

# Does improved information improve incentives?\*

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## Abstract

This paper studies the value of more precise signals on agent performance in an optimal contracting model with endogenous effort. With limited liability, the agent’s wage is increasing in output only if output exceeds a threshold, else it is zero regardless of output. If the threshold is sufficiently high, the agent only beats it, and is rewarded for increasing output through greater effort, if there is a high noise realization. Thus, a fall in output volatility reduces effort incentives – information and effort are substitutes – offsetting the standard effect that improved information lowers the cost of compensation. We derive conditions relating the incentive effect to the underlying parameters of the agency problem.

KEYWORDS: executive compensation, limited liability, options, risk management, relative performance evaluation.

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# 1 Introduction

Since the seminal contributions of Holmstrom (1979) and Shavell (1979), the moral hazard literature has shown that superior information on agent performance can improve the principal's payoff. This result has implications for many contracting applications, such as compensation, financing, insurance, and regulation. However, information is also costly. Thus, to determine whether the principal should invest in information, we must analyze what its benefits depend on – in particular, relate them to the underlying parameters of the agency problem. Doing so will identify the situations in which information is most valuable, and thus its acquisition most justified.

Analyzing the benefits of information requires an optimal contracting approach. Assuming a particular contracting form may lead to misleading results – for example, when contracts are suboptimal, adding noise to the contract may be desirable, suggesting that precision has strictly negative value. As is well-known, solving for the optimal contract in a general setting is highly complex (e.g., Grossman and Hart, 1983). We consider the standard framework of risk neutrality and limited liability, originally analyzed by Innes (1990) and widely used in a number of settings (e.g., Biais, Mariotti, Rochet, and Villeneuve, 2010; Clementi and Hopenhayn, 2006; DeMarzo and Fishman, 2007a, 2007b; DeMarzo and Sannikov, 2006). This framework allows for an optimal contracting approach and involves the agent receiving a call option, as observed in practice.<sup>1</sup>

We model the option contract as based on output and information as affecting output volatility, but the model is virtually identical if the contract is instead based on a separate performance signal and information affects the volatility of this signal. We start by assuming a general output distribution and an endogenous implemented effort level. An increase in information precision (fall in volatility), in the sense of a mean-preserving spread of output, has two effects, each with a clear economic interpretation. First, a fall in volatility reduces the value of the option and thus the expected wage: the direct effect. Second, it changes the agent's effort incentives: the incentive effect. While the direction of the direct effect is unambiguous – the standard intuition that information reduces the cost of contracting – the incentive effect can either be positive or negative, i.e., effort and precision can be complements or substitutes. The heart of this paper analyzes this incentive effect. Our most general result is to derive a

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<sup>1</sup>While options are not the only instruments used in practice, Dittmann and Maug (2007) find that the payoff structure provided by a CEO's overall compensation package resembles an option.

condition that determines the sign of this effect, and thus whether effort and precision are complements (precision increases incentives) or substitutes (precision decreases incentives). The condition is simple and easy to check, as we show with a simple example.

We then consider the common case of output distributions with a location parameter, i.e., effort shifts the location of the distribution without affecting its shape, as in the normal and uniform distributions. This allows us to relate the sign of the incentive effect to the strike price of the option. We show that the incentive effect is positive if and only if the strike price is below a threshold, and negative otherwise. The intuition is as follows. Since the wage is positive only if output exceeds the strike price, increasing effort increases the wage only if output ends up higher than the strike price – if output still ends up below, the agent receives zero, regardless of output and thus his effort. Increasing precision moves probability mass from the tails towards the center of the distribution. If the strike price is low, this change moves mass from below to above the strike price, increasing the probability that output exceeds it. In simple language, the agent thinks “If I work harder, I’ll get paid more unless I get so unlucky that output falls below the target. Now that precision is higher, I’m unlikely to suffer bad enough luck, so it’s worth it for me to work harder.” Thus, effort and precision are complements, and so the principal’s benefit from increasing precision is even higher than when focusing on the direct effect alone.

On the other hand, if the strike price is high, increasing precision shifts mass from above to below the strike price, reducing the probability that output exceeds it. In simple language, the agent thinks “The target is so high that, even if I did work, I wouldn’t meet it unless I also got lucky. Now that precision is higher, I’m unlikely to get lucky enough to meet the target, so there’s no point in working.” Thus, effort and precision are substitutes, and so the principal’s benefit from increasing precision is lower than when focusing on the direct effect alone. As a result, the net benefit of precision may be insufficient to justify its cost.

Since the strike price is endogenous, we next relate the incentive effect to the underlying parameters of the agency problem – specifically the cost function. In any contracting setting, the effect of the cost function on the strength of incentives (here, captured by the option’s delta and thus the strike price) depends on whether the implemented effort level is fixed or endogenous. With endogenous effort, a less convex cost of effort typically leads to the principal implementing a higher effort level and

thus offering stronger incentives; with fixed effort, a lower cost of effort means that weaker incentives are needed to implement the given effort level. We therefore analyze both cases. Where effort is endogenous, the sign of the incentive effect depends on the convexity of the cost function. If the cost function is sufficiently convex, inducing effort is costly and so the principal implements a low effort level. As a result, the strike price is high relative to the effort level, and so precision reduces incentives. This result contrasts intuition that information should be more valuable when the agency problem is strong. In addition, an analysis focusing only on the direct effect of precision, and ignoring the incentive effect, would suggest that the value is highest when the option is at-the-money – i.e., a moderate initial strike price and a moderate agency problem.

A fixed effort level arises with a binary effort space, which is often used for tractability. In addition to being tractable, a fixed effort level is also realistic if the benefits of effort are large relative to the costs, as with CEOs of large firms (Edmans and Gabaix, 2011), because the principal always wishes to implement full productive efficiency. The sign of the incentive effect depends on the cost of effort: where the cost is high, the principal must give the agent strong incentives to induce effort. These incentives are provided by a low strike price, and so precision increases incentives.

It is limited liability, and not risk neutrality, that is key to our results. We show that the incentive effect similarly depends on the strike price when the agent is risk-averse. Regardless of whether the agent is risk-neutral or risk-averse, when there is limited liability, the optimal contract pays the agent zero when output is below a threshold. Thus, he is only rewarded for marginal increases in effort if output exceeds the threshold, the probability of which depends on output volatility.

We also analyze the case in which the principal can renege on any pre-announced level of precision after the agent has exerted effort. Then, there is no incentive effect (the agent ignores the initial announcement when selecting effort) and so the principal only considers the direct effect when choosing the final level of precision. Thus, her benefit from increasing precision is higher *ex ante* than *ex post* if and only if the incentive effect is positive. In this case, she may wish to commit to a high level of precision *ex ante*. We also show that the level of precision chosen by the principal generally differs from the socially optimal level, and may exceed it. This result suggests that regulations to increase information disclosure may move us further from the social optimum. In addition, if the agent had bargaining power and chose precision, he would typically select a different level from the principal, so control rights matter for efficiency.

Our results have a number of implications for compensation contracts. Most importantly, they highlight that information may not improve incentives, contrary to conventional wisdom that more precise signals make incentive provision easier. They also identify the settings in which investing in information is optimal for the principal. Using the fixed effort model as an example, when incentives are strong (weak) to begin with, e.g., for CEOs (rank-and-file managers), an increase in precision increases (reduces) incentives. One way in which the principal can invest in information is to engage in relative performance evaluation (“RPE”). There is very little evidence that RPE is used for rank-and-file managers, and only modest evidence of its usage for CEOs.<sup>2</sup> Bebchuk and Fried (2004) interpret this rarity as evidence that CEO contracts are inefficient. However, to evaluate this argument, we need to identify the settings in which the value of information is smallest, and compare them to the cases in which RPE is particularly absent in reality. That RPE is more common for CEOs than managers is consistent with the above prediction.

In addition to the gains from precision, our analysis also studies the impact of exogenous changes in precision, such as changes in stock market efficiency. For example, an increase in volatility raises (lowers) the incentives of agents with out-of-the-money (in-the-money) options. If firms recontract in response, CEOs with in-the-money options should receive the highest increase in incentives.

As Innes (1990) showed, in addition to compensation, the model can also be applied to a financing setting where the agent (entrepreneur) raises financing from the principal (investor), in which case the contract is risky debt, and the strike price represents its face value. Our model sheds light on the settings in which the investor’s incentive to reduce output volatility is highest. As with investing in information, doing so is potentially costly – implementing a risk management system is expensive, and imposing covenants can stifle investment. An analysis based on the direct effect would suggest that risk management is most valuable for firms at the bankruptcy threshold, as then the value of debt is most sensitive to volatility. This is consistent with standard intuition that risk management incentives are increasing in loan size (up to the bankruptcy threshold), because the lender has more at stake. This intuition is incomplete because it ignores the incentive effect. When the face value of debt is low, equity is in-the-money and the incentive effect is positive. As a result, risk management raises

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<sup>2</sup>While Aggarwal and Samwick (1999) and Murphy (1999) document almost no use, more recent evidence by Albuquerque (2009), Gong, Li, and Shin (2011), and De Angelis and Grinstein (2017) find evidence of RPE. See Edmans, Gabaix, and Jenter (2017) for a review of the evidence on RPE.

effort incentives, further increasing its value over and above the direct effect. Thus, surprisingly, risk management may be more valuable for firms that are some distance from bankruptcy, and when the investor has little debt at stake. Separately, the model suggests that the entrepreneur's initial wealth affects the value of precision. Where wealth is low, the entrepreneur needs to raise debt with a high face value which, as is known, reduces effort incentives. Our results suggest that increases in precision may further exacerbate the incentive problems originating from low initial wealth, since the incentive effect is then negative.

Dittmann, Maug, and Spalt (2013) also consider the effect on effort when analyzing a specific form of increased precision – indexing stock and options – and similarly show that indexation may weaken incentives. They use a quite different setting, reflecting the different aims of each paper. Their goal is to calibrate real-life contracts, and so they fix the implemented effort level, restrict the contract to comprising salary, stock, and options, and hold stock constant when changing the contract to restore the agent's incentives upon indexation. They acknowledge that the actual savings from indexation will be different if the principal recontracts optimally. Our primary goal is theoretical. We take an optimal contracting approach, where the contract adjusts optimally to changes in precision, and the implemented effort level is endogenous.

The interaction between effort and (mean-preserving) risk has been studied by other papers. In Holmstrom and Milgrom (1987), risk has no direct effect on effort (unlike in our paper), but greater risk reduces the optimal pay-performance sensitivity and thus incentives. In this sense, precision always increases effort, unlike in our paper. Gjesdal (1982) shows that incentives can increase with a different type of risk – giving the agent a stochastic contract where, for a given output level, the agent receives a lottery rather than a wage. If the agent's utility is non-separable in the wage and his action, higher effort may directly reduce the agent's risk aversion; giving him a lottery incentivizes him to exert effort to reduce his aversion to the lottery. In our paper, a deterministic contract is optimal (as is common in reality) and risk instead involves changing the precision of the performance measure, allowing our model to apply to risk management and monitoring. The channel through which risk affects effort is also very different.

This paper proceeds as follows. Section 2 presents the model. The main results are presented in Section 3, where we study the effects from increased signal precision and relate the incentive effect to the underlying parameters of the agency problem. Section 4 discusses applications and alternative modeling assumptions, and Section 5

concludes. The proofs are presented in Appendix A.

## 2 The model

We consider a standard principal-agent model with risk neutrality and limited liability, as in Innes (1990). At time  $t = -1$ , the principal offers a contract to the agent. At  $t = 0$ , the agent chooses effort  $e$  from a non-empty, compact subset of the real line  $\mathcal{E}$ . The agent's cost of exerting effort  $e$  is  $C(e)$ , where  $C(\cdot)$  is continuous and increasing. As is standard, effort can refer not only to working rather than shirking, but also choosing projects to maximize firm value rather than private benefits, or not diverting cash flows. We normalize the lowest effort level to 0 and its cost to  $C(0) = 0$ . At  $t = 1$ , output  $q$  is realized.

