Supplementary Material to "Perfect Competition in Markets with Adverse Selection"

Eduardo M. Azevedo and Daniel Gottlieb*

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*Azevedo: Wharton, eazevedo@wharton.upenn.edu. Gottlieb: Washington University in St. Louis, dgottlieb@wustl.edu.

C Strategic Foundations when Firms Offer Menus of Contracts

We now extend the results from Online Appendix A to the case where firms can offer menus of contracts. We consider exactly the same preferences as in Online Appendix A, but now each firm can offer a menu of contracts. We will see that, when firms are not very differentiated and capacity is sufficiently small, pure-strategy equilibria exist, and profits per unit sold of every contract are low. In particular, even though firms could cross-subsidize contracts, they choose not to in equilibrium. The reason is that a firm that tries to sell some contracts at a loss ends up with a disproportionate demand for those unprofitable contracts, making this a bad strategy. Thus, the results in this section justify the no cross-subsidization assumption in our definition of a competitive equilibrium and show that the analysis of Online Appendix A is robust to firms offering menus of contracts.

We now formally state the assumptions and results, and then discuss the assumptions. Throughout this section, fix an economy $E = [\Theta, X, \mu]$ and a perturbation $(E, \bar{X}, \eta)$. To simplify notation, take the total mass of consumers $\mu(\Theta) + \eta(\bar{X})$ to equal 1. Assume that preferences are quasilinear, so that $U(x, p, \theta) = u(x, \theta) - p$.

Consider Bertrand competition between differentiated firms selling varieties of each contract. Each firm $i \in \{1, 2, \cdots, n\}$ chooses a subset of contracts to offer, and at which price. We will simplify the model by assuming that each firm offers the entire set of contracts. This is without loss of generality because offering a strict subset of contracts is dominated by offering that subset, plus selling the other contracts at a sufficiently high price. Consumers have logit demand with semi-elasticity parameter σ and errors at the firm-product level.

Consider the case where all firms but one set the same price vector p. It is sufficient to define demand in these situations because we consider symmetric equilibria. Assume that firm i sets a price vector P, while all other firms set prices p. Then, the share of standard types θ purchasing contract x from firm i is almost everywhere equal to

$$S(P, p, x, \theta) = \frac{e^{\sigma \cdot (u(x,\theta) - P(x))}}{\sum_{x' \in X} (n-1) \cdot e^{\sigma \cdot (u(x',\theta) - p(x'))} + e^{\sigma \cdot (u(x',\theta) - P(x'))}}.$$

For a behavioral type $\theta = x$, this share equals

$$S(P, p, x, x) = \frac{e^{-\sigma \cdot P(x)}}{(n-1) \cdot e^{-\sigma \cdot p(x)} + e^{-\sigma \cdot P(x)}},$$

and 0 for all other products. The quantity of product x supplied by the firm is

$$Q(P, p, x) = \int_{\theta} S(P, p, x, \theta) \, d(\mu + \eta).$$

The total quantity is denoted by $\bar{Q}(P,p) = \sum_{x\in X} Q(P,p,x)$.

Each firm has constant returns to scale up to a capacity limit of k consumers and infinite costs of supplying to more than k consumers. Profits on contract x from a firm setting prices P while all other firms price according to p equal

$$\Pi(P,p,x) = \int_\theta (P(x) - c(x,\theta)) \cdot S(P,p,x,\theta)\, d(\mu+\eta). \tag{C1}$$

The firm's total profits $\bar{\Pi}(P,p)$ equal the sum of profits on all contracts if $\bar{Q}(P,p) \le k$ and $-\infty$ otherwise.

A **symmetric Bertrand equilibrium** under parameters (n,k,σ) is a vector p^* such that, for all $x \in \bar{X}$,

$$p^* \in \arg\max_{P\ge 0} \bar{\Pi}(P,p^*).$$

Define the constants

$$\bar{c} = \max_{(x,\theta)} c(x,\theta)$$

and

$$\bar{\eta} = \min_{x\in\bar{X}} \eta(\{x\}).$$

Let $|X|$ denote the number of contracts in X. The proposition below shows that an equilibrium exists if there are enough firms to serve the market of each variety ($n\cdot k > 1$), the semi-elasticity parameter is high enough ($\sigma > 1/\bar{c}$), and firms have small enough capacity ($k < \bar{\eta}/4$ and $k < b/\sigma$):

Proposition C1. *There exists a constant b such that a symmetric Bertrand equilibrium p^* exists if $n\cdot k > 1$, $\sigma > 1/\bar{c}$, $k < \bar{\eta}/4$, and*

$$k < \frac{b}{\sigma}. \tag{C2}$$

Moreover, total profits per unit sold equal

$$\frac{\bar{\Pi}(p^*,p^*)}{\bar{Q}(p^*,p^*)} = \frac{n}{n-1}\cdot\frac{1}{\sigma},$$

and the profit per unit of each contract sold are bounded by

$$\left|\frac{\Pi(p^*,p^*,x)}{Q(p^*,p^*,x)} - \frac{1}{\sigma}\right| \le 6|X|\frac{\bar{c}}{\bar{\eta}}\cdot k.$$

This result extends Proposition A1 to the case where firms offer menus of contracts. Consumer preferences are the same as in Online Appendix A, with the key difference here being that each firm can offer a menu of contracts. In particular, we continue to use logit error terms for preference shocks, which considerably simplify the statement of the results. Nevertheless, the proof only depends on bounds of the semi-elasticity and curvature of demand, and not on fine details of the logit functional form, as in Proposition A1, and as will be clear in the proof below. Moreover, the proof does not depend on cross-derivatives of demand across products converging to zero. What is important is that the own derivatives grow faster than cross-derivatives, so that a firm that tries to gain by offering cross-subsidies ends up facing a high demand for its money-losing products.

Proposition C1 differs from Proposition A1 because we consider a different game. The conditions for existence are more stringent in Proposition C1. While capacity has to be of the order of the semi-elasticity parameter, there is no simple formula for the constant in this relationship, unlike in Proposition A1. Moreover, Proposition C1 imposes additional, not very restrictive, conditions on parameters. As bounds on profits, both propositions show that per-unit profits are small for all contracts, and the bounds are of similar order of magnitude. Crucially, the bounds work for all contracts, which shows that the limit of this strategic model does not feature cross-subsidies between contracts.

C.1 Outline of the Proof of Proposition C1

The proof is based on the first and second derivatives of profits and on the first-order conditions of firms. To gain some intuitive understanding of the proof, note that the first derivative of profits with respect to the price of a contract is approximately equal to

$$\frac{d}{dP(x)}\bar{\Pi}(P,p) \approx (\frac{1}{\sigma} - \frac{\Pi(P,p,x)}{Q(P,p,x)}) \cdot \sigma Q(P,p,x),$$

and the second derivative is approximately equal to

$$\frac{d^2}{dP(x)^2}\bar{\Pi}(P,p) \approx (-\frac{2}{\sigma} + \frac{\Pi(P,p,x)}{Q(P,p,x)}) \cdot \sigma^2 Q(P,p,x).$$

The key points in Proposition C1 are established in three claims. Claim 3 establishes the existence of a candidate equilibrium price vector. The proof works by demonstrating the existence of equilibrium in a game where each player controls the price of one product, and the proof is similar to the existence proof in the one-contract case. Claim 6 demonstrates that profits are low in the candidate equilibrium. The proof uses the first-order conditions of firms, which implies that profits per unit are approximately equal to $1/\sigma$ (as can be seen from the approximate formula for the first derivative of profits). This proof is similar to the one-contract case.

The most difficult point is established in Claim 11, which shows that the candidate equilibrium is an equilibrium. The proof is by contradiction. If the candidate equilibrium is not an equilibrium, then there exists a best response that yields higher profits. We use the firm's first-order conditions to show that, in both the candidate equilibrium and in the best response, profits per unit sold are approximately equal to $1/\sigma$ (this is suggested by the approximate formula for the first derivative of profits above). Moreover, the profit function is strictly concave at the candidate equilibrium prices (this intuition is consistent with the approximate formula for the second derivative of profits above). This implies that there exists a price that is a convex combination of the best response and the candidate equilibrium that yields lower profits than the candidate equilibrium, and where the profit function is not strictly concave. We apply the Gershgorin circle theorem to the Hessian matrix of profits and show that it implies that profits per unit are higher than approximately $2/\sigma$ at these prices (this intuition can be grasped from the approximate formula for the second derivative of profits). We reach a contradiction by showing that it is impossible for profits per unit to vary so much in this range.

Even though the proof is based on this intuitive argument, it involves several steps, where we carefully bound the necessary approximations and establish auxiliary results. We divided the proof into subsections, so that readers understand the purpose of the auxiliary results. The intermediate results are presented as a series of claims, and we collect them in the proof of the proposition at the end.

C.2 Preliminary Definitions

Throughout the proof, whenever there is no risk of confusion, we use the shorthand S_x for $S(P, p, x)$, Q_x for $Q(P, p, x)$, Π_x for $\Pi(P, p, x)$, c_x for $c(x, \theta)$, P_x for $P(x)$ and p_x for $p(x)$.

We begin by defining a number of constants that will be used in the proof. Let

$$\delta = \frac{k}{\bar{\eta}},$$

$$\bar{P} = \frac{1}{1-k}\bar{c} + \frac{1}{1-k}\frac{1}{\sigma},$$

$$\underline{P} = \frac{1}{\sigma} - k\bar{c},$$

and

$$\bar{\lambda} = (1-\delta)^{-1} \cdot (4|X| + 2)\bar{P}\delta.$$

We take note that the assumptions that $\delta = k/\bar{\eta} \leq k \leq 1/4$ and $1/\sigma \geq \bar{c}$, which were made in the proposition statement, imply that

$$\bar{P} \leq 2\frac{1}{1-1/4}\bar{c} = \frac{8}{3}\bar{c} \leq 3\bar{c}.$$

Finally, we use a value for the constant b in the proof that is low enough so that bounds (C8), (C12), (C17), (C18), (C23), and (C25) stated below hold.

Throughout the proof we will use the fact that, if $\bar{Q}(P,p) \leq k$, then, for all contracts x and types θ,

$$S(P,p,x,\theta) \leq \delta.$$

This bound follows from the same argument as in Claim 1.

C.3 Derivatives of Market Shares and Profits

The proof uses derivatives of market shares and profits with respect to prices. To simplify the exposition, we collect these formulas here, and define error terms ξ, which we will bound in the course of the proof. In the derivative formulas, contract y is different than contract x.

