), MacDuffie et al. (2007), and Sako (2004)). Since the mid-80's, the industrial structure underwent a wave of outsourcing and "de-verticalization" that transformed the industry (Sturgeon et al. 2008). Driven by a need to reduce cost in a globalized market, as well as a conviction that they should emulate the modular structure of the computer industry, producers increasingly outsourced part of the manufacturing. They spun-off some of the subsidiaries producing intermediate parts. Some suppliers merged giving rise to the "mega-suppliers" supplying complex modules to multiple producers (Jacobides et al. 2016). For example, in the 1990s, Nissan announced it would source components from one of Toyota's supplier, Denso (going against a long-standing norm that suppliers should not be shared between two rival supply chains or *keiretsu*). Denso had lower costs thanks to Toyota's large market share which provided greater economies of scale.

If, as we argue in this paper, shared suppliers are obstacles to innovation, why do car manufacturers rely so much on them? The outsourcing wave took place jointly with a move towards greater standardization of intermediate inputs (Ahmadjian et al. 2001). Shared suppliers are co-effective when intermediate inputs have become standardized and innovation is minimal. Yet, Jacobides et al. (2016) use case studies and historical research to argue that manufacturers overlooked contractual risks. They put themselves at risk of surrendering power to mega-suppliers, who became strategic bottlenecks[18]. Soon, the limits of the paradigm became apparent, and manufacturers became wary of shared suppliers.

The following quote, from a Fiat executive, highlights that manufacturers believed suppliers lacked incentives to innovate: "It's all a question of money – suppliers can't imagine spending lots of money. The mega-suppliers want only big volume, they want to stick with processes they know. Their short-term incentive is to stay focused on components. [...] They are not likely to offer us their latest technologies if that threatens their existing investments – this can be a barrier to our innovation." (ibid.). According to Jacobides et al. (ibid.), manufacturers feared that the rents from product differentiation could be eroded by shared suppliers transfering technology to competitors. As a result, manufacturers attempted to exercise a high degree of control on suppliers.

The state of affairs described above can be contrasted with somewhat savvier management of supplier relationships by Japanese auto-makers. As mentioned earlier, Nissan broke the long-standing Japanese *keiretsu* norm of not sharing key suppliers by starting to source from Denso, Toyota's main supplier of electronics. However, when it became clear that electronics were a central and complex component of car technologies, Toyota invested heavily in its internal capacity to manufacture electronics in order to lessen its reliance on Denso and to better monitor and control its dealings with its supplier, who now had split loyalties

[18] The situation was different for the computer industry thanks to the higher degree of modularity in the product architecture (Baldwin et al. 2000).

(Ahmadjian et al. 2001).

These case studies highlight that shared suppliers pose strategic problems in the process of innovation. What are the consequences for the low-carbon transition? With the realization that the traditional combustion-engine-based car is responsible for a significant share of greenhouse gases emissions, the car manufacturing sector is in a period of ferment with many alternative power-train technologies under testing (Sierzchula et al. 2012). Our model suggests that this multitude of technology directions exacerbates the strategic problems documented above, as the "mega-suppliers" have neither the incentive nor the capacity to make the requisite complementary investments, especially given the risk arising from the uncertainty in technological directions.

Although most major manufacturers have announced ambitious plans for new clean products, investments in these new models still is limited and driven by compliance with regulatory mandates. For example, Wells et al. (2012) argue that current electric vehicles tend to be of inferior quality because the architecture of most models has not been sufficiently adapted to the new requirement of batteries. Critically, the industry has not scaled up its production and sales to the level hoped for by the Obama administration when it decided to make sizable investments in the battery supply-chain (Canis 2013).

According to our model, we would expect that the players best positioned to make innovative and successful investments in alternative vehicles would be either firms with long-term relationships with their main suppliers, capable of co-design via relational contracts, such as Toyota, or vertically integrated firms. As a case in point, Tesla, arguably the most innovative and successful producer of electric vehicles, is a new entrant, free from linkages with the historical network of suppliers. Dyer et al. (2015) explain that Tesla initially tried to establish links with the global supply chain to reduce costs, but having manufacturing so spread out led to "massive coordination problems". In response, they chose to manufacture most components in-house. This helped them bring electric cars to the market because the pace of change was otherwise too fast for suppliers to follow. So far, however, electric cars have yet to reach the status of mass-production. To do so, incumbents and their suppliers have to switch as well.

Institutions to coordinate innovation in industrial networks Echoing earlier work in strategic management and business history (Chandler 1990; Teece 1986), our model suggests that vertical integration can solve the coordination problem that we have identified. However, many forces push the firm away from vertical integration (labor costs, conquest of distant markets, shareholder pressure). Hence, it is important to find ways of working with critical suppliers towards innovative products. Our model sheds light on why some institutions can help in this regard. The switch to producing significantly greener innovations is likely to take place only if producers coordinate to create rents and economies of scope for their shared suppliers. Thus an institution that helps coordinate expectations of different players about the direction of technological change would help increase ambition and firms' collective capabilities, as argued by the literature on institutions of innovation (Ansell 2000; Mazzucato 2011; O'Riain 2004). We also find that it would complement economic incentives to innovate.

