Bounded Rationality in Rules of Price Adjustment and the Phillips Curve*

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Abstract

This paper presents a model of endogenous bias in rules of price adjustment that allows one to analyse the behaviour of inflation and output continuously throughout the entire spectrum of rationality, from one end to the other. Specifically, it proposes an alternative microfoundation for both the New Keynesian sticky-price and the sticky-information Phillips Curve by considering a possibility where price setters are constrained by the length of the time horizon κ over which they can form rational expectations, and they use the growth of past prices at the rate of the central bank's inflation target as a heuristic alternative in place of their own expectations beyond this horizon. Three interesting results emerge. Firstly, how price setters form inflation expectations and whether these expectations are accurate or heterogeneous do not matter when they are able to gather information or change prices more frequently. Secondly, should policymakers expect private agents to similarly adopt the inflation target as a nominal anchor for their own expectations, then even the choice of this numerical target could prove to be pivotal to output stabilization. Thirdly, larger degrees of bounded rationality increase the persistence of inflation, and, under sticky-information, raise the possibility of discontinuous jumps and oscillatory dynamics of inflation and real output.

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1 Introduction

Why does the relationship between inflation and output weaken, and what exactly is the role of inflation expectations? The fact that these are sometimes unclear poses a challenge to the microfoundations of the new Keynesian Phillips Curve (NPKC). While reflecting on Milton Friedman's influential 1967 presidential address to the American Economic Association that set the stage for rational expectations (RE), Mankiw and Reis (2017) recognized the growing tendency within the profession to lean away from this workhorse model and towards other forms that do not impose such a demanding assumption, at least in the short run. Perhaps one might speculate that a more robust model of expectations could potentially be found at the interaction between new Keynesian economics and behavioural economics, given a more *realistic* portrayal of the economic agent.

This, however, is not a new endeavour. Rather than follow the new Keynesian microfoundation that proposes a purely forward-looking inflation dynamics model (Coibion, Gorodnichenko and Kamdar, 2017), Galí and Gertler (1999) introduced a model where agents are simultaneously forward and backward-looking and Milani (2005) explored a behaviour of Bayesian learning. Ball (2000) further examined a near-rationality model of price adjustment where agents have rational inattention, and they pay attention only to information on inflation and nothing else when forming their expectations. These are all 'behavioural models' to model the NPKC more realistically[1] by relaxing the demanding assumption of rational expectations, especially when omniscient price setters are rare. In fact, this is the motivation behind the theory of *sticky-information*. Proposed by Mankiw and Reis (2002) to replace the sticky-price model, they model the cost of information gathering into rules of price adjustment and finds that the sticky-information Phillips Curve is capable of producing inflation and output dynamics that are more consistent with U.S. data.

This paper acts on this foundation to present a model of endogenous bias in dynamic price adjustment where price setters without rational expectations rely on the central bank's inflation target as a heuristic anchor. It is therefore a novel attempt to present a model of inflation expectations where agents in this model look to the central bank for their own optimal pricing behaviour when their rationality is bounded. There is thus a bias, endogenous to the agent, whenever there is a departure from what prices would be set optimally under rational expectations. This paper will endeavour to use Tversky and Kahneman's (1974) three judgment

[1] Fuhrer and Moore (1995) showed that this canonical model is empirically inconsistent with inflation persistence and Ball (1994a) showed that it is theoretically challenged by the disinflationary boom.

heuristics - *representativeness*, *availability* and *anchoring* - to justify such a form of heuristic behaviour when agents depart from rational expectations, and hence, attempts to introduce a channel to which behavioural economics *may* impact dynamic responses of inflation and output. Therefore, it approaches the literature on Phillips Curves from a different angle. The conjecture for the model is as follows:

Price setters maximize utility by forming rational expectations of their future desired adjustment price as much as they can, until they are constrained by the time horizon (κ) beyond which they can no longer form such expectations due to its increasingly complex nature. They subsequently use the growth of past prices at the rate[2] *of the inflation target as a heuristic alternative in place of their own expectations beyond this horizon.* In other words, their expectation $E_t(P^*_{t+\kappa+1})$[3] is replaced with $P_{t-1}(1+\pi^T)^{\kappa+2}$, with π^T being the notation for the central bank's inflation target.

While there could be many constraints that prevent monopolistic firms from forming rational expectations in reality, this paper focuses only on the length of the forecast horizon over which one is able to do so. The motivation here lies in a somewhat odd mismatch between empirical and theoretical macroeconomics: there have been many papers (such as Gavin and Mandal (2001), Mankiw, Wolfers and Reis (2003) and Jonung and Lindén (2010)) written on the heterogeneity of expectations or some biases in inflation forecasts over different time periods (or even over a period of 12 months) that present clear evidence against the unbounded forecast horizon. Yet, most of the theoretical work on models of inflation expectations continue to use some discounted affine function of forecast horizons that sum to infinity. How does the Phillips Curve behave when this is no longer the case? This paper thus stands in contrast with the approach taken by current literature, which is to discount the importance[4] of expectations further ahead by assigning geometrically declining weights to expectations formed into perpetuity. By further relaxing the standard definition of an omniscient *homo economicus*, this paper subsequently explores a microfoundation established by price setters who rely on heuristic principles to 'reduce the complex tasks' of assessing their future expectations of prices, which, although will not be a precise estimate, is the best they can do given that they have 'bounded' or 'procedural' rationality (Simon, 1978).

[2]Or perhaps even at some multiple of the inflation target, as section II will show that the model can be easily adapted.

[3]$P^*_{t+j} = P_{t+j} + \alpha Y_{t+j}$. P^*_{t+j} is the firm's desired price at period $t+j$. This implies that firms raise prices in times of booms and lower them during recessions, with $0 < \alpha < 1$ representing some real rigidities.

[4]Expectations of what the firm's desired prices would be in the far future is less important than those in the near future as the firm gets to update their prices periodically.

This proposed heuristic principle is a hybrid that incorporates two recent progresses in both behavioural and monetary economics, and the design is anything but arbitrary. Firstly, the rule is motivated in part by Gabaix's (2014) sparsity-based utility-maximization model where agents build simplified models of understanding the world, but replaces certain parameters which they are unable to acquire due to their cognitive limits or costs of mental processing. However, I take an alternative approach by using the *time horizon* as a basis of the agent's reparameterization in an attempt to align the model more closely with how a rational agent would behave in the world through the eyes of Kahneman and Tversky. Hence, this contains traces of Galí and Gertler's (1999) simultaneously forward and backward-looking model, except that agents are backward-looking in *expectation* that some past prices will grow at the rate of the inflation target and therefore can be used an appropriate reference point. Secondly, the rule further reasons that since most central banks have adopted inflation targets into their loss functions[5] and forward guidance has become a recent hallmark of monetary policy, it would be rational for agents to set prices using the central bank's inflation target as a nominal anchor when their inflation expectations are uncertain. In fact, this is as if one is using the inflation target to solve the famous time inconsistency problem, except through the lens of price setters at a micro level. As the paper will show subsequently, the role of the inflation target in the proposed rule of price adjustment becomes crucial in stabilizing inflation and output in the presence of realistic monetary policy shocks. By modelling the inflation target into the rule of the price adjustment, the Phillips Curve now becomes an explicit function of the central bank's credibility. The following subsection provides a more thorough justification for the proposed pricing heuristic.

Acknowledging the progress made by both sides of the profession, this paper derives a framework that may be capable of analyzing the dynamics of inflation and output continuously throughout the *entire spectrum of rationality*, from one end to the other depending on the severity of the bias. This is how the paper hopes to contribute to existing literature. To derive this framework, this paper incorporates the rule of price adjustment into Calvo's (1983) staggered-pricing model and Mankiw and Reis' (2002) sticky-information model, as both are the two more widely embraced models of inflation expectations (Dupor, Kitamura and Tsuruga, 2006). While the sticky-price model is flawed, it serves as a useful benchmark when one ventures to depart from full rational expectations. The sticky-information model, on the other hand, provides a more realistic foundation to study what happens if agents have constraints that prevent them from acquiring rational expectations *even if* they are able to gather information costlessly. In fact, this paper is a departure from the theory of sticky-information. Both models will subsequently be used to

[5]See Svensson (1996), Bernanke and Mishkin (1997) with regards to the efficacy of the inflation target as a solution to the inflation bias problem.

study the role of endogenous bias on a gradient going from a strict to semi-strict RE model.

By writing the proposed rule of price adjustment into both of these models, this paper will show in section II that it is possible to derive models of both Phillips Curves that are now explicit functions of both κ and the central bank's inflation target. Each value of κ can subsequently be mapped into a specific dynamic path and an impulse response function for prices. These will be used in section III to study the behaviour of inflation and output in response to exogenous shocks. In order to compare the results to some benchmark where κ is initially infinity, this paper will model the exact same shocks used by Mankiw and Reis (2002) and study the deviation in behaviour when κ decreases from infinity. I subsequently study these dynamics when κ is initially zero and when it increases from zero. Monetary policy shocks will be used to motivate the role of the inflation target. Section IV examines the theory further by studying the interactions between κ and the initial parameters in both the sticky-price and sticky-information model, and hence develops the framework fully. Ultimately, this framework hopes to answer two questions: Firstly, if the current full-rationality model of expectations leaves one with some gaps in understanding the curious relationship between inflation and output, can one better do so by surveying this relationship from the other end of bounded rationality? Secondly, supposing that is possible to do so, can these new Keynesian models of inflation be used as a vehicle for behavioural economics to inform monetary policy in any way? Summarizing the results from the impulse responses and using them to comment on these two questions, section V concludes.

1.1 Motivations for proposed design of price adjustment

One might be tempted to ask: why might price setters use the growth of past prices at the rate of the inflation target as a heuristic alternative for their own expectations of future price levels? When they form future expectations $E_t(P^*_{t+j})$ to as far as they can until they become constrained by the time horizon κ, why not set all $E_t(P^*_{t+\kappa}) = E_t(P^*_{t+\kappa+1})$ to be a constant function, rather than being backward-looking and set $P_{t-1}(1+\pi^T)^{\kappa+2} = E_t(P^*_{t+\kappa+1})$? Do *homo sapiens* alternate from *system 2* to *system 1* once some threshold (such as κ) has been reached?

To be sure, there are many different ways in which one can model how expectations are formed and how prices are set conditioned on these expectations. In most cases, such as the theory of sticky-information or sticky-prices, the model rests on a core assumption that price setters are constrained by some exogenous variation that deter them from forming rational expectations of some unknown price vector. Under sticky-information, for instance, this arises from the slow rate of information diffusion. In proposing the behaviour described above, this paper claims that the length of the time period ahead which one intends to form expectations over is a appropriate *internal constraint*, which, similar to the exogenous constraints, is equally capable of preventing one from forming rational expectations, even if one has information or the opportunity to change prices. The focus, however, is to motivate an alternative behaviour to circumvent this constraint. To do so, one might venture into fields of psychology, behavioural economics and finance where there have been overwhelming evidences that persistently distinguished *homo economicus* from *homo sapiens* (Lo, 2013). In their 1974 *magnum opus* "*Judgment under Uncertainty: Heuristics and Biases*", Tversky and Kahneman laid out three different judgment heuristics that capture how *homo sapiens* would likely behave under certain circumstances. Paraphrasing using their own words (italicized), these are:

1. **Representativeness**: *in which probabilities are assessed by the degree to which A resembles B.* If A represents B or is a subset of B, and price setters know the probability of B occurring at each period, then they can infer what the probability of A occurring is.

2. **Availability**: *in which probabilities are assessed by the probability of past events or the ease to which similar occurrences can be brought to mind.*

3. **Adjustment from an anchor**: *in which probabilities are assessed by making estimates starting from an initial value, that might be the result of some partial computation.*

Can one contextualize these judgment heuristics in the microfoundations of dynamic price adjustment to motivate the *behavioural rule*? It would perhaps be appropriate to do so if the behavioural model is centered upon some degree of backward-lookingness. This is a rational course of action for the price-setter, who, in lacking the cognitive tools to navigate the complexity that is ahead of him, turns to his experience or the information contained in some realized prices as a guide. In fact, this paper takes the view that Tversky and Kahneman's set of judgment heuristics (henceforth TKC) are relevant only if price setters are *allowed* to be backward-looking. This provides the justification for using past prices, one period before[6] as a heuristic instrument.

Consider using the TKC to evaluate the following: is $E_t(p^*_{t+\kappa})$ *representative* of $E_t(p^*_{t+\kappa+1})$? This could be true, especially if prices are sticky and are unlikely to change between an interval of one period. Then, one might argue that one's expectation $E_t(p^*_{t+\kappa})$ is *as good as* $E_t(p^*_{t+\kappa+1})$. However, the strength of this argument weakens if one were to argue that $E_t(p^*_{t+\kappa})$ is as good as

[6]Assume that price setters only refer to prices realized *one period before* as this vector contains the most recent information about the state of the economy.

$\{E_t(p^*_{t+\kappa+1})...E_t(p^*_{t+\kappa+q})\}$ $\forall$ $q \in (1,\infty)$ because the approximation of expectation over the price vector between two periods that are longer apart begins to fail: even if the central bank targets a zero inflation rate such that price levels remain constant, it is possible that the economy could be hit with a shock in the future that is unobserved at time t.

In comparison, using $P_{t-1}(1+\pi^T)^{\kappa+2}$ as a representation for $E_t(p^*_{t+\kappa+1})$ is a better approximation if one were to assume that price setters demonstrate *satisficing* behaviour by seeking to minimize their mean squared errors (MSE) of price forecast rather than seeking to target the price level perfectly: rather than exercise their discretion in forming forecasts, which may be inconsistent to the literature on rational inattention, they simply follow a rule that suggests raising prices by the rate of the inflation target. For price setters who are unable to observe P^*_{t+j} and hence form expectations $E_t(P^*_{t+j})$, it is a rational alternative to believe that $\frac{P^*_{t+j}}{P^*_t} = (1+\pi^T)^j$, especially since P^*_{t+j} is the aggregate price level that the central bank indirectly targets through the inflation target. This is further appropriate if the time horizon κ over which price setters are able to form rational expectations is *long enough* such that the eventual realized inflation beyond this period is close to the target. Then, the justification for this rule simply rests on having the central bank to be credible. Implicitly, this proposes a case of profit-satisficing for the price-setter that occurs whenever the central bank minimizes its loss function.