Using the logit demand we can derive the formulas for derivatives of market shares.

$$\frac{d}{dP(x)}S_x = -\sigma S_x + \sigma S_x^2 = -\sigma S_x(1-S_x). \tag{C3}$$

$$\frac{d}{dP(y)}S_x = \sigma S_x S_y.$$

The derivative of profits equals

$$\frac{d}{dP(x)}\bar{\Pi} = \int S_x - \sigma(P_x - c_x)\cdot S_x + \sigma \sum_{x'\in X}(P_{x'} - c_{x'})S_{x'}S_x \, d(\mu+\eta). \tag{C4}$$

This can be simplified as

$$\frac{d}{dP(x)}\bar{\Pi} = Q_x - \sigma\Pi_x + \xi_x,$$

where

$$\xi_x = \sum_{x'\in X}\int (P(x') - c(x',\theta))\cdot\sigma S_x S_{x'}\, d(\mu+\eta).$$

The second derivative of profits with respect to the price of a contract equals

$$\frac{d^2}{dP(x)^2}\bar{\Pi} = -\sigma Q_x - \sigma\frac{d}{dP(x)}\bar{\Pi} + \xi_{xx}, \tag{C5}$$

where

$$\xi_{xx} = \sigma \int S_x^2 - 2\sigma(P_x - c_x)S_x^2 + 2\sigma \sum_{x' \in X} (P_{x'} - c_{x'})S_{x'}S_x^2 \, d(\mu + \eta).$$

The second cross-derivative of profits equals

$$\frac{d^2}{dP(x)\, dP(y)} \bar{\Pi} = \xi_{xy}, \tag{C6}$$

where

$$\begin{aligned} \xi_{xy} \quad &= \quad \sigma \int S_x S_y - \sigma(P_x - c_x) \cdot S_x S_y + 2\sigma \sum_{x' \in X} (P_{x'} - c_{x'}) S_{x'} S_x S_y \, d(\mu + \eta) \\ & \quad -\sigma^2 \int (P_y - c_y) S_y S_x \, d(\mu + \eta). \end{aligned}$$

The derivative of quantity equals

$$\frac{d}{dP(x)} \bar{Q} = -\sigma Q_x + \xi_{Q,x}, \tag{C7}$$

where

$$\xi_{Q,x} = \sigma \int \sum_{x' \in X} S_x S_{x'} \, d(\mu + \eta).$$

C.4 Existence of a Candidate Equilibrium

Claim 1. Assume that $\bar{Q}(P, p) < k$. If $P(x) > \bar{P}$ and $P(x') \leq P(x)$ for all contracts x', then $\partial_P \bar{\Pi}(P, p) < 0$. If $P(x) < \underline{P}$ and $P(x) \leq P(x')$ for all contracts x', then $\partial_P \bar{\Pi}(P, p) > 0$. Moreover, if $\underline{P} \leq P(x) \leq \bar{P}$, then, for all types θ,

$$|P(x) - c(x, \theta)| \leq \bar{P}.$$

Proof. The first derivative of profits is

$$\frac{d}{dP(x)} \bar{\Pi}(P, p) = \int S_x [1 - \sigma P_x + \sigma c_x + \sigma \sum_{x' \in X} P_{x'} S_{x'} - \sigma \sum_{x' \in X} c_{x'} S_{x'}] \, d(\mu + \eta).$$

Assume first that $P(x) > \bar{P}$ and $P(x) \geq P(x')$ for all x'. Then

$$\begin{aligned} \frac{d}{dP(x)} \bar{\Pi}(P, p) \quad &\leq \quad \int S_x [1 - \sigma P_x + \sigma \bar{c} + \sigma \sum_{x' \in X} P_x S_{x'}] \, d(\mu + \eta) \\ &\leq \quad \int S_x [1 - \sigma(1 - k) P_x + \sigma \bar{c}] \, d(\mu + \eta) < 0. \end{aligned}$$

Here the first inequality follows from $P_{x'} \leq P_x$, $c_{x'} \geq 0$, and $c_x \leq \bar{c}$, the second inequality follows from $\sum_{x'} S_{x'} \leq k$, and the last inequality from $P_x > \bar{P}$.

Consider now the case where $P(x) < \underline{P}$ and $P(x) \leq P(x')$ for all x'. Then

$$\begin{aligned} \frac{d}{dP(x)}\bar{\Pi}(P,p) &\geq \int S_x[1 - \sigma P_x - \sigma \sum_{x' \in X} \bar{c} S_{x'}]\, d(\mu + \eta) \\ &\geq \int S_x[1 - \sigma P_x - \sigma \bar{c} k]\, d(\mu + \eta) > 0. \end{aligned}$$

The first inequality follows from dropping positive terms, and from $c_{x'} \leq \bar{c}$, the second inequality follows from $\sum_{x'} S_{x'} \leq k$, and the last inequality follows from $P_x < \underline{P}$.

The bound for $|P(x) - c(x, \theta)|$ follows from

$$\bar{P} \geq \underline{P} - c(x,\theta) \geq -(1+k)\bar{c} \geq -\bar{P}.$$

□

Claim 2. If $Q(P, p, x) \leq k$ and $0 \leq P(x) \leq \bar{P}$ for all x, then

$$|\xi_{xx}(P,p)| < \sigma Q(P,p,x).$$

Proof. We have

$$\begin{aligned} |\xi_{xx}(P,p)| &= |\sigma \int S_x^2 - 2\sigma(P_x - c_x)S_x^2 + 2\sigma \sum_{x' \in X} (P_{x'} - c_{x'}) S_{x'} S_x^2 \, d(\mu+\eta)| \\ &\leq \sigma \int \delta S_x + 2\sigma \bar{P} \delta S_x + 2\sigma |X| \bar{P} \delta^2 S_x \, d(\mu + \eta) \\ &= (1 + 2\sigma \bar{P} + 2\sigma\delta|X|\bar{P}) \cdot \delta \cdot \sigma Q(P,p,x). \end{aligned}$$

The second line follows from the triangle inequality, $S_{x'} \leq \delta$, and $|P_{x'} - c_{x'}| \leq \bar{P}$. The last line follows from evaluating the integral. To obtain the result, note that $1/\sigma < \bar{c}$, $\bar{P} < 3\bar{c}$, $\delta \leq \frac{1}{4}$, and $\delta\sigma < b/\bar{\eta}$ imply that

$$\begin{aligned} (1 + 2\sigma\bar{P} + 2\sigma\delta|X|\bar{P}) \cdot \delta &< (1/\sigma + 2\bar{P} + 2\delta|X|\bar{P}) \cdot \sigma\delta \\ &< (\bar{c} + 6\bar{c} + \frac{3}{2}|X|\bar{c}) \cdot \frac{1}{\bar{\eta}} \cdot b. \end{aligned}$$

Hence, the claim holds as long as b is small enough so that

$$(7 + \frac{3}{2}|X|) \cdot \frac{\bar{c}}{\eta} \cdot b \leq 1. \tag{C8}$$

□

Claim 3. Consider a game where there are $|X|$ players for each of the n firms. Each player chooses the price of a product. The player receives a payoff equal to the entire profits of her firm, but capacity constraints are set for each product separately. Formally, denote by $\tilde{\Pi}(P_0, p)$ the payoff that a player choosing the price of contract x_0 gets if she sets a price of P_0 and all other players set prices according to a vector $p = (p(x))_{x \in X}$. We have

$$\tilde{\Pi}(P_0, p, x_0) = \sum_{x' \in X} \Pi((P_0, p_{-x_0}), p, x)$$

if

$$Q((P_0, p_{-x_0}), p, x_0) \leq k$$

and the payoff is $-\infty$ otherwise. A symmetric equilibrium of this game is a vector $p^* = (p^*(x))_{x \in X}$ such that, for all $x_0 \in X$ and $P_0 \in \mathbb{R}$,

$$\tilde{\Pi}(P_0, p^*, x_0) \geq \tilde{\Pi}(P_0', p^*, x_0).$$

This game has a symmetric equilibrium p^* with $0 \leq p^*(x) \leq \bar{P}$ for all $x \in X$.

Proof. The key part of the proof is showing that if $x_0 \in X$, $\underline{P} \leq P_0 \leq \bar{P}$, $\underline{P} \leq p(x) \leq \bar{P}$ for all x, and

$$\partial_{P_0} \tilde{\Pi}(P_0, p, x_0) = 0$$

then

$$\partial_{P_0 P_0} \tilde{\Pi}(P_0, p, x_0) < 0.$$

To see this note that Equation (C5) implies that

$$\partial_{P_0 P_0} \tilde{\Pi}(P_0, p, x_0) = -\sigma Q + \xi_{xx}.$$

Claim 2 implies that this expression is strictly negative.

Given this fact, the argument is the same as in the existence proof in Proposition A1. Define a payoff function that equals $\tilde{\Pi}$ if it is greater than $-\infty$, and

$$\tilde{\Pi}(\tilde{P}_0(p, x), p, x) - (\tilde{P}_0(p, x) - P_0)$$

otherwise, where $\tilde{P}_0(p, x)$ is the lowest price for which profits are greater than $-\infty$. Consider the game with this profit function and strategies restricted to $[\underline{P}, \bar{P}]$. The fact that all points with $\partial_{P_0} \tilde{\Pi} = 0$ have strictly negative second derivatives implies that the payoff function is quasi-concave. Because the payoff function is continuous, a symmetric equilibrium exists

(Fudenberg and Tirole, 1991 p. 34 Theorem 1.2). Claim 1 implies that this is an equilibrium of the game with the original payoff function and unrestricted action spaces. □

C.5 Useful Bounds

Henceforth, we refer to the symmetric equilibrium of the game in Claim 3 as the candidate equilibrium, denoted by p^*.

