Welfare Costs of Catastrophes: Lost Consumption and Lost Lives

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Abstract: Most of the literature on the economics of catastrophes assumes that such events cause a reduction in the stream of consumption, as opposed to widespread fatalities. Here we show how to incorporate death in a model of catastrophe avoidance, and how a catastrophic loss of life can be expressed as a welfare-equivalent drop in consumption. We examine how potential fatalities affect the policy interdependence of catastrophic events and “willingness to pay” (WTP) to avoid them. Using estimates of the “value of a statistical life” (VSL), we find the WTP to avoid major pandemics, and show it is large (10% or more of annual consumption) and partly driven by the risk of macroeconomic contractions. Likewise, the risk of pandemics significantly increases the WTP to reduce consumption risk. Our work links the VSL and consumption disaster literatures.

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1 Introduction

The welfare costs of unpredictable catastrophic events have been addressed in a variety of studies. In most, the events are “generic,” in that the cause is unspecified, and each event reduces GDP and consumption by some (random) amount. One can think of these events as destroying a random part of the capital stock or reducing its productivity, and reducing output accordingly. Welfare costs arise in part from the direct impact of the events on consumption and in part from their unpredictability.\(^1\) In Martin and Pindyck (2015), we considered different types of catastrophic events (e.g., nuclear terrorism, a climate catastrophe, or a financial crisis), explored their policy interdependence, and developed a rule for determining which events it is optimal to avert. But as with generic disasters, we assumed that the impact of each type of catastrophic event is a reduction in consumption.

Some catastrophic events kill people but cause little or no drop in the consumption of those who survive. Perhaps the best example—and the one we focus on in this paper—is a major pandemic that spreads uncontrollably. Most of the damage from such an event would be the deaths of a significant fraction of the population. Indeed, the Spanish Flu pandemic of 1918–1919 infected about 20 percent and killed about 4 to 5 percent of the populations of Europe and the U.S., but had a minimal impact on GDP and the consumption of survivors. Thus an equally virulent pandemic today might kill more than ten million people in the U.S. alone. Epidemiologists have argued that new and more virulent pandemics are very likely to occur in the next couple of decades, and because of modern travel, could kill an even larger fraction of the population.\(^2\) Indeed, as we write this paper, the COVID-19 coronavirus has developed into a worldwide pandemic that is spreading rapidly.

We explore the welfare implications of the threat of pandemics, but in the context of a world also threatened by “generic” consumption disasters. As usual, we measure the benefit from averting either threat in terms of “willingness to pay” (WTP), i.e., the maximum fraction of consumption society would be willing to sacrifice, now and throughout the future, to eliminate the threat. We examine the WTP to avoid future pandemics, and ask how that WTP is affected by ongoing economic uncertainty. In particular, what is the WTP to avoid

\(^1\)Barro (2006, 2009), Martin (2008), Pindyck and Wang (2013), and others refer to (generic) catastrophes as “consumption disasters.” The potential for such catastrophes can help explain the equity premium and low risk-free rate “puzzles.” As discussed below, Barro (2006, 2009), Barro and Ursua (2008) and others use historical data for a panel of countries to estimate the mean arrival rate and impact distribution of these events; Martin (2008) and Pindyck and Wang (2013) use data on aggregate economic and financial variables to get the mean arrival rate and impact distribution as outputs of a general equilibrium model.

a pandemic, the WTP to avoid consumption disasters, and the WTP to avoid both?

Why include consumption disasters in a study of the threat of pandemics? First, as we showed in Martin and Pindyck (2015), with multiple potential catastrophes, simple cost-benefit analysis breaks down: because they are “non-marginal,” considering them in isolation can lead to sub-optimal policies, even if the catastrophes occur independently of one another. Second, general economic uncertainty is a form of “background risk” that raises the WTP to avert a pandemic, and we want to determine by how much. Third, the welfare effects of consumption disasters provides a scale that can help us evaluate the importance of averting pandemics. And finally, combining the two in one model is natural because the framework we develop works by relating the welfare effects of fatalities to losses of consumption.³

Here we show how to incorporate death in a model of catastrophe avoidance, and how a catastrophic loss of life can be expressed as a welfare-equivalent drop in consumption. We want to find the WTP to avert a catastrophe that would kill some fraction \( \psi \) of the population (chosen at random), leaving the consumption of those who live unchanged, as opposed to reducing the consumption of everyone by \( \psi \). As one would expect, the WTP to avert the deaths of some fraction of the population is much greater than the WTP to avert a drop in consumption by the same fraction; a drop in consumption causes a marginal loss of utility, whereas death causes a total loss (albeit for only a fraction of the population).

We proxy for the value of a life lost by the “value of a statistical life” (VSL). The VSL is the marginal rate of substitution between wealth (or consumption, or discounted lifetime consumption) and the probability of survival. Thus it is a local measure; we might expect an individual (or society) to be willing to pay more than the VSL to avoid certain death. Its use is appropriate in our context, however, because for each individual even a severe pandemic implies a small probability of death. Furthermore, the VSL is used widely in public policy applications. Many studies have estimated the VSL using data on risk-of-death choices made by individuals, and typically find numbers in the range of 3 to 10 times average lifetime income or consumption. (So as society becomes richer, the VSL will increase.) Here we link the VSL literature to the literature on consumption disasters.

In Martin and Pindyck (2015) we assumed that there are \( N \) types of potential catastrophes, all of which would cause a drop in consumption. We now allow for just two types of catastrophes, one of which is a generic consumption disaster and the other is an event (e.g.,

³Barro (2014) develops a model with two catastrophes, a consumption disaster and also an environmental catastrophe (e.g., from climate change), with different probabilities of occurring. But he assumes the two catastrophes have the same impact distribution and simply adds the probabilities, in effect modeling a single catastrophe, but with a greater likelihood of occurring than the consumption disasters in the earlier papers.
a pandemic) that kills people. Thus the WTP to avoid the second type results from lives saved, as opposed to consumption saved.4

Determining the WTP to avert a catastrophe that kills people introduces a fundamental issue: How should society value the very existence of people, alive now and potentially in the future, relative to the consumption enjoyed by those people? We want to be clear that we do not have anything new to say about the hard philosophical question regarding the value of human life. To our knowledge, there is no consensus regarding the social value of more or fewer people, although economists tend to value changes in population asymmetrically. Increases in population are seen by some as “good” (on purely ethical grounds, but also by contributing to technological change), and by some as “bad” (because of crowding, congestion, and environmental stress). Based on social norms, decreases in population are seen as purely “bad,” in that we go to great lengths to save lives and prevent life-threatening disasters.5 Consistent with these social norms, the model we develop treats deaths caused by catastrophes as a pure “bad” that reduces social welfare. However, the model leaves open the welfare effects of “natural” increases (or decreases) in population.

Using middle-of-the-road estimates of the VSL and conservative estimates of the likelihood and impact distribution, we find the WTP to avert future pandemics to be large — more than 10% of consumption. Also, a good portion of that WTP comes from “background risk,” here the threat of consumption catastrophes, which we assess using estimates from Barro and Ursúa (2008). Likewise, the threat of pandemics substantially affects the WTP to avert consumption catastrophes. This is another example of the policy interdependence of major catastrophic threats.

In the next section we lay out our framework for measuring the welfare loss from catastrophes that kill people, but leave the consumption of survivors unchanged. In Sections 3 and 4 we present a fully dynamic model with two types of catastrophes that arrive independently as Poisson processes, one that causes a (random) drop in consumption and the other a (random) number of deaths. We specify distributions for the random impacts of each type, and we find the WTPs to avert each and to avert both. In Section 5 we calibrate the model using parameter values consistent with the recent literatures on pandemics and macroeconomic contractions, calculate the WTPs, and discuss the policy implications. Section 6 concludes.

4As is clear from Barro and Ursúa (2008), some of the greatest “consumption disasters” during the past century were the result of major wars. Those wars caused vast numbers of deaths (military and civilian), the welfare losses from which were likely greater than the losses from reduced consumption.

5But see Young (2005) and Voigtländer and Voth (2013), who show how pandemics and plagues can raise welfare over time. For estimates of the negative population externality associated with climate change, see Bohn and Stuart (2015).
2 Framework

We assume that at time $t$, $N_t$ identical consumers are alive. Utility for each of them comes only from consumption, and we denote this utility by $u(C_t)$. People occasionally die from “natural” (non-catastrophic) causes, and new people are born.

Without any catastrophes, real per-capita consumption, $C_t$, grows at the constant rate $g$, and we normalise so that at time $t = 0$, $C_0 = 1$. Likewise the population grows at the constant net (of natural deaths) rate $n$, so $N_t^* = N_0 e^{nt}$, where $N_t^*$ denotes the population when there are no catastrophe-induced deaths, and we normalise so $N_0 = 1$.

We assume that catastrophes occur as Poisson events with a known mean arrival rate, and can occur repeatedly. We consider two types of catastrophes. The first type causes a permanent drop in log consumption by a random fraction $\phi$ for the entire population (so that $\phi$ is roughly the fraction by which total consumption falls). Thus if the catastrophic event first occurs at time $t_1$, $C_t = e^{gt}$ for $t < t_1$ and then falls to $C_t = e^{-\phi + gt}$ at $t = t_1$. Let $\lambda_c$ denote the mean arrival rate of this type of event.