Here, price setters know that past, realized prices one period before is not a perfect anchor for them to form expectation of some prices in the future. However, this does not matter to them insofar as over the long run both converges. For example, $P_{t-1}(1+\pi^T)^{\kappa+2}$ is mathematically a linear trend extrapolated off a point P_{t-1} from the business cycle, with κ on the x-axis. Then, depending on whether P_{t-1} lies on a peak or the trough, this extrapolated trend could either be above or below the *actual* trend. This, however, is not a concern to our price setters, who instead only care about their long run convergence. This requires the assumption that the inflation target is set to be equal to the long run average growth rate of prices, and so having prices to grow at the rate of the inflation target regardless of its position at the business cycle will allow for *convergence to the mean*. This, is satisficing behaviour. Setting $E_t(p^*_{t+\kappa})$ as an approximation for $\{E_t(p^*_{t+\kappa+1})...E_t(p^*_{t+\kappa+q})\}$ risks resulting in the two abovementioned trends diverging forever, with the peril here being the argument that price setters are willing to tolerate such a large *mean squared error* (MSE).

What about the criteria of availability under TKC? How do one assess the probability of *prices*? But once one realizes that this is but a matter of inflation forecasting, the question becomes: how often do the same (or certain) rates of inflation occur, over how many time periods and what is its associated mean and variance? These questions matter because persistence of inflation is likely to be *representative*: to forecast what inflation will be tomorrow, one can simply refer to the past for answers. If this is so, then P_{t-1} naturally becomes a suitable factor in the forecast of $P^*_{t+\kappa}$. Furthermore, this paper is inclined to take the view that it is the mean of inflation that matters to price setters more, rather than its variability. There are two good reasons established on the grounds of satisficing behaviour once again: (1) forecasting the long run average of inflation of tomorrow $\bar{\pi}_{t+1}$ is *easier* than forecasting a point estimate π_{t+1} and (2) given rational inattention, it is unlikely that price setters will actively gather all associated information on the economy in an attempt to precisely forecast inflation. Then, given (1) and (2), price setters actually do not need to bother with inflation forecasts if they can equate the mean of inflation with the inflation target over the medium to long run! This is not a thoroughly demanding assumption, especially since the hallmark of monetary policy today lies in the central bank's commitment to keep prices low and stable. With these premises, price setters expect past prices P_{t-1} to grow at the rate of the inflation target *on average*, and this becomes an approximation for $E_t(p^*_{t+k+1})$.

For practical reasons, setting $P_{t-1}(1+\pi^T)^{\kappa+2} = E_t(P^*_{t+\kappa+1})$ may even allow for richer dynamics of the impulse responses which could be extremely helpful in demonstrating the theory. If one were to instead pursue an alternative where the price setter forms expectations for as far as he can and fixes them for the subsequent horizon beyond which he can no longer form expectations, this may be no different from the benchmark case of price adjustment under full rational expectations, with the exception of having the price adjustment rule being split into its usual Calvo summation of expected prices and some constant. One may thus expect the impulse responses under this alternative to be of little difference when compared to the benchmark.

2 Reparameterizing the Phillips Curve with κ

2.1 Deriving a model of the sticky-price new Keynesian Phillips Curve with bounded rationality

I begin by modifying the microfoundation of Calvo's (1983) staggered-pricing model using the proposed rule of price adjustment explained in the introduction. In this original model, opportunity to change prices arrive stochastically at a rate of λ, where $0 \leq \lambda \leq 1$. When price setters have the opportunity to change prices, they set their adjustment price X_t[7] to be equal to some

[7] All prices P_t, adjustment price X_t and money M_t are expressed in logs throughout this paper.

weighted average of current and expected optimal prices P^*_{t+j} in the future as follows:

$$X_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_t(P^*_{t+j}) \tag{1}$$

Observe that the adjustment price at period t is purely a function of expectations formed *today* of what the future desired price will be in perpetuity. This presupposes that the price setter is capable of forming expectations over *any* time horizon, however long it is. I relax this demanding assumption and propose that the price setter does not have such capabilities. Instead, I propose that they behave like rational agents and form rational expectations of their desired prices as much as possible, until they reach a process where they can no longer do so as a result of their cognitive limits. Hence, the rule of price adjustment is rewritten in the following manner:

$$X_t = \lambda \sum_{j=0}^{\kappa} (1-\lambda)^j E_t(P^*_{t+j}) + \lambda \sum_{j=\kappa+1}^{\infty} (1-\lambda)^j P_{t-1}(1+\pi^T)^{j+1} \tag{2}$$

Under this rule, price setters can only form rational expectations of what their desired prices P^*_t are over a time horizon that is κ periods long, and they use the growth of past prices at the rate of the central bank's inflation target as a heuristic alternative in place of their own expectations beyond this horizon. Observe that this rule is not a complete deviation away from Calvo's model. It continues to presuppose that price setters behave in a forward-looking manner as much as they can. In fact, it requires them to be strictly forward-looking for as long as the time horizon over which they form expectations is within their cognitive boundaries. This rule also assumes credibility of the central bank in order to substantiate the choice of π^T as an anchor for inflation. This is thus a departure from how current literature has approached the issue (see Afrouzi and Yang (2016), Milani (2005) and Galí and Gertler (1999)). This microfoundation further suggests that some backward-looking behaviour only exists for some κ, and the position of κ in turn determines the geometric weights that past prices have in setting prices today. As with Calvo's original staggered-pricing model, aggregate prices P_t today continue to be a weighted average of all prices that firms have set in the past:

$$P_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j X_{t-j} \tag{3}$$

Using the law of iterated expectations and rearranging the algebra, which I leave to the appendix, it is possible to combine equations (2) and (3) to obtain the following model of inflation:

$$\pi_t = \frac{\alpha\lambda^2}{1-\lambda(1-\psi_\kappa)} Y_t + \frac{1-\lambda}{1-\lambda(1-\psi_\kappa)} E_t(\pi_{t+1}) + \frac{\psi_\kappa \lambda^2}{1-\lambda(1-\psi_\kappa)} P_t \tag{4}$$

such that

$$\psi_\kappa = \frac{\lambda(1-\lambda)^{\kappa+1}(1+\pi^T)^{\kappa+2}}{\lambda - \pi^T(1-\lambda)} \tag{5}$$

For the rest of this paper, I refer to equation (4) as the *κ-augmented sticky-price Phillips Curve.* On an *a priori* basis, this model of staggered pricing suggests that output gaps and expectations may contribute less to inflation depending on the size of ψ_κ. An important point to note here is that this modified Phillips Curve is in fact nested in the original model: if ψ_κ is 0, the κ-augmented model reduces to the canonical sticky-price model given by:

$$\pi_t = \frac{\alpha\lambda^2}{1-\lambda}Y_t + E_t(\pi_{t+1}) \tag{6}$$

When will ψ_κ be equal to 0? If one were to make the strict assumption that $\lambda \neq 1$, then the value of ψ_κ as 0 can never be obtained by having $\lambda = 1$. This is a reasonable assumption, given that opportunity for price adjustment almost never arrives with full certainty at each period. As such, the value of ψ_κ now becomes contingent on the value of κ. Then, observe that as κ increases to ∞, $(1-\lambda)^{\kappa+1}$ geometrically declines faster then $(1+\pi^T)^{\kappa+2}$ increases. This holds if one were to assume that $\lambda > \pi^T$, which is a suitable assumption to make. With inflation targets of most central banks centered arbitrarily at 2%, $\lambda \leq \pi^T$ suggests that firms change prices less than once every 50 quarters! This is a rather absurd assumption. For now, this paper will endeavour to defer a more thorough discussion on the role of the inflation target π^T to section 3.5, where shocks to monetary policy are analysed in a setting where private agents respond by forming expectations using the inflation target as a nominal anchor. Hence, one might as well treat π^T tentatively as zero.[8] As long as $\lambda > \pi^T$, ψ_κ reduces to 0 for a large enough κ as shown by taking the limits:

$$\lim_{\kappa\to\infty} \frac{\lambda(1-\lambda)^{\kappa+1}(1+\pi^T)^{\kappa+2}}{\lambda - \pi^T(1-\lambda)} = 0 \tag{7}$$

This result must necessarily hold. Having κ to be equal to ∞ essentially brings one back to the workhorse model where price setters are able to form expectations over an indefinite period, and hence, equation (4) must revert back to (6).

By incorporating the bias in the rule of price adjustment, the canonical Phillips Curve is now an explicit function of both κ (and the inflation target π^T), which in turn determines the extent to which output gaps and expectations contribute to inflation. This provides a theoretical model to analyse the behaviour of inflation when κ ranges from zero from one end of spectrum to ∞ at the other. Should price setters have a shorter horizon over which they can compute rational expectations, the role of inflation expectations and output matter less for actual inflation today as a significant share of the price-setting rule is accounted for by the use of heuristics. What may be of greater interest is the additional P_t term that now features in the sticky-price model. As section III will show, this changes the solution for a dynamic path of prices. One can immediately

[8] As section III will illustrate, zero is a suitable parameter value for π^T also due to the fact that the path of money M_t (either in levels or in growth rates) is set to zero ex-post to the exogenous shocks.

infer that inflation and output dynamics under the κ-augmented model will deviate from those of the benchmark as long as $\kappa \neq \infty$.

2.2 Deriving a model of the sticky-information Phillips Curve with bounded rationality

As an alternative proposal to replace the sticky-price model, Mankiw and Reis (2002) writes a model of price adjustment where the rate of information arrival, γ, is slow. As not everyone receives the most updated information about the state of the economy, only a fraction updates their prices optimally and the rest continue to set prices based on past information. This assumes that whenever price setters acquires information, they behave in a manner that is consistent with rational expectations. The price adjustment rule of the benchmark follows:

$$X_t = E_{t-j}(P_t^*) \tag{8}$$

With aggregate price levels being a weighted average of all prices in the economy:

$$P_t = \gamma \sum_{j=0}^{\infty} (1-\gamma)^j E_{t-j}(P_t^*) \tag{9}$$

This model of price adjustment motivates a microfoundation where price setters still form rational expectations, but these are expectations formed ex-ante to the information that arrives later. However, depending on the rate of information arrival, price setters may have to rely on some expectations that were formed at the beginning of time. Like Calvo's staggered-pricing model, this presupposes that there are no bounds to the horizon over which expectations are formed. Applying the same behavioural conjecture, this assumption is relaxed and replaced with the following:

$$X_t = \begin{cases} E_{t-j}(P_t^*) & \text{if } j \leq \kappa \\ P_{t-j}(1+\pi^T)^j & \text{if } j > \kappa \end{cases}$$

Similar to the microfoundation of the κ-augmented sticky-price model, this rule of price adjustment suggests that price setters anchor themselves on the inflation target as a guide to how prices would grow in the future. What is different here is the recency of past prices that price setters choose as a reference point. Here, the sticky-information model differs from the sticky-price model in the *past* expectations matter for inflation today, rather than *future* expectations. As a result, only prices occurring a period before the most outdated expectation represent the *next-best* knowledge that the price setter has about the state of the economy. This results in an adaptive behaviour that is more backward-looking into the past as compared to that for the κ-augmented sticky-price model. With this rule of price adjustment, overall prices in the economy are then pinned down by the following:

$$P_t = \gamma \sum_{j=0}^{\kappa} (1-\gamma)^j E_{t-j}(P_t^*) + \gamma \sum_{j=\kappa+1}^{\infty} (1-\gamma)^j P_{t-j}(1+\pi^T)^j \tag{10}$$

With some tedious algebra, which I once again leave to the appendix, inflation can now be expressed as:

$$\pi_t = (\frac{\alpha\gamma}{1-\gamma})Y_t + \gamma \sum_{j=0}^{\kappa} (1-\gamma)^j E_{t-j-1}(\pi_t + \alpha\Delta Y_t) \\ + \gamma \sum_{j=\kappa+1}^{\infty} (1-\gamma)^j (1+\pi^T)^j \left(\pi_{t-j} + (\frac{\gamma}{1-\gamma})P_{t-j}\right) \tag{11}$$

For the rest of this paper, I refer equation (11) as the *κ-augmented sticky-information Phillips Curve*. By and large, this model of inflation resembles the benchmark sticky-information model. Inflation today is a result of some output gap, sum of past inflation expectations formed up to κ periods into the past and some past inflation and prices realized more than $\kappa + 1$ periods ago in the past. Crucially, past expectations of current inflation only matter until a certain time horizon $\kappa + 1$ from the past (with a lower bound defined by $t - \kappa - 1$) whereas they extend back to the beginning of time in the benchmark. This is consistent with the bias under bounded rationality, where price setters are only permitted rational expectations over a limited time horizon. Beyond this horizon, what matters more for inflation today is past inflation. As a result, this model of price adjustment introduces inflation inertia and a variant of adaptive inflation by design similar to Galí and Gertler's (2004) backward-looking rule of thumb model, except that price setters in this model only turn to backward-looking behaviour conditionally when they need to rely on the inflation target.

As the paper will show in section III, it is further useful to study the model dynamics by considering the value of κ at its extremes. Suppose $\kappa=\infty$. This is to say that price setters can make forecasts of what optimal prices are over a time period that is infinitely large. This behaviour thus implies the absence of any pricing bias. It is no surprise then, that equation (11) reduces to the baseline model of the sticky-information Phillips Curve as all coefficients $(1-\gamma)$ of past inflation π_{t-j} and past prices P_{t-j-1} are raised to the power of infinity. Under the assumption that $\gamma \neq 1$ and $\gamma > \pi^T$, the entire summation term of past inflation and price variables become 0. This retrieves the benchmark model of sticky-information given by:

$$\pi_t = (\frac{\alpha\gamma}{1-\gamma})Y_t + \gamma \sum_{j=0}^{\infty} (1-\gamma)^j E_{t-j-1}(\pi_t + \alpha\Delta Y_t) \tag{12}$$

On the other hand, suppose instead that κ=0. For the sticky-information model this suggests the unique case where price setters can only form rational expectations for one period ahead:

$$\pi_t = (\frac{\alpha\gamma}{1-\gamma})Y_t + \gamma E_{t-1}(\pi_t + \alpha\Delta Y_t) + \gamma\sum_{j=2}^{\infty}(1-\gamma)^j(1+\pi^T)^j\left(\pi_{t-j} + (\frac{\gamma}{1-\gamma})P_{t-j}\right) \tag{13}$$

The reason that price setters continue to form expectations even when $\kappa = 0$ lies in the theory of sticky-information. Given that information arrives with a lag, the expectations formed over one period ex-post to information arrival does not necessarily reflect the best state of knowledge. In this aspect, price setters optimizes with a lag as well. Subsequently, equation (13) indicates that past expectations of inflation only matter for inflation up to one period ago, and there is a larger role assumed by adaptive inflation.