Claim 4. For any contract x and price vector P such that $P_x \geq \underline{P}$ and

$$\bar{Q}(P, p^*) \leq k,$$

we have that, for any type θ,

$$|S(P, p^*, x) - S((P_x, p^*_{-x}), p^*, x, \theta)| \leq \delta \cdot \max\{S(P, p^*, x), S((P_x, p^*_{-x}), p^*, x, \theta)\}.$$

Moreover,

$$|Q(P, p^*, x) - Q((P_x, p^*_{-x}), p^*, x, \theta)| \leq \delta \cdot (Q(P, p^*, x) + Q((P_x, p^*_{-x}), p^*, x, \theta)),$$

$$\frac{1-\delta}{1+\delta} \leq \frac{Q(P, p^*, x)}{Q((P_x, p^*_{-x}), p^*, x, \theta)} \leq \frac{1+\delta}{1-\delta},$$

$$|\frac{\Pi(P, p^*, x)}{Q(P, p^*, x)} - \frac{\Pi((P_x, p^*_{-x}), p^*, x, \theta)}{Q((P_x, p^*_{-x}), p^*, x, \theta)}| \leq \delta \frac{1+\delta}{1-\delta}(|P_x| + \frac{\Pi(P, p^*, x)}{Q(P, p^*, x)}).$$

If, moreover, $|\underline{P}| \leq \bar{c}$, then

$$|\frac{\Pi(P, p^*, x)}{Q(P, p^*, x)} - \frac{\Pi((P_x, p^*_{-x}), p^*, x, \theta)}{Q((P_x, p^*_{-x}), p^*, x, \theta)}| \leq \delta \frac{1+\delta}{1-\delta}(\bar{c} + 2\frac{\Pi(P, p^*, x)}{Q(P, p^*, x)}),$$

and

$$|\Pi(P, p^*, x) - \Pi((P_x, p^*_{-x}), p^*, x, \theta)| \leq \delta \cdot |P_x| \cdot (Q(P, p^*, x) + Q((P_x, p^*_{-x}), p^*, x, \theta)).$$

Proof. We have that

$$S(P, p^*, x) - S((P_x, p^*_{-x}), p^*, x, \theta)$$

equals

$$
\begin{aligned}
& \frac{e^{\sigma(u(x,\theta)-P(x))}}{(n-1)\cdot\sum_{x'\in X} e^{\sigma(u(x',\theta)-p^*(x'))} + e^{\sigma(u(x,\theta)-P(x))} + \sum_{x'\neq x} e^{\sigma(u(x',\theta)-P(x'))}} \\
& - \frac{e^{\sigma(u(x,\theta)-P(x))}}{(n-1)\cdot\sum_{x'\in X} e^{\sigma(u(x,\theta)-p^*(x))} + e^{\sigma(u(x,\theta)-P(x))} + \sum_{x'\neq x} e^{\sigma(u(x',\theta)-p^*(x'))}} \\
= \; & \frac{e^{\sigma(u(x,\theta)-P(x))}}{(n-1)\cdot\sum_{x'\in X} e^{\sigma(u(x,\theta)-p^*(x))} + e^{\sigma(u(x,\theta)-P(x))} + \sum_{x'\neq x} e^{\sigma(u(x',\theta)-P(x'))}} \\
& \cdot \frac{\sum_{x'\neq x} e^{\sigma(u(x',\theta)-p^*(x'))} - \sum_{x'\neq x} e^{\sigma(u(x',\theta)-P(x'))}}{(n-1)\cdot\sum_{x'\in X} e^{\sigma(u(x,\theta)-p^*(x))} + e^{\sigma(u(x,\theta)-P(x))} + \sum_{x'\neq x} e^{\sigma(u(x',\theta)-p^*(x'))}}.
\end{aligned}
$$

This expression is bounded below by

$$-(\sum_{x'\neq x} S(P,p^*,x',\theta))\cdot S((P_x,p^*_{-x}),p^*,x,\theta)$$

and above by

$$(\sum_{x'\neq x} S(p^*,p^*,x,\theta))\cdot S(P,p^*,x,\theta).$$

This implies the desired bound for market shares. The bounds for the absolute value of differences between quantity and profits follow from the bound for market shares and the triangle inequality. The bound on the ratio of quantities follows from the bound for differences in quantities and an algebraic manipulation. □

Claim 5. Consider a vector of prices P such that $\bar{Q}(P,p^*)\leq k$ and $P(x)\geq \underline{P}(x)$ for all contracts x. Then, for any contract x and type θ,

$$|(P(x)-c(x,\theta))\cdot S(P,p^*,x,\theta)|\leq 2\bar{P}\delta. \tag{C9}$$

Moreover, for all x and y in $\bar{X}$,

$$
\begin{aligned}
|\xi_x(P,p^*)| &\leq 2|X|\bar{P}\cdot\delta\cdot\sigma Q_x, \\
|\xi_{xx}(P,p^*)| &\leq (1/\sigma+4\bar{P}+4|X|\bar{P}\delta)\cdot\delta\sigma\cdot\sigma Q_x, \\
|\xi_{xy}(P,p^*)| &\leq (1/\sigma+4\bar{P}+4|X|\bar{P}\delta)\cdot\delta\sigma\cdot\sigma Q_x, \text{ and} \\
|\xi_{Q,x}(P,p^*)| &\leq \delta\cdot\sigma Q_x.
\end{aligned}
$$

Proof. If $P(x)\leq p^*(x)$, then inequality (C9) follows from $|P(x)-c(x,\theta)|\leq\bar{P}$, because $\underline{P}\leq P(x)\leq\bar{P}$, and $S(P,p^*,x,\theta)\leq\delta$ because $\bar{Q}(P,p^*)\leq k$. If $P(x)\geq p^*(x)$, we can bound market shares by

$$
\begin{aligned}
S(P, p^*, x, \theta) &< \frac{e^{\sigma(u(x,\theta)-P(x))}}{(n-1)\cdot e^{\sigma(u(x,\theta)-p^*(x))}} \\
&= e^{-\sigma(P(x)-p^*(x))}\cdot S(p^*(x), p^*, x, \theta)\cdot \frac{n}{n-1} \\
&\leq e^{-\sigma(P(x)-p^*(x))}\cdot \delta \cdot \frac{n}{n-1}.
\end{aligned}
$$

If profits are negative, they must be bounded by $-\bar{P}\cdot\delta$ because $P(x) - c(x,\theta) \geq -\bar{P}$. If profits are positive, they are bounded above by

$$P(x)\cdot e^{-\sigma(P(x)-p^*(x))}\cdot \delta \cdot \frac{n}{n-1}. \tag{C10}$$

This is a unimodal function of $P(x)$ with a maximum at $P(x) = 1/\sigma$. Thus, if $1/\sigma \leq p^*(x)$, profits are bounded by

$$p^*(x)\cdot\delta\cdot\frac{n}{n-1} \leq \bar{P}\cdot\delta\cdot\frac{n}{n-1}.$$

If $1/\sigma > p^*(x)$, this is bounded above by substituting $P(x) = 1/\sigma$ into expression (C10), which is no greater than

$$\frac{1}{\sigma}\cdot\delta\cdot\frac{n}{n-1}.$$

The bound (C9) then follows from $1/\sigma \leq \bar{P}$ and $n/(n-1) \leq 2$. The other bounds follow from application of the triangle inequality and bound (C9) to the formulas for the error terms ξ. $\square$

C.6 Profits are Low

Claim 6. If all firms set prices according to p^*, then total profits per unit sold equal

$$\frac{\bar{\Pi}(p^*, p^*)}{\bar{Q}(p^*, p^*)} = \frac{n}{n-1}\cdot\frac{1}{\sigma}.$$

Profits of per unit sold of each contract are bounded by

$$\left|\frac{\Pi(p^*, p^*, x)}{Q(p^*, p^*, x)} - \frac{1}{\sigma}\right| \leq 2|X|\bar{P}\delta.$$

Proof. The first-order condition $\partial_{P(x)}\bar{\Pi} = 0$ can be written, using equation (C4), as

$$\frac{\Pi_x}{Q_x} = \frac{1}{\sigma} + \frac{\xi_x}{\sigma Q_x}.$$

Claim 5 implies the bound on the profits of each contract. To obtain the formula for total profits we add Q_x multiplied by the equation above for each x, and divide by $\bar{Q}$ obtaining

$$\begin{aligned}\frac{\bar{\Pi}}{\bar{Q}} &= \frac{1}{\sigma} + \frac{1}{\sigma}\frac{1}{\bar{Q}} \cdot \sum_x \xi_x \\ &= \frac{1}{\sigma} + \frac{1}{\bar{Q}} \cdot \sum_{x,x'} \int (P(x') - c(x',\theta)) \cdot S_x S_{x'}\, d(\mu+\eta) \\ &= \frac{1}{\sigma} + \bar{\Pi}.\end{aligned}$$

Substituting $\bar{Q} = 1/n$ and rearranging we have the desired expression. □

Claim 7. If P is a best-response to p^* then, for any contract x,

$$\frac{1}{\sigma} - 2|X|\bar{P}\delta \le \frac{\Pi(P,p^*,x)}{Q(P,p^*,x)} \le \frac{1}{\sigma} + (1+\delta)\bar{\lambda} + 2|X|\bar{P}\delta.$$

Proof. If P is a best response, then there exists a Lagrange multiplier $\lambda \ge 0$ such that

$$\nabla_P \Pi(P,p^*,x) - \lambda \nabla_P Q(P,p^*,x) = 0.$$

Using the formulas for the derivatives of profits and quantities we have that, for all contracts x,

$$(1+\lambda\sigma)Q_x - \sigma\Pi_x = \lambda\xi_{Q,x} - \xi_x.$$

This can be rearranged as

$$\frac{\Pi_x}{Q_x} = \frac{1}{\sigma} + \lambda\left(1 - \frac{\xi_{Q,x}}{\sigma Q_x}\right) + \frac{\xi_x}{\sigma Q_x}. \tag{C11}$$

Consider first the case where the quantity constraint is not binding, so that $\lambda = 0$. We have

$$\frac{\Pi(P,p^*,x)}{Q(P,p^*,x)} = \frac{1}{\sigma} + \frac{\xi_x(P,p^*,x)}{\sigma Q(P,p^*,x)}.$$

Claim 5 then implies the desired bounds.