The need to coordinate industrial actors echoes studies of the Defense Advanced Research Projects Agency (DARPA) (Fuchs 2010) and discussions about how to replicate it in the energy sector (Anadon et al. 2014; Bonvillian et al. 2011; Fuchs 2009; VanAtta 2007). Fuchs (2010) shows how DARPA facilitates coordination among competitors. She describes DARPA's technology policy as "embedded government agents" that supports the coordination of technology development across a vertically fragmented industry. DARPA's actions consist in bringing together established vendors with academics and start-ups with the goal to support knowledge-sharing within industry, and between competitors.

A quote from an industry participant at a DARPA seminar illustrates the incentives we analyze in our model: "You just can't make anything happen in industry (today) on your own, because it's completely impossible. You have to find a partner, you have to convince your competition this is the right thing to do. You're guiding people [your competitors], ... and they ask, 'Why are you helping me with this?,' and the fact is you give them information so the suppliers are in the right place to help you." Here, the industry actor quoted by Fuchs (2010) clearly makes reference to the importance of coordinating supply chains, and in particular shared suppliers, in the hope of fostering technological change in the industry.

Finally, our model also shows that shared suppliers are key nodes in the network, capable of triggering change. Indeed, investment by the shared supplier can induce complementary investments by the other players. Thus, a subsidy or other form of incentive to push the shared supplier may be particularly effective in triggering wider investments (see Sakovics et al. 2012, for a similar logic in resolving coordination problems). Our model thus provides a formal analysis for Dani Rodrik's claim that an effective industrial policy must take into account what he calls "coordination externalities", i.e. the fact that any player choosing to innovate takes risks of not being followed by partners, while making it easier for partners to follow (Rodrik 1996, 2014).

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A Proofs

We first establish two intermediate results, Remark A.1 and A.2 below, which will be useful to establish other results. Since the producers are symmetric, we express proofs and results for producer 1 WLOG.

Remark A.1. $\forall \hat{z}_1, \quad \frac{dp_1^*(\hat{z}_1)}{d\hat{z}_1}|_{\hat{z}_2} \geq 0$

Proof.

$$\begin{aligned}\frac{d\Pi_1}{dp_1} = 0 &\Leftrightarrow sq_1 + (sp_1 - c_p)\frac{\partial q_1}{\partial p_1} = 0 \\ &\Leftrightarrow sq_1 + (sp_1 - c_p)\frac{1}{\mu}q_1(\frac{q_1}{M} - 1) = 0 \\ &\Leftrightarrow \frac{\mu}{1 - \frac{q_1}{M}} + \frac{c_p}{s} - p_1 = F(p_1, \hat{z}_1, p_2, \hat{z}_2) = 0\end{aligned}$$

In turn, by the implicit function theorem:

$$\begin{aligned}\frac{dp_1^*}{d\hat{z}_1} &= -\frac{\frac{\partial F}{\partial \hat{z}_1}}{\frac{\partial F}{\partial p_1}} = -\frac{\frac{\mu}{M}\frac{1}{(1-\frac{q_1}{M})^2}\frac{\partial q_1}{\partial \hat{z}_1}}{-1 + \frac{\mu}{M}\frac{1}{(1-\frac{q_1}{M})^2}\frac{\partial q_1}{\partial p_1}} = \frac{\frac{\partial q_1}{\partial \hat{z}_1}}{\frac{M}{\mu}(1 - \frac{q_1}{M})^2 - \frac{\partial q_1}{\partial p_1}} \\ &= \frac{\frac{\beta}{\mu}q_1(1 - \frac{q_1}{M})}{\frac{M}{\mu}(1 - \frac{q_1}{M})^2 + \frac{1}{\mu}q_1(1 - \frac{q_1}{M})} = \frac{\beta q_1}{M - q_1 + q_1} = \frac{\beta}{M}q_1 > 0 \qquad \text{(A.1)}\end{aligned}$$

□

Remark A.2. $\forall \hat{z}_1, \quad \frac{dq_1}{d\hat{z}_1}|_{\hat{z}_2} \geq 0$

Proof. Total derivative of demand:

$$\frac{dq_1}{d\hat{z}_1} = \frac{\partial q_1}{\partial \hat{z}_1} + \frac{\partial q_1}{\partial p_1}\frac{dp_1^*}{d\hat{z}_1} + \frac{\partial q_1}{\partial p_2}\frac{dp_2^*}{d\hat{z}_1}$$

This gives:

$$\frac{dq_1}{d\hat{z}_1} = \beta q_1\Big(1 - \frac{q_1}{M}\Big) + \frac{1}{\mu}q_1\Big(\frac{q_1}{M} - 1\Big)\frac{\beta}{M}q_1 - \frac{1}{M\mu}q_1 q_2\frac{\beta q_1 q_2}{M^2(1-\frac{q_2}{M})}$$

Simplifying:

$$\frac{dq_1}{d\hat{z}_1} = \beta q_1(1 - \frac{q_1}{M})^2 - \frac{1}{M^3}(q_1 q_2)^2\frac{\beta}{(1-\frac{q_2}{M})}$$

Since $M - q_1 \geq q_2$, we have:

$$\begin{aligned}\frac{dq_1}{d\hat{z}_1} &\geq \beta\frac{q_1}{M^2}q_2^2 - \frac{1}{M^2}(q_1 q_2)^2\frac{\beta}{(M-q_2)}\\ &= \frac{\beta}{M^2}q_1 q_2^2(\frac{M-q_2-q_1}{M-q_2}) \geq 0\end{aligned}$$