2.3 Motivations for proposed microfoundation

Up to this juncture, the paper has presented a variant of the Phillips curve motivated by both sticky-prices and sticky-information under a set of behaviour that is discrete in motion: first, a strictly forward-looking behaviour as one would expect in the benchmark full rational expectations model and second, some backward-looking rule of thumb with weights conditioned on the time horizon κ. Here, this paper recognises that the assumption of full rational expectations does not *strictly* require the price setter to form such expectations over an infinite time horizon, as the adjustment process imposes weights on these expectations that geometrically decline to zero anyway. This suggests that a working model does not require price setters to be able to form such expectations over some horizon κ such that $\kappa = \infty$. However, notice that the rule of price adjustment proposed earlier attaches geometrically declining weights to the heuristic term in a similar manner and therefore continues to give a non-trivial role to rational expectations in this microfoundation. What is of a larger significance here is the inclusion of some reference-dependent expectations that has long been identified as a cornerstone of loss-averse utility functions. This would be relevant if one were to examine some utility function of price setters who are loss averse[9], such that they prefer to target some 'general growth' in price levels than forming their own stochastic expectations. As Tversky and Kahneman (1991) famously noted, one's preferences change endogenously according to where the point of reference is. Why not apply the same strand of thought to the formation of expectations? Surely as self-fulfilling equilibriums of hyperinflation and deflations would show, the expectations that one forms of prices are very much dependent on the nature of the environment that one is in. In this sense,

[9] See '*A Model of Reference-Dependent Preferences*' by Kőszegi and Rabin (2006).

this paper has proposed the heuristic in the price adjustment rule to resemble some form of adaptive behaviour.

In '*Disagreement about inflation expectations*', Mankiw et al. (2004) illustrated using 50 years of survey data on median inflation expectations (12 months ahead) in the U.S. that a central bias exists in the inflation forecasts by both professional economists and households, and there is significant evidence to reject the claim of rational expectations for both. What is relevant to this paper here is that *knowing* more about the economy - as one could claim about technocrats - certainly does not help one in making better predictions. Therefore, motivating a theory of bounded rationality in rules of price adjustment naturally builds on the literature and what the profession has already learned about the formation of expectations. More importantly, perhaps what is less agreed upon within the profession is the upper bound that κ should take in place of ∞ in the workhorse model as a *standard*. Whether having such a standard is important, and whether or not it matters if κ is not modelled as ∞ is precisely the goal of section III.

3 The dynamic behaviour of inflation and output under varying degrees of price setter foresight

Having derived the κ-augmented Phillips Curves for both sticky-prices and sticky-information, I examine the behaviours of inflation and output in the presence of macroeconomic shocks and compare them to the results given by the baseline model as outlined by Mankiw and Reis (2002). *Note that the shocks are the same except for the introduction of an inflation target by an independent central bank*:

- Macroeconomic shock 1: *an unexpected fall in the level of aggregate demand by 10% at period 0.* That is, $M_t = -log(0.9)$ for $t \leq 0$ and $M_t = 0$ for $t \geq 0$. *The inflation target π^T is unchanged at 0.0% in all periods.*
- Macroeconomic shock 2: *an unexpected fall in the rate of money growth from 2.5% to 0% per period at period 0.* Thus, $M_t = 0.025(t+1)$ for $t \leq -1$ and $M_t = 0$ for $t \geq 0$. *The inflation target π^T is lowered from 2.5% to 0.0% at period 0.*
- Macroeconomic shock 3: *an announced disinflation at period $t = -8$ of the same magnitude as (2). While the announcement of the forthcoming change inflation target π^T occurs at period $t = -8$, it is only lowered from 2.5% to 0.0% at period $t = 0$.*

As the intention here is to study the extent to which the business cycle behaves differently in response to the same shock, the same parameter[10] values in Mankiw and Reis (2002) are used but different solutions for the dynamic paths are solved under the κ-augmented Phillips Curves. This is where the paper takes another different direction: instead of having a single parameter λ (or γ, for the sticky-information case) to pin down the these solutions, there is now an additional parameter, κ, that concurrently specifies the deviation from some benchmark in the ideal world where information and opportunity to change prices arrive with full certainty at each period. Then, by adjusting the values of κ in relation to λ and vice versa on an incremental basis, this section introduces a framework that models a behaviour of inflation and output precisely for each degree of bias. To achieve this, the dynamics of inflation and output when κ is ∞ and when κ deviates from ∞ will be first illustrated, followed by the results when κ is zero and when κ increases from zero. By motivating this study from both extremes, the hope is that one can learn more about the limits of this behavioural model.

3.1 Solving for a dynamic path under the κ-augmented sticky-price Phillips Curve

In order to construct a dynamic path for prices under the modified sticky-price and sticky-information Phillips Curve, I use the exact same specifications in Mankiw and Reis's original paper for a model of aggregate demand given by:

$$M_t = P_t + Y_t \tag{14}$$

This model of aggregate demand can be combined with the κ-augmented sticky-price model in (4) to yield the following expectational difference equation:

$$E_t(P_{t+1}) + \left(\frac{\psi_\kappa \lambda^2}{(1-\lambda)} - (1 + \beta + \frac{\beta}{\mu})\right) P_t + \frac{\beta}{\mu} P_{t-1} = -\beta M_t \tag{15}$$

with parameters μ, β and ψ_κ defined as:

$$\mu = \frac{\alpha\lambda^2}{1 - \lambda(1 - \psi_\kappa)} \tag{16}$$

$$\beta = \frac{\alpha\lambda^2}{1 - \lambda} \tag{17}$$

$$\psi_\kappa = \frac{\lambda(1-\lambda)^{\kappa+1}(1+\pi^T)^{\kappa+2}}{\lambda - \pi^T(1-\lambda)} \tag{18}$$

[10] According to the benchmark models, $\alpha = 0.1$, $\lambda = \gamma = 0.25$, $\rho = 0.5$. The residual for monetary policy shocks is chosen to have a standard deviation of 0.007 based on historical estimates of monetary aggregates M1 and M2 between 1960 and 1999. Refer to Mankiw and Reis (2002) pages 1302 and 1308 for a more thorough discussion.

A dynamic path for prices given by the model of the sticky-price new Keynesian Phillips Curve can thus be obtained by solving for the expectational difference equation in (15) such that:

$$P_t = \theta P_{t-1} + (\frac{\alpha\lambda^2}{1-\lambda(1-\psi_\kappa)})\theta\sum_{i=0}^{\infty}\left((\frac{1-\lambda}{1-\lambda(1-\psi_\kappa)})\theta\right)^i E_t(M_{t+i}) \tag{19}$$

This equation itself pins down the dynamic path for all prices and subsequently inflation. The path of output can then be obtained from equation (14) easily. A full proof is deferred to the appendix. Observe that as $\kappa \to \infty$, price setters approach a world without bias: they have full rational expectations. ψ_κ subsequently approaches 0, allowing us to obtain the impulse response of the benchmark model given by:

$$P_t = \theta P_{t-1} + (1-\theta)^2\sum_{i=0}^{\infty}(\theta)^i E_t(M_{t+i}) \tag{20}$$

At this juncture, notice that equations (19) and (20) each produces a different dynamic path of prices (and, of output) that differ for two reasons. First, observe that the impulse response of prices under the model with bounded rationality looks exactly like the impulse response of the benchmark model, except that each of the terms in the summation operator are multiplied by some scaling factor $\neq 1$. Second, the key parameter that determines the dynamic path is θ, and its value differs for both equations. In the model under bounded rationality, θ is chosen as the smaller of the two positive roots from the coefficient of LP_t^* (L and F are lag and forward operators respectively) following expectational difference equation:

$$\left(F^2 + \left(\frac{\psi_\kappa\lambda^2}{1-\lambda} - (1+\beta+\frac{\beta}{\mu})\right)F + \frac{\beta}{\mu}\right)LP_t^* = -\beta M_t^* \tag{21}$$

rather than the expectational difference equation in the benchmark case as given by:

$$\left(F^2 - (2+\beta)F + 1\right)LP_t^* = -\beta M_t^* \tag{22}$$

Subsequently, θ in the modified model is obtained without a loss of generality as the smaller of the following two positive roots:

$$\theta = \frac{-\frac{\psi_\kappa\lambda^2}{1-\lambda} + (1+\beta+\frac{\beta}{\mu}) \pm \sqrt{\left(\frac{\psi_\kappa\lambda^2}{1-\lambda} - (1+\beta+\frac{\beta}{\mu})\right)^2 - \frac{4\beta}{\mu}}}{2} \tag{23}$$

While θ determines the dynamic paths of prices and output, the deviation from its benchmark value now rests on the value of ψ_κ. Only when $\psi_\kappa = 0$, one retrieves the original value of θ given by the benchmark model. In turn, this occurs when $\kappa = \infty$, or when price setters have unlimited foresight and can formulate rational forecasts over an infinite time horizon. Expectedly, the dynamic properties of the model are now determined by the values of both λ and κ, such that θ is only defined when:

$$\left(\frac{-\psi_\kappa \lambda^2}{1-\lambda} - (1+\beta+\frac{\beta}{\mu})\right)^2 - \frac{4\beta}{\mu} \geq 0 \tag{24}$$

With some tedious algebra, which I once again leave to the appendix, the above inequality can be equivalently expressed as an inequality held between two functions $f(.)$ and $g(.)$ where

$$f(\kappa, \lambda) \geq g(\kappa, \lambda) \tag{25}$$

such that

$$f(\kappa, \lambda) = (\psi_\kappa + \frac{\alpha\lambda}{1-\lambda})^2 = \left(\frac{\lambda(1-\lambda)^{\kappa+1}(1+\pi^T)^{\kappa+2}}{\lambda - \pi^T(1-\lambda)} + \frac{\alpha\lambda}{1-\lambda}\right)^2 \tag{26}$$

$$g(\kappa, \lambda) = \frac{4}{1-\lambda}(\psi_\kappa - \alpha) = \frac{4}{1-\lambda}\left(\frac{\lambda(1-\lambda)^{\kappa+1}(1+\pi^T)^{\kappa+2}}{\lambda - \pi^T(1-\lambda)} - \alpha\right) \tag{27}$$

This relation subsequently imposes a restriction on the possible values that κ and λ can simultaneously take in the sticky-price model. As the paper would illustrate in section 3.4 below, this restriction limits the realism of the sticky-price model in the context of bounded rationality, as there are only a certain range of λ that defines κ, which subsequently implies that there is only a restricted range of real solutions for the path of prices.

3.2 Solving for a dynamic path under the κ-augmented sticky-information Phillips Curve

Similar to the sticky-price model, the sticky-information model also imposes constraints on parameter values of κ. However, this constraint is no longer between γ and κ but instead between t and κ. This is due to prices being set by two different groups of price setters in the model:

$$P_t = \gamma \sum_{j=0}^{t} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] + \gamma \sum_{j=t+1}^{\infty} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] \tag{28}$$

Price setters who are aware of the demand shock that occurs at $t = 0$ strictly form expectatons of the new path of prices ex-post beginning from $t = 0$, while those who are not aware of the demand shock have ex-ante expectations that are formed strictly before $t = 0$. It is straightforward to see that the upper bound of ∞ the second term is changed to κ under bounded rationality and the above is written with a heuristic as follows:

$$\begin{aligned} P_t = \gamma \sum_{j=0}^{t} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] \\ + \gamma \sum_{j=t+1}^{\kappa} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] + \gamma \sum_{j=\kappa+1}^{\infty} \varphi^j P_{t-j} \end{aligned} \tag{29}$$

where $\varphi = (1-\gamma)(1+\pi^T)$. Notice that price setters who hold ex-ante expectations are now constrained by their bounded rationality and no longer have past expectations that are formed since the beginning time. This imposes a restriction $t \leq \kappa - 1$. Notice further that κ does not feature in the first summation operator. This implicitly assumes that the constraint $t \leq \kappa - 1$ being satisfied, such that price setters who form ex-ante expectations do so within their cognitive bounds. What happens if $t > \kappa$? The above equation must then be written in the following manner instead:

$$P_t = \gamma \sum_{j=0}^{\kappa} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + +\alpha M_t] + \gamma \sum_{j=\kappa+1}^{\infty} \varphi^j P_{t-j} \tag{30}$$

Observe that all expectation terms are ex-post to the shock at $t = 0$, with the furthest possible expectation formed in the past given by $E_{t-\kappa}P_t$. These are all expectations formed by price setters who are *aware* of the new path of aggregate demand. Price setters who had ex-ante expectations can no longer form expectations as the time period t is now too far away from $t = 0$. As a result, they set prices based on the heuristic expressed by the second term. Leaving the full derivations to the appendix, the impulses responses are thus summarized by the following:

$$P_t = \begin{cases} \dfrac{-log(0.9)\left((1-\gamma)^{t+1} - (1-\gamma)^{\kappa+1}\right) + \gamma \displaystyle\sum_{j=\kappa+1}^{\infty} \varphi^j P_{t-j}}{1-(1-\alpha)\left(1-(1-\gamma)^{t+1}\right)} & \text{for } t \leq \kappa - 1 \\ \zeta P_{t-\kappa-1} + \varphi P_{t-1} & \text{if } t > \kappa \end{cases}$$

where $\zeta = \frac{\gamma}{\omega}\varphi^{\kappa+1}$ and $\omega = 1-(1-\alpha)[1-(1-\gamma)^{\kappa+1}]$ are constants for a given κ. This provides the path for prices under policy experiment 1. Notice that for $t > \kappa$, prices today are some weighted average of *yesterday's* prices and prices $\kappa - 1$ *periods ago*. A large κ subsequently gives less Iight to past prices (as an anchor) $\kappa - 1$ periods ago. This is consistent with price setters approaching the perfect benchmark, resulting in past prices becoming more redundant in serving its role as an anchor. The dynamic paths of prices for experiment 2 and 3 are solved in a similar manner in subsection 4.5 of the appendix.