The principal does not observe the agent's effort, but observes the realization of output. Output is continuously distributed according to a cumulative distribution function ("CDF")  $F_\theta(q|e)$  with a continuous probability density function ("PDF")  $f_\theta(q|e)$  that satisfies the monotone likelihood ratio property ("MLRP"): for any  $e_H > e_L$  and any  $\theta$ ,  $\frac{f_\theta(q|e_H)}{f_\theta(q|e_L)}$  is strictly increasing in  $q$ .<sup>3</sup> Intuitively, MLRP means that higher outputs indicate higher effort.

To ensure existence of an optimal contract, we assume that output has a finite mean  $\mathbb{E}_\theta[q|e] < \infty$  and the integral  $\int_X^\infty q f_\theta(q|e) dq$  is a continuous function of  $e$  for each  $X \in \mathbb{R}$ . Moreover, to simplify notation, we assume that the CDF  $F_\theta(q|e)$  is differentiable with respect to  $\theta$ .

The parameter  $\theta$  orders the precision of the output distribution in the sense of a mean-preserving spread ("MPS"). Formally, for any  $\theta \geq \theta'$ ,  $\mathbb{E}_\theta[q|e] = \mathbb{E}_{\theta'}[q|e]$ , and

$$F_\theta(q|e) \left\{ \begin{array}{l} \leq \\ \geq \end{array} \right\} F_{\theta'}(q|e) \text{ for } q \left\{ \begin{array}{l} < \\ > \end{array} \right\} q_e. \quad (1)$$

for some  $q_e$ .<sup>4</sup> Since a higher  $\theta$  removes noise from the distribution of output without affecting its mean,  $\theta$  represents how informative output  $q$  is about effort  $e$ .

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<sup>3</sup>Note that MLRP implies first-order stochastic dominance ("FOSD"):  $F_\theta(q|e)$  is strictly decreasing in  $e$  for each fixed  $(\theta, q)$ . Thus, throughout the paper, we use "effort increases output" as a shorthand for "effort improves the distribution of output in the sense of FOSD."

<sup>4</sup>This definition follows Machina and Rothschild (2008) and states that the distributions differ by a single MPS. This notion implies second-order stochastic dominance ("SOSD"), which allows distributions to differ by a sequence of MPSs.

The precision parameter  $\theta$  may be chosen by the principal, or result from exogenous changes such as a reduction in economic uncertainty. Our goal is to analyze the value of information and its effect on incentives, which applies to either setting.<sup>5</sup>

The agent is paid a “wage”  $W_\theta(q)$  and the principal receives a “profit”  $R_\theta(q) = q - W_\theta(q)$ . The agent is risk-neutral and so maximizes his expected wage

$$\mathbb{E}_\theta [W_\theta(q) | e] = \int_{-\infty}^{\infty} W_\theta(q) f_\theta(q|e) dq,$$

less the cost of effort. His reservation utility is zero and there is no discounting.

Following Innes (1990), we make two assumptions on the set of feasible contracts. First, there is a limited liability constraint on the agent:

$$W_\theta(q) \geq 0 \quad \forall q. \tag{2}$$

Second, a monotonicity constraint ensures the principal’s payoff is non-decreasing in output:

$$\eta \geq W_\theta(q + \eta) - W_\theta(q) \tag{3}$$

for all  $\eta > 0$ . Innes (1990) justifies this constraint on two grounds. First, if it did not hold, the principal would have incentives to sabotage output. Second, if it did not hold, the agent would gain more than one-for-one for increases in output. Thus, he would have incentives to borrow on his own account to increase output.

The principal wishes the agent to exert effort level  $e^*$ , and so the contract must satisfy the following incentive compatibility constraint:

$$e^* \in \arg \max_e \mathbb{E}_\theta [W_\theta(q) | e] - C(e). \tag{4}$$

Following standard arguments, the participation constraint will be slack and can be ignored in the analysis that follows. The principal is also risk-neutral and thus chooses a contract  $W_\theta(q)$  and an effort level  $e^*$  to maximize her expected profit  $R_\theta(q)$  subject to the limited liability (2), monotonicity (3), and incentive (4) constraints.

We discuss three features of our modeling setup. First, following Innes (1990), the principal contracts on output  $q$  and so changes in the precision of the performance

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<sup>5</sup>While we consider the effect of changing the volatility of output, Chaigneau, Edmans, and Gottlieb (2017) derive necessary and sufficient conditions for whether the addition of a new signal has strictly positive value under contracting constraints.



measure change the volatility of output, as with risk management. We have also analyzed a model in which  $q$  is non-contractible, as in Baker (1992). For example, the principal may not have a contractible objective (e.g., a non-profit firm or government agency, or a private firm with no traded stock); the agent may be only one employee in a team and only the team’s output is observable (e.g., a public firm where the employee has very little effect on the stock price); effort may contribute to a long-term project such as R&D where, by the time verifiable output is fully realized, the agent may no longer be with the firm; or the effort may have non-quantifiable outcomes, such as corporate social responsibility initiatives or actions with externalities on other divisions (see also Malenko (2016)). In this model, there is a separate signal  $s = q + \eta$  on which contracts could be written. Thus, the precision of the signal  $s$  could be affected without changing output volatility, and so this model applies to improving monitoring technology or filtering out noise.<sup>6</sup> All results continue to hold (because of risk neutrality, changing the volatility of output has no effect), but the notation is more complex due to the introduction of an additional variable. Appendix B analyzes an alternative framework where both output  $q$  and a separate signal  $s$  are contractible, and changes in precision change the volatility of  $s$  but not  $q$ . In the core model, since the “signal” equals “output”, we will use these terms interchangeably.

Second, in Innes (1990), the agent offers the contract and maximizes his utility subject to the principal’s participation constraint. Since it is the principal who will typically invest in information, we model her as offering the contract so that she captures the surplus and thus the benefits from precision. Appendix F considers the agent offering the contract and choosing precision. Third, while Innes (1990) assumes continuous effort and the first-order approach (“FOA”), we do not impose any such structure on effort for our most general results.

## 2.1 The optimal contract

This section solves for the optimal contract holding precision  $\theta$  fixed. The analysis is similar to Innes (1990). Our main results will come in Section 3, which analyzes the gains from increasing  $\theta$ .

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<sup>6</sup>In that model, there is a similar justification for the monotonicity constraint (3). Rather than borrowing on his own account to increase the signal, the agent could exert effort to manipulate the signal. If the marginal cost of increasing the signal by 1 is  $\mu$ , the contract slope will equal  $\mu$  (any greater slope will induce manipulation). Thus, all the results will hold except that the contract slope is now  $\mu$  rather than 1.

Lemma 1 below establishes that the optimal payment to the agent  $W_\theta(\cdot)$  is a call option on  $q$  with strike price  $X_\theta$ . Alternatively, the optimal payment to the principal  $R_\theta(\cdot)$  is risky debt with face value  $X_\theta$ .

**Lemma 1** (*Optimal contract*) *For each  $\theta$ , there exists an optimal contract with*

$$W_\theta(q) = \max\{0, q - X_\theta\}, \quad (5)$$

$$R_\theta(q) = \min\{q, X_\theta\}, \quad (6)$$

for some  $X_\theta$ .

As in Innes (1990), the intuition is as follows. The absolute value of the likelihood ratio is highest in the tails of the distribution of  $q$ , so output is most informative about effort in the tails. The principal cannot incentivize the agent in the left tail by giving negative wages (due to limited liability), and so she incentivizes him in the right tail by giving high wages. With an upper bound on the slope, the optimal contract involves call options on  $q$  with the maximum feasible slope, i.e.,  $\frac{\partial W_\theta}{\partial q}(q) = 1$ . Since the agent's positive wage for high output cannot be offset by a negative wage for low output, his expected wage  $\mathbb{E}_\theta [W_\theta(q) | e]$  strictly exceeds his reservation utility of zero, and so he enjoys rents.

Substituting the contract (5) into the incentive constraint (4), the effort that the agent chooses when offered a contract with strike price  $X_\theta$  is given by

$$e_\theta(X_\theta) \in \arg \max_e \int_{X_\theta}^{\infty} (q - X_\theta) f_\theta(q|e) dq - C(e). \quad (7)$$

In general,  $e_\theta(X_\theta)$  may not be single-valued. Whenever this is the case, we follow the standard approach of choosing the effort level preferred by the principal.

### 3 The value of information

This section calculates the value of information to the principal, by studying the effect of changes in signal precision on the principal's profits. Section 3.1 shows that the total effect of precision contains both a direct and incentive effect, and provides a condition that determines the sign of the incentive effect that holds for all output distributions. Section 3.2 shows that, for distributions with a location parameter, the sign of the

incentive effect depends on the strike price of the option. Section 3.3 relates the initial strike price of the option – and thus the incentive effect and the value of information – to the underlying parameters of the agency problem.

### 3.1 The incentive effect

We initially assume that the effort function  $e_\theta(X_\theta)$  is differentiable with respect to  $\theta$ . This is for simplicity of exposition and transparency of intuition; we drop this assumption later.

Differentiating the principal’s expected profits with respect to precision yields:

$$\underbrace{\frac{d}{d\theta} \mathbb{E}_\theta [R_\theta(q) | e_\theta(X_\theta)]}_{\text{Total Effect}} = \underbrace{\frac{\partial}{\partial \theta} \mathbb{E}_\theta [R_\theta(q) | e_\theta(X_\theta)]}_{\text{Direct Effect}} + \underbrace{\frac{d}{dX_\theta} \mathbb{E}_\theta [R_\theta(q) | e_\theta(X_\theta)] \frac{\partial X_\theta}{\partial \theta}}_{\text{Zero by Envelope Theorem}} + \underbrace{\frac{\partial}{\partial e} \mathbb{E}_\theta [R_\theta(q) | e_\theta(X_\theta)] \frac{\partial e_\theta(X_\theta)}{\partial \theta}}_{\text{Incentive Effect}} \quad (8)$$

The first term is the *direct effect*. Holding constant the strike price and effort level, an increase in output precision reduces the value of the agent’s option  $W$  and increases the principal’s expected profit  $R$ . Due to limited liability, the principal’s profit,  $\min\{q, X_\theta\}$ , is concave in  $q$ ; the agent’s option,  $\max\{0, q - X_\theta\}$ , is convex in  $q$ . Since  $\theta$  orders the distribution of  $q$  in terms of SOSD, the expected profit (wage) is increasing (decreasing) in  $\theta$ . This reduction in pay is the benefit of precision highlighted by Bebchuk and Fried (2004) in their argument that the lack of RPE is inefficient. In the Holmstrom (1979) setting of a risk-averse agent, an increase in precision reduces the risk borne by the agent and thus allows the principal to lower the expected wage. In our setting of risk neutrality and limited liability, precision directly reduces the expected wage by lowering the value of the option.

The second term is the effect of re-optimizing the strike price on profits, which is zero by the envelope theorem – the principal had already optimized it for the initial level of precision. The third term is the *incentive effect*, which arises because the increase in precision affects the agent’s incentive to exert effort. An increase in effort raises profits ( $\frac{\partial}{\partial e} \mathbb{E} [R_\theta(q) | e_\theta(X_\theta)] > 0$ ), but whether precision increases or decreases effort is ambiguous. Our goal is to analyze the determinants of this effect and relate them to the underlying parameters of the agency problem.

Even when  $\frac{\partial e_\theta(X_\theta)}{\partial \theta} < 0$  and the incentive effect counteracts the direct effect, it can never outweigh it. The total effect is always weakly negative from revealed preference: if reducing precision reduced the principal's profit, she would have added in randomness to the contract, and so the initial contract would not have been optimal.

