Using the aa index over the last 14 solar cycles to characterize extreme Geomagnetic activity

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Key Points:
- We present a new method that parameterizes extremes of 14 solar cycles of the $aa$ geomagnetic index
- We find a 4% (28%) chance of at least one great (severe) storm per year over 14 solar cycles
- A DST perturbation weaker than $−1,000$ nT Carrington storm is in the same occurrence rate distribution as other superstorms since 1868

Supporting Information:
- Supporting Information S1

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Citation:

Using the $aa$ Index Over the Last 14 Solar Cycles to Characterize Extreme Geomagnetic Activity

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Abstract  Geomagnetic indices are routinely used to characterize space weather event intensity. The $D_{st}$ index is well resolved but is only available over five solar cycles. The $aa$ index extends over 14 cycles but is highly discretized with poorly resolved extremes. We parameterize extreme $aa$ activity by the annual-averaged top few percent of observed values, show that these are exponentially distributed, and they track annual $D_{st}$ index minima. This gives a 14-cycle average of ~4% chance of at least one great ($D_{st} < −500$ nT) storm and ~28% chance of at least one severe ($D_{st} < −250$ nT) storm per year. At least one $D_{st} = −809$ [$−663, −955$] nT event in a given year would be a 1:151 year event. Carrington event estimate $D_{st} \sim −850$ nT is within the same distribution as other extreme activity seen in $aa$ since 1868 so that its likelihood can be deduced from that of more moderate events. Events with $D_{st} \leq −1,000$ nT are in a distinct class, requiring special conditions.

Plain Language Summary  Here we use measurements of disturbances in the Earth’s magnetic field that go back to 1868, and we present a novel way of analyzing the data to identify the largest magnetic storms going back some 80 years longer than has been done before. As a result, we are able to state the chance of at least one superstorm occurring in a year. We find that on average there is a 4% (28%) chance of at least one great (severe) storm per year and a 0.7% chance of a Carrington class storm per year, which can be used for planning the level of mitigation needed to protect critical national infrastructure.

1. Introduction

Extreme space weather events significantly disrupt systems for power distribution, aviation, communication, and satellites; they are driven by large-scale plasma structures emitted from the solar corona, but their impact depends on a variety of factors (Baker & Lanzerotti, 2016). Quantifying the chance of occurrence of extreme space weather events is essential to planning the resilience of vulnerable systems to catastrophic failure. Events that lead to geomagnetically induced currents that affect power grids are more likely close to solar maximum and in the descending phase of the solar cycle, but importantly, they can occur at all other times in the solar activity cycle (Thomson et al., 2010). The number of major solar eruptions varies with the approximately 11 year cycle of solar (sunspot) activity and with the amplitude of each solar cycle which is unique (Thomay, 2015). A particular concern is the possibility of a Carrington-class event, named after the unique (Hathaway, 2015). A particular concern is the possibility of a Carrington-class event, named after the

Due to their rarity, amplitude and occurrence rates of space weather superstorms are challenging to quantify; it requires modeling based on the few observed large events. There have been a number of statistical studies, most of which rely on observations since the beginning of the space age. Estimates based on extrapolating a power law event distribution (Riley, 2012) suggest a 12% probability of a Carrington-class event in any given solar cycle but are highly uncertain (Riley & Love, 2016). A lognormal event distribution yields a much lower probability, again with a wide confidence interval (Love et al., 2015). Estimates based on Extreme Value Theory (Thomson et al., 2011) also suggest that the probability can be much lower (Elvidge & Angling, 2018; Siscoe, 1976; Silbergleit, 1996, 1999; Tsubouchi & Omura, 2007). More moderate storms provide a larger set of observations. When storms across different solar cycles are aggregated, there is a well-established correlation between occurrence rate and solar activity (Tsubouchi & Omura, 2007; Tsurutani et al., 2006).
Both solar wind driving (Tindale & Chapman, 2016, 2017) and geomagnetic activity (Chapman et al., 2018; Hush et al., 2015; Lockwood, Owens, et al., 2018) track the differences in the level of activity at different phases of distinct solar cycles and between cycles of different intensity.