3.3 Theoretical results in the presence of macroeconomic shocks when κ deviates from ∞

Having solved for the dynamic path of prices for both the κ-augmented sticky-price and sticky-information model, the paper now turns to illustrating the business cycle in response to macroeconomic shocks and how it evolves throughout the the entire spectrum as κ is allowed to deviate from ∞. Three interesting results immediately emerge when these impulse responses are compared to the benchmark[11]:

Result 1: *When price setters are only capable of forming rational expectations ahead for a limited planning horizon, inflation demonstrates a larger degree of persistence and takes longer to return to equilibrium after an exogenous shock.* Throughout this paper, inflation persistence will be defined as *the impulse response having a gentler gradient on its path of return*[12]. Here, inflation persistence is endogenous to the price adjustment rule rather than the Phillips curve[13]. This is hardly a surprising result, especially given that the backward-looking nature of price adjustment is now provided by the heuristic bias. Note, however, that a deviation from full rational benchmark here is still unable to generate a hump-shaped response of inflation under the sticky-price model. This is also unsurprising. Consistent with Woodford (2003), this is due to agents being able to acquire full forward-looking behaviour for *as long a horizon as they can* even under the proposed rule of price adjustment motivated by this paper.

Result 2: *The path of inflation is no longer smooth under the sticky-information model.* Rather, in the presence of pricing bias, inflation *may* jump discontinuously after some periods of the shock before oscillating back to equilibrium.

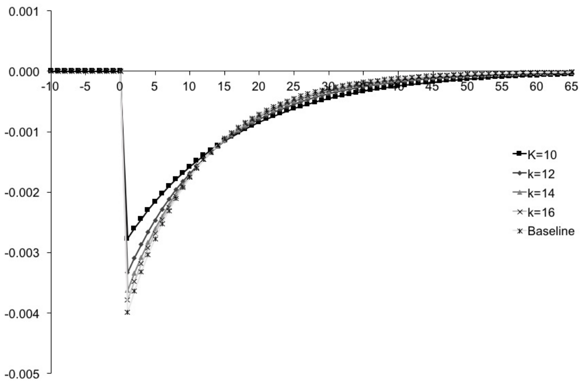
Result 3: *Under the case when $\kappa \leq 10$, announced disinflation is equally capable of causing a boom, contradicting the results given by Mankiw and Reis (2002) and how central banks conduct monetary policy in reality.* All impulse responses of inflation and output are given as follows:

[11]That is, when $\kappa = \infty$.

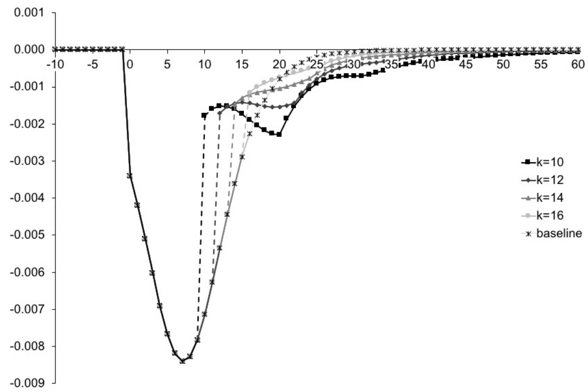
[12]This definition makes it easy to identify persistence just from a simple inspection of the impulse responses themselves.

[13]Fuhrer and Moore (1995) models inflation persistence with $\pi_t = \frac{1}{2}\pi_{t-1} + \frac{1}{2}E_t\pi_{t+1} + cY_t$ where the backward-lookingness is modelled explicitly.

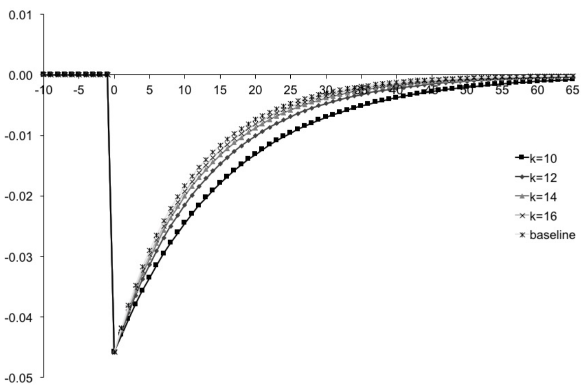
3.3.1 Macroeconomic shock #1 when κ deviates from ∞: an unexpected fall in aggregate demand



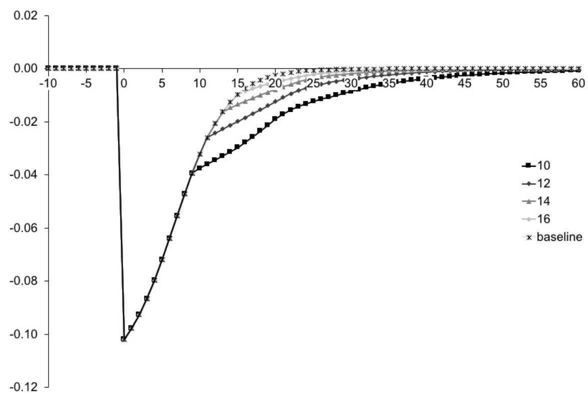
Inflation under sticky-prices



Inflation under sticky-information

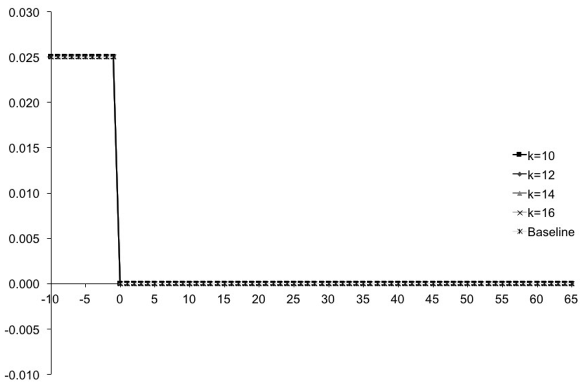


Output under sticky-prices

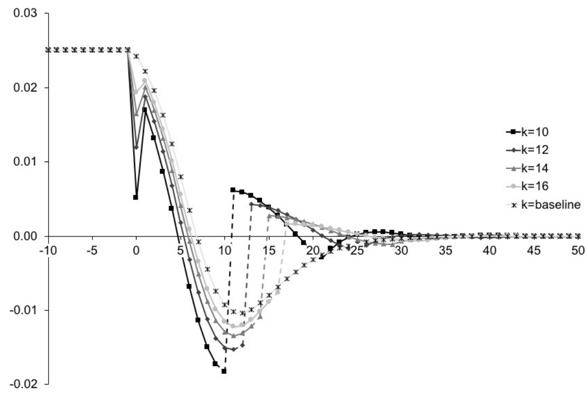


Output under sticky-information

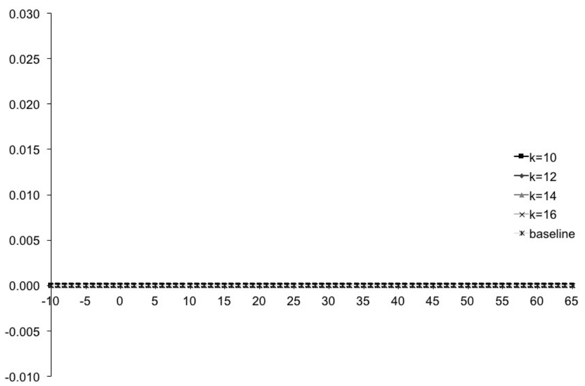
3.3.2 Macroeconomic shock #2 when κ deviates from ∞: an unexpected fall in the rate of money growth from 2.5% to 0% per period at period 0



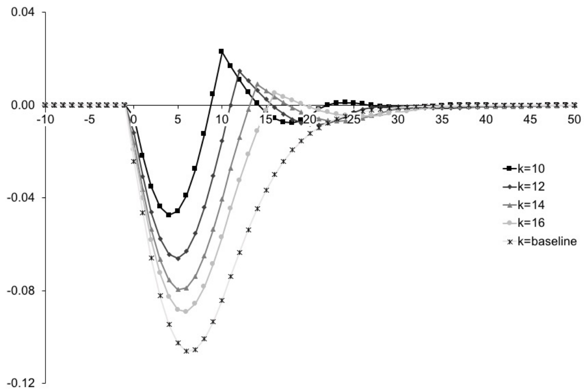
Inflation under sticky-prices



Inflation under sticky-information

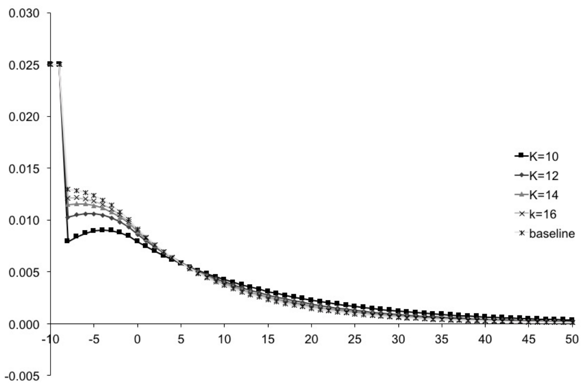


Output under sticky-prices

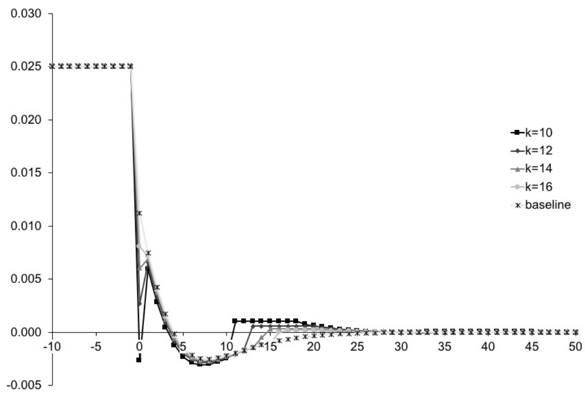


Output under sticky-information

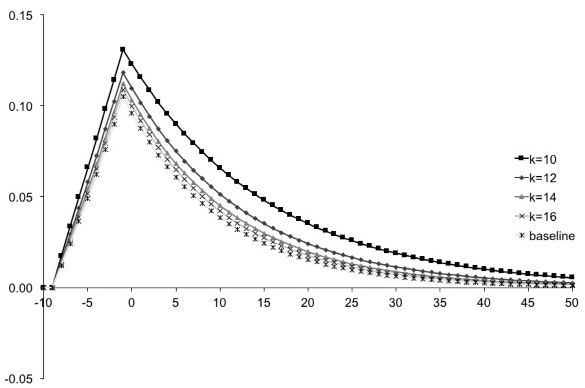
3.3.3 Macroeconomic shock #3 when κ deviates from ∞: an announced disinflation at period $t = -8$ of the same magnitude as (2)



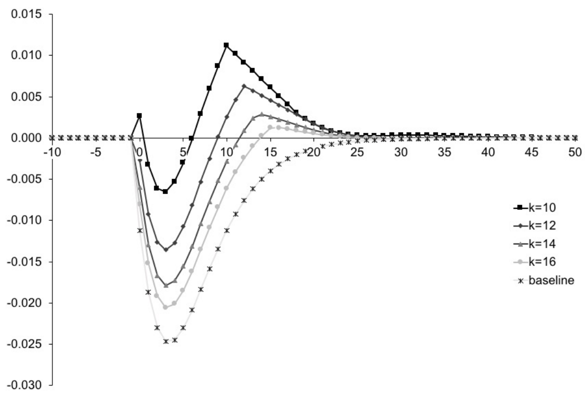
Inflation under sticky-prices



Inflation under sticky-information



Output under sticky-prices



Output under sticky-information

3.4 Theoretical results in the presence of macroeconomic shocks when κ deviates from 0

Having illustrated the behaviour of inflation and output when κ was initially infinity and when κ deviates from this full-rational expectations benchmark, this paper now takes a position to study their behaviour from the other end of the spectrum: that is, when κ is first 0 and when κ starts deviating from zero. At this end of the extreme, $\kappa = 0$ corresponds to the hypothetical case when price setters cannot form rational expectations at all, and becomes thoroughly backward-looking conditioned on the inflation target. Yet, as this paper will show, both models of inflation expectations augmented with κ does not yield entirely oscillatory dynamics[14] as one would expect from the purely backward-looking model. What is unique about κ approaching the limit of 0 is that the impulse responses under both sticky-prices and sticky-information have to be modified further.

Under the κ-augmented sticky-price model, $\kappa = 0$ imposes strict bounds on the values of λ. This in turn determines the value of θ that pins down the dynamic path of prices. This is a result of having to satisfy the following set constraints first introduced in secion 3.1:

$$f(\kappa,\lambda) \geq g(\kappa,\lambda) \tag{31}$$

$$\frac{\theta\mu}{\beta} < 1 \tag{32}$$

such that

$$f(\kappa,\lambda) = (\psi_\kappa + \frac{\alpha\lambda}{1-\lambda})^2 = \left(\frac{\lambda(1-\lambda)^{\kappa+1}(1+\pi^T)^{\kappa+2}}{\lambda - \pi^T(1-\lambda)} + \frac{\alpha\lambda}{1-\lambda}\right)^2 \tag{33}$$

$$g(\kappa,\lambda) = \frac{4}{1-\lambda}(\psi_\kappa - \alpha) = \frac{4}{1-\lambda}\left(\frac{\lambda(1-\lambda)^{\kappa+1}(1+\pi^T)^{\kappa+2}}{\lambda - \pi^T(1-\lambda)} - \alpha\right) \tag{34}$$

While both equations (31) and (32) pin down the dynamic properties of the κ-augmented sticky-price model, they also introduce a theoretical challenge to the model by limiting the values that both κ and λ can simultaneously take. As the value of κ decreases from ∞ and approaches 0, the range of values that λ can take narrows to [0.88, 0.99]. The sticky-price model then implies that a dynamic path of prices will now only exist for a very high frequency of price change, which could well be inconsistent with empirical data. Table 1 below illustrates the relationship between κ, λ and the subsequent solution for θ. It thus becomes apparent that it is no longer as straightforward to compare the impulse responses for $\kappa = 0$ and for small deviations away from this lower bound, as a chosen value of $\lambda \notin [0.88, 0.99]$ for deviations of κ above 0 will not yield a solution for the path of prices. In order to subsequently provide a meaningful benchmark for

[14] Mankiw and Reis (2002) also illustrates the impulse responses of a model of adaptive expectations in the form of $\pi_t = (\frac{\alpha^2\lambda}{1-\lambda})Y_t + \pi_{t-1}$ to the shocks elaborated in this paper.

