

Novel Approaches to Coherency Conditions in Dynamic LDV  
Models:  
Quantifying Financing Constraints  
and a Firm's Decision and Ability to Innovate

by

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Abstract

We develop novel methods for establishing coherency conditions in Static and Dynamic Limited Dependent Variables (LDV) Models. We propose estimation strategies based on Conditional Maximum Likelihood Estimation for simultaneous LDV models without imposing recursivity. Monte-Carlo experiments confirm substantive Mean-Squared-Error improvements of our approach over other estimators.

We analyse the impact of financing constraints on innovation: *ceteris paribus*, a firm facing binding finance constraints is substantially less likely to undertake innovation, while the probability that a firm encounters a binding finance constraint more than doubles if the firm is innovative. A strong role for state dependence in dynamic versions of our models is also established.

**Keywords:** Financing Constraints; Innovation; Dynamic Limited Dependent Variable Models; Joint Bivariate Probit Model; Econometric Coherency Conditions; State Dependence.

**JEL Classifications:** C51, C52, C15

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# 1 Introduction

In this paper, we investigate the fundamental identification issue of *coherency* of Limited Dependent Variable (LDV) models with endogeneity and flexible temporal and contemporaneous correlations in the unobservables. An LDV model was defined originally by [Gourieroux et al., 1980] to be “coherent” if it implies a valid function from the unobservables that drive the model to the observed limited dependent variables. We develop two novel methods for establishing coherency, which have intuitive interpretations and are easy to implement and generalize. These methods lead to estimation strategies based on Conditional Maximum Likelihood Estimation (CMLE) for simultaneous LDV models without imposing unnecessarily restrictive assumptions. They also allow us to establish the coherency of several Dynamic LDV models that until now, it was impossible to determine whether they were coherent or incoherent using traditional methods.

We focus our discussion of the coherency problem in LDV models by using the Simultaneous LDV Model with Two Binary Responses. In this model, limited dependent variables  $y_1$  and  $y_2$  are jointly determined through filter functions  $\tau_1(\cdot)$  and  $\tau_2(\cdot)$  operating on latent variables  $y_1^*$  and  $y_2^*$  respectively:

$$y_{1it} = \tau_1(y_{1it}^* \equiv [h_1(x'_{1it}\beta_1, y_{2it}\gamma) + \epsilon_{1it}]) \quad (1)$$

$$y_{2it} = \tau_2(y_{2it}^* \equiv [h_2(x'_{2it}\beta_2, y_{1it}\delta) + \epsilon_{2it}]) \quad (2)$$

The (possibly non-linear) functions  $h_1(\cdot)$  and  $h_2(\cdot)$  are known up to parameter vectors  $\beta_1$  and  $\beta_2$  and the two interaction coefficients  $\gamma$  and  $\delta$ . The interaction terms  $y_{2it}\gamma$  and  $y_{1it}\delta$  appear in the respective latent variables  $y_{1it}^*$  and  $y_{2it}^*$ . Let  $x_{1it}$  and  $x_{2it}$  denote the vectors of exogenous factors for each side of the model. The parameter vector to be estimated is  $\theta \equiv (\beta_1', \beta_2', \gamma, \delta, \sigma_1^2, \sigma_2^2, \rho)'$  where  $\rho \equiv \text{correlation}(\epsilon_{1it}, \epsilon_{2it})$ . In the most general case, the sample is a panel data set indexed by  $i = 1, \dots, N$  and  $t = 1, \dots, T$ .

The existing econometric literature has established as the typical coherency condition to be:  $\gamma \cdot \delta = 0$ , i.e., no reverse interaction terms are allowed among the two endogenous variables. This condition, which is termed “recursivity,” is sufficient for the joint distribution  $(y_{1it}, y_{2it} | x_1, x_2, \theta)$  to be well-specified. [Gourieroux et al., 1980] explain the condition in terms of there being a valid function from  $(\epsilon_{1it}, \epsilon_{2it})$  to the observable endogenous variables  $(y_{1it}, y_{2it})$ . [Lewbel, 2007] establishes necessary and sufficient conditions for coherency by approaching the problem as requiring a valid reduced form system for  $(y_{1it}, y_{2it})$ . For example, if  $\delta = 0$  then the RF for  $y_{2it}$  is:

$$y_{2it} = \tau_2(h_2(x'_{2it}\beta_2) + \epsilon_{2it})$$

and hence the RF for  $y_{1it}$  is given by:

$$y_{1it} = \tau_1(h_1(x'_{1it}\beta_1, \gamma \cdot (\tau_2 h_2(x'_{2it}\beta_2) + \epsilon_{2it})) + \epsilon_{1it})$$

In practice there are many situations where assuming recursivity may be too

restrictive.<sup>2</sup> For instance, let us consider the case where we want to study the interactions between innovation by firms and the financial constraints they may face. Obviously, the propensity to innovate may be affected by financial constraints, and at the same time, innovative firms are likely to face specific financial constraints: innovation affects survival of firms (see [Audretsch, 1995] and [Klette and Kortum, 2004]), asset intangibility is higher for innovative firms which lowers their collateral value and due to their innovative nature informational asymmetries with external investors are more pronounced. While such issues may be investigated based on survey data and self-assessed measures of financial constraints on firms, the estimation of such models is complicated by the problem of coherency. Recursivity corresponds to the key identifying assumption that innovation does not affect financial distress directly ( $\delta = 0$ ). On a priori grounds, this assumption seems particularly dubious since innovation may lead to more profits and thus relax financial constraints (corresponding to  $\delta > 0$ ). An alternative possibility is that innovation may lead to higher investment in intangible assets thus reinforcing binding financial constraints (corresponding to  $\delta < 0$ ). Both possibilities violate the traditional coherency condition.

Our main contributions can be summarized as follows: first, we show how to establish coherency of static and dynamic LDV models without imposing recursivity. Second, this novel approach leads to less strict conditions. It is shown how to establish coherency without model recursiveness through the use of (a) endogeneity in terms of latent variables and/or (b) sign restrictions on model parameters.

Our approach overcomes three major sources of confusion in the analysis of coherency: first, incoherency may be one of two distinct types, which we term below “empty region incoherency” vs. “overlap incoherency.” Second, a given econometric model may exhibit simultaneously *both* types of incoherency and hence it is not a model property — see the model of subsection 2.2 for an illustration. This point is not made clear in the analysis of [Chesher and Rosen, 2014]. Third, the traditional approaches focus on establishing *sufficient* conditions for coherency, while our methods allow us to prove that they *may not be necessary*. In addition, our approach exhibits two major improvements over existing methods: firstly, they are intuitive; secondly, they can be generalized. Secondly, they can be extended to considerably more complicated LDV models, especially in cases where the models are allowed to contain intertemporal endogeneity of the type considered in [Falcetti and Tudela, 2008]. In Subsection 2.7.1 we establish for the first time the coherency of the Panel Univariate Probit model with State Dependence and in Subsection 2.7.2 the coherency of the Panel Bivariate Probit model with State Dependence.

We also develop and summarize the results of a set of extensive Monte-Carlo experiments, which confirm very substantive Mean-Squared-Error estimation improvements of the CMLE approach over estimators that make overly restrictive coherency assumptions about the Data Generating Process (DGP). The fact that our novel approach for the first time eliminates the need to assume model recursivity is quite

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<sup>2</sup>Exceptions of this do exist: for example in industrial organization a two-agent discrete game may be employed to model the strategic interactions between firms in a duopoly setup. If one firm is a Stackelberg leader while the other is a follower, a recursive model may be applicable, even though the analogy is not precise.

important for the economic problem studied in the empirical application.

Our empirical results are quite striking: *ceteris paribus*, we estimate that a firm that faces a binding finance constraint is approximately 30% less likely to undertake innovation, while the probability that a firm encounters a binding finance constraint more than doubles if the firm is classified as innovative. Finally, we establish a strong role for state dependence in dynamic versions of our models: firms tend to innovate continuously rather than occasionally and past financial difficulties are correlated with the present ones even after conditioning on important firm characteristics. Moreover, it seems that firms with current but also past innovative experiences are more likely to find it difficult to finance their current projects.

Section 2 explains the coherency issue in LDV models and proposes two novel approaches for establishing coherency without imposing recursivity. Section 3 summarizes the results from the Monte-Carlo experiments. Section 4 presents the empirical application, which quantifies the interaction between financial constraints and firm innovation. Section 5 concludes.

## 2 Econometric Coherency in LDV Models

To analyze the problem of econometric coherency we use two discrete-response LDV models.

### 2.1 The Joint Bivariate Binary Probit Model

The first case we focus on here is the binary threshold crossing response model in which:

$$\tau_j(z) \equiv \mathbf{1}(z > 0)$$

where  $\mathbf{1}(z > 0)$  is the *indicator function* defined by:  $\mathbf{1}(z > 0) \equiv \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z \leq 0 \end{cases}$ . In terms of the two latent variables  $y_1^*$  and  $y_2^*$  and the observed binary indicators  $y_1$  and  $y_2$ , and suppressing the observation indices:

$$y_1 = \begin{cases} 1 & \text{if } y_1^* \equiv x_1' \beta_1 + \gamma y_2 + \epsilon_1 > 0 \\ 0 & \text{if } y_1^* \equiv x_1' \beta_1 + \gamma y_2 + \epsilon_1 \leq 0 \end{cases} \quad (3)$$

$$y_2 = \begin{cases} 1 & \text{if } y_2^* \equiv x_2' \beta_2 + \delta y_1 + \epsilon_2 > 0 \\ 0 & \text{if } y_2^* \equiv x_2' \beta_2 + \delta y_1 + \epsilon_2 \leq 0 \end{cases} \quad (4)$$

In the Empirical application of Section 4.2, we employed this model to study the impact of financing constraints on a firm's decision and ability to innovate in a panel data context.<sup>3</sup>

The specific version of model becomes:

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<sup>3</sup>A related application of this setup in International Finance is the Banking and Currency Crises Model of [Falcetti and Tudela, 2008] where  $(C_{it}, B_{it})$  refer to Currency and Banking Crises respectively. Their model is recursive, in that Currency crises are allowed to depend on Banking crises but not vice-versa.

$$I_{it} = \begin{cases} 1 & \text{if } I_{it}^* \equiv x_{it}^I \beta^I + \gamma F_{it} + \epsilon_{it}^I > 0 \\ 0 & \text{if } I_{it}^* \equiv x_{it}^I \beta^I + \gamma F_{it} + \epsilon_{it}^I \leq 0 \end{cases} \quad (5)$$

$$F_{it} = \begin{cases} 1 & \text{if } F_{it}^* \equiv x_{it}^F \beta^F + \delta I_{it} + \epsilon_{it}^F > 0 \\ 0 & \text{if } F_{it}^* \equiv x_{it}^F \beta^F + \delta I_{it} + \epsilon_{it}^F \leq 0 \end{cases} \quad (6)$$

Analytically,  $(I_{it}, F_{it}) \in \{(1, 1), (1, 0), (0, 1), (0, 0)\}$  such that:

$(I_{it}, F_{it})$	$I_{it}^*$	$F_{it}^*$
(1, 1)	$x'_{1it}\beta_1 + \gamma + \epsilon_{1it} > 0$	$x'_{2it}\beta_2 + \delta + \epsilon_{2it} > 0$
(1, 0)	$x'_{1it}\beta_1 + \epsilon_{1it} > 0$	$x'_{2it}\beta_2 + \delta + \epsilon_{2it} < 0$
(0, 1)	$x'_{1it}\beta_1 + \gamma + \epsilon_{1it} < 0$	$x'_{2it}\beta_2 + \epsilon_{2it} > 0$
(0, 0)	$x'_{1it}\beta_1 + \epsilon_{1it} < 0$	$x'_{2it}\beta_2 + \epsilon_{2it} < 0$

For a typical  $it$  observation, the probability  $Prob(I_{it}, F_{it}|X, \theta)$  is thus characterized by the constraints on the unobservables:

$$(a^I, a^F)' < (\epsilon^I, \epsilon^F)' < (b^I, b^F)'$$

through the configuration:

$I_{it}$	$F_{it}$	$a^I$	$b^I$	$a^F$	$b^F$
1	1	$-x_{it}^I \beta^I - \gamma$	$\infty$	$-x_{it}^F \beta^F - \delta$	$\infty$
1	0	$-x_{it}^I \beta^I$	$\infty$	$-\infty$	$-x_{it}^F \beta^F - \delta$
0	1	$-\infty$	$-x_{it}^I \beta^I - \gamma$	$-x_{it}^F \beta^F$	$\infty$
0	0	$-\infty$	$-x_{it}^I \beta^I$	$-\infty$	$-x_{it}^F \beta^F$

In general, in the absence of coherency conditions, there will be *overlaps* and/or *gaps* in the domain of  $(\epsilon_{1it} + x'_{1it}\beta_1, \epsilon_{2it} + x'_{2it}\beta_2)$ . These would be ruled out by the aforementioned sufficient coherency condition.<sup>4</sup>

<sup>4</sup>A related LDV model that does **not** exhibit similar coherency difficulties is the bivariate probit model with **latent variable** interactions (as opposed to limited variable interactions). Specifically:

$$\begin{aligned} y_{1it} &= \tau_1 (y_{1it}^* \equiv [h_1(x'_{1it}\beta_1, y_{2it}^*\gamma) + \epsilon_{1it}]) \\ y_{2it} &= \tau_2 (y_{2it}^* \equiv [h_2(x'_{2it}\beta_2, y_{1it}^*\delta) + \epsilon_{2it}]) \end{aligned}$$

Then:

$$\begin{aligned} y_1^* &= x_1\beta_1 + y_2^*\gamma + \epsilon_1 \\ y_2^* &= x_1\beta_1 + y_1^*\delta + \epsilon_2 \end{aligned}$$

and

$$\begin{aligned} y_1^* &= x_1\beta_1 + \gamma \cdot [x_2\beta_2 + y_1^*\delta + \epsilon_2] + \epsilon_1 \\ y_2^* &= x_2\beta_2 + \delta \cdot [x_1\beta_1 + y_2^*\gamma + \epsilon_1] + \epsilon_2 \end{aligned}$$

Hence  $y_1^* = RF_1$  and  $y_2^* = RF_2$ , allowing us to obtain  $y_1 = \tau(RF_1)$  and  $y_2 = \tau(RF_2)$ . We thus see that it is considerably more straightforward to establish coherency identification of LDV models with latent variable interactions as opposed to limited variable interactions.

## 2.2 The Traditional Approach to Coherency Conditions

The second model we focus on to analyze econometric coherency, is a slightly more complicated simultaneous LDV model, namely the *binary & trinomial ordered probit model* of [Hajivassiliou and Ioannides, 2007]. This model studies interactions between liquidity and employment constraints on individual households indexed by  $i$  at a given point in time indexed by  $t$ . The reason we select this model is because it can exhibit simultaneously *both* types of incoherency (overlaps and gaps). This is critical because it will allow us to devise estimation strategies that overcome certain types of incoherency.