For a given contract, if the agent's effort level increases (decreases) with precision, we say that the incentive effect is positive (negative) and that effort and precision are complements (substitutes). Differentiating the incentive constraint (4) with respect to  $e$  and  $\theta$ , the incentive effect is positive if and only if

$$\frac{\partial^2}{\partial e \partial \theta} \mathbb{E}_\theta [W_\theta(q) | e] \geq 0. \quad (9)$$

and negative if the reverse inequality holds.

Integrating by parts, we can rewrite the agent's expected wage as follows:

$$\mathbb{E}[W_\theta(q) | e, \theta] = \mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta} F_\theta(q|e) dq. \quad (10)$$

Precision does not affect the mean output  $\mathbb{E}[q|e]$ , and effort does not affect the strike price  $X_\theta$ . Thus, only the cross-partial of the third term on the right-hand side ("RHS") of (10) is non-zero. Therefore, precision increases (decreases) effort, i.e., effort and precision are complements (substitutes), if

$$\frac{\partial^2}{\partial \theta \partial e} \int_{-\infty}^X F_\theta(q|e) dq \geq (\leq) 0 \quad \forall e, X. \quad (11)$$

We now formalize the above argument and generalize it to cases where the effort function is not differentiable with respect to precision.<sup>7</sup> To allow for this case, and thus to accommodate situations where the FOA fails or the effort space is discrete, Definition 1 replaces the derivatives that determine the total and direct effects by their discrete counterparts. Let  $\Pi(\theta) \equiv \mathbb{E}_\theta[R_\theta(q)|e_\theta(X_\theta)]$  denote the principal's expected profit when precision equals  $\theta$ .

**Definition 1** *The incentive effect of precision is positive at  $\theta$  if*

$$\frac{\Pi(\theta') - \Pi(\theta)}{\theta' - \theta} \geq \frac{\mathbb{E}_{\theta'}[R_\theta(q)|e_\theta(X_\theta)] - \mathbb{E}_\theta[R_\theta(q)|e_\theta(X_\theta)]}{\theta' - \theta} \quad (12)$$

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<sup>7</sup>Since the envelope theorem only holds at differentiability points of the value function, the decomposition (8) may not be well defined if the agent's effort  $e_\theta(X_\theta)$  is not differentiable in  $\theta$ .

for all  $\theta' > \theta$  in an open neighborhood of  $\theta$ , and negative at  $\theta$  if

$$\frac{\Pi(\theta') - \Pi(\theta)}{\theta' - \theta} \leq \frac{\mathbb{E}_{\theta'}[R_\theta(q)|e_\theta(X_\theta)] - \mathbb{E}_\theta[R_\theta(q)|e_\theta(X_\theta)]}{\theta' - \theta} \quad (13)$$

for all  $\theta' < \theta$  in an open neighborhood of  $\theta$ .

The left-hand side (“LHS”) of (12) and (13) corresponds to the total effect: the total change in profits when precision changes from  $\theta$  to  $\theta'$ . The RHS is the direct effect: the change in profits when when precision changes from  $\theta$  to  $\theta'$  but we hold the strike price and effort fixed. The incentive effect is the difference. Definition 1 is analogous to the decomposition in (8), except that it considers sub- and super-gradients to incorporate situations where  $e_\theta(X_\theta)$  is not differentiable.

Proposition 1 generalizes the conditions in (11), for the incentive effect to be positive and negative, without assuming a differentiable effort function:

**Proposition 1** (*Effect of precision on incentives*) *The incentive effect of precision is positive if, for all  $e_H > e_L$  and all  $X$ ,*

$$\frac{\partial}{\partial \theta} \int_{-\infty}^X [F_\theta(q|e_H) - F_\theta(q|e_L)] dq \geq 0. \quad (14)$$

*The incentive effect of precision is negative if, for all  $e_H > e_L$  and all  $X$ ,*

$$\frac{\partial}{\partial \theta} \int_{-\infty}^X [F_\theta(q|e_H) - F_\theta(q|e_L)] dq \leq 0. \quad (15)$$

Proposition 1 provides a sufficient condition for the incentive effect to be positive that is both general and simple.<sup>8</sup> The condition is general: it can be applied under any output distribution and any assumption about the effort space. In particular, we do not require the first-order approach to hold or the effort space to be an interval. The condition is also simple and thus easy to check: while the expected wage contains several terms (equation (10)), Proposition 1 requires us to check only one term. Appendix B provides an analogous condition for the case in which both output and a separate signal are contractible, and changes in precision affect the distribution of the signal rather than output.

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<sup>8</sup>Formally, condition (14) states that  $\int_{-\infty}^X F_\theta(q|e)$  is supermodular in  $(e, \theta)$ , whereas (15) states that it is submodular in  $(e, \theta)$ . As usual, supermodularity (submodularity) can be interpreted as  $e$  and  $\theta$  being complements (substitutes).

Example 1 illustrates the results from Proposition 1 in a simple case where effort has a closed-form solution:

**Example 1** *Suppose the cost of effort is quadratic  $C(e) = \frac{e^2}{2}$  and that output belongs to a linear distribution family (Hart and Holmstrom, 1987):*

$$F_\theta(q|e) = eF_\theta^H(q) + (1 - e)F_\theta^L(q), e \in [0, 1],$$

where  $F_\theta^H(q)$  and  $F_\theta^L(q)$  are CDFs. In this case, we can write the agent's optimal effort in closed form,

$$e_\theta(X) = \mathbb{E}[q|e = 1] - \mathbb{E}[q|e = 0] + \int_{-\infty}^X [F_\theta^H(q) - F_\theta^L(q)] dq,$$

which allows us to obtain the conditions from Proposition 1 directly:

$$\frac{\partial e_\theta(X_\theta)}{\partial \theta} = \int_{-\infty}^{X_\theta} \frac{\partial}{\partial \theta} [F_\theta^H(q) - F_\theta^L(q)] dq.$$

### 3.2 The incentive effect and the strike price

Proposition 1 gives sufficient conditions for the incentive effect to be positive or negative regardless of the strike price of the option. An important limitation, however, is that for most distributions,  $\frac{\partial}{\partial \theta} \int_{-\infty}^X [F_\theta(q|e_H) - F_\theta(q|e_L)] dq$  is positive for some values of  $X$  and negative for other values of  $X$ , and so neither (14) nor (15) holds. In these cases, the sign of the incentive effect will depend on the strike price. This section relates the incentive effect to the strike price for output distributions with a location parameter.

We henceforth decompose output as follows:

$$q = e + \varepsilon, \tag{16}$$

where  $\varepsilon$  is continuously distributed according to a PDF  $g_\theta$  and CDF  $G_\theta$  with full support on an interval of the real line. Equation (16) is without loss of generality, since we can always define “noise”  $\varepsilon$  as the difference between effort and output. In practice, noise can result from a market or industry shock, the contribution of other agents, or measurement error.

When output has a location parameter, the noise distribution  $g_\theta$  is not a function of  $e$ : exerting effort shifts the distribution of output rightward without affecting its shape.

As a result, distributions with a location parameter are commonly used in agency settings; examples include the normal, uniform, logistic, and Laplace distributions. Then,  $f_\theta(q|e) \equiv g_\theta(q - e)$  and  $F_\theta(q|e) \equiv G_\theta(q - e)$  denote the PDF and CDF of output conditional on effort  $e$ . As before, we assume that effort increases output in the sense of MLRP and that  $\theta$  orders the precision of output in terms of a MPS. For distributions with a location parameter, the former entails that, for any  $e_H > e_L$  and any  $\theta$ ,  $\frac{g_\theta(q - e_H)}{g_\theta(q - e_L)}$  is strictly increasing in  $q$ . The latter entails that the mean of  $\varepsilon$  is the same for all  $q$  and that there exists  $\hat{\varepsilon}$  such that

$$\frac{\partial G_\theta}{\partial \theta}(\varepsilon) \begin{cases} \leq \\ \geq \end{cases} 0 \text{ for } \varepsilon \begin{cases} < \\ > \end{cases} \hat{\varepsilon}. \quad (17)$$

For tractability, we assume that the FOA is valid: the effort space is an interval and that the agent's incentive constraint (7) can be replaced by its first-order condition:<sup>9</sup>

$$\frac{d}{de} \mathbb{E}_\theta [W_\theta(q) | e] = C'(e) \quad (18)$$

With the FOA, the agent's effort, implicitly defined by (18), is a differentiable function of  $\theta$ , which simplifies the analysis of the impact of  $\theta$  on effort.

Proposition 2 shows that the sign of the incentive effect depends on whether the strike price of the option is above or below a fixed threshold.

**Proposition 2** (*Effect of information on the strike price*) *Suppose the FOA is valid. There exists  $\hat{X}$ , independent of  $\theta$ , such that:*

(i) *if  $X_\theta < \hat{X}$ , effort and precision are complements and the incentive effect of precision is positive;*

(ii) *if  $X_\theta > \hat{X}$ , effort and precision are substitutes and the incentive effect of precision is negative.*

The intuition is as follows. The agent's marginal benefit of effort is its positive effect on the value of his option. When output has a location parameter, an increase in effort by one unit shifts the distribution of output by \$1. Since the agent receives a positive wage if and only if  $q \geq X_\theta$ , increasing effort only increases the wage if output

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<sup>9</sup>To verify the validity of the FOA, it suffices to check that, at the strike price chosen by the principal under this relaxed program, the agent's payoff is a single-peaked function of effort. A sufficient condition for the validity of FOA in our setting is  $\sup_e \{g_\theta(\varepsilon)\} < \inf_e \{C''(e)\}$ , which we will assume throughout this section.

ends up higher than the strike price – if output still ends up below the strike price, then he receives no return for his effort. Recall that increasing precision moves probability mass from the tails towards the center of the distribution. If  $X_\theta$  is low, raising  $\theta$  shifts mass from below to above  $X_\theta$ , increasing the probability that output exceeds the strike price. On the other hand, if  $X_\theta$  is high, raising  $\theta$  shifts mass from above to below  $X_\theta$ , reducing the probability that output exceeds the strike price.

Importantly, the threshold is independent of  $\theta$ . The threshold will depend on the effort level that the principal wishes to implement; hypothetically, this effort level might depend on  $\theta$ . However, effort is a smooth function of  $\theta$ . Since the threshold separates the areas where effort is increasing and decreasing in  $\theta$ , effort is constant in  $\theta$  at the threshold, and so the threshold is independent of  $\theta$ . This independence is important to allow us to sign the incentive effect as precision changes.<sup>10</sup>

Appendix C shows that similar results hold when the agent is risk-averse, and so the driver of our results is limited liability, rather than risk aversion. With limited liability, regardless of whether the agent is risk-neutral or risk-averse, the optimal contract pays zero when output is below a threshold (regardless of the level of output) and a positive amount, increasing in output, when output exceeds the threshold. (The only difference is that, with risk aversion, the contract may not be linear above the threshold, and so the contract may not be an option). Thus, regardless of the agent’s utility function, he is only rewarded for marginal increases in output if output ends up greater than the threshold, as only then is his wage strictly increasing in output. As a result, precision affects incentives by affecting the likelihood that output exceeds the threshold.

### 3.3 The incentive effect and the cost of effort

Proposition 2 has shown that the sign of the incentive effect depends on the strike price  $X_\theta$ . This result is particularly useful since it is often difficult to solve agency models, i.e., obtain the optimal contract and effort level as functions of the exogenous parameters. Proposition 2 allows us to understand the incentive effect – and in particular the intuition for why it depends on the strike price – without fully solving for these quantities.

