

Perfect Competition in Markets with Adverse Selection*

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Abstract

This paper proposes a perfectly competitive model of a market with adverse selection. Prices are determined by zero-profit conditions, and the set of traded contracts is determined by free entry. Crucially for applications, contract characteristics are endogenously determined, consumers may have multiple dimensions of private information, and an equilibrium always exists. Equilibrium corresponds to the limit of a differentiated products Bertrand game. We apply the model to establish theoretical results on the equilibrium effects of mandates. Mandates can increase efficiency but have unintended consequences. With adverse selection, an insurance mandate reduces the price of low-coverage policies, which necessarily has indirect effects such as increasing adverse selection on the intensive margin and causing some consumers to purchase less coverage.

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1 Introduction

Policy makers and market participants consider adverse selection¹ a first-order concern in many markets. These markets are often heavily regulated, if not subject to outright government provision, as in social programs like unemployment insurance and Medicare. Government interventions are typically complex, involving the regulation of contract characteristics, personalized subsidies, community rating, risk adjustment, and mandates.² However, most models of competition with adverse selection take contract characteristics as given, limiting the scope of normative and even positive analyses of these policies.

Standard adverse selection models face three limitations. The first limitation arises in the [Akerlof \(1970\)](#) model, which, following [Einav et al. \(2010a\)](#), is used by most of the recent applied work.³ The Akerlof lemons model considers a market for a single contract with exogenous characteristics, making it impossible to consider the effect of policies that affect contract terms.⁴ In contrast, the [Spence \(1973\)](#) and [Rothschild and Stiglitz \(1976\)](#) models do allow for endogenous contract characteristics. However, they restrict consumers to be heterogeneous along a single dimension,⁵ despite evidence on the importance of multiple dimensions

¹Traditionally, a market is said to be adversely selected if one side of the market has private information, and the least desirable informed trading partners are those who are most eager to trade. The classic example is an used car market, where sellers with the lowest quality cars are those most willing to sell them. See [Akerlof \(1970\)](#).

²[Van de Ven and Ellis \(1999\)](#) survey health insurance markets across eleven countries with a focus on risk adjustment (cross-subsidies from insurers who enroll cheaper consumers to those who enroll more expensive ones). Their survey gives a glimpse of common regulations. There is risk adjustment in seventeen out of the eighteen markets. Eleven of them have community rating, which forbids price discrimination on some observable characteristics, such as age or preexisting conditions. Private sponsors also use risk adjustment and limit price discrimination. For example, large corporations in the United States typically offer a restricted number of insurance plans to their employees. Benefits consulting firms often advise companies to risk-adjust contributions due to concerns about adverse selection. This is in contrast to setting uniform employer contributions, what is known as a “fixed-dollar” or “flat rate” model ([Pauly et al., 2007](#); [Cutler and Reber, 1998](#)).

³Recent papers using this framework include [Handel et al. \(2015\)](#), [Hackmann et al. \(2015\)](#), [Mahoney and Weyl \(forthcoming\)](#), and [Scheuer and Smetters \(2014\)](#).

⁴Many authors highlight the importance of taking the determination of contract characteristics into account and the lack of a theoretical framework to deal with this. [Einav and Finkelstein \(2011\)](#) say that “abstracting from this potential consequence of selection may miss a substantial component of its welfare implications [...]. Allowing the contract space to be determined endogenously in a selection market raises challenges on both the theoretical and empirical front. On the theoretical front, we currently lack clear characterizations of the equilibrium in a market in which firms compete over contract dimensions as well as price, and in which consumers may have multiple dimensions of private information.” According to [Einav et al. \(2009\)](#), “analyzing price competition over a fixed set of coverage offerings [...] appears to be a relatively manageable problem, characterizing equilibria for a general model of competition in which consumers have multiple dimensions of private information is another matter. Here it is likely that empirical work would be aided by more theoretical progress.”

⁵[Chiappori et al. \(2006\)](#) highlight this shortcoming: “Theoretical models of asymmetric information typically use oversimplified frameworks, which can hardly be directly transposed to real-life situations. Rothschild and Stiglitz’s model assumes that accident probabilities are exogenous (which rules out moral hazard), that only one level of loss is possible, and more strikingly that agents have identical preferences which are moreover

of private information.⁶ Moreover, the Spence model suffers from rampant multiplicity of equilibria, while the Rothschild and Stiglitz model often has no equilibrium.⁷

In this paper, we develop a competitive model of adverse selection. The model incorporates three key features, motivated by the central role of contract characteristics in policy and by recent empirical findings. First, the set of traded contracts is endogenous, allowing us to study policies that affect contract characteristics.⁸ Second, consumers may have several dimensions of private information, engage in moral hazard, and exhibit deviations from rational behavior such as inertia and overconfidence.⁹ Third, equilibria always exist and yield sharp predictions. Equilibria are inefficient, and even simple interventions can raise welfare (measured as total surplus). Nevertheless, standard regulations have important unintended consequences once we take firm responses into account.

The key idea is to consistently apply the price-taking logic of the standard [Akerlof \(1970\)](#) and [Einav et al. \(2010a\)](#) models to the case of endogenous contract characteristics. Prices of traded contracts are set so that every contract makes zero profits. Moreover, whether a contract is offered depends on whether the market for that contract unravels, exactly as in the [Akerlof](#) single-contract model. For example, take an insurance market with a candidate equilibrium in which a policy is not traded at a price of \$1,100. Suppose that consumers would start buying the policy were its price to fall below \$1,000. Consider what happens as the price of the policy falls from \$1,100 to \$900 and buyers flock in. One case is that buyers are bad risks, with an average cost of, say, \$1,500. In this case, it is reasonable for the policy not to be traded because there is an adverse selection death spiral in the market for the policy. Another case is that buyers are good risks, with an expected cost of, say, \$500. In that case, the fact that the policy is not traded is inconsistent with free entry because any firm who entered the market for this policy would earn positive profits.

We formalize this idea as follows. The model takes as given a set of *potential* contracts and a distribution of consumer preferences and costs. A contract specifies all relevant characteristics, except for a price. Equilibrium determines both prices and the contracts that

perfectly known to the insurer. The theoretical justification of these restrictions is straightforward: analyzing a model of “pure,” one-dimensional adverse selection is an indispensable first step. But their empirical relevance is dubious, to say the least.”

⁶See [Finkelstein and McGarry \(2006\)](#), [Cohen and Einav \(2007\)](#), and [Fang et al. \(2008\)](#).

⁷According to [Chiappori et al. \(2006\)](#), “As is well known, the mere definition of a competitive equilibrium under asymmetric information is a difficult task, on which it is fair to say that no general agreement has been reached.” See also [Myerson \(1995\)](#).

⁸We study endogenous contract characteristics in the sense of determining, from a set of potential contracts, the ones that are traded and the ones that unravel as in [Akerlof \(1970\)](#), [Einav et al. \(2010a\)](#), and [Handel et al. \(2015\)](#). Unraveling is a central concern in the adverse selection literature. However, contract and product characteristics depend on many other factors, even when there is no adverse selection. This is a broader issue that we do not explore.

⁹See [Spinnewijn \(2015\)](#) on overconfidence, [Handel \(2013\)](#) and [Polyakova \(2014\)](#) on inertia, and [Kunreuther and Pauly \(2006\)](#) and [Baicker et al. \(2012\)](#) for discussions of behavioral biases in insurance markets.

are traded. A weak equilibrium is a set of prices and an allocation such that all consumers optimize and prices equal the average cost of supplying each contract. There are many weak equilibria because this notion imposes little discipline on which contracts are traded. For example, there are always weak equilibria where no contracts are bought because prices are high, and prices are high because the expected cost of a non-traded contract is arbitrary.

We make an additional requirement that formalizes the idea that entry into non-traded contracts is unprofitable. We require equilibria to be robust to a small perturbation of fundamentals. Namely, equilibria must survive in economies with a set of contracts that is similar to the original, but with a finite number of contracts, and with a small mass of consumers who demand all contracts and have low costs. The definition avoids pathologies related to conditional expectation over measure zero sets because all contracts are traded in a perturbation, much like the notion of a proper equilibrium in game theory (Myerson, 1978). The second part of our refinement is similar to the one used by Dubey and Geanakoplos (2002) in a model of competitive pools.

Competitive equilibria always exist and make sharp predictions in a wide range of applied models that are particular cases of our framework. The equilibrium matches standard predictions in the models of Akerlof (1970), Einav et al. (2010a), and Rothschild and Stiglitz (1976) (when their equilibrium exists). Besides the price-taking motivation, we give strategic foundations for the equilibrium, showing that it is the limit of a game-theoretic model of firm competition, which is similar to the models commonly used in the empirical industrial organization literature. We discuss in detail the relationship between our equilibrium and standard solution concepts below.

To exemplify the importance of contract characteristics and different dimensions of heterogeneity, we illustrate our framework in a calibrated health insurance model based on Einav et al. (2013), and study policy interventions. Consumers have four dimensions of private information, giving a glimpse of equilibrium behavior beyond standard one-dimensional models. There is moral hazard, so that welfare-maximizing regulation is more nuanced than simply mandating full insurance. We calculate the competitive equilibrium with firms offering contracts covering from 0% to 100% of expenditures. There is considerable adverse selection in equilibrium, creating scope for regulation. We calculate the equilibrium under a mandate that requires purchase of insurance with actuarial value of at least 60%. Figure 1 depicts the mandate's impact on coverage choices. A model that does not take firm responses into account would simply predict that consumers who originally bought less than 60% coverage would migrate to the least generous policy. In equilibrium, however, the influx of cheaper consumers into the 60% policy reduces its price, which in turn leads some of the consumers who were purchasing more comprehensive plans to reduce their coverage. Taking equilibrium effects into account, the mandate has important unintended consequences. The mandate

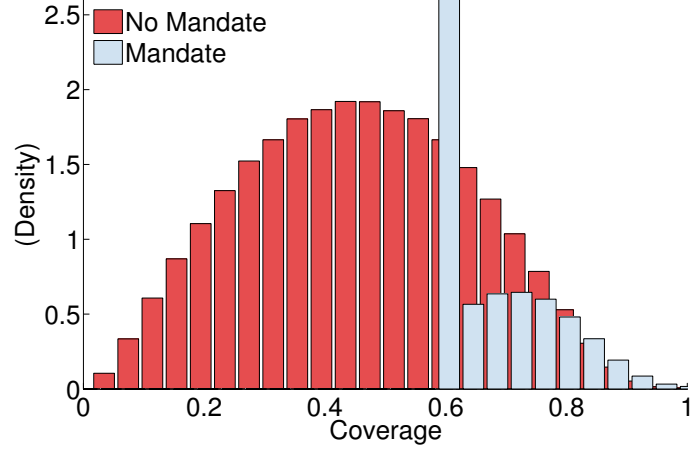


Figure 1: Equilibrium effects of a mandate.

Notes: The figure depicts the distribution of coverage choices in the numerical example from Section 5. In this health insurance model, consumers choose contracts that cover from 0% to 100% of expenses. The dark red bars represent the distribution of coverage in an unregulated equilibrium. The light gray bars represent coverage in equilibrium with a mandate forcing consumers to purchase at least 60% coverage. In the latter case, about 85% of consumers purchase the minimum coverage, and the bar at 60% is censored.

forces some consumers to increase their purchases to the minimum quality standard but also increases adverse selection on the intensive margin.

We derive theoretical comparative statics results on the effects of a mandate, that do not rely on the particular functional forms of the illustrative calibration. We show that increasing the minimum coverage of a mandate lowers the price of low-quality coverage by an amount approximately equal to a measure of adverse selection in the original equilibrium, due to the inflow of cheap consumers. Moreover, the mandate’s direct effect on prices implies that the mandate necessarily has knock-on effects, as in the illustrative calibration.

Finally, under our model, there is room for welfare-enhancing government intervention. For example, in the illustrative calibration, the mandate considerably increases consumer surplus, despite its unintended consequences. Moreover, policies that involve subsidies in the intensive margin can generate considerably higher consumer surplus than a simple mandate. We leave a detailed analysis of these issues for future work ([Azevedo and Gottlieb, in preparation](#)).

2 Model

2.1 The Model

We consider competitive markets with a large number of consumers and free entry of identical firms operating at an efficient scale that is small relative to the market. To model the gamut of behavior relevant to policy discussions in a simple way, we take as given a set of potential contracts, preferences, and costs of supplying contracts.¹⁰ We restrict attention to a group of consumers who are indistinguishable with respect to characteristics over which firms can price discriminate.

Formally, firms offer **contracts** (or products) x in X . Each consumer wishes to purchase a single contract. Consumer **types** are denoted θ in Θ . Consumer type θ derives utility $U(x, p, \theta)$ from buying contract x at a price p , and it costs a firm $c(x, \theta) \geq 0$ measured in units of a numeraire to supply it. Utility is strictly decreasing in price. There is a positive mass of consumers, and the **distribution of types** is a measure μ .¹¹ An **economy** is defined as $E = [\Theta, X, \mu]$.

2.2 Clarifying Examples

The following examples clarify the definitions, limitations of the model, and the goal of deriving robust predictions in a wide range of selection markets. Parametric assumptions in the examples are of little consequence to the general analysis, so some readers may prefer to skim over details. We begin with the classic [Akerlof \(1970\)](#) model, which is the dominant framework in applied work. It is simple enough that the literature mostly agrees on equilibrium predictions.

Example 1. (Akerlof) Consumers choose whether to buy a single insurance product, so that $X = \{0, 1\}$. Utility is quasilinear,

$$U(x, p, \theta) = u(x, \theta) - p, \tag{1}$$

and the contract $x = 0$ generates no cost or utility, $u(0, \theta) \equiv c(0, \theta) \equiv 0$. Thus, it has a price of 0 in equilibrium. All that matters is the joint distribution of willingness to pay $u(1, \theta)$ and costs $c(1, \theta)$, which is given by the measure μ .

A competitive equilibrium in the Akerlof model has a compelling definition and is amenable to an insightful graphical analysis. Following [Einav et al. \(2010a\)](#), let the demand curve $D(p)$ be the mass of consumers with willingness to pay higher than p , and let $AC(q)$ be the aver-

¹⁰This is similar to [Veiga and Weyl \(2014, 2016\)](#) and [Einav et al. \(2009, 2010a\)](#).

¹¹The relevant σ -algebra and detailed assumptions are described below.

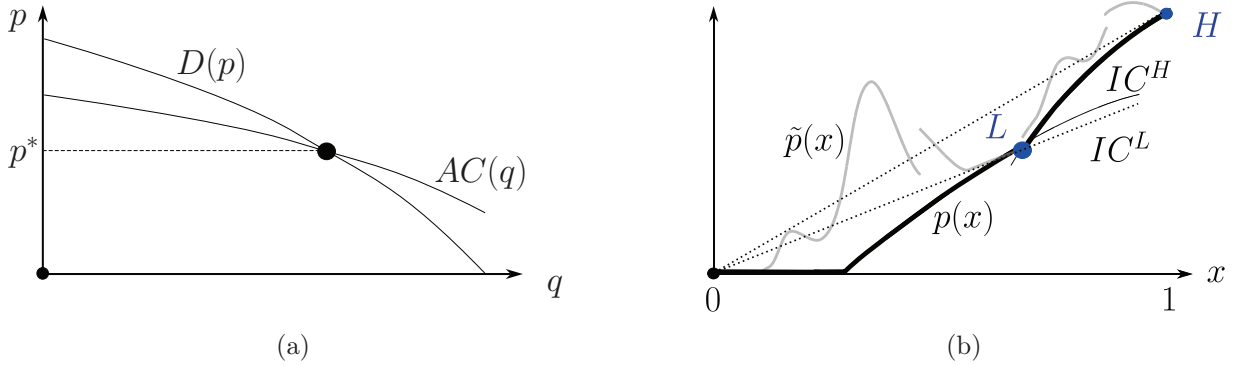


Figure 2: Weak equilibria in the (a) Akerlof and (b) Rothschild and Stiglitz models.

