Government Financing of R&D: A Mechanism Design Approach[1]

Saul Lach [2] Zvika Neeman [3] Mark Schankerman [4]

June 26, 2020

[1]We are very grateful to two referees and the Editor for constructive comments and suggestions. We have also benefitted from comments by Gidon Azaryev, Alberto Galasso, Konrad Stahl, David Weiss, participants at 2014 Bank of Finland/CEPR conference, Bocconi, Copenhagen Business School, EARIE 2018, IDC Herzeliya, Hebrew University, University of Haifa, Valencia, and Yale. Lach and Neeman gratefully acknowledge financial support from the ISF under grant No. 1508/15, and from the Maurice Falk Institute for Economic Research in Israel. Schankerman gratefully acknowledges the Sackler Fellowship for research visits to Tel Aviv University.

[2]The Hebrew University of Jerusalem and CEPR. E-mail: Saul.Lach@mail.huji.ac.il

[3]Tel Aviv University, Email: zvika@tauex.tau.ac.il

[4]London School of Economics and Political Science, Sackler Senior Visiting Professor at Tel Aviv University, and CEPR. E-mail: m.schankerman@lse.ac.uk

Abstract

We study how to design an optimal government loan program for risky R&D projects with positive externalities. With adverse selection, the optimal government contract involves a high interest rate but nearly zero co-financing by the entrepreneur. This contrasts sharply with observed loan schemes. With adverse selection and moral hazard (two effort levels), the optimal policy consists of a menu of at most two contracts, one with high interest and zero self-financing, and a second with a lower interest plus co-financing. Calibrated simulations assess welfare gains from the optimal policy, observed loan programs, and a direct subsidy to private venture capital firms. The gains vary with the size of the externalities, cost of public funds, and effectiveness of the private VC industry.

KEYWORDS: mechanism design, innovation, R&D, entrepreneurship, additionality, government finance, venture capital.
JEL CODES: D61, D82, O32, O38.

1 Introduction

Innovation, and the knowledge externalities it generates, is the primary source of economic growth. These externalities are central to policy debates over how best to promote sustained growth and competitiveness.[1] Accordingly, governments around the world invest large amounts of public resources to support private sector R&D, through direct support (loans/grants), tax subsidies, and schemes targeting high-technology startups. In the U.S., for example, total government support for private R&D was about \$36.6 billion in 2014, or 11.5% of private R&D. About two-thirds of this funding was in the form of direct support, the remainder as tax subsidies.[2]

In this paper we use mechanism design methods to study the optimal structure of government loans for R&D startups and to show how the optimal design depends on key features of the economic environment. Our approach is motivated by the following key observation. Government funds are socially costly, so to the extent possible, governments should confine their support to startups that generate a sufficiently large positive externality. However, not all high externality projects should be supported. Those with sufficiently high probabilities of success will be supported anyway by the private market which cares only about the expected private return, and those with sufficiently low success probabilities are not worthy of support. This implies that the government should focus on projects that generate a large externality and have an intermediate probability of success. In this sense, the optimal government policy needs to 'target the middle.'[3]

In order to analyze the optimal design of R&D loan programs, we develop a static model in which risk neutral entrepreneurs have risky projects that generate positive externalities. Entrepreneurs have limited internal funds to finance their projects, and face a competitive private venture capital market which provides both finance and 'advice and network connections' that enhance a project's probability of success. In order to focus on externalities, we simplify the description of the private finance market by assuming that venture capital firms are able to solve the adverse selection and moral hazard problems, so that they know the success probability of projects.[4]

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[1] There is an extensive empirical literature documenting that R&D spillovers are large and pervasive, and thus that the market generates underinvestment in innovation – confirmed by the fact that the social rate of return to R&D is much larger than the private return. For an early review see Griliches (1992). For a recent study of both positive spillovers and negative business stealing effects from R&D, see Bloom, Schankerman and Van Reenen (2013) who show that the social rate of return to R&D is much larger than the private return.

[2] See OECD (2019), "R&D Tax Incentives: United States," www.oecd.org/sto/rd-tax-stats -united-states.pdf

[3] This paper focuses on R&D spillovers as the main economic justifiation for government support. Another possible reason is capital market imperfections. There is empirical evidence that cash flow constraints affect capital and R&D investment. The classic reference is Fazzari, Hubbard and Petersen (1989). For a more recent review, see Hall and Lerner (2010).

[4] In practice, VC firms use sophisticated, contingent contracts to overcome this informational asymmetry, but this is outside the scope of our model.

The risk neutral government, however, does not have any information about these probabilities.

In our model, R&D projects are characterised by three features: a probability of success, private returns and an externality. Both the success probability and social returns vary across projects; for simplicity, we assume private returns to be common to all projects but this restriction is relaxed later.

We assume that the government has an unbiased signal of the social returns and has two instruments at its disposal: the interest rate on the loan and a co-financing (matching funds) requirement.[5] Specifically, the government supports projects in the following way. When an entrepreneur applies for a loan, the government obtains an unbiased signal about the externality of the project, and offers a menu of loan contracts to the entrepreneur, which are repaid upon success. The entrepreneur chooses one contract from the menu. The menu consists of pairs of an interest rate and a self-financing requirement, *conditional on the size of the project externality*. A loan with an interest rate equal to minus one is akin to a grant. Matching-loan schemes are used by many countries, but our specification has the additional feature that the co-payment requirement is allowed to depend on the externality generated by the project.[6]

Two core objectives shape the design of the optimal R&D loan policy. The first is to minimise redundancy, i.e., to not support projects that would anyway be funded by the private sector because public funds are costly. The second is to maximise the 'additionality' of government funding, i.e., ensure that entrepreneurs implement all, and only, those projects that generate positive expected social returns. The first objective requires that high probability projects be screened out. The second requires that very risky projects also be excluded because their expected social returns will not justify undertaking them. One would like to design support policies that are both additional and non-redundant. However, for reasons we will explain later, the optimal policy may not always maximise additionality or minimise redundancy. This implies that policy design, and *ex post* evaluation of existing schemes, should not be based exclusively on just one of these criteria.

For simplicity, we assume that an entrepreneur may apply for financing from the private market or from the government, but not from both. We first derive the welfare maximising policy with adverse selection, where projects differ in terms of risk, but without moral hazard – i.e., project success probabilities are exogenous. We show that the optimal policy approximates 'first-best' efficiency and involves selecting exactly those projects that are socially profitable but will not be financed by the capital market.

The optimal contract is to set the interest rate as close as possible to the *ex post* rate of return of the project with the highest probability of success that would still not be supported by the private market, together with a co-financing rate that approxi-

[5] In practice, government loan programs for R&D startups vary along three main dimensions: whether grants or loans are used, the interest rate charged in the case of loans, and the co-financing requirements from the applicant for both grants and loans.

[6] As will become clear in the theoretical analysis, the menu of loan contracts we study is equivalent to a menu of equity contracts, where the government's equity share depends on the project externality, plus a co-payment requirement.

mates zero. Using a high interest rate –in the limit, the *ex post* rate of return – reduces redundancy. The low co-financing requirement increases the set of projects applying for government support, which increases additionality. Under this optimal policy, the entrepreneur bears (almost) no risk in the event the project fails. We call this policy the 'zero liability contract.'

It is worth noting that this contract design differs sharply from the typical R&D loan schemes observed in the real world, which have significant co-financing requirements but zero or negative interest rates (for more information, see `https://rio.jrc.ec.europa.eu/en`). The optimal policy is also very different from the commonly observed pure grant schemes (equivalent to a loan with an interest rate of minus 100%).

When we introduce moral hazard into the model, allowing the entrepreneur to choose between two effort levels, the optimal policy potentially changes sharply. We show that the optimal policy consists of at most two contracts: one is the *zero liability contract,* which is the same as in the case with no moral hazard; the other is characterised by a lower interest rate in order to provide incentives to the entrepreneur to undertake effort, together with a higher co-financing requirement. We call this policy the *maximum outlay contract.*

An alternative way to promote innovation by startups and other small firms, which some governments have adopted, is to provide a subsidy to private venture capital firms (Hellman and Schure, 2010). While the focus of our paper is the design of an optimal loan policy, we also derive the optimal venture capital subsidy policy and compare it to the optimal loan policy in simulations of the model. A direct government loan policy has both advantages and disadvantages relative to a venture capital subsidy. The main advantage is that the government selects projects taking into account the externalities they generate, whereas private venture capital firms do not. On the other hand, VC firms are assumed to have better information on riskiness of projects as compared to government. In addition, VC firms not only provide finance but also advisory services and network connection that increase the probability of project success. These trade-offs imply that the choice between a government R&D loan policy and support for private venture capital finance will depend on the size of project externalities and the effectiveness of the venture capital sector, both of which are likely to vary across countries and perhaps also across sectors.

We simulate the model, using parameters calibrated from various data sources, to illustrate how the optimal government R&D loan policy varies with three key parameters: the cost of public funds, the size of project externalities, and the effectiveness of VCs in enhancing project success. We also assess the welfare gains from using the optimal policy, relative to policies commonly observed in practice, and to the alternative of an optimal direct subsidy to private venture capital firms. We find that the optimal R&D loan policy generates significant welfare gains relative to the private market alone and relative to observed loan policies. The direct VC subsidy can give a higher welfare gain per dollar of government expenditure than the optimal loan policy, but only in those cases where the optimal loan is the maximum outlay contract. The key policy message is that the optimal approach to government support for R&D start-ups depends on these three features of the economic environment. As such, our analysis and simulations show that a 'one size fits all' approach is not appropriate.

From a theoretical perspective, the problem we analyze is a mechanism design problem with type-dependent participation constraints and moral hazard.[7] Three features of the optimal loan policy we develop are worth noting. First, the optimal solution is 'simple' in the sense that it consists of at most two alternatives (at most one for each level of induced effort, two in our model), even though there is a continuum of types. This is unusual in the mechanism design literature.[8] The feature that generates this simplified mechanism is that the optimal policy involves 'targeting the middle': the high types (projects with high probability of success) will be funded by the private market and the low types do not justify public financing because their expected social gains are negative. We show that if a given type prefers one government loan over another, then so do all higher types. Because of the incentive compatibility constraint, offering another contract to the higher type involves leaving more rent to the entrepreneur and there is no social payoff to doing that.[9] Second, we also show that, under a mild restriction which is empirically relevant, the optimal policy actually consists of *only one* contract. Whether it is the zero liability or maximum outlay contract depends on parameter values, in particular the size of the project externality and the cost of public funds. Third, our conclusion that the optimal solution consists of at most two contracts is robust to the introduction of two-dimensional uncertainty, where there is both asymmetric information about the project probability of success and the private return when successful. We are not aware of any examples in the mechanism design literature for which this is the case.

1.1 Related Literature

Two recent papers study the impact of using direct grants and indirect fiscal instruments to support innovation. First, applying regression discontinuity analysis, Howell (2017) shows that seed grants from the Small Business Innovation Research Program in the United States significantly improve the chances of small high-technology companies to secure venture capital funding and enhance their subsequent perfor-

[7]Myerson (1982) introduced and proved the Revelation Principle for generalized principal-agent problems with a privately informed agent. Julien (2000) applies these results to monopolistic and competitive nonlinear pricing.

[8]The optimal mechanism in our model is a single (linear) contract for each effort level. Laffont and Tirole (1986) provide an early example of mechanism design with adverse selection and moral hazard that generates a menu of linear incentive contracts. For discussion of 'simple' mechanisms and how to achieve them, see Hurwicz (1973), Wilson (1985), Dasgupta and Maskin (2000), and Bergemann and Morris (2005).

[9]This intuition is analogous to the one that underlies the famous "no-haggling" result in monopoly pricing that implies the optimality of menus with a single contract (Myerson, 1981; Riley and Zeckhauser, 1983). Samuelson (1984) and, more recently, Bergemann et al. (2018) applied a similar intuition to explain why in the presence of an additional constraint, the optimal menu may consist of up to two contracts. Here, the additional constraint ensures that the second contract induces full effort from some entrepreneurs' types (note that there is no need to require that the first contract induces partial effort, which would introduce yet another constraint, because an entrepreneur who exerts no effort cannot get any government support anyway).

mance. In a very different type of analysis, Acemoglu et al. (2018) develop and estimate a macroeconomic model of firm-level innovation and productivity growth that incorporates heterogeneous firms and entry and exit, and then use it to simulate various counterfactual fiscal policies. Among other results, they show that an optimal R&D subsidy (equivalent to 39% of R&D) generates a 1.22% welfare gain. In their framework, the subsidy induces adverse selection effects on incumbent firms and entrants. When expressed in comparable terms, our simulations imply that the optimal R&D loan policy generates somewhat larger welfare gains – between 1.73% and 2.42%. Our optimal R&D loan policy is based on a mechanism that is designed to avoid negative selection effects and maximize welfare. Given that our optimal policy is targeted in this way, larger welfare gains are to be expected.