The above statistical studies are feasible for indices which are well resolved in amplitude, such as $D_{ST}$. Whereas most indices, such as $D_{ST}$, are only available over the last five solar cycles, the aa index extends across 14 solar cycles—it is the longest almost continuous record of changes in magnetic field across the Earth’s surface. Given the variability in the amplitude of different solar cycles, it is desirable to obtain event occurrence rates for this longer sample. However, the aa index is by construction based on combining observations that are logarithmically discretized in amplitude, and thus, individual records of the 3 hr aa index will have uncertainties that are both significant and nontrivial to estimate (Bubenik & Fraser-Smith, 1977).

In this Letter we propose a parameterization of extreme aa activity using averages of the annual top few percent of observed records. Our goal is to use aa to obtain a proxy for $D_{ST}$ extremes that have occurred over the last 150 years. Our methodology is as follows. We first show that there is a good linear correlation between the annual average of the top few percent values of aa and the annual $D_{ST}$ minimum seen over the last five solar cycles. This establishes a linear “mapping” between the annual average of the top few percentage values of aa and the annual $D_{ST}$ minimum. We next use this linear mapping to convert these 150 annual averages of the top few percent of aa values into proxy $D_{ST}$ extremes. This gives us 150 estimates for the annual minimum $D_{ST}$ that occurred over the last 14 solar cycles of activity. This record then provides an estimate of how many years have included superstorm activity over the last 14 cycles, where superstorm activity is categorized in terms of the largest annual event crossing a typical threshold minimum $D_{ST}$ level. We find that the largest samples are exponentially distributed. We can then determine the range of minimum $D_{ST}$ that would occur if this distribution applied to the next largest record in excess of these 150 estimates, that is, a 1:151 year event. The Carrington event is also characterized in terms of its excursion in $D_{ST}$, and estimates vary considerably (Hayakawa et al., 2019; Siscoe et al., 2006; Tsurutani et al., 2003). We compare these estimates with the range of minimum $D_{ST}$ for a 1:151 event inferred from the 14 solar cycle proxy $D_{ST}$ extremes record. This provides an assessment of whether the Carrington event was a more intense version of the other superstorms that have occurred since 1868 or whether it was in a class of its own, which would require the concurrence of special conditions in the corona and solar wind and at the Earth. Only if it is the former can we use the set of observed storms to try to predict how likely such an event is in the future.

2. The Data Sets

Geomagnetic indices are derived from ground-based magnetometer observations (Mayaud, 1980) and are widely used to indicate the intensity of space weather events. The $D_{ST}$ index (Sugiura, 1964; Sugiura & Kamei, 1991) measures low-latitude global variations in the horizontal component of the geomagnetic field, thus representing the strength of the equatorial ring current. The $D_{ST}$ index is available (World Data Center for Geomagnetism et al., 2015) since 1957, so that we can directly compare the aa index to $D_{ST}$ over the last five solar cycles.