Consider now the case where the capacity constraint binds, so that $\lambda > 0$. There exists at least one contract x for which $Q(P,p^*,x) > Q(p^*,p^*,x)$, and therefore $P(x) < p^*(x)$. This implies that $P(x) \ge \underline{P}$ so that Claim 4 applies, and

$$\begin{aligned}\Pi_x &\le \Pi((P_x.p^*_{-x}),p^*,x) + 2\bar{P}\delta Q(P,p^*,x) \\ &\le \Pi(p^*,p^*,x) + 2\bar{P}\delta Q(P,p^*,x).\end{aligned}$$

The second inequality follows from the fact that p^* is an equilibrium of the game in Claim 3. Thus,

$$\frac{\Pi_x}{Q_x} \leq \frac{\Pi(p^*, p^*, x) + 2\bar{P}\delta Q(P, p^*, x)}{Q(P, p^*, x)} = \frac{\Pi(p^*, p^*, x)}{Q(p^*, p^*, x)} + 2\bar{P}\delta.$$

By Claim 6 we have

$$\frac{\Pi_x}{Q_x} \leq \frac{1}{\sigma} + (2|X| + 2)\bar{P}\delta.$$

Applying this bound to equation (C11), we get

$$\frac{1}{\sigma} + (2|X| + 2)\bar{P}\delta \geq \frac{1}{\sigma} + \lambda(1 - \frac{\xi_{Q,x}}{\sigma Q_x}) + \frac{\xi_x}{\sigma Q_x}.$$

Rearranging this expression,

$$\lambda \leq (1 - \frac{\xi_{Q,x}}{\sigma Q_x})^{-1} \cdot ((2|X| + 2)\bar{P}\delta - \frac{\xi_x}{\sigma Q_x}).$$

Applying the bounds in Claim 5,

$$\lambda \leq \bar{\lambda}.$$

Equation (C11) and Claim 5 imply that the per-unit profits of any contract are bounded above by

$$\begin{aligned}\frac{\Pi_x}{Q_x} &\leq \frac{1}{\sigma} + \bar{\lambda}(1 - \frac{\xi_{Q,x}}{\sigma Q_x}) + \frac{\xi_x}{\sigma Q_x} \\ &\leq \frac{1}{\sigma} + (1 + \delta)\bar{\lambda} + 2|X|\bar{P}\delta.\end{aligned}$$

The lower bound follows from equation (C11), the fact that λ is weakly positive, and that $\bar{\lambda} \geq 0$ by because $\delta < 1$. □

C.7 Sufficient Conditions for Concavity of the Profit Function

Claim 8. Consider a vector of prices P such that $\bar{\Pi}(P, p^*) > -\infty$ and $\partial_{P(x)}\bar{\Pi}(P, p^*) = 0$ for all x in X. Then $\bar{\Pi}$ is strictly concave in P at (P, p^*)

Proof. The formulas for the derivatives of the profit function imply that

$$\partial_{P(x)P(x)}\bar{\Pi} + \sum_{y \neq x} |\partial_{P(x)P(y)}\bar{\Pi}| = -\sigma Q_x + \xi_{xx} + \sum_{y \neq x} |\xi_{xy}|.$$

Therefore,

$$\partial_{P(x)P(x)}\bar{\Pi} + \sum_{y \neq x} |\partial_{P(x)P(y)}\bar{\Pi}| \leq -\sigma Q_x + \sum_{x' \in X} |\xi_{xx'}|.$$

Claim 5 implies that

$$\partial_{P(x)P(x)}\bar{\Pi} + \sum_{y\neq x} |\partial_{P(x)P(y)}\bar{\Pi}| \leq (-1 + |X| \cdot (1/\sigma + 4\bar{P} + 4|X|\bar{P}\delta) \cdot \delta\sigma)\sigma Q_x.$$

The fact that $1/\sigma < \bar{c}$, $\bar{P} < 3\bar{c}$, $\delta \leq 1/4$, and $\delta\sigma < b/\bar{\eta}$ imply that the right hand side can be bounded above, so that

$$\partial_{P(x)P(x)}\bar{\Pi} + \sum_{y\neq x} |\partial_{P(x)P(y)}\bar{\Pi}| < (-1 + |X| \cdot (\bar{c} + 12\bar{c} + 3|X|\bar{c}) \cdot \frac{b}{\bar{\eta}})\sigma Q_x.$$

Moreover, the right-hand side of this expression is strictly negative as long as we take b small enough so that

$$(13|X| + 3|X|^2) \cdot \frac{\bar{c}}{\bar{\eta}} \cdot b < 1. \tag{C12}$$

The Gershgorin circle theorem implies that all of the eigenvalues of the Hessian matrix of $\bar{\Pi}$ at (P, p^*) are strictly negative. Therefore, $\bar{\Pi}$ is strictly concave in P at (P, p^*). □

Claim 9. Consider a vector of prices P such that $\bar{\Pi}(P, p^*) > -\infty$ and

$$\frac{\Pi_x}{Q_x} < \frac{2}{\sigma} - |X|(\frac{1}{\sigma} + 6\bar{P} + 4|X|\bar{P}\delta) \cdot \delta\sigma \cdot \frac{1}{\sigma}$$

for all contracts x. Then $\bar{\Pi}$ is strictly concave in P at (P, p^*).

Proof. The formulas for the derivatives of the profit function imply that

$$\begin{aligned}
\partial_{P(x)P(x)}\bar{\Pi} + \sum_{y\neq x} |\partial_{P(x)P(y)}\bar{\Pi}| &= -2\sigma Q_x + \sigma^2\Pi_x - \sigma\xi_x + \xi_{xx} + \sum_{y\neq x} |\xi_{xy}| \\
&= (-\frac{2}{\sigma} + \frac{\Pi_x}{Q_x} - \frac{\xi_x}{\sigma Q_x} + \frac{1}{\sigma}\frac{\xi_{xx}}{\sigma Q_x} + \sum_{y\neq x} \frac{1}{\sigma}\frac{|\xi_{xy}|}{\sigma Q_x}) \cdot \sigma^2 Q_x.
\end{aligned}$$

Claim 5 implies that this expression is bounded above by

$$(-\frac{2}{\sigma} + \frac{\Pi_x}{Q_x} + 2|X|\bar{P} \cdot \delta\sigma \cdot \frac{1}{\sigma} + |X|(\frac{1}{\sigma} + 4\bar{P} + 4|X|\bar{P}\delta) \cdot \delta\sigma \cdot \frac{1}{\sigma}) \cdot \sigma^2 Q_x,$$

or

$$(\frac{\Pi_x}{Q_x} - (2 - |X|(\frac{1}{\sigma} + 6\bar{P} + 4|X|\bar{P}\delta) \cdot \delta\sigma) \cdot \frac{1}{\sigma}) \cdot \sigma^2 Q_x.$$

The assumption in the claim's statement implies that this expression is strictly negative. The Gershgorin circle theorem then implies that all eigenvalues of the Hessian matrix of the profit function are negative at (P, p^*). Thus, profits are strictly concave at (P, p^*). □

C.8 Monotonicity of the Profit per Unit Ratio

Claim 10. Define the function

$$r : \mathbb{R} \times X \to \mathbb{R}$$

as

$$r(\rho, x) = \frac{\Pi((\rho, p^*_{-x}), p^*, x)}{Q((\rho, p^*_{-x}), p^*, x)}.$$

Let I_x be the set

$$\{\rho : \underline{P} \le \rho \le 2\bar{P}, \exists P_{-x} \in [\underline{P}, \infty)^{|X|-1} : \bar{Q}((\rho, P_{-x}), p^*) \le k\}.$$

Then, for all x, I_x is an interval and $r(\rho, x)$ is strictly increasing in I_x.

Proof. The fact that I_x is an interval follows from $\bar{Q}$ being strictly decreasing in ρ.

To establish the monotonicity of $r(\rho, x)$, we explicitly calculate its derivative. By the quotient rule, and by the formulas for the derivatives of profits and quantities, the derivative of $r(\rho, x)$ with respect to x equals

$$\begin{aligned}
\frac{d}{d\rho} r(\rho, x) &= \frac{\partial_\rho \bar{\Pi}}{Q} - \frac{\Pi}{Q}\frac{\partial_\rho Q}{Q} \\
&= \frac{Q - \sigma\Pi}{Q} + \frac{\xi_x}{Q} + \frac{\Pi}{Q}\frac{\sigma Q}{Q} - \frac{\Pi}{Q}\frac{\xi_{Q,x}}{Q} \\
&= 1 + (\frac{\xi_x}{Q} - \frac{\Pi}{Q}\frac{\xi_{Q,x}}{Q}). \qquad \text{(C13)}
\end{aligned}$$

To show that this derivative is strictly positive, we need to establish bounds for some of the terms in expression (C13). We first bound the market share of a firm setting prices of (ρ, p^*_{-x}). If ρ is in I_x, there exists P_{-x} such that

$$\bar{Q}((\rho, P_{-x}), p^*) \le k.$$

Therefore, for any type θ and contract y,

$$S((\rho, P_{-x}), p^*, y, \theta) \le \delta$$

Claim 4 implies that

$$S((\rho, p^*_{-x}), p^*, y) - S((\rho, P_{-x}), p^*, y, \theta) \le \delta(S((\rho, p^*_{-x}), p^*, y, \theta) + S((\rho, P_{-x}), p^*, y, \theta)).$$

Thus,

$$S((\rho, p^*_{-x}), p^*, y, \theta) \le \frac{1+\delta}{1-\delta} S((\rho, P_{-x}), p^*, y, \theta) \le \frac{1+\delta}{1-\delta}\delta.$$

This implies, by the same argument in the proof of Claim 5, that

$$\frac{|\xi_y((\rho, p^*_{-x}), p^*)|}{Q((\rho, p^*_{-x}), p^*, y)} \leq 2|X|\bar{P} \cdot \delta\sigma \cdot \frac{1+\delta}{1-\delta} \tag{C14}$$

and

$$\frac{|\xi_{Q,y}((\rho, p^*_{-x}), p^*)|}{Q((\rho, p^*_{-x}), p^*, y)} \leq \delta\sigma \cdot \frac{1+\delta}{1-\delta}. \tag{C15}$$

We then bound the ratio $r(\rho, x)$. By the definition of profits it follows that

$$\rho - \bar{c} \leq \frac{\Pi((\rho, p^*_{-x}), p^*, x)}{Q((\rho, p^*_{-x}), p^*, x)} \leq \rho.$$

Because $\underline{P} \leq \rho \leq 2\bar{P}$, we have

$$\underline{P} - \bar{c} \leq \frac{\Pi((\rho, p^*_{-x}), p^*, x)}{Q((\rho, p^*_{-x}), p^*, x)} \leq 2\bar{P}.$$

The fact that $|\bar{P} - \bar{c}| \leq \bar{P}$ implies that

$$|\frac{\Pi((\rho, p^*_{-x}), p^*, x)}{Q((\rho, p^*_{-x}), p^*, x)}| \leq 2\bar{P}. \tag{C16}$$

Finally, we can use these bounds to show that the derivative of r is strictly positive. Applying bounds (C14), (C15), and (C16) to equation (C13), we have that

$$\begin{aligned} \frac{d}{d\rho} r(\rho, x) \quad &\geq \quad 1 - 2|X|\bar{P} \cdot \delta\sigma \cdot \frac{1+\delta}{1-\delta} - 2\bar{P} \cdot \delta\sigma \cdot \frac{1+\delta}{1-\delta} \\ &= 1 - (2|X| + 2)\bar{P} \cdot \frac{1+\delta}{1-\delta} \cdot \delta\sigma. \end{aligned}$$

The bounds $\bar{P} < 3\bar{c}$, $\delta\sigma < b/\bar{\eta}$, and $\delta < 1/4$ imply that the right-hand side of this expression is bounded below by

$$1 - (2|X| + 2) \cdot 3\bar{c} \cdot \frac{5}{3} \cdot \frac{1}{\bar{\eta}} \cdot b.$$

Thus, the claim holds if we take b to be small enough so that

$$10(1 + |X|) \cdot \frac{\bar{c}}{\bar{\eta}} \cdot b < 1. \tag{C17}$$

□

C.9 The Candidate Equilibrium is an Equilibrium

Claim 11. p^* is an equilibrium.