□

Proof of Remark 1

Proof. Consider the stage 2 profit function (induced by equilibrium prices):

$$\Pi_1^*(z_1, z_2) = \Big(sp_1^*(z_1, z_2) - c_p\Big)q_1^*(z_1, z_2) - R_p z_1 \tag{A.2}$$

At equilibrium, we have: $p_1^* = \frac{c_p}{s} + \frac{\mu}{1-q_1^*/M} \Leftrightarrow q_1^* = M\Big(1 - \frac{s\mu}{sp_1^* - c_p}\Big)$

Hence, we can rewrite Eq. A.2 as:

$$\Pi_1^*(z_1, z_2) = M\Big(sp_1^*(z_1, z_2) - c_p - \mu s\Big) - R_p z_1$$

Taking the derivative with respect to z_1 and knowing that $\frac{dp_1^*}{z_1} = \frac{\beta}{M}q_1 > 0$ from Eq. A.1:

$$\frac{d\Pi_1^*(z_1, z_2)}{dz_1} = Ms\frac{dp_1}{dz_1} - R_p = \beta s q_1 - R_p. \tag{A.3}$$

We know that q_1 monotonically increases with z_1 (Remark A.2). q_1 therefore takes values between a minimum, call it q_1^0, when $z_1 = 0$, and up to M when z_1 goes to infinity[19]. If $\beta s M < R_p$, then $\frac{d\Pi_1^*(z_1,z_2)}{dz_1}$ is always negative and the highest possible profits will always be for $z_1 = 0$: there is no incentives for more radical innovation. We make the assumption that $R_p < \beta s M$ to rule this out. On the contrary, if $R_p < \beta s q_1^0$, $\frac{d\Pi_1^*(z_1,z_2)}{dz_1}$ is always positive and highest profits are reached for $z_1 = \mathbf{Z}$. In the last case, when $\beta s q_1^0 < R_p < \beta s M$, there exists a value $\tilde{z}_1(z_2) > 0$ above which $\frac{d\Pi_1^*(z_1,z_2)}{dz_1}$ is positive, meaning profits increase monotonically. This means that the profit function of the producer increases monotonically with z_1 beyond some threshold value $\tilde{z}_1$, and becomes higher than the value at $z_1 = 0$ after another threshold $\underline{z}_1$. This threshold value $\underline{z}_1$ does in fact depend on z_2 the innovation level of the other player. We can therefore define the function $\underline{z}_1(z_2)$ denoting the minimum level of innovation for firm 1 so that profits become larger than under $z_1 = 0$, given z_2 the value chosen by firm 2. In the same way, we can define $\underline{z}_2(z_1)$. Thus, either $\underline{z}_1(z_2) \in [0, Z]$, or $\Pi_1^*(z_1, z_2) < \Pi_1^*(0, z_2)$ for $z_1 \in [0, Z]$. □

Proof of Result 1

Proof. Given Remark 1, both firms profits from innovating at level z iff:

$$\begin{cases} \Pi_1(0, z) \leq \Pi_1(z, z) \\ \Pi_2(0, z) \leq \Pi_2(z, z) \end{cases} \Leftrightarrow \begin{cases} \underline{z}_1(z) \leq z \\ \underline{z}_2(z) \leq z \end{cases} \tag{A.4}$$

The existence and location of equilibria thus depend on the set of points defined by $\underline{z}_1(z)$ and $\underline{z}_2(z)$. In what follows, we further characterize those points as a function of the parameters of the model. We start from the definition of $\underline{z}_1(z)$ and $\underline{z}_2(z)$, that is any z such

[19] There will be a different q_1^0 for every z_2. The smallest q_1^0 will be for $z_2 = Z$

that $\Pi_1(0,z) = \Pi_1(z,z)$ and $\Pi_2(0,z) = \Pi_2(z,z)$. From this, we derive:

$$\begin{aligned}
\Pi_1(0,z) = \Pi_1(z,z) &\Leftrightarrow (sp_1^*(0,z) - c_p)q_1^*(0,z) = (sp_1^*(z,z) - c_p)q_1^*(z,z) - R_p z \\
&\Leftrightarrow \frac{s\mu}{1 - q_1^*(0,z)/M} q_1^*(0,z) = \frac{s\mu}{1 - q_1^*(z,z)/M} q_1^*(z,z) - R_p z \\
&\Leftrightarrow z = \frac{sM\mu}{R_p}\left(\frac{\frac{q_1^*(z,z)}{M}}{1 - \frac{q_1^*(z,z)}{M}} - \frac{\frac{q_1^*(0,z)}{M}}{1 - \frac{q_1^*(0,z)}{M}}\right) \equiv T(z) \qquad \text{(A.5)}
\end{aligned}$$

$T(z)$ is a map and we search for fixed points of this map. We note that $z = 0$ is a solution to Equation A.5. $z = 0$ is the only solution if and only if T is a contraction map. This means that, for all $z > 0$, $\Pi_1(0,z) > \Pi_1(z,z)$. There is, in this case, no equilibrium at the technology frontier where both firms innovate. In that case, if $Z > \underline{z}_1(0)$, we have the anti-coordination game where one of the two firms innovates and not the other ($(0,Z)$ or $(Z,0)$, else neither does (the equilibrium is $(0,0)$).