One limitation is that, because the strike price  $X_\theta$  is endogenous, Proposition 2 does

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<sup>10</sup>For example, suppose precision increased and we observed that the strike price rose. If the original incentive effect were negative, we can conclude that it remains negative, but we would be unable to do so if the threshold depended on  $\theta$ .



not relate the incentive effect to the underlying parameters of the agency problem. We now relate the incentive effect to the cost of effort. Before doing so, we note that the strike price  $X_\theta$  reflects the strength of incentives: a lower  $X_\theta$  corresponds to a higher option delta and thus stronger incentives. Generally, the link between the strength of incentives and the cost of effort depends on whether the implemented effort level is endogenous or fixed (see the survey of Edmans and Gabaix (2016), Section 3.3). When the implemented effort level is endogenous, a less convex cost of effort means that it is cheaper to provide incentives; as a result, the principal typically induces higher effort and raises incentives to do so (see, e.g., Holmstrom and Milgrom, 1987). When the implemented effort level is fixed, a lower cost of effort means that fewer incentives are needed to implement this effort level.

Both the endogenous and fixed effort level may be appropriate in different circumstances. For example, Edmans and Gabaix (2011) show that, if the agent is a CEO who has a multiplicative impact on firm value and the firm is large, the benefits of effort (increased firm value) swamp the costs (increased incentives) since firm value is much larger than the CEO's salary. In this case, the fixed effort level corresponds to full productive efficiency (taking all value-adding projects) and the principal wishes to implement it regardless of the cost of effort. Moreover, a fixed effort level arises in binary effort models, which are often used for tractability. We thus study both frameworks, starting with the endogenous-effort model that we have analyzed thus far.

### 3.3.1 Endogenous effort level

When effort is endogenous, the key feature of the cost of effort function is its convexity. This convexity parametrizes the severity of the agency problem: The higher the convexity, the more rapidly the marginal cost of effort rises with the level of effort, and so the greater the incentives the principal must provide to increase effort.

Let  $\kappa$  parametrize the convexity of the cost of effort,  $\frac{\partial^3 C}{\partial e^2 \partial \kappa}(e; \kappa) > 0$ . A typical example is the case of quadratic costs  $C(e; \kappa) = \frac{\kappa e^2}{2}$ , where  $\frac{\partial^2 C}{\partial e^2}(e; \kappa) = \kappa$ . We now index the strike price by  $X_{\theta, \kappa}$  and the effort level by  $e_{\theta, \kappa}$ .

Proposition 3 shows that the incentive effect is positive if and only if convexity  $\kappa$  is below a threshold.

**Proposition 3** (*Effect of cost function on incentive effect, endogenous effort*) *Suppose the FOA is valid. There exists  $\bar{\kappa}$  such that the incentive effect is positive (negative) if*

$\kappa < (>) \bar{\kappa}$ .

The intuition for Proposition 3 is as follows. Recall that  $\hat{\varepsilon}$  is the inflection point of  $\frac{\partial G_\theta}{\partial \theta}(\varepsilon)$  defined in (17). If  $X_{\theta,\kappa} - e_{\theta,\kappa} \leq \hat{\varepsilon}$ , precision increases incentives by moving mass from below the strike price to above it; if  $X_{\theta,\kappa} - e_{\theta,\kappa} \geq \hat{\varepsilon}$ , precision reduces incentives by moving mass from above the strike price to below it. Thus, to determine the effect of convexity  $\kappa$  on effort incentives, we must determine how it affects the difference between the initially optimal strike price and the initially optimal effort:  $X_{\theta,\kappa} - e_{\theta,\kappa}$ .

An increase in convexity  $\kappa$  directly reduces the effort chosen by the agent  $e_{\theta,\kappa}$ . The effect on the optimal strike price  $X_{\theta,\kappa}$  is ambiguous: since it is more costly to provide incentives when effort costs are more convex, the principal may reduce incentives by increasing the strike price, or she may compensate for the decrease in effort by providing higher incentives through a lower strike price. Even if the principal chooses to reduce the strike price, the increase in incentives is always outweighed by the direct reduction in effort from an increase in convexity – if it were not, the principal should have chosen a lower strike price before, when the incentive effect of doing so was higher since the cost of effort was less convex. The proof also shows that the fall in effort exceeds any fall in the strike price. Thus, when convexity is high,  $X_{\theta,\kappa} - e_{\theta,\kappa} \geq \hat{\varepsilon}$  and so precision reduces incentives.

Example 2 below illustrates this intuition in a simple model that admits a closed-form solution:

**Example 2** *Suppose the cost function is quadratic  $C(e) = \frac{\kappa e^2}{2}$  and that noise  $\varepsilon$  is uniformly distributed in  $[-\frac{1}{\theta}, \frac{1}{\theta}]$ , so that  $\hat{\varepsilon} = 0$ . The first-order condition that determines the agent's effort choice yields:*

$$e = \frac{1 - \theta X}{2\kappa - \theta}, \quad (19)$$

*and the second-order condition is satisfied as long as  $\theta < 2\kappa$ . Suppose this inequality holds so that the FOA is valid. Substituting (19) into the principal's profit function and maximizing with respect to the strike price, we obtain the optimal strike price and effort:*

$$X_{\theta,\kappa}^* = \frac{1}{\theta} - \frac{1}{\kappa} + \frac{\theta}{2\kappa^2}, \quad e_{\theta,\kappa}^* = \frac{\theta}{2\kappa^2}. \quad (20)$$

*Note that, while effort is always decreasing in  $\kappa$ , the strike price is not monotonic: it decreases (increases) in  $\kappa$  if  $\kappa \leq (\geq) \theta$ . Nevertheless,  $X_{\theta,\kappa}^* - e_{\theta,\kappa}^* = \frac{1}{\theta} - \frac{1}{\kappa}$  is increasing*

in  $\kappa$ . Setting  $X_{\theta,\kappa}^* - e_{\theta,\kappa}^*$  equal to  $\hat{\varepsilon} = 0$ , we find that, consistent with Proposition 3, the incentive effect is positive if  $\kappa \leq \theta$  and negative if  $\kappa \geq \theta$ .

### 3.3.2 Fixed effort level

We now relate the incentive effect to the cost of effort in the case of a fixed effort level. Formally, suppose the effort space is  $\mathcal{E} = \{e, \bar{e}\}$ , where  $e = \underline{e}$  (“shirking”) costs zero and  $e = \bar{e}$  (“working”) costs  $C > 0$ . The principal wishes to implement  $e = \bar{e}$ .<sup>11</sup>

The incentive constraint is now

$$\mathbb{E}[W_\theta(q) | \bar{e}] - \mathbb{E}[W_\theta(q) | \underline{e}] \geq C, \quad (21)$$

and we refer to the LHS of (21) as effort incentives. Recall that, from Lemma 1, the optimal contract is an option.

Differentiating the principal’s expected profits with respect to precision yields:

$$\frac{d}{d\theta} \mathbb{E}[R_\theta(q) | \bar{e}] = \underbrace{\frac{\partial}{\partial \theta} \mathbb{E}[R_\theta(q) | \bar{e}]}_{\text{direct effect}} + \underbrace{\frac{d}{dX_\theta} \mathbb{E}[R_\theta(q) | \bar{e}] \frac{\partial X_\theta}{\partial \theta}}_{\text{incentive effect}}.$$

The direct effect is unchanged, but the incentive effect is different from the case where effort is endogenous. Since effort is now fixed at  $\bar{e}$ , the principal instead responds to a change in precision by changing the strike price. Intuitively, when precision reduces effort incentives (i.e., it tightens the incentive constraint (21)), the principal must lower the strike price to increase the delta and restore incentives ( $\frac{dX_\theta}{d\theta} \leq 0$ ). This lower strike price increases the expected wage and lowers the principal’s profit ( $\frac{d}{dX_\theta} \mathbb{E}[R_\theta(q) | \bar{e}] > 0$ ).<sup>12</sup> Conversely, when precision increases incentives (i.e., it relaxes the incentive constraint), the principal can increase the strike price without violating incentive compatibility ( $\frac{dX_\theta}{d\theta} \geq 0$ ), thus reducing the expected wage. The incentive effect is thus now the effect of precision on profits through changes in the strike price, rather than changes in the effort level.

As in the general model of Section 3.1, the direct effect is always positive but the

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<sup>11</sup>The principal wishes to implement high effort if  $X_\theta - \int_{-\infty}^{X_\theta} F_\theta(q|\bar{e})dq \geq \mathbb{E}[q|\underline{e}]$ , where the strike price  $X_\theta$  is implicitly determined by the incentive constraint:  $\mathbb{E}[q|\bar{e}] - \mathbb{E}[q|\underline{e}] + \int_{-\infty}^{X_\theta} [F_\theta(q|\bar{e}) - F_\theta(q|\underline{e})] dq = C$ .

<sup>12</sup>With endogenous effort,  $\frac{d}{dX_\theta} \mathbb{E}[R_\theta(q) | \bar{e}] = 0$ : a higher strike price reduces the value of the option and increases the expected profit, but also reduces effort and reduces the expected profit. By the envelope theorem, these two effects cancel out. With fixed effort, the second effect does not exist.

incentive effect is ambiguous. We again relate it to the severity of the agency problem. With continuous effort, this severity was parametrized by the convexity of the cost function. With binary effort, this severity is captured by the cost of effort  $C$ . Lemma 2 below shows that the strike price falls with  $C$ . Intuitively, when effort is costly, a high option delta is needed to induce effort, which corresponds to a low strike price.

**Lemma 2** (*Effect of effort cost on strike price*) *The strike price  $X_\theta$  is strictly decreasing in the cost of effort  $C$ .*

Recall that Proposition 2 established that the incentive effect is positive if  $X_\theta < \widehat{X}$  and negative if  $X_\theta > \widehat{X}$  (while derived for the case of endogenous efforts, it is straightforward to verify the result for fixed efforts). Thus, the incentive effect is positive when the strike price is low, i.e., incentives are strong. In turn, Proposition 3 showed that, for an endogenous effort level, incentives are strong when the agency problem is mild (the cost of effort is less convex). Lemma 2 now shows that, for a fixed effort level, incentives are strong when the agency problem is severe (the cost of effort  $C$  is high). Thus, the incentive effect is positive when the agency problem is severe. Proposition 4 states this result formally.

**Proposition 4** (*Effect of cost function on incentive effect, fixed effort*) *There exists a constant  $\widehat{C}$  such that, if  $C > (<) \widehat{C}$ , the incentive effect of precision is positive (negative) and  $\frac{dX_\theta}{d\theta} > (<) 0$ .*

While Proposition 2 guarantees that a single cutoff  $\widehat{X}$  separates the regions where the incentive effect is positive and negative (for continuous effort), we do not know in general where this cutoff lies. A benefit of the fixed effort model is that, since we know the implemented effort level, we can relate the cutoff to it. Indeed, Corollary 1 shows that, when the distribution is symmetric (as with the normal, uniform, logistic, and Laplace distributions),  $\widehat{X}$  lies half-way between the expected output when the agent works  $\bar{e}$  and the expected output when he shirks  $\underline{e}$ , i.e.,  $\widehat{X} = \frac{\bar{e} + \underline{e}}{2}$ . Thus, we do not need to solve for the optimal strike price as a function of the exogenous parameters of the model and take derivatives with respect to  $\theta$  to sign the incentive effect. It is sufficient to observe whether the strike price is above or below the threshold  $\frac{\bar{e} + \underline{e}}{2}$ : the strike price is a “sufficient statistic” for the direction of the incentive effect.

**Corollary 1** (*Symmetric distributions*) *Suppose that  $G_\theta$  is symmetric around zero. Then, the incentive effect is positive (negative) if  $X_\theta < (>) \frac{\bar{e} + \underline{e}}{2}$ .*

We now discuss the intuition using symmetric distributions in the location-scale family, that is, output distributions with a CDF that can be written as  $F_\sigma(q|e) = G\left(\frac{q-e}{\sigma}\right)$ . Examples include the normal, uniform, and logistic distributions. Such distributions clarify the intuition since precision  $\theta$  can be parametrized by volatility  $\sigma = \frac{1}{\theta}$ , which allows us to discuss the intuition using the familiar concept of the option “vega”: the sensitivity of its value to  $\sigma$ .