The second type consists of catastrophes which result in the death of a random fraction $\psi$ of the population, where the distribution of deaths is also random, i.e., each individual has a probability $\psi$ of dying. This type of catastrophe, however, leaves unchanged the consumption path for those who remain alive. Thus the first occurrence of a “death” event causes the population to fall to $N_t = e^{-\psi + nt}$ but leaves $C_t$ unchanged.\(^6\) Let $\lambda_d$ denote the mean arrival rate of this type of event. For now we make no assumptions regarding the distributions for $\phi$ and $\psi$ (and these distributions need not be the same).

A person who is living gets utility $u(C_t)$, but what about a person who dies? We denote that person’s utility by $v(C)$. This is quite general; $v(C)$ could be a constant, something independent of $C$, or some function of $C$. Why should $v(C)$ be anything other than zero? In part because people can (and do) make bequests to children and others, and those bequests typically depend on income and hence consumption. Likewise, governments can redistribute wealth (and hence consumption) after death, as in Rosen (1988). We will derive an expression for $v(C)$ shortly.

The total welfare for society is the welfare of those living plus the welfare of those who died, and we assume that each is proportional to the number of people in that category. The

\(^6\)Some of the literature on the value of life makes the opposite assumption that the wealth of the deceased goes to the living, so that their consumption will rise; see, e.g., Rosen (1988). One might argue instead that a death catastrophe will lower $C_t$ by causing critical labor shortages. We could allow for this, but at the cost of complicating the analysis.
number of people alive is \( N_t \) and the number who have died is \( N_t^* - N_t \), so total welfare is:

\[
V_0 = \mathbb{E} \int_0^\infty N_t u(C_t) e^{-\delta t} dt + \mathbb{E} \int_0^\infty (N_t^* - N_t) v(C_t) e^{-\delta t} dt ,
\]

(1)

where \( \delta \) is the (common) rate of time preference.

We now make a key assumption: \( u(C_t) \) exhibits constant relative risk aversion (CRRA), i.e., \( u(C_t) = C_t^{1-\eta}/(1-\eta) \), where \( \eta \) is the index of relative risk aversion. Unless noted otherwise, we will assume that \( \eta > 1 \). This is consistent with the finance and macroeconomics literatures, which put \( \eta \) in the range of 2 to 5. Then (1) becomes:

\[
V_0 = \mathbb{E} \left\{ \int_0^\infty e^{-\delta t} \left[ \frac{N_tC_t^{1-\eta}}{1-\eta} + (N_t^* - N_t) v(C_t) \right] dt \right\} ,
\]

(2)

Note that with \( \eta > 1 \), \( V_0 < 0 \), suggesting that a lower natural rate of population growth \( n \) raises total social welfare. But of course we could have added an arbitrary constant to the utility function, i.e., written it as \( u(C_t) = C_t^{1-\eta}/(1-\eta) + B \), and make \( B \) sufficiently large so that \( u(C_t) > 0 \), in which case a lower \( n \) would lower total welfare. As \( B \) cannot be determined based on observables, it is not meaningful to ask whether higher or lower population growth would raise or lower welfare.\(^7\) On the other hand, the WTPs we calculate below are meaningful, and in particular are independent of the positive constant \( B \).

Later, when we calibrate and apply the model, we will be more specific about the nature of these two types of catastrophes (e.g., a “death” event will be a pandemic). For now we simply assume that one causes a drop in \( C_t \) and the other a drop in \( N_t \). We will want to calculate the WTP to avert each type, and the WTP to avert both.\(^8\)

### 2.1 The Quick and the Dead

To handle catastrophes that cause death, we must aggregate the welfare of those who remain alive after the event and the welfare of those who die.

What is the welfare loss for a person that dies? One approach to this question is to simply assume that the consumption of those who die falls to zero. This might make sense for CRRA utility with \( \eta < 1 \), because then \( u(C) = C^{1-\eta}/(1-\eta) = 0 \) when \( C = 0 \). But we have assumed that \( \eta > 1 \), so utility is negative, and unbounded as \( C \to 0 \).

\(^7\)Note that there is no explicit population externality in eqn. (2). We could introduce such an externality by writing individual utility as \( u_t = C_t^{1-\eta} N_t^\alpha/(1-\eta) \), so the externality is positive (negative) if \( \alpha > (\alpha < 0) \). Then \( N_t \) in eqn. (2) is replaced by \( N_t^{1+\alpha} \), so \( \partial V_0/\partial N > (\partial V_0/\partial N < 0 \) if \( \alpha > (\alpha < 0) \). Population externalities are not the focus of this paper, so we will not introduce one.

\(^8\)We focus on completely averting a catastrophe, but as shown in Martin and Pindyck (2015), our framework also allows for partially alleviating the catastrophe, e.g., by reducing its mean arrival rate.
An alternative approach, sometimes referred to as “critical-level utilitarianism,” is to find a minimum consumption level $C_m$ just high enough to sustain (tolerable) life, and then measure utility relative to $u(C - C_m)$. (See Broome (2004) and Blackorby et al. (2005).) But as a matter of calibration, we see no obvious way to determine $C_m$. More importantly, in our model $C$ could fall below $C_m$, and we need a way to compare this to an outcome in which $C$ falls just to $C_m$.

Another and perhaps simplest approach is to assume that upon death, utility (as opposed to consumption) falls to zero, i.e., the individual loses the utility she enjoyed while alive. But with CRRA utility, this can considerably underestimate the welfare loss from death.

To see this, consider an individual who lives two periods and enjoys the same consumption $C_0 = 1$ in each period. Ignore discounting and assume $\eta = 2$, so her welfare is $V = -2C_0^{-1} = -2$. But what if she dies just before the second period? Does she simply lose the utility she otherwise would have gained from consuming 1 unit? Upon death her consumption goes to zero and $U(0) = -\infty$, which implies a loss far greater than $-1$. To see why the loss must be greater than the utility she otherwise would have received in Period 2, suppose her consumption in Period 2 fell not to zero, but only by 75%, i.e., to 0.25. Then Period 2 utility would be $-4$ and her welfare change would be $-4 - (-1) = -3$, a much greater loss than the utility otherwise gained from consuming 1 unit. But consuming only 25% of “normal” consumption is still (for most people) far preferable to death.

Nonetheless, the usual approach in the literature is to assume that upon death utility falls to zero. For example, Hall and Jones (2007), in their study of the value of life-extending health expenditures, do this by adding a positive constant $b$ to the utility function:

$$u(C) = \frac{1}{1-\eta}C^{1-\eta} + b.$$  

For Hall and Jones (2007) the constant $b$ is essential because it allows them to show that as income and consumption increase, the marginal utility of an extra dollar of consumption falls relative to the marginal benefit (in terms of an increase in the probability of survival) from an extra dollar of health care spending.\(^9\) In their case, death corresponds to a drop in consumption from $C_0$ to a value $\omega$ such that $u(\omega) = 0$, i.e., $\omega = [(\eta - 1)b]^{1/(1-\eta)}$.

\(^9\)Becker et al. (2005) also use this formulation, normalise utility at death to be zero, and take $\eta \approx 0.8$ so that $b < 0$. With $\eta > 1$, a value of $b$ sufficiently large so that $u(C) > 0$ over any relevant range of $C$ is also needed to ensure that indifference curves between consumption and life expectancy slope down. See Pratt and Zeckhauser (1996). In a related paper, Jones (2016) develops a representative agent model in which research effort can lead to consumption growth, but also the chance of a catastrophe that will kill the agent, whose utility then falls to zero. Arrow and Priebsch (2014) propose a generalised hyperbolic absolute risk aversion utility function which is bounded for any value of $C$.  

\[\]
We are interested in the loss of welfare resulting from death, but not a comparison of marginal benefits from additional health care spending, so we can retain the CRRA utility function without adding a constant. We can then treat death using the same framework used to treat the utility loss from a drop in consumption. To do this we adopt the VSL measure that is widely used in public policy.

2.2 The Value of Life

To determine the welfare effects of fatalities, we need to know the value of a life (or more precisely, the value of a life lost). There is a large literature on this topic, which focuses on the “value of a statistical life” (VSL), defined as the marginal rate of substitution between wealth (or future lifetime consumption) and the probability of survival. Thus the VSL tells us how much an individual (or society) would be willing to pay in terms of a small decrease in wealth or consumption in return for a small increase in the probability of survival. It does not tell us how much an individual or society would pay to avoid certain death, or a significant probability of death, which might be much more than the VSL. It also need not aggregate consistently; a VSL estimate applied to a country’s entire population can easily exceed the present value of the country’s projected GDP over the next, say, 40 years.

Many studies have sought to estimate the VSL using data on risk-of-death choices made by individuals, such as the decision to take a riskier but higher-paying job rather than a safer one. (See, e.g., Viscusi (1993) and Cropper and Sussman (1990).) The range of estimates is wide: 3 to 10 times lifetime income or lifetime consumption, with an average value of around 7. Consistent with this average value, the U.S. Environmental Protection Agency set the VSL at about $9 million when it conducted cost-benefit analyses of proposed policies.

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10 Hugonnier et al. (2018) introduce the value of life “calculated at Gunpoint” (GPV), i.e., the WTP to avoid certain death. They connect the VSL to GPV by valuing the flow of income (when alive) along an optimal path, and estimate both using data on health and health insurance expenditures for about 8000 people in 2013. They find a VSL of about $8.4 million, but a GPV of only $447 thousand.