These two papers differ from ours in their objective and approach. However, our paper is related to Howell (2017) and others in that we focus on direct instruments to foster innovation (loans and grants) in a partial equilibrium framework, and to recent macroeconomic models such as Acemoglu et al. (2018) in that we offer a quantitative welfare evaluation of government R&D support policies. These studies, including ours, highlight the importance of assessing the innovation and welfare effects of different policy instruments.

A number of empirical studies, some based on survey data and others adopting more formal econometric methods, have analysed the 'additionality' of existing R&D subsidies and loan schemes. These include Takalo, Tanayama and Toivanen (2013 and 2017) who develop structural models of R&D to estimate the welfare effects of R&D subsidies. Non-structural econometric studies include Busom (2000), Klette, Moen and Griliches (2000), Wallsten (2000), Lach (2002), and Gonzalez, Jaumandreu and Pazo (2005). Most of these studies find evidence of additionality from government subsidies, but they also reveal substantial variation in the degree of additionality across programs.[10] This naturally raises the question of how the design of support programs affects additionality and, more generally, how loan (and grant) programs should be structured to maximise welfare.

To our knowledge, there is almost no research that addresses this important question. One recent, closely related paper by Akcigit, Hanley and Stantcheva (2016) studies the optimal design of R&D subsidies and corporate taxation as a dynamic mechanism design with asymmetric information and externalities. However, the setting and the focus of their paper is very different from ours, in part because they study different instruments and do not incorporate a role for private venture capital financing. We view our paper as complementary to theirs, and part of a broader research agenda that focuses on the design of R&D policies rather than evaluating existing programs.

Our analysis of the optimal design of R&D loan policy is set in the context of a private VC market which constitutes the alternative source of funding for entrepreneurs' R&D projects. In modelling the VC market (in Section 2.2), we draw on a rich theoretical and empirical literature on the role venture capital firms play and how they

[10] For general discussion of additionality, see OECD (2006).

structure contracts to minimise the problems of adverse selection and moral hazard (Gompers, 1995; Lerner, 1995; Kaplan and Stromberg, 2002, 2004; for a review of the literature, see Da Rin, Hellmann and Puri, 2012). Among other things, this literature emphasises that VC firms provide more than just finance; they also provide 'advice' and a network of connections that enhance the probability of success of the start-up projects they support, and this is reflected in the price entrepreneurs pay for VC affiliation (Hellman and Puri, 2002; Hsu, 2005). In addition, the literature emphasises the dynamic structure of contracts – in particular, the use of contingent, performance-based cash flow rights, control rights and governance structures.

The rest of the paper is organized as follows. Section 2 presents the setup of the model with the private venture capital market. In Section 3 we derive the optimal policy when there is adverse selection but no moral hazard. Section 4 introduces moral hazard, and shows that this materially changes the structure of the optimal policy. We also briefly discuss extensions to the model. Section 5 presents the optimal venture capital subsidy and characterizes its properties. In Section 6 we present simulations to assess the welfare performance of different policies against the benchmark of the optimal policy with moral hazard and to compare the optimal loan policy to the optimal venture capital subsidy. We conclude with a brief summary and implications for policy. All proofs are relegated to an Appendix (additional computational details and tables are in a series of online appendices).

2 Model

2.1 Definitions and Assumptions

We consider a model where a risk neutral government faces a large number of risk neutral entrepreneurs. Each entrepreneur has a project that generates both private and social benefits, and the government has to determine whether and how to support these projects.

An entrepreneur's project is characterised by a pair (p, s) where p is the project's probability of success as explained below, and s is the (non-negative) externality it generates. A successful project generates a commonly known (private) return $R > 1$.[11] If the project fails, the private return and social contribution are both zero. The cost of the project is normalized to 1; it is assumed to be commonly known. We decompose this cost into two additive components: c_I is the cost of developing the idea and prototype for the project ('inspiration') and c_p is the cost of further development ('perspiration') that enhances the project's probability of success but is not necessary for the project to succeed.

The parameter $p \in [0, 1]$ denotes the project's probability of success if the entrepreneur exerts 'full effort' at cost $c_I + c_P \equiv 1$. If the entrepreneur only exerts the

[11] The assumption that R is commonly known is relaxed in Section 4.2.1. It simplifies the analysis but does not affect our main results.

'partial effort' c_I, then the project's probability of success is kp for some $k \in [0, 1]$. Entrepreneurs have funds $\overline{b} \leq 1$ of their own. We assume that they are able to finance the first (inspiration) stage of the project on their own, i.e., $c_I \leq \overline{b}$. If $\overline{b} < 1$, an entrepreneur cannot complete the project without partial funding by a VC or the government in the amount of $1 - \overline{b}$.

If the entrepreneur is funded and advised by a venture capitalist, the project's probability of success is scaled up by a factor of $\beta \geq 1$ or $\frac{1}{p}$, whichever is smaller, provided the entrepreneur exerts full effort.[12] It is assumed that this enhancement does not apply if the entrepreneur only exerts partial effort. The probability of success of an entrepreneur who is funded by the government and exerts full effort is p. We assume that the probability p is known by the entrepreneur and observable to VCs, but not to the government. Since agents are risk neutral, this is equivalent to allowing the project's p to be drawn from a distribution whose mean is known by the entrepreneur and VCs.

We distinguish between two cases: one where the entrepreneur's effort c_P is observable to the VCs and the government, and the other where it is not. If c_P is observable, there is no moral hazard. We analyze this simpler case in Section 3. The case where c_P is unobservable involves moral hazard. One way this can arise is that the entrepreneur may choose to divert the external funds she receives and not exert the additional effort c_P. The severity of the moral hazard problem facing the entrepreneur is increasing in k because a higher k makes the option of exerting partial effort more attractive. We analyze this more realistic case in Section 4.

For most of the analysis, we impose no restriction on the relationship between k and c_I. But in the simulations section, we focus on the case where $k = c_I$ to simplify the analysis. When $k = c_I$, the percentage increase in cost in moving from partial to full effort is equal to the percentage increase in the associated probability of success. We call this 'constant returns to effort'.

Finally, we assume that the government observes a signal about the externality from the project, $\sigma \in [0, \infty)$, which we normalize to be such that $\sigma \equiv E[s|\sigma]$.[13] Because σ provides the best estimate of the unobserved s and the government is risk neutral, no loss of generality is involved by simply replacing s by σ below. Thus, we assume that a project is characterised by a pair (p, σ) instead of (p, s), and that the government believes that p and σ are drawn from a commonly known joint distribution, which we write as $F_\sigma(p)$. In the model we do not make any assumptions about the correlation between p and σ, so the correlation between expected private and total social returns, pR and $p(R + \sigma)$, is unrestricted.

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[12]Thus, the probability of success of projects with a small p is multiplied by β, and that of projects with a larger p, for which $\beta p \geq 1$, increases to 1.

[13]Since only the (risk neutral) government cares about these externalities anyway, our model is formally equivalent to one in which the public returns of projects, if successful, are publicly known.

2.2 The Venture Capital Market

Our primary objective is to study the optimal design of government loan policies for R&D, not the VC market. But we want to place the analysis in the context of a stylised depiction of the alternative private financing opportunities entrepreneurs face. For this purpose, we adopt a simplified characterisation of the venture capital market. Since our model is static, we cannot capture the dynamic features of contingent contracting that are observed in the VC industry. However, we incorporate two important characteristics of observed venture capital markets.

First, we assume that the advice and networks VC's provide (in addition to capital funds) enhance the probability of success of supported projects. Second, we assume that VC firms provide capital in return for an equity stake in the project, which is realised if the project succeeds. As we show below, the equity stake will vary across projects and depend on their probability of success, because competition among VC's drives expected profits to zero.[14]

Finally, we assume that VC firms know the probability of success for each project. In other words, we assume, for purposes of simplification, that the contractual provisions the VC firms actually use (but which we do not model here) work effectively to solve the VC's adverse selection problem. While we recognise that this description of the VC market is highly stylised, it allows us to focus on the optimal design of public support under the information constraints the government faces.

In our setup, from a welfare perspective, the VC market has two advantages over the government loan. First, VC firms have an informational advantage in that they are assumed to know the probability of success for each project, which is unknown to the government. Second, VC involvement enhances the success probability by providing technical, marketing advice and network connections. On the other hand, the government has the advantage of taking into account the externality generated by the project. Since they are profit maximising, VC firms do not factor this externality into their evaluation of potential projects.

As mentioned already, we assume entrepreneurs have internal funds in the amount $\overline{b}$, where $c_I \leq \overline{b} \leq 1$, so they are able to finance the first (inspiration) stage of the project on their own. In addition, they have access to a perfectly competitive venture capital market in which they can obtain financial support of $1 - \overline{b}$ that allows them to complete the development and commercialisation of the project. In addition, the VC increases the project's probability of success from kp (with partial effort) to $\min(\beta p, 1)$ with full effort. The parameter β captures the effectiveness of the VC in its advisory and networking role.

The VC assesses the success probability p, and asks for an equity share $\alpha(p)$ that ensures it can break even on its investment, provided the entrepreneur exerts full

[14]Note two points. First, in practice, VC firms charge for their management services through a percentage levy per dollar of capital invested (typically 2%) for each round of financing. We can easily incorporate this fee into the model, but it does not change any of our results. For simplicity, we drop it from the analysis. Second, the value of the equity stake from the successful projects (after repayment of initial capital invested) is shared by the venture capital managers and the investors who fund the VC, but this plays no role in our model.

effort. If the project succeeds, the payoff to the VC is $\alpha(p)R$; if it fails, the return to the project is zero and the VC and entrepreneur do not recoup their costs.

Since the VC market is competitive, the zero expected profit condition is

$$\alpha\left(p\right)\min(\beta p, 1)R - (1+\varrho)(1-\bar{b}) = 0$$

where ϱ is the risk-adjusted normal rate of return, which we normalize to zero. It follows that the zero-profit equity share for the project is

$$\alpha\left(p\right) = \frac{1-\overline{b}}{\min(p\beta, 1)R}.$$

Because the VC cannot take an equity stake greater than one, it will refuse to support projects whose $p < \frac{1-\overline{b}}{\beta R}$ on which it cannot break even, even if it has right to the entire return of the project.

Observe that the VC would only invest in a project if the entrepreneur is induced to exert full effort. This is because, given the equity stake $\alpha\left(p\right)$, a VC would lose money on its investment if it lends $1-\overline{b}$ to an entrepreneur who is not induced to exert full effort.[15,16]

This analysis yields the following *moral hazard constraint*:

$$\min(\beta p, 1)(1-\alpha(p))R + \left(1-\overline{b}\right) - 1 \geq kp(1-\alpha(p))R + \left(1-\overline{b}\right) - c_I.$$

The left hand side is the expected payoff to the entrepreneur from exerting full effort: a project is successful with probability $\min(\beta p, 1)$ and generates a return $(1-\alpha(p))R$ to the entrepreneur; the sum $1-\overline{b}$ is obtained from the VC; and -1 is the entrepreneur's cost of full effort. The right hand side is similar except for the fact that, with partial effort, the probability of success decreases to kp and the cost of the entrepreneur's effort decreases to c_I. The moral hazard constraint simplifies to

$$p \geq \frac{1-c_I}{R(\beta-k)} + \frac{1-\overline{b}}{\beta R}$$

[15]Of course, since the VC observes p, it could offer a different equity stake to entrepreneurs who exert partial effort. While the VC cannot directly observe effort, it can compute the critical value p^* above which the entrepreneur would exert full effort. It could offer funding of $1-\overline{b}$ in exchange for the zero-profit equity stake $\alpha(p)$ for $p \geq p^*$ and funding of c_I with a different $\alpha(p)$ for $p < p^*$ that allows it to break even on those projects. However, the entrepreneur exerting partial effort does not require VC support since $c_I \leq \overline{b}$, and she would be indifferent to taking the offer or using her own funds. We assume that in cases of indifference she self-finances.