Notes: Panel (a) depicts demand $D(p)$ and average cost $AC(p)$ curves in the Akerlof model, with quantity on the horizontal axis, and prices on the vertical axis. The equilibrium price of contract $x = 1$ is denoted by p^* . Panel (b) depicts two weak equilibria of the Rothschild and Stiglitz model, with contracts on the horizontal axis and prices on the vertical axis. IC^L and IC^H are indifference curves of type L and H consumers. The dashed lines depict the contracts that give zero profits for each type. L and H denote the contract-price pairs chosen by each type in these weak equilibria, which are the same as in [Rothschild and Stiglitz \(1976\)](#) when their equilibrium exists. The bold curves $p(x)$ (black) and $\tilde{p}(x)$ (gray) depict two weak equilibrium price schedules. $p(x)$ is an equilibrium price, but $\tilde{p}(x)$ is not.

age cost of the q consumers with highest willingness to pay.¹² An equilibrium in the [Akerlof \(1970\)](#) and [Einav et al. \(2010a\)](#) sense is given by the intersection between the demand and average cost curves, depicted in [Figure 2a](#). At this price and quantity, consumers behave optimally and the price of insurance equals the expected cost of providing coverage. If the average cost curve is always above demand, then the market unravels and equilibrium involves no transactions. \square

This model is restrictive in two important ways. First, contract terms are exogenous. This is important because market participants and regulators often see distortions in contract terms as crucial. In fact, many of the interventions in markets with adverse selection regulate contract dimensions directly, aim to affect them indirectly, or try to shift demand from some type of contract to another. It is impossible to consider the effect of these policies in the Akerlof model. Second, there is a single non-null contract. This is also restrictive. For example, [Handel et al. \(2015\)](#) approximate health insurance exchanges by assuming that they offer only two types of plans (corresponding to $x = 0$ and $x = 1$), and that consumers

¹²Under appropriate assumptions, the definitions are

$$\begin{aligned} D(p) &= \mu\{\theta : u(1, \theta) \geq p\} \\ AC(q) &= \mathbb{E}[c(1, \theta) | \mu, u(1, \theta) \geq D^{-1}(q)]. \end{aligned}$$

are forced to choose one of them.¹³ Likewise, [Hackmann et al. \(2015\)](#) and [Scheuer and Smetters \(2014\)](#) lump the choice of buying any health insurance as $x = 1$.

The next example, the [Rothschild and Stiglitz \(1976\)](#) model, endogenously determines contract characteristics. However, preferences are stylized. Still, this model already exhibits problems with existence of equilibrium, and there is no consensus about equilibrium predictions.

Example 2. (Rothschild and Stiglitz) Each consumer may buy an insurance contract in $X = [0, 1]$, which insures her for a fraction x of a possible loss of l . Consumers differ only in the probability θ of a loss. Their utility is

$$U(x, p, \theta) = \theta \cdot v(W - p - (1 - x)l) + (1 - \theta) \cdot v(W - p),$$

where $v(\cdot)$ is a Bernoulli utility function and W is wealth, both of which are constant in the population. The cost of insuring individual θ with policy x is $c(x, \theta) = \theta \cdot x \cdot l$. The set of types is $\Theta = \{L, H\}$, with $0 < L < H \leq 1$. The definition of an equilibrium in this model is a matter of considerable debate, which we address in the next section. \square

We now illustrate more realistic multidimensional heterogeneity with an empirical model of preferences for health insurance used by [Einav et al. \(2013\)](#).

Example 3. (Einav et al.) Consumers are subject to a stochastic health shock l and, after the shock, decide the amount e they wish to spend on health services. Consumers are heterogeneous in their distribution of health shocks F_θ , risk aversion parameter A_θ , and moral hazard parameter H_θ .

For simplicity, we assume that insurance contracts specify the fraction $x \in X = [0, 1]$ of health expenditures that are reimbursed. Utility after the shock equals

$$CE(e, l; x, p, \theta) = [(e - l) - \frac{1}{2H_\theta}(e - l)^2] + [W - p - (1 - x)e],$$

where W is the consumer's initial wealth. The privately optimal health expenditure is $e = l + H_\theta \cdot x$, so, in equilibrium,

$$CE^*(l; x, p, \theta) = W - p - l + l \cdot x + \frac{H_\theta}{2} \cdot x^2.$$

[Einav et al. \(2013\)](#) assume constant absolute risk aversion (CARA) utility before the health

¹³In accordance with the Affordable Care Act, health exchanges offer bronze, gold, silver and platinum plans, with approximate actuarial values ranging from 60% to 90%. Within each category, plans still vary in important dimensions such as the quality of their hospital networks. Silver is the most popular option, and over 10% of adults were uninsured in 2014.

shock, so that ex-ante utility equals

$$U(x, p, \theta) = \mathbb{E}[-\exp\{-A_\theta \cdot CE^*(l; x, p, \theta)\} | l \sim F_\theta].$$

For our numerical examples below, losses are normally distributed with mean M_θ and variance S_θ^2 , which leaves four dimensions of heterogeneity.¹⁴ Calculations show that the model can be described with quasilinear preferences as in equation (1), with willingness to pay and cost functions

$$\begin{aligned} u(x, \theta) &= x \cdot M_\theta + \frac{x^2}{2} \cdot H_\theta + \frac{1}{2}x(2-x) \cdot S_\theta^2 A_\theta, \text{ and} \\ c(x, \theta) &= x \cdot M_\theta + x^2 \cdot H_\theta. \end{aligned} \tag{2}$$

The formula decomposes willingness to pay into three terms: average covered expenses xM_θ , utility from overconsumption of health services $x^2H_\theta/2$, and risk-sharing $x(2-x) \cdot S_\theta^2A_\theta/2$. Since firms are responsible for covered expenses, the first term also enters firm costs. Overconsuming health services, which is caused by moral hazard, costs firms twice as much as consumers are willing to pay for it. Moreover, the risk-sharing value of the policy is increasing in coverage, in the consumer's risk aversion, and in the variance of health shocks. However, because firms are risk neutral, the risk-sharing term does not enter firm costs. \square

The example illustrates that the framework can fit multidimensional heterogeneity in a more realistic empirical model. Moreover, it can incorporate ex-post moral hazard through the definitions of the utility and cost functions. The model can fit other types of consumer behavior, such as ex-ante moral hazard, non-expected utility, overconfidence, or inertia to abandon a default choice. It can also incorporate administrative or other per-unit costs on the supply side. Moreover, it is straightforward to consider more complex contract features, including deductibles, copays, stop-losses, franchises, network quality, and managed restrictions on expenses.

In the last example, and in other models with complex contract spaces and rich heterogeneity, there is no agreement on a reasonable equilibrium prediction. Unlike the Rothschild and Stiglitz model, where there is controversy about what the correct prediction is, in this case the literature offers almost no possibilities.

¹⁴Because of the normality assumption, losses and expenses may be negative in the numerical example. We report this parametrization because the closed form solutions for utility and cost functions make the model more transparent. In the supplementary material, we calibrate a model with log-normal loss distributions and nonlinear contracts and finds similar qualitative results.

2.3 Assumptions

The assumptions we make are mild enough to include all the examples above, so applied readers may wish to skip this section. On a first read, it is useful to keep in mind the particular case where X and Θ are compact subsets of Euclidean space, utility is quasilinear as in equation (1), and u and c are continuously differentiable. These assumptions are considerably stronger than what is needed, but they are weak enough to incorporate most models in the literature. We begin with technical assumptions.

Assumption 1. (*Technical Assumptions*) X and Θ are compact and separable metric spaces. Whenever referring to measurability we will consider the Borel σ -algebra over X and Θ , and the product σ -algebra over the product space. In particular, we take μ to be defined over the Borel σ -algebra.

Note that X and Θ can be infinite dimensional, and the distribution of types can admit a density with infinite support, may be a sum of point masses, or a combination of the two. We now consider a more substantive assumption. Let $d(x, x')$ denote the distance between contracts x and x' .

Assumption 2. (*Bounded Marginal Rates of Substitution*) There exists a constant L with the following property. Take any $p \leq p'$ in the image of c , any x, x' in X , and any $\theta \in \Theta$. Assume that

$$U(x, p, \theta) \leq U(x', p', \theta),$$

that is, that a consumer prefers to pay more to purchase contract x' instead of x . Then, the price difference is bounded by

$$p' - p \leq L \cdot d(x, x').$$

That is, the willingness to pay for an additional unit of any contract dimension is bounded. The assumption is simpler to understand when utility is quasilinear and differentiable. In this case it is equivalent to the absolute value of the derivative of u being uniformly bounded.

Assumption 3. (*Continuity*) The functions U and c are continuous in all arguments.

Continuity of the utility function is not very restrictive because of Berge's Maximum Theorem. Even with moral hazard, utility is continuous under standard assumptions. Continuity of the cost function is more restrictive. It implies that we can only consider models with moral hazard where payoffs to the firm vary continuously with types and contracts. This may fail if consumers change their actions discontinuously with small changes in a contract. Nevertheless, it is possible to include some models with moral hazard in our framework. See [Kadan et al. \(2014\)](#) Section 9 for a discussion of how to define a metric over a contract space, starting from a description of actions and states.

3 Competitive Equilibrium

3.1 Weak Equilibrium

We now define a minimalistic equilibrium notion, a weak equilibrium, requiring only that firms make no profits and consumers optimize. A vector of **prices** is a measurable function $p : X \rightarrow \mathbb{R}$, with $p(x)$ denoting the price of contract x . An **allocation** is a measure α over $\Theta \times X$ such that the marginal distribution satisfies $\alpha|\Theta = \mu$. That is, $\alpha(\{\theta, x\})$ is the measure of θ types purchasing contract x .¹⁵ We are often interested in the expected cost of supplying a contract x and use the following shorthand notation for conditional moments:

$$\mathbb{E}_x[c|\alpha] = \mathbb{E}[c(\tilde{x}, \tilde{\theta})|\alpha, \tilde{x} = x].$$

That is, $\mathbb{E}_x[c|\alpha]$ is the expectation of $c(\tilde{x}, \tilde{\theta})$ according to the measure α and conditional on $\tilde{x} = x$. Note that such expectations depend on the allocation α . When there is no risk of confusion we omit α , writing simply $\mathbb{E}_x[c]$. Similar notation is used for other moments.

Definition 1. The pair (p^*, α^*) is a **weak equilibrium** if

1. For each contract x , firms make no profits. Formally,

$$p^*(x) = \mathbb{E}_x[c|\alpha^*]$$

almost everywhere according to α^* .

2. Consumers select contracts optimally. Formally, for almost every (θ, x) with respect to α^* , we have

$$U(x, p^*(x), \theta) = \sup_{x' \in X} U(x', p^*(x'), \theta).$$

This is a price-taking definition, not a game-theoretic one. Consumers optimize taking prices as given, as do firms, who also take the average costs of buyers as given. We do not require that all consumers participate. This can be modeled by including a null contract that costs nothing and provides zero utility.

A weak equilibrium requires firms to make zero profits on every contract. This is a substantial economic restriction, as it rules out cross-subsidies between contracts. In fact, there are competitive models, such as in [Wilson \(1977\)](#) and [Miyazaki \(1977\)](#), where firms earn zero profits overall but can have profits or losses on some contracts. It is possible to micro-found the requirement of zero profits on each contract with a strategic model with

¹⁵This formalization is slightly different than the traditional way of denoting an allocation as a map from types to contracts. We take this approach because different consumers of the same type may buy different contracts in equilibrium, as in [Chiappori et al. \(2010\)](#).

differentiated products, as discussed in Section 4.2. Intuitively, in this kind of model, a firm that tries to cross-subsidize contracts is undercut in contracts that it taxes and is left selling the contracts that it subsidizes.

We only ask that prices equal expected costs almost everywhere.¹⁶ In particular, weak equilibria place no restrictions on the prices of contracts that are not purchased. As demonstrated in the examples below, this is a serious problem with this definition and the reason why a stronger equilibrium notion is necessary.

3.2 Equilibrium Multiplicity and Free Entry

We now illustrate that weak equilibria are compatible with a wide variety of outcomes, most of which are unreasonable in a competitive marketplace.

Example 2'. (Rothschild and Stiglitz - Multiplicity of Weak Equilibria) We first revisit Rothschild and Stiglitz's (1976) original equilibrium. They set up a Bertrand game with identical firms and showed that, when a Nash equilibrium exists, it has allocations given by the points L and H in Figure 2b. High-risk consumers buy full insurance $x_H = 1$ at actuarially fair rates $p_H = H \cdot l$. Low-risk types purchase partial insurance, with actuarially fair prices reflecting their lower risk. The level of coverage x_L is just low enough so that high-risk consumers do not wish to purchase contract x_L . That is, L and H are on the same indifference curve IC^H of high types.

Note that we can find weak equilibria with the same allocation. One example of weak equilibrium prices is the curve $p(x)$ in Figure 2b. The zero profits condition is satisfied because the prices of the two contracts that are traded, x_L and x_H , equal the average cost of providing them. The optimization condition is also satisfied because the price schedule $p(x)$ is above the indifference curves IC^L and IC^H . Therefore, no consumer wishes to purchase a different contract.

However, many other weak equilibria exist. One example is the same allocation with the prices $\tilde{p}(x)$ in Figure 2b. Again, firms make no profits because the prices of x_H and x_L are actuarially fair, and consumers are optimizing because the price of other contracts is higher than their indifference curves.

There are also weak equilibria with completely different allocations. For example, it is a weak equilibrium for all consumers to purchase full insurance, and for all other contracts to be priced so high that no one wishes to buy them. This does not violate the zero profits condition because the expected cost of contracts that are not traded is arbitrary. This weak equilibrium has full insurance, which is the first-best outcome in this model. It is also a weak

¹⁶The reason is that conditional expectation is only defined almost everywhere. Although it is possible to understand all of our substantive results without recourse to measure theory, we refer interested readers to Billingsley (2008) for a formal definition of conditional expectation.

equilibrium for no insurance to be sold, and for prices of all contracts with positive coverage to be prohibitively high. Therefore, weak equilibria provide very coarse predictions, with the Bertrand solution, full insurance, complete unraveling, and many other outcomes all being possible. \square

In a market with free entry, however, some weak equilibria are more reasonable than others. Consider the case of $H < 1$ and take the weak equilibrium with complete unravelling. Suppose firms enter the market for a policy with positive coverage, driving down its price. Initially no consumers purchase the policy, and firms continue to break even. As prices decrease enough to reach the indifference curve of high-risk consumers, they start buying. At this point, firms make money because risk averse consumers are willing to pay a premium for insurance. Therefore, this weak equilibrium conflicts with the idea of free entry. A similar tâtonnement eliminates the full-insurance weak equilibrium. If firms enter the market for partial insurance policies, driving down prices, they do not attract any consumers at first. However, once prices decrease enough to reach the indifference curve of low-risk consumers, firms only attract good risks and therefore make positive profits.

The same argument eliminates the weak equilibrium associated with $\tilde{p}(x)$. Let $x_0 < x_L$ be a non-traded contract with $\tilde{p}(x_0) > p(x)$. Suppose firms enter the market for x_0 , driving down its price. Initially no consumers purchase x_0 , and firms continue to break even. As prices decrease enough to reach $p(x_0)$, the L types become indifferent between purchasing x_0 or not. If they decrease any further, all L types purchase contract x_0 . At this point, firms lose money because average cost is higher than the price.¹⁷ The price of x_0 is driven down to $p(x_0)$, at which point it is no longer advantageous for firms to enter. In fact, this argument eliminates all but the weak equilibrium with price $p(x)$ and the allocation in Figure 2b.