The agent’s effort incentives stem from the difference in the value of two options. If he works, he receives an option-when-working worth  $\mathbb{E}[W_\sigma(q)|\bar{e}]$ ; if he shirks, he receives an option-when-shirking worth  $\mathbb{E}[W_\sigma(q)|\underline{e}]$ . His effort incentives are given by the difference, i.e.,

$$\mathbb{E}[W_\sigma(q)|\bar{e}] - \mathbb{E}[W_\sigma(q)|\underline{e}]. \quad (22)$$

Thus, the incentive effect of increasing volatility  $\sigma$  (reducing precision  $\theta$ ) is given by

$$\frac{\partial}{\partial \sigma} \{ \mathbb{E}_\sigma[W(q)|\bar{e}] - \mathbb{E}_\sigma[W(q)|\underline{e}] \} \Big|_{W(q)=W_\sigma(q)}. \quad (23)$$

An increase in volatility raises the value of each option, because the vega of an option is positive, but does so to different degrees since the options have different vegas. The incentive effect of increasing volatility is thus the vega of the option-when-working,  $\frac{\partial}{\partial \sigma} \mathbb{E}_\sigma[W(q)|\bar{e}]$ , minus that of the option-when-shirking,  $\frac{\partial}{\partial \sigma} \mathbb{E}_\sigma[W(q)|\underline{e}]$ .

The vega of an option is highest for an at-the-money option (see Claim 1 in Appendix D<sup>13</sup>), and declines when the option moves either in- or out-of-the-money. Thus, the vega of the option-when-working is highest at  $X = \bar{e}$  and the vega of the option-when-shirking is highest at  $X = \underline{e}$ . If the initial strike price is  $X_\sigma = \hat{X} = \frac{\bar{e} + \underline{e}}{2}$ , both options are out-of-the-money by  $\frac{\bar{e} - \underline{e}}{2}$ . They thus have the same vega (see Claim 2 in Appendix D), and so effort incentives are independent of  $\sigma$ . We thus have  $\frac{dX_\sigma}{d\sigma} = 0$ .

When  $X_\sigma < \hat{X}$ , the option-when-shirking is closer to at-the-money, and so it has a higher vega. An increase in  $\sigma$  raises the value of the option-when-shirking more than the option-when-working and thus reduces effort incentives. Thus, the strike price must be lowered to restore effort incentives, and so  $\frac{dX_\sigma}{d\sigma} < 0$ . When  $X_\sigma > \hat{X}$ , the option-when-working is closer to at-the-money than the option-when-shirking. An increase in  $\sigma$  raises effort incentives, and so  $\frac{dX_\sigma}{d\sigma} > 0$ .

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<sup>13</sup>It is well-known that for lognormal distributions, the vega is highest for at-the-money options (as maturity approaches zero). Claim 1 extends this result to all distributions with a location and scale parameter.

Note that the LHS of (23) is related to the option's vanna.<sup>14</sup> The vanna of an option is given by  $\frac{\partial^2 \mathbb{E}[W]}{\partial q \partial \sigma}$  – the cross-partial of its value with respect to both output  $q$  and volatility  $\sigma$ , or alternatively the derivative of its delta ( $\frac{\partial \mathbb{E}[W]}{\partial q}$ ) with respect to volatility  $\sigma$ . When an option is in-the-money, its vanna is negative. Its delta – and thus the agent's effort incentives – decreases with volatility, and thus increase with precision: effort and precision are complements. In contrast, when an option is out-of-the-money, its vanna is positive, and so effort and precision are substitutes.

**Graphical illustration** We now demonstrate graphically the direct and incentive effects, to illustrate the importance of taking the incentive effect into account when calculating the value of information. We consider the common case of a normal distribution, which is symmetric. Figure 1 illustrates how the direct and incentive effects change with the severity of the moral hazard problem (parametrized by  $C$ ). As is standard for graphs of option values, the figure contains the strike price  $X$  on the  $x$ -axis; since  $X$  is strictly decreasing in  $C$  (Lemma 2), there is a one-to-one mapping between  $X$  and  $C$ .

To understand Figure 1, the direct effect,  $\frac{\partial \mathbb{E}[W_\sigma(q)|\bar{e}]}{\partial \sigma}$ , is the vega of the option-when-working. It tends to zero as the strike price approaches either  $-\infty$  or  $\infty$ , and is greatest when the option is at-the-money, i.e.,  $X = 1$ . The incentive effect,  $\frac{\partial \mathbb{E}[W_\sigma(q)|\bar{e}]}{\partial X_\sigma} \frac{dX_\sigma}{d\sigma}$ , is positive for  $X < \hat{X}$  and thereafter negative; when  $X$  crosses above  $\hat{X}$  it becomes increasingly negative but then returns to zero. The total effect  $\frac{d\mathbb{E}[W_\sigma(q)|\bar{e}]}{d\sigma}$  combines these effects. While the direct effect is initially increasing in  $X$ , this is outweighed by the fact that the incentive effect is initially decreasing in  $X$ . Thus, in Figure 1, the total gains from precision are monotonically decreasing in  $X$ .

An analysis focusing purely on the direct effect would suggest that the value of information is greatest when the initial option is at-the-money, which corresponds to a moderate strike price and a moderate cost of effort. In contrast, considering the total effect shows that, for a fixed effort level, the value of information is monotonically increasing in the severity of the agency problem. Appendix E shows analytically that this monotonic effect is general to the normal distribution, rather than applying only to the specific parameter values chosen in Figure 1.

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<sup>14</sup>The difference is that the vanna is defined for local changes in output  $q$ , but equation (23) concerns potentially non-local changes in effort and thus output.

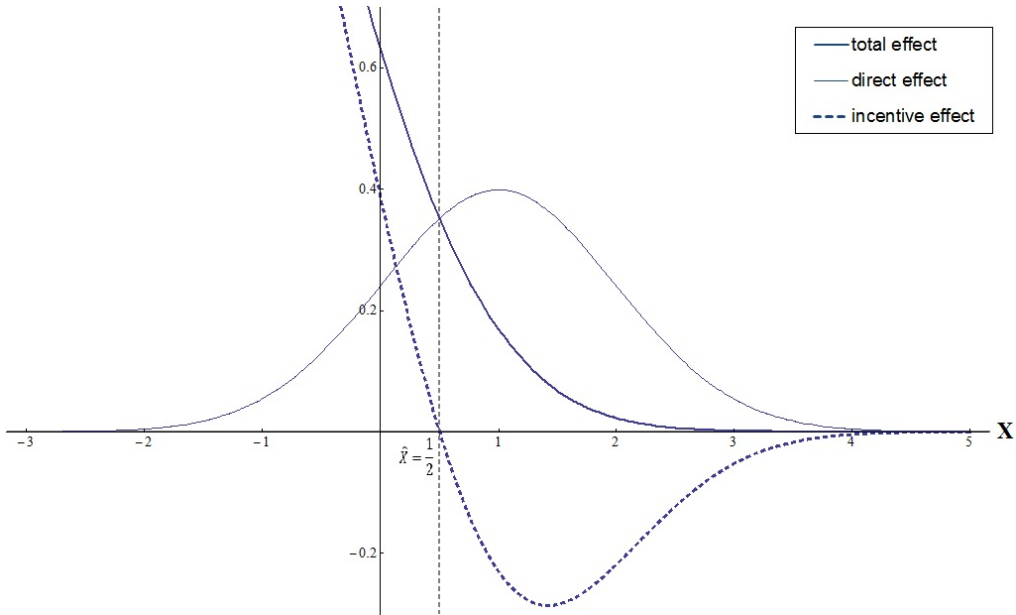


Figure 1: Total and partial derivative of expected pay with respect to  $\sigma$  for a range of values of  $X$ , for  $\underline{e} = 0$ ,  $\bar{e} = 1$ , and  $\sigma = 1$ .

**Ex ante and ex post incentives** We end this section by contrasting the principal's ex ante and ex post incentives to invest in precision. Our analysis thus far assumes that the principal chooses precision ex ante, i.e., before the agent chooses effort, and can commit to this choice. She thus takes into account the effect of precision on both the value of the agent's option (the direct effect) and his effort (the incentive effect). We now consider the case in which the principal cannot commit to an initial level of precision, but instead chooses precision ex post, i.e., after the agent has exerted effort but before output is realized. Any level of precision announced before the agent has exerted effort is not credible as the agent will rationally anticipate that the principal will change precision to the level that maximizes her payoff ex post, and so any pre-announced level is irrelevant.

When precision is chosen ex post, the principal's marginal benefit of precision corresponds to the direct effect only, since the strike price and the agent's effort have already been chosen.<sup>15</sup> Thus, the difference between the principal's ex ante and ex post benefits from precision is given entirely by the incentive effect. If the incentive

<sup>15</sup>By backwards induction, the principal will select a strike price that induces the agent to work given the precision level that he will choose ex post.

effect is positive, the principal would choose a higher level of precision when the choice is made ex ante than ex post. In this case, the principal would like to commit to a high level of precision if such commitment is possible. Conversely, if the incentive effect is negative, the principal would like to commit to a low level of precision.

## 4 Discussion

### 4.1 Applications

This section discusses applications of our results, starting with compensation contracts. Most importantly, our results show that, when employers decide whether to increase the precision with which they monitor agent performance, they should consider the effect on the agent's incentives. This effect can be positive or negative, and so the total benefits of precision can be markedly higher or lower than an analysis focused on the direct effect alone. For all output distributions with a location parameter, and regardless of whether the effort level is fixed or endogenous, the incentive effect is positive when the strike price is low. In turn, this threshold is low when the effort level is fixed and agency problems are severe, or when the effort level is endogenous and agency problems are mild.

Thus, the model provides guidance on where the principal should invest in precision, within a firm. For example, if the fixed effort model applies, agents with high-powered incentives (such as CEOs) should be evaluated more precisely than those with low-powered incentives (such as rank-and-file managers). Relatedly, the model has implications for the optimality of RPE, which is costly as it involves forgoing the benefits of pay-for-luck documented by prior research (e.g., Oyer, 2004; Axelson and Baliga, 2009; Gopalan, Milbourn, and Song, 2010; Hoffman and Pfeil, 2010; Hartman-Glaser and Hébert, 2016). The results suggest that RPE need not be optimal, as it can reduce effort incentives. This effect is particularly likely where incentives are low-powered to begin with, consistent with RPE being even rarer for rank-and-file managers than for executives.

In addition, Proposition 2 suggests that exogenous changes in volatility (see Gormley, Matsa, and Milbourn (2013) and De Angelis, Grullon, and Michenaud (2017) for natural experiments) or stock market efficiency will have different effects on the incentives of agents depending on the moneyness of their outstanding options. This result



applies both within a firm and across firms. In particular, increases in precision will lower (raise) the incentives of CEOs with out-of-the-money (in-the-money) options. Thus, where CEOs have out-of-the-money options, firms may wish to reduce the strike prices to restore incentives. Option repricing is documented empirically by Brenner, Sundaram, and Yermack (2000), although they do not study if it is prompted by falls in volatility.<sup>16</sup> Relatedly, Bebchuk and Fried (2004) advocate out-of-the-money options because they only reward a manager upon good performance. It is already known that a disadvantage of such options is that they provide weaker incentives compared to in-the-money options, due to their lower deltas; our model shows that this disadvantage is increasing in the level of precision.