11 For an overview of the VSL and its measurement and use, see Andersson and Treich (2009), Ashenfelter (2006), and Viscusi (2018). The VSL has other issues. For example, the macro/finance literatures put the coefficient of relative risk aversion, $\eta$, well above 1. Estimates of the income elasticity of the VSL are 0.5 to 0.6; see, e.g., Viscusi and Aldy (2003). Kaplow (2005) shows that this low income elasticity is inconsistent with $\eta > 1$. Also, the VSL is a partial equilibrium measure that ignores transfers across society (if I live to 100, the young will have to work harder to support me); see, e.g., Arthur (1981).

12 U.S. Environmental Protection Agency (2014), page 7-8: “EPA currently recommends a default central VSL of $7.9 million (in 2008 dollars) to value reduced mortality for all programs and policies.” This translates to $9.7 million in 2017 dollars. In 2017, U.S. per capita consumption was about $41,000, which corresponds to lifetime consumption of roughly $1,600,000 (i.e., a factor of 40). The average VSL estimate of 7 times lifetime consumption yields $11.2 million. Rohlfs et al. (2015), using airbag regulations during the 1990s
The VSL can be understood as follows. Let $w$ represent wealth or lifetime consumption, and let $p$ be the \textit{ex ante} probability of death (so $1 - p$ is the probability of survival). Suppose an individual can reduce $p$ at the cost of some reduction in $w$. Let $u(w)$ be the individual’s utility if alive, and $v(w)$ her utility if dead, with $u(w) > v(w)$ and $u'(w) > v'(w)$. The VSL measures the trade-off between wealth (or lifetime consumption) and the probability of survival, i.e., it is the marginal rate of substitution between $w$ and $1 - p$:\footnote{To get eqn. (3), note that expected utility is $V = (1 - p)u(w) + pv(w)$. Setting the total derivative $dV = (\partial V/\partial p)dp + (\partial V/\partial w)dw = 0$ yields $dw/dp$.}

\[ VSL = -\frac{dw}{d(1 - p)} = \frac{dw}{dp} = \frac{u(w) - v(w)}{(1 - p)u'(w) + pv'(w)}. \tag{3} \]

Note that $u(w)$ and $v(w)$ are measured in utils, and $u'(w)$ and $v'(w)$ are measured in utils per dollar, so the VSL is measured in dollars. The VSL is a cardinal measure, invariant to affine transformations of $u$ or $v$. Doubling the VSL implies a doubling of the compensation required to incur an incremental increase in the risk of death. Consistent with the literature, we take the VSL to be an observable multiple of wealth: $VSL = sw$.

The VSL is increasing in the \textit{ex ante} probability of death $p$; if $p$ is high there is less incentive to limit spending to reduce $p$ because it is unlikely the individual will survive and have the opportunity to enjoy whatever wealth remains. (Pratt and Zeckhauser (1996) call this the “dead anyway” effect.) In our case, the \textit{ex ante} probability of death, whether from natural causes or a catastrophe, is low. Likewise, most empirical studies of the VSL are based on the behavior of populations for which $p$ varies, but over a small range and a very low base value. Thus it is reasonable to evaluate the VSL at $p = 0$, which we do below.

We emphasise that our reliance on VSL is limited to the calibration of $s$. This is appropriate despite VSL being a marginal measure because, in our calibrations, even a severe pandemic implies a small probability of death for each individual member of society. The WTP calculations we carry out below do not rely on marginal logic other than through their dependence on the calibration of $s$. Analogously, in Martin and Pindyck (2015), our justification for calibrating risk aversion to (say) 2 is implicitly based on studies that come up with similar numbers based on marginal logic, but once risk aversion is calibrated our calculations were “global” rather than “local” (i.e., marginal).

\footnote{To get eqn. (3), note that expected utility is $V = (1 - p)u(w) + pv(w)$. Setting the total derivative $dV = (\partial V/\partial p)dp + (\partial V/\partial w)dw = 0$ yields $dw/dp$.}

and used car prices, find a median VSL between $9 and $11 million, and Aldy (2019), using employment decisions of married couples, estimates the VSL to be between $9 and $13 million. But Greenstone \textit{et al.} (2012), using variation in occupation-specific mortality risks and occupation-specific re-enlistment bonuses, found that soldiers’ re-enlistment decisions implies a VSL of only $3 to $4 million.
2.3 The Quick and the Dead Once Again

We can now complete eqn. (2) by deriving an expression for $v(C_t)$, utility in the “dead” state. To do this, we will think of $u(w)$ as the utility associated with a wealth level of $w$ in the hands of a living person, and $v(w)$ as the (social) utility associated with the same wealth if the person dies and their assets are redistributed, either via intentional bequests to children or others, or through some other reallocation mechanism. As $s$ is observable and we can approximate $p \approx 0$, from eqn. (3) the relationship between $v(w)$ and $u(w)$ must satisfy

$$v(w) = u(w) - swu'(w).$$

Each person’s utility while alive is ultimately derived from a flow of consumption, $C_t$:

$$\text{Expected utility} = \mathbb{E} \int_0^\infty \frac{1}{1 - \eta} C_t^{1-\eta} e^{-\delta t} dt.$$  

As discussed below, we assume that consumption growth is i.i.d., which implies that expected utility is proportional to $C_t^{1-\eta}/(1 - \eta)$ and that consumption and wealth are proportional to one another (Martin (2008), Barro (2009)). Thus we can simply work with utility $u(w) = w^{1-\eta}/(1 - \eta)$. As long as we are consistent in whether we work with wealth or consumption, the constant of proportionality itself is unimportant, given the invariance of the VSL to affine transformations.

Equation (4) implies that we must have

$$v(w) = \frac{w^{1-\eta}}{1 - \eta} - sw^{1-\eta} = \frac{w^{1-\eta}}{1 - \eta} \left[ 1 + s (\eta - 1) \right]$$

$$= \frac{1}{1 - \eta} \left\{ w \left[ 1 + s (\eta - 1) \right] \frac{1}{1 - \eta} \right\}^{1-\eta}.$$  

It follows immediately that the social welfare loss associated with a death is equivalent to the loss that would occur following a drop in wealth for a living person from $w$ to $\varepsilon w$, where $\varepsilon$ is given by:

$$\varepsilon = [1 + s(\eta - 1)]^{\frac{1}{1-\eta}} < 1.$$  

Or, if thinking in consumption terms, it is equivalent to the loss following a drop in consumption from $C$ to $\varepsilon C$. This means that we can write utility in the “dead” state as

$$v(C) = (\varepsilon C)^{1-\eta}/(1 - \eta) = [1 + s(\eta - 1)] C^{1-\eta}/(1 - \eta),$$

and eqn. (2) becomes

$$V_0 = \mathbb{E} \left\{ \int_0^\infty e^{-\delta t} \left[ \frac{N_t C_t^{1-\eta}}{1 - \eta} + \frac{(N_t^* - N_t)(\varepsilon C_t)^{1-\eta} - N_t^*}{1 - \eta} \right] dt \right\}.$$
As explained above, estimates of $s$ vary, but the average is around 7, and we will use that value in much of what follows. Thus if $\eta = 2$, $\varepsilon = 1/(s + 1) = .125$, i.e, death is equivalent in welfare terms to an 88\% drop in wealth or consumption. If $\eta = 3$, $\varepsilon = .27$, and if $\eta = 4$, $\varepsilon = .42$. Note that $\varepsilon$ is increasing in $\eta$ because a larger value of $\eta$ implies a larger utility loss from any given reduction in $w$ or $C$. The utility loss in (3) is fixed by the VSL, so the welfare-equivalent reduction in consumption upon death must be smaller if $\eta$ is larger.

Note that in our model the social welfare loss from a death is equivalent to a percentage drop in wealth, which implies that this loss depends on the person’s wealth level. In fact, the VSL itself is increasing in wealth $w$, because a wealthier individual has more utility to lose should she die. This would be a problem if our objective were to make interpersonal (or cross-country) comparisons, or to estimate à la Hall and Jones (2007) how the relative marginal benefit of health care spending varies with income and consumption. We address a simpler problem by assuming in eqn. (1) that prior to a catastrophe consumption is the same for everyone, so total welfare is proportional to the population.

Finally, it is important to stress that these results depend critically on our assumption of CRRA utility. Others have estimated the value of life using more general preferences, and in particular recursive (non-separable) preferences, which allows for the separation of risk aversion and the elasticity of intertemporal substitution. Córdoba and Ripoll (2017), for example, use Epstein–Weil–Zin (EWZ) recursive preferences to estimate the value of gains in longevity between 1970 and 2005 across 144 countries. Compared to the CRRA-based estimates in Becker et al. (2005), they find a larger value of life in poor countries, but not rich ones. (The reason is that there are diminishing returns to survival in the EWZ model, so the value of life in poor countries, with shorter life spans, is larger.) And Bommier and Villeneuve (2012) use EWZ preferences to estimate how the VSL varies as a function of age. However, we ignore income or age variation, treat the population as homogeneous, and focus on a catastrophic loss of life, so we avoid the added complication that the EWZ framework entails.