If T is not a contraction map, then there are more fixed points. The dominant term in $T(z)$ is $\frac{sM\mu}{R_p}\frac{\frac{q_1^*(z,z)}{M}}{1-\frac{q_1^*(z,z)}{M}}$. This is equal to $\frac{sM\mu}{R_p}\frac{1}{1+e^{-\frac{1}{\mu}(\beta z - p^*(z,z))}}$, a logistic function. Hence T behaves like a logistic function and $T(z) = z$ can at most have 3 roots: $z = 0$, ζ^L and ζ^U.

In the case where $T(Z) = z$ has three solutions, we can distinguish between three cases:

- $Z \in [\zeta^L, \zeta^U]$. Then we know that $Z > \underline{z}_1(Z)$ and $Z > \underline{z}_2(Z)$, so the best response of both actors is to innovate maximally, i.e. at Z, so (Z,Z) is the unique symmetric Nash equilibrium.

- $Z < \zeta^L$, then $Z < \underline{z}_1(Z)$ and $Z < \underline{z}_2(Z)$, so Z cannot constitute a symmetric Nash equilibrium. In this case, we can distinguish between the situation where $Z > \underline{z}_1(0)$, in which case the best response of firm j to the other firm not innovating is to innovate at level Z, yielding once again the anti-coordination equilibria. If on the other hand, $Z < \underline{z}_1(0)$ and $Z < \underline{z}_2(0)$, the best response of both actors is to not innovate, and the

Nash equilibrium is $(0, 0)$.

- If $Z > \zeta^U$, then $Z < \underline{z}_1(Z)$ and $Z < \underline{z}_2(Z)$ so it cannot be a symmetric Nash equilibrium. Yet, $Z > \underline{z}_1(0)$. Hence, if firm 2 does not innovate, then firm 1' best response is to innovate at level Z, and vice versa. This yields two anti-coordination equilibria $(Z, 0)$ and $(0, Z)$.

What parameter values determine whether $T(z)$ has roots other than $z = 0$ and therefore support the equilibrium where both firms innovate at the technology frontier? Denote z^1 the point where $T'(z^1) = 1$. We seek the combination of parameters such that $T(z^1) = z^1$, such that the line z is just tangent to $T(z)$ in one point, marking the transition between the regime where $z = 0$ is the single root and the regime where there are three roots. Unfortunately, it is not possible to arrive at an analytical expression for z^1 and thereby for the relation between parameters at the transition due to the fact that q^* is defined implicitly. However, we can identify the relevant set of parameters. These are the ratio M/R_p, and the parameters that affect the equilibrium price and the equilibrium market share: β, μ, c_p and s. Of these, we have singled out M, R_p and β as influenced by policy. Importantly, M and R_p enter only as a ratio, and as a linear prefactor in Eq. A.5, and do not affect the term $\frac{\frac{q_1^*(z,z)}{M}}{1-\frac{q_1^*(z,z)}{M}} - \frac{\frac{q_1^*(0,z)}{M}}{1-\frac{q_1^*(0,z)}{M}})$. Hence, we can define the condition as $\frac{M}{\underline{\mathrm{R}}_p} = g(\beta, \mu, c, s)$, where $\frac{M}{\underline{\mathrm{R}}_p}$ is the minimum value of the market size to upfront cost ratio given β that is compatible with mutual innovation. g then is defined as:

$$g(\beta, \mu, c_p, s) = \frac{1}{s\mu} \frac{z^1}{\frac{\frac{q_1^*(z^1,z^1)}{M}}{1-\frac{q_1^*(z^1,z^1)}{M}} - \frac{\frac{q_1^*(0,z^1)}{M}}{1-\frac{q_1^*(0,z^1)}{M}}} \tag{A.6}$$

□

Result 2

This result is a Corollary of Result 1. Now each competing product is produced by two players (a supplier and final producer), with profit functions that have the same functional form but potentially different cost parameters. Since they make independent decisions, in addition to the conditions inducing the final producers to innovate (specified in Result 1), we must consider a similar set of conditions for the suppliers and how they overlap with that of the final producers.

Under the first condition in the table outlining the equilibria in Result 1, we have $M/R_p > g(\beta, \mu, c_p, s)$ AND $M/R_s > g(\beta, \mu, c_s, 1-s)$. This means that from both the point of view of suppliers and of producers, the region of profitable mutual innovation is non-empty. For the producers, it is delineated by $[\zeta_p^L, \zeta_p^U]$ and for the suppliers it is delineated by $[\zeta_s^L, \zeta_s^U]$. Due to the fact that investments are perfect complements, the region of profitable mutual innovation from the point of view of the whole supply-chains is given by $[\zeta^L, \zeta^U] = [max(\zeta_p^L, \zeta_S^L), min(\zeta_p^U, \zeta_s^U)]$. If Z lies within that interval, it is an equilibrium for all to innovate at level Z. As in Result 1, if $Z < \underline{z}(0)$, no innovation occurs. If $0 < Z < \zeta^L$ or $Z > \zeta^U$, only one of the two supply-chains can innovate. Furthermore, if either of the two inequalities $M/R_p > g(\beta, \mu, c_p, s)$ and $M/R_s > g(\beta, \mu, c_s, 1-s)$ fail, then the region of profitable mutual innovation disappears (as usual, because investments from across the supply-chain are perfect complements), and we have the same anti-coordination equilibria as in 1 if $Z > \underline{z}(0)$, and no innovation otherwise.