We focus on the 3-hourly resolution aa index over the last 14 solar cycles, from 1868 to the present. This will be analyzed alongside the daily sunspot number which is available for the same time period. The aa index is constructed (Mayaud, 1972) from the $K$ indices determined at two antipodal observatories (invariant magnetic latitude 50°) to provide a quantitative characterization of magnetic activity, which is homogeneous through the whole series. A key consideration for this study is that the aa index (units, nT) is discretized in amplitude (Bubenik & Fraser-Smith, 1977) since the underlying K index (Bartels et al., 1939) is a quasi-logarithmic 0–9 integer scale that characterizes the maximum positive and negative magnetic deviations that occur during each 3 hr period at a given observatory. Due to its longevity, the index has also recently required some corrections. The response seen by a magnetometer to geomagnetic activity depends on the station’s location with respect to the auroral oval. A scale factor for each station is applied to the scale of threshold values used to convert the observed continuous values into quantized $K$ values. This scale factor is adjusted for each station to allow for its location and characteristics such that the $K$ value is a standardized measure of the level of geomagnetic activity, irrespective of the location of the observation. The Mayaud (1980) original scheme assumes that this scale factor does not change with time. This does not account for secular changes in the intrinsic geomagnetic field that have occurred over the 150 years of the aa index, which introduce a drift in the individual stations and “steps” in value as stations are changed.
are discussed in detail and corrected for in Lockwood, Chambodut, et al. (2018). These corrections are typically less than 10 nT in magnitude, and while this is important for estimates of the overall long-term change in \(a\alpha\), it is a relatively small (and we will see, within uncertainties) perturbation on typical superstorm values. Lockwood, Finch, et al. (2018) extended this work to correct for hemispheric asymmetry using a model of the time-of-year and time-of-day response functions of the stations. They have produced a homogenized 3-hourly \(a\alpha\) index utilizing these corrections. We have repeated the analysis here for both the homogenized and original (“classic”) versions of the \(a\alpha\) index, and key plots that use the homogenized \(a\alpha\) index in the main sections of the Letter are reproduced using the “classic” (ISGI) \(a\alpha\) index in the supporting information (SI). The homogenized \(a\alpha\) index is available to end 2017, and our analysis extends up to this date, giving 150 calendar years of data.

3. The \(a\alpha\) Index Compared to \(D_{ST}\) at Large Values

As the \(a\alpha\) index is nonlinearly and nonuniformly discretized in amplitude, we need to explore to what extent it can be used to characterize superstorms. We can see this by comparing it to \((-)D_{ST}\), which is a well-established measure of geomagnetic storm intensity. The \(D_{ST}\) index is well sampled in amplitude, and therefore, its maximum value does provide a meaningful estimate of superstorm intensity. Semilog rank

**Figure 1.** Rank order plots at the minima, maxima, and declining phases of the last five solar cycles plotting data records for the classic \(a\alpha\) index (a–c), the homogenized (Lockwood, Chambodut, et al., 2018; Lockwood, Finch, et al., 2018) \(a\alpha\) index (d–f), and \(-D_{ST}\) index (g–i). The time interval from which data are used to form each rank order plot is indicated in the inset, overplotted on the daily sunspot number. Colors indicate the solar cycles 20 (blue), 21 (red), 22 (green), 23 (orange), and 24 (purple).
order plots (Sornette, 2003) provide a method to display the behavior of a set of values, particularly where they are large to extreme. The observations $x_k$ are sorted in descending amplitude and plotted (ordinate) versus their rank $k$ (abscissa); that is, the largest observed value is Rank 1, the next largest, Rank 2, and so on. Figure 1 compares rank order plots of the data records for $(-)D_{ST}$ with that for classic and homogenized $aa$ for the solar maximum interval, the solar minimum interval, and the declining phase of each of the last five solar cycles for which $D_{ST}$ is available. We identify the intervals of solar minimum, solar maximum, and the declining phases by applying a single algorithm across the entire time series as detailed in the SI. In Figure 1 it is immediately apparent that the classic $aa$ amplitude is strongly discretized at the high values, whereas $(-)D_{ST}$ resolves them. Figure 1 plots the individual data points, and the homogenized $aa$ index shown in Figures 1d–1f is less discretized in appearance (Lockwood, Finch, et al., 2018) than the classic $aa$ as the individual data points have been adjusted using time- and station-dependent scale factors as discussed above. While this does correct $aa$ for secular changes, it cannot recover the information lost by the original discretization, on a quasi-logarithmic scale, involved in constructing the $K$ indices that underlie the $aa$ index. Therefore, the $aa$ maximum value (within a given interval or event) does not quantify the extrema of geomagnetic disturbances very well. As a consequence, $aa$ is not readily amenable to standard analysis techniques for extracting and quantifying the statistical properties of events or bursts. Thus, while the Peak Over Threshold method has been successfully applied in quantifying the statistics of events in $D_{ST}$ using Extreme Value Theory (e.g., Tsubouchi & Omura, 2007), it cannot simply be applied to the $aa$ index. For this reason we will focus on yearlong averages of the largest 0.5% and 5% $aa$ records seen in each year as an estimate of the relative level of extreme activity captured by the $aa$ index. Figure 1 verifies that the large $aa$ and $(-)D_{ST}$ records do indeed both follow the variation within and between solar cycles in the same manner despite the discretization present in the $aa$ index. We can hence use $aa$ to provide an indication of the variation in the extremes of geomagnetic activity over the last 14 solar cycles.