### 2.4 Welfare Loss

Initially, static welfare is $U_0 = N_0 C_0^{1-\eta}/(1-\eta)$. With $\eta > 1$ so $U_0 < 0$, it might appear that a catastrophe-induced drop in $N_0$ raises welfare, but that is not the case. Setting $C_0 = N_0 = 1$, 

\[ \text{Think of } D(w) = v(w) - u(w) \text{ as the “cost of death,” so expected utility is } V = u(w) - pD(w) \text{ and we can rewrite eqn. (3) as } \frac{dw}{dp} = \frac{D(w)}{u'(w) + pD'(w)}. \] 

$D(w)$ is increasing in $w$, as is the VSL. (Thus Bill Gates has more to lose by dying than most other people.)

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14 Think of $D(w) = v(w) - u(w)$ as the “cost of death,” so expected utility is $V = u(w) - pD(w)$ and we can rewrite eqn. (3) as $\frac{dw}{dp} = \frac{D(w)}{u'(w) + pD'(w)}$. $D(w)$ is increasing in $w$, as is the VSL. (Thus Bill Gates has more to lose by dying than most other people.)
if a fraction $\psi$ of the population dies, welfare becomes

$$U_N = U_0 - \psi \frac{1 - \varepsilon^{1-\eta}}{1 - \eta} < U_0$$

The adjustment term $\psi(1 - \varepsilon^{1-\eta})/(1 - \eta)$ captures the welfare loss associated with the fraction of the population that has died, and ensures that $U_N < U_0$. From (5), $\varepsilon^{1-\eta} = 1 + s(\eta - 1)$, so $U_N = [1 + (\eta - 1)\psi s]/(1 - \eta) < U_0$ as long as $s > 0$. And if instead consumption falls by a fraction $\phi$, welfare becomes $U_C = (1 - \phi)^{1-\eta}/(1 - \eta) < U_0$.

We can also compare the welfare loss for an event that kills a fraction $\phi$ of the population with the loss for an event that reduces the consumption of everyone by the same fraction. Let LR denote the ratio of the first welfare loss to the second. Using (5), the (annual) welfare loss for each person who dies is $L_d = \varepsilon - u(C_0)[(\eta - 1)]/(1 - \eta) = -s$. Since a fraction $\phi$ die, the total loss is $-\phi s$. This is independent of $\eta$, because the drop in consumption (from 1 to $\varepsilon$) is constrained to yield a welfare loss equal to the VSL. If instead the consumption of everyone falls by the same fraction $\phi$, the welfare loss is $L_c = [(1 - \phi)^{1-\eta}]/(1 - \eta)$. (This loss depends on $\eta$ because the drop in consumption is fixed.)

The ratio of the welfare loss from death to the loss from reduced consumption is then:

$$LR = L_d/L_c = \frac{(\eta - 1)\phi s}{(1 - \phi)^{1-\eta} - 1}$$

If $s = 7$ and $\eta = 2$, for “low” values of $\phi$, e.g., $\phi = .1$, the welfare loss from death is more than six times the welfare loss from destruction. (As $\phi \to 0$, LR $\to s$, i.e., LR approaches the VSL.) The ratio is large because compared to a drop in consumption, death causes a much larger loss of utility, albeit for only a fraction $\phi$ of the population.

How large would the drop in consumption for all members of society have to be to yield the same welfare loss as an event that kills a fraction $\phi$? Denoting the equivalent drop in consumption by $\phi_c$, using (5), and setting $u[(1 - \phi_c)C_0] - u(C_0) = \phi[u(\varepsilon C_0) - u(C_0)]$,

$$\phi_c = 1 - [s\phi(\eta - 1) + 1]^{1/\eta}$$

If $s = 7$ and $\phi = .05$, when $\eta = 2$, $\phi_c = .26$, and when $\eta = 4$, $\phi_c = .21$. (Setting $\phi = .05$ corresponds roughly to the Spanish Flu of 1918-19.) If $\phi = .1$, $\phi_c = .41$ when $\eta = 2$ and .31 when $\eta = 4$. So $\phi_c$ is 3 to 5 times as large as $\phi$ when $\phi \leq .10$. However, the multiple is smaller when $\phi$ is large, and can be less than 1 if $\phi$ is sufficiently large. (If $\phi = .8$ and $\eta = 4$, $\phi_c = .62$.) Once again, the VSL constrains $L_d$, but $L_c$ is unconstrained as $\phi_c$ increases.
3 A Dynamic Model

To determine the welfare loss from a drop in per-capital consumption \( C_t \) or a drop in population \( N_t \), we need to describe how these events can unfold over time. We thus turn to a fully dynamic model, and specify the processes for \( C_t \) and \( N_t \). We assume that catastrophic events that reduce \( C_t \) occur as Poisson arrivals with mean arrival rate \( \lambda_c \), and the impact of the \( k \)th arrival, \( \phi_k \), is i.i.d. across realisations \( k \). Then the process for consumption is:

\[
c_t = \log C_t = g_t - \sum_{k=1}^{Q(t)} \phi_k
\]

where \( g \) is the normal growth rate of per capita consumption, and \( Q(t) \) is a Poisson counting process with known mean arrival rate \( \lambda_c \). When the \( k \)th catastrophic event occurs, per-capita consumption is multiplied by the random variable \( e^{-\phi_k} \). (We are scaling consumption so that \( C_0 = 1 \) and hence \( c_0 = 0 \).) We can then define the cumulant-generating function (CGF),

\[
\kappa_{C,t}(\theta) \equiv \log E e^{c_t \theta} = \log E \sum_{k=1}^{Q(t)} \phi_k \equiv \log E C_t^\theta.
\]

We showed in Martin and Pindyck (2015) that since log consumption follows a Lévy process, the CGF is linear in \( t \)—that is, \( \kappa_{C,t}(\theta) = \kappa_{C,1}(\theta)t \)—and we can write

\[
\kappa_C(\theta) \equiv \kappa_{C,1}(\theta) = g \theta + \lambda_c \left( \mathbb{E} e^{-\theta \phi} - 1 \right),
\]

where \( \phi \) is a representative of the (i.i.d.) \( \phi_k \).

We assume that \( N_t \), the population alive at time \( t \), follows an analogous process:

\[
n_t = \log N_t = n_t - \sum_{k=1}^{X(t)} \psi_k,
\]

where \( n \) is the natural rate of population growth, \( X(t) \) is a Poisson counting process with arrival rate \( \lambda_d \) that determines how many “death events” have happened by time \( t \), and \( \psi_k \) measures the size of the \( k \)th event. Thus when the \( k \)th event occurs, \( N_t \) is multiplied by the random variable \( e^{-\psi_k} \). We scale \( N_t \) so that \( N_0 = 1 \), and we define the corresponding CGF,

\[
\kappa_{N,t}(\theta) \equiv \log E e^{n_t \theta} \equiv \log E N_t^\theta.
\]

\[\text{We could easily make the normal growth of consumption stochastic, i.e., replace } g_t \text{ in (6) by any Lévy process } g_t \text{ (e.g., a Brownian motion). Then } g \theta \text{ in (7) becomes } g(\theta), \text{ the CGF of } g_1. \text{ The normal rate of population growth can likewise be stochastic. For simplicity, we limit uncertainty to catastrophic events.}\]
Once again, the CGF scales linearly with $t$, so we write

$$\kappa_N(\theta) \equiv \kappa_{N,1}(\theta) = n\theta + \lambda_d \left( \mathbb{E} e^{-\theta \psi} - 1 \right)$$  \hfill (9)

where $\psi$ is a representative of the (i.i.d.) $\psi_k$. Thus

$$\mathbb{E} N_t = e^{\kappa_N(1)t} = e^{nt + \lambda_d (\mathbb{E} e^{-\psi} - 1)t} = e^{nt - \tilde{\lambda}_d t}.$$ 

The term $\tilde{\lambda}_d = \lambda_d (1 - \mathbb{E} e^{-\psi})$ can be interpreted as the adjusted death arrival rate, taking into account that only a fraction of the population dies when a death event occurs. Depending on the relative sizes of $n$ and $\tilde{\lambda}_d$, the population may grow or shrink in expectation: if disasters are sufficiently frequent (large $\lambda_d$) or cataclysmic (small $\mathbb{E} e^{-\psi}$), and population growth $n$ is sufficiently low, then the expected population is declining in $t$, in which case $\kappa_N(1) < 0$.

We will use an asterisk to denote the absence of catastrophes. So if there are no death catastrophes ($\lambda_d = 0$), the population evolves as $N^*_t = e^{nt}$, and the CGF for $N_t$ is $\kappa^*_N(\theta) = n\theta$. Likewise, if there are no consumption catastrophes ($\lambda_c = 0$), consumption evolves as $C^*_t = e^{gt}$, and the CGF for $C_t$ is $\kappa^*_C(\theta) = g\theta$.

Recall that the social welfare loss associated with a death is equivalent to the loss that would occur from a drop in consumption to a fraction $\varepsilon$ of what it was before, i.e., from $C_t$ to $\varepsilon C_t$, where $\varepsilon$ is given by eqn. (5). So if no catastrophes are averted, total welfare is

$$V = \mathbb{E} \left\{ \int_0^\infty e^{-\delta t} \left[ \frac{N^*_t C^*_t}{1-\eta} + \frac{(N^*_t - N_t)\varepsilon}{1-\eta} C^*_t \right] dt \right\},$$

where $(N^*_t - N_t)$ is the number of people that have died.