In addition to the equilibria that already appear in Result 1, we have several more equilibria that arise because suppliers must coordinate with their respective final producer. Indeed, in addition to the competitive innovation game, we also have a vertical coordination game. Hence, in all cases $(0, 0, 0, 0)$ is a possible equilibrium because any of the two suppliers should not innovate if their respective producer is not innovating, and conversely for the producers. For the same reason, we have two additional anti-coordination equilibria in the favorable case that supports (Z, Z, Z, Z): $(Z, Z, 0, 0)$ is an equilibrium because supplier 2 doesn't want to innovate if producer 2 doesn't innovate, and vice versa (even though they could both agree to a combined move to Z), the same logic applying to the equilibrium

$(0, 0, Z, Z)$.

Proof of Remark 2

The critical point is that z now is *common* to all players, including to both competing final producers. The derivative of the shared supplier's profit with respect to the common z is:

$$\frac{d\Pi_s}{dz} = 2s\frac{dp_1^*}{dz}q_1(z, p_1^*(z)) + (2sp_1^*(z) - c_s)\frac{dq_1}{dz} - R_s = 0 \equiv F_S \tag{A.7}$$

Here $\frac{dp_1^*}{dz}$ and $\frac{dq_1}{dz}$ are with respect to the common z, affecting both competitors and therefore differs from the derivatives given in Remarks A.1 and A.2, where z_1 and z_2 were taken to vary independently. To derive them, we start with the partial derivative of q_1 with respect to z:

$$\begin{aligned}\frac{\partial q_1}{\partial z} &= M\frac{\frac{\beta}{\mu}e^{(\beta z - p_1^*)/\mu}(e^{(\beta z - p_1^*)/\mu} + e^{(\beta z - p_2^*)/\mu} + 1) - \frac{\beta}{\mu}e^{(\beta z - p_1^*)/\mu}(e^{(\beta z - p_1^*)/\mu} + e^{(\beta z - p_2^*)/\mu})}{(1 + e^{(\beta z - p_1^*)/\mu} + e^{(\beta z - p_2^*)/\mu})^2} \\ &\Rightarrow \frac{\beta}{\mu}q_1(1 - \frac{q_1 + q_2}{M})\end{aligned} \tag{A.8}$$

We now use Equation A.8 to derive $\frac{dp_1^*}{dz}$, using the function F_S given by Eq. A.7.

$$\begin{aligned}\frac{dp_1^*}{dz} &= -\frac{\frac{\partial F_S}{\partial z}}{\frac{\partial F_S}{\partial p}} = -\frac{\frac{\mu}{M}\frac{1}{(1-\frac{q_1}{M})^2}\frac{\partial q_1}{\partial z}}{-1 + \frac{\mu}{M}\frac{1}{(1-\frac{q_1}{M})^2}\frac{\partial q_1}{\partial p_1}} = \frac{\frac{\partial q_1}{\partial z}}{\frac{M}{\mu}(1 - \frac{q_1}{M})^2 - \frac{\partial q_1}{\partial p_1}} \\ &= \frac{\beta q_1(1 - \frac{q_1+q_2}{M})}{M(1 - \frac{q_1}{M})^2 + q_1(1 - \frac{q_1}{M})}\end{aligned} \tag{A.9}$$

In comparison to Eq. A.1, we see that the price will not increase monotonically with z and instead will reach a plateau (since the numerator of Equation A.9 tends to 0).

Using Equation A.9, we can get the total derivative $\frac{dq_1}{dz}$:

$$\begin{aligned}\frac{dq_1}{dz} &= \frac{\partial q_1}{\partial z} + \frac{dp_1^*}{dz}\frac{\partial q_1}{\partial p_1} + \frac{dp_2^*}{dz}\frac{\partial q_1}{\partial p_2} \\ &= \frac{\beta}{\mu} q_1 (1 - \frac{q_1+q_2}{M}) + \frac{\beta q_1 (1 - \frac{q_1+q_2}{M})}{M(1-\frac{q_1}{M})^2 + q_1(1-\frac{q_1}{M})} (\frac{q_1}{\mu}(\frac{q_1+q_2}{M} - 1)) \end{aligned} \tag{A.10}$$

Inserting Equation A.9 and A.10 in Equation A.7, we get:

$$\begin{aligned}\frac{d\Pi_s}{dz} = \beta q_1 (1 - \frac{q_1+q_2}{M}) \Big(\frac{2(1-s)q_1}{M-q_1} + \\ \frac{1}{\mu}(2sp_1^*(z) - c_s)(1 - \frac{1}{M-q_1}) \Big) - R_s \end{aligned} \tag{A.11}$$

The prefactor in Equation A.11 goes to 0 as z increases, so eventually, this derivative becomes negative. By the mean value theorem, it must therefore be equal to 0 for some value z at which it reaches a maximum.

Proof of Remark 3

Since $\Pi_s(z) = \Pi_1(z)$, we know that $\Pi_s(\zeta^L) = 0$ and $\Pi_s(\zeta^U) = 0$ and for $z \notin (\zeta^L, \zeta^U)$, we have $\Pi_s(z) < 0$. Hence z_s^{max} must be contained in the interval $[\zeta^L, \zeta^U]$.