[16]For simplicity, we assume that the VC does not enhance the probability of success unless the entrepreneur exerts full effort. However, even if $\beta > 1$ with partial effort, the qualitative analysis is unaffected as long as the entrepreneur's payoff function remains increasing and convex in p. This will hold as long as the value of β is larger when the entrepreneur exerts full effort than with partial effort (this follows from the definition of $U_P(p)$ below).

for projects with $p < \frac{1}{\beta}$. When $p \geq \frac{1}{\beta}$, the moral hazard constraint is satisfied if $R \geq 1 - \bar{b} + \frac{1-c_I}{1-k}$. We assume that this inequality holds. As explained in the simulation section, the calibrated value of R based on observed data easily satisfies this constraint. This inequality also ensures that $\alpha(p) \leq 1$.

In addition, it is important to note that VCs would only lend to projects with nonnegative expected value, i.e., $\beta pR - 1 \geq 0$ or equivalently $p \geq \frac{1}{\beta R}$. An entrepreneur with a lower p would not be interested in a loan if it intends to exert full effort. For simplicity, we assume that any project that satisfies the moral hazard constraint also generates a nonnegative expected value with full effort. This requires that $\frac{1}{\beta R} \leq \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}$ or equivalently $\beta(1 - \bar{b} - c_I) + \bar{b}k \geq 0$. Given the calibrated parameters we use (based on various data sources; Appendix A for details), this inequality is easily satisfied.[17]

An entrepreneur who is denied VC support can still develop the project on her own with partial effort and obtain expected payoff $kpR - c_I$. An entrepreneur prefers taking VC funding with an equity stake of $1 - \alpha(p)$ and exerting full effort, to developing the project on her own with only partial effort, if and only if:

$$\min(p\beta, 1)(1 - \alpha(p))R + \left(1 - \bar{b}\right) - 1 \geq kpR - c_I.$$

It is straightforward to verify that this inequality is satisfied if p satisfies the moral hazard constraint.

Summarising these results, the following proposition characterises the set of projects that are developed without government intervention.

Proposition 1. *(Sorting in the Private Market) Entrepreneurs of type* $p \in \left[0, \frac{c_I}{kR}\right)$ *abandon their projects. Entrepreneurs of type* $p \in \left[\frac{c_I}{kR}, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right)$ *develop their projects on their own and exert only partial effort. However, if* $\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R} < \frac{c_I}{kR}$ *then entrepreneurs of type* $p \in \left[0, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right)$ *abandon their projects. Entrepreneurs of type* $p \in \left[\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}, 1\right]$ *develop their projects with VC funding of size* $1 - \bar{b}$*, for which they grant the VC an equity stake* $\alpha(p)$*. These entrepreneurs exert full effort.*

Denote the payoff to the entrepreneur from developing its project by $U_P(p)$. Using the formula above for the equity stake $\alpha(p)$, Proposition 1 implies that U_P is

[17] The model can also be solved without this assumption. However, we would need to distinguish between the two cases. We prefer not to do it because this would complicate the analysis without adding any important insights.

given by[18]

$$U_P(p) = \begin{cases} 0 & \text{if } p \in \left[0, \frac{c_I}{kR}\right) \\ kpR - c_I & \text{if } p \in \left[\frac{c_I}{kR}, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right) \\ \min\{\beta p, 1\}R - 1 & \text{if } p \in \left[\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}, 1\right] \end{cases}$$

This analysis shows that, absent government intervention, entrepreneurs with $p < \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}$ will not be able to obtain financing for their projects from the private market and these projects are either abandoned or only implemented with partial effort. Entrepreneurs with $p < \frac{c_I}{kR}$ would not want to develop their projects on their own even with partial effort. However, to the extent that some of these projects increase social welfare, the government would be interested in helping to fund them. In addition, some of the projects with $p \in \left[\frac{c_I}{kR}, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right)$, which are only implemented with partial effort, may also increase social welfare and warrant government support to induce full effort.

The VC model presented in this section is obviously very stylised, though we think it captures some basic features that are observed in the real world. Its role is simply to provide a reasonable description of the private market within which to analyze the design of government support of start-ups and conduct simulations, which is our main focus. It is worth noting, however, that *any model* of the private market that induces convex expected payoffs – such that entrepreneurs with small p's drop their projects, those with intermediate p's implement their projects but exert only partial effort, and those with large p's implement their projects and exert full effort – would deliver similar results as far as the analysis of government support is concerned.

2.3 Government Funding

We assume that the government supports projects in the following way. When an entrepreneur applies for a loan, the government obtains a signal σ about the externality of the project, and offers a menu of conditional loan contracts to the entrepreneur of $1 - b_\sigma$ at interest rate r_σ. Each conditional loan enables the entrepreneur to implement the project, and she chooses (at most) one of the conditional loan contracts offered. An entrepreneur who selects a loan of size $1 - b_\sigma$ needs to raise an amount b_σ from her own or borrowed funds. We restrict attention to cases where $b_\sigma \leq \bar{b}$. We emphasize that our specification has the feature that the co-payment requirement b_σ and interest

[18]However, if $\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R} < \frac{c_I}{kR}$ then

$$U_P(p) = \begin{cases} 0 & \text{if } p \in \left[0, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right) \\ \min\{\beta p, 1\}R - 1 & \text{if } p \in \left[\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}, 1\right] \end{cases}$$

rate r_σ are allowed to depend on the externality generated by the project, σ. As we show below, the optimal menu consists of at most one pair (b_σ, r_σ) for each level of induced effort. For notational simplicity, in what follows we omit the subscript σ.[19]

The cost of public funds is $1+\lambda$ where $\lambda \geq 0$. In what follows, we refer to λ as the shadow price of public funds. Consider a project (p, σ) that receives government support in the form of a loan of size B at interest rate $r \leq \frac{R}{B} - 1$ (this inequality ensures the entrepreneur can pay back the loan to the government if the project succeeds), and where the entrepreneur exerts full effort. This project generates expected social welfare

$$W(p, \sigma, b, r) \quad = \quad p(R+\sigma) - 1 - \lambda(1-b)(1-p(1+r)). \tag{1}$$

With probability $1-p$ the project fails, generates no return, and costs $b + (1-b)(1+\lambda)$. With probability p the project is successful, and generates a social return of $R+\sigma$, at a cost $b + (1-b)(1+\lambda) - \lambda(1-b)(1+r)$. Note that, if the project succeeds, the social cost $\lambda(1-b)(1+r)$ is offset by the entrepreneur's payback and is not incurred. In expectation, this yields the expression in (1). If the entrepreneur takes the loan but only exerts partial effort, the expression is similar except that p is replaced by kp, and -1 is replaced by $-c_I$.

The expected social welfare of a project in which the entrepreneur exerts full effort with VC support, but without a government loan, is

$$W(p, \sigma) \quad = \quad \min(\beta p, 1)(R+\sigma) - 1 \tag{2}$$

If the entrepreneur receives no support from either a VC or the government and exerts only partial effort, then the probability of success is kp and the cost of effort is c_I so expected social welfare is[20]

$$W(p, \sigma) \quad = \quad kp(R+\sigma) - c_I \tag{3}$$

Entrepreneurs who can obtain VC support, or who would develop their project on their own with partial effort, may nevertheless accept a government loan. When this happens, the government loan does *not* generate additional innovation. In this case, government support of these projects is 'redundant' because the set of projects being implemented and the effort exerted by entrepreneurs is not changed by the support program. The only exception is when the entrepreneur would have exerted partial effort but exerts full effort with the government loan.

This suggests that in order to maximise expected social welfare the government should try to fund only those projects that will not be financed by the private market.

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[19] We do not analyse the case where an entrepreneur is funded both by a VC firm and the government. Generalizing the model in this way would raise informational complexities that would seriously complicate the model – in particular, whether the VC would have the incentive and/or the ability to credibly signal the project's success probability to the government. It would also raise issues of how the blended finance arrangement would affect the ability of the VC firm to implement its performance-based contingent contract provisions without agreement of the government funder. A proper treatment of this issue requires a separate analysis. We leave this for future research.

[20] Recall that VCs do not support entrepreneurs who are not induced to exert full effort.

However, as we will show, this is only part of the story because avoiding redundancy can restrict the set of projects being implemented, but some of these projects may have large externalities that justify government support. In other words, it may be impossible to generate additionality without also incurring some redundancy.

3 Analysis of Optimal Policy without Moral Hazard

In this section we solve for the optimal policy in the case in which both VCs and the government can verify the entrepreneur's investment of c_P, and thus there is no moral hazard. For simplicity we assume that $k \leq c_I$, which implies weakly increasing returns to effort. The purpose of this assumption is clarified in footnote 21 below.

Under this assumption, neither VCs nor the government would be interested in supporting projects implemented with partial effort (which entrepreneurs can anyway do on their own without outside support). VCs have no interest in supporting partial effort because of our assumption that VCs require full effort to realize the benefit of their advice (enhancing the success probability); and if the government can generate positive expected social welfare with support of only partial entrepreneurial effort, then it is also possible to do it with full effort, with even larger expected social welfare.[21]

In this case, it can be shown that the function U_P is given by[22]

$$U_P(p) = \begin{cases} 0 & \text{if } p \in \left[0, \frac{1}{\beta R}\right) \\ \\ \min\{\beta p, 1\}R - 1 & \text{if } p \in \left[\frac{1}{\beta R}, 1\right] . \end{cases}$$

Entrepreneurs of type $p \in [0, \frac{1}{\beta R})$ abandon their projects, and entrepreneurs of type $p \in [\frac{1}{\beta R}, 1]$ develop their projects with VC funding of size $1 - \overline{b}$, and pay the VC an equity stake $\alpha(p)$. These entrepreneurs exert full effort.

The analysis of this case is intuitive and simple, and we believe it is interesting in its own right. We start by describing the first-best solution in this case, i.e., the

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[21]Equation (1) and the text below it imply that the maximal social welfare that is generated by government support of a project with partial effort is $pk(R+\sigma) - c_I - \lambda(c_I - b)(1 - pk(1+r)) = pk(R+\sigma) - c_I(1+\lambda) + pk\lambda R$. And, the maximal social welfare that is generated by government support of a project with full effort is $p(R+\sigma) - 1 - \lambda(1-b)(1-p(1+r)) = p(R+\sigma) - (1+\lambda) + p\lambda R$. With partial and full effort, non-negative social welfare requires that $p \geq \frac{c_I(1+\lambda)}{(1+\lambda)kR+k\sigma}$ and $p \geq \frac{1+\lambda}{(1+\lambda)R+\sigma}$, respectively. Therefore, if $k \leq c_I$, the social welfare associated with full effort is larger than that associated with partial effort for every $p \geq \frac{1+\lambda}{(1+\lambda)R+\sigma}$.

[22]When there is no moral hazard, the entrepreneur can either exert partial effort on its own and obtain $pkR - c_I$, exert full effort with VC support and obtain $\min\{\beta p, 1\}R - 1$, or drop the project. This means that $U_P(p) = \max\{pkR - c_I, \min\{\beta p, 1\}R - 1, 0\}$. As mentioned above, we focus on the case where the return to the entrepreneur from full effort is larger than from partial effort for every $p \geq \frac{c_I}{kR}$, which requires that $\beta \geq 1 \geq \frac{k}{c_I}$ as assumed.

optimal solution if the government could observe p. In many cases, the first-best is only a theoretical benchmark that cannot be implemented in practice, but here the first-best is (approximately) attainable if the entrepreneur faces no moral hazard.