Proof. The proof is by contradiction. To reach a contradiction, assume that p^* is not an equilibrium. We will bound profits per unit at three points and show that these bounds contradict the monotonicity established in Claim 10. Then there exists a price vector $\hat{P}$ that is a best response to p^* such that $\bar{\Pi}(\hat{P}, p^*) > \bar{\Pi}(p^*, p^*)$.

Consider the function

$$\pi : [0,1] \to \mathbb{R}$$

such that

$$\pi(t) = \bar{\Pi}((1-t) \cdot p^* + t \cdot \hat{P}, p^*).$$

The function π is continuous because the set of prices for which $\bar{Q}$ is below k is convex. The function π is strictly concave and attains a local maximum at $t = 0$, by Claim 8. Moreover, $\pi(0) < \pi(1)$. Thus, the function π has a local minimum at some $\tilde{t}$ in $(0, 1)$. Let

$$\tilde{P} = (1-\tilde{t}) \cdot p^* + \tilde{t} \cdot \hat{P}.$$

The fact that $\tilde{t}$ is a local minimum of π implies that π is not strictly concave at $\tilde{t}$. This implies that $\bar{\Pi}$ is not strictly concave at $\hat{P}$. Claim 9 implies that there exists a contract x such that

$$\frac{\Pi(\tilde{P}, p^*, x)}{Q(\tilde{P}, p^*, x)} \geq \frac{2}{\sigma} - |X|(\frac{1}{\sigma} + 6\bar{P} + 4|X|\bar{P}\delta) \cdot \delta\sigma \cdot \frac{1}{\sigma}.$$

Claim 4 implies that

$$\begin{aligned}\frac{\Pi((\tilde{P}_x, p^*_{-x}), p^*, x)}{Q((\tilde{P}_x, p^*_{-x}), p^*, x)} \quad &\geq \quad \frac{2}{\sigma} - |X|(\frac{1}{\sigma} + 6\bar{P} + 4|X|\bar{P}\delta) \cdot \delta\sigma \cdot \frac{1}{\sigma} \\ &\quad -\delta\frac{1+\delta}{1-\delta}(\bar{c} + \frac{4}{\sigma} + 2|X|(\frac{1}{\sigma} + 6\bar{P} + 4|X|\bar{P}\delta) \cdot \delta\sigma \cdot \frac{1}{\sigma}).\end{aligned}$$

Using that $1/\sigma < \bar{c}$, $\bar{P} < 3\bar{c}$, and $\delta\sigma < b/\bar{\eta}$, we have that the right-hand side of this expression is bounded below by

$$\begin{aligned}& \frac{2}{\sigma} - |X|(\bar{c} + 18\bar{c} + 3|X|\bar{c}) \cdot \frac{1}{\bar{\eta}}b \cdot \frac{1}{\sigma} - \frac{1}{4}\frac{5}{3}(\bar{c} + 4\bar{c} + 2|X|\bar{c} + 36|X|\bar{c} + 6|X|^2\bar{c}) \cdot \frac{1}{\bar{\eta}} \cdot b \cdot \frac{1}{\sigma}) \\ = \quad & (2 - (\frac{25}{12} + \frac{209}{6}|X| + \frac{11}{2}|X|^2) \cdot \frac{\bar{c}}{\bar{\eta}} \cdot b) \cdot \frac{1}{\sigma}.\end{aligned}$$

We take b to be small enough so that

$$(\frac{25}{12} + \frac{209}{6}|X| + \frac{11}{2}|X|^2) \cdot \frac{\bar{c}}{\bar{\eta}} \cdot b < \frac{1}{2}. \tag{C18}$$

This implies the first bound that we will use in the proof:

$$\frac{\Pi((\tilde{P}_x, p^*_{-x}), p^*, x)}{Q((\tilde{P}_x, p^*_{-x}), p^*, x)} > (1 + \frac{1}{2}) \cdot \frac{1}{\sigma}. \tag{C19}$$

We now derive the second bound used in the proof. Claim 6 implies that

$$\begin{aligned}\frac{\Pi(p^*, p^*, x)}{Q(p^*, p^*, x)} &\leq \frac{1}{\sigma} + 2|X|\bar{P}\delta. \\ &< \frac{1}{\sigma} + 3|X| \cdot \frac{\bar{c}}{\bar{\eta}} \cdot b \cdot \frac{1}{\sigma}.\end{aligned}$$

Bound (C18) implies that

$$\frac{\Pi(p^*, p^*, x)}{Q(p^*, p^*, x)} < (1 + \frac{1}{2}) \cdot \frac{1}{\sigma}. \tag{C20}$$

Claim 7 implies that

$$\frac{\Pi(\hat{P}, p^*, x)}{Q(\hat{P}, p^*, x)} \leq \frac{1}{\sigma} + (1+\delta)\bar{\lambda} + 2|X|\bar{P}\delta. \tag{C21}$$

Claim 4 then implies that

$$\begin{aligned}\frac{\Pi((\hat{P}_x, p^*_{-x}), p^*, x)}{Q((\hat{P}_x, p^*_{-x}), p^*, x)} &\leq \frac{1}{\sigma} + (1+\delta)\bar{\lambda} + 2|X|\bar{P}\delta \\ &\quad + \delta\frac{1+\delta}{1-\delta}(\bar{c} + \frac{2}{\sigma} + 2(1+\delta)\bar{\lambda} + 4|X|\bar{P}\delta).\end{aligned}$$

We can bound the right-hand side of this expression using the fact that $1/\sigma < \bar{c}$, $\delta < 1/4$, $\delta\sigma < b/\bar{\eta}$, and $\bar{P} < 3\bar{c}$. The right-hand side is bounded above by

$$\frac{1}{\sigma} + \frac{5}{4}\bar{\lambda} + 6|X|\frac{\bar{c}}{\bar{\eta}} \cdot b \cdot \frac{1}{\sigma} + \frac{5}{3}(\bar{c} + 2\bar{c} + \frac{5}{2}\bar{\lambda} + 3|X|\bar{c}) \cdot \frac{1}{\bar{\eta}} \cdot b \cdot \frac{1}{\sigma}. \tag{C22}$$

To bound this expression, note that $\bar{\lambda}$ is bounded by

$$4(4|X| + 2)\frac{\bar{c}}{\bar{\eta}} \cdot b \cdot \frac{1}{\sigma}$$

and by

$$(4|X|+2)\bar{c}.$$

Substituting these two bounds for $\bar{\lambda}$ we find that expression (C22) is bounded above by

$$(1+5(4|X|+2)\cdot\frac{\bar{c}}{\bar{\eta}}\cdot b+(6|X|+5+\frac{5}{3}\frac{5}{2}(4|X|+2)+5|X|)\cdot\frac{\bar{c}}{\bar{\eta}}\cdot b)\cdot\frac{1}{\sigma}.$$

If we take b to be small enough so that

$$(\frac{55}{3}+\frac{143}{3}|X|)\cdot\frac{\bar{c}}{\bar{\eta}}\cdot b<\frac{1}{2}, \tag{C23}$$

it follows that.

$$\frac{\Pi((\hat{P}_x,p^*_{-x}),p^*,x)}{Q((\hat{P}_x,p^*_{-x}),p^*,x)}\leq(1+\frac{1}{2})\cdot\frac{1}{\sigma}. \tag{C24}$$

We wish to use these bounds to find a contradiction with Claim 10. To do so, we first have to show that $\hat{P}(x)\leq 2\bar{P}$. We have that

$$\hat{P}(x)-\bar{c}\leq\frac{\Pi(\hat{P},p^*,x)}{Q(\hat{P},p^*,x)}.$$

Applying inequality (C21), we have

$$\begin{aligned}\hat{P}(x) &\leq \bar{c}+\frac{1}{\sigma}+(1+\delta)\bar{\lambda}+2|X|\bar{P}\delta\\ &< \bar{P}+\frac{5}{4}\bar{\lambda}+2|X|\bar{P}\delta.\end{aligned}$$

Using the bounds for $\bar{\lambda}$, $\bar{P}$ and δ, we have

$$\begin{aligned}\hat{P}(x) &< \bar{P}+5(4|X|+2)\cdot\frac{\bar{c}}{\bar{\eta}}\cdot b\cdot\frac{1}{\sigma}+6|X|\frac{\bar{c}}{\bar{\eta}}\cdot b\cdot\frac{1}{\sigma}\\ &< \bar{P}+(10+26|X|)\cdot\frac{\bar{c}}{\bar{\eta}}\cdot\bar{c}\cdot b.\end{aligned}$$

This is smaller than $2\bar{P}$ if the latter term is smaller than $\bar{c}$. This is true as long as b is small enough so that

$$(10+26|X|)\cdot\frac{\bar{c}}{\bar{\eta}}\cdot b<1. \tag{C25}$$

We can now complete the proof. Bounds (C19), (C20), and (C24) imply that

$$\frac{\bar{\Pi}((\rho,p^*_{-x}),p^*,x)}{\bar{Q}((\rho,p^*_{-x}),p^*,x)}$$

is not monotone in ρ, contradicting Claim 10. □

C.10 Proof of Proposition C1

We can now collect these results in the proof of the proposition.

Proposition C1. Existence of an equilibrium follows from Claim 11, with b chosen to satisfy the necessary bounds. The profit bounds follow from Claim 6 and the fact that

$$2|X|\bar{P}\delta \leq 6|X|\frac{\bar{c}}{\bar{\eta}} \cdot k.$$

□

D Robustness of the Set of Competitive Equilibria

This section discusses the robustness of the set of competitive equilibria to small changes in the distribution of consumer preferences and costs. The sensitivity of equilibria with respect to fundamentals is a particularly relevant issue with adverse selection, where it is possible that the introduction of consumers with very high costs creates equilibria where no contracts are traded, as in Akerlof (1970) and Hendren (2013, 2014).