Proof of Result 3

This is a corollary of Results 1 and 3.

We know that $0 < \zeta^L < z_s^{max} < \zeta^U$. Therefore, if Z is within the region $[\zeta^L, \zeta^U]$, the supplier chooses her optimum z_s^{max} unless Z is lower. So the equilibrium is given by $min(Z, z_s^{max})$. We don't need to consider the case $Z > \zeta^U$ because in that case $z_s^{max} < Z$ and so z_s^{max} determines the outcome. If $Z < \zeta^L$, then Z is the upper limit since $z_s^{max} > Z$. Hence, we get the same two possibilities as in Result 1: if $Z < \underline{z}(0)$, the equilibrium is $(0,0,0)$ and if $\zeta^L > Z > \underline{z}_1(0)$, then we have the anti-coordination equilibria $(Z, 0, Z)$ and $(0, Z, Z)$. Finally, we always have the possibility that all actors fail to coordinate on a positive level of innovation and so $(0,0,0)$ is always an additional equilibrium.

Proof of Remark 4

To establish these results, we use the implicit function theorem, and therefore seek to find the signs of $\frac{\partial F_s}{\partial M}$, $\frac{\partial F_s}{\partial \beta}$ and $\frac{\partial F_s}{\partial R_s}$. To do so, we will use an approximation for F_s. Comparing Equations A.8 and A.9, we see that $\frac{\partial q_1}{\partial z_s} >> \frac{dp_1^*}{dz_s}$ since they both have the same numerator but $\frac{dp_1^*}{dz_s}$ has a much larger denominator. Hence, we can approximate A.7 by considering that $\frac{dq_1}{dz_s} \approx \frac{\partial q_1}{\partial z_s}$ and that the term involving $\frac{dp_1^*}{dz_s}$ is dominated by $\frac{dq_1}{dz_s}$. This justifies using the following approximation:

$$F_s \approx (2sp_1^* - c_s)\frac{\beta}{\mu}q_1^*(1 - \frac{2q_1^*}{M}) - R_s \tag{A.12}$$

where the $2q_1$ comes from the fact that we are in the symmetric case where $q_1 + q_2 = 2q_1$. p_1^* stands for $p_1^*(z_s^*)$ and q_1^* stands for $q_1(z_s^*, p_1^*(z_s^*))$.

Now we study $\frac{\partial F_s}{\partial M}$. Using $q_1 = MP_1$ where P_1 is the market share of the producer, we get:

$$\frac{\partial F_s}{\partial M} \approx \frac{\partial}{\partial M}(2sp_1^* - c_s)\frac{\beta}{\mu}MP_1^*(1 - 2P_1^*) - R_s \tag{A.13}$$

$$= (2sp_1^* - c_s)\frac{\beta}{\mu}P_1(1 - 2P_1) \geq 0 \tag{A.14}$$

This establishes that $\frac{dz_s^{max}}{dM} \geq 0$.

We now study $\frac{\partial F_s}{\partial \beta}$.

$$\frac{\partial F_s}{\partial \beta} \approx 2s\frac{dp_1^*}{d\beta}\frac{\beta}{\mu}q_1^*(1 - \frac{2q_1^*}{M}) + \\ (2sp_1^* - c_s)\frac{q_1}{\mu}(1 - \frac{2q_1}{M}) + (2sp_1^* - c_s)\frac{\beta}{\mu}\frac{\partial q_1}{\partial \beta}(\frac{M - 4q_1^*}{\mu M}) \tag{A.15}$$

We have that $\frac{dp_1^*}{d\beta} = \frac{\mu}{2M(1-q_1/M)^2}\frac{\partial q_1}{\partial \beta}$. Hence the first term in the above equation is of the order $1/M^2$, while the other two terms are of the order $1/M$. We thus ignore the first term, as the overall sign will be dominated by the sign of the other two terms. In addition, $\frac{\partial q_1}{\partial \beta} = \frac{z_s^{max}}{\mu} q_1(1 - \frac{2q_1}{M})$. Hence:

$$\frac{\partial F_s}{\partial \beta} \approx (2sp_1^* - c_s)\frac{q_1^*}{\mu}(1 - \frac{2q_1^*}{M}) + (2sp_1^* - c_s)\frac{q_1^*}{\mu}(1 - \frac{2q_1^*}{M})\frac{\beta z_s^*}{\mu}\frac{M - 4q_1^*}{\mu M} \tag{A.16}$$

At z_s^{max}, we know that $(2sp_1^* - c_s)q_1^* - R_s z_s^{max} > 0$. Hence $z_s^* < \frac{(2sp_1^* - c_s)q_1^*}{R_s}$. Additionally, the FOC ($F_s = 0$) gives us $R_s \approx (2sp_1^* - c_s)\frac{\beta}{\mu}q_1^*(1 - \frac{2q_1^*}{M})$. So we can further write

$$z_s^* < \frac{(2sp_1^* - c_s)q_1^*}{(2sp_1^* - c_s)\frac{\beta}{\mu}q_1^*(1 - \frac{2q_1^*}{M})} = \frac{1}{\frac{\beta}{\mu}(1 - \frac{2q_1^*}{M})}.$$

Putting this into Equation A.16, we get:

$$\frac{\partial F_s}{\partial \beta} < (2sp_1^* - c_s)\frac{q_1^*}{\mu}(1 - \frac{2q_1^*}{M}) + (2sp_1^* - c_s)\frac{q_1^*}{\mu}(1 - \frac{4q_1^*}{M}) \tag{A.17}$$

$$= (2sp_1^* - c_s)\frac{q_1^*}{\mu}(2 - \frac{6q_1^*}{M}) < 0 \tag{A.18}$$

This is negative because at equilibrium, $q_1^* > 1/3$. This establishes that $\frac{dz_s^{max}}{d\beta} < 0$.