Because $C_t$ is an exponential Lévy process, it evolves independently of $N_t$. This makes it easy to calculate the expectations inside the integral above. In particular, $\mathbb{E}(N_t C_t^{1-\eta}) = \mathbb{E} N_t \mathbb{E} C_t^{1-\eta} = \mathbb{E} N_t e^{\kappa_N(1)t} e^{\kappa_C(1-\eta)t}$ and $\mathbb{E} \left[ (N^*_t - N_t)\varepsilon C_t^{1-\eta} \right] = (e^{\kappa^*_N(1)t} - e^{\kappa_N(1)t}) \varepsilon e^{\kappa_C(1-\eta)t}$. Substituting these expressions into the integral above,

$$V = \frac{1}{1-\eta} \left\{ \frac{1 - \varepsilon^{1-\eta}}{\delta - \kappa_N(1) - \kappa_C(1-\eta)} + \frac{\varepsilon^{1-\eta}}{\delta - \kappa^*_N(1) - \kappa_C(1-\eta)} \right\}. \hfill (10)$$

To interpret this equation, note that the second term captures the welfare associated with the guaranteed consumption stream $\varepsilon C_t$ (which is “received” via bequests, etc., even after death). We can think of $\delta - \kappa^*_N(1) = \delta - n$ as the social rate of time preference (i.e., discount rate on future utility) associated with this consumption stream: it adjusts, via $\kappa^*_N(1) = n$, for the fact that the population is increasing (so that the larger is $n$, the lower is the discount rate, and because $V < 0$, the lower is total welfare). The first term captures the welfare
associated with the additional ongoing consumption stream \((1 - \varepsilon)C_t\) received by those who are still alive (i.e., those alive receive \((1 - \varepsilon)C + \varepsilon C = C\)). Since there is a risk of death, this latter consumption stream is discounted at the higher rate \(\delta - \kappa_N(1) > \delta - \kappa_N^*(1)\).\(^{16}\)

We want to find the WTPs to avert each type of catastrophe individually, and the WTP to avert both. Averting a consumption catastrophe corresponds to setting \(\lambda_c = 0\). The WTP \(w_c\) is defined as the fraction of consumption that people would be prepared to sacrifice permanently in order to avert the catastrophe. We define the WTPs to avert the death catastrophe \((w_d)\), and to avert both catastrophes \((w_{c,d})\), similarly.

**Result 1.** The WTPs to avert the consumption catastrophe, death catastrophe, or both catastrophes, satisfy, respectively,

\[
\begin{align*}
(1 - w_c)^{1 - \eta} &= A \times B \times C \quad (11) \\
(1 - w_d)^{1 - \eta} &= C \quad (12) \\
(1 - w_{c,d})^{1 - \eta} &= A \times C \quad (13)
\end{align*}
\]

where

\[
A = \frac{\delta - \kappa_N^*(1) - \kappa_C^*(1 - \eta)}{\delta - \kappa_N^*(1) - \kappa_C(1 - \eta)}
\]

\[
B = \frac{\delta - \kappa_N(1) - \kappa_C(1 - \eta)}{\delta - \kappa_N(1) \varepsilon^{1 - \eta} - (1 - \varepsilon^{1 - \eta}) \kappa_N^*(1) - \kappa_C^*(1 - \eta)}
\]

\[
C = \frac{\delta - \kappa_N(1) \varepsilon^{1 - \eta} - (1 - \varepsilon^{1 - \eta}) \kappa_N^*(1) - \kappa_C(1 - \eta)}{\delta - \kappa_N(1) - \kappa_C(1 - \eta)}
\]

**Proof.** Averting the consumption catastrophe corresponds to replacing \(\kappa_C(1 - \eta)\) by \(\kappa_C^*(1 - \eta) \equiv g(1 - \eta)\). If it is averted at the cost of a permanent loss of a fraction \(w_c\) of consumption, welfare is

\[
V_c = \frac{(1 - w_c)^{1 - \eta}}{1 - \eta} \left\{ \frac{1 - \varepsilon^{1 - \eta}}{\delta - \kappa_N(1) - \kappa_C^*(1 - \eta)} + \frac{\varepsilon^{1 - \eta}}{\delta - \kappa_N^*(1) - \kappa_C^*(1 - \eta)} \right\}.
\]

(17)

The WTP is found by equating (10) to (17); doing so gives equation (11).

To find the WTP to avoid only the death catastrophe, note that if a fraction \(w_d\) of consumption is sacrificed to avert, welfare is

\[
V_d = E \left\{ \int_0^\infty e^{-\delta t} \left( \frac{(1 - w_d)^{1 - \eta} N_t C_t^{1 - \eta}}{1 - \eta} \right) dt \right\} = \frac{(1 - w_d)^{1 - \eta}}{1 - \eta} \left\{ \frac{1}{\delta - \kappa_N^*(1) - \kappa_C(1 - \eta)} \right\}.
\]

(18)

\(^{16}\)The expressions \(1/[\delta - \kappa_N(1) - \kappa_C(1 - \eta)]\) and \(1/[\delta - \kappa_N^*(1) - \kappa_C(1 - \eta)]\) can be interpreted as the valuation ratios of the two consumption streams, as shown by Martin (2013). Because \(V < 0\), a higher value of \(\delta\) implies a higher value of \(V\). However, we treat \(\delta\) as fixed and ignore how it might be determined.
Equation (12) follows by setting (10) equal to (18). Lastly, if a fraction \( w_{c,d} \) of consumption is sacrificed to avert both catastrophes, welfare is

\[
V_{c,d} = \frac{(1 - w_{c,d})^{1 - \eta}}{1 - \eta} \left\{ \frac{1}{\delta - \kappa_N^*(1) - \kappa_C^*(1 - \eta)} \right\}. 
\] (19)

Equating (10) and (19) gives equation (13) for \( w_{c,d} \).

Below, we determine precise values for \( w_c, w_d, \) and \( w_{c,d} \) in some quantitative examples. But we have the following result which holds in general.

**Result 2.** Independent of the distributions of catastrophe sizes, or their frequency, we have

\[
\max\{w_c, w_d\} < w_{c,d} < w_c + w_d - w_c w_d.
\]

**Proof.** As \( \eta > 1 \), we have \( A > 1 \) and \( B < 1 < C \), using the facts that \( \kappa_C^*(1 - \eta) < \kappa_C(1 - \eta) \), \( \kappa_N^*(1) > \kappa_N(1) \), and \( \varepsilon^{1-\eta} > 1 \). We also have \( BC > 1 \), as the expressions for \( B \) and \( C \) can be rewritten

\[
B = 1 - \frac{(\varepsilon^{1-\eta} - 1) (\kappa_N^*(1) - \kappa_N(1))}{\delta - \kappa_N(1) \varepsilon^{1-\eta} + (\varepsilon^{1-\eta} - 1) \kappa_N^*(1) - \kappa_C^*(1 - \eta)}
\]

and

\[
\frac{1}{C} = 1 - \frac{(\varepsilon^{1-\eta} - 1) (\kappa_N^*(1) - \kappa_N(1))}{\delta - \kappa_N(1) \varepsilon^{1-\eta} + (\varepsilon^{1-\eta} - 1) \kappa_N^*(1) - \kappa_C(1 - \eta)},
\]

whence \( B > 1/C \). These inequalities give the claimed results.

In particular, the WTPs do not add—\( w_{c,d} \neq w_c + w_d \)—so that the two catastrophes are interdependent.\(^{17}\)

### 4 Impact Distributions and WTPs

Our results so far are quite general, in that the CGFs of eqns. (7) and (9) apply to any probability distributions for the impacts \( \phi \) and \( \psi \). In order to apply the model, we will assume that \( \phi \) and \( \psi \) are exponentially distributed. This distribution has often been used to model catastrophic events, and considerably simplifies the expressions for the WTPs:

\[
\begin{align*}
    f_{\phi}(x) &= \beta_c e^{-\beta_c x} \quad \text{for } x \geq 0 \\
    f_{\psi}(x) &= \beta_d e^{-\beta_d x} \quad \text{for } x \geq 0.
\end{align*}
\]

\(^{17}\)The inequality in Martin and Pindyck (2015) (page 2957) that applied to two types of consumption catastrophes (1 and 2) said only that \( w_{1,2} < w_1 + w_2 \); in fact the stronger result that \( w_{1,2} < w_1 + w_2 - w_1 w_2 \) also applies in that context.
This is equivalent to assuming that \( z_c = e^{-\phi} \) and \( z_d = e^{-\psi} \), the surviving (post-impact) fractions of consumption and people, follow the power distributions \( b(z_c) = \beta_c z_c^{\beta_c - 1} \) for \( 0 \leq z_c \leq 1 \) and \( b(z_d) = \beta_d z_d^{\beta_d - 1} \) for \( 0 \leq z_d \leq 1 \). Note that \( E(\phi) = 1/\beta_c \) and \( E(z_c) = E e^{-\phi} = \beta_c/(\beta_c + 1) \), and similarly for \( \psi \) and \( z_d \). Thus large values of \( \beta_c \) and \( \beta_d \) imply small expected impacts, i.e., small values of \( E(\phi) \) and \( E(\psi) \) and large values of \( E(z_c) \) and \( E(z_d) \).