A second application is to financing contracts, where the principal (investor) receives debt with face value of  $X_\theta$ , and the entrepreneur (agent) holds equity – a call option on firm value with a strike price equal to the face value of debt. This application is relevant for both mature firms, and also young firms since they frequently raise debt and the entrepreneur holds levered equity, as shown by Robb and Robinson (2014).<sup>17</sup> Our results shed light on the investor’s incentives to reduce output volatility via risk management. Such risk management has several interpretations: the investor can implement risk management herself since she retains control rights on output; she stipulates in the contract that the entrepreneur implement the above measures; or she has a menu of projects that she can finance and thus can choose project risk. Standard intuition would suggest that these incentives are increasing in the size of her debt claim, and thus her value-at-risk, but this intuition ignores the effect of risk management on effort. If the initial face value is low ( $X_\theta < \hat{X}$ ), a fall in output volatility raises effort incentives. This reinforces the direct effect of risk management, that it increases the value of the investor’s risky debt due to its concave payoff. Thus, surprisingly, risk management may be more valuable for firms that are some distance from bankruptcy, and when the investor has little debt at stake. In contrast, if the initial face value is high ( $X_\theta > \hat{X}$ ), risk management reduces effort incentives, offsetting the direct effect.

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<sup>16</sup>Acharya, John, and Sundaram (2000) also study the repricing of options theoretically, although in responses to changes in the mean rather than volatility of the signal.

<sup>17</sup>This is the original interpretation of the contract in Innes (1990). In his model, the agent has the bargaining power; under the financing interpretation of our model, the principal continues to hold the bargaining power so that she captures the surplus and so has incentives to invest in precision. (Appendix F analyzes the case in which the agent has bargaining power and chooses precision).

## 4.2 Binding participation constraint

In the core model, the agent's participation constraint is slack and he earns rents. The direct effect arises because information reduces the value of the option and thus the agent's rents. This section considers the case in which the agent's participation constraint binds and so the principal must restore the value of the option (by reducing the strike price) to maintain the agent's participation following an increase in precision. Even though the direct effect is fully offset by the reduction in the strike price, we will show that the total effect of information is typically non-zero.

Now assume that the agent's reservation utility is given by  $\bar{U}$  rather than zero. As in the core model, the optimal contract is an option,<sup>18</sup> and so the agent's participation constraint is now given by

$$\int_{X_\theta}^{\infty} (q - X_\theta) f_\theta(q|e^*)dq \geq \bar{U} + C(e^*). \quad (24)$$

An increase in precision reduces the value of the agent's option on the LHS of the participation constraint (24). When the participation constraint is binding, the principal must reduce  $X_\theta$  to maintain the agent's participation.<sup>19</sup> This reduction must fully offset the direct effect so that the participation constraint continues to hold with equality, i.e.,  $\frac{\partial X_\theta}{\partial \theta}$  must satisfy

$$\frac{d}{dX_\theta} \mathbb{E}_\theta [W_\theta(q) | e_\theta(X_\theta)] \frac{\partial X_\theta}{\partial \theta} = - \frac{\partial}{\partial \theta} \mathbb{E}_\theta [W_\theta(q) | e_\theta(X_\theta)],$$

where the RHS is the direct effect in (8). Indeed, if the effort level were fixed ( $e^* = \bar{e}$ ), the value of information to the principal would be exactly zero. Total surplus is constant at  $\mathbb{E}[q|\bar{e}]$ , since effort is constant. With a binding participation constraint, the agent's

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<sup>18</sup>The proof of Lemma 1 can be adapted by letting the initial non-option contract also satisfy the agent's participation constraint. Then the corresponding option contract constructed in the proof has the same expected value as the initial non-option contract, and it leaves both the principal's and agent's objective functions unchanged, since the effort level is the same. Because the agent chooses effort optimally and can achieve the same expected utility by choosing the same effort level under the new option contract as under the initial contract, he is better off under the option contract with the higher effort level than under the initial contract with the initial effort level. Thus, with a binding participation constraint, the initial non-option contract remains dominated by an option contract.

<sup>19</sup>Since the agent chooses effort optimally, we know from the envelope theorem that the total effect of a change in  $X_\theta$  on the agent's expected utility net of effort cost is simply equal to the partial effect, holding effort constant.

utility is constant at  $\bar{U} + C(\bar{e})$ . Since the principal's profit equals total surplus minus the agent's utility, it would also be constant. Intuitively, when the agent's participation constraint binds, precision cannot be used to reduce the cost of compensation, and so has no value to the principal.

However, when the implemented effort level is endogenous, the value of information is typically non-zero. The fall in the strike price required to maintain the agent's participation increases the option's delta and thus effort incentives. We call this the "participation effect". The total value of information to the principal in (8) is now given by:

$$\underbrace{\frac{d}{d\theta} \mathbb{E}_\theta [R_\theta(q) | e_\theta(X_\theta)]}_{\text{Total Effect}} = \underbrace{\frac{\partial}{\partial e} \mathbb{E}_\theta [R_\theta(q) | e_\theta(X_\theta)] \frac{\partial e_\theta(X_\theta)}{\partial \theta}}_{\text{Incentive Effect}} + \underbrace{\frac{\partial}{\partial e} \mathbb{E}_\theta [R_\theta(q) | e_\theta(X_\theta)] \frac{\partial e_\theta(X_\theta)}{\partial X_\theta} \frac{\partial X_\theta}{\partial \theta}}_{\text{Participation Effect}}. \quad (25)$$

There is no longer a direct effect. When the agent's participation constraint is binding, the value of information to the principal stems entirely from its effect on effort – it cannot be used to reduce the agent's rents.

The "effort effect"  $\frac{de}{d\theta}$  now has two components. These can be more clearly seen as follows:

$$\frac{de}{d\theta} = \frac{\partial e}{\partial \theta} + \frac{\partial e}{\partial X} \frac{\partial X}{\partial \theta}. \quad (26)$$

First, as in the core model, there is the "incentive effect"  $\frac{\partial e}{\partial \theta}$ , which can be positive or negative depending on whether effort and precision are complements or substitutes. Second, there is an additional "participation effect", which is strictly positive: since  $\frac{\partial X}{\partial \theta} < 0$  and  $\frac{\partial e}{\partial X} < 0$ ,  $\frac{\partial e}{\partial X} \frac{\partial X}{\partial \theta} > 0$ . Thus, if effort and precision are not substitutes, the incentive effect is non-negative and so precision increases effort due to the positive participation effect. Even if effort and precision are substitutes, effort can still rise, if the negativity of the incentive effect is outweighed by the positivity of the participation effect.

Note that a similar result holds if we instead assume Nash bargaining, where the agent captures a fraction  $\alpha$  of the total surplus. The participation effect is replaced by a "bargaining effect": following an increase in precision, the strike price falls to maintain the agent's share of total surplus, which in turn increases effort incentives.

Appendix F studies the opposite case in which the principal's participation constraint binds. Specifically, the agent (entrepreneur) has full bargaining power and

chooses precision. He raises an amount  $I > 0$  from the principal (investor) to fund a project which produces output  $q$ , and so the principal's expected payoff must equal  $I$ . There is a similar participation effect to this section, and the value of information again stems entirely from its effect on effort, which remains (26).

In addition to demonstrating robustness, the analysis delivers two additional results. First, the value of precision depends on the entrepreneur's initial wealth. The amount  $I$  can be thought of as the amount of new financing that the entrepreneur must raise, net of his initial wealth. Thus, if wealth is low,  $I$  is high, which leads to a high face value of debt. It is already known that this reduces effort incentives, since part of the gains from effort go to the investor. Appendix F shows that increases in precision may further exacerbate the incentive problems originating from low initial wealth, since the incentive effect is then negative. In contrast, firms with abundant internal wealth have high incentives to increase precision. Second, the level of precision will be different than when it is chosen by the principal. The party that chooses precision internalizes the change in effort triggered by an increase in precision only to the extent that it affects him/her, depending on the initial contract. For example, with a low face value of debt  $X_\theta$ , the principal has close-to-safe debt and so benefits little from changes in effort, while the agent is close to the residual claimant. Thus, the agent is affected more by the effort effect (26) than the principal. As a result, if the effort effect is positive, he will choose a higher level of precision. Due to these differential benefits from precision, control rights matter – the chosen level of information depends on which party has bargaining power and chooses precision.

### 4.3 Social welfare and regulation

Appendix G analyzes the effect of precision on social welfare and, in particular, whether the principal overinvests or underinvests relative to the social optimum. Total surplus depends only on effort; the wage is a pure transfer from the principal to the agent with no effect on total surplus. Since the principal does not take the agent's utility into account when choosing the level of precision, she typically does not choose the socially optimal level. This has implications for the optimality of regulation.

For example, when effort is fixed, total surplus is independent of precision. Nevertheless, the principal has an incentive to increase precision to reduce the agent's wage, and so she always overinvests in precision compared to the social optimum. This result is interesting since most regulation increases the precision of performance metrics, such

as reporting and disclosure requirements or mandatory audits. When effort is endogenous, the principal overinvests in precision relative to the social optimum whenever precision reduces the agent's utility and underinvests in precision whenever it increases the agent's utility.<sup>20</sup>

## 5 Conclusion

This paper uses an optimal contracting model to study the value of information – a more precise signal of agent performance – to the principal. We show that increasing signal precision has two effects on the principal's profit. The first is the direct effect: reducing signal volatility lowers the value of the agent's option, and unambiguously increases profit. This is the standard effect of precision considered by arguments that information is valuable to the principal. Our paper focuses on a second, indirect effect: reducing signal volatility changes the agent's effort incentives. Crucially, this effect may be negative and harm the principal – effort and precision may be substitutes – offsetting the benefits of the direct effect.

We derive a general condition that determines whether effort and precision are complements or substitutes, and thus the sign of the incentive effect, that holds for any output distribution and does not assume the first-order approach. When the output distribution has a location parameter, and the first-order approach is valid, we show that this condition is satisfied – precision increases incentives – if and only if the strike price of the option is low. We then relate the strike price of the option – and thus the incentive effect of effort – to the underlying parameters of the agency problem, to determine the conditions under which information is most valuable to the principal. When the effort level is endogenous, the principal will choose a low effort level and thus a high strike price (relative to this effort level) when the cost of effort is sufficiently convex – i.e., the agency problem is severe – as then incentive provision is difficult. When the effort level is fixed, as may be the case for CEOs of large firms, the strike price is high when the cost of effort is low – i.e., the agency problem is mild – as then weak incentives are needed to implement the fixed effort level.

In a compensation setting, our results have implications for the situations in which

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<sup>20</sup>Unlike in the fixed effort case, it is possible for an increase in precision to augment the agent's utility. This may occur if it optimal for the principal to significantly reduce the strike price to implement a higher effort level.

information on agent performance is most valuable, and how firms should recontract in response to changes in the signal precision. In a financing setting, they have implications for the value of risk management. In particular, the incentives to manage risk may, surprisingly, be high when the face value of debt is low. Even though the investor has little skin in the game, risk management is especially valuable due to its positive effect on the agent's incentives.

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# A Proofs

## Proof of Lemma 1

The proof adopts Lemma 1 from Matthews (2001) to a setting with a continuum of signals and general supports. Let  $(W^*, e^*)$  be a feasible contract and consider the option contract  $W^O = \max\{0, q - X\}$  where the strike price  $X$  is chosen so that both contracts have the same expected payment under effort  $e^*$ :

$$\int_{-\infty}^{\infty} W^*(q) f(q|e^*) dq = \int_{-\infty}^{\infty} W^O(q) f(q|e^*) dq. \quad (27)$$

It is straightforward to show that the contract  $W^O$  exists and is unique. We will show that replacing  $W^*$  by  $W^O$  increases effort and raises the principal's expected profit.