3.1 First-Best

If the government can observe p, it should only support a project if it would not otherwise be funded, that is

$$p < \frac{1}{\beta R}$$

and if the project generates a positive expected social welfare with government support through some conditional loan contract (b, r):

$$p\,(R+\sigma) - 1 - \lambda\,(1-b)\,(1-p\,(1+r)) \geq 0. \tag{4}$$

An entrepreneur will accept a government loan at interest rate r with self-financing requirement b if it makes a nonnegative payoff

$$p\,(R-(1-b)\,(1+r)) + (1-b) - 1 \geq 0, \tag{5}$$

and it cannot do better on its own, possibly with VC support. We assume that the interest rate charged by the government on any conditional loan it makes is smaller than or equal to $R-1$. Otherwise the entrepreneurs that the government targets – those with $p < \frac{1}{\beta R}$ – would make negative profits by accepting the loan.[23]

To summarise, the constraint that a conditional loan contract (b, r) generates positive expected welfare can be rewritten as:

$$p\,(R+\sigma) - 1 - \lambda + \lambda\,(b + p\,(1-b)\,(1+r)) > 0.$$

The necessary condition that the entrepreneur accepts a contract (b, r) – the participation constraint – can be rewritten as:

$$b + p(1-b)(1+r) \leq pR.$$

The fact that public funds are costly ($\lambda \geq 0$) implies that maximising expected welfare, subject to the entrepreneur's participation constraint, implies that $b + p\,(1-b)\,(1+r)$ should be set as high as possible, and therefore equal to pR. Using this fact and solving for p in (4), treating it as an equality, yields $p = \frac{1}{R+\frac{\sigma}{1+\lambda}}$. Entrepreneurs with $p < \frac{1}{R+\frac{\sigma}{1+\lambda}}$ should be excluded by the government because they generate negative expected welfare. Entrepreneurs with $p \geq \frac{1}{R+\frac{\sigma}{1+\lambda}}$ should be supported unless $p \geq \frac{1}{\beta R}$, in which case they would anyway be supported by VCs.[24]

[23]The entrepreneur's expected payoff from a conditional loan B with interest rate $r > R-1$ is $p(R - B(1+r)) + B - 1 \leq p(R-BR) + B - 1 = (1-B)(pR-1)$. This is negative for $p < \frac{1}{\beta R}$.

[24]Of course, if $\frac{1}{\beta R} < \frac{1}{R+\frac{\sigma}{1+\lambda}}$ then the government should not support any entrepreneur. This occurs when the externality $\sigma \leq (\beta-1)(1+\lambda)R$ is too small to justify government support. We proceed under the assumption that this is not the case here.

There are many (b,r) contracts that satisfy the equation $b + p(1-b)(1+r) = pR$ for any given probability p. Of particular note is the contract $(b,r) = (0, R-1)$ because it is independent of the value of p, and it satisfies this equation for every value $\frac{1}{R+\frac{\sigma}{1+\lambda}} < p < \frac{1}{\beta R}$. This suggests that the government may be able to set b and r optimally even without being able to observe p. We show this in the next section.

3.2 Optimal Policy without Moral Hazard

Suppose that the government cannot observe p. The problem with the contract $(b,r) = (0, R-1)$ is that it induces an expected payoff of zero to the entrepreneur, regardless of its type. Therefore, such a contract may be picked by entrepreneurs with $p < \frac{1}{R+\frac{\sigma}{1+\lambda}}$, which would reduce welfare.

We now describe a *family of contracts* $\{b_\varepsilon, r_\varepsilon\}_{\varepsilon>0}$, parametrized by ε, such that each contract induces an increasing payoff to the entrepreneur that is linear in p and equal to 0 at the point $p = \frac{1}{R+\frac{\sigma}{1+\lambda}}$. This means that entrepreneurs with $p < \frac{1}{R+\frac{\sigma}{1+\lambda}}$ will refuse contracts in this family. Furthermore, the slope of the induced payoff is decreasing in ε so that the entrepreneur's payoff from this contract decreases to zero as ε tends to zero. This implies that, as ε tends to zero, the contract $(b_\varepsilon, r_\varepsilon)$ described below approximates the first-best outcome.

Define

$$r_\varepsilon = R - 1 - \varepsilon; \;\; b_\varepsilon = \frac{\varepsilon(1+\lambda)}{\sigma + \varepsilon(1+\lambda)}.$$

It is easy to verify that the expected payoff to an entrepreneur of type p from accepting the contract $(b_\varepsilon, r_\varepsilon)$ is

$$\frac{\varepsilon((1+\lambda)R + \sigma)}{\varepsilon(1+\lambda)+\sigma}p - \frac{\varepsilon(1+\lambda)}{\sigma+\varepsilon(1+\lambda)}.$$

This payoff function is linear in p, is equal to zero at $p = \frac{1}{R+\frac{\sigma}{1+\lambda}}$, and its slope decreases to zero with ε as required.

An entrepreneur p prefers the contract $(b_\varepsilon, r_\varepsilon)$ to developing the project with VC support or dropping the project if and only if

$$p(R - (1-b_\varepsilon)(1+r_\varepsilon)) + (1-b_\varepsilon) - 1 \geq \max\{\beta pR - 1, 0\},$$

which is equivalent to

$$\frac{1}{R+\frac{\sigma}{1+\lambda}} \leq p < \frac{1}{1 + r_\varepsilon + \frac{(\beta-1)R}{1-b_\varepsilon}}.$$

The fact that the denominator $1 + r_\varepsilon + \frac{(\beta-1)R}{1-b_\varepsilon}$ increases to βR as ε decreases to zero implies that, by choosing a mechanism $(b_\varepsilon, r_\varepsilon)$ with a small $\varepsilon > 0$, the government can minimise the set of 'redundant' projects $p \in \left[\frac{1}{\beta R}, \frac{1}{1+r_\varepsilon+\frac{(\beta-1)R}{1-b_\varepsilon}}\right]$ that would have been developed anyway with VC support. Moreover, because r_ε approaches $R-1$ as ε approaches 0, the government can extract almost the entire rent from each participating entrepreneur. It follows that a mechanism $(b_\varepsilon, r_\varepsilon)$ with a small $\varepsilon > 0$ allows the government to approximate the first-best solution.

However, note that the *specific* contract $(b_\varepsilon, r_\varepsilon)$ will depend on the values of λ and σ which define the lower bound of p at which the entrepreneur would accept the contract, as given above. We call such a $(b_\varepsilon, r_\varepsilon)$ contract the "zero liability contract" because, as ε decreases to zero, the potential liability (loss) incurred by an entrepreneur who takes it decreases to zero as well.

Proposition 2. *(Optimal Policy w/o Moral Hazard) It is possible to approximate the first-best solution with a conditional loan contract $(b_\varepsilon, r_\varepsilon)$ that tends to the zero liability contract $(b, r) = (0, R-1)$ as ε tends to zero.*[25]

The economic intuition for this result is that the optimal policy charges a high interest rate to induce entrepreneurs with $p \geq \frac{1}{\beta R}$ to prefer developing their projects on their own: the higher the interest rate (smaller ε) the smaller the set of subsidized projects that would have been financed by the market. But increasing the interest rate also reduces the set of socially desirable projects that would accept the government's contract. To induce such entrepreneurs to seek a government loan, the government increases the size of the loan so as to make it just profitable for them to implement their projects. Notice also that, somewhat paradoxically, the optimal policy calls for (almost) fully funding the supported projects (b is approximately equal to zero) even though public funds are more expensive than private funds.

The mechanism described here has one unattractive property: it leaves almost no rent for the entrepreneur and thus gives no incentive to exert greater effort. This is not a problem if the entrepreneur's effort is verifiable or if the project's probability of success is exogenous. However, if the entrepreneur's unverifiable effort affects the probability of success, we need the optimal mechanism to incorporate this moral hazard. We address this issue in the next section.

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[25] It is possible to implement the first-best solution exactly with the following direct revelation mechanism: ask entrepreneurs to report their type p. If an entrepreneur reports a type $p \in \left[\frac{1}{R+\frac{\sigma}{1+\lambda}}, \frac{1}{\beta R}\right]$, then offer a loan with interest rate $r = R-1$ and self-financing requirement $b = 0$. If the reported type p lies outside this interval, do not offer any loan. It is straightforward to verify that this mechanism is incentive compatible and ex-post efficient, and thus implements the first-best outcome. However, this mechanism may be difficult to implement because, apart from requiring entrepreneurs to report their type which may be difficult to do in practice, entrepreneurs with types $p \in \left[0, \frac{1}{\beta R}\right]$ are indifferent between reporting their types truthfully or not, but it is crucial for the efficiency of the mechanism that they report their types truthfully. We are grateful to Phil Reny for this observation.

4 Analysis of Optimal Policy with Moral Hazard

4.1 Optimal Policy

We now analyze the case with moral hazard, where the entrepreneur can exert additional effort to increase the probability of success. The timing of moves is as follows: an entrepreneur learns its probability of success p and decides whether to make an initial investment of c_I. Next the entrepreneur decides whether to seek funding either from a VC or to obtain a government loan that would help it complete its project and, if it receives additional funding, whether to exert full effort. The payoff to the entrepreneur is whatever remains after the VC takes its share or the entrepreneur repays its government loan. The loan is not repaid if the project fails.

As explained above, we assume the government offers entrepreneurs to choose whatever their preferred combination (b_σ, r_σ) of self financing requirement and interest rate from a menu of such choices $\{(b_\sigma, r_\sigma)\}$. We denote the government contract that maximises entrepreneur p's payoff by $(b_\sigma(p), r_\sigma(p))$. Again, for notational simplicity, we henceforth omit the subscript σ.

Let $U_G(p)$ denote the payoff to entrepreneur p if she chooses the government contract $(b(p), r(p))$. Observe that

$$U_G(p) \equiv \max\{p(R-(1-b(p))(1+r(p)))-b(p),\ kp(R-(1-b(p))(1+r(p)))+1-b(p)-c_I\}$$

where the first and second terms in the braces describe the expected payoff to entrepreneur p under government contract $(b(p), r(p))$ when she exerts full and partial effort, respectively. Note that $U_G(p)$ may be smaller than the expected payoff to entrepreneur p in the private market, $U_P(p)$, in which case type p prefers either to obtain VC funding, to develop its project on its own, or to drop the project.

Moral Hazard Constraint: A government contract (b, r) induces full effort from an entrepreneur of type p who receives government support if

$$p(R-(1-b)(1+r))-b \geq\ kp(R-(1-b)(1+r))+1-b-c_I. \tag{6}$$

This inequality is satisfied if and only if

$$p \geq \frac{1-c_I}{1-k} \cdot \frac{1}{R-(1-b)(1+r)}. \tag{7}$$

Participation Constraint: A government contract (b, r) induces an entrepreneur of type p to accept a government loan if the expected payoff to the entrepreneur under the government contract with either full or partial effort is larger than or equal to what the entrepreneur can obtain in the private market:

$$U_G\,(p) \geq U_P(p).$$

Incentive Compatibility Constraint: A menu of government contracts $\{(b(p), r(p))\}_{p \geq p^*}$ is incentive compatible for types $p \geq p^*$ if each type $p \geq p^*$ is induced to choose the government contract $(b(p), r(p))$, when its choice is restricted to only government contracts, and to exert full effort.

Since the government contract $(b(p), r(p))$ denotes the preferred choice of entrepreneur p from the menu of contracts, incentive compatibility merely implies that the menu offered by the government induces full effort for all types $p \geq p^*$ who choose a government contract. It does *not* imply that types $p < p^*$ who choose a government contract would exert full effort, and it does *not* imply that types $p \geq p^*$ would necessarily choose a government contract. Indeed, they may well prefer VC support.

The following characterisation of incentive compatible menus follows from standard arguments in mechanism design.

Proposition 3. *A menu $\{(b(p), r(p))\}_{p \geq p^*}$ is incentive compatible for types $p \geq p^*$ if and only if the induced expected payoff function of the entrepreneur $p\,(R - (1 - b(p))\,(1 + r(p))) - b(p)$ is monotone increasing and convex for types $p \geq p^*$, and type p^* is induced to exert full effort.*

The government's objective is to choose an incentive compatible menu $\{b(p), r(p)\}_{p \geq p^*}$ that maximises expected welfare, taking into account two considerations: (1) projects with $p \geq \frac{c_I}{kR}$ will be financed by the entrepreneurs themselves (possibly with VC support), if they do not obtain a government loan; and (2) among entrepreneurs who choose any government contract, some (those with $p \geq p^*$) may exert full effort while others (with $p < p^*$) may exert only partial effort.