3.3 Definition and Existence of an Equilibrium

We now define an equilibrium concept that formalizes the free entry argument. Equilibria are required to be robust to small perturbations of a given economy. A perturbation has a large but finite set of contracts approximating X . The perturbation adds a small measure of behavioral types, who always purchase each of the existing contracts and impose no costs on firms. The point of considering perturbations is that all contracts are traded, eliminating the paradoxes associated with defining the average cost of non-traded contracts.

We introduce, for each contract x , a behavioral consumer type who always demands contract x . We write x for such a behavioral type and extend the utility and cost functions as $U(x, p, x) = \infty$, $U(x', p, x) = 0$ if $x' \neq x$, and $c(x, x) = 0$. For clarity, we refer to non-behavioral types as standard types.

¹⁷To see why, note that L types buy x_L at an actuarially fair price. Therefore, they would only purchase less insurance if firms sold it at a loss.

Definition 2. Consider an economy $E = [\Theta, X, \mu]$. A perturbation of E is an economy with a finite set of contracts $\bar{X} \subseteq X$ and a small mass of behavioral types demanding each contract in \bar{X} . Formally, a **perturbation** (E, \bar{X}, η) is an economy $[\Theta \cup \bar{X}, \bar{X}, \mu + \eta]$, where $\bar{X} \subseteq X$ is a finite set, and η is a strictly positive measure over \bar{X} .¹⁸

The next definition says that a sequence of perturbations converges to the original economy if the set of contracts fills in the original set of contracts and the total mass of behavioral consumers converges to 0.

Definition 3. A sequence of perturbations $(E, \bar{X}^n, \eta^n)_{n \in \mathbb{N}}$ converges to E if

1. Every point in X is the limit of a sequence $(x^n)_{n \in \mathbb{N}}$ with each $x^n \in \bar{X}^n$.
2. The total mass of behavioral types $\eta^n(\bar{X}^n)$ converges to 0.

We now define what it means for a sequence of equilibria of perturbations to converge to the original economy.

Definition 4. Take an economy E and a sequence of perturbations $(E, \bar{X}^n, \eta^n)_{n \in \mathbb{N}}$ converging to E , with weak equilibria (p^n, α^n) . The **sequence of weak equilibria** $(p^n, \alpha^n)_{n \in \mathbb{N}}$ converges to a **price-allocation pair** (p^*, α^*) of E if

1. The allocations α^n converge weakly to α^* .
2. For every sequence $(x^n)_{n \in \mathbb{N}}$ with each $x^n \in \bar{X}^n$ and limit $x \in X$, $p^n(x^n)$ converges to $p^*(x)$.¹⁹

We are now ready to define an equilibrium.

Definition 5. The pair (p^*, α^*) is an **equilibrium** of E if there exists a sequence of perturbations that converges to E and an associated sequence of weak equilibria that converges to (p^*, α^*) .

The most transparent way to understand how equilibrium formalizes the free entry idea is to return to the [Rothschild and Stiglitz](#) model from [example 2](#). Recall that there is a weak equilibrium where no one purchases insurance and prices are high. But this is not an equilibrium. A perturbation cannot have such high-price equilibria because, if standard

¹⁸Both an economy and its perturbations have a set of types contained in $\Theta \cup X$ and contracts contained in X . To save on notation, we extend distributions of types to be defined over $\Theta \cup X$ and allocations to be defined over $(\Theta \cup X) \times X$. With this notation, measures pertaining to different perturbations are defined on the same space.

¹⁹In a perturbation, prices are only defined for a finite subset \bar{X}^n of contracts. The definition of convergence is strict in the sense that, for a given contract x , prices must converge to the price of x for any sequence of contracts $(x^n)_{n \in \mathbb{N}}$ converging to x .

types do not purchase insurance, prices are driven to 0 by behavioral types. Likewise, the weak equilibrium corresponding to \tilde{p} in Figure 2b is not an equilibrium. Consider a contract x_0 with $\tilde{p}(x_0) > p(x_0)$. In any perturbation, if prices are close to \tilde{p} , then only behavioral types would buy x_0 . But this would make the price of x_0 equal to 0 because the only way to sustain positive prices in a perturbation is by attracting standard types. In fact, equilibria of perturbations sufficiently close to E involve most L types purchasing contracts similar to x_L , and most H types purchasing contracts similar to x_H . The price of any contract $x_0 < x_L$ must make L types indifferent between x_0 and x_L . There is a small mass of L types purchasing x_0 to maintain the indifference. If prices were lower, L types would flood the market for x_0 , and firms would lose money. If prices were higher, no L types would purchase x_0 . The only equilibrium is that corresponding to $p(x)$ in Figure 2b (this is proven in Corollary 1).

The mechanics of equilibrium are similar to the standard analysis of the Akerlof model from example 1. In the example depicted in Figure 2a, the only equilibrium is that associated with the intersection of demand and average cost.²⁰ This is similar to the way that prices for x_L and x_H are determined in example 2. If the average cost curve were always above the demand curve, the only equilibrium would be complete unraveling. This is analogous to the way that the market for contracts other than x_L and x_H unravels.

There are two ways to think about the equilibrium requirement. One is that it consistently applies the logic of Akerlof (1970) and Einav et al. (2010a) to the case where there is more than one potential contract. This is similar to the intuitive free entry argument discussed in Section 3.2. Another interpretation is that the definition demands a minimal degree of robustness with respect to perturbations, while paradoxes associated with conditional expectation do not occur in perturbations. This rationale is similar to proper equilibria (Myerson, 1978).

We now show that equilibria always exist.

Theorem 1. *Every economy has an equilibrium.*

The proof is based on two observations. First, equilibria of perturbations exist by a standard fixed-point argument. Second, equilibrium price schedules in any perturbation are uniformly Lipschitz. This is a consequence of the bounded marginal rate of substitution (Assumption 2). The intuition is that, if prices increased too fast with x , no standard types would be willing to purchase more expensive contracts. This is impossible, however, because a contract cannot have a high equilibrium price if it is only purchased by the low-cost behavioral types. We then apply the Arzelà–Ascoli Theorem to demonstrate existence of equilibria.

²⁰There are other weak equilibria in the example in Figure 2a, but the only equilibrium is the intersection between demand and average cost. For example, it is a weak equilibrium for no one to purchase insurance, and for prices to be very high. But this is not an equilibrium. The reason is that, in a perturbation, behavioral types make the average cost curve well-defined for all quantities, including 0. The perturbed average cost curve is continuous, equal to 0 at a quantity of 0, and slightly lower than the original. As the mass of behavioral types shrinks, the perturbed average cost curve approaches its value in the original economy. Consequently, the only equilibrium is the standard solution, where demand and average cost intersect.

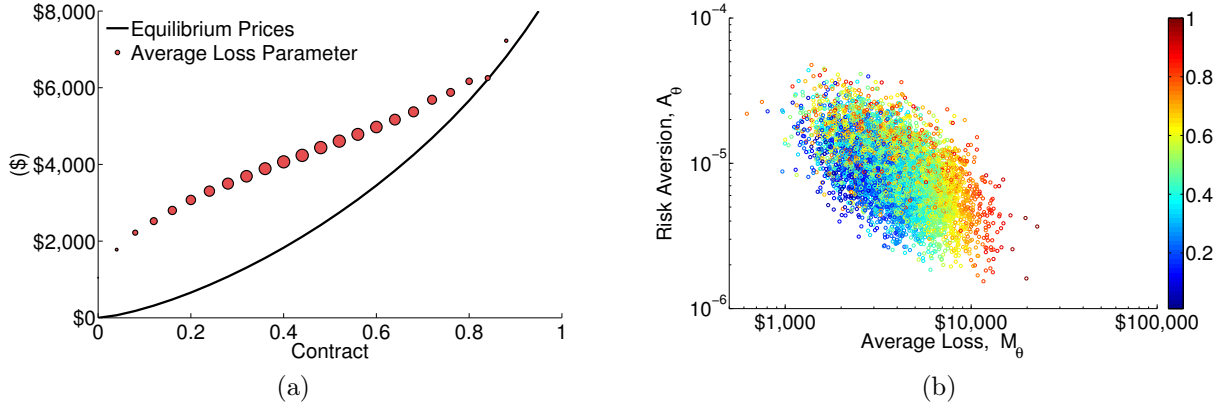


Figure 3: Equilibrium prices (a) and demand profile (b) in the multidimensional health insurance model from example 3.

Notes: Panel (a) illustrates equilibrium prices and quantities in example 3 under benchmark parameters. The solid curve denotes prices. The size of the circles represent the mass of consumers purchasing each contract, and its height represents the average loss parameter of such consumers, that is $\mathbb{E}_x[M]$. Panel (b) illustrates the equilibrium demand profile. Each point represents a randomly drawn type from the population. The horizontal axis represents expected health shock M_θ , and the vertical axis represents the absolute risk aversion coefficient A_θ . The colors represent the level of coverage purchased in equilibrium.

Existence only depends on the assumptions of Section 2.3. Therefore, equilibria are well-defined in a broad range of theoretical and empirical models. Equilibria exist not only in stylized models, but also in rich multidimensional settings. Figure 3 plots an equilibrium in a calibration of the Einav et al. model (example 3). Equilibrium makes sharp predictions, displays adverse selection, with costlier consumers purchasing higher coverage, and consumers sort across the four dimensions of private information. We return to this example below.

4 Discussion

This section establishes consequences of competitive equilibrium, and discusses the relationship to existing solution concepts.

4.1 Equilibrium Properties

We begin by describing some properties of equilibria.

Proposition 1. *Let (p^*, α^*) be an equilibrium of economy E . Then:*

1. *The pair (p^*, α^*) is a weak equilibrium of E .*

2. For every contract $x' \in X$ with strictly positive price, there exists (θ, x) in the support of α^* such that

$$U(x, p^*(x), \theta) = U(x', p^*(x'), \theta) \quad \text{and} \quad c(x', \theta) \geq p(x').$$

That is, every contract that is not traded in equilibrium has a low enough price for some consumer to be indifferent between buying it or not, and the cost of this consumer is at least as high as the price.

3. The price function is L -Lipschitz, and, in particular, continuous.
4. If X is a subset of Euclidean space, then p^* is Lebesgue almost everywhere differentiable.

The proposition shows that equilibria have several regularity properties. They are weak equilibria. Moreover, equilibrium prices are continuous and differentiable almost everywhere. Finally, the price of an out-of-equilibrium contract is either 0 or low enough that some type is indifferent between buying it or not. In that case, the cost of selling to this indifferent type is at least as high as the price. Intuitively, these are the consumer types who make the market for this contract unravel.²¹

With these properties, we can solve for equilibrium in the Rothschild and Stiglitz model:

Corollary 1. *Consider example 2. If $H < 1$, the unique equilibrium is the price p and allocation in Figure 2b. If $H = 1$, the market unravels with equilibrium prices of $x \cdot l$ and low types purchasing no insurance.*

The corollary shows that equilibrium coincides with the Riley (1979) equilibrium and with the Rothschild and Stiglitz (1976) equilibrium when it exists.²² Therefore, competitive equilibrium delivers the standard results in the particular cases of Akerlof (1970) and Rothschild

²¹These conditions are necessary but not sufficient for an equilibrium. The reason is that the existence of a type satisfying the conditions in Part 2 of the proposition does not imply that the market for a contract x would unravel in a perturbation. This may happen because there can be other types who are indifferent between purchasing x or not, and some of them may have lower costs. It is simple to construct these examples in models similar to Chang (2010) or Guerrieri and Shimer (2015).

²²There is some controversy over whether the Riley (1979) equilibrium is reasonable and whether other notions, such as the Wilson (1977) equilibrium, are more compelling. The Riley allocation has been criticized because it is constrained Pareto inefficient when there are few H types (Crocker and Snow, 1985a), and because the equilibrium does not depend on the proportion of each type and changes discontinuously to full insurance when the measure of H types is 0 (Mailath et al., 1993). Although our solution concept inherits these counter-intuitive predictions, we see it as reasonable, especially in the richer settings in which we are interested, for two reasons. First, the assumptions made by Rothschild and Stiglitz are extreme and counter-intuitive. Namely, they assume that there are only two types of consumers, and that consumers are heterogeneous along a single dimension. Thus, the counter-intuitive results are driven not only by the equilibrium concept but also by counter-intuitive assumptions. We give some evidence that the Rothschild and Stiglitz setting is atypical in the supplementary material. We show that, under certain assumptions, generically, the set of competitive equilibria varies continuously with fundamentals. Moreover, whenever there is some pooling (as in example 3), equilibrium depends on the distribution of types. Second, our

and Stiglitz (1976). Moreover, simple arguments based on Proposition 1 can be used to solve models with richer heterogeneity, such as Netzer and Scheuer (2010), where the analysis of game-theoretic solution concepts is challenging.

4.2 Strategic Foundations

Our equilibrium concept can be justified as the limit of a strategic model, which is similar to the models used in the empirical industrial organization literature. This relates our work to the literature on game-theoretic competitive screening models and the industrial organization literature on adverse selection. Moreover, the assumptions on the strategic game clarify the limitations of our model and the situations where competitive equilibrium is a reasonable prediction.

We consider such a strategic setting in the Online Appendix A. We start from a perturbation (E, \bar{X}, η) . Each contract has n differentiated varieties, and each variety is sold by a different firm. Consumers have logit demand with semi-elasticity σ . Firms have a small efficient scale. To capture this in a simple way, we assume that each firm can only serve up to a fraction k of consumers. Firms cannot turn away consumers, as with community rating regulations.²³ The key parameters are the number of varieties of each contract n , the semi-elasticity of demand σ , and the maximum scale of each firm k .

We consider symmetric Bertrand-Nash equilibria, where firms independently set prices. Proposition A1 shows that Bertrand-Nash equilibria exist as long as firm scale is sufficiently small and there are enough firms selling each product to serve the whole market. The maximum scale that guarantees existence is of the order of the inverse of the semi-elasticity. Therefore, equilibria exist even if demand is close to the limit of no differentiation. At a first blush, this result seems to contradict the finding that the Rothschild and Stiglitz model often has no Nash equilibrium (Riley, 1979). The reason why Bertrand-Nash equilibria exist is that the profitable deviations in the Rothschild and Stiglitz model rely on firms setting very low prices and attracting a sizable portion of the market. However, this is not possible

model produces intuitive predictions and comparative statics in our calibrated example in Section 5. While we see our framework as a reasonable first step to study markets with rich consumer heterogeneity, it would be interesting to explore alternative equilibrium notions in settings with rich heterogeneity. For example, it would be interesting to generalize the Wilson (1977) equilibrium to such settings. Moreover, we caution readers that, while we seek to propose a useful framework that can be applied more generally, we do not seek to resolve the debate about whether the Riley (1979) or the Wilson (1977) allocations are more reasonable in the Rothschild and Stiglitz example. Nevertheless, we believe that exploring alternative equilibrium notions in settings with more realistic assumptions on preferences can contribute to understanding what equilibrium notions produce useful predictions in these settings.

²³Guerrieri et al. (2010) considered a directed search model where firms can turn away consumers, and established that, as search frictions vanish, their equilibria converges to the competitive equilibrium in the Rothschild and Stiglitz model. However, equilibria of this kind of model do not converge to competitive equilibrium in general. For further discussion see the Online Appendix.

if firms have small scale and cannot turn consumers away. Besides establishing existence of a Bertrand-Nash equilibrium, Proposition A1 shows that profits per contract are bounded above by a term of order $1/\sigma$ plus a term of order k .