Define two latent dependent variables  $y_{1it}^*$  and  $y_{2it}^*$ . The first denotes the propensity of individual  $i$  in period  $t$  to be liquidity constrained and the second its propensity to face employment hour constraints. The corresponding limited dependent variables are denoted by  $y_{1it}$  and  $y_{2it}$ . Dropping the  $it$  subscripts for simplicity, the model is defined by:

$$y_1 = \begin{cases} 1 & \text{if } y_1^* > 0 \text{ (liquidity constraint binding),} \\ 0 & \text{if } y_1^* \leq 0 \text{ (liquidity constraint not binding).} \end{cases}$$

$$y_2 = \begin{cases} -1 & \text{if } y_2^* \leq \lambda^- \text{ (overemployed)} \\ 0 & \text{if } \lambda^- \leq y_2^* < \lambda^+ \text{ (voluntarily employed)} \\ +1 & \text{if } \lambda^+ \leq y_2^* \text{ (under-/unemployed).} \end{cases}$$

where the latent variables are given by:

$$y_1^* = \mathbf{1}(y_2^* < \lambda^-)\gamma_{11} + \mathbf{1}(\lambda^- < y_2^* < \lambda^+)\gamma_{12} + x_1'\beta_1 + \epsilon_1$$

$$y_2^* = \mathbf{1}(y_1^* > 0)\delta + x_2\beta_2 + \epsilon_2$$

Since  $(S, E)$  lie in  $\{0, 1\} \times \{-1, 0, 1\}$ , the 6 possible configurations may be enumerated as follows:

$S$	$E$	$y_1^*$	$y_2^*$
0	-1	$\gamma_{11} + x_1\beta_1 + \epsilon_1 < 0,$	$x_2\beta_2 + \epsilon_2 < \lambda^-$
0	0	$x_1\beta_1 + \epsilon_1 < 0,$	$\lambda^- < x_2\beta_2 + \epsilon_2 < \lambda^+$
0	+1	$\gamma_{12} + x_1\beta_1 + \epsilon_1 < 0,$	$\lambda^+ < x_2\beta_2 + \epsilon_2$
1	-1	$\gamma_{11} + x_1\beta_1 + \epsilon_1 > 0,$	$\delta + x_2\beta_2 + \epsilon_2 < \lambda^-$
1	0	$x_1\beta_1 + \epsilon_1 > 0,$	$\lambda^- < \delta + x_2\beta_2 + \epsilon_2 < \lambda^+$
1	+1	$\gamma_{12} + x_1\beta_1 + \epsilon_1 > 0,$	$\lambda^+ < \delta + x_2\beta_2 + \epsilon_2$

In terms of the unobservables, the probability of a  $(y_1, y_2)$  observed pair is equivalent to the probability:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} < \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} < \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

where  $(\epsilon_1, \epsilon_2)' \sim N(0, \Sigma_\epsilon)$ , and  $a$  and  $b$  are given by:

$S$	$E$	$a_1$	$a_2$	$b_1$	$b_2$
0	-1	$-\infty$	$-\infty$	$-(\gamma_{11} + x_1\beta_1)$	$\lambda^- - x_2\beta_2$
0	0	$-\infty$	$\lambda^- - x_2\beta_2$	$-x_1\beta_1$	$\lambda^+ - x_2\beta_2$
0	+1	$-\infty$	$\lambda^+ - x_2\beta_2$	$-(\gamma_{12} + x_1\beta_1)$	$+\infty$
1	-1	$-(\gamma_{11} + x_1\beta_1)$	$-\infty$	$+\infty$	$\lambda^- - \delta - x_2\beta_2$
1	0	$-x_1\beta_1$	$\lambda^- - \delta - x_2\beta_2$	$+\infty$	$\lambda^+ - \delta - x_2\beta_2$
1	+1	$-(\gamma_{12} + x_1\beta_1)$	$\lambda^+ - \delta - x_2\beta_2$	$+\infty$	$+\infty$

Using traditional arguments, we obtain that a sufficient condition for coherency of the model is:

$$(\gamma_{11} + \gamma_{12})\delta = 0 \text{ and } \gamma_{11}\gamma_{12}\delta = 0.$$

To verify this condition, suppose  $(S, E) = (0, 0)$ . This rules out  $(S, E) = (0, -1)$  because  $x_2\beta_2 + \epsilon_2 > \lambda^-$ , and rules out  $(S, E) = (1, 0)$  because  $x_1\beta_1 + \epsilon_1 < 0$ .

But  $(1, -1)$  is not ruled out if the coherency conditions do not hold, since  $\gamma_{11}$  could be sufficiently negative and  $\delta$  sufficiently positive to imply the  $(1, -1)$  conditions.

Similarly, the  $(1, 1)$  possibility cannot be ruled out in the absence of the coherency conditions, since  $\gamma_{12}$  and  $\delta$  can be sufficiently positive.

Such logical inconsistencies are prevented if either (a)  $\delta = 0$  or (b)  $\gamma_{11}$  and  $\gamma_{12}$  are simultaneously 0.

### 2.3 Extending the Traditional Approach to Coherency

The traditional approaches to model coherency suffer from several major difficulties. Firstly, derivations of formal conditions using the traditional approach lack intuition. Secondly, the derived conditions are impossible to generalize and verify in moderately more complicated LDV models, especially in cases where the models are allowed to contain intertemporal endogeneity of the type considered in [Falcetti and Tudela, 2008]. Similarly, in case the joint binary probit model (3)-(4) is extended intertemporally, as for example in the empirical dynamic application in Section 4.2, the coherency condition is impossible to generalize and verify using the traditional analysis of the previous subsection. Thirdly, in practice non-triangular or reverse triangular cases are the most interesting from an economic point of view.<sup>5</sup> Finally, the traditional approaches focus on establishing *sufficient* conditions for coherency, while our methods allow us to prove that they are *not necessary*.

To overcome the first two difficulties, alternative ways for establishing coherency are developed here, that are both intuitive and straightforward, as well as much more generalizable. In addition, our methods allow us to resolve the last two difficulties leading to estimation based on CMLE for much more interesting practical applications. It is shown in the next Section how to establish coherency without model

<sup>5</sup>Exceptions of this do exist: for example in industrial organization a two-agent discrete game may be employed to model the strategic interactions between firms in a duopoly setup. If one firm is a Stackelberg leader while the other is a follower, a recursive model may be applicable, even though the analogy is not precise.

recursiveness through the use of (a) endogeneity in terms of latent variables and/or (b) sign restrictions on model parameters.

### 2.3.1 Novel Approach 1: Graphical

Let us illustrate the first approach using the Liquidity-Employment constraints application of [Hajivassiliou and Ioannides, 2007]. This graphical approach was first included in the LSE working paper [Hajivassiliou, 2002] and was presented at the CRETE Conference in Syros in 2003. It should also be noted that our graphical approach presented here is related to that of [Tamer, 2003] who studied the problem of coherency in bivariate discrete models for games with multiple equilibria.

Figure 1 gives the 6 possible regimes  $(y_1 \times y_2) = \{1, 0\} \times \{-1, 0, 1\}$  in terms of the two latent variables  $y_1^*$  and  $y_2^*$  and the possible configurations in terms of parameters  $\bar{\lambda}$ ,  $\underline{\lambda}$ ,  $\delta$ ,  $\gamma_{11}$ , and  $\gamma_{12}$ .  $y_1^*$  is on the horizontal axis and  $y_2^*$  on the vertical. The figure makes clear the role of the coherency condition (a)  $\delta = 0$  or (b)  $\gamma_{11} = \gamma_{12} = 0$ : in general, regions  $R2$  and  $R6$  exhibit double-counting (cross-hatched area), as well as a white rectangle remains which makes the six regions not mutually exhaustive. These two logical incoherencies disappear when either  $\delta = 0$  and/or  $\gamma_{11} = \gamma_{12} = 0$  hold.

[Figure 1 approximately here.]

We develop further our graphical approach here and use it to highlight the fundamental distinction between two types of incoherency, the first corresponding to overlap regions in latent variables space, while the second to empty regions. A critical fact that this model illustrates is that a particular model may **simultaneously** exhibit **incoherencies of both kinds**, *empty region incoherency* as well as *overlapping region incoherency*. This is a critical point that is not well understood in previous work, e.g., [Chesher and Rosen, 2014]. In fact, the terminology adopted by those authors, calling overlapping region incoherency as “model incompleteness” and empty region inchoherency as proper “model inchoherency,” exacerbates this confusion in somehow giving the impression that a model can only exhibit one of the two irregularities.

We then show below that under prior sign restrictions on model parameters, incoherencies of the empty region type can be eliminated by relying on suitable parameter sign restrictions through the use of Conditional MLE.

### 2.3.2 Novel approach 2: DGP From First Principles

Despite the usefulness of the graphical approach of the previous section to LDV problems with two latent variables, the method is very unwieldy or inapplicable to higher dimensional cases. To cover such problems, we develop a second approach to incoherency, which consists of designing a data-generating algorithm (hypothetical or implemented on a computer) to simulate random draws from an LDV model’s structure. Again let us use the Liquidity-Employment Constraints application of [Hajivassiliou and Ioannides, 2007] to illustrate the method. We draw  $\epsilon_1$  and  $\epsilon_2$  under the joint bivariate normal distribution with zero mean vector and variance-covariance



matrix  $\Sigma_\epsilon$ , and given  $x'_1\beta_1$  and  $x'_2\beta_2$  we attempt to generate  $y_1^*$  and  $y_2^*$ . This is straightforward provided the coherency condition holds: If (a)  $\delta = 0$ , then latent  $y_2^*$  can be drawn, then LDV  $y_2$ , which together with  $\epsilon_1$  and  $x'_1\beta_1$  determines the right hand side of  $y_1^*$ , thus allowing  $y_1$  to be drawn. Similarly, if (b)  $\gamma_{11} = \gamma_{12} = 0$ , then  $y_1^*$  can be drawn from the first equation based on  $\epsilon_1$  and  $x'_1\beta_1$ , which determines  $y_1$ , thus giving  $y_2^*$  and hence  $y_2$ . In general it is impossible, however, to devise such a data generation mechanism in case the coherency condition does not hold.

This approach is related to the [Gourieroux et al., 1980] condition that a function exist from  $\epsilon_1, \epsilon_2$  to  $y_1, y_2$ . It is also related to [Lewbel, 2007] in that coherency translates to there being a valid reduced form for the endogenous variables.

As we will show in section 2.7, the approach extends naturally to cases with intertemporal endogeneities in panel LDV models, and can be used to prove the coherency of the classic multiperiod panel probit with state dependence ([Heckman, 1981a]), as well as the intertemporal endogeneity versions of the models in Section 4.2 with explicit dynamic effects.

## 2.4 Identification Under Prior Sign Restrictions

The graphical approach we developed in the previous section highlights two distinct cases of incoherency: the first type of incoherency corresponds to regions of the observed endogenous variables of the model being *overlapping*, while the second to regions that are *empty*. We show that empty region incoherency can be overcome through conditional maximum likelihood (CMLE) of truncating the LDVs to lie outside the incoherency regions.<sup>6</sup> Our CMLE approach can also be motivated through the DGP approach for establishing coherency that we discussed in the previous subsection. In that case, we need to consider DGPs truncated to lie on a specific region of the latent variables space. A specific method for achieving this is given in [Hajivassiliou, 2008].

It is useful to highlight here the similarities and differences to the analysis in [Tamer, 2003], who also used a graphical approach to resolve an incomplete simultaneous discrete response model for a homogeneous two-agent discrete game of entry. Since the two rival firms in his setting were assumed identical, any incoherency arising was necessarily of the indeterminate type — see our two subcases 2.4.1 and 2.4.2, where the interaction terms  $\gamma$  and  $\delta$  are of the same sign. Consequently, the possibility of the interaction terms being of opposite sign was not under focus in his analysis and hence the applicability of CMLE to resolve those cases was not considered. It is also useful to note that our approach for establishing coherency through the use of prior sign restrictions developed here is related to the recent approach by [Uhlig, 2005] for Vector Autoregression identification under prior sign restrictions on impulse response functions.<sup>7</sup> [Dagenais, 1997] also makes a distinction between al-

<sup>6</sup>We also explain below that overlapping region incoherency *cannot* be transformed into empty region incoherency by redefining one of the observed binary LDVs to its complement.

<sup>7</sup>We are indebted to Alain Trognon for pointing out the potential of parameter sign restrictions overcoming incoherency of the “empty region” type, and to Hashem Pesaran for bringing to our attention Uhlig’s work on sign identification.

ternative types of incoherency regions.<sup>8</sup> We stress again that the approach and terminology in [Chesher and Rosen, 2014] is likely to lead to the incorrect belief that a model may exhibit only a single type of incoherency, either of the empty region- or overlap- type. Such misunderstanding would prevent the CMLE solution we develop below.

It is also critical at this point to explain why recent methodologies developed for econometric partially identified models with multiple equilibria *cannot* solve the coherency problems of the type we study here. There is a fundamental reason why the works of [Berry and Tamer, 2006], [Ciliberto and Tamer, 2009], [Beresteanu et al., 2011] and [de Paula and Tang, 2012], which follow on the pioneering approach of [Tamer, 2003], are not applicable to our models: these works require simultaneous games with *multiple decision makers making a simultaneous decision*. In the absence of these two ingredients, we believe that our CMLE approach is the only available solution.

In addition, contrary to [Tamer, 2003] and the four cited papers that followed him, our approach here can be extended to study the coherency of *dynamic* LDV econometric models. We develop these extensions in section 2.7 below.

We illustrate the CMLE approach for establishing coherency through prior sign restrictions by using the joint binary probit model:<sup>9</sup>

$$I = \begin{cases} 1 & \text{if } I^* \equiv x_1' \beta_1 + \gamma F + \epsilon_1 > 0 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

$$F = \begin{cases} 1 & \text{if } F^* \equiv x_2' \beta_2 + \delta I + \epsilon_2 > 0 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

Obviously, there exist **four cases** based on the signs of  $\gamma$  and  $\delta$ . These are presented in the four figures that follow.

#### 2.4.1 Case 1: $\gamma > 0, \delta > 0$ — overlapping regions, incoherency

[Figure 2 approximately here.]

#### 2.4.2 Case 2: $\gamma < 0, \delta < 0$ — overlapping regions, incoherency

[Figure 3 approximately here.]

#### 2.4.3 Case 3: $\gamma > 0, \delta < 0$ — empty regions, coherency through conditioning

[Figure 4 approximately here.]

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<sup>8</sup>Unfortunately his work remains incomplete and unpublished due to his untimely death.

<sup>9</sup>For the first equation,  $I^*$  is used for the latent and  $I$  for the observed LDV as a mnemonic to the *Innovation* side of the model of Section 4.2 below. Similarly, for the second equation we use  $F^*$  and  $F$  as a mnemonic to *Financing Constraints*.