Given these distributions for \( \phi \) and \( \psi \), the CGFs are

\[
\kappa_C(1-\eta) = g(1-\eta) - \frac{\lambda_c (1-\eta)}{\beta_c + (1-\eta)} \quad \text{and} \quad \kappa_N(1) = n - \frac{\lambda_d}{\beta_d + 1}
\]

As always, \( \kappa_C^*(1-\eta) = g(1-\eta) \) and \( \kappa_N^*(1) = n \).

We can substitute these CGFs into the expressions for the three factors \( A \), \( B \), and \( C \) that are multiplied together in (11) to determine each of the WTPs. Define

\[
\rho \equiv \delta - n + g(\eta - 1)
\]

One can think of \( \rho \) as the discount rate on future total consumption (as opposed to \( \delta - n \), the discount rate on future utility). It accounts for the growth of total consumption via \( n \) and also the decline in marginal utility of per capita consumption via \( g \). Also define

\[
\lambda'_c \equiv \frac{\lambda_c (\eta - 1)}{\beta_c + 1 - \eta} \\
\lambda'_d \equiv \frac{\lambda_d}{\beta_d + 1}
\]

One can think of \( \lambda'_c \) and \( \lambda'_d \) as risk- and impact-adjusted arrival rates for the two types of catastrophes. For example, increasing \( \beta_d \) reduces the expected impact of a death catastrophe, which is welfare-equivalent to reducing its expected arrival rate. And \( \lambda'_c \) further adjusts for risk aversion; increasing \( \eta \) increases the utility loss from a catastrophe that reduces consumption, which is welfare-equivalent to increasing its expected arrival rate.

Substituting the CGFs, \( \lambda'_c \), \( \lambda'_d \), and \( \rho \equiv \delta - n + g(\eta - 1) \) into (14), (15), and (16), the factors \( A \), \( B \), and \( C \) become:

\[
A = \frac{\rho}{\rho - \lambda'_c} \\
B = \frac{\rho + \lambda'_d}{\rho + \lambda'_d e^{1-\eta}} \\
C = \frac{\rho + \lambda'_d e^{1-\eta} - \lambda'_c}{\rho + \lambda'_d - \lambda'_c}
\]

The next result uses these expressions for \( A \), \( B \), and \( C \) to get the WTPs.
**Result 3.** Under the distributional assumptions made in this section,

\[ w_c = 1 - \left[ \frac{(\rho - \lambda'_c)(\rho + \lambda'_d \varepsilon^{1-\eta})(\rho + \lambda'_d - \lambda'_c)}{\rho(\rho + \lambda'_d)(\rho + \lambda'_d \varepsilon^{1-\eta} - \lambda'_c)} \right]^{\frac{1}{\eta-1}} \]

\[ w_d = 1 - \left[ \frac{(\rho - \lambda'_d - \lambda'_c)}{\rho(\rho + \lambda'_d \varepsilon^{1-\eta} - \lambda'_c)} \right]^{\frac{1}{\eta-1}} \]

\[ w_{c,d} = 1 - \left[ \frac{(\rho - \lambda'_c)(\rho + \lambda'_d - \lambda'_c)}{\rho(\rho + \lambda'_d \varepsilon^{1-\eta} - \lambda'_c)} \right]^{\frac{1}{\eta-1}} \]

Proof. The result follows on using the expressions for \( A \), \( B \), and \( C \) in the relationships \((1 - w_c)^{1-\eta} = ABC\), \((1 - w_d)^{1-\eta} = C\), and \((1 - w_{c,d})^{1-\eta} = AC\). \(\square\)

Note that \( w_c < 1 \) (and \( w_{c,d} < 1 \)) only if \( \rho > \lambda'_c \). Recall that \( \rho \) is a discount rate on future consumption, but ignoring consumption catastrophes. Thus one can think of \( \lambda'_c \) as a depreciation rate that accounts for the risk- and impact-adjusted arrival rate of consumption catastrophes, and thereby reduces the expected future welfare from consumption. If the net discount rate \( \rho - \lambda'_c \) is zero, the welfare loss from the catastrophes is unbounded, pushing the WTP to avoid the catastrophes to one.

Also note that \( \partial w_d/\partial \lambda'_c > 0 \) and \( \partial w_c/\partial \lambda'_d > 0 \), i.e., an increase in the likelihood of catastrophe \( i \) increases the WTP to avert catastrophe \( j \). Each potential catastrophe creates “background risk” that raises the WTP to avert the other catastrophe. As we will see in the next section when we examine pandemics and consumption contractions, these effects of “background risk” can be substantial.

To determine which catastrophes (if any) should be averted, we would also need to know the cost of averting each one, expressed as a permanent tax on consumption at rates denoted by \( \tau_c \) and \( \tau_d \). We would then calculate the net (of taxes) welfare of doing nothing \((W_0)\), averting only consumption catastrophes \((W_c)\), averting only death catastrophes \((W_d)\), and averting both \((W_{c,d})\), and choose the one that is highest. Because a death catastrophe divides the population into two groups with different levels of consumption, Result 2 of Martin and Pindyck (2015) does not hold. Thus we must calculate the net welfare for each possible policy to find the optimal one.

Using eqns. (10), (17), (18), (19) and the expressions above for the CGFs, the net welfare for each policy is:

\[ W_0 = \frac{1}{1 - \eta} \left[ -s(\eta - 1) + \frac{s(\eta - 1) + 1}{\rho + \lambda'_d - \lambda'_c} \right] \]

\[ W_c = \frac{(1 - \tau_c)^{1-\eta}}{1 - \eta} \left[ \frac{\rho + \lambda'_d(s(\eta - 1) + 1)}{\rho(\rho + \lambda'_d)} \right] \]
\[ W_d = \frac{(1 - \tau_d)^{1-\eta}}{(1-\eta)(\rho - \lambda'_c)} \]
\[ W_{c,d} = \frac{(1 - \tau_c)^{1-\eta}(1 - \tau_d)^{1-\eta}}{(1-\eta)(1 - \eta)^2} \]

An increase in \( \lambda_d \) (and thus \( \lambda'_d \)) reduces \( W_c \), net welfare when the consumption catastrophe is eliminated, and an increase in \( \lambda_c \) (and thus \( \lambda'_c \)) reduces \( W_d \), net welfare when the death catastrophe is eliminated. This is the loss of welfare created by “background risk,” here the risk created by the second catastrophe. (And of course an increase in \( \lambda_d \) or \( \lambda_c \) also reduces \( W_0 \), net welfare when neither catastrophe is eliminated.)

5 Lost Consumption vs. Lost Lives

We turn now to the threat of major pandemics and its interdependence with the threat of macroeconomic contractions. What is the WTP to avoid each, and the connection between the two? To address these questions we need values for the mean arrival times \( \lambda_d \) and \( \lambda_c \), the impact parameters \( \beta_d \) and \( \beta_c \), and the parameters \( s, g, n, \delta \) and \( \eta \).

5.1 Pandemics

According to an assessment by the Centers for Disease Control and Prevention (CDC), the threat of a “mega-virus” that could cause widespread fatalities is considerable: “While we can’t predict exactly when or where the next epidemic or pandemic will begin, we know one is coming.” The main reasons: “Increased risk of infectious pathogens spilling over from animals to humans; development of antimicrobial resistance; spread of infectious diseases through global travel and trade; acts of bioterrorism; and weak public health infrastructures.”

Our experience with major pandemics is (fortunately) limited, making the calibration of \( \lambda_d \) and \( \beta_d \) somewhat speculative. (However, as we write, the COVID-19 coronavirus pandemic is spreading around the world.) During the past century the Spanish Flu was by far the worst pandemic, but there were several others, though not as severe. For example, the 1957-1958 H2N2 pandemic killed 1.1 million worldwide and 116,000 in the U.S. (for a U.S. fatality rate of .0064); the 1968 H3N2 pandemic killed about 1 million worldwide and 100,000 in the U.S. (for a U.S. fatality rate of .0006); and the 2009 H1N1 pandemic killed

\(^{18}\text{https://www.cdc.gov/globalhealth/healthprotection/fieldupdates/winter-2017. The CDC is especially concerned with the possibility of a pandemic resulting from mutations of the Asian H7N9 avian influenza virus: “...the pandemic potential of this virus is concerning. Influenza viruses constantly change and it is possible that this virus could gain the ability to spread easily and sustainably among people, triggering a global outbreak of disease.” See https://www.cdc.gov/flu/avianflu/h7n9-virus.htm.}\)
12,469 in the U.S. And then there is HIV/AIDS, which killed more than 700,000 in the U.S. since the epidemic began in 1985, and worldwide close to a million people in 2017 alone.

Based on these numbers and assuming that the likelihood and potential virulence has remained about the same, we would set \( \lambda_d = .02 \) (i.e., an event on average every 50 years and a 33-percent chance of at least one more in the next 20 years) and \( \beta_d = 24 \) (consistent with the roughly 4-percent U.S. mortality rate of the Spanish Flu). These numbers would imply that the probability of a pandemic over the next 20 years with at least a 4-percent (8-percent) mortality rate is .14 (.05).\(^{19}\) However, well before the arrival of COVID-19, epidemiologists have argued that major pandemics have become much more likely and could be much more virulent. For example, Jones et al. (2008) analyse 335 “emerging infectious diseases” from 1940 through 2004, and show sharp increases in the numbers of events over time, after controlling for reporting bias.\(^{20}\) Although these studies do not provide specific estimates of the current likelihood of a major pandemic or possible mortality rates, a doubling of \( \lambda_d \) to .04, while holding \( \beta_d \) fixed at 24, is consistent with the literature. This would imply a 55-percent chance of another pandemic (of some size) in the next 20 years, and implies that the probability of a pandemic occurring in the next 20 years with at least a 4 percent (8 percent) mortality rate is .26 (.10).