Proposition A2 then shows that, for a sequence of parameters satisfying the conditions for existence and with semi-elasticity converging to infinity, Bertrand-Nash equilibria converge to a competitive equilibrium. Thus, competitive equilibrium corresponds to the limit of this game-theoretic model.

A limitation of this result is that each firm offers a single contract, as opposed to a menu of contracts. In particular, the strategic model rules out the possibility that firms cross-subsidize contracts, which is a key requirement of our equilibrium notion. To address this, we generalize our convergence results to the case where firms offer menus of contracts in the supplementary material. This generalization shows that, even if firms can cross-subsidize contracts, in equilibrium they do not do so, and earn low profits on all contracts.

These results have four implications. First, convergence to competitive equilibrium is relatively brittle because it depends on the Bertrand assumption, on the number of varieties and maximum scale satisfying a pair of inequalities, and on semi-elasticities growing at a fast enough rate relative to those parameters. This is to be expected because existing strategic models lead to very different conclusions with small changes in assumptions. Second, although convergence depends on special assumptions, it is not a knife-edge case. There exists a non-trivial set of parameters for which equilibria are justified by a strategic model.

Third, our results relate two types of models in the literature. Our strategic model is closely related to the differentiated products models in the industrial organization literature, such as [Starc \(2014\)](#), [Decarolis et al. \(2012\)](#), [Mahoney and Weyl \(forthcoming\)](#), and [Tebaldi \(2015\)](#). Our results show that our competitive equilibrium corresponds to a particular limiting case of these models. This implies that the models of [Riley \(1979\)](#) and [Handel et al. \(2015\)](#) are also limiting cases of the differentiated products models, because their equilibria coincide with ours in particular cases, as discussed below.

Finally, the sufficient conditions give insight into situations where competitive equilibrium is reasonable. Namely, when there are many firms, efficient firm scale is small relative to the market, and firms are close to undifferentiated. The results do not imply that markets with adverse selection are always close to perfect competition. Indeed, market power is often an issue in these markets (see [Dafny, 2010](#), [Dafny et al., 2012](#), and [Starc, 2014](#)). Nevertheless, the sufficient conditions are similar to those in markets without adverse selection: the presence of many, undifferentiated firms, with small scale relative to the market (see [Novshek and Sonnenschein, 1987](#)).

4.3 Unravelling and Robustness to Changes in Fundamentals

It is possible that there is no trade in one or all competitive equilibria. This is illustrated in Corollary 1 and in other particular cases of our model. For example, with one contract (example 1), there is no trade if average cost is always above the demand curve, as in Akerlof’s classic example. Hendren (2013) gives a no-trade condition in a binary loss model with a richer contract space.

Unravelling examples such as those in Hendren (2013) raise the question of whether competitive equilibria are too sensitive to small changes in fundamentals. For example, consider an Akerlof model as in example 1, with a unique equilibrium, which has a positive quantity. Suppose we add a positive but small mass of a type who values every non-null contract more than all other types, say \$1,000,000, and has even higher costs, say \$2,000,000. This change in fundamentals creates a new equilibrium where all contracts cost \$1,000,000 and no contracts are traded (although there may be other equilibria close to the original one).

We examine the robustness of the set of equilibria with respect to fundamentals in the supplementary material. We give examples in the one-contract case where adding a small mass of high-cost types introduces a new equilibrium with complete unraveling. However, competitive equilibria have two important generic robustness properties. First, generically, equilibria with trade are never considerably affected by the introduction of a small measure of high-cost types. Second, generically, small changes to demand and average cost curves lead to small changes in the set of equilibria. That is, the only way to produce large changes in equilibrium predictions is to considerably move average cost or demand curves. In particular, the \$1,000,000 example only works because it considerably changes expected costs conditioning on the consumers who have sufficiently high willingness to pay. This would not be possible if, for example, the original model already had consumers with high willingness to pay. Finally, the supplementary material includes a formal result showing that the latter robustness property holds with many potential contracts.

4.4 Equilibrium Multiplicity and Pareto Ranked Equilibria

Competitive equilibria may not be unique. This is the case, for example, in the Akerlof model (Example 1) when average cost and demand cross at multiple points. This example is counterintuitive because equilibria are Pareto ranked, so market participants may attempt to coordinate on the Pareto superior equilibrium. Moreover, only the lowest-price crossing of average cost and demand is an equilibrium under the standard strategic equilibrium concept in Einav et al. (2010a). Thus, in applications, a researcher may choose to select Pareto dominant equilibria, as commonly done in dynamic oligopoly models and cheap talk games.

While this selection is sometimes compelling, we note that multiple equilibria are a standard feature of Walrasian models. There is experimental evidence that multiple equilibria are observed in competitive markets where supply is downward sloping (Plott and George, 1992). In markets with adverse selection, Wilson (1980) pointed out the potential multiplicity of equilibria, and Scheuer and Smetters (2014) used multiple equilibria to study how market outcomes depend on initial conditions.²⁴

4.5 Relationship to the Literature

Our price-taking approach is reminiscent of the early work by Akerlof (1970) and Spence (1973). Multiplicity of weak equilibria is well-known since Spence’s (1973) analysis of labor market signaling.

The literature addressed equilibrium multiplicity in three ways. One strand of the literature employed game-theoretic equilibrium notions and restrictions on consumer heterogeneity, typically in the form of ordered one-dimensional sets of types. This is the case in the competitive screening literature, initiated with Rothschild and Stiglitz’s (1976) Bertrand game, which led to the issue of non-existence of equilibria. Subsequently, Riley (1979) showed that Bertrand equilibria do not exist for a broad (within the one-dimensional setting) class of preferences, including the standard Rothschild and Stiglitz model with a continuum of types. Wilson (1977), Miyazaki (1977), Riley (1979), and Netzer and Scheuer (2014), among others, proposed modifications of Bertrand equilibrium so that an equilibrium exists. It has long been known that the original Rothschild and Stiglitz game has mixed strategy equilibria, but only recently Luz (2013) has characterized them.²⁵

The literature on refinements in signaling games shares the features of game-theoretic equilibrium notions and restrictive type spaces. In order to deal with the multiplicity of price-taking equilibria described by Spence, this literature modeled signaling as a dynamic game. However, since signaling games typically have too many sequential equilibria, Banks and Sobel (1987), Cho and Kreps (1987), and several subsequent papers proposed equilibrium refinements that eliminate multiplicity.

²⁴Moreover, game theorists debate whether selecting Pareto dominant equilibria is reasonable, and when well-motivated refinements produce this selection (see Chen et al., 2008 for a discussion of this issue in cheap talk models). Unfortunately, these refinements do not immediately select Pareto efficient equilibria in our model. The most closely related paper is Ambrus and Argenziano (2009), who apply “Nash equilibrium in coalitionally rationalizable strategies” to a two-sided markets model. Their refinement guarantees, for example, that consumers do not all coordinate on an inferior platform. However, in Example 1, the coordination failure depends on both consumer and firm behavior. Moreover, firms are indifferent between all equilibria, because they earn zero profits. Thus, the Ambrus-Argenziano approach does not rule out the Pareto dominated equilibria in our setting.

²⁵There has also been work on this type of game with nonexclusive competition. Attar et al. (2011) show that nonexclusive competition leads to outcomes similar to the Akerlof model. The game we consider in Section 4.2 is related to the search models of Inderst and Wambach (2001) and Guerrieri et al. (2010).

Another strand of the literature considers price-taking equilibrium notions, like our work, but imposes additional structure on preferences, such as [Bisin and Gottardi \(1999, 2006\)](#), following work by [Prescott and Townsend \(1984\)](#). Most closely related to us is the work of [Dubey and Geanakoplos \(2002\)](#) and [Dubey et al. \(2005\)](#). [Dubey and Geanakoplos \(2002\)](#) introduced a general equilibrium model where consumers have different endowments in different states of the world and may join “competitive pools” to share risk. They write the Rothschild and Stiglitz setup as a particular case of their model. [Dubey et al. \(2005\)](#) considered a related model with endogenous default and non-exclusive contracts. Both papers address multiplicity of equilibria with a refinement where an “external agent” makes high deliveries to each pool in every state of the world. This refinement is similar to our approach in the case of a finite number of contracts. There are three main differences with respect to our work. First, we consider more general preferences, which can accommodate richer preference heterogeneity as in [example 3](#). Moreover, our model dispenses the specification of pools and endowments, making it considerably easier to work with. Second, we allow for continuous sets of contracts, as in [examples 2 and 3](#). To do so, we generalize the equilibrium refinement, make the key assumption of bounded marginal rates of substitution, and develop the proof strategy of [Theorem 1](#), which allows us to tackle the problem of defining this kind of refinement and of proving existence with infinite sets of contracts. Third, we introduce new analytical techniques by analyzing our examples directly in the limit, enabling novel applied results such as [Propositions 2 and 3](#).

[Gale \(1992\)](#), like us, considers general equilibrium in a setting with less structure than the insurance pools. However, he refines his equilibrium with a stability notion based on [Kohlberg and Mertens \(1986\)](#). More recent contributions have considered general equilibrium models where firms can sell the right to choose from menus of contracts ([Citanna and Siconolfi, 2014](#)).

Our results are related to this previous work as follows. In standard one-dimensional models with ordered types, our unique equilibrium corresponds to what is usually called the “least-costly separating equilibrium.” Thus, our equilibrium prediction is the same as in models without cross-subsidies, such as [Riley \(1979\)](#), [Bisin and Gottardi \(2006\)](#), and [Rothschild and Stiglitz \(1976\)](#) when their equilibrium exists. It also coincides with [Banks and Sobel \(1987\)](#) and [Cho and Kreps \(1987\)](#) in the settings they consider. It differs from equilibria that involve cross-subsidization across contracts, such as [Wilson \(1977\)](#), [Miyazaki \(1977\)](#), [Hellwig \(1987\)](#), and [Netzer and Scheuer \(2014\)](#). Our equilibrium differs from mixed strategy equilibria of the [Rothschild and Stiglitz \(1976\)](#) model, even as the number of firms increases. This follows from the [Luz \(2013\)](#) characterization. In the case of a pool structure and finite set of contracts, our equilibria are the same as in [Dubey and Geanakoplos \(2002\)](#).

Although our equilibrium coincides with the Riley equilibrium in particular settings, our

equilibrium exists, is tractable, and has strategic foundations in settings where the Riley equilibrium may not exist. Our predictions are the same as the Riley equilibrium in two important particular cases. One is [Riley's \(1979\)](#) original setup with ordered types, and the other is [Handel et al.'s \(2015\)](#) model, where types come from a more realistic empirical health insurance model and are not ordered, but there are only two contracts. In particular, our strategic foundations results lend support to the predictions in these models. We note that, with multidimensional heterogeneity, existence of Riley equilibrium can only be guaranteed with restrictions on preferences (see [Azevedo and Gottlieb, 2016](#) for a simple example where a Riley equilibrium does not exist).

Another strand of the literature considers preferences with less structure. [Chiappori et al. \(2006\)](#) consider a very general model of preferences within an insurance setting. This paper differs from our work in that they consider general testable predictions without specifying an equilibrium concept, while we derive sharp predictions within an equilibrium framework. [Rochet and Stole \(2002\)](#) consider a competitive screening model with firms differentiated as in [Hotelling \(1929\)](#), where there is no adverse selection. Their Bertrand equilibrium converges to competitive pricing as differentiation vanishes, which is the outcome of our model. However, [Riley's \(1979\)](#) results imply that no Bertrand equilibrium would exist if one generalizes their model to include adverse selection.

[Einav et al. \(2010a\)](#), [Handel et al. \(2015\)](#), and [Veiga and Weyl \(2016\)](#) consider endogenous contract characteristics in a multidimensional framework. [Einav et al. \(2010a\)](#) and [Handel et al. \(2015\)](#) consider settings where consumers must purchase one of two insurance products and use the Riley and Akerlof equilibrium concepts. This is a clever way to endogenously determine what contracts are traded, albeit at the cost of a simple contract space (two products), and the assumption that consumers are forced to buy one of the products. A natural interpretation of our work is that we build on their insights, while allowing for richer contract spaces. [Veiga and Weyl \(2016\)](#) consider an oligopoly model of competitive screening in the spirit of [Rochet and Stole \(2002\)](#), but where each firm offers a single contract. Contract characteristics are determined by a simple first-order condition, as in the [Spence \(1975\)](#) model. Moreover, their model can incorporate imperfect competition. Our numerical results suggest that our model and [Veiga and Weyl's](#) agree on many qualitative predictions. For example, insurance markets provide inefficiently low coverage, and increasing heterogeneity in risk aversion seems to attenuate adverse selection.

The key difference is that [Veiga and Weyl's](#) model has a single traded contract, while our model endogenously determines the set of traded contracts. In their model, when competitive equilibria exist,²⁶ all firms offer the same contract.²⁷ In contrast, a rich set of contracts is

²⁶Perfectly competitive equilibria do not always exist in their model. In a calibration they find that perfectly competitive symmetric equilibria do not exist, and equilibria only exist with very high markups.

²⁷This is so in the more tractable case of symmetrically differentiated firms. In general, the number of

offered in our equilibrium. For example, in the case of no adverse selection (when costs are independent of types), our equilibrium is for firms to offer all products priced at cost, which corresponds to the standard notion of perfect competition. A colorful illustration is tomato sauce. The [Veiga and Weyl \(2016\)](#) model predicts that a single type of tomato sauce is offered cheaply, with characteristics determined by the preferences of average consumers. In contrast, our prediction is that many different types of tomato sauce are sold at cost. Italian style, basil, garlic lover, chunky, mushroom, and so on. In a less gastronomically titillating example, insurers offer a myriad types of life insurance: term life, universal life, whole life, combinations of these categories, and many different parameters within each category. Our results on the convergence of Bertrand equilibria suggest that the two models are appropriate in different situations. Their model of perfect competition seems more relevant when there are few firms, which are not very differentiated, the fixed cost of creating a new contract is high, and it is a good strategy for firms to offer products of similar quality as their competitors. That is, when firms herd on a particular type of contract.

5 Equilibrium Effects of Mandates

5.1 Illustrative Calibration

To illustrate the equilibrium concept and equilibrium effects of policy interventions, we calibrated the multidimensional health insurance model from example 3 based on [Einav et al.’s \(2013\)](#) preference estimates from employees in a large US corporation.²⁸

We considered linear contracts and normal losses, so that willingness to pay and costs are transparently represented by equation (2). Consumers differ along four dimensions: expected health shock, standard deviation of health shocks, moral hazard, and risk aversion. We assumed that the distribution of parameters in the population is log-normal.²⁹ Moments of the type distribution were calibrated to match the central estimates of [Einav et al. \(2013\)](#) with two exceptions. We reduced average risk aversion because linear contracts involve losses in a much wider range than the contracts in their data. Lower risk aversion better matched the substitution patterns in the data because constant absolute risk aversion models do not work well across different ranges of losses ([Rabin, 2000](#) and [Handel and Kolstad, forthcoming](#)). The other exception is the log variance of moral hazard, which we vary in our simulations.³⁰

contracts offered is no greater than the number of firms.

²⁸Our simulations are not aimed at predicting the outcomes in a particular market as in [Aizawa and Fang \(2013\)](#) and [Handel et al. \(2015\)](#). Such simulations would take the [Einav et al. \(2013\)](#) estimates far outside the range of contracts in their data, so even predictions about demand would rely heavily on functional form restrictions.

²⁹Note that the set of types is not compact in our numerical simulations. Restricting the set of types to a large compact set does not meaningfully impact the numerical results.