For this case, coherency can be achieved by conditioning to lie outside the “empty” region of Figure 4, which has conditioning probability:

$$1 - Prob(-\gamma < \epsilon_1 + x'_1\beta_1 < 0, 0 < \epsilon_2 + x'_2\beta_2 < -\delta)$$

The estimation method that implements this is CMLE

#### 2.4.4 Case 4: $\gamma < 0, \delta > 0$ — empty regions, coherency through conditioning

[Figure 5 approximately here.]

For this case also, coherency is achieved by conditioning to lie outside the “empty” region of Figure 5. The conditioning probability is:

$$1 - Prob(0 < \epsilon_1 + x'_1\beta_1 < -\gamma, \delta < \epsilon_2 + x'_2\beta_2 < 0)$$

and the appropriate estimation method is CMLE.

#### 2.4.5 Can Overlapping Regions Incoherency be Overcome through LDV Redefinition?

We have shown that in general, in the absence of coherency conditions, there will be *overlaps* and/or *gaps* in the domain of  $(\epsilon_1 + x'_1\beta_1, \epsilon_2 + x'_2\beta_2)$ . At this point, a researcher might be tempted to propose that the incoherency cases with overlapping regions (Cases 1 and 2 above) may be overcome by redefining one of the two limited dependent variables to their complement. According to this reasoning, since the incoherency is caused in these cases because  $\gamma$  and  $\delta$  are of the same sign, and since changing  $y_2$ , say, to its complement  $y_2^N \equiv (1 - y_2)$  would result in  $\delta^N \equiv -\delta$ , then coherency would be achieved since then  $\gamma \cdot \delta^N < 0$ .

Such reasoning would be incorrect, however. We analyze here this idea and show that such a redefinition would *maintain* the overlapping-region incoherency. This is because the  $y_2^N \equiv (1 - y_2)$  redefinition would also switch the sign of  $\gamma$  and hence  $\gamma^N \cdot \delta^N > 0$  just as  $\gamma \cdot \delta > 0$ .

Let us return to the bivariate binomial probit (7) and (8). Suppose we have incoherency because we believe  $\gamma > 0$  (in our application below translating to binding finance constraints expected to raise the chance of innovation  $I$ ) and that  $\delta > 0$  (innovative firms face a higher chance that the banks will refuse them a loan). So  $\gamma \cdot \delta > 0$ . This is Case 1 analyzed in subsection 2.4.1 as represented by Figure 2, and corresponding to the constraints on the unobservables:

$$(a^1, a^2)' < (\epsilon^1, \epsilon^2)' < (b^1, b^2)'$$

such that:

$I$	$F$	$a^1$	$b^1$	$a^2$	$b^2$	Shading	Region
1	1	$-x'_1\beta_1 - \gamma$	$\infty$	$-x'_2\beta_2 - \delta$	$\infty$	horizontal	R1
1	0	$-x'_1\beta_1$	$\infty$	$-\infty$	$-x'_2\beta_2 - \delta$	////////	R2
0	1	$-\infty$	$-x'_1\beta_1 - \gamma$	$-x'_2\beta_2$	$\infty$	\\\\\\\\	R3
0	0	$-\infty$	$-x'_1\beta_1$	$-\infty$	$-x'_2\beta_2$	vertical	R4

Now consider the transformed model with  $NF$  **instead of**  $F$ . This transformation still gives an overlapping region in the transformed variables, and hence corresponds to an incoherent model. To see this, proceed as follows:

In terms of the two latent variables  $I^*$  and  $NF^* = -F^*$  and the observed binary indicators  $I$  and  $NF = 1 - F$ , and suppressing the observation index:

$$I = \begin{cases} 1 & \text{if } I^* \equiv x'_1\beta_1 + \gamma^N NF + \epsilon_1 > 0 \\ 0 & \text{if } I^* \equiv x'_1\beta_1 + \gamma^N NF + \epsilon_1 \leq 0 \end{cases} \quad (9)$$

$$NF = \begin{cases} 1 & \text{if } NF^* \equiv x'_2\beta_2^N + \delta^N I + \epsilon_2^N > 0 \\ 0 & \text{if } NF^* \equiv x'_2\beta_2^N + \delta^N I + \epsilon_2^N \leq 0 \end{cases} \quad (10)$$

Given this transformation, we expect that  $\gamma^N < 0$  (high  $NF$  means not very binding constraints so cause dampening of  $I$ ) and that  $\delta^N < 0$  (firms who have high  $I$  i.e., innovate, raise the chance the banks will refuse them a loan so low  $NF$ ). So  $\gamma^N \cdot \delta^N > 0$ . See Figure 6.

[Figure 6 approximately here.]

For a typical  $i$  observation, the probability  $Prob(y_{1i}, y_{2i} | X, \theta)$  is characterized by the constraints on the unobservables:

$$(a^1, a^2)' < (\epsilon_1, \epsilon_2^N)' < (b^1, b^2)'$$

through the configuration:

$I$	$NF$	$a^1$	$b^1$	$a^2$	$b^2$	Shading	Region
1	0	$-x'_1\beta_1$	$\infty$	$-\infty$	$-x_2'^N\beta_2 - \delta^N$	horizontal	R1
1	1	$-x'_1\beta_1 - \gamma^N$	$\infty$	$-x_2'^N\beta_2 - \delta^N$	$\infty$	////////	R2
0	0	$-\infty$	$-x'_1\beta_1$	$-\infty$	$-x_2'^N\beta_2$	\\\\\\\\	R3
0	1	$-\infty$	$-x'_1\beta_1 - \gamma^N$	$-x_2'^N\beta_2$	$\infty$	vertical	R4

## 2.5 Efficient Estimation through Conditional Maximum Likelihood for Empty Region Incoherency

The optimal parametric estimation approach for the models with empty region incoherency (Cases 3 and 4 above) will be *conditional maximum likelihood (CMLE)*, employing the appropriate likelihood contributions that characterize correctly the

necessary conditioning through truncation that ensures that the LDVs stay out of the empty region of incoherency. For example, assuming independence across observations  $\{it\}$ , the likelihood contribution in Case 3 will be:

$$l_{it} = \frac{Prob(\epsilon_1, \epsilon_2 : I = 1(I^* > 0) \ \& \ F = 1(F^* > 0))}{(1 - Prob(-\gamma < \epsilon_1 + x'_1\beta_1 < 0, 0 < \epsilon_2 + x'_2\beta_2 < -\delta))}$$

while for Case 4:

$$l_{it} = \frac{Prob(\epsilon_1, \epsilon_2 : I = 1(I^* > 0) \ \& \ F = 1(F^* > 0))}{(1 - Prob(0 < \epsilon_1 + x'_1\beta_1 < -\gamma, \delta < \epsilon_2 + x'_2\beta_2 < 0))}$$

These likelihood contributions make it clear why approaches that ignore the coherency issue are inconsistent in general: the inconsistency would arise because the conditioning probability expressions in the denominator are functions of the underlying parameters and data, and hence affect critically the evaluation of the correct likelihood function.

It is important to remember that fact that the likelihood contributions depend on denominator probabilities characterizing the support of the underlying truncated distributions that are also functions of parameters and data does *not* make the CMLE estimation problem irregular. The earliest example where such likelihood problems were studied formally is [Amemiya, 1973] for models of censoring and truncation. The uniform consistency, asymptotic normality, and efficiency of the CMLE estimators for the empty region incoherency Cases 3 and 4 can be established using methods in [Amemiya, 1973] and in works that followed.

## 2.6 Estimation for Overlap Region Incoherency

We note that Cases 1 and 2 (with same sign of the interaction coefficients  $\gamma$  and  $\delta$ ) may be handled in an analogous fashion *provided it is assumed first* that the Data Generating Process (DGP) that overcomes the overlapping-regions incoherency is one where  $(\epsilon_{1i}, \epsilon_{2i})$  are drawn from an unrestricted bivariate normal distribution and then any draws falling into the overlap region are rejected. The analogous CMLE approach for handling overlap regions incoherency is presented here *for completeness only* because (a) on a priori grounds the  $\gamma$  and  $\delta$  of our Financing Constraints/Firm Innovation application are of opposite sign, a theoretical feature that is confirmed by our empirical findings; and (b) CMLE is arguably much less unambiguous and clear-cut to apply it to overlap regions incoherency compared to the implied truncation in the cases of empty regions incoherency.

To find the correct likelihood contributions in Cases 1 and 2, first define:

$$\begin{aligned} p_{11}^* &\equiv Prob(I^* > 0, F^* > 0) \\ p_{10}^* &\equiv Prob(I^* > 0, F^* \leq 0) \\ p_{01}^* &\equiv Prob(I^* \leq 0, F^* > 0) \\ p_{00}^* &\equiv Prob(I^* \leq 0, F^* \leq 0) \end{aligned}$$

Then, note that:

$$p_{11}^* + p_{10}^* + p_{01}^* + p_{00}^* = S > 1$$

where  $S - 1 \equiv d$ , the probability of the overlap region. In Case 1, the overlap occurs between regions (1, 1) and (0, 0), while for Case 2 between regions (1, 0) and (0, 1). Consequently, assuming an Accept/Reject DGP out of the overlap region, the likelihood contribution for observation  $\{it\}$  for Case 1 is:

$$l_{it} = \begin{cases} p_{11} \equiv \text{Prob}(I = 1 \& F = 1) = (p_{11}^* - d)/(2 - S) \\ p_{10} \equiv \text{Prob}(I = 1 \& F = 0) = p_{10}^*/(2 - S) \\ p_{01} \equiv \text{Prob}(I = 0 \& F = 1) = p_{01}^*/(2 - S) \\ p_{00} \equiv \text{Prob}(I = 0 \& F = 0) = (p_{00}^* - d)/(2 - S) \end{cases}$$

while for Case 2:

$$l_{it} = \begin{cases} p_{11} \equiv \text{Prob}(I = 1 \& F = 1) = p_{11}^*/(2 - S) \\ p_{10} \equiv \text{Prob}(I = 1 \& F = 0) = (p_{10}^* - d)/(2 - S) \\ p_{01} \equiv \text{Prob}(I = 0 \& F = 1) = (p_{01}^* - d)/(2 - S) \\ p_{00} \equiv \text{Prob}(I = 0 \& F = 0) = p_{00}^*/(2 - S) \end{cases}$$

As mentioned at the outset, this Accept/Reject DGP approach for overcoming *overlapping regions incoherency* is logically less clear-cut and more ambiguous. One may consider instead alternative schemes for the overlapping regions case which are particularly suitable for specific economic applications — see, for example, the game-theoretic models of entry analyzed by [Tamer, 2003] and the works cited above that followed him.

## 2.7 Establishing the Coherency of Panel LDV Models with Intertemporal Endogeneities using DGP Approach

Extending the analysis to a panel data set, we explain how the probability of a pair  $(S_{it}, E_{it})$  in subsection 2.2 and a pair  $(y_{1it}, y_{2it})$  in subsection 2.1, can be represented in terms of the linear inequality:

$$(a_1, a_2)' < (\epsilon_1, \epsilon_2)' < (b_1, b_2)'$$

where the error vector has a flexible autocorrelation structure. For example, one-factor random effect assumptions will imply an equicorrelated block structure on  $\Sigma_\epsilon$ , while our most general assumption of one-factor random effects *combined with* an AR(1) process for each error implies that  $\Sigma_\epsilon$  combines equicorrelated and Toeplitz-matrix features. Consequently, the approach incorporates fully (a) the contemporaneous correlations in  $\epsilon_{it}$ , (b) the one-factor plus AR(1) serial correlations in  $\epsilon_i$ , and (c) the dependency of  $S_{it}$  on  $E_{it}$  and vice versa. The coherency issue expands naturally to the panel sequence of data, by thinking of each (correlated) time-period for a given individual  $i$  as a **distinct probit equation** and then dealing with the independent cross-section of equations across individuals. Details of the analysis can be found in [Hajivassiliou, 2007].

### 2.7.1 Dynamic Model 1: Univariate Panel Data Probit with State Dependence

Our hypothetical DGP method presented in Subsection 2.3.2 for establishing coherency is now applied to the canonical panel data Probit model with state dependence, first analyzed by [Heckman, 1981a]. The model is defined by:

$$\begin{aligned}
y_{iT} &= \mathbf{1}(\lambda y_{i,T-1} + x_{iT}\beta + \epsilon_{iT} > 0) \\
y_{i,T-1} &= \mathbf{1}(\lambda y_{i,T-2} + x_{i,T-1}\beta + \epsilon_{i,T-1} > 0) \\
&\vdots \\
y_{i2} &= \mathbf{1}(\lambda y_{i1} + x_{i2}\beta + \epsilon_{i2} > 0) \\
y_{i1} &= \mathbf{1}(x_{i1}\xi_1 + \dots + x_{iT}\xi_T + u_{i1} > 0)
\end{aligned}$$

The equation for  $t = 1$  is a generalization of the [Barghava and Sargan, 1982] approach. Let  $\Sigma \equiv VCov(\epsilon_{iT}, \dots, \epsilon_{i1}, u_{i1})$ . Imposing one-factor random effect assumptions will imply an equicorrelated block structure on the top left  $T - 1 \times T - 1$  block of  $\Sigma$ , while more general assumptions of one-factor random effects *combined with* an AR(1) or ARMA(p,q) processes for each  $\epsilon$  error implies that  $\Sigma$  combines equicorrelated and Toeplitz-matrix parts. The last row and last column of  $\Sigma$  giving the variance of  $u_{1i}$  and its covariances with all  $\epsilon_{it}$  allow the flexibility stipulated by [Heckman, 1981b].

Define the Cholesky lower triangular times upper triangular factorization of  $\Sigma = CC'$ . Given the assumed normality, the error vector can be written:

$$(\epsilon'_i, u_{1i})' = C\nu_i \quad \nu_i \sim N(0_T, I_T)$$

**Theorem 1:** *The Univariate PD Probit Model with State Dependence defined above is coherent.*

**Proof:** (using the DGP approach)

Let us begin with the simplified case of the initial condition being exogenous:

$$y_{iT} = \mathbf{1}(\lambda y_{i,T-1} + x'_{iT}\beta + \epsilon_{iT} > 0) \tag{11}$$

$$y_{i,T-1} = \mathbf{1}(\lambda y_{i,T-2} + x'_{i,T-1}\beta + \epsilon_{i,T-1} > 0) \tag{12}$$

$$\vdots \tag{13}$$

$$y_{i2} = \mathbf{1}(\lambda y_{i1} + x'_{i2}\beta + \epsilon_{i2} > 0) \tag{14}$$

$$y_{i1} = \text{exogenous} \tag{15}$$

Suppose first the  $\epsilon_{it}$  has the one-factor (equicorrelated) error components structure  $\epsilon_{it} = \alpha_i + \nu_{it}$ . Conditional on  $\alpha_i$ , these  $T - 1$  equations are independent (since they only depend on the i.i.d.  $\nu_{it}$ s). Hence draw an  $\alpha_i$  and an independent  $\nu_{i2}$ . Then

use the exogenous  $y_{i1}$  outcome to generate  $y_{i2}$ . This completes equation 14 which allows to move sequentially to generating  $y_{i3}$ , then  $y_{i4}$ , etc. until  $y_{iT}$  is generated. This establishes the coherency of the model.