Let  $e^O$  be an optimal effort for the agent when he is offered the option contract:

$$e^O \in \arg \max_{e \in E} \int_{-\infty}^{\infty} W^O(q) f(q|e) dq - C(e).$$

Since the agent chooses  $e^*$  when offered  $W^*$  and  $e^O$  when offered  $W^O$ , we must have:

$$\begin{aligned} \int_{-\infty}^{\infty} W^O(q) f(q|e^O) dq - C(e^O) &\geq \int_{-\infty}^{\infty} W^O(q) f(q|e^*) dq - C(e^*), \\ \int_{-\infty}^{\infty} W^*(q) f(q|e^*) dq - C(e^*) &\geq \int_{-\infty}^{\infty} W^*(q) f(q|e^O) dq - C(e^O). \end{aligned}$$

Combining these two inequalities, we obtain

$$\int_{-\infty}^{\infty} [W^O(q) - W^*(q)] [f(q|e^O) - f(q|e^*)] dq \geq 0. \quad (28)$$

Since both contracts have the same expected value under effort  $e^*$  and the option contract pays the lowest feasible amount for  $q < X$  and has the highest possible slope for  $q > X$ , there exists  $\bar{q} \geq X$  such that

$$W^O(q) \left\{ \begin{array}{l} \leq \\ \geq \end{array} \right\} W^*(q) \text{ for all } q \left\{ \begin{array}{l} \leq \\ \geq \end{array} \right\} \bar{q}. \quad (29)$$

We will now show by contradiction that  $e^* \leq e^O$ . Suppose that  $e^* > e^O$ . Then:

$$\begin{aligned}
0 &\leq \int_{-\infty}^{\infty} [W^O(q) - W^*(q)] \left[ \frac{f(q|e^O)}{f(q|e^*)} - 1 \right] f(q|e^*) dq \\
&= \int_{-\infty}^{\infty} [W^O(q) - W^*(q)] \frac{f(q|e^O)}{f(q|e^*)} f(q|e^*) dq - \underbrace{\int_{-\infty}^{\infty} [W^O(q) - W^*(q)] f(q|e^*) dq}_0 \\
&= \int_{-\infty}^{\bar{q}} [W^O(q) - W^*(q)] \frac{f(q|e^O)}{f(q|e^*)} f(q|e^*) dq + \int_{\bar{q}}^{\infty} [W^O(q) - W^*(q)] \frac{f(q|e^O)}{f(q|e^*)} f(q|e^*) dq \\
&< \int_{-\infty}^{\bar{q}} [W^O(q) - W^*(q)] \frac{f(\bar{q}|e^O)}{f(\bar{q}|e^*)} f(q|e^*) dq + \int_{\bar{q}}^{\infty} [W^O(q) - W^*(q)] \frac{f(\bar{q}|e^O)}{f(\bar{q}|e^*)} f(q|e^*) dq \\
&= \frac{f(\bar{q}|e^O)}{f(\bar{q}|e^*)} \int_{-\infty}^{\infty} [W^O(q) - W^*(q)] f(q|e^*) dq = 0
\end{aligned}$$

where the first line divides and multiplies the expression inside the integral in (28) by  $f(q|e^*)$ , the second line adds a term that equals zero (from (27)), the third line splits the integral between outputs lower and higher than  $\bar{q}$ , and the fourth line uses MLRP and (29). These inequalities give us a contradiction ( $0 < 0$ ), showing that  $e^* \leq e^O$ .

To conclude the proof, we need to show that the principal's profits from offering the option contract  $(W^O, e^O)$  are higher than with the original contract  $(W^*, e^*)$ :

$$\int [q - W^O(q)] f(q|e^O) dq \geq \int [q - W^*(q)] f(q|e^*) dq.$$

Subtracting  $\int [q - W^O(q)] f(q|e^*) dq$  from both sides, gives:

$$\int [q - W^O(q)] [f(q|e^O) - f(q|e^*)] dq \geq \int [W^O(q) - W^*(q)] f(q|e^*) dq = 0,$$

where the expression on the RHS equals zero by (27). Rearranging this expression, it follows that the principal profits from the replacement if

$$\int [q - W^O(q)] f(q|e^O) dq \geq \int [q - W^O(q)] f(q|e^*) dq,$$

which is true because  $q - W^O(q) = \min\{q, X\}$  is an increasing function of  $q$  and  $f(q|e^O)$  first-order stochastically dominates  $f(q|e^*)$  ( $e^O \geq e^*$  and FOSD is implied by MLRP).

### Proof of Proposition 1

We start with two auxiliary lemmas that will be useful for the main proof. Lemmas 3 and 4 show that the incentive effect is positive if  $e_\theta(X_\theta)$  is increasing in  $\theta$  and negative

if  $e_\theta(X_\theta)$  is decreasing in  $\theta$ . The principal's program is:

$$V(\theta) \equiv \max_{X_\theta} R(X_\theta, \theta), \quad (30)$$

where  $R(X_\theta, \theta) \equiv X_\theta - \int_{-\infty}^{X_\theta} F_\theta(q|e_\theta(X_\theta))dq$ . Let  $X_\theta^* \in \arg \max_{X_\theta} R(X_\theta, \theta)$ .

**Lemma 3** *Suppose  $e_\theta(X)$  is a non-decreasing function of  $\theta$  at  $X = X_\theta^*$  in an interval  $[\theta, \theta + \Delta)$ . Then,*

$$\frac{V(\theta') - V(\theta)}{\theta' - \theta} \geq - \frac{\int_{-\infty}^{X_\theta^*} [F_{\theta'}(q|e_{\theta'}(X_\theta^*)) - F_\theta(q|e_{\theta'}(X_\theta^*))] dq}{\theta' - \theta} \quad (31)$$

for all  $\theta' \in [\theta, \theta + \Delta)$ . Moreover, if  $V(\theta)$  is right-hand differentiable, then  $V'(\theta+) \geq - \int_{-\infty}^{X_\theta^*} \frac{\partial F_\theta}{\partial \theta}(q|e_\theta(X_\theta^*))dq$ .

**Proof.** Since  $X_\theta^*$  is solution of program (30),  $V(\theta') \geq R(X_\theta^*, \theta')$  and  $V(\theta) = R(X_\theta^*, \theta)$ . Therefore, for any  $\theta' > \theta$ ,

$$\frac{V(\theta') - V(\theta)}{\theta' - \theta} \geq \frac{R(X_\theta^*, \theta') - R(X_\theta^*, \theta)}{\theta' - \theta}. \quad (32)$$

Substituting the expression for  $R$ , we have

$$\begin{aligned} R(X_\theta^*, \theta') - R(X_\theta^*, \theta) &= \int_{-\infty}^{X_\theta^*} [F_\theta(q|e_\theta(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*))] dq \\ &= \int_{-\infty}^{X_\theta^*} \left[ \underbrace{F_\theta(q|e_\theta(X_\theta^*)) - F_\theta(q|e_{\theta'}(X_\theta^*))}_{+} + F_{\theta'}(q|e_{\theta'}(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*)) \right] dq \\ &\geq - \int_{-\infty}^{X_\theta^*} [F_{\theta'}(q|e_{\theta'}(X_\theta^*)) - F_\theta(q|e_{\theta'}(X_\theta^*))] dq, \end{aligned}$$

where the inequality used the fact that  $F_\theta(q|e)$  is decreasing in  $e$  (FOSD) and  $e_\theta(X_\theta^*) \leq e_{\theta'}(X_\theta^*)$ . Substituting back in (32), establishes (31). For the second claim, take the limit as  $\theta' \searrow \theta$ . ■

**Lemma 4** *Suppose  $e_\theta(X)$  is a non-increasing function of  $\theta$  at  $X = X_\theta^*$  in an interval  $(\theta - \Delta, \theta]$ . Then,*

$$\frac{V(\theta') - V(\theta)}{\theta' - \theta} \leq - \frac{\int_{-\infty}^{X_\theta^*} [F_\theta(q|e_{\theta'}(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*))] dq}{\theta' - \theta} \quad (33)$$

for all  $\theta' \in (\theta - \Delta, \theta]$ . Moreover, if  $V(\theta)$  is left-hand differentiable, then  $V'(\theta-) \leq -\int_{-\infty}^{X_\theta^*} \frac{\partial F_\theta}{\partial \theta}(q|e_\theta(X_\theta^*))dq$ .

**Proof.** Since  $X_\theta^*$  solves the maximization program in (30),  $V(\theta') \geq R(X_\theta^*, \theta')$  and  $V(\theta) = R(X_\theta^*, \theta)$ . Therefore, for any  $\theta' < \theta$ ,

$$\frac{V(\theta') - V(\theta)}{\theta' - \theta} \leq \frac{R(X_\theta^*, \theta') - R(X_\theta^*, \theta)}{\theta' - \theta}. \quad (34)$$

From the definition of  $R$ , we have

$$\begin{aligned} R(X_\theta^*, \theta') - R(X_\theta^*, \theta) &= \int_{-\infty}^{X_\theta^*} [F_\theta(q|e_\theta(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*))] dq \\ &= \int_{-\infty}^{X_\theta^*} \left[ \underbrace{F_\theta(q|e_\theta(X_\theta^*)) - F_\theta(q|e_{\theta'}(X_\theta^*))}_{+} + F_\theta(q|e_{\theta'}(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*)) \right] dq, \\ &\geq -\int_{-\infty}^{X_\theta^*} [F_\theta(q|e_{\theta'}(X_\theta^*)) - F_{\theta'}(q|e_{\theta'}(X_\theta^*))] dq \end{aligned}$$

where we used FOSD and  $e_{\theta'}(X) \geq e_\theta(X_\theta)$  to establish that  $F_\theta(q|e_\theta(X_\theta^*)) \geq F_\theta(q|e_{\theta'}(X_\theta^*))$ . Substituting in (34), establishes (33). The second claim takes the limit as  $\theta' \nearrow \theta$ . ■

The full proof of Proposition 1 now follows. The agent's best-response correspondence is

$$e_\theta(X_\theta) \equiv \arg \max_e \int_{X_\theta}^{\infty} (q - X_\theta) f_\theta(q|e) dq - C(e). \quad (35)$$

By the maximum theorem,  $e_\theta(X_\theta)$  is non-empty and compact-valued. Recall that the agent chooses the effort preferred by the principal whenever  $e_\theta(X_\theta)$  is not single-valued. We now show that the principal will strictly prefer the highest one. To see this, recall from (10) that expected wage can be written:

$$\mathbb{E}[W_\theta(q)|e, \theta] = \mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta} F_\theta(q|e) dq. \quad (36)$$

Suppose the agent is indifferent between efforts  $e_H > e_L$ . Then,

$$\mathbb{E}[q|e_H] - \mathbb{E}[q|e_L] = \int_{-\infty}^{X_\theta} [F_\theta(q|e_L) - F_\theta(q|e_H)] dq + C(e_H) - C(e_L) > 0,$$

where the inequality follows from  $e_H > e_L$ ,  $C' > 0$ , and FOSD. Using this inequality,

we obtain

$$\mathbb{E}_\theta[R_\theta(q) | e_H] = X_\theta - \int_{-\infty}^{X_\theta} F_\theta(q|e_H)dq > X_\theta - \int_{-\infty}^{X_\theta} F_\theta(q|e_L)dq = \mathbb{E}_\theta[R_\theta(q) | e_L],$$

showing that the principal has a higher profit under  $e_H$  than under  $e_L$ .

Using the expression for the agent's wage from (36), the agent's effort is the largest element of

$$\arg \max_e \mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta} F_\theta(q|e)dq - C(e).$$

The result then follows from Topkis's theorem, since the objective function satisfies increasing (decreasing) differences if  $\frac{\partial^2}{\partial \theta \partial e} \left[ \int_{-\infty}^X F_\theta(q|e)dq \right] \geq (\leq) 0$  for all  $e, X$ .