Proof of Result 4

Proof. We first show that the maximizer z_s^{max} that arises in the equilibrium $(z_s^{max}, z_s^{max}, z_s^{max})$ decreases with θ. Then we will show that as θ increases, the equilibrium can shift to $(0, 0, 0)$. By the envelope theorem, $\frac{dz_s^{max}}{d\theta} = \frac{\partial^2 E[\Pi_s]/\partial\theta\partial z_s}{-\partial^2 E[\Pi]^S/\partial z_s \partial z_s}$, where the derivatives are estimated at z_s^{max}. Since at z_s^{max} the denominator is positive, the sign is determined by the sign of the cross-derivative.

As before, both producers produce the same quantity at the same price, which we denote $q_1^*(z_s)$ and $p_1^*(z_s)$. Also denote $C^c(z_s)$ the cost under successful coordination and $C^m(z_s)$ the cost under miscoordination.

$$\frac{\partial E[\Pi_s]}{\partial z_s} = 2s\left(\frac{\partial p_1^*}{\partial z_s}q_1(z_s) + p_1^*\frac{dq_1^*}{dz_s}\right) - R_s - (1-\theta)\frac{\partial C^c}{\partial q_1}\frac{dq_1^*}{dz_s} - \theta\left(\frac{\partial C^m}{\partial q_1}\frac{dq_1^*}{dz_s} + \frac{\partial C^m}{\partial \rho}\frac{d\rho}{dz_s}\right)$$

$$\Rightarrow \frac{\partial^2 E[\Pi_s]}{\partial\theta\partial z_s} = -\underbrace{\left(\frac{\partial C^m}{\partial q_1} - \frac{\partial C^c}{\partial q_1}\right)}_{>0}\underbrace{\frac{dq_1^*}{dz_s}}_{\geq 0} - \underbrace{\frac{\partial C^m}{\partial \rho}}_{<0}\underbrace{\frac{\partial \rho}{\partial z_s}}_{<0} < 0 \tag{A.19}$$

The sign of each term is evident given previous results, except for $\frac{\partial C^m}{\partial q} - \frac{\partial C^c}{\partial q}$, which is positive because $\frac{\partial C^m}{\partial q} - \frac{\partial C^c}{\partial q} = (2^{k/(1-\sigma z_s^{max})} - 2^k)kq_1^{k-1} > 0$.

Within the region in which the $(z_s^{max}, z_s^{max}, z_s^{max})$ solution holds ($z_s^{max} \geq \zeta^L$ and $E\Pi(z_s^{max}) > \Pi(0)$), we therefore have that z_s^{max} decreases with θ. But θ also changes the size of that region. First, since z_s^{max} decreases as the chance of miscoordination increases, it could fall under ζ^L (which does not vary with θ, switching the NE to, at best, $\zeta^L, \zeta^L, \zeta^L$). Second, by the envelope theorem, $\frac{dE\Pi(z_s^{max},\theta)}{d\theta} = \frac{\partial E\Pi(z_s^{max}(\theta),\theta)}{\partial\theta}$. This is $-C^c(z_s^{max}) + C^m(z_s^{max} > 0$ since costs under miscoordination are higher than under coordination. Hence, the profits decrease as the chance of miscoordination increases. In particular, the profits can drop under $\Pi_i(0)$, switching the NE to $(0, 0, 0)$. These possible discrete changes in the NE lead to the same conclusion that an increase in the chance of miscoordination decreases the equilibrium value of the innovation. □

Figure A.1 illustrates this result.

Proof of Result 5

Proof. Consider a generic function $f(x, y)$. Define $\underline{\mathrm{x}}(y)$ such that $f(\underline{\mathrm{x}}, y) = 0$. Then:

- the combination of $f_y(x) > 0$ and $f_x(y) > 0$ in the vicinity of $\underline{\mathrm{x}}(y)$, form a sufficient condition for $\frac{d\underline{\mathrm{X}}}{dy} < 0$
- the combination of $f_y(x) < 0$ and $f_x(y) < 0$ in the vicinity of $\underline{\mathrm{x}}(y)$, form a sufficient condition for $\frac{d\underline{\mathrm{X}}}{dy} < 0$