There are two situations where small perturbations to fundamentals can have large effects on the set of equilibria. First, it may be that average cost and demand curves intersect each other tangentially, so that a small change in average cost shifts the equilibrium far from the initial point. Economies where average cost and demand curves intersect tangentially are knife-edge, however. Second, it may be that small perturbations in fundamentals leads to large changes in the average cost curve. We illustrate these situations in the context of the one-contract Akerlof (1970) model in Section D.1.

In Section D.2, we formally establish a robustness result that allows for multiple contracts, albeit under more restrictive conditions than our general model. We show that the equilibrium price correspondence is continuous with respect to fundamentals under certain conditions. The key condition is that changes in fundamentals do not drastically change average cost curves.

D.1 The one-contract case

Consider the Akerlof model from example 1. There is a single non-null contract, with demand and average cost curves $D(\cdot)$ and $AC(\cdot)$. It is helpful to consider the effect of a change in the fundamentals in two steps: how it affects demand and average costs, and how the resulting change in demand and average costs affects equilibria. There are three important observations.

1. Equilibria of the Akerlof model are not always robust to small changes in average cost and demand curves.

To see this, consider Figure D1a. In this example, a small upward shift in the average cost curve eliminates the interior equilibrium. Note, however, that this is only possible because the average cost and demand curves intersect each other tangentially, which is a knife-edge situation.

2. Large changes in the average cost curve can have large effects on the set of equilibria.

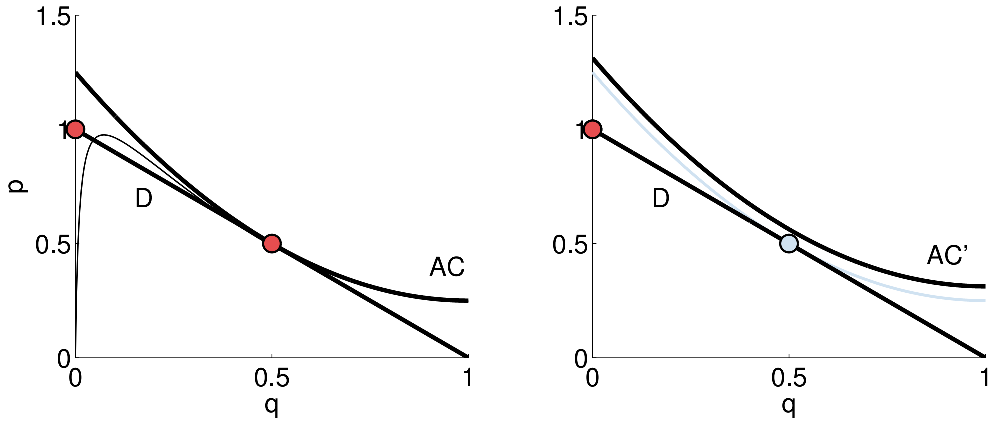
This point is illustrated in Figure D1b. The left panel depicts demand and average cost curves with an interior equilibrium. In the right panel we added a small measure of types with very high costs and very high demand for insurance. The original interior equilibrium continues to exist. However, after the change, there are two additional equilibria, one with complete unravelling and another with almost complete unravelling. This is only possible if we make large changes in the average cost curve, changing the conditional distribution of costs in the tail of willingness to pay.

Adding a small measure of high-cost types cannot have a large impact on the average cost curve when there is a positive mass of consumers with even higher willingness to pay. Consider an example where willingness to pay is uniformly distributed between \$0 and \$1,000,000, and there is a unique interior equilibrium as in Figure D2a. If we add a small measure of types with willingness to pay of \$1,200,000 and costs of \$1,300,000, the model will have a new equilibrium with full unravelling and a price of \$1,300,000. However, if the distribution of willingness to pay were uniform between \$0 and \$2,000,000, a sufficiently small measure of types with willingness to pay of \$1,200,000 would not change the set of equilibria very much.

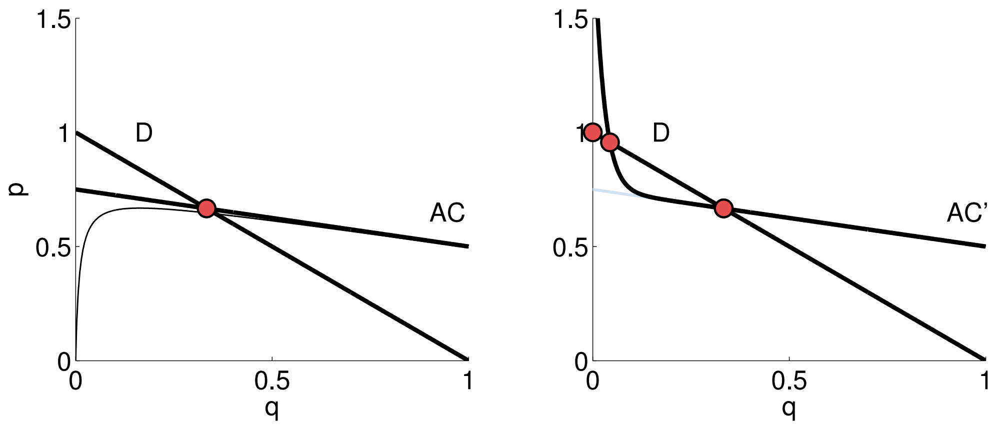
A last observation is that the new equilibria are less plausible from a tâtonnement perspective. Scheuer and Smetters (2014) propose a tâtonnement procedure to study the stability of equilibria in markets with adverse selection. Their model works similarly to models of network effects in industrial organization. According to their definition, the equilibrium with full unravelling and the original equilibrium are stable, while the new equilibrium with almost full unravelling is unstable. Even though the full unravelling equilibrium is stable, its basin of attraction is small because initial conditions above the intermediate equilibrium converge to the original equilibrium.

3. Generically, the set of equilibria is robust to small changes in average cost and demand.

Finally, we note that in a typical case the set of equilibria changes continuously with small changes in average cost and demand curves. This is illustrated in Figure D2. The lower panels depict a market with an interior equilibrium. A small decrease in average cost lowers the equilibrium price, but the change is continuous with respect to the reduction in average



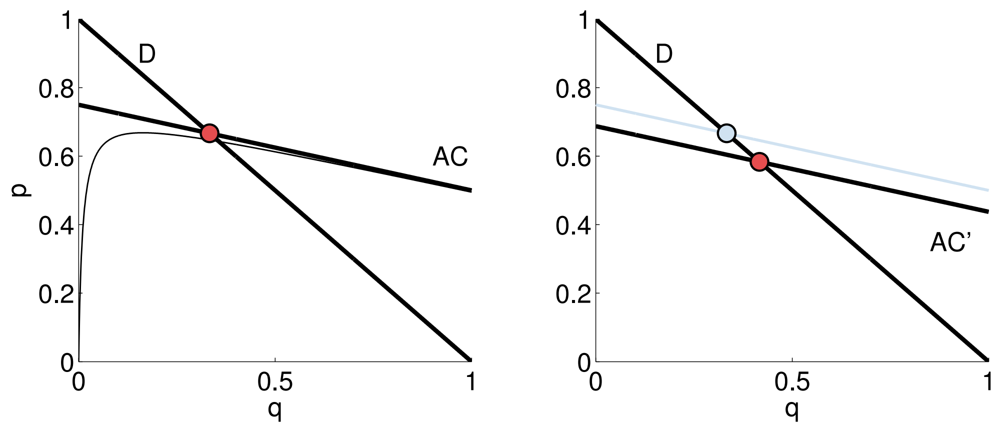
(a)



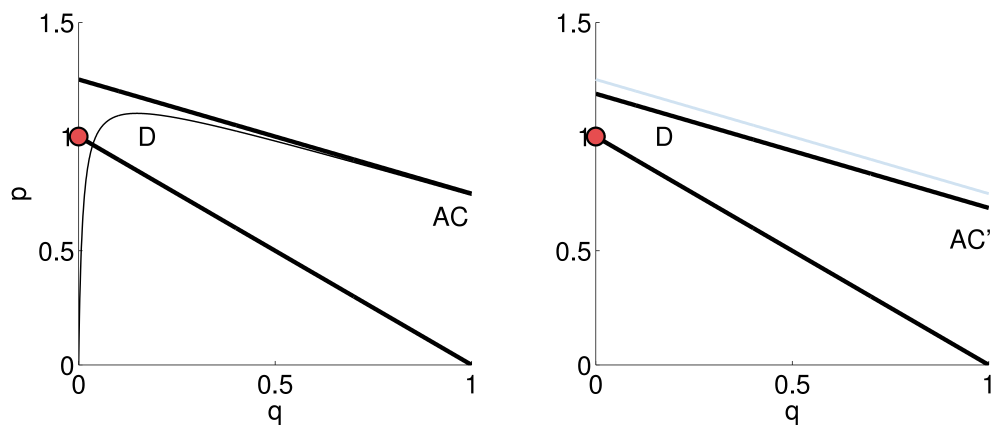
(b)

Figure D1: Cases where the set of equilibria is not robust to changes in fundamentals.

Notes: Each subfigure ((a) and (b)) depicts how equilibria of the Akerlof model respond to changes in the average cost curve. The left panel depicts original demand and average cost curves as the thick lines, and the average cost curve in a perturbation with 1% of behavioral consumers as the thin line. Equilibria are depicted by dark red dots. The right panel depicts demand and average cost curves after a change in fundamentals by the thick dark lines. The thin blue line depicts average cost before the change. Equilibria after the change are depicted by dark red dots, and any points that were equilibria before the change but not after are depicted as light blue dots.



(a)



(b)

Figure D2: Cases where the set of equilibria is robust to changes in fundamentals.

Notes: See the notes in Figure D1.

cost. The lower panels depict a market where full unravelling is the unique equilibrium, and remains so with small changes in the average cost curve. In the next section, we show that this generic robustness property holds even when there is more than one potential contract.

D.2 Robustness with many contracts and formal result

We now state a formal result showing that, generically, the set of equilibria is robust to small changes in the distribution of types. The result formalizes the points made in the examples in Figure D2 and extends them to the case of multiple contracts. The key substantial assumptions are that average cost and demand curves vary continuously with fundamentals, and the set of changes in fundamentals is rich enough. We also make technical assumptions to focus on the main ideas and simplify the mathematical exposition. We assume a finite set of contracts, smoothness of demand and average cost curves, and that the support of the set of valuations does not change with fundamentals.