Proposition 4. *(Optimal Policy with Moral Hazard) The optimal menu of government contracts consists of at most two contracts: one that induces full effort from some entrepreneurs who would exert partial or no effort otherwise, and another that induces partial effort from some entrepreneurs who would not have implemented the project otherwise.*

The intuition for this result is as follows: As explained in Section 3.1, because public funds are costly, the maximisation of social welfare requires that entrepreneurs' payoffs be minimised. Proposition 3 shows that the incentive compatibility of government contracts implies that their induced payoffs to entrepreneurs are increasing and convex. The smallest possible increasing and convex function $p\,(R - (1 - b(p))\,(1 + r(p))) - b(p)$ is linear, which is the payoff that is induced by a single government contract. The fact that, if a certain type p is induced to exert full effort by some government contract then so do all higher types $p' > p$, implies that there is no need for more than one government contract that induces full effort. A similar argument shows that there is no need for more than one government contract that induces partial effort.[26]

[26] As mentioned in the introduction, this intuition is analogous to the one that underlies the famous "no-haggling" result in monopoly pricing (Myerson, 1981; Riley and Zeckhauser, 1983).

Proposition 5. *The optimal government contract that induces partial effort is the "zero liability contract", $(b_\varepsilon, r_\varepsilon) \approx \left(0, \frac{R}{c_I} - 1\right)$, which attracts low probability projects that are welfare-increasing but not privately profitable. The optimal government contract that induces full effort is a "maximum outlay contract" $(\overline{b}, r)$ for some interest rate r.*

The zero liability contract offers a loan of c_I (more precisely, $c_I - b_\varepsilon$) and induces projects that would not otherwise be implemented to be partially implemented. This contract thus generates *additionality at the extensive margin*. The self-financing requirement under this contract is approximately equal to zero. In contrast, the maximum outlay contract that induces full effort from some entrepreneurs who would not exert full effort otherwise is a (b, r) contract, in which the self-financing requirement b is set equal to the upper bound $\overline{b}$. This contract generates *additionality both at the extensive and intensive margins*.

The intuition for the optimality of the zero liability contract is identical to the one offered in the case without moral hazard. Namely, the zero liability contract induces the entrepreneur to exert partial effort while yielding no socially costly rent. The intuition for the optimality of the maximum outlay contract stems from the fact that the expected payoff to the entrepreneur under a government contract (b, r) is decreasing in $b + p(1-b)(1+r)$ if the entrepreneur exerts full effort (or $b + pk(1-b)(1+r)$ with partial effort). Optimality requires that the socially costly rent to the entrepreneur be minimized, and thus that $b + p(1-b)(1+r)$ (or $b + pk(1-b)(1+r)$) be maximized. The moral hazard constraint implies an upper bound on the value of $(1-b)(1+r)$. It therefore follows that optimality implies that b should be set at its upper bound, $b = \overline{b}$, while the interest rate r is adjusted so that $(1-b)(1+r)$ is set equal to the upper bound induced by the moral hazard constraint.

Figure 1 depicts the entrepreneur's expected payoff from VC funding, and from a menu that includes two contracts: the zero liability contract, and some (not necessarily optimal) maximum outlay contract. As the figure shows, the maximum outlay contract, which is introduced in order to induce full effort, is also chosen by some entrepreneur's types who only exert partial effort, and would have also exerted partial effort if this contract was not offered. This redundancy means that a maximum outlay contract may be very costly for the government. It would be offered only if k is relatively small (moral hazard is less severe), as the extra effort has a high payoff, and if the set of such projects is small, which depends on the density of $F(p)$.

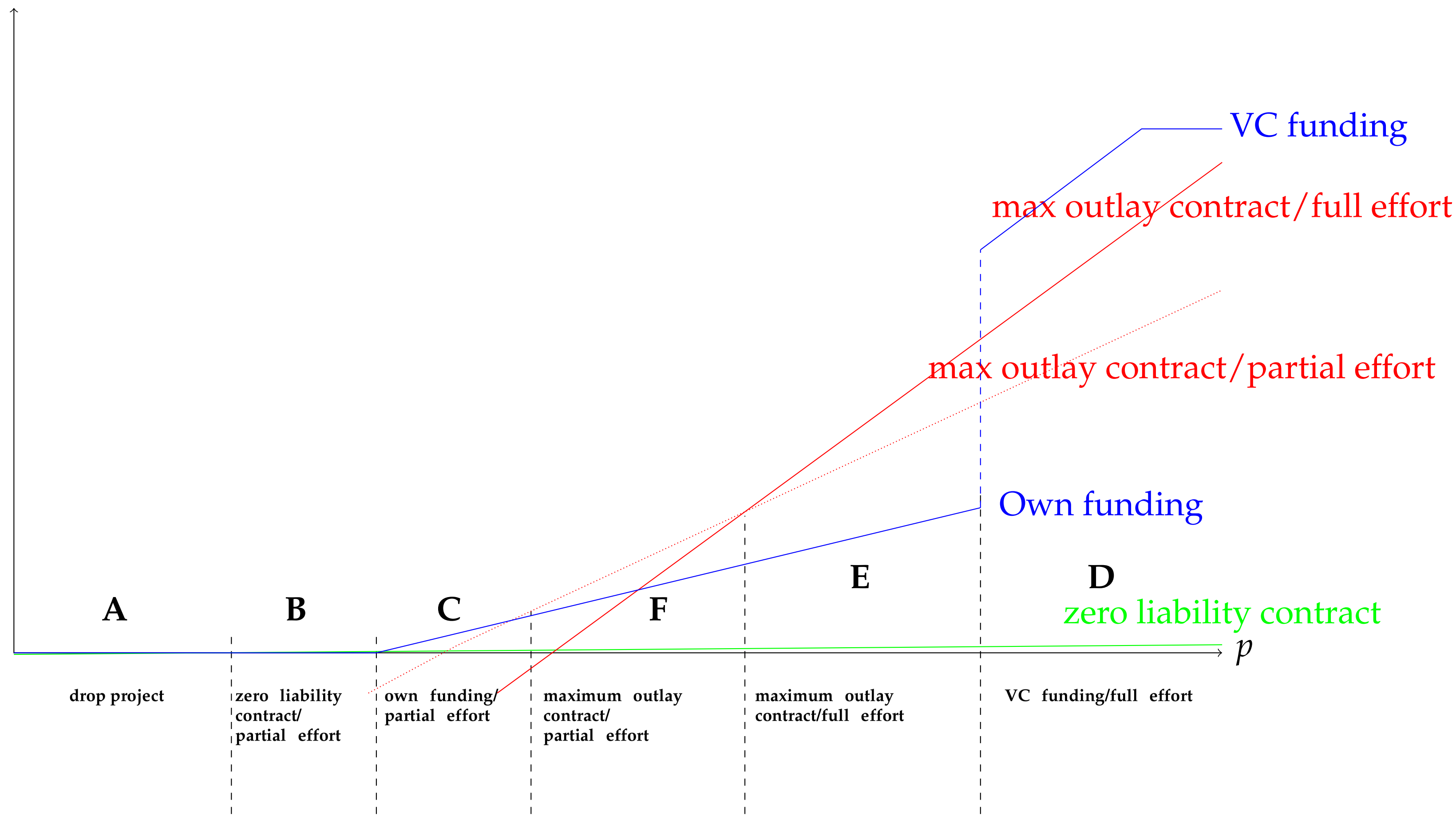


Figure 1: Expected returns to entrepreneurs with own funding, VC funding, zero liability contract, and maximum outlay contract with full and partial effort

Whether the government prefers to offer only one of the contracts or both depends on which policy generates higher welfare, which in turn depends on the parameters of the model: the values of k, private returns R, externality σ, and shadow price of public funds λ (as well as $\bar{b}$ and the distribution $F(p)$). However, we can show a stronger result. Under a relatively weak assumption, it is optimal for the government to offer only one contract.

Proposition 6. *If $\overline{b} + c_I \leq 1$, then the optimal menu includes only one contract, either a zero liability contract, or a maximum outlay contract.*

Given the calibrated parameters we use (Appendix A), the inequality $\overline{b} + c_I \leq 1$ is easily satisfied. We discuss the conditions under which each contract is offered in the next sub-section and in the simulations.

4.2 Analysis of Sorting and its Implications

The government loan and private market contracts generate endogenous sorting of entrepreneurs, as we discussed in the previous Section and depicted in Figure 1. This sorting has implications for the project additionality and welfare, which we discuss in this Section.

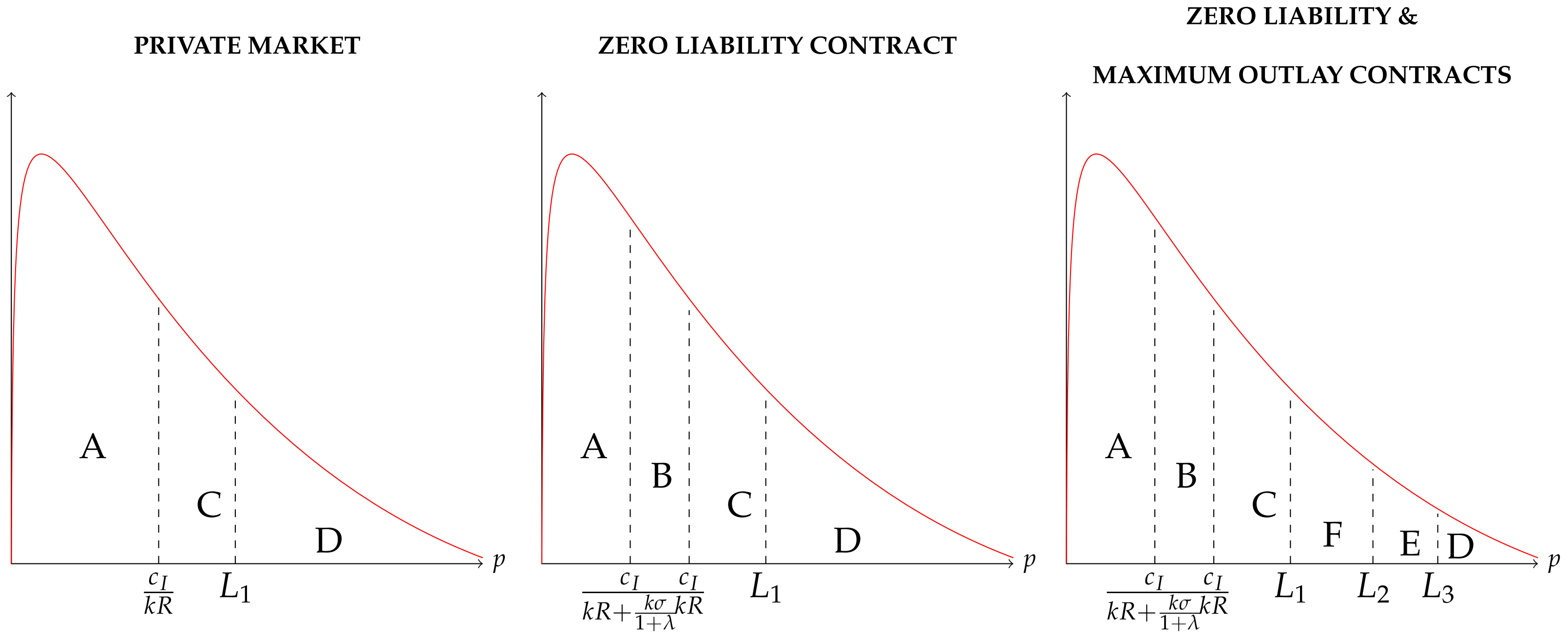
The first two graphs in Figure 2 display the sorting of projects induced by the private market and the zero liability contract (the labelling of the regions depicted in Figure 2 correspond to the labels in Figure 1). Note that the zero liability contract induces some projects that were not implemented by the private market to be partially implemented, and thus generates *project additionality* at the extensive margin. These new projects are those with low success probabilities, $p \in \left(\frac{1}{R+\frac{\sigma}{1+\lambda}}, \frac{c_I}{kR}\right)$, represented in Region B. This project additionality increases with the externality σ, and declines with the shadow price of public funds λ. This zero liability contract does not fund projects that would have been implemented by the private market, i.e., *it does not generate any redundancy.*

The welfare gain generated by the zero liability contract (relative to the private market) can be written as

$$\int_{\sigma} \int_{\frac{1}{R+\frac{\sigma}{1+\lambda}}}^{\frac{c_I}{kR}} [kp(R+\sigma) - (1+\lambda)(c_I - kpR]\, dF_{\sigma}(p) > 0$$

As this welfare gain is positive, the zero liability contract *always* improves upon the private market, whether or not it is optimal. The welfare gain increases with σ, but the effect of λ is ambiguous.[27]

[27]With respect to σ, this is correct provided an increase in σ does not reduce the density of p in the relevant interval too much. The welfare gain declines with λ if $c_I \geq k$, and thus holds under constant returns to effort $k = c_I$.