³⁰See Online Appendix B for details on the calibration and computational procedures and the supple-

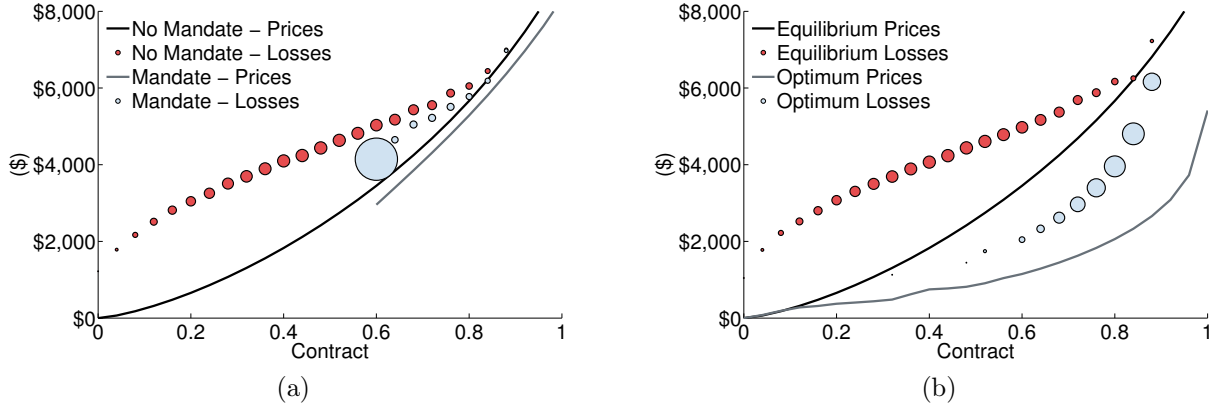


Figure 4: Equilibrium prices with a 60% mandate (a) and optimal prices (b).

Notes: The graphs plot equilibria of the multidimensional health insurance model from example 3. In both graphs, the solid curve denotes prices. The size of the circles represent the mass of consumers purchasing each contract, and its height represents the average loss parameter of such consumers, that is $\mathbb{E}_x[M]$.

To calculate an equilibrium, we used a perturbation with 26 evenly spaced contracts and added a mass equal to 1% of the population as behavioral consumers. We then used a fixed-point algorithm. In each iteration, consumers choose optimal contracts taking prices as given. Prices are adjusted up for unprofitable contracts and down for profitable contracts. Prices consistently converge to the same equilibrium for different initial values.

The equilibrium is depicted in Figure 3a. It features adverse selection in the sense that consumers who purchase more coverage have higher average losses. As Figure 3b illustrates, consumers sort across contracts in accordance to their preferences, and those with a higher expected loss and higher risk aversion tend to buy more coverage. However, even for the same levels of risk aversion and expected loss, different consumers choose different contracts due to other dimensions of heterogeneity.

Although there is adverse selection, equilibrium does not feature a complete “death spiral,” where no contracts are sold. In some other cases, however, the support of traded contracts is a small subset of all contracts (for example, in our calibration with non-linear contracts in the supplementary material). Whenever this is the case, buyers with the highest willingness to pay for each contract that is not traded value it below their own average cost (Proposition 1). That is, the markets for non-traded contracts are shut down by an Akerlof-type death spiral.

mentary material for calibrations with more realistic nonlinear contracts similar to those in Einav et al. (2013).

5.2 Policy Interventions in the Illustrative Calibration

This section investigates the effect of a mandate requiring consumers to purchase at least 60% coverage. Equilibrium is depicted in Figures 1 and 4a. With the mandate, about 85% of consumers get the minimum coverage. Moreover, some consumers who originally chose policies with greater coverage switch to the minimum amount after the mandate. In fact, the mandate increases the fraction of consumers who buy 60% coverage or less, as only 80% of consumers did so before the mandate.

The reason why some consumers reduce their coverage is that the mandate exacerbates adverse selection on the intensive margin. With the mandate, many low-cost consumers purchase the minimum coverage. This reduces the price of the 60% policy, attracting consumers who were originally purchasing more generous policies. In equilibrium, consumers sort across policies so that prices are continuous (as must be the case by Proposition 1). This leads to a lower but steeper price schedule, so that some consumers choose less coverage.

Consider now the welfare measure consisting of total consumer and producer surplus. Despite the unintended consequences, the mandate increases welfare in the baseline example by \$140 per consumer. This illustrates that competitive equilibria are inefficient (in the sense of not maximizing total surplus), and that even coarse policy interventions can have large benefits.³¹

We calculated the optimal price schedule for a regulator that maximizes welfare, can use cross subsidies, but does not possess more information than firms (Figure 4b). The optimal price schedule is much flatter than the unregulated market or the mandate. That is, optimal regulation involves subsidies across contracts, aimed at reducing adverse selection on the intensive margin. Optimal prices increase welfare by \$279 from the unregulated benchmark.

We considered variations of the model to understand whether the results are representative. Expected coverage and welfare are reported in Table 1 for different sets of contracts and log variances of moral hazard. Equilibrium behavior is robust to both changes. For example, a 60% mandate in a market with 0%, 60%, and 90% policies also increases welfare. In all cases, optimal regulation considerably increases welfare with respect to the 60% mandate.³²

Finally, the variance in moral hazard does not have a large qualitative impact on equi-

³¹While we focus on total surplus as a welfare measure, as does much of the applied literature, constrained Pareto efficiency is also an important concept in the study of markets with adverse selection. Crocker and Snow (1985a) show that, in the Rothschild and Stiglitz model, the Miyazaki-Wilson equilibrium is constrained Pareto efficient in that its allocations maximize the low-risk type's utility subject to incentive and zero profits constraints. The Riley equilibrium, which coincides with competitive equilibrium in the Rothschild and Stiglitz model, is only constrained efficient when it coincides with Miyazaki-Wilson. Therefore, in our model, equilibria may be constrained Pareto inefficient. As Einav et al. (2010b) have shown, mandates may decrease welfare.

³²We also replicate the result in Handel et al. (2015) and Veiga and Weyl (2014) that the markets with only 60% and 90% contracts almost completely unravel, suggesting that our results are not driven by details of the parametric model.

Table 1: Welfare and coverage under different scenarios

X	$\sigma_H^2 = 0.28$				$\sigma_H^2 = 0.98$			
	Equilibrium		Efficient		Equilibrium		Efficient	
	Welfare	$E[x]$	Welfare	$E[x]$	Welfare	$E[x]$	Welfare	$E[x]$
$[0, 1]$	0	0.46	279	0.8	0	0.43	366	0.84
$[0.60, 1]$	140	0.62	280	0.8	191	0.61	363	0.84
$0, 0.90$	101	0.66	256	0.9	131	0.63	355	0.9
$0.60, 0.90$	128	0.62	263	0.83	175	0.61	355	0.9
$0, 0.60, 0.90$	63	0.53	263	0.83	86	0.51	355	0.9

Notes: The table reports the welfare gain relative to an unregulated market with $X = [0, 1]$ (normalized to 0). When the set of contracts includes an interval we added a contract for every 0.04 coverage. Welfare is optimized with a tolerance of 1% gain in each iteration. Due to this tolerance, calculated welfare under efficient pricing is slightly higher with $X = [0.60, 1]$ than with $X = [0, 1]$, but we know theoretically that these are at most equal.

librium but considerably changes optimal regulation. For example, when $X = [0, 1]$, the optimal allocation in the high moral hazard variance scenario gives about 84% coverage to all consumers, which is quite different from the rich menu in Figure 4b. The reason is that consumers with higher moral hazard tend to buy more insurance, but it is socially optimal to give them less insurance. Therefore, a regulator may give up screening consumers.³³ More broadly, this numerical result shows that the relative importance of different sources of heterogeneity can have a large impact on optimal policy. Therefore, taking multiple dimensions of heterogeneity into account is important for government intervention.

5.3 Theoretical Results

To clarify the main forces behind the calibration findings, we derive two comparative statics results on the effects of increasing a mandate's minimum coverage. First, if there is selection, the mandate necessarily has knock-on effects. The intuition is that the mandate changes relative prices, which induces consumers to change their choices. For example, if there is adverse selection, the inflow of cheap consumers decreases the price of low-quality coverage,

³³To understand why the regulator may prefer not to screen consumers, notice that the first-best coverage for consumer θ can be calculated by equating marginal utility and marginal cost, which gives:

$$x = \frac{A_\theta S_\theta^2}{A_\theta S_\theta^2 + H_\theta}.$$

This expression is decreasing in the moral hazard parameter. All things equal, the social planner prefers to provide less coverage to consumers who are more likely to engage in moral hazard. However, consumers with higher moral hazard parameters always wish to purchase more insurance. Hence, if all heterogeneity is in moral hazard, the planner prefers not to screen consumers and, instead, assigns the same contract to everyone. This phenomenon has been described by Guesnerie and Laffont (1984) in one-dimensional screening models, who call it non-responsiveness.

inducing some consumers who are not directly affected by the mandate to change their choices. Second, we give a sufficient statistics formula for the effect on the price of low-quality coverage. The formula predicts the sign and magnitude of the change, while using only a small amount of data from the original equilibrium. The formula predicts, in particular, that prices go down if there is adverse selection.

5.3.1 Knock-on Effects

Consider economies where the set of contracts is an interval $X = [m + dm, 1]$ with $0 < m \leq m + dm < 1$. Utility is quasilinear and higher contracts are better and more costly. A regulator mandates a level of minimum coverage $m + dm$. We are interested in how equilibrium changes as the regulator changes dm , increasing the minimum coverage. Consider, for every sufficiently small $dm \geq 0$, an equilibrium (p_{dm}, α_{dm}) .

Instead of making parametric assumptions, we require some regularity conditions on the original equilibrium. Assume that the marginal distribution of contracts according to α_{dm} is represented by a distribution G_{dm} . We denote G_0 by G , p_0 by p , and α_0 by α . G has a point mass at minimum coverage with $G(m) > 0$, p_{dm} is continuous, and both G and p are continuously differentiable at m . Consumer choices are described by a function $\hat{x}(\theta, dm)$. That is, the allocation α_{dm} is

$$\alpha_{dm}(F) = \mu(\{\theta : (\theta, \hat{x}(\theta, dm)) \in F\}).$$

We assume that consumers who purchased minimum coverage for $dm = 0$ continue to do so after minimum coverage increases, and that the original optimal choice is unique for consumers purchasing sufficiently low coverage.

Define the intensive margin selection coefficient at minimum coverage as

$$S_I(m) = p'(m) - \mathbb{E}_m[mc].$$

This coefficient corresponds to the cost increase per additional unit of coverage minus the average marginal cost of a unit of coverage. In other words, $S_I(m)$ is the increase in costs due to selection. This coefficient is positive if, locally around the contract m , consumers who purchase more coverage are more costly, and it is negative if consumers who purchase more coverage have lower costs. Thus, $S_I(m)$ is closely related to the positive correlation test of [Chiappori and Salanié \(2000\)](#).³⁴ It is natural to say that there is adverse selection around m if $S_I(m)$ is positive, and advantageous selection if $S_I(m)$ is negative.

³⁴Under certain regularity conditions, $S_I(m)$ can be written as the increase in cost of purchasing contract m for consumers who purchase slightly higher contracts, clarifying the connection with the positive correlation test.

The next result shows that, if there is selection, mandates must have knock-on effects, as in the unintended consequences found in the calibrations.

Proposition 2. (*Knock-on Effects of Mandates*) *Consider the effects of a small increase in minimum coverage, and assume that there is selection in the sense that the intensive margin selection coefficient at m , $S_I(m)$, is not zero.*

Then there are changes in the relative prices of contracts. Moreover, there is a positive mass of consumers who change their choices beyond the direct effect of the mandate. That is, there is a positive mass of consumers whose choice after the mandate is not their preferred contract in $[m + dm, 1]$ under pre-mandate prices p .

Proposition 2 shows that a mandate affects prices and coverage decisions, beyond the direct effect of restricting coverage choices. To understand the intuition, consider the case of adverse selection, when $S_I(m) > 0$. The mandate drives cheap consumers into the contract with the minimum coverage, so the direct effect of the mandate is to reduce the price of the minimum coverage contract. If consumers who previously purchased better contracts did not change their choices after the mandate, the prices of these better contracts would remain the same (since each contract must break even). But this would imply that prices are discontinuous, contradicting Proposition 1.

5.3.2 Sufficient Statistics Formula for the Effect of Mandates on Prices

The next result requires some regularity conditions on how equilibrium changes with dm . We assume that equilibrium prices and allocations vary smoothly, consumer types are smoothly distributed, and consumers change their choices continuously.

We formalize these assumptions as follows. $p_{dm}(x)$ is a smooth function of x and dm . $\hat{x}(\theta, dm)$ is continuous, and is smooth when $\hat{x} > m + dm$. G_{dm} has a point mass at minimum coverage with $G_{dm}(m + dm) > 0$ and is otherwise atomless with smooth density g_{dm} .

For each consumer θ and contract x , define the intensive margin elasticity of substitution as

$$\epsilon(x, \theta) = \frac{1}{x} \cdot \frac{mu(x, \theta)}{\partial_{xx}u(x, \theta) - p''(x)}.$$

This elasticity represents, for consumers choosing an interior optimum, the percent change in optimal coverage given a one percent increase in marginal prices. We assume that the joint distribution of elasticities, costs, and marginal costs is atomless and varies continuously with contracts. That is, the joint distribution of $(\epsilon(x, \theta), c(x, \theta), mc(x, \theta))$ conditional on α and a contract x is represented by a smooth density $h(\cdot|x)$. Moreover, $h(\cdot|x)$ varies smoothly with x for $x > m$.

Proposition 3. (Effect of Mandates on Equilibrium Prices) Consider the effects of a small increase in minimum coverage from m to $m + dm$. The change in prices close to minimum coverage equals the negative of the intensive margin adverse selection coefficient plus an error term, that is

$$\lim_{x \rightarrow m} \partial_{dm} p_{dm}(x)|_{dm=0} = -S_I(m) + \xi,$$

where the error term ξ is given by equation (9) in the Appendix.

If there is adverse selection, and if the error term ξ is small, then the level of prices goes down, pushed by the inflow of cheaper consumers who originally purchased minimum coverage. The error term ξ is small if there are many consumers initially purchasing minimum coverage so that $g(m)/G(m)$ is small.

The proposition provides a sufficient statistics formula for how much the price of low-quality coverage changes with the introduction of a mandate. The intuition is that prices are shifted by the inflow of consumers who are constrained to purchase minimum coverage in the original equilibrium. In particular, if there is adverse selection, these consumers are cheaper and push down the price of low-quality coverage.³⁵

6 Conclusion

This paper considers a competitive model of adverse selection, which has a well defined equilibrium in settings with rich heterogeneity and complex contract spaces and has strategic foundations. Competitive equilibrium extends the [Akerlof \(1970\)](#) and [Einav et al. \(2010a\)](#) models beyond the case of a single contract, endogenously determining which contracts are traded with supply and demand.

An interesting set of questions is to what extent competitive equilibria are inefficient, and what kinds of government interventions can restore efficiency. The illustrative calibration shows by example that equilibria can be inefficient (in the sense of not maximizing total surplus), and that even simple policies like mandates can considerably increase efficiency. Moreover, optimal policies that also address adverse selection on the intensive margin can

³⁵We can gauge the accuracy of the approximate formula in the calibrated 60% mandate example, where $S_I(m)$ equals 5385 (measured in \$/100% coverage), so that there is a large amount of adverse selection at the lowest level of coverage. Proposition 3 predicts that each 1% increase in minimum coverage should decrease prices by \$54. To test this, we calculated the equilibrium of an economy with minimum coverage set at 64% instead of 60%. The price of the contract offering 64% coverage went down by \$183, which is close to the $0.04 \cdot S_I(m)$, or \$215, as predicted by Proposition 3. The approximation depends on there being a large mass of consumers purchasing minimum coverage. To evaluate the robustness of the formula, we simulated an increase in minimum coverage from 40% to 44%, where only 55% of consumers purchase minimum coverage. The decrease in prices predicted by Proposition 3 is \$135, while the actual change is \$80. While this approximation is less accurate, it is still useful, given the low data requirements of the formula.

further increase efficiency. This is in concert with the view of regulators, who often implement policies aimed at affecting contract characteristics, and with [Einav et al. \(2009\)](#), who have suggested that these characteristics may be important. We leave a detailed analysis of optimal interventions and how they relate to commonly used policies to future work.