4. Historical Space Weather Activity

Figure 2 plots the level of extreme activity captured by the homogenized $aa$ index versus annual average sunspot number from 1868–2017 inclusive, corresponding to the last 14 solar cycles. We parameterize extreme activity in $aa$ by annual averages of the largest 0.5% (top panels) and the largest 5% (center panels) and compare this with the average of all records (bottom panels). The averages are performed over nonoverlapping calendar years. Figures 2a–2c show the parameter space explored by $aa$ and sunspot number over the last 14 solar cycles. Figure 2c reproduces the well-known result (Feynman, 1982) that time averages of $aa$ always exceed a baseline value which increases linearly with averaged sunspot number. A baseline can also be seen in the annual averages of the largest 0.5% and the largest 5% $aa$ values.

We use the data from the last five solar cycles to obtain an approximate mapping between values of extreme activity in $D_{ST}$ and $aa$ parameterized as above. We expect from Figure 1 that the large to extreme records of $aa$ will track those of $D_{ST}$. As discussed above, the amplitude of $D_{ST}$ is well resolved, so that we can consider the single observed minimum $D_{ST}$ record that occurs in any given calendar year as a measure of the most severe storm that occurred in that year. Figure 3 overplots versus time the nonoverlapping calendar year annual averages of the largest 0.5% of the homogenized $aa$ index with the maximum of $(-)D_{ST}$ that occurs in the same calendar year. We see that these quantities do track each other, albeit imperfectly. Figure 2d plots (blue dots) these same quantities against each other; that is, the nonoverlapping calendar year annual averages of the largest 0.5% of the homogenized $aa$ index are plotted versus the maximum of $(-)D_{ST}$ that occurs in each calendar year as a scatter plot. Figures 2e and 2f plot the analogous scatter plots for annual averages of the largest 5% and annual averages of $aa$. Since the $aa$ index is derived from observatory $K$ index values, it has an upper bound, whereas $D_{ST}$ is unbounded. If the observed values of $aa$ over the last five solar cycles (where we have contemporaneous $D_{ST}$) explored this upper bound, we would see a saturation or “pile up” in $aa$ when plotted versus $D_{ST}$. We do not see any evidence of saturation in Figures 2d and 2e) and therefore perform a least squares linear regression fit which is plotted as the solid black line, the .95 confidence bounds are indicated by dotted lines. The $r$-squared coefficient of determination (which indicates the proportionate amount of variation in the response variable explained by the variable in the linear regression model) for each fit is given on the panels. Nonoverlapping calendar year annual averages of the largest 0.5% of the homogenized $aa$ index (Figure 2d) are well described by the linear least squares fit to annual minimum $D_{ST}$ with $r$-squared coefficient of determination $r = 0.81$. The coefficients of this fitted line $a(x − b)$ are (with 95% confidence intervals) $a = 0.87, [0.76, 0.99]$ and $b = −43.12 [−79.48, −6.76]$. The fit is reasonable, $r = 0.76$ for
Figure 2. Panels (a–c) plot each value (black *) of the average of the largest 0.5%, largest 5%, and all homogenized aa index records in each calendar year, versus average sunspot number, for all observations 1868–2017 inclusive. The annual (calendar year) intervals are nonoverlapping. Panels (d–f) plot (blue dots) the subset of the nonoverlapping calendar year aa averages versus the maximum value of $-D_{ST}$ that occurred in the same yearlong window, taken over the last five solar cycles. In each panel the solid black line plots the least squares fit and the dotted lines, the 0.95 confidence level of the fit; the $r^2$-squared coefficient for each fit is given on the panels. The green lines use this fit to map between $D_{ST}$ thresholds of $-250$ and $-500$ nT and corresponding aa values.
Figure 3. Comparison between $(-)D_{ST}$ and homogenized $aa$ across the last five solar cycles. The average of the largest 0.5% homogenized $aa$ index records in each calendar year (*) is plotted alongside the maximum $(-)D_{ST}$ (o) record that occurred in that year. The calendar year samples are nonoverlapping.