A: Not Implemented **B**:Extensive Additionality **C**:Own Funding with PE **D**: VC Funding **E**: Intensive Additionality
F: Redundancy $L_1 = \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}$

Figure 2: Sorting under the private market, zero liability contract and both the zero liability and maximum outlay contracts together

The third graph displays the sorting generated when both the zero liability and (arbitrary) maximum outlay contract, $(\bar{b}, r)$, is offered (note that the specific thresholds L_2 and L_3 depend on the interest rate in the $(\bar{b}, r)$ contract, and thus are not shown). Unlike the zero liability contract, the maximum outlay contract shifts some projects from partial to full effort. Thus it generates project additionality at the intensive margin (Region E).[28] However, it also creates redundancy, as projects previously funded by the entrepreneur's own funds with partial effort now shift to the government $(\bar{b}, r)$ contract and only exert partial effort (Region F). This makes it costly, as compared to the zero liability contract.

As shown, the zero liability contract always improves welfare relative to the private market. However, this is not generally true for an arbitrary maximum outlay contract because it may generate redundancy. This is more costly when λ is high. On the other hand, the maximum outlay contract has the advantage that it induces intensive (and possibly extensive) additionality, and this welfare gain increases in σ. These observations suggest that this contract is likely to be optimal only when λ is low and σ is high enough. The simulations in Section 6 confirm this conclusion.

Also note that the zero liability contract does not maximize the set of projects being implemented. This implies that in those cases where the zero liability contract is optimal, project additionality is not being maximised. This finding means that project additionality and redundancy are *not* necessarily appropriate criteria to evaluate the effectiveness of R&D support schemes, which is the approach taken in the policy literature.

4.3 Extensions

4.3.1 Uncertainty about private returns

In this section we show that our main result – that the optimal contract includes at most one contract that induces full effort and another contract that induces partial effort (Proposition 4) – is robust to adding asymmetric information about the success payoff, R. To analyse this, suppose that R has two possible values, $R_H > R_L > 1$. Entrepreneurs and VCs know the realisation of R but the government does not.

Suppose that the government offers two alternative contracts, (b, r) and (b', r'), and that the entrepreneur does not have an outside option. It is easy to show that if type (p, R_L) prefers the contract (b, r) to (b', r'), then so does type (p, R_H), and vice versa.[29] This fact implies that the government cannot screen entrepreneurs based

[28]In Figure 2, which corresponds to the payoffs depicted in Figure 1, the maximum outlay contract does not increase the number of implemented projects, and thus does not generate extensive additionality. But this can happen in the general case, as it depends on where the red lines in Figure 1 (payoffs with the maximum outlay contracts with partial and full effort) intersect the y-axis.

[29]Type (p, R_L) prefers government contract (b, r) to contract (b', r') if and only if $p(R_L - (1-b)(1+r)) - b \geq p(R_L - (1-b')(1+r')) -$..This This holds if and only if $p(R_H - (1-b)(1+r)) - b \geq p(R_H - (1-b')(1+r')) - b'$ which holds if and only if type (p, R_H) prefers government contract (b, r) to contract (b', r').

on R, and therefore the optimal government policy involves only two contracts, as before, one to induce full effort and another to induce partial effort. The next proposition shows that this result also holds if we allow the entrepreneur to have an outside option to drop the project or obtain private VC funding.

Proposition 7. *Suppose that R has two values, $R_H > R_L > 1$. The optimal menu of government contracts consists of at most two contracts: one that induces full effort from some entrepreneurs who would not exert full effort otherwise, and another that induces partial effort from some entrepreneurs who would not exert partial effort otherwise.*

For simplicity, this argument was made for the case in which R can have two possible realisations, but it also applies to any number of possible realisations larger than two.

4.3.2 Government budget constraint

We have assumed that the government does not face a budget constraint. Introducing a constraint does not fundamentally alter the analysis, if there is a continuum of projects (in terms of p and σ). In order to maximise expected welfare, the government should simply rank projects by the welfare per dollar of government money invested and then fund them in descending order until the budget is exhausted. Of course, to do this the government must first compute the optimal policy for each σ, as discussed before.

However, if there is a discrete number of indivisible projects, this criterion may create a 'knapsack problem'. Specifically, it may be the case that the project which generates the highest expected welfare per dollar invested requires a large investment that prevents the government from investing in other projects, whereas the project with the second highest expected welfare per dollar invested is cheaper and allows for more welfare enhancing investments. However, this is a computational, rather than a conceptual, problem that can be addressed with existing algorithms.

5 Optimal VC Subsidy

Our paper focuses on the optimal design of government R&D loans. However, in the simulation analysis that follows, we want to compare the welfare performance of the optimal loan policy against an alternative policy of direct subsidies to the private VC market. In this section we describe this policy and characterise the optimal VC subsidy to be used in the simulations.

We assume that the government provides a subsidy at rate δ on every dollar invested by private VC's. Then VC profits are

$$\Pi_{VC} = Min\left\{\beta p, 1\right\} \alpha R - (1 - \bar{b})(1 - \delta)$$

Since VC's are assumed to make zero expected profit, the subsidy reduces the equity stake they require. For a project of type p, the equity stake is

$$\alpha(p) = \frac{\left(1-\bar{b}\right)(1-\delta)}{Min\left\{\beta p, 1\right\} R}.$$

The welfare generated by a project of type p, which we denote by $W(p;\delta)$, is the sum of the payoff to the entrepreneur and the expected spillover $p\sigma$ if the entrepreneur exerts full effort (or $kp\sigma$ with partial effort), minus the cost of the subsidy $\delta(1-\bar{b})(1+\lambda)$:

$$W(p,\delta) = \begin{cases} 0 & \text{if } p \in \left[0, \frac{c_I}{kR}\right) \\ kp\left(R+\sigma\right) - c_I & \text{if } p \in \left[\frac{c_I}{kR}, \frac{1-c_I}{(\beta-k)R} + \frac{\left(1-\bar{b}\right)(1-\delta)}{\beta R}\right) \\ \beta p\left(R+\sigma\right) - 1 - \lambda\delta(1-\bar{b}) & \text{if } p \in \left[\frac{1-c_I}{(\beta-k)R} + \frac{\left(1-\bar{b}\right)(1-\delta)}{\beta R}, \frac{1}{\beta}\right) \\ \left(R+\sigma\right) - 1 - \lambda\delta(1-\bar{b}) & \text{if } p \in \left[\frac{1}{\beta}, 1\right] \end{cases} \tag{8}$$

where we omit other parameters of the welfare function, including the cost of public funds. Entrepreneurs in the first interval of p drop their projects, while those in the second interval implement their projects with partial effort which they can fund themselves. Projects supported by VC's are those in the last two intervals.

The optimal subsidy rate is found by maximizing $W(\delta) = \int_0^1 W(p;\delta) dF_\sigma(p)$ with respect to δ, given a value of the externality σ. Note that we do *not* assume that the externality σ and the probability p are independent in this analysis. After some manipulation, we can write the first order condition as

$$\left(\beta-k\right) L(\delta) \left(R+\sigma\right) - \lambda\delta(1-\bar{b}) - \lambda\frac{\left(1-F_\sigma(L)\right)}{dF_\sigma(L)}\beta R - \left(1-c_I\right) = 0$$

where $L(\delta) = \frac{1-c_I}{(1-k)\beta R} + \frac{\left(1-\bar{b}\right)(1-\delta)}{\beta R}$.[30] In general there is no closed form solution for the optimal subsidy δ^*.[31] However, under the assumption that the hazard rate of

[30]The first term is the incremental benefit of the VC subsidy: without VC support the success probability is kp;with VC support it is $max(\beta p, 1)$. Thus $(\beta-k)(R+\sigma)$ is the social gain over the relevant mass of projects given by $p \geq L$. The second term is the cost of the subsidy at the margin, and the third is the inframarginal cost since the subsidy applies to all VC-supported projects.

[31]In the simulation analysis we assume that σ and p are independent. We specify and calibrate a Beta distribution for $F(p)$ and compute the corresponding optimal subsidy. The optimal subsidy rate depends on both the project externality σ and shadow price of public funds λ. If the government cannot condition the VC subsidy on the project externality, then the optimal subsidy rate should be computed using the mean value of σ.We assume this to be the case in the simulations. Conditioning the subsidy on σ would require that the government inspect every project with VC support, and this would be administratively difficult.

the distribution $F_\sigma(p)$ is non-decreasing (in the neighborhood of L), we derive the following comparative statics results:

$$\frac{\partial \delta^*}{\partial \sigma} > 0, \frac{\partial \delta^*}{\partial \lambda} < 0, \frac{\partial \delta^*}{\partial \beta} \gtreqless 0$$

As expected, the optimal VC subsidy rate increases with the project externality and declines with the cost of public funds. The effectiveness of VC firms at enhancing a project's probability of success, β, has an ambiguous effect on the optimal subsidy. The reason is that an increase in β implies a higher probability of success and thus larger expected externality from any funded projects. However, at the same time, the higher β induces more projects with lower p to go to the VCs and this increases the subsidy cost on *infra-marginal* projects since the government cannot condition the subsidy on p.

Unfortunately, it is not possible to show analytically whether welfare is higher with the optimal VC subsidy or the R&D loan contract. In part this is because which R&D contract is optimal – the zero liability or maximum outlay contract – depends on underlying parameters including σ and λ. The simulations in the next section illustrate the comparative static results quantitatively and compares the welfare from the optimal R&D loan policy and the optimal VC subsidy.

6 Simulations

In this section we simulate the model with moral hazard and compute the welfare generated by the optimal policy and typical loan/grant schemes we observe in practice. We also illustrate how their performance varies with parameters of the model. We calibrate the parameters of the model based on a variety of data sources (online Appendix A for details). [32]

6.1 Optimal loan policy vs. private VC market

We begin by comparing performance metrics of the optimal policy relative to the private market. Table 1 presents results for the baseline case of $\beta = 1.12$ (i.e., VC support increases the success probability by 12%) for different values of the shadow price of public funds, λ, and different levels of externality as measured by the ratio of social

[32] The required parameters includes the private returns to a project R, distribution of success probabilities $F(p)$, project externality σ, shadow price of public funds λ, project costs with partial effort c_I, entrepreneur's funds as a share of project cost with full effort $\bar{b}$, and the effectiveness of VC's in enhancing a project's success probability β.

to private returns to R&D.[33] We indicate whether the optimal policy is the zero liability contract, or the maximum outlay contract (in the latter, we include the optimal interest rate, r).

Table 1 highlights several important features. First, in nearly all cases the *optimal policy is the zero liability contract*. This conclusion holds across a wide range of values for the cost of public funds and size of the externality. The reason is that the zero liability contract produces no redundancy, while the maximum outlay, or (b, r), contract involves redundancy which is socially costly unless λ is very low.

Second, the maximum outlay contract is optimal only when λ is low *and* the externality is large, as shown in Panels D and E. It is interesting to note that, in those cases, the optimal policy takes the form of *a full grant* ($r = -1$).[34] Though this contract entails redundancy and a large government outlay, this is not too costly at low λ. With this contract, all projects are implemented and some projects that were implemented with partial effort switch to full effort, both of which are more valuable when σ is high. But while large externalities can make it worth incurring this social cost at moderate levels of λ, at high levels of λ the zero liability contract is again optimal. It is also noteworthy that, even in the three cases where the zero liability contract is *not* optimal, the welfare gains from using it, relative to the private market, are still substantial (27% and 35%, not shown).

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[33] Online Appendix Tables B1 and B2 present the tables for the case $\beta = 1.0$, where VCs do not enhance project success, and $\beta = 1.24$ where VCs are more effective. The qualitative results in these cases are similar to the baseline specification, but the welfare gains from the optimal (zero liability) contract are smaller when the private VC market is more effective at enhancing project success.