Now allow for a general  $\Sigma \equiv VCov(\epsilon_{iT}, \dots, \epsilon_{i2}) = CC'$ . Given that we assume Gaussianity and dropping the  $i$  index, we obtain:

$$\begin{aligned} y_T &= \mathbf{1}(\lambda y_{T-1} + x_T\beta + c_{T1}\nu_1 + c_{T2}\nu_2 + \dots + c_{T,T-1}\nu_{T-1} + c_{TT}\nu_T > 0) \\ y_{T-1} &= \mathbf{1}(\lambda y_{T-2} + x_{T-1}\beta + c_{T-1,1}\nu_1 + c_{T-1,2}\nu_2 + \dots + c_{T-1,T-1}\nu_{T-1} > 0) \\ &\vdots \\ y_2 &= \mathbf{1}(\lambda y_1 + x_2\beta + c_{22}\nu_2 + c_{21}\nu_1 > 0) \\ y_1 &= \text{exogenous} \end{aligned}$$

Given a random draw of  $\nu_{i1}, \dots, \nu_{iT}$ , an unambiguous rule gives sequentially  $y_{i1} \rightarrow y_{i2} \rightarrow \dots y_{iT-1} \rightarrow y_{iT}$ . Hence, the above defines a recursive DGP which establishes the coherency of the model.

Finally, consider the more general case when  $y_{i1}$  cannot be assumed as exogenous. We then supplement the system with an initial condition equation:

$$y_{i1} = \mathbf{1}(x_{i1}\xi_1 + \dots + x_{iT}\xi_T + u_{i1}) > 0 \quad (16)$$

The following remarks are in order: First note that (16) is a generalization of the [Barghava and Sargan, 1982] approach. Second, one-factor random effect assumptions will imply an equicorrelated block structure on the top left  $T - 1 \times T - 1$  block of  $\Sigma$ , while more general assumptions of one-factor random effects *combined with* an AR(1) or ARMA(p,q) processes for each  $\epsilon$  error implies that  $\Sigma$  combines equicorrelated and Toeplitz-matrix parts. The last row and last column of  $\Sigma$  giving the variance of  $u_{i1}$  and its covariances with all  $\epsilon_{it}$  allow the flexibility stipulated by [Heckman, 1981a]. The only modification now necessary is to change the initial condition equation to:

$$y_{i1} = \mathbf{1}(x_{i1}\xi_1 + \dots + x_{iT}\xi_T + c_{11}\nu_{i1} > 0)$$

This recursive representation again establishes the coherency of the model: given a random draw of  $\nu_{i1}, \dots, \nu_{iT}$ , an unambiguous DGP rule can be defined to establish sequentially  $y_{i1} \rightarrow y_{i2} \rightarrow \dots y_{iT-1} \rightarrow y_{iT}$ .

### 2.7.2 Dynamic Model 2: Bivariate Panel Data Probit with State Dependence

*Parameter mnemonics:*

- Exogenous variable coefficients:  $\beta, \theta$
- Simultaneous interaction terms:  $\gamma, \delta$



- Own state dependence:  $\lambda_y, \lambda_w$
- Cross state dependence:  $\zeta_w, \zeta_y$

$$\begin{aligned} y_{it} &= \mathbf{1}(x'_{it}\beta + \lambda_y y_{i,t-1} + \gamma w_{it} + \zeta_w w_{i,t-1} + \epsilon_{it} > 0) \\ w_{it} &= \mathbf{1}(z'_{it}\theta + \lambda_w w_{i,t-1} + \delta y_{it} + \zeta_y y_{i,t-1} + u_{it} > 0) \end{aligned}$$

$$\begin{aligned} y_{iT} &= \mathbf{1}(x'_{iT}\beta + \lambda_y y_{i,T-1} + \gamma w_{iT} + \zeta_w w_{i,T-1} + \epsilon_{iT} > 0) \\ w_{iT} &= \mathbf{1}(z'_{iT}\theta + \lambda_w w_{i,T-1} + \delta y_{iT} + \zeta_y y_{i,T-1} + u_{iT} > 0) \end{aligned}$$

$$\begin{aligned} y_{i,T-1} &= \mathbf{1}(x'_{i,T-1}\beta + \lambda_y y_{i,T-2} + \gamma w_{i,T-1} + \zeta_w w_{i,T-2} + \epsilon_{i,T-1} > 0) \\ w_{i,T-1} &= \mathbf{1}(z'_{i,T-1}\theta + \lambda_w w_{i,T-2} + \delta y_{i,T-1} + \zeta_y y_{i,T-2} + u_{i,T-1} > 0) \end{aligned}$$

$\vdots$

$$\begin{aligned} y_{i2} &= \mathbf{1}(x'_{i2}\beta + \lambda_y y_{i1} + \gamma w_{i2} + \zeta_w w_{i1} + \epsilon_{i2} > 0) \\ w_{i2} &= \mathbf{1}(z'_{i2}\theta + \lambda_w w_{i1} + \delta y_{i2} + \zeta_y y_{i1} + u_{i2} > 0) \end{aligned}$$

$y_{i1}$

$w_{i1}$

**Lemma 1:** *Without any restrictions on the  $\gamma, \delta$  parameters or the distribution of  $(\epsilon, u)$ , the General Bivariate PD Probit Model with State Dependence above is not coherent.*

**Proof:**

$$\begin{aligned} y_{it} &= \mathbf{1}(x'_{it}\beta + \lambda_y y_{i,t-1} + \gamma w_{it} + \zeta_w w_{i,t-1} + \epsilon_{it} > 0) \\ w_{it} &= \mathbf{1}(z'_{it}\theta + \lambda_w w_{i,t-1} + \delta y_{it} + \zeta_y y_{i,t-1} + u_{it} > 0) \end{aligned}$$

Given  $y, w$  from period  $t-1$ , the  $\lambda$  and  $\zeta$  terms are determined on the latent variable terms for period  $t$  (defining the event arguments of the indicator functions).

Together with unrestricted values of the random shocks and the exogenous variables of period  $t$ , everything in the event conditions is determined, except the simultaneous interaction terms  $\gamma, \delta$ .

- But since the interaction terms appear both as conditioning variables on the RHS as well as dependent variable dummies on the LHS, they cannot be determined unambiguously. Hence, no complete DGP can be defined from  $\epsilon, u$  to  $y, w$ .

**Theorem 2:** *The General Bivariate PD Probit Model with State Dependence above is coherent without any restrictions on the  $\lambda, \zeta$  state dependence parameters or the distribution of  $(\epsilon, u)$ , if the simultaneous interaction terms satisfy  $\gamma \cdot \delta = 0$ , i.e., the model is triangular.*

**Proof:** Assume that  $\gamma \cdot \delta = 0$  because  $\gamma = 0$ .

$$\begin{aligned} y_{it} &= \mathbf{1}(x'_{it}\beta + \lambda_y y_{i,t-1} + \zeta_w w_{i,t-1} + \epsilon_{it} > 0) \\ w_{it} &= \mathbf{1}(z'_{it}\theta + \lambda_w w_{i,t-1} + \delta y_{it} + \zeta_y y_{i,t-1} + u_{it} > 0) \end{aligned}$$

Given  $y, w$  from period  $t-1$ , the  $\lambda$  and  $\zeta$  terms are determined on the latent variable terms for period  $t$  (defining the event arguments of the indicator functions). Together with unrestricted values of the random shocks and the exogenous variables of period  $t$ , everything in the event condition of the  $y_t$  is determined, since there no simultaneous interaction term is present on the RHS (as  $\gamma = 0$ ).

Entering the  $y_t$  value in the interaction term on the RHS of the  $w_t$  equation, everything in its event condition is now determined, which fixes  $w_t$ .

Hence, a complete DGP can be defined sequentially from the errors to the observables:  $y_{i1}, w_{i1} \rightarrow y_{i2}, w_{i2} \rightarrow \dots y_{iT-1}, w_{iT-1} \rightarrow y_{iT}, w_{iT}$ .

The proof for the  $\delta = 0$  case is perfectly symmetric and will not be repeated.

**Theorem 3:** *The General Bivariate PD Probit Model with State Dependence above is coherent without any restrictions on the  $\lambda, \zeta$  state dependence parameters, if:*

- (i) *the simultaneous interaction terms are of opposite signs, i.e.,  $\gamma \cdot \delta < 0$  and*
- (ii) *the distribution of  $(\epsilon, u)$  satisfies  $F(\epsilon_t, u_t | \epsilon_{-t}, u_{-t}) = F(\epsilon_t, u_t | \epsilon_{<t}, u_{<t})$  and the error r.v.s  $(\epsilon, u)$  are restricted on rectangular regions that are determined recursively.*

• **Proof:** Assume that  $\gamma \cdot \delta < 0$  because  $\gamma < 0, \delta > 0$ .

$$\begin{aligned} y_{it} &= \mathbf{1}(x'_{it}\beta + \lambda_y y_{i,t-1} + \gamma w_{it} + \zeta_w w_{i,t-1} + \epsilon_{it} > 0) \\ w_{it} &= \mathbf{1}(z'_{it}\theta + \lambda_w w_{i,t-1} + \delta y_{it} + \zeta_y y_{i,t-1} + u_{it} > 0) \end{aligned}$$

Given  $y, w$  from period  $t-1$ , the  $\lambda$  and  $\zeta$  terms are determined on the latent variable terms for period  $t$  (defining the event arguments of the indicator functions).

Given the exogenous variables of period  $t$ , the event conditions of  $y_t, w_t$  are determined except (a) the interaction terms  $\gamma, \delta$  and (b) the error terms.

In the absence of condition (ii), the model would exhibit “empty region incoherency” as defined above. Employing the graphical approach of the Static Bivariate Probit above, defines the necessary rectangular exclusion region (drawn white) for the support of the truncated Gaussian:

$$\begin{aligned} 0 &< \epsilon_{it} + x'_{it}\beta_t + \lambda_y y_{i,t-1} + \zeta_w w_{i,t-1} < -\gamma \\ \delta &< u_{it} + z'_{it}\theta + \lambda_w w_{i,t-1} + \zeta_y y_{i,t-1} < 0 \end{aligned}$$

Based on the underlying uniform rv's drawn at the start of the DGP, the truncated Gaussian  $\epsilon, u$  are drawn to satisfy the identifying rectangle restrictions using the probability integral transform method defined in [Hajivassiliou, 2008].

Hence, the model under conditions (i) and (ii) is coherent, since a complete DGP could be defined sequentially from the errors to the observables:  $y_{i1}, w_{i1} \rightarrow y_{i2}, w_{i2} \rightarrow \dots y_{iT-1}, w_{iT-1} \rightarrow y_{iT}, w_{iT}$ .

The proof for  $\gamma \cdot \delta < 0$  because  $\gamma > 0, \delta < 0$  is exactly symmetric and will not be repeated.

### 2.7.3 Extensions to Bivariate Multinomial Ordered Probit

[Hajivassiliou, 2007] discusses how to extend the analysis to the case of two simultaneous (bivariate) *ordered probit equations with multiple regions*. We refer the interested reader to that study.

### 3 Monte-Carlo Evidence on the Performance of CMLE

#### 3.1 Overview

As we showed in Subsection 2.4, we obtain a coherent non-recursive model with interaction dummies included on both sides, provided we believe the feedback terms have opposite signs on the two sides. Note that it is sufficient to consider only the  $\gamma \geq 0$ ,  $\delta \leq 0$  case, since the reverse can always be subsumed by redefining both dependent binary variables to their complements  $y'_{it} \equiv (1 - y_{it})$ .

We performed extensive Monte Carlo experiments, designed to illustrate the consequences of adopting existing and novel estimation strategies for the problem of this paper. The experiments confirm that the CMLE approach under sign restrictions derived above provides reliable, consistent and efficient estimates of the underlying parameters including the two interaction terms. In contrast, the existing traditional approaches (unrestricted MLE ignoring possible incoherency and MLE that incorrectly assumes recursivity of the system) give seriously misleading and inconsistent results. The interested reader is referred to the online companion paper [Hajivassiliou, 2008] for an extensive presentation of the Monte Carlos summarized here and detailed analysis and findings.<sup>10</sup> We give a brief summary of the key findings:

- The Truncated CMLE proposed in this paper performs very satisfactorily, being the only consistent estimator for the reverse feedback cases, and with only small sacrifices in terms of efficiency in the recursive DGPs when it is not strictly necessary.
- The linear probability estimators perform very badly in all cases with endogenous interaction terms, thus suggesting that the inherent non-linearities of the bivariate probits cannot be safely ignored.

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<sup>10</sup>The cited study considered *nine* estimation approaches:

- (a) Incorrectly forcing the old coherency condition to hold, i.e., assuming recursivity when in fact both feedback terms are present (estimators **E-TRWN**=assuming  $\delta = 0$  and **E-TRNW**=assuming  $\gamma = 0$ );
- (b) unrestricted likelihood estimation, which ignores the resulting incoherency due to the empty or overlap region(s) (estimator **E-INCO**);
- (c) restricted likelihood estimation conditioning on the data lying outside the empty region(s) of incoherency (estimators **E-SQPM**=assuming  $(\gamma \geq 0, \delta \leq 0)$  and **E-SQMP**=assuming  $(\gamma \leq 0, \delta \geq 0)$ );
- (d) restricted likelihood estimation conditioning on the data lying outside the overlap region(s) of incoherency (estimators **E-SQPM**=assuming  $(\gamma \geq 0, \delta \geq 0)$  and **E-SQMP**=assuming  $(\gamma \leq 0, \delta \leq 0)$ ); and
- (e) LPOLS: (linear probability) ordinary least squares estimation of each binary probit equation ignoring the possible endogeneity of the interaction terms; and LP2SLS: applying two-stage least squares recognizing that the two interaction terms on the RHS of each probit equation can be endogenous.

In the cited study, six “true” models were generated, depending on whether interaction terms were allowed on one or both sides. In each case, the nine estimators **E-TRWN**, **E-TRNW**, **E-INCO**, **E-SQPM**, **E-SQMP**, **E-SQPP**, **E-SQMM**, **LPOLS**, and **LP2SLS** were calculated.

Apart from confirming the excellent performance of the Truncated MLE approach adopted here, the study also confirmed that application of linear probability methods to the bivariate binary probit model typically leads to very unreliable findings, even if such methods attempt to take account of the endogeneity of the direct and reverse interaction effects.

- Truncated CMLE also works well for the overlap region incoherency cases.
- Unrestricted likelihood estimation ignoring the resulting incoherency due to the empty or overlap region(s) is by far the worst performing estimator, dominated even by equation by equation univariate estimators which estimate the two equations separately while ignoring the other side of the model.

We then proceed with the design and implementation of the experiments.