It would be interesting to test how well our competitive equilibrium notion predicts behavior in markets with adverse selection and to test it against alternative models. For example, in the case of one dimension of heterogeneity, there is considerable controversy over what a reasonable equilibrium notion is, despite many alternatives such as those proposed by [Riley \(1979\)](#) and [Miyazaki \(1977\)-Wilson \(1977\)](#). Unfortunately, these equilibria are defined in more restrictive settings, so one cannot compare predictions in richer settings like our calibrated example. It would be interesting to extend these equilibrium notions to richer settings and compare their predictions to competitive equilibria.

References

- Aizawa, Naoki and Hanming Fang**, “Equilibrium Labor Market Search and Health Insurance Reform,” *NBER Working Paper 18698*, 2013.
- Akerlof, George A.**, “The Market for “Lemons”: Quality Uncertainty and the Market Mechanism,” *Quarterly Journal of Economics*, 1970, pp. 488–500.
- Ambrus, Attila and Rossella Argenziano**, “Asymmetric networks in two-sided markets,” *American Economic Journal: Microeconomics*, 2009, 1 (1), 17–52.
- Attar, Andrea, Thomas Mariotti, and François Salanié**, “Nonexclusive Competition in the Market for Lemons,” *Econometrica*, 2011, 79 (6), 1869–1918.
- Azevedo, Eduardo M and Daniel Gottlieb**, “Non-Existence of Riley Equilibrium in Markets with Adverse Selection,” *Mimeo, University of Pennsylvania*, 2016.
- **and** –, “Optimal Regulation of Markets with Adverse Selection,” *Mimeo, University of Pennsylvania*, in preparation.
- Baicker, Katherine, Sendhil Mullainathan, and Joshua Schwartzstein**, “Behavioral Hazard in Health Insurance,” *NBER Working Paper 18468*, 2012.
- Banks, Jeffrey S and Joel Sobel**, “Equilibrium Selection in Signaling Games,” *Econometrica*, 1987, pp. 647–661.
- Billingsley, Patrick**, *Probability and Measure*, John Wiley & Sons, 2008.

- Bisin, Alberto and Piero Gottardi**, “Competitive Equilibria with Asymmetric Information,” *Journal of Economic Theory*, 1999, *87* (1), 1–48.
- **and** –, “Efficient Competitive Equilibria with Adverse Selection,” *Journal of Political Economy*, 2006, *114* (3), 485–516.
- Chang, Briana**, “Adverse Selection and Liquidity Distortion in Decentralized Markets,” *Mimeo, Wisconsin School of Business*, 2010.
- Chen, Ying, Navin Kartik, and Joel Sobel**, “Selecting Cheap-Talk Equilibria,” *Econometrica*, 2008, *76* (1), 117–136.
- Chiappori, Pierre-André and Bernard Salanié**, “Testing for Asymmetric Information in Insurance Markets,” *Journal of Political Economy*, 2000, *108* (1), 56–78.
- , **Bruno Jullien, Bernard Salanié, and Francois Salanié**, “Asymmetric Information in Insurance: General Testable Implications,” *Rand Journal of Economics*, 2006, *37* (4), 783–798.
- , **Robert J McCann, and Lars P Nesheim**, “Hedonic Price Equilibria, Stable Matching, and Optimal Transport: Equivalence, Topology, and Uniqueness,” *Economic Theory*, 2010, *42* (2), 317–354.
- Cho, In-Koo and David M Kreps**, “Signaling Games and Stable Equilibria,” *Quarterly Journal of Economics*, 1987, pp. 179–221.
- Citanna, Alessandro and Paolo Siconolfi**, “Refinements and Incentive Efficiency in Walrasian Models of Insurance Economies,” *Journal of Mathematical Economics*, 2014, *50*, 208–218.
- Cohen, Alma and Liran Einav**, “Estimating Risk Preferences from Deductible Choice,” *American Economic Review*, 2007, *97* (3), 745–788.
- Crocker, Keith J. and Arthur Snow**, “The Efficiency of Competitive Equilibria in Insurance Markets with Asymmetric Information,” *Journal of Public Economics*, 1985a, *26* (2), 207–219.
- Cutler, David M and Sarah Reber**, “Paying for Health Insurance: The Trade-off between Competition and Adverse Selection,” *Quarterly Journal of Economics*, 1998, *113* (2).
- Dafny, Leemore, Mark Duggan, and Subramaniam Ramanarayanan**, “Paying a Premium on Your Premium? Consolidation in the Us Health Insurance Industry,” *American Economic Review*, 2012, *102* (2), 1161–1185.

Dafny, Leemore S, “Are Health Insurance Markets Competitive?,” *American Economic Review*, 2010, *100* (4), 1399–1431.

Decarolis, Francesco, Maria Polyakova, and Stephen Ryan, “The Welfare Effects of Supply Side Regulations in Medicare Part D,” *Mimeo, Stanford University*, 2012.

Dubey, Pradeep and John Geanakoplos, “Competitive Pooling: Rothschild-Stiglitz Reconsidered,” *Quarterly Journal of Economics*, 2002, pp. 1529–1570.

– , – , **and Martin Shubik**, “Default and Punishment in General Equilibrium,” *Econometrica*, 2005, pp. 1–37.

Einav, L, A Finkelstein, and J Levin, “Beyond Testing: Empirical Models of Insurance Markets.,” *Annual Review of Economics*, 2009, *2*, 311–336.

Einav, Liran, Amy Finkelstein, and Mark R Cullen, “Estimating Welfare in Insurance Markets Using Variation in Prices,” *Quarterly Journal of Economics*, 2010, *125* (3), 877–921.

– , – , **and Paul Schrimpf**, “Optimal Mandates and the Welfare Cost of Asymmetric Information: Evidence from the UK Annuity Market,” *Econometrica*, 2010, *78* (3), 1031–1092.

– , – , **Stephen P Ryan, Paul Schrimpf, and Mark R Cullen**, “Selection on Moral Hazard in Health Insurance,” *American Economic Review*, 2013, *103* (1), 178–219.

– **and** – , “Selection in Insurance Markets: Theory and Empirics in Pictures,” *Journal of Economic Perspectives*, 2011, *25* (1), 115–138.

Fang, Hanming, Michael P Keane, and Dan Silverman, “Sources of Advantageous Selection: Evidence from the Medigap Insurance Market,” *Journal of Political Economy*, 2008, *116* (2), 303–350.

Finkelstein, Amy and Kathleen McGarry, “Multiple Dimensions of Private Information: Evidence from the Long-Term Care Insurance Market,” *American Economic Review*, 2006, *96* (4), 938–958.

Gale, Douglas, “A Walrasian Theory of Markets with Adverse Selection,” *The Review of Economic Studies*, 1992, *59* (2), 229–255.

Guerrieri, Veronica and Robert Shimer, “Markets with Multidimensional Private Information,” *Mimeo, University of Chicago*, 2015.

- , – , and **Randall Wright**, “Adverse Selection in Competitive Search Equilibrium,” *Econometrica*, 2010, *78* (6), 1823–1862.
- Guesnerie, Roger and Jean-Jacques Laffont**, “A Complete Solution to a Class of Principal-Agent Problems with an Application to the Control of a Self-Managed Firm,” *Journal of Public Economics*, 1984, *25* (3), 329–369.
- Hackmann, Martin B, Jonathan T Kolstad, and Amanda E Kowalski**, “Adverse Selection and an Individual Mandate: When Theory Meets Practice,” *American Economic Review*, 2015, *105* (3), 1030–66.
- Handel, Benjamin, Igal Hendel, and Michael D Whinston**, “Equilibria in Health Exchanges: Adverse Selection Vs. Reclassification Risk,” *Econometrica*, 2015, *83* (4), 1261–1313.
- Handel, Benjamin R**, “Adverse Selection and Inertia in Health Insurance Markets: When Nudging Hurts,” *American Economic Review*, 2013, *103* (7), 2643–2682.
- and **Jonathan T Kolstad**, “Health Insurance for Humans: Information Frictions, Plan Choice, and Consumer Welfare,” *American Economic Review*, forthcoming.
- Heinonen, Juha**, *Lectures on Analysis on Metric Spaces*, Springer, 2001.
- Hellwig, Martin**, “Some Recent Developments in the Theory of Competition in Markets with Adverse Selection*,” *European Economic Review*, 1987, *31* (1), 319–325.
- Hendren, Nathaniel**, “Private Information and Insurance Rejections,” *Econometrica*, 2013, *81* (5), 1713–1762.
- Hotelling, Harold**, “Stability in Competition,” *Economic Journal*, 1929, *39* (153), 41–57.
- Inderst, Roman and Achim Wambach**, “Competitive Insurance Markets under Adverse Selection and Capacity Constraints,” *European Economic Review*, 2001, *45* (10), 1981–1992.
- Kadan, Ohad, Philip Reny, and Jeroen Swinkels**, “Existence of Optimal Mechanisms in Principal-Agent Problems,” *Mimeo, University of Chicago*, 2014.
- Kohlberg, Elon and Jean-Francois Mertens**, “On the Strategic Stability of Equilibria,” *Econometrica*, 1986, pp. 1003–1037.
- Kunreuther, Howard and Mark Pauly**, *Insurance Decision-Making and Market Behavior*, Now Publishers Inc, 2006.

- Luz, Vitor F.**, “Characterization and Uniqueness of Equilibrium in Competitive Insurance,” *Mimeo, Yale University*, 2013.
- Mahoney, Neale and E Glen Weyl**, “Imperfect Competition in Selection Markets,” *Review of Economics and Statistics*, forthcoming.
- Mailath, George J, Masahiro Okuno-Fujiwara, and Andrew Postlewaite**, “Belief-Based Refinements in Signalling Games,” *Journal of Economic Theory*, 1993, 60 (2), 241–276.
- Miyazaki, Hajime**, “The Rat Race and Internal Labor Markets,” *Bell Journal of Economics*, 1977, pp. 394–418.
- Myerson, Roger B**, “Refinements of the Nash Equilibrium Concept,” *International Journal of Game Theory*, 1978, 7 (2), 73–80.
- , “Sustainable Matching Plans with Adverse Selection,” *Games and Economic Behavior*, 1995, 9 (1), 35–65.
- Netzer, Nick and Florian Scheuer**, “Competitive Screening in Insurance Markets with Endogenous Wealth Heterogeneity,” *Economic Theory*, 2010, 44 (2), 187–211.
- and —, “A Game Theoretic Foundation of Competitive Equilibria with Adverse Selection,” *International Economic Review*, 2014, 55 (2), 399–422.
- Novshek, William and Hugo Sonnenschein**, “General equilibrium with free entry: a synthetic approach to the theory of perfect competition,” *Journal of Economic Literature*, 1987, 25 (3), 1281–1306.
- Pauly, Mark V, Olivia S Mitchell, and Yuhui Zeng**, “Death Spiral or Euthanasia? The Demise of Generous Group Health Insurance Coverage,” *Inquiry*, 2007, pp. 412–427.
- Plott, Charles R and Glen George**, “Marshallian vs. Walrasian stability in an experimental market,” *The Economic Journal*, 1992, 102 (412), 437–460.
- Polyakova, Maria**, “Regulation of Insurance with Adverse Selection and Switching Costs: Evidence from Medicare Part D,” *Mimeo, Stanford University*, 2014.
- Prescott, Edward C and Robert M Townsend**, “Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard,” *Econometrica*, 1984, pp. 21–45.
- Rabin, Matthew**, “Risk Aversion and Expected-Utility Theory: A Calibration Theorem,” *Econometrica*, 2000, 68 (5), 1281–1292.

- Riley, John G**, “Informational Equilibrium,” *Econometrica*, 1979, pp. 331–359.
- Rochet, Jean-Charles and Lars A Stole**, “Nonlinear Pricing with Random Participation,” *Review of Economic Studies*, 2002, *69* (1), 277–311.
- Rothschild, Michael and Joseph Stiglitz**, “Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information,” *Quarterly Journal of Economics*, 1976, *90* (4), 629–649.
- Scheuer, Florian and Kent A Smetters**, “Could a Website Really Have Doomed the Health Exchanges? Multiple Equilibria, Initial Conditions and the Construction of the Fine,” *NBER Working Paper 19835*, 2014.
- Spence, A Michael**, “Monopoly, Quality, and Regulation,” *Bell Journal of Economics*, 1975, pp. 417–429.
- Spence, Michael**, “Job Market Signaling,” *Quarterly Journal of Economics*, 1973, pp. 355–374.
- Spinnewijn, Johannes**, “Unemployed but Optimistic: Optimal Insurance Design with Biased Beliefs,” *Journal of the European Economic Association*, February 2015, *13* (1), 130–167.
- Starc, Amanda**, “Insurer Pricing and Consumer Welfare: Evidence from Medigap,” *The Rand Journal of Economics*, 2014, *45* (1), 198–220.
- Tebaldi, Pietro**, “Estimating Equilibrium in Health Insurance Exchanges: Price Competition and Subsidy Design under the ACA,” *Mimeo, Stanford University*, 2015.
- Van de Ven, Wynand PMM and Randall P Ellis**, “Risk Adjustment in Competitive Health Plan Markets,” in A J Culyer and Joseph P Newhouse, eds., *Handbook of Health Economics*, 1999.
- Veiga, André and E Glen Weyl**, “The Leibniz Rule for Multidimensional Heterogeneity,” *Mimeo, University of Chicago*, 2014.
- and —, “Product Design in Selection Markets,” *Quarterly Journal of Economics*, May 2016, *131* (2), 1007–1056.
- Wilson, Charles**, “A Model of Insurance Markets with Incomplete Information,” *Journal of Economic Theory*, 1977, *16* (2), 167–207.
- , “The Nature of Equilibrium in Markets with Adverse Selection,” *Bell Journal of Economics*, 1980, pp. 108–130.

Appendix: Proofs

Existence of Equilibrium

The proof of Theorem 1 follows from three lemmas. The first uses a standard fixed-point argument to show that every perturbation has a weak equilibrium, the second establishes that price vectors in any perturbation are uniformly Lipschitz, and the third uses this fact to show that every sequence of weak equilibria of perturbations has a converging subsequence. Fix an economy $E = [\Theta, X, \mu]$.

Lemma 1. *Every perturbation has an equilibrium.*

Proof. Preliminaries.

Consider the perturbation (E, \bar{X}, η) . Let $\bar{\alpha} \in \Delta((\Theta \cup \bar{X}) \times \bar{X})$ denote the allocation of behavioral types. That is, for each $x \in \bar{X}$,

$$\bar{\alpha}(x, x) = \eta(x),$$

and $\bar{\alpha}$ has no mass in the complement of these points. Letting α denote the allocation of standard types, we will write an allocation as $\alpha + \bar{\alpha}$, which has support contained in $(\Theta \cup \bar{X}) \times \bar{X}$ and $\alpha|_{\Theta} = \mu$.

Let A be the set of all allocations for standard types with the topology of weak convergence of measures. That is,

$$A = \{\alpha \in \Delta((\Theta \cup \bar{X}) \times \bar{X}) : \text{support}(\alpha) \subseteq \Theta \times \bar{X}, \alpha|_{\Theta} = \mu\}.$$

Let P be the set of all prices vectors in (convex closure of the image of c) $^{\bar{X}}$ with the standard Euclidean topology.