Consider a set of economies

$$\{E(z) : z \in Z\}$$

indexed by an Euclidean vector of parameters in some open subset of Euclidean space Z. The economies only differ in their distribution of types, so that $E(z) = [X, \Theta, \mu(z)]$. X is finite, and there exists a null contract $x = 0$ that costs 0.

Assume that, at any price, the set of types who are indifferent between two contracts has measure 0. In particular, demand for and total cost of supplying a contract are uniquely defined given prices. Denote demand and total cost of supplying contract x given prices of non-null contracts $p = (p(x))_{x \in X \setminus \{0\}}$ and parameter vector z as

$$D_x(p|z) = \int 1 \, d\alpha$$

and

$$C_x(p|z) = \int c(x, \theta) \, d\alpha,$$

where α is any allocation in which almost all consumers choose optimal contracts given the price vector $(0, p)$. Note that demand and cost functions are defined for vectors of prices of non-null contracts, with the prices of the null contract equal to 0. This definition will simplify notation below.

Assume that there exists $\bar{p}(x)$ such that demand for contract x is strictly positive if and only if $p(x) \in [0, \bar{p}(x))$. In particular, the set of prices where demand for all contracts is strictly positive is

$$P = \times_{x \in X \setminus \{0\}} [0, \bar{p}(x)).$$

Let $\bar{P}$ be the closure of P. Proposition 1 part 2 implies that all equilibrium prices are in $\{0\} \times \bar{P}$.

Define the average cost of supplying contract x given prices p in P and parameter vector z as

$$AC_x(p|z) = \frac{C_x(p|z)}{D_x(p|z)}.$$

Denote by D, C, and AC the vectors of demand, costs, and average costs for non-null contracts $x \in X \backslash \{0\}$. Assume that AC can be continuously extended to $\bar{P} \times Z$, and that D, C, and AC are smooth functions over $\bar{P} \times Z$.

Define the equilibrium price correspondence as

$$P^*(z) = \{p^* : p^* \text{ is an equilibrium price of } E(z)\}.$$

We assume that, if p^* is an equilibrium price, then, for all non-null contracts, $p^*_x \geq \underline{p}$ for some constant $\underline{p} > 0$.

Our goal is to state a result that is true for typical economies. To formalize this statement, we need to assume that $E(z)$ varies with z in a sufficiently rich way so that $E(z)$ is typical for most values of z. We assume that

$$\partial_z (AC_x(p, z))_{x \in A}$$

has rank $|A|$ for all $z \in Z$, $A \subseteq X \backslash \{0\}$, and $p \in \bar{P}$. With these definitions, we have the following result:

Proposition D2. *For almost every z,*

1. *$P^*(z)$ consists of a finite number of isolated points, and*

2. *P^* is continuous at z.*

D.3 Proof of the proposition

The proof uses differential topology techniques, which give conditions for the solutions of systems of equations to vary continuously with parameters. We refer the reader to Guillemin and Pollack (2010) for a textbook treatment. We will use their definitions of transversality and stable properties of maps (pp. 28, 34).

The proof applies differential topology results to two maps, T_A and $\tilde{T}_A$, which we define below. The key steps are established in two lemmas. Lemma D1 shows that certain regularity properties are satisfied for most values of z. Lemma D2 shows that, under these regularity properties, equilibrium prices are a subset of the roots of T_A.

The two maps used in the proof are defined as follows. Let $A \subseteq X\backslash\{0\}$ be a set of non-null contracts. Define the map

$$\tilde{T}_A : \times_{x\in A}(\underline{p}, \bar{p}(x)) \times Z \to \mathbb{R}^A \times \mathbb{R}^{X\backslash(A\cup\{0\})}$$

as follows. The input to $\tilde{T}$ is a vector $p = (p(x))_{x\in A}$ of prices for the contracts in A and $z \in Z$. To describe $\tilde{T}$, let $\tilde{p}$ be a price vector with $\tilde{p}(x) = p(x)$ for $x \in A$, and $\tilde{p}(x) = \bar{p}(x)$ for contracts $x \in X\backslash(A \cup \{0\})$. $\tilde{T}_A(p|z)$ is a vector, and its component associated with contract x equals

$$\tilde{T}_A(p|z)(x) = AC_x(\tilde{p}|z) - \tilde{p}(x).$$

Let T_A be a map

$$T_A : \times_{x\in A}(\underline{p}, \bar{p}_x) \times Z \to \mathbb{R}^A$$

equal to the coordinates of $\tilde{T}_A$ corresponding to the contracts in A.

Let Z_r be the set of $z \in Z$ such that:

1. $T_A(\cdot|z)$ intersects $\{0\}$ transversally for all subsets A of non-null contracts, and
2. If $T_A(p^*_A|z) = 0$, and if $\tilde{p}$ is a price vector with $\tilde{p}(x) = p^*_A(x)$ for $x \in A$ and $\tilde{p}(x) = \bar{p}(x)$ for all non-null contracts $x \notin A$, then for all non-null contracts $x \notin A$ we have $AC_x(\tilde{p}|z) \neq \bar{p}(x)$.

The first lemma shows that most values of z satisfy these regularity properties.

Lemma D1. *Z_r is open and $Z\backslash Z_r$ has measure* 0.

Proof. Property (2) in the definition of Z_r is equivalent to $\tilde{T}_A(\cdot|z)$ not attaining the value 0 for any $A \subsetneq X\backslash\{0\}$. The map $\tilde{T}_A(\cdot|z)$ is transversal to $\{0\}$ if and only if it does not attain the value 0 because the dimension of its image is larger than the dimension of its domain. Therefore, property (2) is equivalent to the map $\tilde{T}_A(\cdot|z)$ being transversal to $\{0\}$ for all $A \subsetneq X\backslash\{0\}$. Therefore, $z \in A$ if and only if all maps $T_A(\cdot|z)$ and $\tilde{T}_A(\cdot|z)$ are transversal to $\{0\}$.

The rank assumption on $\partial_z(AC_x(p, z))_{x\in A}$ implies that the maps $T_A(\cdot|\cdot)$ and $\tilde{T}_A(\cdot|\cdot)$ are transversal to $\{0\}$ for all A. Thom's Transversality Theorem implies that the maps $T_A(\cdot|z)$ and $\tilde{T}_A(\cdot|z)$ are transversal to $\{0\}$ for almost all $z \in Z$ (Guillemin and Pollack, 2010 p. 68). Moreover, both maps are transversal in an open set of values of z because transversality is a stable property (Guillemin and Pollack, 2010 p. 35). □

Lemma D2. *(Generic equivalence between equilibria and roots) Take $z \in Z_r$ and $p^* \in \bar{P}$, and let*

$$A = \{x \in X\backslash\{0\} : D_x(p^*|z) > 0\}.$$

We have that $(0,p^*)$ *is an equilibrium price of* $E(z)$ *if and only if*

$$T_A((p^*(x))_{x\in A}|z) = 0 \tag{D26}$$

and $AC_x(p^*) > \bar{p}(x)$ *for all non-null contracts* x *in* $X\backslash A$.

Proof. Part 1 (only if).

Assume that p^* is an equilibrium price of $E(z)$. Then, p^* is also a weak equilibrium, so equation (D26) holds. Moreover, by the definition of an equilibrium, there exists a sequence of perturbations $(E(z), X, \eta^k)_{k\in\mathbb{N}}$ converging to the original economy $E(z)$, with a sequence of equilibria $(p^k)_{k\in\mathbb{N}}$ converging to p^*. The zero profits condition for x implies that

$$AC_x(p^k|z) = p^k(x)\cdot(1+\frac{\eta_x^k}{D_x(p^k|z)}) \geq p^k(x).$$

Taking the limit, we have $AC_x(p^*|z) \geq \bar{p}_x$. The fact that z is in Z_r implies that $AC_x(p^*|z) > \bar{p}_x$.

Part 2 (if).

Assume that p^* satisfies the conditions in the statement of the lemma. To prove that $(0,p^*)$ is an equilibrium price in $E(z)$, we must construct a sequence of perturbations $(E(z), X, \eta^k)_{k\in\mathbb{N}}$ converging to the original economy $E(z)$ and a sequence $(p^k)_{k\in\mathbb{N}}$ where $(0,p^k)$ is an equilibrium of the perturbation $(E(z), X, \eta^k)$ and $\left(p^k\right)_{k\in\mathbb{N}}$ converges to p^*.

We define p^k as follows. Let

$$A = \{x \in X\backslash\{0\} : p^*(x) < \bar{p}(x)\}.$$

For non-null contracts $x \notin A$ set

$$p^k(x) = \bar{p}(x) - 1/k.$$

Note that $T_A(\cdot|z)$ maps a neighborhood of $(p^*(x))_{x\in A}$ diffeomorphically into a neighborhood of 0. Therefore, there exists a sequence $((p^k(x))_{x\in A})_{k\in\mathbb{N}}$ such that

$$AC_x(p^k|z) - p^k = 1/k$$

for all sufficiently large k and $x \in A$. Moreover, $p^k(x)$ converges to $p^*(x)$.

Let

$$\eta_x^k = \frac{AC_x(p^k|z) - p^k(x)}{p^k(x)}\cdot D_x(p^k|z) \tag{D27}$$

for non-null contracts and $\eta_x^k = 1/k$ for the null contract. For contracts $x \in A$ we have

$$\eta_x^k = \frac{1}{k \cdot p^k(x)} \cdot D_x(p^k|z).$$

This is positive and converges to 0 because $p^*(x) \geq \underline{p}(x)$. For non-null contracts $x \notin A$, expression (D27) is positive for sufficiently high k because $AC_x(p^*|z) > p^*(x)$. The expression converges to 0 because $p^*(x) = \bar{p}(x) > 0$ and $D_x(p^*|z) = 0$. Therefore, all η_x^k are strictly positive and converge to 0. This implies that $(E(z), X, \eta^k)_{k \in \mathbb{N}}$ is a sequence of perturbations converging to the original economy.

Equation (D27) implies that, at prices $(0, p^k)$ and at any allocation where consumers optimize, firms make 0 profits in the perturbation $(E(z), X, \eta^k)$. Therefore, $(0, p^k)$ is an equilibrium price of the perturbation $(E(z), X, \eta^k)$. This implies that $(0, p^*)$ is an equilibrium of $E(z)$. □

Proof of the proposition. By Lemma D1, we only need to demonstrate the desired properties for $z \in Z_r$.