### 3.2 Design of Monte-Carlo Experiments

The experiments were designed to illustrate the importance of coherency on the following *nine* estimation approaches:

(Est1&Est2) likelihood estimation that incorrectly forces the old coherency condition to hold, i.e., assuming recursivity when in fact both feedback terms are present (estimators **E-TRWN**=assuming  $\delta = 0$  and **E-TRNW**=assuming  $\gamma = 0$ );

(Est3) unrestricted likelihood estimation, which ignores the resulting incoherency due to the empty or overlap region(s) (estimator **E-INCO**);

(Est4&Est5) restricted likelihood estimation conditioning on the data lying outside the empty region(s) of incoherency (estimators **E-SQPM**=assuming  $(\gamma \geq 0, \delta \leq 0)$  and **E-SQMP**=assuming  $(\gamma \leq 0, \delta \geq 0)$ );

(Est6&Est7) restricted likelihood estimation conditioning on the data lying outside the overlap region(s) of incoherency (estimators **E-SQPP**=assuming  $(\gamma \geq 0, \delta \geq 0)$  and **E-SQMM**=assuming  $(\gamma \leq 0, \delta \leq 0)$ ).

(Est8&Est9) LPOLS: (linear probability) ordinary least squares estimation of each binary probit equation ignoring the possible endogeneity of the interaction terms; and LP2SLS: applying two-stage least squares recognizing that the two interaction terms on the RHS of each probit equation can be endogenous.

We generate six “true” models:

- |  |  |
|--|--|
| 1. DGP – TRWN( $\delta = 0$ )                    | 4. DGP – SQMP ( $\gamma \leq 0, \delta \geq 0$ ) |
| 2. DGP – TRNW( $\gamma = 0$ )                    | 5. DGP – SQPP ( $\gamma \geq 0, \delta \geq 0$ ) |
| 3. DGP – SQPM ( $\gamma \geq 0, \delta \leq 0$ ) | 6. DGP – SQMM ( $\gamma \leq 0, \delta \leq 0$ ) |

To simulate data from these six models, it is necessary to devise a methodology for generating standard Gaussian variates truncated to lie outside an interval  $[\underline{\lambda}, \bar{\lambda}]$ . The following algorithm achieves this: Let  $z \sim N(0, 1)$  and define  $\tau \sim z | \{z \notin [\underline{\lambda}, \bar{\lambda}]\}$ . Then  $cdf(z) : F(z) = \Phi(z)$  and

$$cdf(\tau) : F(\tau) = \begin{cases} \frac{\Phi(z)}{1-\Phi(\bar{\lambda})+\Phi(\underline{\lambda})} & \text{if } z < \underline{\lambda}, \\ \frac{\Phi(\underline{\lambda})}{1-\Phi(\bar{\lambda})+\Phi(\underline{\lambda})} & \text{if } \underline{\lambda} < z \leq \bar{\lambda}, \\ \frac{\Phi(z)-\Phi(\bar{\lambda})+\Phi(\underline{\lambda})}{1-\Phi(\bar{\lambda})+\Phi(\underline{\lambda})} & \text{if } z > \bar{\lambda}. \end{cases}$$

The procedure is exact for a univariate  $z$  truncated on  $\{z \notin [\underline{\lambda}, \bar{\lambda}]\}$ , but it will not

work for higher dimensions.<sup>11</sup>

Based on data generated from each of the six DGPs in turn, we calculate the nine estimators E-TRWN, E-TRNW, E-INCO, E-SQPM, E-SQMP, E-SQPP, E-SQMM, LPOLS, and LP2SLS.

The generating equations are:

$$\begin{aligned} ystar1 &= x1[nobs, kx1] * beta1 + gamma * y2 + eps1, & y1 &= 1(ystar1 > 0) \\ ystar2 &= x2[nobs, kx2] * beta2 + delta * y1 + eps2, & y2 &= 1(ystar2 > 0) \end{aligned}$$

where  $x1$  is a  $nobs \times kx1$  matrix and  $x2$  is a  $nobs \times kx2$  matrix.

### 3.2.1 Case DGP-TRWN: $\gamma$ unrestricted, $\delta = 0$

$$\begin{aligned} ystar1 &= x1[nobs, kx1] * beta1 + gamma * y2 + eps1, & y1 &= 1(ystar1 > 0) \\ ystar2 &= x2[nobs, kx2] * beta2 + eps2, & y2 &= 1(ystar2 > 0) \end{aligned}$$

Given the recursivity of the  $\gamma \cdot \delta = 0$  restriction in this case,  $ystar2$  is generated first, which gives  $y2$ . This is then plugged into the RHS of the  $ystar1$  equation thus allowing  $ystar1$  and  $y1$  to be obtained. The symmetric case DGP-TRNW is handled analogously.

### 3.2.2 Case DGP-SQPM: $\gamma \geq 0, \delta \leq 0$

$$0 \leq eps1 + x1 * b1 \leq gamma, -delta \leq eps2 + x2 * beta2 \leq 0 \quad (17)$$

Accept-reject methods are used to generate the data so that these restrictions are satisfied. The symmetric DGP-SQMP case is handled analogously.<sup>12</sup>

### 3.2.3 Case DGP-SQPP: $\gamma \geq 0, \delta \geq 0$

Accept-reject methods are used to generate the data so that these restrictions are satisfied, as well as for the symmetric case DGP-SQMM.

## 3.3 Implementation of the Monte-Carlo Experiments

We performed 32 Monte-Carlo experiments, indexed by MCxyz as follows:

	$\delta$	$\gamma$				$x_{11}$	$x_{12}$	$x_{13}$	$x_{21}$	$x_{22}$	$x_{23}$
x=1	0	0		$\rho_{\epsilon_1, \epsilon_2}$	$z = 1$	const	$\chi^2(1)$	Bernoulli(0.7)	const	$x_{12}$	DoubleExponentialSS
x=2	0.8	0	y=1	0.3	$z = 2$	const	$\chi^2(1)$	Bernoulli(0.9)	const	$x_{12}$	DoubleExponentialSS
x=3	0.8	1	y=2	-0.3	$z = 3$	const	$\chi^2(1)$	Bernoulli(0.7)	const	$x_{12}$	DoubleExponentialLS
x=4	0.8	-1			$z = 4$	const	$\chi^2(1)$	Bernoulli(0.9)	const	$x_{12}$	DoubleExponentialLS

<sup>11</sup> For DGPs with higher dimensions, the leading alternative procedures are Acceptance-Rejection and Gibbs resampling — see [Hajivassiliou and McFadden, 1998] for discussion.

<sup>12</sup> See also [Hajivassiliou, 2008] for an exact algorithm for generating draws from truncated normal distributions restricted to lie on region (17).

where *DoubleExponential* stands for a Double Exponential distribution with mean 0 with asymmetric two sides, *SS* for “small skewness” and *LS* with “large skewness.” Each random data set had 2000 observations and 200 Monte-Carlo replications were generated. The true beta parameters were set at:  $\beta_1 = (0.8, -0.5, -0.3)'$  and  $\beta_2 = (-0.3, 0.7, -0.4)'$ .<sup>13</sup> The four regime probabilities and their row and column sums across the 32 Monte-Carlo experiments that we performed were as follows:

		$Y_2 = 1$		$Y_2 = 0$				
	$Y_1 = 1$	$p_{11}$		$p_{10}$		$p_{1\cdot}$		
	$Y_1 = 0$	$p_{01}$		$p_{00}$		$p_{0\cdot}$		
		$p_{\cdot 1}$		$p_{\cdot 0}$				
Across Experiments	$p_{11}$	$p_{10}$	$p_{01}$	$p_{00}$	$p_{0\cdot}$	$p_{1\cdot}$	$p_{\cdot 1}$	$p_{\cdot 0}$
minimum	0.120	0.141	0.065	0.166	0.266	0.333	0.444	0.382
average	0.318	0.220	0.217	0.245	0.462	0.538	0.535	0.466
maximum	0.552	0.330	0.410	0.307	0.667	0.734	0.618	0.556

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<sup>13</sup>The full tables presenting the detailed Monte-Carlo results in terms of various estimation criteria (root-mean-squared error, absolute bias, absolute median bias, variance, interquartile range, and nine-decile range) can be found in the online companion paper [Hajivassiliou, 2008].

## 4 Empirical Application: Quantifying the Interactions between Financial Constraints and Firm Innovation

A large strand of the theoretical literature shows how investment is affected by informational asymmetries about the quality of the investment to be financed or relating to the behaviour of entrepreneurs. Such imperfections increase the cost of external finance and therefore, firms may be credit constrained. Due to their specificities inducing large informational asymmetries, high risk in terms of probability of failure, unpredictability in R&D returns and poor collateral, innovative firms are more likely to face agency issues and to be financially constrained ([Holmstrom, 1989]). Most of the earlier papers studying the link between firm level financial factors and R&D investment build on the work of [Bond and Meghir, 1994] and [Bond and van Reenen, 2007] on the financing of investment. These papers obtain mixed evidence of such binding constraints on innovation (e.g., [Brown et al., 2009], [Brown et al., 2012], and see [Lerner and Hall, 2010] for a survey). Empirical evidence of the impact of financial constraints on the behaviour of firms is however not easy to obtain, essentially because the notional demand of firms for external finance is not observed directly (see [Hottenrott and Peters, 2012] for a test based on the use of a hypothetical payment received by the firm).

### 4.1 Direct Measures of Innovation and Financial Constraints

Instead, in this paper the existence of constraints is not deduced indirectly through the common arguments above nor identified through changes in financing supply conditions, but is directly measured by employing real data on the encountering of binding financing constraints as reported by firms in surveys by the European Union, as well as in a French survey about the financing of innovation. See Data Appendix for details.

Due to the serious drawbacks of indirect approaches, direct measures of financial difficulties reported by firms can be useful, but very few surveys collect such information. For instance, [Guiso, 1998] uses a direct qualitative measure given by a survey run by Banca d'Italia. In this paper, a firm is characterized as credit constrained "if at the rate of interest prevailing in the loan market, it would like to obtain a larger amount of loans but cannot". Such a precise definition of credit constrained firms is obtained thanks to the survey used where three questions are asked about access to credit (i) whether at the current market interest rate the firm wish a larger amount of credit, (ii) whether the firm would be willing to obtain more credit, (iii) whether the firm has applied for credit but has been turned down by the financial intermediary. Thanks to this information, the probability to be credit constrained is estimated which leads to the finding that low-tech firms are less likely to be financially constrained than high-tech firms. [Hottenrott and Peters, 2012] rely on hypothetical questions in a firm survey where firms are asked to imagine that they receive additional cash exogenously and indicate how they would spend it.

In the survey we use (FIT, Financement de l'Innovation Technologique) firms are asked whether some of their innovative projects were delayed, abandoned or non



started because of (i) unavailability of new financing, (ii) searching and waiting for new financing, (iii) too high cost of finance. We define as financially constrained firms with hampered innovative projects because of one of these three reasons so that our direct indicator of financial constraints takes into account both quantity rationing and higher cost of finance.<sup>14</sup> Given that this is a qualitative self-assessed measure of financial constraint, we checked that it correlates strongly with quantitative balance sheet variables related to the financial health of firms (See Table 3). Innovation in this survey is defined according to the technological innovation in the Oslo Manual (OECD). This is then a qualitative self-assessed and survey-based information which was implemented to overcome some other shortcomings of traditional measures (R&D, patents), see [Mohnen, 2019] for a detailed discussion on these issues.

## 4.2 Empirical Application

Using the econometric machinery developed in Section 4 that allows us to estimate joint binary probit equations with interaction terms on both sides, we apply those methods to the key issue of Being Innovative vs. Binding Financing Constraints interactions.

We take as our starting point the results obtained by [Savignac, 2008] who studied the impact of financial constraints on the decision to innovate by investigating the impact of financial constraints on innovation through a recursive model that did not allow for the probability of a binding finance constraint to depend on whether or not the firm is innovative. The propensity to innovate is explained by the traditional determinants of innovation presented above (firm size and market power, technology push, latent consumer demand).<sup>15</sup> We account for financial constraints through our qualitative indicator reflecting the financial difficulties encountered by firms to conduct their innovative projects.<sup>16</sup>

In sum, we model the probability that a firm decides to be innovative as:<sup>17</sup>

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<sup>14</sup>For summary descriptive statistics, see Table 2 in the Data Appendix.

<sup>15</sup>See Table 4 for variable definitions and Table 5 for descriptive statistics.

<sup>16</sup>For the importance of endogeneity in this setting, see [Mohnen and Roller, 2005] for an example of another study that finds the “paradox” of a positive correlation between financial constraints and innovation.

<sup>17</sup>Main determinants of the propensity for a firm to innovate are known to be its size, its market power and its environment ([Cohen and Levin, 1989]).

The positive correlation between innovation and firm size is largely exposed in the literature (see [Cohen and Klepper, 1996]). Large firms can amortize sunk costs caused by their innovative activities and are able to diversify the risk incurred by innovation by running simultaneously several investment projects at the same time. And finally, large established firms are less likely to be financially constrained as they are able to generate cash-flow and to raise external funds.

Regarding the link between innovation and competition, the Schumpeterian theory argues that market power and innovation are positively correlated whereas Arrow’s theory shows that the gains to innovate are larger in an ex-ante competitive market. [Aghion et al., 2005] try to solve this puzzle and propose an inverted U shape relationship between innovation and competition : in a competitive environment, firms are incited to innovate to gain market power and increase their profits, but when competition becomes hard, the followers can be discouraged to innovate.

Other factors affecting innovative behaviour are driven by the firm environment (technological push, latent consumer demand perceived by the firm). See among others [Crepon et al., 1998] or

$$\begin{aligned} \text{Prob}(\text{Innovate?}) = & \mathbf{f}(\text{Financial Constraints,} \\ & \text{Size, Market Power,} \\ & \text{Technological Opportunities,} \\ & \text{Latent Consumer Demand for New Products, ...}) \end{aligned} \quad (18)$$

We model the investment outcomes as depending on the discrete *outcome* from the other side (on  $y_{2it}$  of equation 2).

To close the system, we now define also the probability of a binding financing constraint, which is assumed to have as an important determinant the (binary) decision of whether or not the firm chooses to be innovative:

$$\begin{aligned} \text{Prob}(\text{Binding Financing Constraint?}) = & \mathbf{f}(\text{Innovation?,} \\ & \text{Size,} \\ & \text{Guarantees or Collateral,} \\ & \text{Profit Margin,} \\ & \text{Banking Debt Structure,} \\ & \text{Internal Financing, ...}) \end{aligned} \quad (19)$$

The key idea modelled by this equation is that prospective lenders will try to assess the creditworthiness of the applicant firm in the face of incomplete information. In particular, they do not know the precise riskiness of assets so they attempt to infer that using observable characteristics of firms. In the face of such uncertainty, it makes sense for lenders to be more cautious granting loans to innovative firms since they present a higher inherent (but not directly observed) risk.