[34] For the reported parameter values, when the maximum outlay contract is optimal, the associated interest rate is actually *more* than a full grant (i.e. $r < -1$). In these cases we set it at $r = -1$. The reason it can be more than a full grant is that for low enough cost of public funds and large enough externality, it can be welfare improving for the government to *pay entrepreneurs* to undertake projects in order to secure the externalities. From a theoretical perspective, this implies that the set of projects entreprenreurs bring to the table depends on how large the payment $r < -1$ – i.e., it makes the distribution $F(p)$ endogenous, which is beyond the scope of our model. Also, from a political perrspective, such a policy is likely to be difficult to implement.

Table 1: Performance metrics of optimal policy, β = 1.12

1+λ	Optimal policy	(1) Welfare gain (%)	(2) % projects implemented by private market	(3) % additional projects implemented by optimal policy
Panel A: Social/Private Rate of Returns = 1.1				
1.25	Zero Liability	0.4	20.0	3.4
1.5	Zero Liability	0.4	20.0	2.8
1.75	Zero Liability	0.3	20.0	2.4
2	Zero Liability	0.3	20.0	2.1
Panel B: Social/Private Rate of Returns = 1.5				
1.25	Zero Liability	6.7	20.0	18.4
1.5	Zero Liability	5.7	20.0	15.6
1.75	Zero Liability	5.0	20.0	13.5
2	Zero Liability	4.4	20.0	11.9
Panel C: Social/Private Rate of Returns =2				
1.25	Zero Liability	17.5	20.0	36.8
1.5	Zero Liability	15.4	20.0	32.3
1.75	Zero Liability	13.8	20.0	28.7
2	Zero Liability	12.5	20.0	25.8
Panel D: Social/Private Rate of Returns = 2.5				
1.25	Maximum Outlay, r = -1	56.4	20.0	80.0
1.5	Zero Liability	24.9	20.0	47.0
1.75	Zero Liability	22.8	20.0	43.0
2	Zero Liability	21.1	20.0	39.5
Panel E: Social/Private Rate of Returns =3				
1.25	Maximum Outlay, r = -1	86.9	20.0	80.0
1.5	Maximum Outlay, r = -1	53.1	20.0	80.0
1.75	Zero Liability	30.7	20.0	54.6
2	Zero Liability	28.8	20.0	51.3

Notes: Column 1 shows the percentage increase in welfare generated by the optimal policy relative to the welfare generated by the private market only. Column 2 shows the percentage of projects implemented by the private market. Column 3 shows the percentage of additional projects implemented by the optimal policy.

Third, the optimal zero liability generates substantial welfare gains, unless the externality is very small (as in Panel A). Not surprisingly, the gains strongly increase with the size of the externality and decline with the cost of public funds. The welfare gains range from a low of about 4.4% to a high of 30.7%.

In this paper we model the financing of R&D projects in a partial equilibrium setting. Thus the welfare gains reported above correspond to the 'R&D sector', not the aggregate economy. By way of comparison, Acemoglu et al. (2018) use an estimated macroeconomic growth model to assess welfare gains from various innovation-related fiscal policies. In particular, they show that an R&D subsidy for incumbent firms, equivalent to 14% of their R&D (1% of GDP), increases welfare in their model by 0.6%. For the optimal (uniform) subsidy in their model, which is equivalent to 39% of R&D, the increase is 1.22%. In order to compare their aggregate welfare gains to those in Table 1, we need to account for the fact that ours relate only to the R&D

sector, and to account for the differences in the scale (government cost) of our optimal loan policy and the subsidies considered by the Acemoglu et al. study. Making these adjustments, the welfare gains from the optimal zero liability contract in Panel C are equivalent to roughly 0.62% to 0.87% (depending on λ), as compared to their estimate of 0.6% for the 14% subsidy. When compared to their 1.22 welfare gains from their 39% optimal subsidy, our estimated welfare gains are 1.73% to 2.42%.[35]

Although their framework and policy instruments are different from those in our paper, the welfare gains are broadly similar. That our welfare gains are somewhat higher may not be surprising since the optimal mechanism is designed to mitigate the adverse selection effects that arise in their model. At the same time, we do not want to overinterpret the simulation results; they should be viewed as illustrative, given the simplicity of the model and the specific calibration of parameters.

6.2 Optimal loan policy vs. other schemes

Finally, we compare the optimal policy to loan schemes typically used by governments and to a direct subsidy to private VC firms. Loan schemes almost always involve a single interest rate and matching requirement, whereas in our optimal policy these features vary with the project externality σ and the shadow price of public funds λ. Typically, observed policies are either full grant schemes or interest-free loans.

Table 2 presents the welfare gains per dollar of cost for different policies, relative to the private market. When externalities are small (Panel A), the zero liability contract generates about 21 cents *net welfare gain* per dollar (i.e., a benefit-cost ratio of 1.21), but this increases sharply with the size of the externality and declines with the cost of public funds. When social returns are twice as large as private returns, the benefit-cost ratio varies from 2.6, for $\lambda = 2$, to 3.7 for $\lambda = 1.25$. By contrast, a full grant or zero-interest loan contract actually *reduces* welfare, unless externalities are very large, implying benefit-cost ratios less than one.

[35]To do this, decompose the change in aggregate welfare into the part generated by the R&D sector and by other sectors, denoted by $\Delta W_A = \Delta W_{R\&D} + \Delta W_O$. But $\Delta W_O = 0$ since we account for *all the externalities* generated by the R&D sector. Thus $\frac{\Delta W_A}{W_A} = \frac{\Delta W_{R\&D}}{W_{R\&D}} \frac{W_{R\&D}}{W_A}$. We assume that $\frac{W_{R\&D}}{W_A}$ is roughly equal to the ratio of R&D sector *output* to GDP, which can be expressed as $\frac{(1+\rho_s)\ R\&D}{GDP}$ where $R\&D$ is the input (expenditure) and ρ_s is the social rate of return to R&D. Setting $\rho_s = 0.55$ from Bloom, Schankerman and Van Reenen (2013), this adjustment factor is 4.25% ($= 1.55 \times 2.74\%$, which is the ratio of R&D/GDP in the U.S. in 2016).

In addition, we adjust for the relative cost of the optimal loan and subsidy policies. Acemoglu et al. report that their R&D subsidy is 14% of R&D, while their optimal subsidy is 39% of R&D. The simulated cost of our optimal zero liability R&D loan policy in Panel C in Table 1 is about 12% of R&D in our model (averaged across values of λ). Thus we scale our welfare gains up by the factor $1.17 = 0.14/0.12$ to compare to their 14% subsidy, and by $3.25 = 0.39/0.12$ for comparison to their optimal subsidy.

Table 2: Effectiveness of support schemes per dollar program cost, β = 1.12

		(1)	(2)	(3)	(4)
1+λ	Optimal policy	Welfare gain per dollar cost			
		Optimal policy	Full grant	Zero interest loan	Constrained optimal VC subsidy
Panel A: Social/Private Rate of Returns = 1.1					
1.25	Zero Liability	0.22	-0.26	-0.26	0.08
1.5	Zero Liability	0.22	-0.38	-0.38	-0.10
1.75	Zero Liability	0.21	-0.47	-0.47	-0.23
2	Zero Liability	0.21	-0.53	-0.54	-0.32
Panel B: Social/Private Rate of Returns = 1.5					
1.25	Zero Liability	1.2	-0.16	-0.17	0.49
1.5	Zero Liability	1.1	-0.30	-0.31	0.24
1.75	Zero Liability	0.95	-0.40	-0.41	0.06
2	Zero Liability	0.88	-0.47	-0.48	-0.07
Panel C: Social/Private Rate of Returns = 2					
1.25	Zero Liability	2.7	0.01	-0.03	1.16
1.5	Zero Liability	2.1	-0.16	-0.19	0.80
1.75	Zero Liability	1.8	-0.28	-0.30	0.54
2	Zero Liability	1.6	-0.37	-0.39	0.35
Panel D: Social/Private Rate of Returns =2.5					
1.25	Maximum Outlay, r = -1	0.23	0.23	0.17	2.06
1.5	Zero Liability	3.3	0.02	-0.02	1.55
1.75	Zero Liability	2.6	-0.12	-0.16	1.18
2	Zero Liability	2.3	-0.23	-0.27	0.91
Panel E: Social/Private Rate of Returns = 3					
1.25	Maximum Outlay, r = -1	0.51	0.51	0.43	3.25
1.5	Maximum Outlay, r = -1	0.26	0.26	0.19	2.54
1.75	Zero Liability	3.7	0.08	0.02	2.03
2	Zero Liability	3.1	-0.05	-0.11	1.65

Notes: Columns 1 to 4 show the difference between the welfare generated by the policy and that generated by the private market divided by the cost of the policy. Self financing b in the full grant and zero interest loan is set to $\bar{b}$. In column 4, the constrained optimal subsidy rate is set to 100% because the unconstrained optimal rate is above 100% (between 125% and 183%). The unconstrained optimal subsidy rate is found by maximising expected social welfare at the value of σ implied by the mean ratio of social to private rates of return estimated by BVS (2013), for each value of λ.

The optimal VC subsidy generates welfare gains for almost all levels of the externality and cost of public funds, but the associated benefit-cost ratio is smaller than for the optimal loan policy.[36] For example, when social returns are twice as large as private returns (Panel C), the benefit-cost ratio for the optimal zero liability contract

[36]For the parameter configurations presented here, the optimal VC subsidy rate $\delta^* > 1$ (it varies from 1.25 to 1.83). This is because the VC's enhance the project success probability significantly (calibrated at 12%), so it can be welfare-improving for the government to raise revenue and actually *pay VC firms* in order to enhance project success. In these cases, we set $\delta = 1$. Paying VC firms ($\delta > 1$) would raise adverse selection effects for VC participants, which are beyond the scope of our analysis, and likely to be politically difficult to implement. This issue is analogous to implementing more than a full grant in the maximum outlay contract (see footnote 30).

is between 2.6 and 3.7, while it is only 1.35 to 2.16 for the (constrained) optimal VC subsidy. At the same time, however, it is worth noting that the VC subsidy does much better than either a full grant or zero interest loan, both of which are widely adopted by governments.

Of course, we recognise that any simulation analysis is limited by the simplicity of the model and the realism of the calibrated parameters. Still, our results at least suggest that loan policies often used by governments – full grants or zero interest loans – may be inferior to the zero liability loan policy or a direct subsidy to VCs. This is especially the case where the social cost of public funds is high and/or externalities are small. While many factors play a role, countries with weaker institutional capacity are likely to have less efficient tax systems, and thus a higher cost of public funds. Unless the externalities are especially high in such countries, the results suggest that typical R&D loan schemes are likely to be ill-suited for developing countries.

Why do R&D loan programs in the real world differ sharply from the theoretically optimal policy? A policy of 'targeting the middle' is likely to be politically less attractive to governments than targeting the 'best' (low risk) projects, as is often done in practice. Being able to show program 'successes' may increase prospects for budgetary support. The social cost of redundancy which such a program entails remains hidden. In addition, the public agency responsible for the program may worry about the government's commitment to fund it in the future, and hedge this risk by choosing profitable projects if they can retain the proceeds. Whatever the reason, our paper indicates that moving to the optimal loan policy (or direct VC subsidy) can potentially generate significant welfare gains.

7 Concluding remarks

We study the optimal design of government loan financing of R&D projects that vary in risk and generate positive externalities. Such programs are often used to support innovation by start-up companies. We show that, when there is adverse selection over project risk, the optimal contract requires a high interest rate but (virtually) zero self-financing. This contrasts sharply with observed policies that use zero or negative interest rates and high self-financing provisions. When we add moral hazard, by allowing the entrepreneur to choose between two effort levels, the optimal policy consists of a menu of at most two contracts – one with high interest/zero self-financing and a second with lower interest but maximum feasible co-financing. Moreover, under a mild assumption, we show that only one contract is optimal.

The simulations of the model indicate that the optimal zero liability policy can generate significant welfare gains, relative to the private market and government policies we typically observe, especially when project externalities are large and the cost of public funds is low. We also find that an optimal direct subsidy to private VC's outperforms either a full grant or zero interest loan policy, both of which are widely used by governments.