the largest 5% (Figure 2e). We need to choose a high threshold in order to isolate the largest events seen in each year of the $aa$ index in order for these to be comparable with the largest annual minimum value of the $D_{ST}$ index. This confirms that the correspondence is not strongly sensitive to the particular choice of high threshold. As we would expect, the correspondence will be poor between the annual averages of $aa$ and the largest annual minimum of $D_{ST}$ and this is indeed the case with $r = 0.4$ (Figure 2f). We therefore focus on the annual averages of the largest few percent of the $aa$ index as the parameter for extreme activity.

We now use this least squares fit to read across between annual averages of the largest few percent of $aa$ records to the corresponding annual $D_{ST}$ minimum ($(-)D_{ST}$ maximum) values that would have been expected to occur over the last 14 solar cycles. Extreme space weather activity is often categorized in terms of $D_{ST}$ crossing a minimum threshold. In Figure 2 we read across (green lines) $Dst$ levels of $-250$ nT, the threshold for “severe” (Riley & Love, 2016) and $-500$ nT, the threshold for “great” (Lakhina & Tsurutani, 2016) geomagnetic storms. $D_{ST}$ levels of $(-250, -500)$ map onto the $aa$ parameters as follows: annual averages of the largest 0.5% of the homogenized $aa$: (255, 473) and annual averages of the largest 5% of the homogenized $aa$ (126, 196). Counting the points that lie above these thresholds in $aa$ indicates that over 150 years, on average at least one great storm occurred in 6 (4%) of those years, and at least one severe storm occurred in 42 (28%) of those years. These estimates average over any solar cycle variation.

We use the least squares fit in Figure 2 to read across from all 150 annual averages of the largest few percent of $aa$ records to the corresponding $D_{ST}$ proxy, that is, the annual $D_{ST}$ minimum ($(-)D_{ST}$ maximum) values that would have been expected to occur over the last 14 solar cycles. These are plotted in Figure 4 as rank order plots. In addition to the 150 annual $D_{ST}$ proxy samples, we have one additional sample that arguably exceeds all 150 values, that is, the Carrington event. The Carrington event estimate will therefore be Rank 1 on this plot. The largest of the 150 annual $D_{ST}$ proxy samples is plotted as Rank 2, the next largest as Rank 3, and so on.

The dependencies seen on rank order plots are simply those of the distribution (Sornette, 2003) since an empirical estimate of the cumulative density function (cdf) $C(x_k)$ is obtained by plotting rank $k$ normalized to the total number of samples, $N$, $C(x_k) = k/N$ versus the samples $x_k$ arranged in ascending order of size. The leading rank observation (Rank 2 here) in 150 annual samples is then a 1/150 year event, and we indicate this and the location of a 1/10 year event across the top of the plot. To estimate the distribution functional form, we have performed a least squares fit of a straight line on this semilog plot to the 100 largest ranked $D_{ST}$ proxy samples. The green lines plot the fitted line $x_k = \beta (\log(k) - b)$ where $k = [2..101]$ is the rank. The r-square values for these fits are high, $r > 0.99$. In Figure 4 the fit parameters with 95% confidence in brackets are $\beta = [-146 [-148, -144]]$ and $b = 5.53 [5.50, 5.56]$. The high r-square value of these fitted lines confirms that the tail of the distribution is well described by an exponential function (Sornette, 2003) $f(x) = (1/\beta)exp(-x/\beta)$. The 95% confidence intervals for this fitted line give an uncertainty that deviates less than 1% from the fitted line. The dominant uncertainty on this plot arises from the variation between different empirical realizations of the cdf (or rank order plot) for which Greenwood (1926) provides an estimate as shown on the figure. Applying this uncertainty to the results from Figure 4 then gives the chance of at least one great $D_{ST} < -500$ nT storm in a given year that is then 4% with uncertainty bounds $[0.9, 7]$, and for a severe, $D_{ST} < -250$ nT storm is 28% $[20, 35]$. The top 10 most active years in the 150 year $aa$ record (plotted as rank $k = 2..11$ on Figure 4) are summarized in Table 1. As we would expect, years in which some of the most severe storms occurred appear here; however, we can now directly rank them and can estimate their percent occurrence likelihood.