Table 1: range of possible λ narrows as κ approaches 0

κ	λ	θ
8	[0.23, 0.99]	[0.9911, 0.0854]
6	[0.28, 0.99]	[1.0009, 0.0854]
4	[0.37, 0.99]	[0.9973, 0.0854]
2	[0.53, 0.99]	[1.0256, 0.0854]
0	[0.88, 0.99]	[1.2663, 0.1710]

the dynamics of inflation and output as κ deviates slightly from 0, this paper will first provide a complete characterisation of these dynamics as λ goes from one end of the spectrum to another. The purpose of this is to allow for an inference of behaviour beyond an arbitrarily chosen value of λ used for subsequent analyses as κ deviates from 0. Interestingly, these results reveal three highly contrasting paths of inflation when κ is kept at 0, but λ is allowed to vary slightly from the lower bound of 0.88.

Path 1: *At the extreme lower bound with $\kappa = 0$, inflation and output exhibits explosive dynamics and essentially becomes a bubble.* Recall that the dynamic path of prices under the sticky-price model is pinned down by the following equation:

$$P_t = \theta P_{t-1} + (\frac{\alpha\lambda^2}{1-\lambda(1-\psi_\kappa)})\theta \sum_{i=0}^{\infty} \big((\frac{1-\lambda}{1-\lambda(1-\psi_\kappa)})\theta\big)^i E_t(M_{t+i}) \tag{35}$$

where a real solution for θ is obtained from solving the quadratic from the following expectational difference equation:

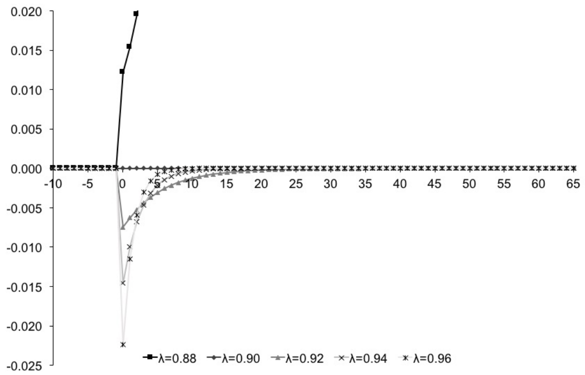
$$\left(F^2 + \left(\frac{\psi_\kappa\lambda^2}{1-\lambda} - (1+\beta+\frac{\beta}{\mu})\right)F + \frac{\beta}{\mu}\right)LP_t^* = -\beta M_t^* \tag{36}$$

When κ is uniquely 0 and λ is 0.88, the equations above yield a real solution for θ being larger than 1. From equation (35), this results in prices today becoming ever higher than yesterday's and thus inflation ends up becoming a bubble.

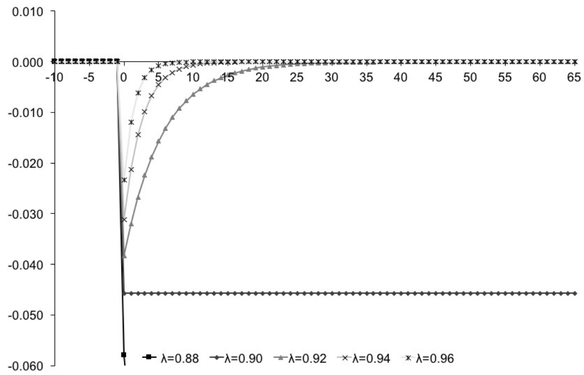
Path 2: *Raising λ slightly to 0.90 with $\kappa = 0$ results in a static path of inflation.* This is a result of θ being exactly 1. As a result of prices not changing after the intial shock, inflation initially falls and returns to zero immediately at period $t = 0$.

Path 3: *The paths of inflation retrieve their benchmark characteristics as λ increases even higher.* This is a result of $\theta < 1$. The impulse responses of both inflation and output are summarized by figures (13) - (18) below:

3.4.1 Macroeconomic shock #1 when κ is 0: an unexpected fall in aggregate demand

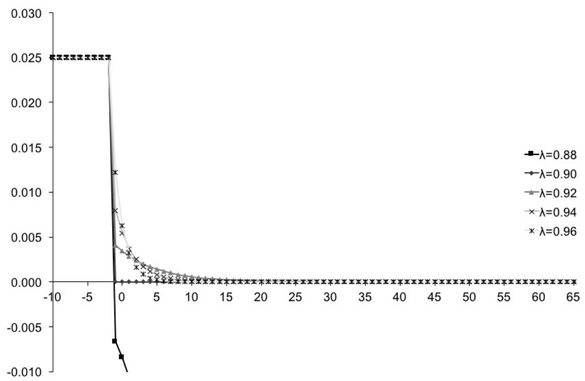


Inflation under sticky-prices

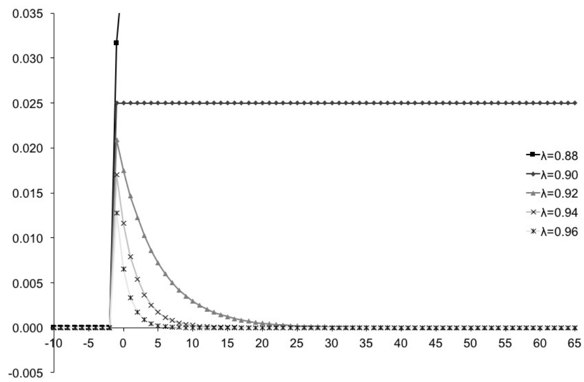


Output under sticky-prices

3.4.2 Macroeconomic shock #2 when κ is 0: an unexpected fall in the rate of money growth from 2.5% to 0% per period at period 0

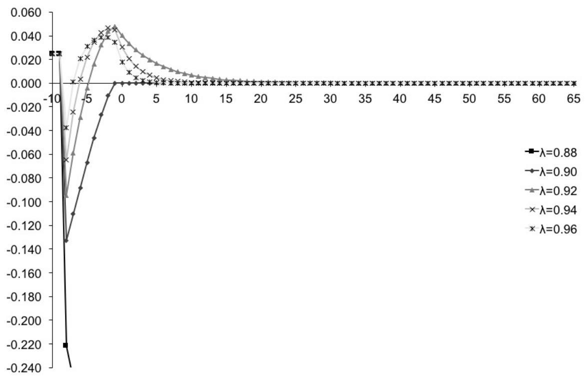


Inflation under sticky-prices

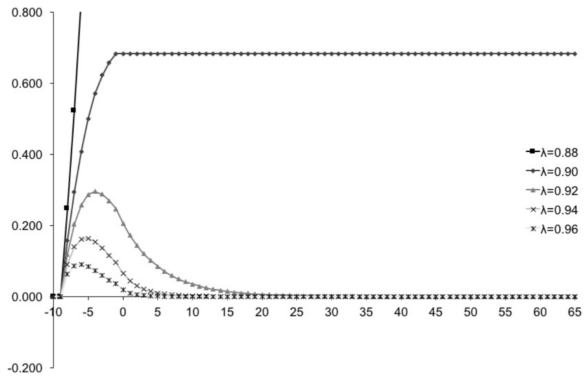


Output under sticky-prices

3.4.3 Macroeconomic shock #3 when κ is 0: an announced disinflation at period $t = -8$ of the same magnitude as (2)



Inflation under sticky-prices



Output under sticky-prices

The impulse responses under sticky-information differs from those under sticky-prices in that no parameter values are restricted in any way by the value of κ. However, the impulse responses become completely different when $\kappa = 0$. Overall prices under sticky-information are instead pinned down by the following equation[15]:

$$P_t = \gamma[(1-\alpha)P_t + \alpha M_t] + \gamma \sum_{j=1}^{\infty} \varphi^j P_{t-j} \tag{37}$$

Observe that prices are now independent from expectations. This holds true by definition and is implied by having $\kappa = 0$. The dynamic path of prices under all three macroeconomic shocks is then given by:

$$P_t = \frac{\alpha\gamma}{1-\gamma(1-\alpha)} M_t + \frac{\gamma}{1-\gamma(1-\alpha)} \sum_{j=1}^{\infty} \varphi^j P_{t-j} \tag{38}$$

Rejecting the equilibrium where inflation is either a bubble or follows some static path, the following impulse responses study the behaviour of inflation and output when κ is allowed to deviate marginally from the extreme end of 0. The parameter values of λ and γ are set to be 0.54. Two particular conclusions stand out:

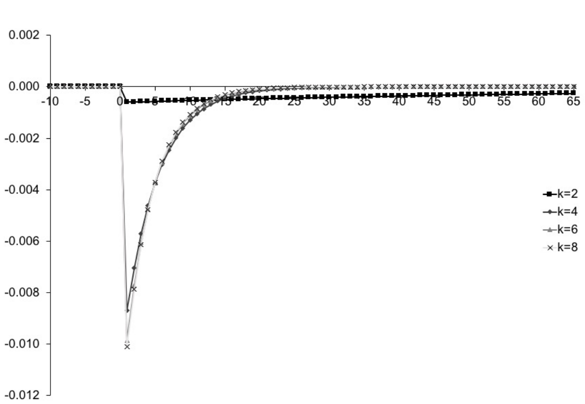
Conclusion 1: *In the presence of recessions, inflation tends to be more resilient but also takes much longer to return to equilibrium after the shock.* Combined with result 1, this implies that as κ varies from ∞ to 0, the response of inflation varies from being most sensitive to being the

[15]Equation (37) is obtained from setting $t = 0$ in the equation $P_t = \gamma \sum_{j=0}^{t} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] + \gamma \sum_{j=t+1}^{\infty} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t]$.

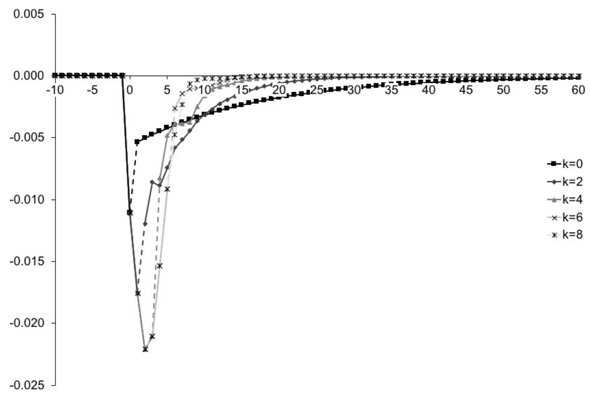
least sensitive. The persistence of inflation, however, increases correspondingly.[16]

Conclusion 2: *Towards the extreme near $\kappa = 0$, the characteristic humped-shaped response of inflation under the sticky-information model disappears and instead starts to resemble that under sticky-prices.* This is not too surprising a result, as the hallmark of the sticky-information lies in how the timing of inflation expectations differs from that under sticky-prices. When κ approaches 0, this difference becomes negligible.

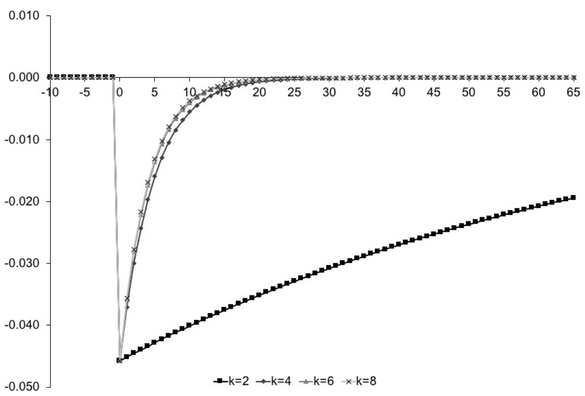
3.4.4 Macroeconomic shock #1 when κ deviates from 0: an unexpected fall in aggregate demand: Here, $\lambda = \gamma = 0.54$



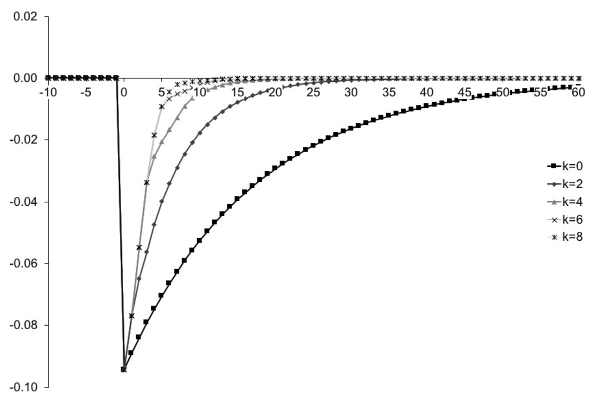
Inflation under sticky-prices



Inflation under sticky-information



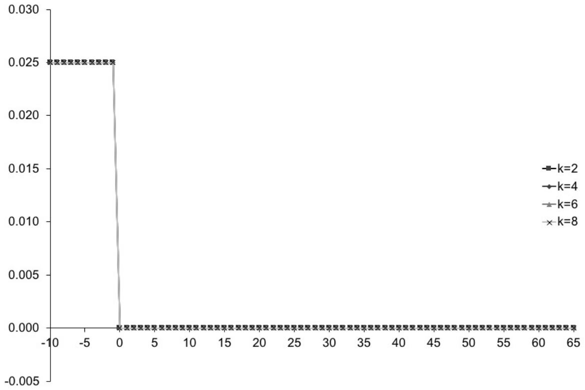
Output under sticky-prices



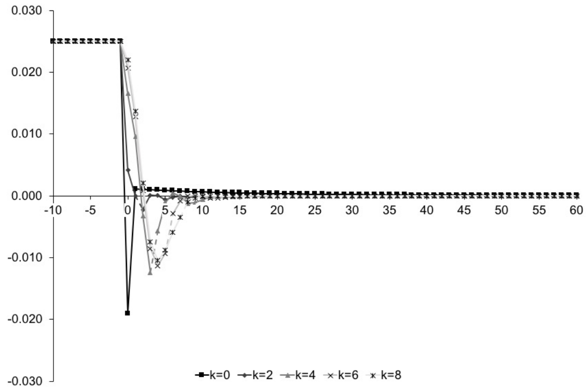
Output under sticky-information

[16] Observe in this simulation that both inflation and output returns to equilibrium relatively faster than the previous string of simulations where κ decreases from ∞. This a result of increasing the value of λ and γ to 0.54 from 0.25.