Given the uncertainties involved, we consider both of these parameterisations: a “low-risk” pandemic scenario roughly matching the past century, i.e., with \( \lambda_d = .02 \) and \( \beta_d = 24 \), and a “high-risk” scenario with \( \lambda_d = .04 \) and \( \beta_d = 24 \).

### 5.2 Consumption Catastrophes

A major recession can be viewed as a pure consumption catastrophe. We want to estimate the frequency of such events (i.e., the Poisson arrival rate \( \lambda_c \)) and their average intensity (i.e., the power distribution parameter \( \beta_c \)). This can be done in two different ways.

\(^{19}\)For Poisson arrivals with mean a arrival rate \( \lambda \) and exponentially distributed impact with parameter \( \beta \), the probability of at least one event with loss \( \geq L \) over the horizon \( T \) is

\[
Pr(T,L) = 1 - \exp[-\lambda T (1 - L)^\beta],
\]

and the expected total loss over the horizon \( T \) is

\[
E(L;T) = 1 - \exp[-\lambda T / (\beta + 1)].
\]

\(^{20}\)Also, in part because of the overuse of antibiotics, strains of bacteria have evolved that are resistant to most or all antibiotics. See Byrne (2008), Kilbourne (2008) and Enserink (2004), and for a detailed discussion of the increased likelihood and severity of pandemics, Harvard Global Health Institute (2018). Fan et al. (2018) point out that most estimates of the costs of pandemics ignore the cost of lost lives. They estimate that cost using age-specific mortality rates following the 1918 Spanish Flu. However, they consider a known pandemic as opposed to the random arrival of pandemics, with an increasing arrival rate.

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The most common approach, used by Barro (2006, 2009), Barro and Ursúa (2008), Nakamura et al. (2013) and others, is to rely on historical data, e.g., a panel of countries over the past century. For example, Barro and Ursúa (2008), whose results we will use, estimate arrival rates and intensity from a panel of 36 countries (for GDP) from 1870 to 2006. One problem with this approach is that these catastrophes are, almost by definition, “rare events,” limiting what we can learn from even a century or more of data. Also, many of the catastrophes that these and other authors have studied are manifestations of just three global events—World Wars I and II and the Great Depression. Nonetheless, given that this approach is so widely used, we will draw upon these studies to calibrate $\lambda_c$ and $\beta_c$.

A second approach is to ask what arrival rate and impact distribution are implied by the behavior of economic and financial variables. Martin (2008) shows, for example, that the economy’s consumption-wealth ratio along with the preference parameters $\delta$ and $\eta$ are sufficient to estimate the welfare effects of eliminating or reducing consumption uncertainty, without knowing the stochastic process for consumption. Pindyck and Wang (2013) develop an equilibrium model of the economy with shocks to the capital stock, which, as here, are Poisson events with known arrival rate and exponential impact distribution. Using data on consumption, investment, output, the risk-free rate, and the first four moments of stock returns, they determine $\lambda_c$ and $\beta_c$, as well as $\eta$ and $\delta$, as calibration outputs. They find $\eta \approx 3.1$, $\delta \approx .05$, $\lambda = .73$ and $\beta = 23.2$, implying that catastrophes are frequent (occurring about every 1.4 years on average) but with a mean loss of only about 4% (so most shocks are small and can be viewed as part of the normal fluctuations in the economy). Backus et al. (2011) use the prices of equity index options to infer the characteristics of consumption contractions; they also find that shocks are more frequent but smaller on average than suggested by estimates obtained from historical consumption data.

To calibrate our model, we will draw upon Barro and Ursúa (2008). Using world GDP data, but only including declines of 10-percent or greater, they estimate an annual arrival rate of $\lambda'_c = .036$ and average decline of 20.8 percent. Thus for all declines, the parameters are $\lambda_c = .079$ and $\beta_c = 7.3$. These numbers imply a contraction every 13 years on average, with an average contraction size of 12 percent. They also imply that the probability of a contraction occurring in the next 20 years of at least 10 percent (25 percent) in magnitude

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21 But as explained in Pindyck and Wang (2013), pp. 320–321, these results are similar to those in Barro and Jin (2011), and those in Barro and Ursúa (2008) using U.S. GDP data. Barro and Jin (2011) and Barro and Ursúa (2008) only include contractions greater than .095 and .10 respectively. Transforming their estimates to account for all contractions yields relatively large numbers for $\lambda_c$ and $\beta_c$, as shown below.

22 $\mathbb{E}(L|L \geq .10) = .208$, so $\beta_c = 7.3$ is the solution to $1 - .9\beta_c/($ $\beta_c + 1) = .208$. Likewise, $\lambda_c = .079$ is the solution to $1 - \exp(-\lambda_c(.9)^7.3) = \lambda'_c = .036 =$ annual probability of 1 event with loss $L \geq .10$. 

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is .52 (.18). We refer to this calibration as BU1.

As an alternative, we also use the Barro and Ursúa (2008) data for GDP contractions of 10-percent or more in the U.S. They document 5 such contractions over 137 years with an average contraction size of 15.5 percent.\textsuperscript{23} Adding the 2008 recession and its estimated contraction size of 10 percent\textsuperscript{24} yields 6 contractions over 149 years, so \( \lambda_c' = .040 \), and an average contraction size of 14.6 percent. Adjusting to allow for contractions of any size yields \( \lambda_c = .29 \) and \( \beta_c = 18.6 \), which implies much more frequent (every 3.4 years on average) but smaller contractions (5 percent on average), and are closer to the results in Pindyck and Wang (2013) and Backus et al. (2011). This calibration (which we refer to as BU2) implies that the probability of a contraction in the next 20 years of at least 10 percent (25 percent) is .56 (.03).

The left-hand panel of Figure 1 plots the probability of a contraction of size \( L \) or greater occurring during the next 20 years for both calibrations, and illustrates how they differ. The average contraction size in BU2 is much smaller than in BU1, but the greater frequency of contractions results in a greater expected cumulative loss over time, as shown by the

\textsuperscript{23}From page 270 and Table C2, the contractions were “10 percent in 1908 and 1914 ..., 12 percent in 1921, 29 percent in 1933, and 16 percent in 1947.”

\textsuperscript{24}See, e.g., Fernald et al. (2017) and Barnichon et al. (2018).
right-hand panel of Figure 1. Thus we treat BU2 as the more “severe” parameterisation.

Figure 1 also includes the pandemic calibrations. It shows the probability of a loss $L$ or greater during the next 20 years and the expected total loss for both the “low-risk” and “high-risk” calibrations, i.e., $(\lambda_d, \beta_d) = (0.02, 24)$ and $(\lambda_d, \beta_d) = (0.04, 24)$. Here the loss is the fraction of the population that dies, so although the expected total loss is much lower than it is for consumption contractions, that does not make the corresponding WTP lower.

5.3 WTPs

The WTPs also depend on $\eta, \delta, g, n$ and the VSL parameter $s$. We use values for $\eta$ and $\delta$ broadly consistent with both the macro/finance and “disaster” literatures. Pindyck and Wang (2013) obtain $\eta \approx 3.1$ and $\delta \approx 0.05$ as a calibration output of their general equilibrium model. Using estimates of $\lambda$ and $\beta$ for historical GDP contractions, Barro and Ursúa (2008) and Barro (2009) find that $\eta \approx 3.5$ or 4 and $\delta \approx 0.05$ are consistent with an unlevered equity premium of 0.05. We will set $\delta = 0.05$ and $\eta = 3$, and examine how the WTPs vary with $\eta$. Historical data for the U.S. puts $g = 0.02$ and $n = 0.01$. We set $s = 7$, which is in the mid-range of most estimates, so $\varepsilon^{1-\eta} = s(\eta - 1) + 1 = 7\eta - 6$. However, estimates of the VSL (and thus $s$) vary widely, so we also examine how the WTPs vary with $s$.

We calculate the WTP to avert each catastrophe and the WTP to avert both, i.e., $w_c, w_d$, and $w_{c,d}$. To gauge the interdependence of the two catastrophes, we also calculate $w'_c$ and $w'_d$, the WTP to avert one but ignoring the presence of the other. We calculate these WTPs for two sets of calibrations: (1) $(\lambda_c, \beta_c) = (0.08, 7.3)$ and $(\lambda_d, \beta_d) = (0.02, 24)$; and (2) $(\lambda_c, \beta_c) = (0.29, 18.6)$ and $(\lambda_d, \beta_d) = (0.04, 24)$. Set (1) can be viewed as relatively “low risk,” in that the total expected losses for both catastrophes are smaller than for set (2).