An important question is whether the Carrington event belongs to the same physical class as the other superstorms. If so, its probable severity and chance of occurrence should be predictable at least in principle, as it will follow that of the other more moderate superstorms. If not, it is in a distinct physical class and past observations of more moderate superstorms may not inform estimates of its chance of occurrence; it
Figure 4. The panels show rank order plots of nonoverlapping annual minimum ($-D_{ST}$) proxy samples derived from (a) the largest 0.5% and (b) the largest 5% of homogenized aa (black stars). The largest of these samples is plotted as Rank 2, the next largest as Rank 3, and so on. We plot as Rank 1 two estimates of the Carrington event: $D_{ST} = -850$ nT (red diamond) and $D_{ST} = -1,760$ nT (red square). The green lines indicates an exponential fit to the largest 100 values, and the r-squared coefficient for each fit is given in the panels. The error bars for the first ranked sample (green error bar) are estimated for an underlying exponential distribution (see text). The 95% confidence level for this empirical realization of the rank order plot is estimated from Greenwood (1926) (blue dashed lines).

is a “Dragon King” (Sornette & Ouillon, 2012). We now determine if estimates for the Carrington event are consistent with the exponential distribution of proxy $D_{ST}$. For an exponential we have (Sornette, 2003) an estimate of the fluctuations between one realization to another for the first ranked sample, it is $\pm \beta$. This is plotted as a green error bar on Rank 1 location of the exponential fit. This gives an estimate $D_{ST} = -809 [-663, -955]$ (using classic aa as shown in the SI, we obtain $D_{ST} = -813 [-667, -959]$). This is the range of values for $D_{ST}$ for this event to be 1 in 151 year event drawn from the same distribution as other extreme activity seen in aa over the last 14 solar cycles. We overplot at Rank 1 the two estimates of the Carrington event (red diamond and square). From Figure 4 we see that the estimate of $D_{ST} = -850$ nT is consistent with the above extrapolation of the exponential fit so that the likelihood of any given year
will be closer correspondence between these two measures. Our estimate that a
to the return period for an event of a specific amplitude. For the most severe and infrequent storms there
that the return period of a level of annual activity that we find here would not be expected to correspond
the largest event in each year. In general, for moderate conditions, there will be several storms per year, so
range determined here for the Rank 1 event. Tsubouchi and Omura (2007) predict an occurrence frequency

Different versions (Tsurutani et al., 2003; Siscoe et al., 2006) of the Burton et al. (1975) equation support these

...exhibiting a Carrington-class event on this scale simply follows the exponential distribution that describes
the other severe storms that have occurred since. However, a value of of $D_{ST} = -1.760$ nT (red square) is in
its own class of behavior; it is far from this exponential distribution tail.