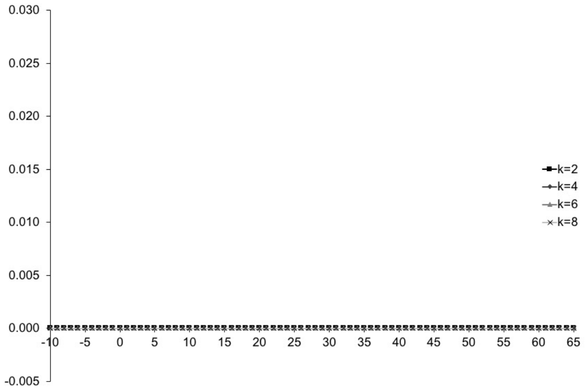
3.4.5 Macroeconomic shock #2 when κ deviates from 0: an unexpected fall in the rate of money growth from 2.5% to 0% per period at period 0: Here, $\lambda = \gamma = 0.54$



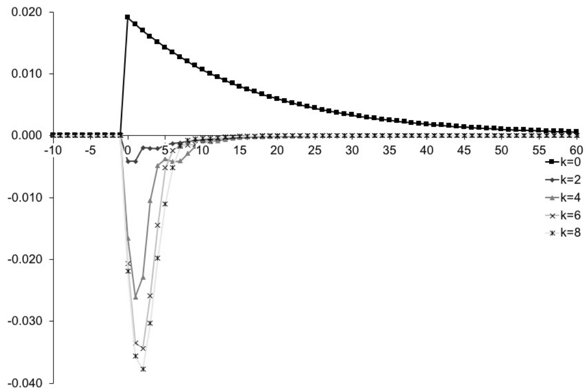
Inflation under sticky-prices



Inflation under sticky-information

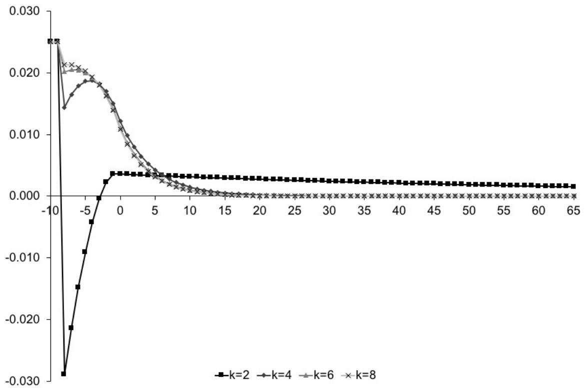


Output under sticky-prices

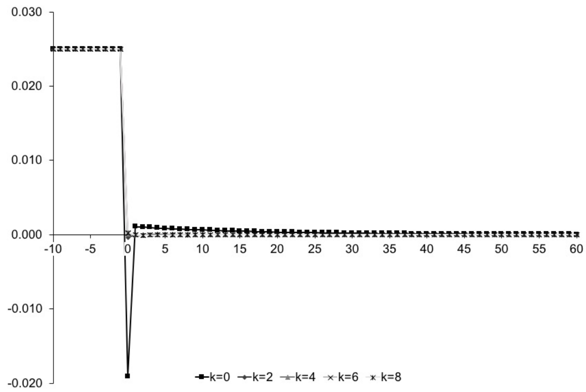


Output under sticky-information

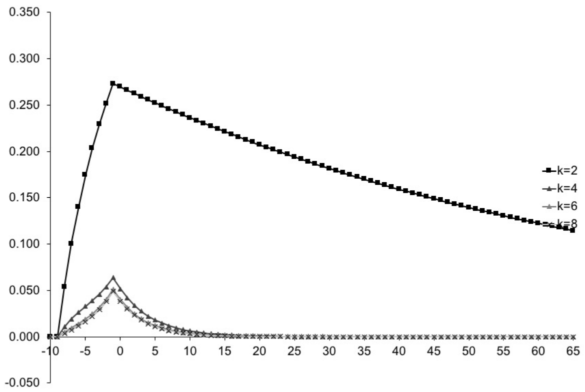
3.4.6 Macroeconomic shock #3 when κ deviates from 0: an announced disinflation at period $t = -8$ of the same magnitude as (2): Here, $\lambda = \gamma = 0.54$



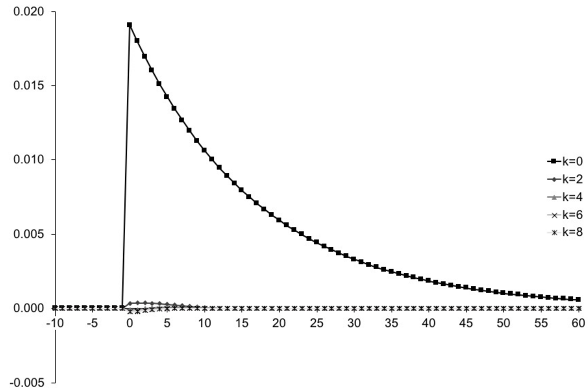
Inflation under sticky-prices



Inflation under sticky-information



Output under sticky-prices



Output under sticky-information

3.5 The role of Central Bank's inflation target in dynamic price adjustment in the presence of monetary policy shocks

Up to this juncture, the focus of this paper has been on κ and how it affects the dynamics of both inflation and output, in terms of both duration and nature of their response to shocks. While much could already be learned about the role of κ from the impulse responses, one might say less with regards to the role of the inflation target in the rule of price adjustment. This is not surprising, as it was set to be 0% in all 3 shocks. Only by exploring the behaviour of impulse responses in the presence of shocks in monetary policy may one be able to motivate the role of the inflation target further. This is because there are now no equations that set the inflation target to 0.0% on an *a priori* basis. Hence, we allow for an additional degree of freedom for the inflation target to take whatever value it has to achieve the goal of policy stabilisation. **Unlike the previous sections, the inflation target is no longer zero.**

In order to illustrate this, I explore the behaviour of inflation and output given a one standard deviation shock in monetary policy as outlined by Mankiw and Reis (2002). While this paper follows the same set-up in order to eventually compare the results with a benchmark, its contribution lies in solving for a different expression of the impulse response, especially since the both models of the Phillips Curve now differ from those of the benchmark and are now functions of the inflation target. The solutions for the impulse responses will then differ according to the extent of bias. I begin by writing the growth of money supply M_t as an AR(1) process as follows:

$$\Delta M_t = \rho \Delta M_{t-1} + \epsilon_t \tag{39}$$

where ϵ_t is a white-noise innovation. Under this model, the level of money supply is non-stationary but the growth rate of money supply is stationary for all $|\rho| < 1$. This essentially requires the absence of an unit root. Given this conjecture, inflation must then follow a stationary process as well. The AR(1) process for inflation and prices can equivalently be written as a

MA(∞) process as follows:

$$\pi_t = \sum_{j=0}^{\infty} \rho^j \epsilon_{t-j} \tag{40}$$

$$P_t = \sum_{\tau=0}^{\infty} \sum_{j=0}^{\infty} \phi_j \epsilon_{t-j-\tau} \tag{41}$$

In order to solve for the impulse responses $\{\phi_\upsilon\}$ of inflation given by the sticky-price model, I substitute the above MA(∞) processes into the dynamic path for prices given by equation (19) solved in section 3.1. This yields the following stochastic equation for inflation π_t:

$$\sum_{\tau=0}^{\infty} \sum_{j=0}^{\infty} \phi_j \epsilon_{t-j-\tau} = \theta \sum_{\tau=0}^{\infty} \sum_{j=0}^{\infty} \phi_j \epsilon_{t-1-j-\tau} + \mu\theta \sum_{i=0}^{\infty} \left(\left(\frac{\mu}{\beta}\right)\theta\right)^i \sum_{j=0}^{\infty} \sum_{\tau=max\{i-j,0\}}^{\infty} \rho^j \epsilon_{t+i-j-\tau} \tag{42}$$

as $E_t\{\epsilon_{t+i-j-\tau}\} = \epsilon_{t+i-j-\tau}$ for all $i-j \leq \tau$ and is zero otherwise. Subsequently, ϕ_j are coefficients to be determined. Matching all coefficients of $\epsilon_{t-\upsilon}$ in this stochastic equation subsequently yields the solution for $\{\phi_\upsilon\}$ such that:

$$\phi_\upsilon = (\theta - 1) \sum_{j=0}^{\upsilon-1} \phi_j + \left(\frac{\mu\theta}{1-\rho}\right) \left(\frac{1}{1 - \frac{\mu}{\beta}\theta} - \frac{\rho^{\upsilon+1}}{1 - \frac{\mu}{\beta}\theta\rho} \right) \tag{43}$$

For the sticky-information model, I redefine the AR(1) processes of prices and money such that:

$$M_t = \sum_{\varrho=0}^{\infty} \sum_{i=0}^{\infty} \rho^i \epsilon_{t-i-\varrho} \tag{44}$$

$$\Delta M_t = \sum_{i=0}^{\infty} \rho^i \epsilon_{t-i} \tag{45}$$

$$P_t = \sum_{\varrho=0}^{\infty} \sum_{i=0}^{\infty} \Psi_i \epsilon_{t-i-\varrho} \tag{46}$$

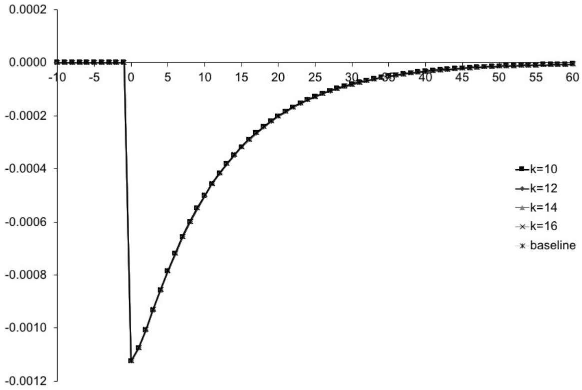
$$\pi_t = \sum_{i=0}^{\infty} \Psi_i \epsilon_{t-i} \tag{47}$$

Substituting the these processes into the κ-augmented sticky-information Phillips Curve outlined in section 2.2 by equation (11), and matching all coefficients of $\epsilon_{t-\upsilon}$ in a similar manner, the full characterization of the stochastic process for inflation $\{\Psi_\upsilon\}$ is given by

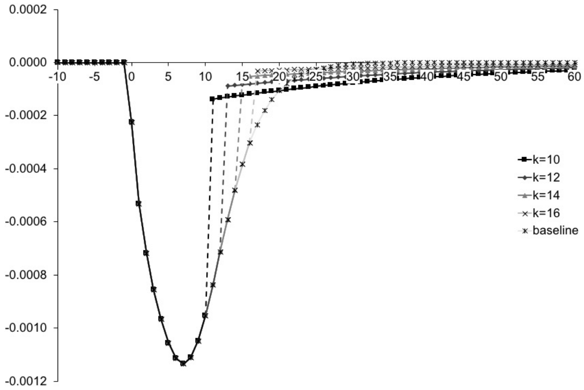
$$\Psi_\upsilon = \frac{\alpha\gamma\left[(1 - \sum_{i=0}^{\upsilon-1} \Psi_i) + \sum_{i=1}^{\upsilon} \rho^i + \rho^\upsilon \sum_{i=1}^{\upsilon} (1-\gamma)^i\right]}{1 - \gamma(1-\alpha) \sum_{i=0}^{\upsilon} (1-\gamma)^i} \quad \text{for} \quad \upsilon \leq \kappa \tag{48}$$

$$\Psi_\upsilon = \frac{\alpha\gamma\left[(1 - \sum_{i=0}^{\upsilon-1} \Psi_i) + \sum_{i=1}^{\upsilon} \rho^i\right] + \Pi}{1 - \gamma(1-\alpha)} \quad \text{for} \quad \upsilon \geq \kappa + 1 \tag{49}$$

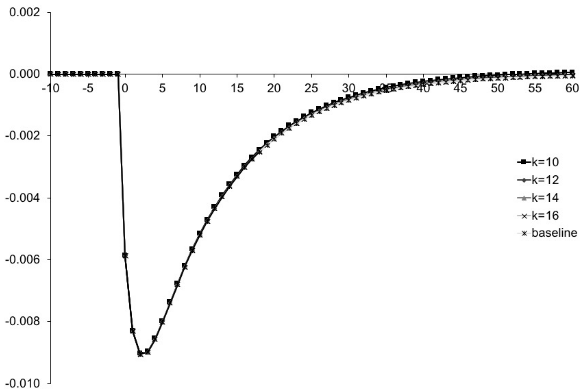
where $\Pi = \gamma \sum_{i=0}^{\upsilon-\kappa-1} \varphi^{\upsilon-i+1}\Psi_i + (1+\pi^T)\gamma^2 \sum_{j=\kappa+1}^{\upsilon} \left(\sum_{i=0}^{\upsilon-j} \Psi_i \right)$ and noting the discontinuity at $\upsilon = \kappa$. For $\lambda = \gamma = 0.25$, the impulse responses for both models of inflation are given as follows:



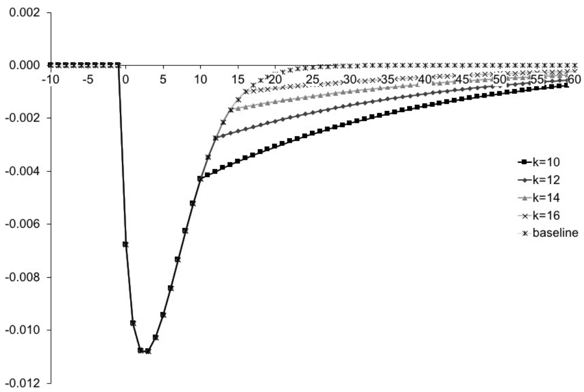
Inflation under sticky-prices



inflation under sticky-information



Output under sticky-prices



Output under sticky-information

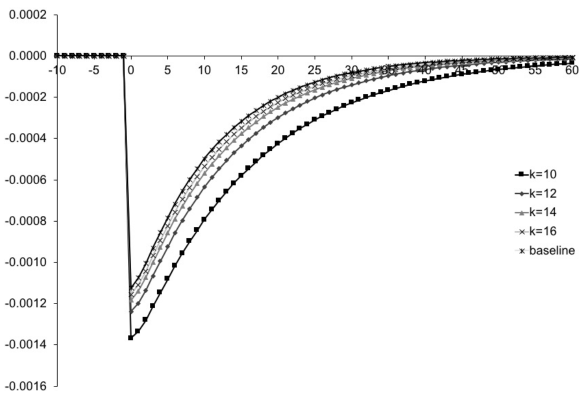
It turns out that this is the set of impulse responses that corresponds to the inflation target π^T being lowered to -25%[17] as a result of the contraction in monetary policy[18]. What happens if the inflation target remains unchanged at 0%? As seen in the alternative set of impulse responses below, output does not return to equilibrium. This is as if there could be monetary non-neutrality even in the long run. This, however, is a violation of Friedman (1968): there cannot be a permanent trade-off between inflation and output! This monetary policy simulation thus reveals two crucial insights behind this paper:

Result 1: *While inflation returns to equilibrium, output does so only if the inflation target is lowered to match the contraction in monetary policy. In fact, the inflation target is now an explicit policy instrument.*

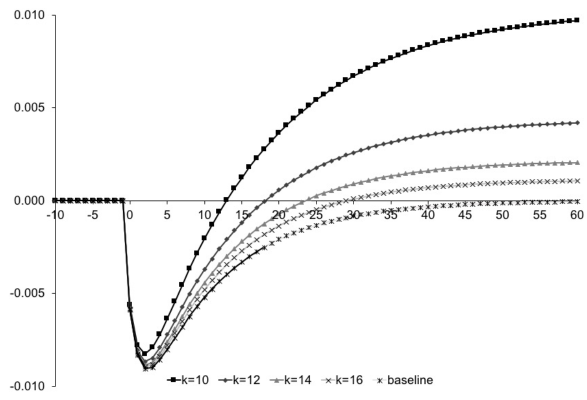
[17]There is no closed form solution for this.

[18]If it were to be an expansion in monetary policy, π^T would be set to be higher at 25% rather than 0.0%.

Intuitively, because all agents in this economy are assumed to be adjusting prices using the inflation target as a heuristic anchor at the limits of their forecast horizon, policy can only be effective if the central bank changes the target in the right direction to guide the price adjustment in that direction as well. In fact, it has a flavour of the *Taylor principle* in the sense that when the frequency of price adjustment is low, the lowering of the inflation target needs to be aggressive for policy to be effective and vice versa[19]. This result potentially builds on two other papers that has been written regarding the conduct of monetary policy. First, Blanchard et. al (2010) proposed raising the inflation target to 4% from 2% in order to avoid the zero lower bound. This suggests that rather than assuming an arbitrary choice of the numerical target for inflation, this choice should be properly discerned and debated. Whilst to a different purpose, this paper equally shows that the specific choice of this numerical target is important to output stabilization, at least under the proposed rule of price adjustment. Second, Reis (2017) examined two episodes where the Fed and the Bank of England *gone long* in the 20^{th} century, and found that targeting the long-term interest rate as a policy instrument for the most part fail to anchor inflation. Yet, what the above result suggests is for the central bank to stabilise output by going long - *in the sense of targeting the long term interest rate indirectly through attaining some target long term inflation rate* - by using the inflation target *as if* it were to be a short-term policy instrument. While a thorough discussion of policy implications is beyond the scope of this paper, it recognizes that the implied consequences might fail in practicality - especially since changing the inflation target as an active instrument may bring back the old time inconsistency problem and put this paper in the cross hairs of the usual 'rules vs discretion' debate.

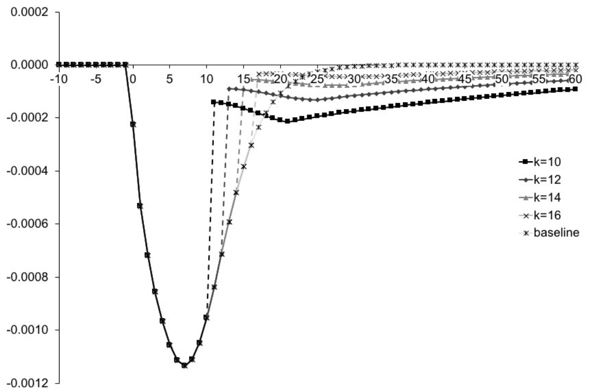


Inflation under sticky-prices

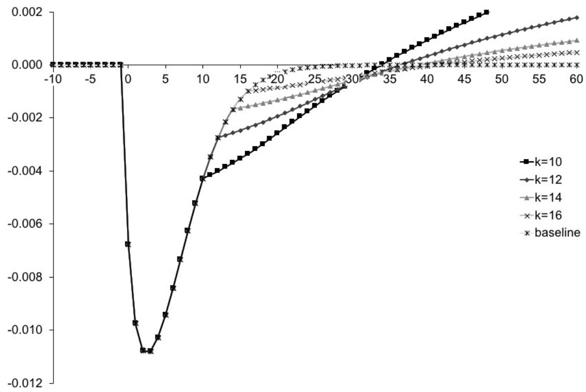


Output under sticky-prices

[19]Refer to sheet 6 of replication file for κ-augmented sticky-information as an example. There, $\kappa = 10$ and γ varies between 0.25 and 0.45. But when γ is 0.45, π^T needs only to be lowered to -10%, as compared to -25% for the case when γ is 0.25.



Inflation under sticky-information



Output under sticky-information

Result 2: *Under the sticky-price model, both the duration and shape of response for inflation and output are the same regardless of the value of κ.*

This implies that whether price setters are capable of forming rational expectations over longer horizons no longer matters when the inflation target is used to anchor expectations. Intuitively, the path of prices converge for all κ because everyone has the same *nominal anchor*; they rely only on the target, rather than making forecasts over the long horizon. Mathematically, the dynamic path of the κ-augmented model converges[20] to that of the canonical model for this inflation target. Under the sticky-information model where information arrives slowly, there is still a role for κ as not all price-setters pay attention to the policy outlook.

4 The dynamic behaviour of inflation and output as we vary parameter values of λ and γ

Having learned much about the role of κ and the inflation target, studying the roles of λ (or γ) remains the only endeavour that is left of this paper in order to develop the proposed theory of price adjustment under bias fully. In the original Calvo model, λ was the parameter that specified the deviation from an economy with fully flexible prices. The higher λ is, the more price setters are able to respond to shocks by re-adjusting prices and one can expect minimal disturbance to output. For the sticky-information model, a higher γ represents that a larger population of price setters are able to gather information, and with this, subsequently acquire rational expectations about some future path of M_t and eventually converge to some Calvo benchmark. Without even illustrating with impulse responses, one can immediately draw a

[20]Check that ψ_κ decreases to 0 when π^T=-0.25. Then, equation (19) reduces to (20).

relationship between these parameters and the behaviour of the business cycle. Yet, what is less clear about λ and γ are the ways in which they may alter certain results that this paper has established thus far with respect to κ. Acknowledging these observations about the short term fluctuations studied throughout this paper, I focus instead on thinking about them as a set questions:

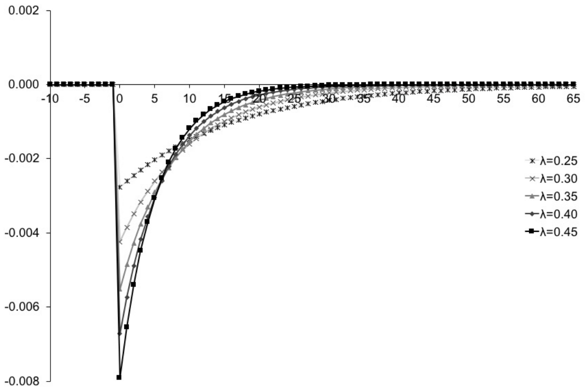
Question 1: *If lowering κ is suggestive of larger inflation persistence, can increasing λ or γ decrease this persistence?*

Question 2: *if lowering κ is suggestive of discontinuous jumps and oscillatory paths of inflation, can increasing γ result some convergence towards a smooth path as one would expect from the benchmark?*

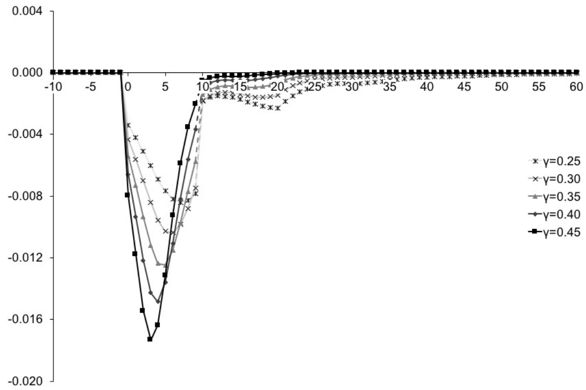
Question 3: *If empirical estimates of the dynamic response of real activity to shocks show a gradual 'hump-shaped' response* (Mankiw and Reis, 2006), *can increasing λ or γ reduce this response in both size and duration that have been augmented by κ?*

As one might expect, the answers to these questions are: *yes*, *yes* and *yes.* If κ is about deviating the model from some full rational expectations benchmark, then λ and γ returns the model to this benchmark despite the fact that the solutions for the impulse responses are still functions of κ. But this is hardly an intellectual victory; in the Calvo economy there is little need to demand for rational expectations if prices are fully flexible, while in the Mankiw-Reis economy the core assumption rests on price setters acquiring full rational expectations *as long as* they gather information. Under sticky-prices, results show that increasing λ mainly affects the convexity and the height of the impulse responses. When price setters are able to change prices more often, there is no need for them to form expectations over a longer time horizon, and so one may be tempted to conclude that the role of expectations and hence, behavioural models, are diminished in an economy where price adjustment is more frequent. Similarly, the role of expectations is reduced in the Mankiw-Reis world when information arrives quickly. Remarkably, the impulse responses now resemble those illustrated previously under the case when κ deviates from ∞ despite the fact that the equation that pins down these responses are all different! This provides a strong case to argue that that the time horizon κ does not matter when information arrives quickly. In the following set of impulse responses, κ is set to 10 while λ (and γ) is allowed to increase:

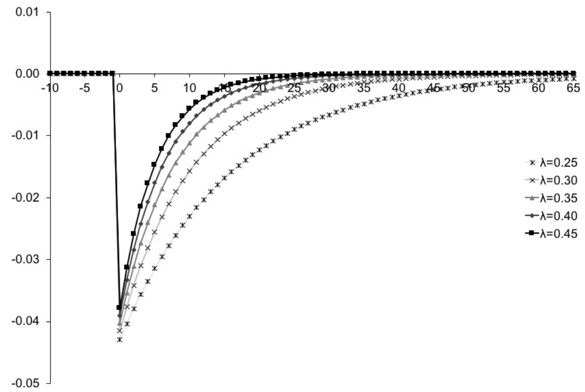
4.0.1 Macroeconomic shock #1 when λ and γ deviates from 0.25: an unexpected fall in aggregate demand: Here, $\kappa = 10$



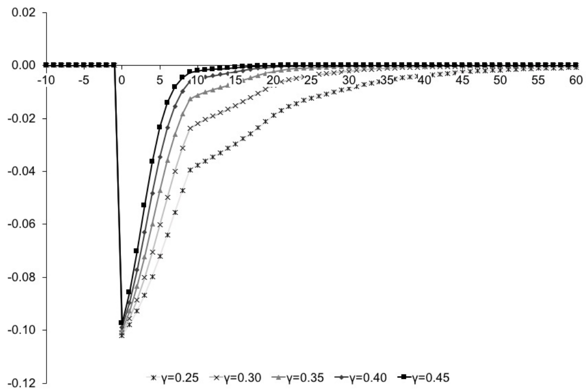
INFLATION UNDER STICKY-PRICES



INFLATION UNDER STICKY-INFORMATION

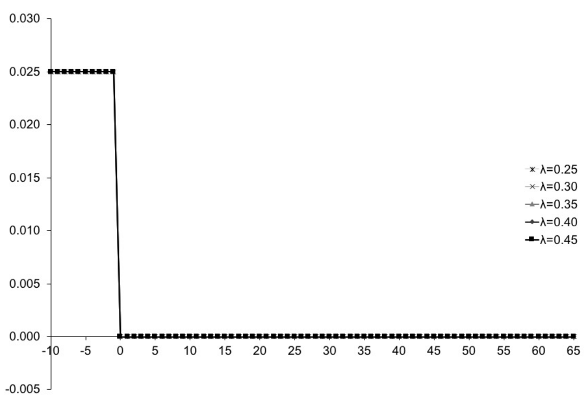


OUTPUT UNDER STICKY-PRICES

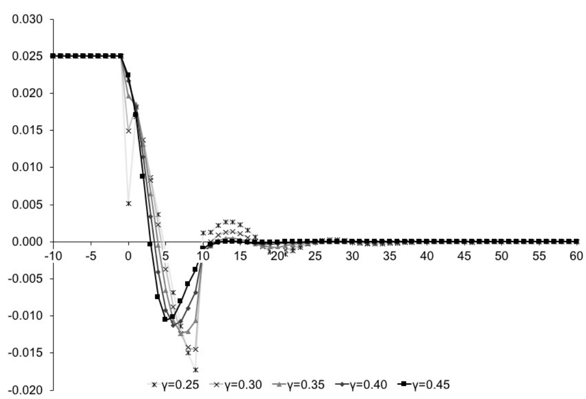


OUTPUT UNDER STICKY-INFORMATION

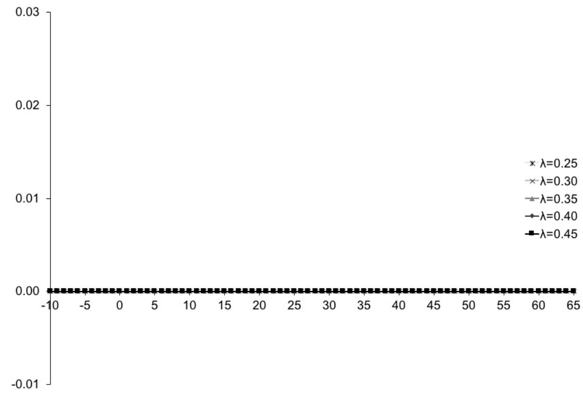
4.0.2 Macroeconomic shock #2 when λ and γ deviates from 0.25: an unexpected fall in the rate of money growth from 2.5% to 0% per period at period 0: Here, $\kappa = 10$