Such a system can be formulated as follows:

$$I_i = \begin{cases} 1 & \text{if } I_i^* \equiv x_i^I \beta^I + \gamma F_i + \epsilon_i^I > 0 \\ 0 & \text{if } I_i^* \equiv x_i^I \beta^I + \gamma F_i + \epsilon_i^I \leq 0 \end{cases} \quad (20)$$

$$F_i = \begin{cases} 1 & \text{if } F_i^* \equiv x_i^F \beta^F + \delta I_i + \epsilon_i^F > 0 \\ 0 & \text{if } F_i^* \equiv x_i^F \beta^F + \delta I_i + \epsilon_i^F \leq 0 \end{cases} \quad (21)$$

The econometric specifications we estimate below belong in three main groups. The first group contains *recursive specifications*, which ignore the possibility that the propensity to innovate may be affected by financial constraints. The second group allows for *reverse interactions*, whereby a firm undertaking actively innovative activities raises significantly the probability of it encountering a binding financing constraint, possibly because potential lenders are particularly wary of granting loans to firms of such type because of the extra riskiness involved. The third group of estimated specifications, investigates *state dependence* in financing and innovation

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[Raymond et al., 2010] for empirical research on the firm level determinants of innovation.

experiences of firms the nature of the available datasets can be exploited to study whether, *ceteris paribus*, past financial distress or innovation failures can affect a firm’s current experiences in these two dimensions.

Though the surveys about innovation we use are not truly longitudinal “panel” sets, the information was collected in multiple biennial waves. We hence restrict our “longitudinal” dataset to two waves in order to limit the reduction of the sample size when merging the waves. Hence, we know whether a particular firm  $i$  has reported binding financing constraints in the past. See the Data Appendix for details about the dataset employed here and the transition tables.

The most general specification that we estimate below, accounting for both reverse and dynamic effects, is:

$$I_{it} = \begin{cases} 1 & \text{if } I_{it}^* \equiv \alpha^I I_{it-1} + x_{it}^{I'} \beta^I + \gamma_0 F_{it} + \gamma_1 F_{it-1} + \epsilon_{it}^I > 0 \\ 0 & \text{if } I_{it}^* \equiv \alpha^I I_{it-1} + x_{it}^{I'} \beta^I + \gamma_0 F_{it} + \gamma_1 F_{it-1} + \epsilon_{it}^I \leq 0 \end{cases} \quad (22)$$

$$F_{it} = \begin{cases} 1 & \text{if } F_{it}^* \equiv \alpha^F F_{it-1} + x_{it}^{F'} \beta^F + \delta_0 I_{it} + \delta_1 I_{it-1} + \epsilon_{it}^F > 0 \\ 0 & \text{if } F_{it}^* \equiv \alpha^F F_{it-1} + x_{it}^{F'} \beta^F + \delta_0 I_{it} + \delta_1 I_{it-1} + \epsilon_{it}^F \leq 0 \end{cases} \quad (23)$$

[Table 1 approximately here.]

### 4.3 Comparative Analysis of Alternative Specifications

Table 1 summarizes succinctly our key empirical results and presents the calculated direct and reverse effects that we obtained. To recapitulate, Models 1 and 2 adopt the existing approach of the past literature of forcing the econometric specification to be triangular with financial constraints allowed to affect the innovation decision, while the financial constraint outcome is assumed independent of being innovative or not. Models 3 and 4 allow the binary interactions on both the Innovation and the Finance sides, proved to be coherent through our Coherency analysis based on prior sign restrictions and estimated through the CMLE approach developed and analyzed in this paper. Model 3 is static with a single cross-section of firms, while Model 4 uses a two-wave, dynamic panel data set.

Apart from the coefficient estimates for the most important exogenous explanatory variables, Table 1 presents also:  $\gamma$ , the coefficient for the financial constraint dummy when entered in the Innovation side;  $\delta$ , the coefficient for the Innovation dummy entered in the Finance side; and for the dynamic Model 4, the coefficients for the lags of the Finance and Innovation dummies. First, note how seriously misleading conclusions were reached by the early strands of the literature, that inappropriately ignored the endogeneity of the Finance dummy: doing so yields a completely counterintuitive  $\gamma$  estimate of +0.55, implying that finance constraints *raise* significantly the likelihood of innovation, confirming [Savnac, 2008]. This positive effect is explained by two sources of bias: a selection bias due to firms not wishing to innovate, which we studied elsewhere and a problem of simultaneity between investment and financing decisions that we tackle below. At the same time, treating the Finance constraint dummy as endogenous gives a range of negative estimates from  $-0.32$  to

-0.56. Since Models 1 and 2 impose a triangular specification, they do not estimate  $\delta$  coefficients for the Innovation dummy in the Finance equation. In contrast, our two CMLE models give statistically very significant  $\delta$  estimates of over 0.6 — as we expected a priori, being innovative raises the probability of facing a binding finance constraint. In the dynamic Model 4, we find very significant state dependence over the two periods of the panel — note the statistical significance of three of the four lagged dummies entered as regressors. Our findings confirm the strong importance of such dynamic terms and establish very significant positive state dependency in our models. Our results show that firms tend to innovate persistently rather than occasionally.

Second, past financial difficulties are positively correlated with current binding financial constraints. As we take into account the experience of a firm concerning innovation, the state dependence of financial constraints seems particularly interesting. Indeed, firms currently implementing innovative projects as well as firms with innovative experience in the past are more likely to find it difficult to finance their current projects.<sup>18</sup>

Third, the probability for a firm to be currently conducting an innovative project is negatively impacted by the current financing difficulties as found in the static regressions but also positively correlated with financing constraints encountered in the past. One possible explanation for this positive correlation could be that financial difficulties mainly impact the beginning of the projects so that innovative projects that were initially hampered by financial difficulties are more likely to be continued when they become more mature. However, additional information on the stage of development of the innovative projects and on their duration would be necessary to further investigate this point. In particular, we are not able to identify whether the firms were continuing in Wave 2 (1997-1999) with projects that were already conducted in Wave 1 (1994-1996).

In order to quantify the importance of our interaction findings about  $\gamma$  and  $\delta$ , we present in Table 6 four estimated probability calculations for each of the two  $I$  and  $F$  sides: (a)  $\widehat{P}_I$ , the average probability of undertaking innovation; (b)  $\widehat{P}_I : F = 0$ , the estimated probability of Innovation given the Finance constraint is not binding; (c)  $\widehat{P}_I : F = 1$ , the estimated probability of Innovation given a binding Finance constraint; and (d)  $\% \Delta \widehat{P}_I : F = 0 \rightarrow 1$ , the percentage change in the estimated probability of changing from  $F = 0$  to  $F = 1$ , while keeping everything else unchanged. For the finance side, the analogous four quantities are:  $\widehat{P}_F$ ,  $\widehat{P}_F : I = 0$ ,  $\widehat{P}_F : I = 1$ , and  $\% \Delta \widehat{P}_F : I = 0 \rightarrow 1$ .

Our estimated probability results are quite striking: for the Innovation equation, we find that *when a firm faces a binding finance constraint, the probability of being*

<sup>18</sup>An important issue discussed frequently in the econometrics literature is the possibility that state dependence may not be an important factor *per se*, but it might appear statistically significant if persistent heterogeneity among individual economic agents is ignored. As [Heckman, 1981a] shows, the two can be identified when a panel data set with more than two waves per individual is available. Since our dynamic sample consists only of two waves, we need to acknowledge the possibility that the strong state dependence we report here may be compounded by unobserved persistent heterogeneity that is not accounted for explicitly.

*innovative falls ceteris paribus by 30% – 40% depending on the version.*<sup>19</sup> Moving to the Finance constraint side, the magnitudes of the results are even more impressive: *all other things equal, a firm being innovative more than doubles the probability of a finance constraint.*<sup>20</sup>

## 5 Conclusions

In this paper we developed two novel methods for establishing coherency conditions in LDV models with endogeneity and flexible temporal and contemporaneous correlations in the unobservables. The first is based on a graphical characterization and the second is based on a hypothetical Monte-Carlo Data Generating Process (DGP) approach. Our novel methods have intuitive interpretations and are easy to implement and generalize. A constructive consequence of the new approaches is that they indicate how to achieve coherency in models traditionally classified as incoherent through the use of prior sign restrictions on model parameters. This allowed us to develop estimation strategies based on CMLE for simultaneous LDV models without imposing recursivity. The proposed CMLE methodology was evaluated through an extensive set of Monte-Carlo experiments. The experiments allowed us also to study the consequences of employing estimators that make overly restrictive coherency assumptions about the DGP. The findings confirmed very substantive improvements by employing the CMLE developed in this paper in terms of estimation Mean-Squared-Error.

Through the CMLE novel approach, we analyzed the existence and impact of financing constraints as a possibly serious obstacle to innovation by firms. We were able to quantify the interaction between financing constraints and a firm’s decision and ability to innovate without forcing the econometric models to be recursive. Direct measures of financing constraints were employed using survey data, which helped us overcome the problems with the traditional approach in the past literature of trying to deduce the existence and impact of financing constraints through the significance of firm wealth variables. We thus obtained direct as well as reverse interaction effects, leading us to conclude that binding financing constraints discourage innovation and at the same time innovative firms are more likely to face binding financing constraints. The empirical results we obtained using CMLE were quite striking: *ceteris paribus*, we found that a firm facing a binding finance constraint is approximately *30% less likely* to undertake innovation, while the probability that a firm encounters a binding finance constraint *more than doubles* if the firm is classified as innovative. In addition, we investigated the importance of state-dependence in dynamic versions of our models and concluded that such issues are critical if direct and reverse interactions between innovation and financing constraints are to be quantified reliably.

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<sup>19</sup>In the erroneous Model 1 that ignores the endogeneity of the Finance constraint, the probability of innovation is predicted to *rise by over 50%* as a result of a binding Finance constraint!

<sup>20</sup>Since Models 1 and 2 do not allow for reverse interactions by excluding the  $I$  dummy from the  $F$  side, they imply  $\% \widehat{\Delta P_F}: I = 0 \rightarrow 1$  equal to zero.

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## 6 Figures

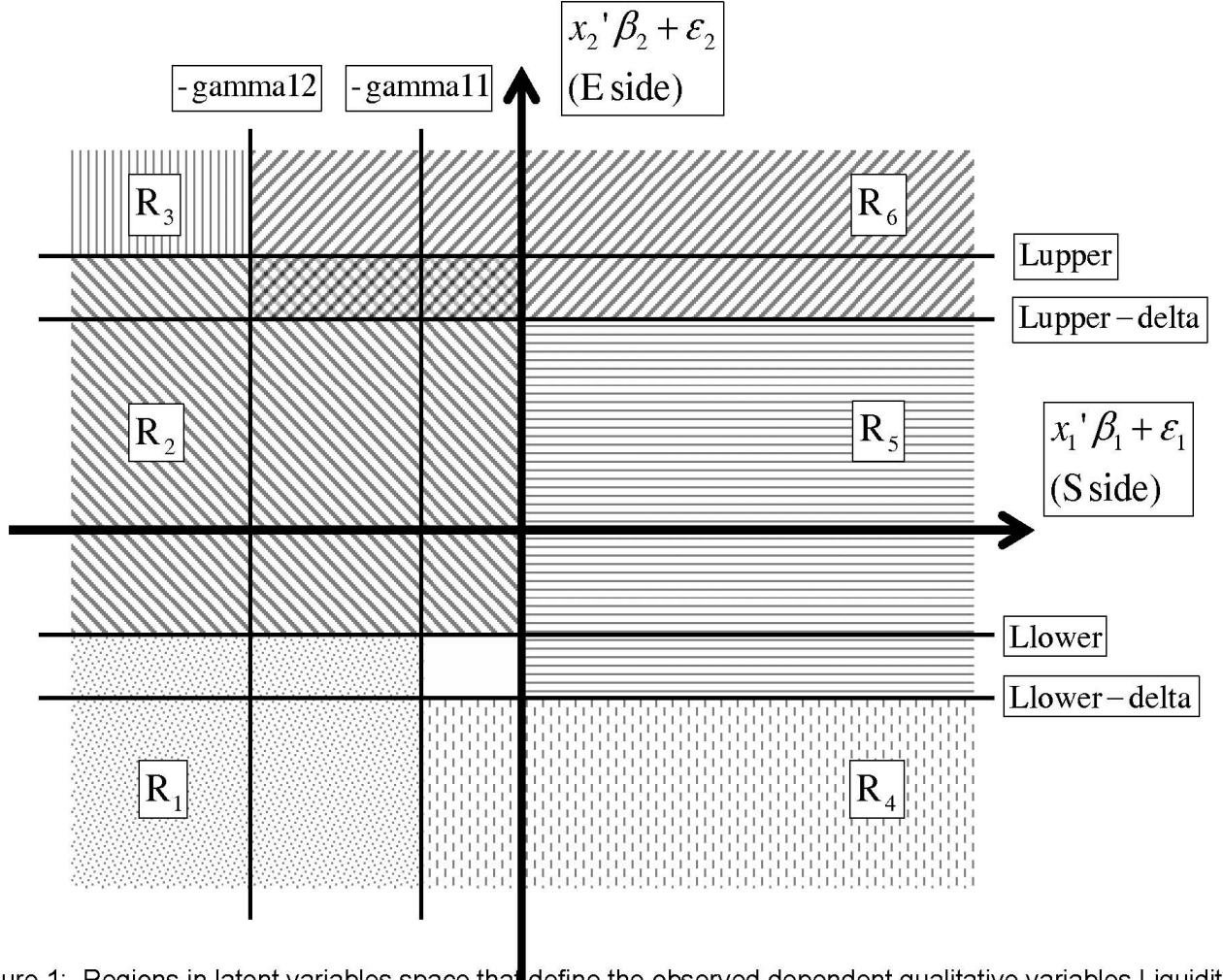


Figure 1: Regions in latent variables space that define the observed dependent qualitative variables Liquidity (binary) and Employment (trinomial) Constraints. Consequently there are six possible regimes. We see two regions of incoherency, one cross-hatched, the other empty. Both regions disappear under the coherency condition that  $\Delta=0$  and/or  $\Gamma_{11}=\Gamma_{12}=0$ .