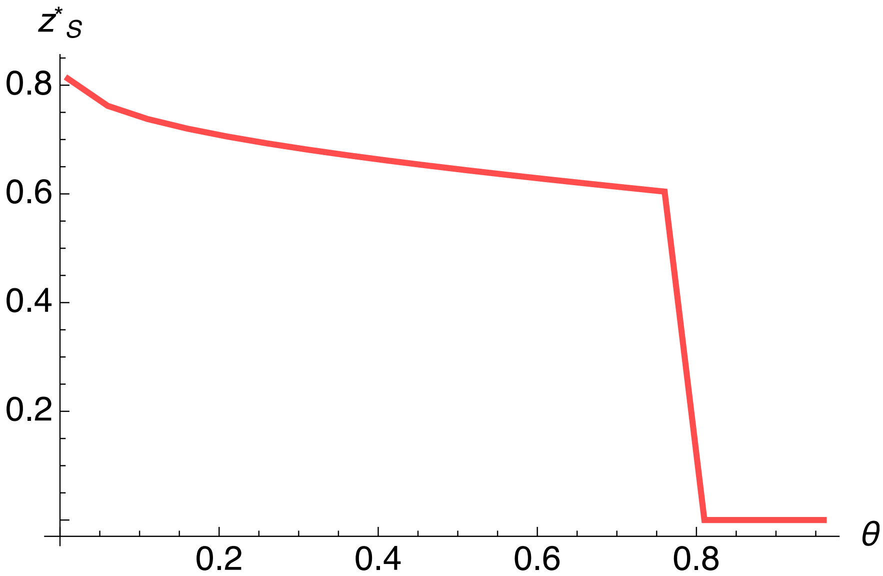

Figure A.1: The effect of θ on z_s^{max}.

- the combination of $f_y(x) < 0$ and $f_x(y) > 0$ in the vicinity of $\underline{\mathrm{x}}(y)$, form a sufficient condition for $\frac{d\underline{\mathrm{X}}}{dy} > 0$
- the combination of $f_y(x) > 0$ and $f_x(y) < 0$ in the vicinity of $\underline{\mathrm{x}}(y)$, form a sufficient condition for $\frac{d\underline{\mathrm{X}}}{dy} > 0$

Hence, to establish the result, it suffices to establish the monotonicity and sign of the derivatives of the supplier's value function $E[\Pi_s^*](\theta, M, \beta, R_s)$ with respect to those four paramters. For this, we use the envelope theorem:

$$\begin{aligned}\frac{dE[\Pi_s^*]}{d\theta} &= \frac{\partial E[\Pi_s](z_s^{max})}{\partial \theta} \\ &= -C^m + C^c < 0 \end{aligned} \tag{A.20}$$

$$\begin{aligned}\frac{dE[\Pi_s^*]}{dR_s} &= \frac{\partial E[\Pi_s](z_s^{max})}{\partial R_s} \\ &= -1 < 0 \end{aligned} \tag{A.21}$$

The combination of Eq. A.20 and A.21 establishes the first part of the result.

$$\frac{dE[\Pi_s^*]}{dM} = \frac{\partial E[\Pi_s](z_s^{max})}{\partial M}$$
$$= sp_1^* \frac{\partial q_1}{\partial M} - \frac{\partial q_1}{\partial M}\frac{\partial E[C]}{\partial q_1} \quad \text{(A.22)}$$

Denote M^c the value of M at which this derivative is equal to 0. We show that at M^c, $E[\Pi_s^*]$ reaches a minimum and is negative, such that $E[\Pi_s^*]$ is monotonically increasing as M increases beyond M^c.

At M^c, $sp_1^* = \frac{\partial E[C]}{\partial q_1}$. Plugging that into the expression for $E[\Pi_s^*]$, we get:

$$E[\Pi_s^*] = \frac{\partial E[C]}{\partial q_1} q_1^* - E[C](q_1^*) - R_s z_s^{max} \approx -R_s z_s^{max} < 0 \quad \text{(A.23)}$$

Hence, Equation A.22 reaches a minimum at a value M^c above which it increases monotonically. Hence in the vicinity of $\underline{\text{M}}$, the derivative is positive. Combining this fact with Equation A.20 establishes the second part of the result.

Very similarly, we have:

$$\frac{dE[\Pi_s^*]}{d\beta} = \frac{\partial E[\Pi_s](z_s^{max})}{\partial \beta}$$
$$= sp^* \frac{\partial q}{\partial \beta} - \frac{\partial q}{\partial \beta}\frac{\partial E[C]}{\partial q} \quad \text{(A.24)}$$

This derivative is equal to 0 at two points: when $sp^* = \frac{\partial E[C]}{\partial q}$ (happening at β^{c_p} and when $\frac{\partial q}{\partial \beta} = 0$ (happening at point β^{c_2}. Beyond β^{c_2}, the demand function q saturates, having included the whole market. At this point, both the demand and profits reach a maximum, and $\frac{\partial q}{\partial \beta} = 0$ and $\frac{dE[\Pi_s^*]}{d\beta}$. In contrast, at β^{c_p}, the profit function reaches a minimum. By the same reasoning as in Equation A.23, the profit function is negative at that point. Suppose $E[\Pi_s^*]$ is positive for β^{c_2} (whether this is true or not depends on parameters governing the relative importance of costs and revenues, such as c_s, M, u_0 etc...). Then by the intermediate value theorem, $\exists \underline{\beta}$ such that $\beta^{c_p} < \beta < \beta^{c_2}$, at which $E[\Pi_s^*] = 0$ and at that point $\frac{dE[\Pi_s^*]}{d\beta} > 0$. The combination of that statement and Equation A.20 establishes the third part of the result. $\square$