### Proof of Proposition 2

With a location parameter,  $F_\theta(q|e) \equiv G_\theta(\varepsilon)$ . Plugging this into (10) yields

$$\mathbb{E}_\theta [W_\theta(q) | e] = \mathbb{E}[q|e] - X_\theta + \int_{-\infty}^{X_\theta - e} G_\theta(\varepsilon)d\varepsilon. \quad (37)$$

Maximizing with respect to effort, we can rewrite the first- and second-order conditions as:

$$1 - G_\theta(X - e_\theta(X)) = C'(e_\theta(X)), \quad (38)$$

$$g_\theta(X - e_\theta(X)) - C''(e_\theta(X)) < 0. \quad (39)$$

Applying the implicit function theorem to (38), it follows that  $\frac{\partial}{\partial X} e_\theta(X) \leq 0$ , and, by MPS  $\left(\frac{\partial G_\theta}{\partial \theta}(\varepsilon) \leq (\geq) 0 \text{ for } \varepsilon < (>) \hat{\varepsilon}\right)$ ,  $\frac{\partial}{\partial \theta} e_\theta(X) \geq (\leq) 0$  if  $X - e_\theta(X) < (>) \hat{\varepsilon}$ . Since  $\lim_{X \searrow -\infty} X - e_\theta(X) = -\infty$ ,  $\lim_{X \nearrow +\infty} X - e_\theta(X) = +\infty$ , and  $X - e_\theta(X)$  is a strictly increasing and continuous function of  $X$ , it follows that there exists a unique  $\hat{X}_\theta$  that solves

$$\hat{X}_\theta - e_\theta(\hat{X}_\theta) = \hat{\varepsilon}. \quad (40)$$

Totally differentiating (40), gives:

$$\frac{d\hat{X}_\theta}{d\theta} \left[ 1 - \frac{\partial e_\theta}{\partial X}(\hat{X}_\theta) \right] = \frac{\partial e_\theta}{\partial \theta}(\hat{X}_\theta).$$

Since  $\frac{\partial e_\theta}{\partial X}(\hat{X}_\theta) \leq 0$  and  $\frac{\partial e_\theta}{\partial \theta}(\hat{X}_\theta) = 0$ , it follows that  $\frac{d\hat{X}_\theta}{d\theta} = 0$ , and so  $\hat{X}_\theta$  is constant in  $\theta$ .

### Proof of Proposition 3

The agent's expected payoff equals:

$$\mathbb{E}[W(q)|e, \theta] - C(e; \kappa) = e - X + \int_{-\infty}^{X-e} G_{\theta}(\varepsilon) d\varepsilon - C(e; \kappa),$$

so the agent's optimal effort is given by the following first-order condition:

$$1 - G_{\theta}(X - e) - \frac{\partial C}{\partial e}(e; \kappa) = 0.$$

The principal's profits equal

$$X - \int_{-\infty}^{X-e_{\theta, \kappa}(X)} G_{\theta}(\varepsilon) d\varepsilon.$$

We introduce the “distance from  $\hat{\varepsilon}$ ” variable  $Z \equiv X - e$ . As seen in the proof of Proposition 2, for the location family,  $\frac{\partial e_{\theta}}{\partial \theta} \geq (\leq) 0$  if  $Z \leq (\geq) \hat{\varepsilon}$ . It suffices to show that the distance from  $\hat{\varepsilon}$  chosen by the principal,  $Z_{\kappa, \theta}^* = X_{\kappa, \theta}^* - e_{\kappa, \theta}^*$ , is increasing in  $\kappa$ .

We can rewrite the principal's profits in terms of  $(Z, e)$  instead of  $(X, e)$ , i.e.,

$$\Pi(Z; \kappa) \equiv Z + e_{\theta, \kappa}^*(Z) - \int_{-\infty}^Z G_{\theta}(\varepsilon) d\varepsilon$$

where  $e_{\theta, \kappa}^*(Z)$  is implicitly determined by:

$$1 - G_{\theta}(Z) - \frac{\partial C}{\partial e}(e; \kappa) = 0.$$

Total differentiation of the previous equality establishes that:

$$\frac{\partial e_{\theta, \kappa}^*(Z)}{\partial Z} = -\frac{g_{\theta}(Z)}{\frac{\partial^2 C}{\partial e^2}(e; \kappa)} < 0, \quad \frac{\partial e_{\theta, \kappa}^*(Z)}{\partial \kappa} = \frac{-\frac{\partial^2 C}{\partial e \partial \kappa}(e; \kappa)}{\frac{\partial^2 C}{\partial e^2}(e; \kappa)} > 0.$$

Note that the principal's profit  $\Pi(Z, \kappa)$  has increasing differences, since

$$\frac{\partial^2 \Pi}{\partial Z \partial \kappa}(Z; \kappa) = \frac{g_{\theta}(Z)}{\left[\frac{\partial^2 C}{\partial e^2}(e; \kappa)\right]^2} \frac{\partial^3 C}{\partial e^2 \partial \kappa}(e; \kappa) > 0.$$

By Topkis's theorem, the solution  $Z_{\kappa, \theta}^* = X_{\kappa, \theta}^* - e_{\kappa, \theta}^*$  is increasing in  $\kappa$ .

## Proof of Lemma 2

The agent works if the incentive constraint holds:

$$\mathbb{E}[W_\theta(q)|\bar{e}] - \mathbb{E}[W_\theta(q)|\underline{e}] \geq C. \quad (41)$$

We define  $X_\theta$  implicitly by the binding incentive constraint:

$$\int_{X_\theta}^{\infty} (q - X_\theta) [f_\theta(q|\bar{e}) - f_\theta(q|\underline{e})] dq = C. \quad (42)$$

It is straightforward to show that  $X_\theta$  exists and is unique. By the implicit function theorem,

$$\frac{dX_\theta}{dC} = \frac{1}{F_\theta(X_\theta|\bar{e}) - F_\theta(X_\theta|\underline{e})} < 0.$$

#### Proof of Proposition 4

As discussed in the main text, the only term in the decomposition of the expected wage (10) with a non-zero cross-partial is  $\int_{-\infty}^{X_\theta} F_\theta(q|e) dq$ . Using the incentive constraint in (21), precision and incentives are complements (substitutes) if and only if

$$\frac{\partial}{\partial \theta} \int_{-\infty}^{X_\theta} [F_\theta(q|\bar{e}) - F_\theta(q|\underline{e})] dq > (<) 0. \quad (43)$$

Since  $F_\theta(q|e) = G_\theta(q - e)$ , we have

$$\begin{aligned} \frac{\partial}{\partial \theta} \int_{-\infty}^{X_\theta} [F_\theta(q|\bar{e}) - F_\theta(q|\underline{e})] dq &= \frac{\partial}{\partial \theta} \left\{ \int_{-\infty}^{X_\theta - \bar{e}} G_\theta(\varepsilon) d\varepsilon - \int_{-\infty}^{X_\theta - \underline{e}} G_\theta(\varepsilon) d\varepsilon \right\} \\ &= - \int_{X_\theta - \bar{e}}^{X_\theta - \underline{e}} \frac{\partial G_\theta}{\partial \theta}(\varepsilon) d\varepsilon. \end{aligned} \quad (44)$$

Therefore, precision and incentives are complements (substitutes) if and only if the expression in (44) is positive (negative). In addition, using (10) we get

$$\frac{\partial}{\partial X_\theta} \{ \mathbb{E}[W_\theta(q)|\bar{e}] - \mathbb{E}[W_\theta(q)|\underline{e}] \} = \frac{\partial}{\partial X_\theta} \int_{-\infty}^{X_\theta} [F_\theta(q|\bar{e}) - F_\theta(q|\underline{e})] dq = F_\theta(X_\theta|\bar{e}) - F_\theta(X_\theta|\underline{e}),$$

which is negative for any  $X_\theta$  by FOSD, so that incentives are decreasing in the strike price. In sum, if precision and incentives are complements (substitutes), then  $X_\theta$  as defined in equation (42) must be increased (decreased) following a rise in  $\theta$ , i.e.,  $\frac{dX_\theta}{d\theta} \geq (<) 0$ .



From the definitions of a MPS in (1) and of  $G_\theta$ , we know that  $\frac{\partial G_\theta}{\partial \theta}$  alternates signs only once, and  $\frac{\partial G_\theta}{\partial \theta} \leq (\geq) 0$  for  $q$  small (large) enough. Therefore, there exists  $\widehat{X}$  such that  $-\int_{X_\theta - \bar{e}}^{X_\theta - \underline{e}} \frac{\partial G_\theta}{\partial \theta}(\varepsilon) d\varepsilon$  is nonnegative for  $X_\theta < \widehat{X}$ , and nonpositive for  $X_\theta > \widehat{X}$ . In sum, precision increases incentives and  $\frac{dX_\theta}{d\theta} \geq 0$  if  $X_\theta < \widehat{X}$ , while precision decreases incentives and  $\frac{dX_\theta}{d\theta} \leq 0$  if  $X_\theta > \widehat{X}$ . Finally, we know from Lemma 2 that the initial strike price  $X_\theta$  is decreasing in  $C$ , which completes the proof.

### Proof of Corollary 1

Recall that the strike price  $X_\theta$  is implicitly defined as the solution to

$$\bar{e} - \underline{e} - \int_{X_\theta - \bar{e}}^{X_\theta - \underline{e}} G_\theta(\varepsilon) d\varepsilon = C.$$

By the implicit function theorem,

$$\frac{dX_\theta}{d\theta} = \frac{\int_{X_\theta - \bar{e}}^{X_\theta - \underline{e}} \frac{\partial G_\theta}{\partial \theta}(\varepsilon) d\varepsilon}{G_\theta(X_\theta - \bar{e}) - G_\theta(X_\theta - \underline{e})}. \quad (45)$$

The denominator is negative; so  $\frac{dX_\theta}{d\theta} \geq (\leq) 0$  if  $\int_{X_\theta - \bar{e}}^{X_\theta - \underline{e}} \frac{\partial G_\theta}{\partial \theta}(\varepsilon) d\varepsilon \leq (\geq) 0$ .

Since  $G_\theta$  is symmetric for any  $\theta$ ,  $G_\theta(\varepsilon) = 1 - G_\theta(-\varepsilon)$  for any  $\theta, \varepsilon$ . In particular,

$$\frac{\partial G_\theta}{\partial \theta}(\varepsilon) = -\frac{\partial G_\theta}{\partial \theta}(-\varepsilon).$$

so, by MPS,  $\frac{\partial G_\theta}{\partial \theta}(x) \geq 0 \Leftrightarrow x \geq 0$ .

Evaluating at  $X_\theta = \frac{\bar{e} + \underline{e}}{2}$ , gives

$$\int_{-\frac{\bar{e} - \underline{e}}{2}}^{\frac{\bar{e} - \underline{e}}{2}} \frac{\partial G_\theta}{\partial \theta}(\varepsilon) d\varepsilon = 0.$$

Note that  $\frac{dX_\theta}{d\theta} \leq 0$  whenever  $X > \bar{e}$ , since, in that case,  $\frac{\partial G_\theta}{\partial \theta}(\varepsilon) \geq 0$  for all  $\varepsilon \in (X - \bar{e}, X - \underline{e})$ . Similarly,  $\frac{\partial G_\theta}{\partial \theta}(\varepsilon) \leq 0$  whenever  $X < \underline{e}$ . Finally, for  $\underline{e} \leq X \leq \bar{e}$ , we have

$$\frac{d}{dX} \left[ \int_{X - \bar{e}}^{X - \underline{e}} \frac{\partial G_\theta}{\partial \theta}(\varepsilon) d\varepsilon \right] = \frac{\partial G_\theta}{\partial \theta}(X - \underline{e}) - \frac{\partial G_\theta}{\partial \theta}(X - \bar{e}) \geq 0,$$

where the inequality follows from the fact that  $X - \underline{e} \geq 0 \geq X - \bar{e}$  implies  $\frac{\partial G_\theta}{\partial \theta}(X - \underline{e}) \geq 0$  and  $\frac{\partial G_\theta}{\partial \theta}(X - \bar{e}) \leq 0$ . Therefore, the numerator in (45) is increasing in  $X$ .