Coherency of Binary+Trinomial Model

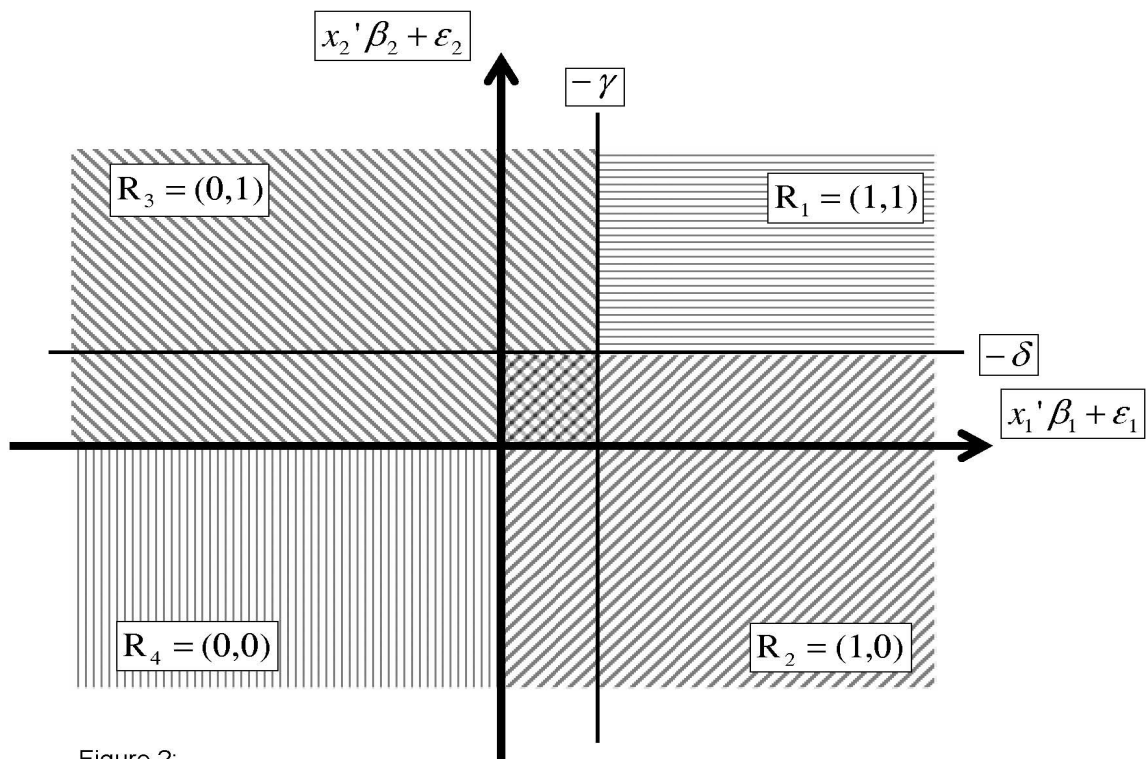


Figure 2:  
Innovation and Finance Constraint bivariate binary model in latent variables space.  
Four implied regimes. Case 1:  $\gamma > 0, \delta > 0$ . Region of incoherency is of the  
overlap type (cross-hatched).

Case 1:  $\gamma > 0, \delta > 0$

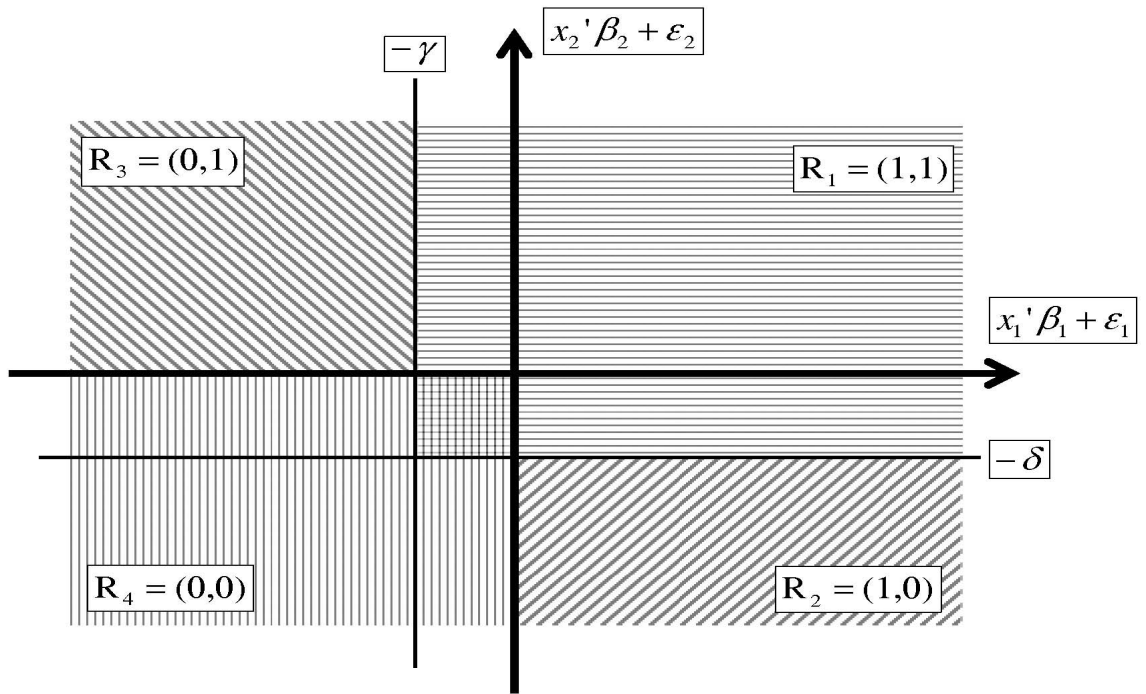


Figure 3:  
Innovation and Finance Constraint bivariate binary model in latent variables space.  
Four implied regimes. Case 2:  $\gamma < 0, \delta < 0$ . Region of incoherency is of the  
overlap type (cross-hatched).

Case 2:  $\gamma < 0, \delta < 0$

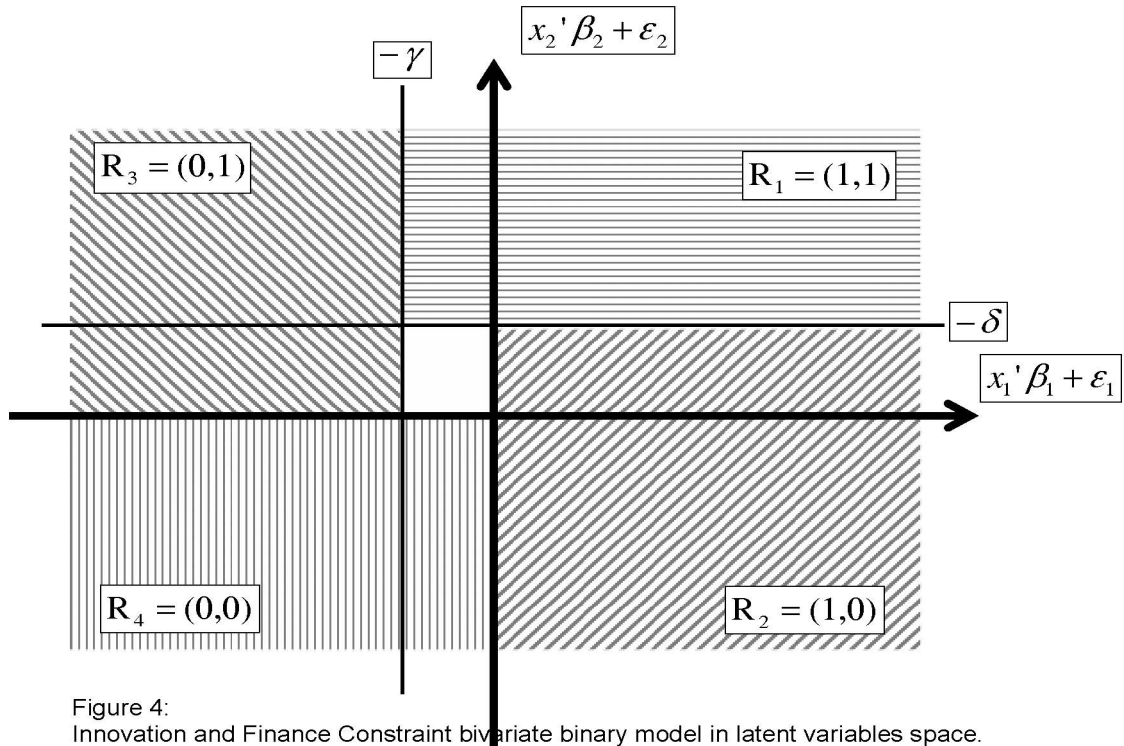


Figure 4:  
Innovation and Finance Constraint bivariate binary model in latent variables space.  
Four implied regimes. Case 3:  $\gamma > 0, \delta < 0$ . Region of incoherency is of the empty  
region type. Hence, Conditional MLE achieves coherency by ruling out the event:  
 $[-\gamma < x_1'\beta_1 + \varepsilon_1 < 0, 0 < x_2'\beta_2 + \varepsilon_2 < -\delta]$

Case 3:  $\gamma > 0, \delta < 0$

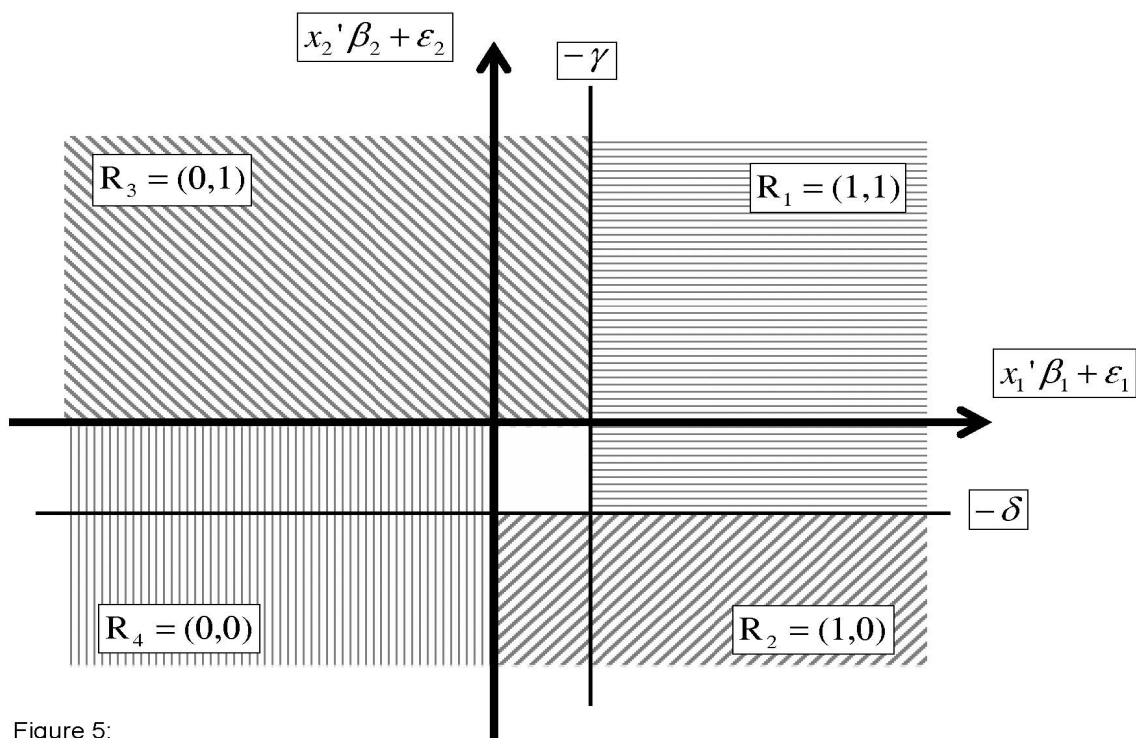


Figure 5:  
Innovation and Finance Constraint bivariate binary model in latent variables space.  
Four implied regimes. Case 4:  $\gamma < 0, \delta > 0$ . Region of incoherency is of the empty region type. Hence, Conditional MLE achieves coherency by ruling out the event:  
 $[0 < \epsilon_1 + x_1\beta_1 < -\gamma, \delta < \epsilon_2 + x_2\beta_2 < 0]$

Case 4:  $\gamma < 0, \delta > 0$

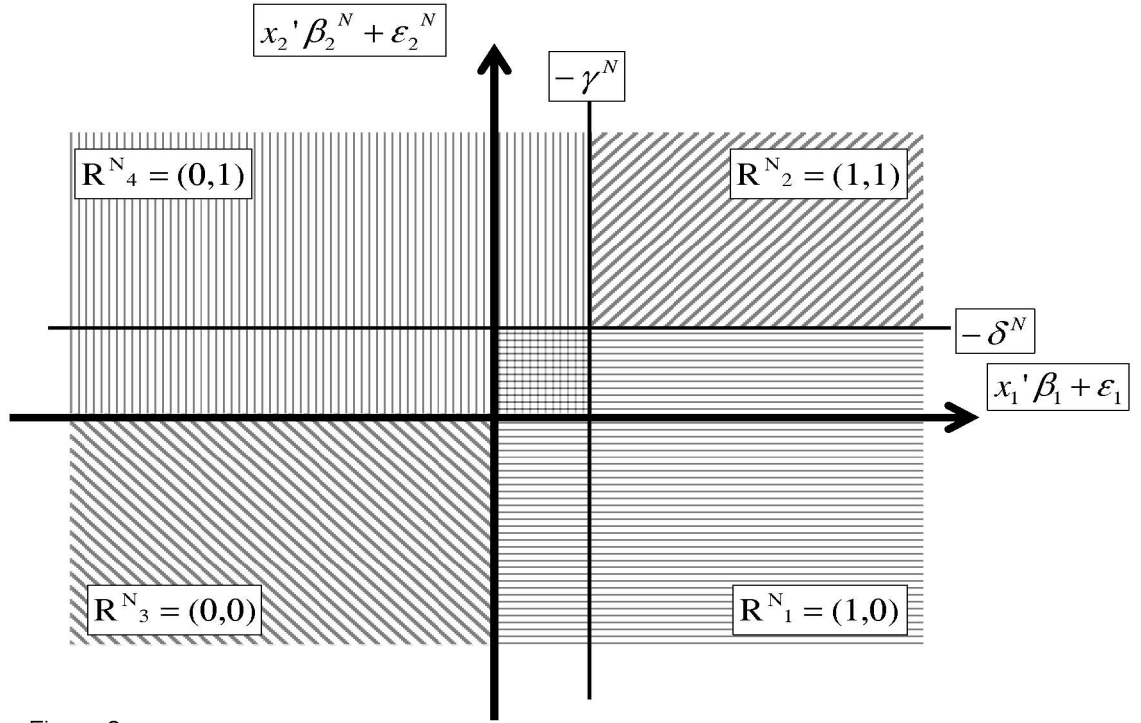


Figure 6:  
 Innovation and Finance Constraint bivariate binary model in latent variables space. Situation identical to Figure 1 (Case 3) but with Innovation negated to its complement (1-Innovation). Both  $\gamma^N$  and  $\delta^N$  switch sign by this redefinition, and hence:  $\gamma^N < 0, \delta^N < 0$ . Consequently, the model remains incoherent with an overlapping (cross-hatched) region.

$$\gamma^N < 0, \delta^N < 0$$

Table 1: Comparative Summary of Empirical Results

All Firms	Model 1 Triangular, Exogenous FC	Model 2 Triangular, Endogenous FC	Model 3 Full Joint Static	Model 4 Full Joint Dynamic
No.of Waves	One	One	One	Two
No.of Firms	1940	1940	1940	1512
INNOVATION EQUATION				
Size	0.33***	0.305	0.183***	0.256***
$\gamma$ (FC dummy)	0.55***	-0.555***	-0.324**	-0.447***
Market Share	-0.01	-0.001***	0.020	0.027
Innov <sub>t-1</sub>	—	—	—	0.829***
FinCon <sub>t-1</sub>	—	—	—	0.301
avg $\widehat{P}_I$	0.418	0.418	0.418	0.543
$\widehat{P}_I: F = 0$	0.384	0.453	0.438	0.554
$\widehat{P}_I: F = 1$	0.601	0.250	0.316	0.377
% $\Delta\widehat{P}_I: F = 0 \rightarrow 1$	56.42	-44.72	-27.93	-31.82
FINANCE EQUATION				
Size	-0.054	-0.002	-0.016	0.035
$\delta$ (Innov dummy)	—	—	0.647***	0.627***
Collateral	0.067	0.030	0.030	0.003
Banking Debt Ratio	0.010***	0.010***	0.015***	0.005
Own Financing Ratio	-0.003**	-0.003***	-0.001**	-0.008***
Profit Margin	-0.007**	-0.008***	-0.002***	-0.007***
Innov <sub>t-1</sub>	—	—	—	0.236**
FinCon <sub>t-1</sub>	—	—	—	0.135***
avg $\widehat{P}_F$	0.160	0.160	0.160	0.060
$\widehat{P}_F: I = 0$	0.160	0.160	0.103	0.029
$\widehat{P}_F: I = 1$	0.160	0.160	0.268	0.102
% $\Delta\widehat{P}_F: I = 0 \rightarrow 1$	0.0	0.0	160.62	252.58
corr(Innov,FinCons)	—	0.572***	0.132**	0.500**
LogLikFunction	-1060-803=-1863	-1853	-1712	-1331 (-1706 imputed)

NOTES:

1. \*\*\*=significant at 1%; \*\*=significant at 5%; \*=significant at 10%.