We define a tâtonnement correspondence

$$T : P \times A \rightrightarrows P \times A.$$

The tâtonnement is defined in terms of two maps,

$$T(p, \alpha) = \Phi(\alpha) \times \Psi(p),$$

where

$$\begin{aligned} \Phi(\alpha) &= \{p \in P : p(x) = E_x[c|\alpha + \bar{\alpha}] \forall x \in \bar{X}\}, \text{ and} \\ \Psi(p) &= \arg \max_{\alpha \in A} \int U(x, p(x), \theta) d\alpha. \end{aligned}$$

That is, given an allocation α , $\Phi(\alpha)(x)$ is the expected cost of supplying contract x . Given p , $\Psi(p)$ is the set of allocations for the standard types where they choose optimally given p .

The fixed points of T correspond to the equilibria of the perturbation. To see this, note that $p \in \Phi(\alpha)$ is equivalent to firms making 0 profits, and $\alpha \in \Psi(p)$ is equivalent to the standard types optimizing. Therefore, (p^*, α^*) is a fixed point of T if and only if $(p^*, \alpha^* + \bar{\alpha})$ is an equilibrium. We will now prove the existence of a fixed point. The proof has three steps.

Step 1: Φ is nonempty, convex valued, and has a closed graph.

To establish that it has a closed graph, consider a sequence $(\alpha^n, p^n)_{n \in \mathbb{N}}$ in the graph of Φ with limit (α, p) . We will show that $p(x)$ is a conditional expectation of cost given $\alpha + \bar{\alpha}$. To see this, take an arbitrary set $S \subseteq \bar{X}$. Let $\tilde{S} = (\Theta \cup \bar{X}) \times S$. We have

$$\begin{aligned} \int_{\tilde{S}} p(x) d(\alpha + \bar{\alpha}) &= \sum_{x \in S} p(x) \cdot [\alpha(\Theta \times x) + \eta(x)] \\ &= \lim_{n \rightarrow \infty} \sum_{x \in S} p^n(x) \cdot [\alpha^n(\Theta \times x) + \eta(x)] \\ &= \lim_{n \rightarrow \infty} \int_{\tilde{S}} p^n(x) d(\alpha^n + \bar{\alpha}) \\ &= \lim_{n \rightarrow \infty} \int_{\tilde{S}} c(x, \theta) d\alpha^n \\ &= \int_{\tilde{S}} c(x, \theta) d\alpha. \end{aligned}$$

The first and third equations follow from decomposing the integral as a sum. The second follows from the convergence of (p^n, α^n) . The fourth is derived from the definition of conditional expectation and the fact that p^n is a conditional expectation of costs under $\alpha^n + \bar{\alpha}$. The fifth follows from the fact that c is continuous and α^n converges weakly to α . Convex-valuedness and non-emptiness follow directly from the definition of Φ .

Step 2: Ψ is nonempty, convex valued, and has a closed graph.

To see that it has a closed graph, consider a sequence $(p^n, \alpha^n)_{n \in \mathbb{N}}$ in the graph of Ψ with limit (p, α) . For any $\alpha' \in A$, we have

$$\int U(x, p^n(x), \theta) d\alpha'(\theta, x) \leq \int U(x, p^n(x), \theta) d\alpha^n(\theta, x).$$

Taking the limit, we have

$$\int U(x, p(x), \theta) d\alpha'(\theta, x) \leq \int U(x, p(x), \theta) d\alpha(\theta, x).$$

The LHS limit follows from the Dominated Convergence Theorem. To see the convergence

of the RHS term, it is helpful to decompose it as

$$\int U(x, p^n(x), \theta) - U(x, p(x), \theta) d\alpha^n(\theta, x) \\ + \int U(x, p(x), \theta) d\alpha^n(\theta, x).$$

The first integrand converges to 0 uniformly in x and θ because \bar{X} is finite, and hence p^n converges uniformly to p , and because the continuous function U is uniformly continuous in the compact set where prices belong to the image of c . Therefore, the first integral converges to 0. The second integral converges to 0 by the continuity of U and weak convergence of α^n to α .

Ψ is nonempty because X is finite, and therefore $U(x, p(x), \theta)$ attains a maximum for every θ . Convexity follows from the definition of Ψ .

Step 3: Existence of a fixed point.

The claims about Φ and Ψ imply that T is convex valued, nonempty, and has a closed graph. We have that the set $P \times A$ is compact, convex and a subset of a locally convex topological vector space. Therefore, by the Kakutani-Glicksberg-Fan Theorem, T has a fixed point. \square

The next result shows that, in a weak equilibrium of a perturbation, prices are a Lipschitz function with constant L . The intuition is that, if prices of similar contracts differed too much, no consumer would be willing to purchase the most expensive contract.

Lemma 2. *Let (p^*, α^*) be a weak equilibrium of a perturbation. Then p^* is an L -Lipschitz function.*

Proof. Consider two contracts x, x' . Assume, without loss of generality, that $p^*(x) > p^*(x')$. In particular, $p^*(x) > 0$, and therefore there exists a standard type θ who prefers x to x' . That is, there exists $\theta \in \Theta$ such that

$$U(x, p^*(x), \theta) \geq U(x', p^*(x'), \theta).$$

The assumption that marginal rates of substitution are bounded then implies

$$|p^*(x) - p^*(x')| \leq d(x, x') \cdot L.$$

\square

The next lemma uses this observation to show that every sequence of perturbations of economy E has a subsequence of equilibria that converges to an equilibrium of E .

Lemma 3. *Consider a sequence of perturbations $(E, \bar{X}^n, \eta^n)_{n \in \mathbb{N}}$ converging to E with weak equilibria $(p^n, \alpha^n)_{n \in \mathbb{N}}$. Then $(p^n, \alpha^n)_{n \in \mathbb{N}}$ has a subsequence that converges to an equilibrium (p^*, α^*) of E . Moreover, p^* is L -Lipschitz.*

Proof. We begin by defining α^* and p^* . First note that the set of allocations is compact. Therefore, without loss of generality, passing to a subsequence, we can take $(\alpha^n)_{n \in \mathbb{N}}$ to converge to a measure $\alpha^* \in \Delta((\Theta \cup X) \times X)$. Moreover, the support of α^* is contained in $\Theta \times X$, and $\alpha^*|_{\Theta} = \mu$.

As for p^* , take for each n a function \tilde{p}^n with domain X , which coincides with p^n in \bar{X} and is L -Lipschitz. Lemma 2 and Theorem 6.2 of Heinenon (2001) p. 43 guarantee the existence of these functions. Without loss of generality, passing to a subsequence, we may take the sequence $(\tilde{p}^n)_{n \in \mathbb{N}}$ to converge pointwise to a limit p^* . Note that, because the sequence $(\tilde{p}^n)_{n \in \mathbb{N}}$ is uniformly L -Lipschitz, it is equicontinuous. By the Arzelà–Ascoli Theorem, the sequence converges uniformly to p^* . This implies convergence in the sense of Definition 4 and the Lipschitz property. \square

Note that the previous lemma directly implies Theorem 1.

Proof of Theorem 1. Take any sequence of perturbations of economy E . By Lemma 3, there exists a subsequence with a converging sequence of equilibria. Hence, the limit of this sequence is an equilibrium of E . \square

Properties of Equilibria

We begin by establishing two of the properties in Proposition 1.

Lemma 4. *Every equilibrium is a weak equilibrium.*

Proof. Consider an economy $E = [\Theta, X, \mu]$ with an equilibrium (p^*, α^*) , and a sequence of perturbations $(E, \bar{X}^n, \eta^n)_{n \in \mathbb{N}}$ converging to E with weak equilibria $(p^n, \alpha^n)_{n \in \mathbb{N}}$ converging to (p^*, α^*) .

To verify that prices are a conditional expectation of the cost, take a measurable set of contracts $S \subseteq X$. Let $\tilde{S} = (\Theta \cup X) \times S$. Let \tilde{p}^n be an L -Lipschitz function extending p^n to X , which exists by the argument in the proof of Lemma 2. We have

$$\begin{aligned} \int_{\tilde{S}} p^*(x) d\alpha^* &= \lim_{n \rightarrow \infty} \int_{\tilde{S}} \tilde{p}^n(x) d\alpha^n \\ &= \lim_{n \rightarrow \infty} \int_{\tilde{S}} c(x, \theta) d\alpha^n \\ &= \int_{\tilde{S}} c(x, \theta) d\alpha^*. \end{aligned}$$

The first equality follows because $(\alpha^n)_{n \in \mathbb{N}}$ converges weakly to α^* , and $(\tilde{p}^n)_{n \in \mathbb{N}}$ converges uniformly to p^* . The second equality follows because p^n is the conditional expectation of c given α^n . The third equation follows because c is continuous and α^n converges weakly to α^* . From this argument, p^* is the conditional expectation of c under the measure α^* .

To see that consumers are optimizing, take an allocation α' . Since (p^n, α^n) are weak equilibria, for all n we have

$$\int_{\Theta \times X} U(x, \tilde{p}^n(x), \theta) d\alpha^n \geq \int_{\Theta \times X} U(x, \tilde{p}^n(x), \theta) d\alpha'.$$

Because $(\tilde{p}^n)_{n \in \mathbb{N}}$ converges uniformly to p^* and U is uniformly continuous on the relevant set, we can take limits on both sides, obtaining

$$\int_{\Theta \times X} U(x, p^*(x), \theta) d\alpha^* \geq \int_{\Theta \times X} U(x, p^*(x), \theta) d\alpha'.$$

Because this inequality holds for any α' , we have that, for α^* -almost every (θ, x) ,

$$U(x, p^*(x), \theta) = \sup_{x' \in X} U(x', p^*(x'), \theta),$$

as desired. □

Lemma 5. *Consider an equilibrium (p^*, α^*) of an economy E . Let x' be a contract with $p^*(x') > 0$. Then there exists (θ, x) in the support of α such that*

$$U(x, p^*(x), \theta) = U(x', p^*(x'), \theta) \tag{3}$$

and

$$c(x', \theta) \geq p^*(x').$$

Proof. Take a sequence of perturbations $(E, \bar{X}^n, \eta^n)_{n \in \mathbb{N}}$ converging to E with equilibria (p^n, α^n) converging to (p^*, α^*) . Take $x'^n \in \bar{X}^n$ converging to x' . Since $p^n(x'^n)$ converges to $p^*(x') > 0$, we must have $p^n(x'^n) > 0$ for sufficiently large n . This implies that there exists a standard type θ^n such that (θ^n, x'^n) is in the support of α^n . Moreover, we can take θ^n so that $c(x'^n, \theta^n) \geq p^n(x'^n)$. We can take a subsequence such that θ^n converges to a type θ because the set of types is compact. Take (θ, x) in the support of α^* , so that x is optimal for θ at prices p^* . Take a sequence $(x^n)_{n \in \mathbb{N}}$ with each $x^n \in \bar{X}^n$ converging to x . Since x'^n is optimal for θ^n in the perturbation, for all sufficiently large n we have

$$U(x'^n, p^n(x'^n), \theta^n) \geq U(x^n, p^n(x^n), \theta^n).$$

Taking the limit, we have

$$U(x', p^*(x'), \theta) \geq U(x, p^*(x), \theta).$$

This implies equation (3) because x is optimal for θ at prices p^* . Moreover, we have

$$c(x'^n, \theta^n) \geq p^n(x'^n).$$

Taking the limit we have $c(x', \theta) \geq p^*(x')$. □

We can now establish Proposition 1.

Proof of Proposition 1. Parts 1, 2, and 3 follow from Lemmas 4, 5, and 3. Part 4 follows from part 3 and Rademacher's theorem. □

Finally, we can use the proposition to derive Corollary 1. The indifference curve of type θ going through (\bar{x}, \bar{p}) is

$$\{(x, p) : U(x, p, \theta) = U(\bar{x}, \bar{p}, \theta)\}.$$

The zero-profits curve for type θ is the set of contracts-price pairs for which firms make no profits, or

$$\{(x, p) : p = x \cdot l \cdot \theta\}.$$

The proof uses two properties of the Rothschild and Stiglitz setting. For each θ , the slope of the indifference curve,

$$\left. \frac{dp}{dx} \right|_{U(x,p,\theta)=U(\bar{x},\bar{p},\theta)} = \frac{l\theta v'(W - p - (1-x)l)}{\theta v'(W - p - (1-x)l) + (1-\theta)v'(W - p)}, \quad (4)$$

is greater than the slope of the zero-profits curve, $l\theta$. Moreover, the slope of the indifference curve is increasing in θ .

Proof of Corollary 1. The proof is divided into four steps.

Step 1. There is no contract $x^* > 0$ that is purchased by a positive mass of both types.

Suppose both types buy $x^* > 0$ with positive probability, so its price exceeds the cost of serving low types:

$$p(x^*) > x^* \cdot l \cdot L.$$

Since low types have flatter indifference curves than high types, they must be indifferent between buying x^* and $x < x^*$, which must have prices weakly below type L 's cost $x \cdot l \cdot L$ (Proposition 1, Property 2). But, since p is continuous, this is not possible for x sufficiently close to x^* .

Step 2. Every traded contract is sold at actuarially fair prices.

The null contract must cost zero, which equals both types' cost. For non-null contracts, step 1 implies that the price of each traded contract must equal the cost of the type purchasing it.

Step 3. If $H = 1$, then $p(x) = l \cdot x$ and all low types purchase the null contract.

From step 2, a contract x^* chosen by the high type costs $p(x^*) = l \cdot x^*$. For high types to pick x^* , any other contract x must cost at least $l \cdot x$. But, at these prices, low types prefer the null contract. Then, by Proposition 1, the price of non-traded contracts lie on the high type's indifference curve: $p(x) = l \cdot x$.

Step 4. If $H < 1$, then high types always buy $x_H = 1$ and low types always buy x_L , where x_L is defined as the point on the low type's zero-profit curve that gives the high type the same utility as the full insurance contract (see Figure 2b).

Let x_H be a contract chosen by type H with positive probability. For type L not to choose x_H , it must be above L 's indifference curve associated with his equilibrium utility. Since lower types have flatter indifference curves, Proposition 1 implies that H is indifferent between x_H and $x \geq x_H$ and prices of all such x are weakly below H 's cost (with equality at x_H). But this is not possible when $x_H < 1$ because indifference curves are steeper than the zero-profits curve. Therefore, high types always buy full insurance: $x_H = 1$.