The $D_{ST}$ excursion that occurred during historical space weather events is challenging to quantify, and as
a consequence, there is considerable diversity in both the values obtained and the methodology used to
obtain them. The $D_{ST} = -1.760$ nT estimate for the Carrington event is a minimum magnetic displacement
in a Bombay magnetogram (Tsurutani et al., 2003), and Lakhina and Tsurutani (2016) discuss supporting
evidence that this is indeed consistent with this $D_{ST}$ value. The Bombay station was fortuitously located near
noon during the peak magnetometer displacement so that the effect of the disturbance field asymmetry is
minimized, and local $H$ component values are close to $D_{ST}$ (see, e.g., Figure 2 of Siscoe et al., 2006). However,
given that $D_{ST}$ is an hourly index, this value has has been interpreted by Siscoe et al. (2006) (see also Cliver
& Dietrich, 2013) as a minimum $D_{ST} \approx -850$ nT based on hourly averages of the Bombay magnetogram.
Different versions (Tsurutani et al., 2003; Siscoe et al., 2006) of the Burton et al. (1975) equation support these
two different estimates. Other observations offer insight: Hayakawa et al. (2019) found that the equatorward
boundary of auroral oval of the Carrington event was comparable with that of other superstorms, suggesting
a $D_{ST}$ value closer to that of Siscoe et al. (2006). Modeling of the “solar storm” of 2012, an intense CME which
did not impact on Earth but was observed at STEREO-A, suggests extreme case scenarios of $D_{ST} = -1,182$ nT
(Baker et al., 2013) and $D_{ST} = -1,150$ nT (Liu et al., 2014). In the 2012 solar storm, the correlated dynamics
of several CMEs created the conditions for an unusually intense event. The analysis in this Letter does not
rule out any of these estimates. Instead, it offers quantitative insight into their interpretation. Events with
$D_{ST} \leq -1,000$ nT are a different class of behavior to other severe storms that have occurred over the last 150
years. They require special conditions which may be physical, observational, or a combination thereof.

We have parameterized extreme space weather activity with annual averages of the top few percent of the
$aa$ index. While this has allowed us to form a distribution from observations over 14 solar cycles, it does not
discriminate the statistics of individual events. This can only be done for time series that are well resolved
in amplitude, such as $D_{ST}$, for which there are a number of studies. We have identified a correspondence
between the annual averages of the top few percent of the $aa$ index and the annual minimum $D_{ST}$, that is,
the largest event in each year. In general, for moderate conditions, there will be several storms per year, so
that the return period of a level of annual activity that we find here would not be expected to correspond
to the return period for an event of a specific amplitude. For the most severe and infrequent storms there
will be closer correspondence between these two measures. Our estimate that a $D_{ST} \sim -850$ nT is an $\sim 1$
in 150 year event is not inconsistent with that of Riley and Love (2016), a 10% $[1,20]$ chance of occurrence per
decade. The $D_{ST}$ excursion $907 \pm 132$ nT Love et al. (2019) estimate for the 1921 event also overlaps with the
range determined here for the Rank 1 event. Tsubouchi and Omura (2007) predict an occurrence frequency

<table>
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<th>Rank</th>
<th>Year</th>
<th>% chance per year</th>
<th>Activity in that year</th>
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<td>0.67 [0, 1.9]</td>
<td>Remarkable storm; Silverman and Cliver (2001), Tables IV, VII</td>
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<td>Fatima; Tables III, IV, VII</td>
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<td>Halloween storms; Weaver and Murtagh (2004), Table III</td>
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<td>1946</td>
<td>2.67 [0.1, 5.2]</td>
<td>Table IV</td>
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<td>1989</td>
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<td>Quebec power outage; MacNeil (2018); Table VII</td>
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<td>1941</td>
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<td>Geomagnetic storm; Love and Coïsson (2016); Tables III, IV</td>
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<td>5.33 [1.7, 8.9]</td>
<td>Remarkable storm; Love et al. (2019) Tables IV, VII</td>
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<td>1960</td>
<td>6.0 [2.2, 9.8]</td>
<td>Table III</td>
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<td>1958</td>
<td>6.67 [2.7, 10.7]</td>
<td>Remarkable storm; Table VII</td>
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</table>

Remarkable storms (geomagnetic perturbation, Table 1 of Tsurutani et al., 2003). Events in Cliver and Svalgaard
(2004), Tables III (fast transit events up to 2003); IV (Greenwich list of great storms up to 1954), and VII (low-latitude
auroras up to 1958). Rank order is derived from annual averaged top 0.5% homogenized $aa$ index values plotted in
Figure 4a. This need not correspond one-to-one with rankings based directly on $D_{ST}$ for individual events.
Mathematical expressions, equations, and symbols are not represented in natural text format.