Let $P_0(z)$ be

$$\{(0, p^*) : T_A((p^*(x))_{x \in A}, z) = 0 \text{ for some } A \subseteq X \backslash \{0\},\ p(x) = \bar{p}(x) \text{ for } x \notin A \cup \{0\}\}.$$

Fix $z_0 \in Z_r$. The transversality of the maps T_A to $\{0\}$ implies that there is a neighborhood Z_0 of z_0 where

$$P_0(z) = \cup_{k=1}^{K} \{(0, \rho_k^*(z))\},$$

where each ρ_k^* is a smooth function. We can take $Z_0 \subseteq Z_r$ because Z_r is open (Lemma D1). Therefore, for any k and any non-null contract x with

$$D_x(\rho_k^*(z_0)|z_0) = 0,$$

we have

$$AC_x(\rho_k^*(z_0)) \neq \bar{p}_x.$$

Consequently, we can take Z_0 so that the sign of

$$AC_x(\rho_k^*(z)) - \bar{p}_x$$

does not depend on z. Lemma D2 then implies that equilibrium prices $P^*(z)$ are the union of $\{(0, \rho_k^*(z))\}$ for which $AC_x(\rho_k^*(z_0)) > \bar{p}_x$ for all non-null contracts x with

$$D_x(\rho_k^*(z_0)|z_0) = 0.$$

This implies that the correspondence P^* consists of a finite number of points and is continuous at z_0.

□

E Calibration with Nonlinear Contracts

We calibrated a model with linear contracts and normally distributed losses in Section 5. We made these simplifying assumptions to obtain a transparent closed-form expression for willingness to pay and costs. Here, we calibrate a model with nonlinear health insurance contracts with characteristics commonly found in practice and a log-normal distribution of losses. The main qualitative results are robust to this more realistic specification.

Preferences and health shocks are the same as in example 3. The only difference is that losses are log-normal. Consumers still have four dimensions of heterogeneity: absolute risk aversion A, moral hazard H, mean losses M and variance of losses S^2. We continue to assume that types are log-normally distributed. In our baseline scenario, we set the moments of the distribution of types to match Einav et al.'s (2013) central estimates. In the calibration with linear contracts, we reduced the value of mean risk-aversion, because Einav et al.'s central estimates implied implausible substitution patterns with linear contracts. However, with the nonlinear contracts we use Einav et al.'s central estimate of absolute risk aversion of 1.9E-3. Parameter values are displayed in Table E1, and we discuss the calibration in Online Appendix B.

Table E2 displays the set of contracts we consider. All contracts have a deductible, up to which consumers pay for all of their expenditures. After the deductible, consumers are responsible for a fraction equal to a copay. Expenses are then bounded by an out-of-pocket maximum. The table reports the (endogenous) expected percentage of losses covered by each contract, calculated for an average consumer and assuming that the consumer makes privately optimal expenditure decisions. This is known as the actuarial value of the policy and is a standard measure of contract generosity.

We selected contracts to represent a broad range of the quality spectrum, much like the calibration with linear contracts. The most generous contract we consider (contract 12) is the most generous contract in Einav et al. (2013)'s data from a large employer among the five new plan options for single employees. It has an actuarial value of 93%. All of the five plan options in Einav et al. (2013) are quite generous. To perform similar analyses as we did with linear contracts, we also included contracts with lower coverage, although this takes us far from the range of the data. We included a typical bronze plan. We used contract parameters from an "average bronze plan" described in `www.healthpocket.com` based on typical 2014-2015 offers in health insurance exchanges (contract 6). This plan has an actuarial value of 60% for an average consumer in our calibration. Moreover, we added a public insurance option for consumers who purchase no insurance (contract 0). The reason is that, with the lognormal loss distribution and CARA preferences, consumers would have negative infinity utility from being uninsured. We followed Kowalski (2014) and assumed that uninsured patients have expenditures above \$30,000 covered by a third party. We assume that the uninsured cannot engage in moral hazard and thus always have expenditures equal to their health shock. Third party expenditures do not affect equilibrium because the price of public insurance is 0, but we included these expenditures in welfare calculations. This turned out to be of little consequence because few consumers are uninsured under the parameters we consider. Finally, we added ten other contracts by linearly interpolating contract characteristics between the most generous plan and the bronze plan, and between the bronze plan and contract 0. We slightly modified these parameters to have round numbers and to space their actuarial values somewhat evenly. Thus, consumers had a rich set of quality choices, but contracts are vertically differentiated, as in the calibration with linear contracts.

We considered the same exercise as in the linear model. We first calculated equilibrium without any government intervention. We then calculated equilibrium with mandated minimum coverage of the bronze contract (contract 6) and a welfare-maximizing allocation. Equilibria were calculated in a perturbation with 1% of behavioral consumers and the same computational procedures as in the linear contracts case.

The results are in Table E3. The results are qualitatively consistent with equilibrium patterns in the linear model. There is considerable adverse selection on the intensive margin. Even though virtually all consumers purchase positive coverage, they pick less generous plans than what would be optimal, and welfare per consumer is \$371 lower than in the optimal allocation. Interestingly, the endogenously determined set of traded contracts is quite narrow, with many contracts not being traded at all. Moreover, the optimum involves a single contract being sold, the phenomenon that we also observed in the linear model with a high variance of the moral hazard parameter σ_H^2. This result suggests that, under these parameters, much of the variation in choices is due to differences in moral hazard. Because the social planner would like to offer less insurance for types with higher moral hazard, it becomes optimal to give up on sorting and offer a single contract.

Under the baseline Einav et al. (2013) parameters, the mandate has almost no effect. The reason is that almost all consumers purchase a contract that is better than the bronze contract, rendering the mandate moot. This is not surprising because their data considers quite generous contracts (the least generous contract has an actuarial value of 77%), and CARA utility does not adequately describe choices over very different domains of losses (Rabin (2000); Handel and Kolstad (forthcoming)). Thus, the variation in tastes is too small under these parameters for many consumers to purchase less than 60% insurance.

To gauge the qualitative effects of a mandate, we considered alternative parameters with more heterogeneity in tastes. Table E3 reports simulations with the log variance of risk aversion increased to 2 (few consumers purchase less than bronze insurance when the variance is much lower). To intuitively understand the amount of preference variation necessary to get dispersion in choices, note that a log variance of 2 implies that a consumer in the top 95th percentile of risk aversion has absolute risk aversion about 100 times higher than a consumer at 5th percentile.

In this last example, the equilibrium effects of the mandate are qualitatively similar to the linear case. Before the mandate, about 22% of consumers purchased bronze coverage or less. After the mandate, many consumers (27%) purchase the minimum contract. That is, the mandate forces some consumers to purchase more insurance. But the mandate also induces some consumers to purchase less, with the fraction purchasing 60% or less going up from 22% to 27%. This unintended consequence is similar to what we found with linear contracts.

In this example, the mandate decreases welfare by 272. This result differs from what we found in the baseline parametrization with linear contracts, where the mandate increased welfare. However, it is consistent with the findings of Einav et al. (2010) that mandates can sometimes increase and sometimes decrease welfare. Finally, our findings are consistent with the idea that regulations aimed at the intensive margin are important. We see that, with more variation in preferences, the optimum calls for some diversity in the set of contracts that are offered, and that subsidies that address selection on the intensive margin can increase welfare (Table E3).

Table E1: Calibrated distribution of consumer types

	A	H	M	S
Mean	1.9E-3	1,330	4,340	24,474
Log covariance				
A	0.25	-0.01	-0.12	0
H		$\sigma^2_{\log H}$	-0.03	0
M			0.20	0
S				0.25

Notes: Consumer types are log normally distributed with the moments in the table. The log variance of moral hazard $\sigma^2_{\log\ \mathrm{H}}$ is set equal to 0.98 and 2.

Table E2: Contracts used in the calibration with nonlinear contracts.

Contract number	Deductible	Coinsurance	Out of pocket maximum	Mean Coverage
0 (public insurance)	30,000	100%	30,000	29%
1	23,000	82%	23,500	34%
2	17,000	65%	17,500	40%
3	12,000	52%	13,000	45%
4	9,200	46%	10,000	51%
5	6,700	39%	7,800	56%
6 (bronze)	5,200	35%	6,400	60%
7	3,300	26%	5,000	66%
8	2,100	20%	4,200	71%
9	1,300	16%	3,500	77%
10	600	13%	2,900	82%
11	300	12%	2,700	87%
12 (Einav et al.)	0	10%	2,500	93%

Notes: Actuarial values equal the expected fraction of covered expenses for an average consumer type under privately optimal behavior.

Table E3: Prices, quantities, and welfare with non-linear contracts.

	$\sigma_A^2 = 0.25$						$\sigma_A^2 = 2$					
	No Mandate		Mandate		Optimum		No Mandate		Mandate		Optimum	
Contract number	Q	P	Q	P	Q	P	Q	P	Q	P	Q	P
0 (public insurance)	0.00	0			0.00	0	0.02	0			0.00	0
1	0.00	0			0.00	10,513	0.00	515			0.00	192
2	0.00	0			0.00	11,655	0.00	944			0.00	196
3	0.00	0			0.00	11,922	0.01	1,337			0.00	351
4	0.00	163			0.00	12,497	0.03	1,692			0.00	402
5	0.00	1,155			0.00	13,121	0.06	2,067			0.00	448
6 (bronze)	0.00	1,878	0.00	1,838	0.00	13,223	0.10	2,415	0.27	2,310	0.00	453
7	0.02	2,745	0.02	2,719	0.00	13,414	0.24	3,009	0.20	2,977	0.00	516
8	0.97	3,447	0.97	3,432	0.00	13,790	0.53	3,724	0.52	3,693	0.00	566
9	0.00	4,339	0.00	4,303	0.00	13,902	0.00	4,627	0.00	4,572	0.46	652
10	0.00	5,160	0.00	5,107	1.00	14,034	0.00	5,463	0.00	5,389	0.54	1,254
11	0.00	5,499	0.00	5,440	0.00	14,397	0.00	5,815	0.00	5,734	0.00	1,667
12 (Einav et al.)	0.00	5,969	0.00	5,904	0.00	14,942	0.00	6,305	0.00	6,216	0.00	2,233
Welfare	0		-6		371		0		-272		250	

Notes: The table reports results without government intervention, with a mandate of at least bronze insurance, and under optimal regulation.

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