Suppose the low type picks x^* with positive probability. From step 2, x^* is sold at the actuarially fair price for L . For type H not to choose x^* , we must have $x^* \leq x_L$. If $x^* < x_L$, then H gets a strictly lower utility from contracts in a neighborhood of x^* than from buying full insurance. Then, type L must be indifferent between all contracts in this neighborhood and prices must be weakly lower than type L 's cost (with equality at x^*). But this is not possible for $x > x^*$ because indifference curves are steeper than the zero-profits curve. Thus, $x^* = x_L$. \square

Equilibrium Effects of Mandates

Proof of Proposition 2. Let g be the density of G . To reach a contradiction assume that, after the increase in minimum coverage, almost all consumers choose a contract that is optimal in the set $[m + dm, 1]$ under the original prices. For sufficiently small dm , for consumers with $\hat{x}(\theta, 0) \leq m + dm$, we have that $\hat{x}(\theta, dm) = m + dm$. This follows from the assumption that optimal choices are unique in the original equilibrium, compactness of the sets of types and contracts, and fact that p_{dm} varies continuously. All other consumers do not change their choices. Thus,

$$p_{dm}(m + dm) = \frac{\int_{x=m}^{m+dm} p(m + dm) \cdot g(x) dx + \mathbb{E}_m[c(m + dm, \theta)] \cdot G(m)}{G(m + dm)}.$$

By Leibniz's rule, the derivative of the numerator at $dm = 0$ equals $p(m) \cdot g(m) + \mathbb{E}_m[mc] \cdot G'(m)$. Using the product rule we have that

$$\partial_{dm} p_{dm}(m + dm)|_{dm=0} = \mathbb{E}_m[mc].$$

Therefore,

$$\partial_{dm} \{p_{dm}(m + dm) - p(m + dm)\}|_{dm=0} = \mathbb{E}_m[mc] - p'(m) = -S_I(m).$$

Using the assumption that the mass of consumers with $\hat{x}(\theta, 0) > m + dm$ and $\hat{x}(\theta, dm) \neq \hat{x}(\theta, 0)$ is 0, we have that

$$p_{dm}(x) = p(x)$$

for $x > m + dm$. This implies that $p_{dm}(\cdot)$ is discontinuous, contradicting Proposition 1. \square

The proof of Proposition 3 uses some additional notation. In this section, conditional expectations \mathbb{E}_x and covariances Cov_x of functions of elasticities, costs, and marginal costs are defined pointwise with respect to h . We denote the right limits of the moments below as

$$\mathbb{E}_m^+[f] = \lim_{x \rightarrow^+ m} \mathbb{E}_x[f], \text{ and}$$

$$\text{Cov}_m^+[f] = \lim_{x \rightarrow^+ m} \text{Cov}_x[f].$$

The proof strategy is to calculate how much prices change, $\partial_{dm} p_{dm}(x)$, based on how consumers change their choices. Consumers change their choices based on changes in prices, $\partial_{dm} p(x)$, and marginal prices, $\partial_{dm} p'_{dm}(x)$. Thus, assuming that consumers optimize and that prices equal average cost gives us a differential equation relating the change in the price function, $\partial_{dm} p_{dm}(x)$, with its derivatives with respect to x . In particular, this differential equation will give us a good approximation for the change in the level of prices close to minimum coverage.

We begin by noting that, by consumers' first-order condition, for any consumer purchasing coverage greater than m we have

$$\partial_{dm} \hat{x}(\theta, dm)|_{dm=0} = \epsilon(\hat{x}(\theta, 0), \theta) \cdot \frac{\partial_{dm} p'_{dm}(\hat{x}(\theta, 0))|_{dm=0}}{p'(\hat{x}(\theta, 0))} \cdot \hat{x}(\theta, 0).$$

Let $I = [m + dm, x]$, where $x > m$. We will first calculate formulas for the change in the demand and total price paid for contracts in I .

Claim 1. For $x > m$,

$$\partial_{dm} G_{dm}(x)|_{dm=0} = -g(x) \cdot \frac{\partial_{dm} p'_{dm}(x)|_{dm=0}}{p'(x)} \cdot x \cdot \mathbb{E}_x[\epsilon].$$

Proof. We have that

$$\begin{aligned} G_{dm}(x) &= \int_{(y,\theta) \in I \times \Theta} \mathbf{1} d\alpha_{dm} \\ &= \int_{(y,\theta)} \mathbf{1}\{\hat{x}(\theta, dm) \leq x\} d\alpha \\ &= \int_{(y,\theta)} \mathbf{1}\{\hat{x}(\theta, dm) - y + y \leq x\} d\alpha, \end{aligned}$$

where $\mathbf{1}$ is the indicator function. Moreover, by the assumption that consumers who originally purchased minimum coverage continue to do so, we have

$$G_{dm}(x) = \int_{(y,\theta) \in (m,1] \times \Theta} \mathbf{1}\{\hat{x}(\theta, dm) - y + y \leq x\} d\alpha + G(m).$$

We can substitute $\hat{x}(\theta, dm) - y$ with the derivative of \hat{x} , and the total error in the integral is bounded above by a term of order dm^2 , because G is atomless for $y > m$. Substituting the derivative we get

$$G_{dm}(x) = \int_{(y,\theta) \in (m,1] \times \Theta} \mathbf{1}\{\partial_{dm} \hat{x}(\theta, 0) \cdot dm + y \leq x\} d\alpha + G(m) + O(dm^2).$$

Substituting the formula for the derivative of x with respect to the elasticity we get

$$G_{dm}(x) = \int_{(y,\theta) \in (m,1] \times \Theta} \mathbf{1}\{\epsilon(y, \theta) \cdot \frac{\partial_{dm} p'_{dm}(y)|_{dm=0}}{p'(y)} \cdot y \cdot dm + y \leq x\} d\alpha + G(m) + O(dm^2).$$

This integrand only depends on the joint distribution of elasticities and contracts. Thus we can evaluate it using the distribution of contracts and the conditional distribution of elasticities. That is,

$$G_{dm}(x) = \int d(\tilde{\epsilon}, \tilde{c}, \tilde{m}c) \int dy g(y) \cdot h(\tilde{\epsilon}, \tilde{c}, \tilde{m}c|y) \cdot \mathbf{1}\{\tilde{\epsilon} \cdot \frac{\partial_{dm} p'_{dm}(y)|_{dm=0}}{p'(y)} \cdot y \cdot dm + y \leq x\} + G(m) + O(dm^2).$$

The inner integral integrates y from m to the implicit solution of

$$\tilde{\epsilon} \cdot \frac{\partial_{dm} p'_{dm}(y)|_{dm=0}}{p'(y)} \cdot y \cdot dm + y = x.$$

Using the implicit function theorem, we can see that the derivative of the upper limit of integration of y with respect to dm evaluated at $dm = 0$ is

$$-\tilde{\epsilon} \cdot \frac{\partial_{dm} p'_{dm}(x)|_{dm=0}}{p'(x)} \cdot x.$$

We can now evaluate the derivative of $G_{dm}(x)$ using Leibniz's rule. We have

$$\begin{aligned} \partial_{dm} G_{dm}(x) &= - \int d(\tilde{\epsilon}, \tilde{c}, \tilde{m}c) g(x) \cdot h(\tilde{\epsilon}, \tilde{c}, \tilde{m}c|x) \cdot \tilde{\epsilon} \cdot \frac{\partial_{dm} p'_{dm}(x)|_{dm=0}}{p'(x)} \cdot x \\ &= -g(x) \cdot \frac{\partial_{dm} p'_{dm}(x)|_{dm=0}}{p'(x)} \cdot x \cdot \mathbb{E}_x[\epsilon]. \end{aligned}$$

□

Claim 2. Define the total expenditures on contracts in I as

$$P_{dm}(x) := \int_{(y,\theta) \in I \times \Theta} p_{dm}(y) d\alpha_{dm}$$

We have that, at $dm = 0$, and $x > m$,

$$\begin{aligned} \partial_{dm} P_{dm}(x)|_{dm=0} &= G(m) \cdot \mathbb{E}_m[mc] + \int_{y=m}^x g(y) \cdot \frac{\partial_{dm} p'_{dm}(y)|_{dm=0}}{p'(y)} \cdot y \cdot \mathbb{E}_y[mc \cdot \epsilon] dy \\ &\quad - g(x) \cdot \frac{\partial_{dm} p'_{dm}(x)|_{dm=0}}{p'(x)} \cdot x \cdot \mathbb{E}_x[c \cdot \epsilon]. \end{aligned}$$

Proof. Because prices equal average costs in equilibrium, we have

$$P_{dm}(x) = \int_{(y,\theta)} c(\hat{x}(\theta, dm), \theta) \cdot \mathbf{1}\{\hat{x}(\theta, dm) \leq x\} d\alpha.$$

We can decompose this integral into

$$\begin{aligned} P_{dm}(x) &= \int_{(y,\theta)} c(y, \theta) \cdot \mathbf{1}\{\hat{x}(\theta, dm) \leq x\} d\alpha \\ &\quad + \int_{(y,\theta)} (c(\hat{x}(\theta, dm), \theta) - c(y, \theta)) \cdot \mathbf{1}\{\hat{x}(\theta, dm) \leq x\} d\alpha. \end{aligned} \tag{5}$$

These two terms decompose the change in total prices paid in two components. The first term of (5) contains the change due to consumers entering or leaving the interval I as prices change. In particular, calculating the derivative of the integral using the same argument as

in Claim 1, gives

$$\partial_{dm} \int_{(y,\theta)} c(y, \theta) \cdot \mathbf{1}\{\hat{x}(\theta, dm) \leq x\} d\alpha|_{dm=0} = -g(x) \cdot \frac{\partial_{dm} p'_{dm}(x)|_{dm=0}}{p'(x)} \cdot x \cdot \mathbb{E}_x[\epsilon \cdot c].$$

The second term of (5) contains the change due to consumers who change their coverage. We can decompose it into consumers who originally purchased minimum coverage, and consumers who purchased an interior level of coverage. That is, the second term of (5) equals

$$\begin{aligned} & \int_{\theta \in \Theta} (c(\hat{x}(\theta, dm), \theta) - c(m, \theta)) \cdot \mathbf{1}\{\hat{x}(\theta, dm) \leq x\} d\alpha|_m(\theta) \\ & + \int_{(y,\theta): y > m} (c(\hat{x}(\theta, dm), \theta) - c(y, \theta)) \cdot \mathbf{1}\{\hat{x}(\theta, dm) \leq x\} d\alpha. \end{aligned} \quad (6)$$

The derivative of the first term of (6) is simple to calculate. We assumed that consumers who originally purchased minimum coverage continue to do so after the increase in minimum coverage. Thus, if minimum coverage increases by dm , these consumers increase their allocations by dm . Therefore, the derivative of the first term with respect to dm evaluated at $dm = 0$ is

$$G(m) \cdot \mathbb{E}_m[mc].$$

The derivative of the second term of (6) is also straightforward. There are order of dm consumers who do not choose an interior bundle or for whom the indicator function is not constant. For each such consumer, the term related to the change in costs is of the order dm . So these consumers do not affect the derivative of the second term. This means that we can calculate the derivative of the second term considering only consumers who always make interior choices and for whom the indicator function is constant. Thus, the derivative of the second term of (6) equals

$$\int_{y=m}^x g(y) \cdot \frac{\partial_{dm} p'_{dm}(y)|_{dm=0}}{p'(y)} \cdot y \cdot \mathbb{E}_y[mc \cdot \epsilon] dy.$$

□

Claim 3. We have

$$\begin{aligned} & \partial_{dm} \{p_{dm}(m + dm) - p(m + dm)\}|_{dm=0} \\ & = -S_I(m) - \frac{g(m)}{G(m)} \cdot \frac{\lim_{x \rightarrow m} \partial_{dm} p'_{dm}(x)|_{dm=0}}{p'(m)} \cdot m \cdot \text{Cov}_m^+[c, \epsilon]. \end{aligned}$$

Proof. We need two intermediate formulas. First, taking the limit of Claim 1 as x converges

to m , we get

$$\lim_{x \rightarrow m} \partial_{dm} G_{dm}(x)|_{dm=0} = -g(m) \cdot \frac{\lim_{x \rightarrow m} \partial_{dm} p'_{dm}(x)|_{dm=0}}{p'(m)} \cdot m \cdot \mathbb{E}_m^+[\epsilon].$$

The left-hand side of this equation is

$$\begin{aligned} & \lim_{x \rightarrow m} \partial_{dm} \left\{ \int_{y=m+dm}^x g_{dm}(y) dy + G_{dm}(m+dm) \right\} |_{dm=0} \\ &= -g(m) + \partial_{dm} G_{dm}(m+dm)|_{dm=0}. \end{aligned}$$

Therefore,

$$\partial_{dm} G_{dm}(m+dm)|_{dm=0} = g(m) - g(m) \cdot \frac{\lim_{x \rightarrow m} \partial_{dm} p'_{dm}(x)|_{dm=0}}{p'(m)} \cdot m \cdot \mathbb{E}_m^+[\epsilon]. \quad (7)$$

Second, taking the limit of Claim 2 as x converges to m we get

$$\begin{aligned} & \lim_{x \rightarrow m} \partial_{dm} P_{dm}(x)|_{dm=0} \\ &= G(m) \cdot \mathbb{E}_m[mc] \\ & \quad - g(m) \cdot \frac{\lim_{x \rightarrow m} \partial_{dm} p'_{dm}(x)|_{dm=0}}{p'(m)} \cdot m \cdot \mathbb{E}_m^+[c \cdot \epsilon]. \end{aligned}$$

The left-hand side of this equation is

$$\begin{aligned} & \lim_{x \rightarrow m} \partial_{dm} \left[\int_{y=m+dm}^x p_{dm}(y) \cdot g_{dm}(y) dy + p_{dm}(m+dm) G_{dm}(m+dm) \right] |_{dm=0} \\ &= -p(m) \cdot g(m) + \partial_{dm} \{ p_{dm}(m+dm) \cdot G_{dm}(m+dm) \} |_{dm=0}. \end{aligned}$$

Therefore,

$$\begin{aligned} & \partial_{dm} \{ p_{dm}(m+dm) \cdot G_{dm}(m+dm) \} |_{dm=0} \\ &= p(m) \cdot g(m) + G(m) \cdot \mathbb{E}_m[mc] \\ & \quad - g(m) \cdot \frac{\lim_{x \rightarrow m} \partial_{dm} p'_{dm}(x)|_{dm=0}}{p'(m)} \cdot m \cdot \mathbb{E}_m^+[c \cdot \epsilon]. \end{aligned} \quad (8)$$

By the product rule we have that

$$\begin{aligned} & G(m) \cdot \partial_{dm} \{ p_{dm}(m+dm) \} |_{dm=0} \\ &= \partial_{dm} \{ p_{dm}(m+dm) \cdot G_{dm}(m+dm) \} |_{dm=0} - p(m) \cdot \partial_{dm} G_{dm}(m+dm)|_{dm=0}. \end{aligned}$$

Substituting equations (7) and (8) we have

$$\begin{aligned}
& G(m) \cdot \partial_{dm} \{p_{dm}(m + dm)\} |_{dm=0} \\
= & G(m) \cdot \mathbb{E}_m[mc] \\
& -g(m) \cdot \frac{\lim_{x \rightarrow m} \partial_{dm} p'_{dm}(x) |_{dm=0}}{p'(m)} \cdot m \cdot (\mathbb{E}_m^+[c \cdot \epsilon] - p(m) \cdot \mathbb{E}_m^+[\epsilon]).
\end{aligned}$$

Using the fact that $p(m) = \mathbb{E}_m^+[c]$ and the definition of covariance we have

$$\begin{aligned}
& G(m) \cdot \partial_{dm} \{p_{dm}(m + dm)\} |_{dm=0} \\
= & G(m) \cdot \mathbb{E}_m[mc] \\
& -g(m) \cdot \frac{\lim_{x \rightarrow m} \partial_{dm} p'_{dm}(x) |_{dm=0}}{p'(m)} \cdot m \cdot \text{Cov}_m^+[c, \epsilon].
\end{aligned}$$

Using this formula and the definition of $S_I(m)$ we have the desired result. \square

We can now establish the proposition.

Proof of Proposition 3. The smoothness of $p_{dm}(x)$ implies that

$$\partial_{dm} \{p_{dm}(m + dm) - p(m + dm)\} |_{dm=0} = \lim_{x \rightarrow m} \partial_{dm} p_{dm}(x) |_{dm=0}.$$

Claim 3 then implies the desired formula for the level effect, with

$$\xi = -\frac{g(m)}{G(m)} \cdot \frac{\lim_{x \rightarrow m} \partial_{dm} p'_{dm}(x) |_{dm=0}}{p'(m)} \cdot m \cdot \text{Cov}_m^+[c, \epsilon]. \tag{9}$$

\square