2. Industry dummies (11) included in both Innovation and Financial Constraint equations.

## 7 Appendix: Data Sources and Constructions

### 7.1 Sources

#### 7.1.1 The FIT survey

The survey “Financement de l’Innovation Technologique” (FIT) was conducted in 2000 by the French Ministry of Industry, in order to obtain statistical information about the financing conditions of innovative projects of manufacturing firms in France.<sup>21</sup> The survey identifies the firms which undertook innovative projects between 1997 and 1999 and gives qualitative information about the financial constraints experienced by firms. 5500 manufacturing companies with 20+ employees were surveyed (excluding agricultural-food and building sectors). The response rate was 85% overall, and 100% among firms with 500+ employees. It is important to note that start-ups and new established firms were not included.

As the Community Innovation Surveys (CIS) compiled by Eurostat, the FIT survey is based upon the technological innovation definition in the Oslo manual (OECD 1997) and is less restrictive than R&D expenditures or patents data.<sup>22</sup>

#### • Definition 1: Innovative firms

A firm is “innovative” if it has introduced or developed a product or process innovation (or was in process of doing so) during the surveyed period. This identification is built on at least one positive answer to the three questions:

1) In 1997, 1998 or 1999, did Your enterprise introduce onto the market any new or significantly improved products for Your enterprise?

2) In 1997, 1998 or 1999, did Your enterprise introduce onto the market any new or significantly improved process for Your enterprise?

3) In 1997, 1998 or 1999, did Your enterprise have projects of new or significantly improved products or processes:

- Which are not yet completed or not yet introduced to the market?
- Which were failures?

#### • Definition 2: Financing constraints

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<sup>21</sup>See [Lhomme, 2002] for details.

<sup>22</sup>The Oslo manual definition was set up to overcome some shortcomings associated with R&D and patents. For instance, innovative activities are not systematically associated with R&D investments and patents are also strategic tools that are not necessarily used by firms to protect innovation. Moreover, the set of innovative firms according to the OECD definition expands for practical reasons, as we need to observe both the innovative behaviour of the firm and its assessment about financial difficulties.

The Community Innovation Surveys (CIS) are conducted in each country by the national statistical entities in order to collect information about the innovative activities of firms. They are based on the same harmonised questionnaire that may be completed at the national level by additional questions. The survey used here (Financement de l’Innovation Technologique, FIT) is different because it is focused on the financing of innovation. However, its methodological framework is the same as the well-known CIS’ one, in particular concerning the definition of innovation and the structure of the questionnaire.



**All surveyed firms** had to answer the following question:

In 1997, 1998 or 1999, what are the obstacles that have prevented your firm to conduct or to start innovative projects (multiple answers possible)?

- Excessive perceived economic risk
- Lack of qualified personnel
- Innovation costs too high
- Lack of sources of finance
- Slowness in the setting up of the financing
- Too high interest rates of the financing
- Excessive get out clause in the shareholder agreement
- Lack of knowledge about ad hoc financial networks
- No obstacle

The firm had to assess the severity of each negative factor (seriously delayed, abandoned, or prevented to be started).

A firm is defined as **financially constrained** when it reported seriously delayed, abandoned or non-started projects because of:

- Too high interest rates of the financing; Lack of sources of finance; or Slowness in the setting up of the financing

### 7.1.2 The Banque de France Balance Sheet Data set

In order to obtain information about the size, economic performance, and financing structure of firms, we use the Banque de France Balance Sheet Data set, or Centrale de Bilans (CdB). This is a database of detailed accounting data of all French companies with 500+ employees, as well as of a fraction of smaller firms, giving a total of around 34,000 companies. It covers about 57% of all industries (by employment), and gives detailed information on financing sources (group financing, internal, etc) and financing expenditures (intangible goods, services, etc.)

We have verified that the direct indicator reported by the firms is in line with the balance sheet data: firms without financial constraints exhibit a better profile than constrained firms in terms of financing structure, risk, and economic performance.

**INSERT TABLE 3 and TABLE 4 ABOUT HERE**

## 7.2 Cross-section vs. Dynamic (Panel) Samples

The cross-section sample results from the matching of FIT and CdB in the 1997-1999 wave, allowing us to recover about 60% of the FIT companies.

### 7.2.1 Our Cross-Section Sample — Wave 1997-1999. (1940 firms)

After some necessary cleaning, our sample contains 1940 firms. The distribution of the firms in our sample according to their innovative behaviour and financing obstacles is given in Table 8 below:

### **7.2.2 Our Dynamic (Panel) Sample — Waves 1994-1996 and 1997-1999. (1512 firms)**

The panel sample is obtained by matching the survey FIT with two other sources: (i) the second French wave of the Community Innovation Survey (CIS2) run by the French Ministry of Industry for Eurostat; and (ii) the balance sheet data set of the Banque de France (Centrale de Bilans). The FIT and CIS2 surveys ask the same questions to identify innovative firms; and very similar questions about financial constraints.<sup>23</sup> The sample obtained by matching FIT, CIS2 and CdB contains 1512 firms. The transitions for innovation and financial constraints between the two surveyed periods are reported in the tables below.

[Tables 6ABC approximately here.]

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<sup>23</sup>Unfortunately, the following wave of the Community Innovation Survey (CIS3) covering 1998-2000 does not include questions about financial constraints and therefore we cannot use it here.

Table 2: Details of the financial obstacles and their consequences

	% of Constrained Firms	Consequences for their Innovative Project(s)		
		delayed	abandoned	non started
Unavailability of new financing	87.74	46.27	10.45	46.27
Searching and waiting for new financing	43.23	35.29	12.13	57.72
Too high cost of finance	22.90	28.17	15.49	57.75

Table 3: Direct indicator and balance sheet ratios

	Constrained				Unconstrained			
	Q1	Q2	Q3	Mean	Q1	Q2	Q3	Mean
Nber of employees	47	112	290	249.7	47	102	243	227.6
Debt/Equity	8.4	50.7	147.9	132.7	7.9	3.4	9.2	55.0
- Long term bank debt/Equity	0.6	21.9	62.3	43.7	1.7	18.6	51.9	33.4
- Short term bank debt/Equity	0.1	15.4	73.5	89.1	0.0	3.7	34.9	21.6
EBITDA/Sales	6.6	15.4	25.4	10.8	11.8	20.8	30.5	20.3
Cash-flow/Total assets	2.9	7.3	11.1	5.8	5.2	8.5	12.2	8.8
Immaterial Inv/Value added	0.4	1.5	3.9	4.6	0.3	1.1	3.0	3.3

Table 4: Definition of the variables

Name	Definition
Dependent variable : $y_{1i}$	=1 if the firm was innovative, =0 otherwise
Explanatory : $x_{1i}$	
Size	log (number of employees)
Market share	$\frac{\text{sales of the firm}}{\text{sales of the sector}} \times 100$
TP1	=1 if the firm's market is technologically not innovative (reference)
TP2	=1 if the firm's market is weakly innovative,
TP3	=1 if the firm's market is moderately innovative
TP4	=1 if the firm's market is strongly innovative
Financial constraints	=1 if the firm faced financial constraints, =0 otherwise
Financial constraints equation	
Dependent variable : $y_{2i}$	=1 if the firm faced financial constraints, =0 otherwise
Explanatory : $x_{2i}$	
Size	log (number of employees)
Collateral	log(tangible assets)
Banking debt ratio	$\frac{\text{Banking debt}}{(\text{Own financing} + \text{Market Financing} + \text{Financial debt})} \times 100$
Own financing ratio	$\frac{\text{Own financing}}{(\text{Own financing} + \text{Market Financing} + \text{Financial debt})} \times 100$
Gross operating profit margin	$\frac{\text{EBITDA}}{\text{Value added}} \times 100$

Sources : Centrale de Bilans (Banque de France), FIT (French Ministry of Industry) and EAE (INSEE)

Table 5: Descriptive statistics

Full sample of 1940 firms

	Mean	Std	Min	Max
Innovation	0.418	0.493	0	1
Size	4.783	1.107	2.890	9.716
Market share	0.177	0.566	0.001	16.15
TP1	0.139	0.312	0	1
TP2	0.416	0.493	0	1
TP3	0.348	0.476	0	1
TP4	0.097	0.297	0	1
Financial constraints	0.160	0.366	0	1
Collateral	71.048	22.698	4.241	302.444
Banking debt ratio	17.678	15.758	0	92.307
Own financing ratio	31.827	24.195	-609.459	90.136
Gross operating profit margin	18.248	19.416	-197.600	76.850

Sources : Centrale de Bilans (Banque de France), FIT (French Ministry of Industry) and EAE (INSEE)

Table 6: Transitions

Legend:

col %
Cell
Count
row %

**Part A: Innov Transitions 1994-6  $\rightarrow$  1997-9**

		1997-1999 (FIT)		
		$I_{it} = 1$	$I_{it} = 0$	Total
1994-1996  (CIS2)	$I_{i,t-1} = 1$	84.45	40.74	42.53
		543	354	897
		60.54	39.46	100
	$I_{i,t-1} = 0$	15.55	59.26	57.47
		100	515	615
		16.26	83.74	100
	Total	100	100	100
		643	869	1512
		42.53	57.47	100

**Part B: FinCons Transitions 1994-6  $\rightarrow$  1997-9**

		1997-1999 (FIT)		
		$F_{it} = 1$	$F_{it} = 0$	Total
1994-1996  (CIS2)	$F_{i,t-1} = 1$	41.32	15.98	20.04
		100	203	303
		33.00	67.00	100
	$F_{i,t-1} = 0$	58.68	84.02	76.96
		142	1067	1209
		67.00	88.25	100
	Total	100	100	100
		242	1270	1512
		16.01	83.99	100

**Part C: 1994-6 → 1997-9 Transitions**

		1997-1999 (FIT)				
		$I_{it} = 1$ and $F_{it} = 1$	$I_{it} = 1$ and $F_{it} = 0$	$I_{it} = 0$ and $F_{it} = 1$	$I_{it} = 0$ and $F_{it} = 0$	Total
1994-1996 (CIS2)	$I_{i,t-1} = 1$ and $F_{i,t-1} = 1$	37.7 58 28.3	15.8 77 37.6	14.8 13 6.3	7.3 57 27.8	13.6 205 100
	$I_{i,t-1} = 1$ and $F_{i,t-1} = 0$	46.1 71 10.3	68.9 337 48.7	30.7 27 3.9	32.9 257 37.1	45.8 692 100
	$I_{i,t-1} = 0$ and $F_{i,t-1} = 1$	7.8 12 12.2	1.4 7 7.1	19.3 17 17.3	7.9 62 63.3	6.5 98 100
	$I_{i,t-1} = 0$ and $F_{i,t-1} = 0$	8.4 13 2.5	13.9 68 13.2	35.2 31 6.0	51.9 405 78.3	34.2 517 100
	Total	100 154 10.2	100 489 32.3	100 88 5.8	100 781 51.7	100 1512 100

## 8 Appendix: Detailed Estimation Results

Table 7: Innovation and Financing Constraints Joint Probit

With Reverse Interaction Effects		
Full sample, nobs=1940		
	Model 3	
	Coeff.	Std.
<b>Innovation Equation</b>		
Constant	-7.235***	0.118
Size	0.183***	0.020
Market share	0.020	0.045
TP4	1.822***	0.183
TP3	1.0110***	0.199
TP2	0.437***	0.176
Financial Constraints	-0.324**	0.255
11 Industry dummies	misc	
<b>Financial Constraint Equation</b>		
Constant	-1.221***	0.241
Firm Innovates	0.647***	0.032
Size	-0.016	0.073
Collateral amount	0.030	0.050
Banking debt ratio	0.015***	0.002
Own financing ratio	-0.001***	0.001
Profit margin	-0.002***	0.002
11 industry dummies	misc	
<i>corr</i> <sub>12</sub>	-0.132***	0.013
Log lik Innovation		
Log lik Fin Constraint		
Log lik Bivariate	-1712	

Table 8: Innovation and Financing Constraints Joint Probit

With Reverse Interaction Effects and Dynamics		
Full sample, nobs=1512		
	Model 4	
	Coeff.	Std.
<b>Innovation Equation</b>		
Constant	-2.441***	0.323
Innov <sub><i>t</i>-1</sub>	0.829***	0.094
Size	0.256***	0.037
Market share	0.027	0.071
TP4	1.461***	0.201
TP3	0.932***	0.156
TP2	0.621***	0.143
Financial Constraints	-0.447***	0.106
Financial Constraints <sub><i>t</i>-1</sub>	0.300	0.123
11 Industry dummies	misc	
<b>Financial Constraint Equation</b>		
Constant	-0.885***	0.311
Firm Innovates <sub><i>t</i></sub>	0.627***	0.022
Firm Innovates <sub><i>t</i>-1</sub>	0.236**	0.133
Financial constraints <sub><i>t</i>-1</sub>	0.135***	0.093
Size	0.035	0.039
Collateral amount	0.003	0.002
Banking debt ratio	0.005	0.003
Own financing ratio	-0.008***	0.002
Profit margin	-0.007***	0.002
11 industry dummies	misc	
<i>corr</i> <sub>12</sub>	0.500**	0.210
Log lik Innovation		
Log lik Fin Constraint		
Log lik Bivariate	-1331	