There are two core policy implications. First, *optimal policies should 'target the middle'*. Low-risk projects are likely to be financed by the private market anyway, so government support is redundant. High-risk projects will not be privately funded but, unless they generate very large externalities, the expected social payoff does not justify supporting them.

Second, *R&D support policies need to be tailored to the economic environment* – one size does not fit all. The size of project externalities, cost of public funds and effectiveness of the private venture capital market are key parameters that affect the optimal policy. If externalities differ across technology fields, the parameters of the policy should ideally vary by field. The same principle applies across countries, where both project externalities and the cost of public funds may vary as well.

Appendix. Proofs of Propositions

Proof of Proposition 1. Follows from the arguments above the statement of Proposition 1. ■

Proof of Proposition 2. Follows from the arguments above the statement of Proposition 2. ■

Proof of Proposition 3. An incentive compatible menu satisfies

$$U_G(p) \equiv p\left(R-(1-b(p))(1+r(p))\right)-b(p) \geq p\left(R-(1-b(p'))(1+r(p'))\right)-b(p')$$

and

$$U_G(p') \equiv p'\left(R-(1-b(p'))(1+r(p'))\right)-b(p') \geq p'\left(R-(1-b(p))(1+r(p))\right)-b(p)$$

for any two types $p > p' > p^*$. It follows that

$$(p-p')\left(R-(1-b(p'))(1+r(p'))\right) \leq U_G(p)-U_G(p') \leq (p-p')\left(R-(1-b(p))(1+r(p))\right)$$

from which it follows that $(1-b(p))(1+r(p))$ is non-increasing in $p \geq p^*$. Dividing the last inequality by $p-p'$ and taking the limit as $p' \searrow p$ implies that the derivative of $U_G(p)$ is equal to $(R-(1-b(p))(1+r(p)))$ whenever it is continuous in p, which because of monotonicity holds a.s. in $p \geq p^*$. And, the fact that the derivative of $U_G(p)$ is non-increasing implies that $U_G(p)$ is convex for $p \geq p^*$.

Conversely, if $U_G(p)$ is convex and type p^* is induced to exert full effort, then the payoff that any type $p' \geq p^*$ obtains from selecting the contract $(b(p), r(p))$ is obtained on a line at the point $(p, U_G(p))$ with slope $U'_G(p)$, at the point p' on that line. Convexity of $U_G(p)$ implies that this payoff lies below $U_G(p')$ which is the payoff that type p' obtains by being truthful.

Finally, rearrangement of the moral hazard constraint (6) shows that a government contract (b, r) induces full effort from type p if and only if

$$p \geq \frac{1-c_I}{1-k} \cdot \frac{1}{R-(1-b)\,(1+r)}.$$

It follows that if p^* is induced to exert full effort under (b, r), then so is every $p \geq p^*$ because convexity of $U_G(p)$ implies that $(1-b(p))\,(1+r(p))$ is nonincreasing in $p \geq p^*$. ■

Proof of Proposition 4. A government contract (b, r) induces an expected payoff to entrepreneurs of $p\,(R-(1-b)(1+r))-b$ that is linear in p. Increasing b pivots this payoff function in the sense that it increases its slope $R-(1-b)(1+r)$ and lowers its intercept $-b$. Increasing b and r in such a way that keeps $(1-b)(1+r)$ fixed shifts the payoff function downwards in a parallel way. Following the last part of the proof of Proposition 3, the moral hazard constraint (6) implies that if p is induced to exert full effort under (b, r), then so is every $p' > p$, and that both increasing b and increasing

b and r in a way that keeps $(1-b)(1+r)$ fixed preserves p's incentive to exert full effort.

Suppose that the optimal menu induces some entrepreneur $p \leq \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}$ who would not exert full effort under the private market to exert full effort. Denote the smallest type that is induced to exert full effort by the optimal menu by p^* and the government contract that is chosen by p^* by (b, r). The fact that p^* is induced to choose the contract (b, r) implies that $p^*(R-(1-b)(1+r)) - b \geq U_P(p^*)$.

Recall that

$$U_P(p) = \begin{cases} 0 & \text{if } p \in \left[0, \frac{c_I}{kR}\right) & \text{project is dropped} \\ kpR - c_I & \text{if } p \in \left[\frac{c_I}{kR}, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right) & \text{project implemented with partial effort} \\ \min\{\beta p, 1\}R - 1 & \text{if } p \in \left[\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}, 1\right] & \text{project implemented with full effort} \end{cases}$$

Distinguish the following three cases:

1. The function $p(R-(1-b)(1+r)) - b$ has a slope smaller than or equal to kR (i.e., flatter than $U_P(p)$ in the interval $\left[\frac{c_I}{kR}, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right)$, and it intersects $U_P(p)$ above or to the right of the point $\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}$ (recall that $U_P(p)$ is discontinuous at $p = \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}$) or lies entirely above $U_P(p)$.
2. The function $p(R-(1-b)(1+r)) - b$ has a slope smaller than or equal to kR and it intersects $U_P(p)$ at a point in the interval $\left[\frac{c_I}{kR}, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right)$.
3. The function $p(R-(1-b)(1+r)) - b$ has a slope larger than kR.

Case 1. b and r can be increased in such a way that keeps $(1-b)(1+r)$ fixed so that the function $p(R-(1-b)(1+r)) - b$ shifts down in a parallel way and the intersection point with $U_P(p)$ moves left. This change satisfies the moral hazard constraint for $p \geq p^*$ and increases social welfare because from (1) it follows that an increase in social welfare requires that $b + p(1-b)(1+r)$ be increased. The suggested change accomplishes this goal without violating the moral hazard constraint. It follows that if the function $p(R-(1-b)(1+r)) - b$ intersects the function $U_P(p)$ above or to the right of the point $\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}$ then either b is increased up to the point where it is equal to $\bar{b}$, or the entrepreneur's payoff function is shifted down so that it intersects $U_P(p)$ on the interval $\left[\frac{c_I}{kR}, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right)$, which is analysed in Case 2 below.

Case 2. Such an intersection necessarily implies that $p(R-(1-b)(1+r)) - b$ is flatter than $U_P(p)$ between $\frac{c_I}{kR}$ and $\frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}$. In this case, it is possible to increase b and decrease r so that $(1-b)(1+r)$ decreases and $b + p^{**}(1-b)(1+r)$ increases, where $p^{**} \leq p^*$ denotes the smallest type that selects the modified contract. The decrease in $(1-b)(1+r)$ ensures that the moral hazard constraint is satisfied for $p \geq p^*$, and the increase in $b + p^{**}(1-b)(1+r)$ ensures that social welfare is increased for $p \geq p^{**}$. Notice that the fact that the new contract generates a larger social welfare implies that it is less attractive to the entrepreneurs, so that some types who accepted the government contract (b,r) may reject the modified contract. However, as shown below, it is possible to induce types who anyway exert partial effort to exert partial effort costlessly using a zero liability contract if this contributes to social welfare, so that the modified contract does not decrease overall efficiency. Thus, it again follows that social welfare is increasing in b, so $b = \bar{b}$ in this case as well.

Case 3. All the types $p > p^*$ exert full effort either under the contract (b,r) or under the private market. So, again there is no need for an additional contract because such a contract cannot increase overall effort, and would generate a larger payoff to the entrepreneurs who select it, at the expense of social welfare.

Finally, observe that a contract that induces full effort from some type $p \leq \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}$ may also be accepted by types $p < p^*$ who would not be induced by it to exert full effort. Nevertheless, the government may still benefit from offering entrepreneurs another contract, which induces only partial effort because such a contract may increase participation from entrepreneurs who otherwise would drop their projects. As explained in the analysis of the problem without moral hazard, this additional contract would require an arbitrarily small self-financing, ε, and a payment of $R - \varepsilon$ upon success, which implies an interest rate of approximately $1 + r = \frac{R}{c_I}$. Such a contract would extract approximately the entire rent of entrepreneurs who would accept it and exert partial effort (approximate rather than exact because the contract has to provide some positive rent to induce participation from those types that generate positive expected social welfare, but not lower types). ■

Proof of Proposition 5. Proposition 4 implies that it is optimal to offer at most two contracts: one that induces only partial effort, and another that induces full effort from some entrepreneur types. As explain in the proof of Proposition 4, the optimal government contract that induces only partial effort is an approximate zero liability contract. Suppose that the optimal contract that induces full effort from some entrepreneur types is given by (b,r). The proof of Proposition 4 shows that if the expected payoff to the entrepreneur under the contract (b,r), which is $p(R-(1-b)(1+r)) - b$, is flatter than the expected payoff to the entrepreneur under the private market on the interval $\left[\frac{c_I}{kR}, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right)$, then it follows that $b = \bar{b}$.

Suppose that the expected payoff to the entrepreneur under the contract (b,r) is steeper than the expected payoff to the entrepreneur under the private market on the interval $\left[\frac{c_I}{kR}, \frac{1-c_I}{R(\beta-k)} + \frac{1-\bar{b}}{\beta R}\right)$, i.e., $R-(1-b)(1+r) > kR$. Denote the smallest type that exerts full effort under (b,r) by p^*. Because public funds are costly, maximizing expected social welfare requires that (b,r) maximize the sum $b + p(1-b)(1+r)$

for types $p \geq p^*$ and $b + pk(1-b)(1+r)$ for types $p < p^*$ (i.e., minimize government funding to induce those projects). This implies that $(1-b)(1+r)$ should be increased so that the moral hazard constraint is binding at p^*, so $(1-b)(1+r) = R - \frac{1}{p^*}\frac{1-c_I}{1-k}$ while b should be simultaneously increased so that $b = \overline{b}$. Notice that, if as $(1-b)(1+r)$ is increased the slope of the entrepreneur's expected payoff under (b, r) drops below kR then we are back in case 2 analysed in the proof of Proposition 4, where we already proved that $b = \overline{b}$. ■

Proof of Proposition 6. If the optimal menu consists of only one contract, the conclusion follows immediately. Now suppose that the optimal menu consists of two contracts. Proposition 5 shows that the two contracts are a zero liability contract and a maximum outlay contract $(\overline{b}, r)$. Because an entrepreneur who chooses the maximum outlay contract can exert either full or partial effort, his expected payoff is $\max\{p(R-(1-\overline{b})(1+r)) - \overline{b}, pk(R-(1-\overline{b})(1+r)) + 1 - \overline{b} - c_I\}$. If $\overline{b} + c_I \leq 1$, this maximum is larger than or equal to zero, which is the entrepreneur's expected payoff under the zero liability contract. Therefore, no entrepreneur would choose the zero liability contract. Finally, Proposition 4 implies there is no need to consider optimal menus with more than two contracts. ■

Proof of Proposition 7. Denote the smallest (p, R_H) type that exerts full effort under the optimal menu of contracts by (p_H^*, R_H) and the contract chosen by this type be (b, r). The argument used in the proof of Proposition 4 can be used to show that there is no need for another contract in order to induce full effort from types (p, R_H) such that $p > p_H^*$.

The contract (b, r) may also be picked by some types (p, R_L). Denote the smallest (p, R_L) type that exerts full effort under (b, r) by (p_L^*, R_L). The argument used in the proof of Proposition 4 implies that there is no need for another contract in order to induce full effort from types (p, R_L) such that $p > p_L^*$. Hence, the only possible reason for introducing another contract is in order to induce full effort from some types (p, R_L) such that $p < p_L^*$.

However, the moral hazard constraint (equation (6) in the text) implies that, for another contract (b', r') to induce full effort, it must be the case that $(1-b')(1+r') \geq (1-b)(1+r)$. This is ruled out by the incentive compatibility constraint, which implies that $(1-b(p))(1+r(p))$ is nonincreasing in p.

There is also no need for more than one contract to induce partial effort. The government has to decide which is best: (1) to offer only one contract that extracts the full rent from types (p, R_L) who would exert partial effort under the first-best contract, and allow types (p, R_H) to capture a positive rent, or (2) to offer a different single contract that extracts the full rent from types (p, R_H) who would then exert partial effort, and exclude types (p, R_L). These are the only two contracts which ensure that types (p, R_L) get approximately zero rent, and any other contract that leaves them positive rent is dominated by one of the above-mentioned contracts, since maximisation of welfare involves minimisation of the rent to entrepreneurs. ■

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