Figure 2 shows the WTPs as functions of $\eta$ (for $s = 7$) and $s$ (for $\eta = 3$) for the “low-risk” set of calibrations, and Figure 3 for the “high-risk” set. Three things stand out. First, the WTPs are large, even for the “low-risk” calibration. Note from Figure 2 that the WTP to avert a pandemic like the Spanish Flu — with an annual likelihood of 2 percent — is around 10% of consumption. The WTP to avert consumption contractions is around 23% of consumption for $\eta = 3$, and higher for $\eta > 3$. This may seem high, but is within the range of estimates by others of the welfare costs of consumption uncertainty. For example, Barro (2009) estimates the welfare cost at about 20% of GDP, Martin (2008) gets an estimate of about 14% (for $\eta = 4$ and $\delta = 0.03$, but about 25% for $\eta = 3$ and $\delta = 0.05$), and Pindyck

\[ \text{For any horizon } T, \text{ the expected total loss, } \mathbb{E}(L; T) = 1 - \exp[-\lambda T/(\beta + 1)], \text{ is increasing in } \lambda/(\beta + 1), \text{ which is } 0.0095 \text{ for BC1 and } 0.0145 \text{ for BC2. We are assuming that contractions are permanent, although Nakamura et al. (2013) have shown that on average about half of the lost GDP is regained during recoveries.} \]
and Wang (2013) get a WTP of 30% (15%) of consumption to eliminate all contractions greater than 5% (10%). For the “high-risk” parameterisations in Figure 3, the WTPs are much higher: 18% (31%) of consumption to eliminate pandemic (consumption) risk.

Second, the interaction between the two catastrophes is strong, although it depends on $\eta$. For $\eta \geq 1.5$, a good part of each WTP results from the “background risk” coming from the threat of the other catastrophe. For example, if $\eta = 3$ and $s = 7$, for the “low-risk” (“high-risk”) parameterisations, consumption risk accounts for about 40 percent (35 percent) of the 10% (18%) WTP to avert pandemics, and pandemic risk accounts for about 13 percent (17 percent) of the 23% (31%) WTP to eliminate consumption risk. This interdependence is even greater for larger values of $\eta$, as illustrated in Figure 4, which shows the ratios $w'_c/w_c$ and $w'_d/w_d$ as functions of $\eta$ for the two calibrations. The dependence on $\eta$ comes about because the threat of Catastrophe $i$ reduces expected future consumption, which increases the future marginal utility of consumption by an amount increasing in $\eta$, and this raises the value of averting Catastrophe $j$, which if it occurs would further reduce consumption.

Third, the dependence of the WTPs on $\eta$ is complicated. In Figure 2 both $w_d$ and $w_c$ decrease and then increase with $\eta$, and in Figure 3, $w_d$ is always decreasing with $\eta$ and $w_c$ is roughly constant. Increasing $\eta$ has two off-setting effects. First, it increases the risk- (and impact-) adjusted arrival rate for the consumption catastrophe, $\lambda'_c$ (but has no effect on the

Figure 2: WTP vs $\eta$ and $s$, $(\lambda_c, \beta_c) = (.08, 7.3)$ and $(\lambda_d, \beta_d) = (.02, 24)$. 
impact-adjusted arrival rate for the death catastrophe, $\lambda'_d$), which increases both $w_d$ and $w_c$. Second, it increases $\rho$, the effective discount rate on future total consumption, $N_t C_t$, and this reduces the WTPs. Thus in general, $\partial w / \partial \eta$ is indeterminate.

### 5.4 Some Implications

The high values of $w_c$ and $w'_c$ support earlier studies that found the welfare costs of (unpredictable) macroeconomic contractions to be large, but they add an important element: These costs are partly due to the threat of other catastrophes, in this case pandemics. Pandemic risk, a problem in its own right, adds to the welfare cost of consumption risk. And pandemics are just one source of “background risk;” here we are ignoring the other potential catastrophes (nuclear and bioterrorism, climate, earthquakes, etc.) examined in Martin and Pindyck (2015). Likewise, macroeconomic risk is an important component of the welfare costs of pandemics. The WTP to avoid future pandemics would be about a third lower if we could somehow eliminate macroeconomic fluctuations.

But even without “background risk” and even for the conservative calibration of Figure 2, the WTP to avoid pandemics is substantial — depending on $\eta$, at least 5 to 10% of consumption. For the U.S. in 2018, this translates to around $1 trillion per year, and would be much higher if we account for consumption risk and use the calibration of Figure 3 (which
Figure 4: Interdependence of WTPs: \( w'_i \) is the WTP to avert \( i \) but ignoring the threat of \( j \).

assumes that pandemics have become more likely and more virulent).

As the right-hand panels of Figures 2 and 3 make clear, these high values for \( w_d \) and \( w'_d \) are dependent on the VSL parameter \( s \). Reducing \( s \) from its base value of 7 to, say, 3 cuts \( w_d \) roughly in half in both calibrations. So is \( s = 7 \) the “correct” value to use for this parameter? It is in the mid-range of estimates and consistent with the VSL values of $9 to $10 million used by U.S. regulatory agencies, but it values the lives of the entire U.S. population at about $3,000 trillion, which is some 4 times greater than the roughly $800 trillion present value of U.S. GDP over the next 40 years. When large numbers of lives are involved — as is the case with pandemics — aggregating individuals’ VSL values (based on small reductions in the risk of death) will overstate society’s WTP to save lives.

However, even if we reduce \( s \) to 2 or 3, the WTP to avert pandemics is high — some 4% to 9% of consumption ($0.5 to $1.3 trillion for the U.S. in 2018), depending on the calibration and on \( \eta \). Is this completely unrealistic? Remember that the WTP is a reservation price, i.e., the maximum fraction of consumption society would be willing to sacrifice to achieve a policy objective. The cost of achieving this objective might be much lower, and indeed for pandemics that appears to be the case. Several studies have concluded that pandemic risk could be greatly reduced (but not eliminated entirely) at a cost well below our WTP numbers. But these studies also make clear that we should be spending far more than we currently are on pandemics, a policy position strongly supported by our results.26

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26See, for example, National Academy of Medicine (2016) and Harvard Global Health Institute (2018). The entire FY 2018 budget for the Centers for Disease Control and Prevention was about $8 billion.
6 Conclusions

In our earlier work we showed that decisions to avert (fully or partially) major catastrophes are interdependent, even if the catastrophes themselves are independent, and we showed how to determine which ones to avert. But as with most (if not all) of the literature, we assumed that catastrophes only cause a reduction in the flow of consumption. Here we have shown how catastrophes that cause death can be incorporated into our framework. In so doing, we link the VSL and consumption disaster literatures.

Because we work with CRRA utility and a constant rate of time preference, consumption and wealth are proportional to each other, and we have shown that the social welfare loss from a death is equivalent to the loss that would occur from a drop in wealth (or consumption) from \( w \) to \( \varepsilon w \) (or \( C \) to \( \varepsilon C \)) for a living person, where \( \varepsilon \) is tied to the VSL via eqn. (5). We use estimates of the VSL to determine the value of \( \varepsilon \) and thus the utility loss from death. We can then find the WTPs to avert a “death” catastrophe \( (w_d) \), a consumption catastrophe \( (w_c) \), and both \( (w_{c,d}) \). Our general results hold for any probability distributions for the random reductions \( \phi \) and \( \psi \) of \( \log C \) and \( \log N \), but if \( \phi \) and \( \psi \) are exponentially distributed, these WTPs are very easy to calculate.

We used our model to explore the welfare implications of the threat of pandemics, a threat that epidemiologists argue is substantial and growing. We did this in the context of macroeconomic uncertainty, and in particular the ongoing threat of sharp consumption contractions. There is a large literature on consumption disasters, which we used to calibrate the parameters for the stochastic process for consumption. The literature on pandemics is more speculative, but sufficient to reasonably calibrate the corresponding parameters. We find the utility loss from a “death” catastrophe is large, as is the WTP to avert it. Using middle-of-the-road estimates of the VSL and conservative estimates of the likelihood and impact distribution, we find the WTP to avert future pandemics to be well over 10% of consumption. We also found that a good portion of the WTP to avert pandemics comes from “background risk,” here the threat of a consumption catastrophe. Likewise, the threat of pandemics substantially affects the WTP to avert consumption catastrophes. This is another example of the policy interdependence of major catastrophic threats.

One could argue that middle-of-the-road estimates of the VSL are unrealistically large when applied to catastrophes that could kill many people. But even a much smaller value for the VSL results in a WTP to avert pandemics that is far large than current expenditures.

An important caveat to this paper is our starting assumption in eqn. (1) that social welfare is proportional to the size of the population \( N \). How should we think about the
welfare effects of changes in population? This question has received considerable attention by economists and philosophers, but there is no consensus regarding the social value of more or fewer people. As a practical matter, society spends considerable resources to save lives, including the lives of the very old and very young, and prevent life-threatening disasters large and small. Consistent with this, a catastrophic loss of life reduces social welfare in our model. (However the welfare effect of a change in the natural rate of population growth is indeterminate.)

A second caveat is that CRRA utility plays a central role in our model, and considerably simplifies the analysis. But CRRA utility is restrictive (it ties the coefficient of relative risk aversion to the elasticity of intertemporal substitution), which is why others have explored the use of EWZ recursive preferences to determine the dependence of the VSL on income or age. But we treat consumers as homogeneous, so it is unclear that much would be gained by adding the complication that recursive preferences entails.

Finally, we have assumed that a “death” catastrophe does not affect the consumption of those who survive. As we are currently learning from COVID-19, a serious pandemic can certainly damage the economy and reduce the consumption of the survivors. Although we have not done so, our model can be expanded to incorporate such spillover effects.

References