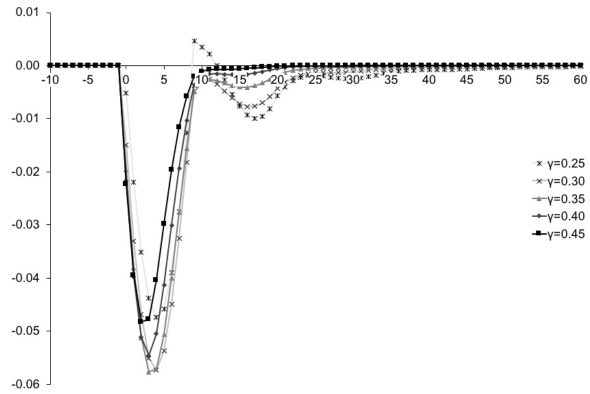


INFLATION UNDER STICKY-PRICES



INFLATION UNDER STICKY-INFORMATION

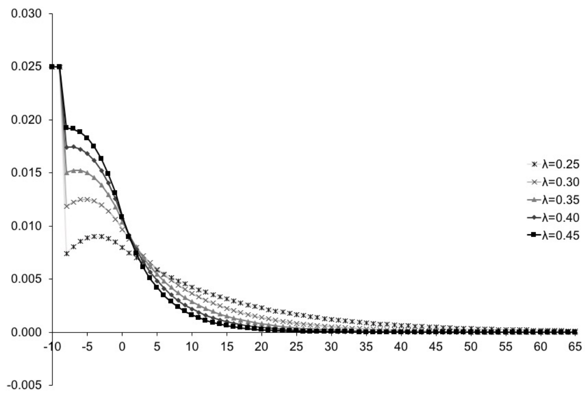


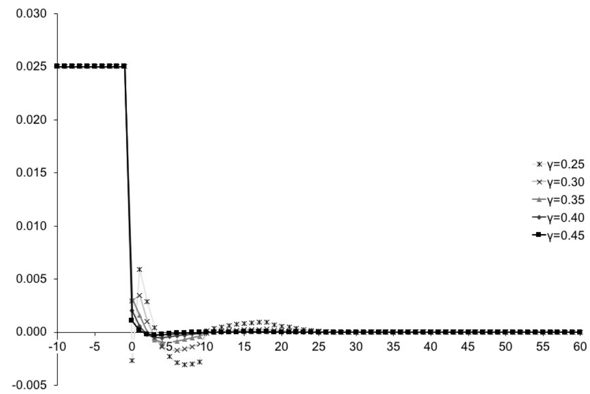


Output under sticky-prices

Output under sticky-information

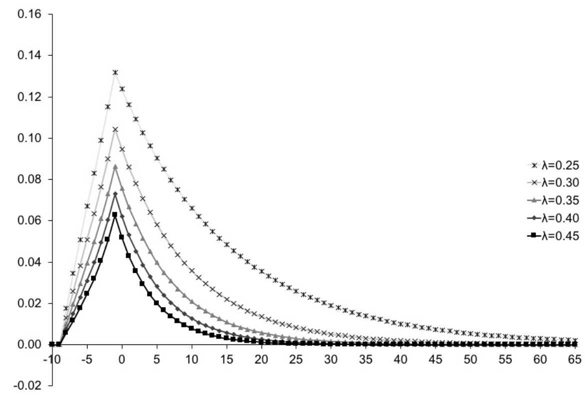
4.0.3 Macroeconomic shock #3 when λ and γ deviates from 0.25: an announced disinflation at period $t = -8$ of the same magnitude as (2): Here, $\kappa = 10$

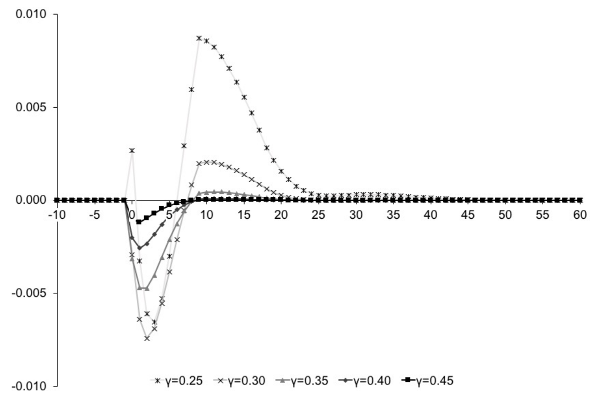




Inflation under sticky-prices

Inflation under sticky-information





Output under sticky-prices

Output under sticky-information

5 Conclusion and future work

In days since the 2008 financial crisis, macroeconomics has been criticized by many for drawing inference based on 'overly simplified' models that rest overwhelmingly on the assumption of rational expectations. This is not quite the case, however, when one examines the amount of active literature prior to the crisis which sought to relax this overly demanding assumption. Perhaps what policymakers need is a behavioural model that is robust enough to accommodate the inconsistencies of human behaviour, and yet still yield results that can be solved using conventional technologies[21] that can be used in a conventional policy setting.

Recognising this priority, this paper extents on the work by Mankiw and Reis (2002) and examines a rule of price adjustment where price setters have *realistic* time horizons over which they can form rational expectations, and, in light of the recent attention both given to, and drawn by central banks through their conduct of forward guidance and inflation targeting, assumes that price setters can equally make use of some past, observable prices and the inflation target as a nominal anchor when their planning horizons are so far ahead that they can no longer justify a well-reasoned expectation of their own. In essence, this is a form of bounded rationality a step further from the theory of sticky-information, which, in turn, is a step away from the new Keynesian, full rational expectations Calvo benchmark. The analysis in this paper shows that it is possible to re-model the Phillips Curve, and present a framework that can illustrate the behaviour of short-run business cycles throughout the *entire spectrum of rationality.* Subsequently, this paper presented several learning points from applying the framework to study the impulse responses of inflation and output to shocks of various natures. *Firstly*, how price setters form inflation expectations and whether these expectations are accurate or heterogeneous do not matter when they are able to gather information or change prices more frequently. *Secondly*, because it is often difficult to figure out the true implications of monetary policy - whether it is due to data uncertainty through lags or the numerous forms of technocratic language that one would expect from central bank communication - policymakers might expect private agents to similarly adopt a nominal anchor for their own expectations, and, as this paper shows, the choice of even this numerical value for such an anchor - such as the inflation target - could prove to be pivotal to output stabilization. *Lastly*, larger extents of bounded rationality increases the persistence of inflation, and, under sticky-information, even allows for discontinuous jumps and oscillatory dynamics of inflation and real output.

With this, can one better understand the fluctuations of the business cycle through the lens of

[21] Such as the method of undetermined coefficients, or minimizing some loss function of the central bank.

bounded rationality? Can behavioural economics inform monetary policy in anyway? This paper believes it can. This opens the door for future work to solve for a general equilibrium with some utility-maximizing, monopolistic price setter, where his behavioural heuristic may be modeled in a continuous, rather than a discrete process. Central banks could further be concerned with the endogeneity of κ: that is, whether their increased actions - both in communication and in the variety of instruments - aid or hinder the formation of expectations. This is pertinent to the work on behavioural models especially since the jury is still out on whether forward guidance is about making predictions of the state of the economy or about the policy instrument itself[22] (Reis, 2018). As this paper has shown, a varying κ ultimately leads to varying degrees of persistence, duration and size of response for both inflation and output. What is the optimal policy response to each case is subject to calibration, and likely will be the form of future research in this area. To this end, incorporating stylized results of behavioural economics into models of how people understand and act on monetary policy will likely pave the way for macroeconomics to progress even further.[23]

[22]Suppose the central bank sets policy according to $i_t = f(s_t) + \epsilon_t$, where ϵ_t is the term that captures policy surprises, s_t is the conjectured state of the economy and $f(.)$ is the policy instrument. Then, is forward guidance about explaining what $f(.)$ is, or is it about communicating predictions for s_t?

[23]This concludes. According to ShareLateX, total word count excluding the appendix is 9952.

6 Appendix: Proofs of solutions presented in the text

6.1 Derivation of the κ-augmented sticky-price new Keynesian Phillips Curve

To derive the modified sticky-price Phillips Curve, start with the following rule of price adjustmented motivated in section II:

$$X_t = \lambda \sum_{j=0}^{\kappa} (1-\lambda)^j E_t(P^*_{t+j}) + \lambda \sum_{j=\kappa+1}^{\infty} (1-\lambda)^j P_{t-1}(1+\pi^T)^{j+1} \tag{50}$$

Taking out the first term and redefining the summation operator, equation (50) can be written as:

$$X_t = \lambda P^*_t + (1-\lambda)\lambda \sum_{j=0}^{\kappa} (1-\lambda)^j E_t(P^*_{t+j+1}) + \lambda \sum_{j=\kappa+1}^{\infty} (1-\lambda)^j P_{t-1}(1+\pi^T)^{j+1} \tag{51}$$

Equivalently, equation (50) can be analogously defined as:

$$X_{t+1} = \lambda \sum_{j=0}^{k} (1-\lambda)^j E_{t+1}(P^*_{t+j+1}) + \lambda \sum_{j=\kappa+1}^{\infty} (1-\lambda)^j P_t(1+\pi^T)^{j+1} \tag{52}$$

Breaking the sum and using the law of iterated expectations, I can substitute equation (52) into (51) to obtain:

$$\begin{aligned} X_t = {} & \lambda P^*_t + (1-\lambda)E_t(X_{t+1}) \\ & - \left((1-\lambda)\lambda \sum_{j=\kappa+1}^{\infty} (1-\lambda)^j P_t(1+\pi^T)^{j+1} - \lambda \sum_{j=\kappa+1}^{\infty} (1-\lambda)^j P_{t-1}(1+\pi^T)^{j+1} \right) \end{aligned} \tag{53}$$

Which can further be simplified into:

$$\begin{aligned} X_t = \lambda P^*_t + (1-\lambda)E_t(X_{t+1}) - \Bigg(\lambda \sum_{j=\kappa+1}^{\infty} (1-\lambda)^j \pi_t (1+\pi^T)^{j+1} \Bigg) & \\ + \lambda^2 \sum_{j=\kappa+1}^{\infty} (1-\lambda)^j P_t(1+\pi^T)^{j+1} & \end{aligned} \tag{54}$$

The general price levels in the economy is then obtained using a weighted average of prices set by firms in the past and the reset prices of firms that had the opportunity to adjust:

$$P_t = \lambda X_t + (1-\lambda)P_{t-1} \tag{55}$$

Rearranging equation (55), I can derive an expression for inflation π_t as follows:

$$X_t = \frac{\pi_t}{\lambda} + P_{t-1} \tag{56}$$

Substituting for X_t in equation (54) using equation (56), I obtain the desired model of the modified Phillips Curve as follows:

$$\pi_t = \frac{\alpha\lambda^2}{1-\lambda}Y_t + E_t(\pi_{t+1}) - (\frac{\lambda}{1-\lambda})(\pi_t)\lambda \sum_{j=\kappa+1}^{\infty}(1-\lambda)^j(1+\pi^T)^{j+1} + (\frac{\lambda^2}{1-\lambda})(P_t)\lambda \sum_{j=\kappa+1}^{\infty}(1-\lambda)^j(1+\pi^T)^{j+1} \quad (57)$$

I can subsequently define ψ_κ to be:

$$\psi_\kappa = \lambda \sum_{j=\kappa+1}^{\infty}(1-\lambda)^j(1+\pi^T)^{j+1} = \frac{\lambda(1-\lambda)^{\kappa+1}(1+\pi^T)^{\kappa+2}}{\lambda-\pi^T(1-\lambda)} \quad (58)$$

Substituting equation (58) into (57) and rearranging, I obtain the desired equation for the new Keynesian Phillips Curve with bounded rationality as seen in the text::

$$\pi_t = \frac{\alpha\lambda^2}{1-\lambda(1-\psi_\kappa)}Y_t + \frac{1-\lambda}{1-\lambda(1-\psi_\kappa)}E_t(\pi_{t+1}) + \frac{\psi_\kappa\lambda^2}{1-\lambda(1-\psi_\kappa)}P_t \quad (59)$$

6.2 Derivation of the κ-augmented sticky-information Phillips Curve

Under the constraints imposed by bounded rationality, overall prices in the economy are given by:

$$P_t = \gamma\sum_{j=0}^{\kappa}(1-\gamma)^j E_{t-j}(P_t^*) + \gamma \sum_{j=\kappa+1}^{\infty}(1-\gamma)^j P_{t-j}(1+\pi^T)^j \quad (60)$$

Taking out the first term and redefining the summation operator, I obtain:

$$P_t = \gamma P_t^* + (1-\gamma)\gamma\sum_{j=0}^{\kappa}(1-\gamma)^j E_{t-j-1}(P_t^*) + \gamma \sum_{j=\kappa+1}^{\infty}(1-\gamma)^j P_{t-j}(1+\pi^T)^j \quad (61)$$

Equation (60) can be analogously defined as:

$$P_{t-1} = \gamma\sum_{j=0}^{\kappa}(1-\gamma)^j E_{t-j-1}(P_{t-1}^*) + \gamma \sum_{j=\kappa+1}^{\infty}(1-\gamma)^j P_{t-j-1}(1+\pi^T)^j \quad (62)$$

An expression for inflation π_t can thus be obtained by subtracting equation (62) from equation (61) as follows:

$$\pi_t = \gamma(P_t + \alpha Y_t) + \gamma\sum_{j=0}^{\kappa}(1-\gamma)^j E_{t-j-1}(\pi_t + \alpha\Delta Y_t) + \gamma \sum_{j=\kappa+1}^{\infty}(1-\gamma)^j \pi_{t-j}(1+\pi^T)^j - \gamma^2\sum_{j=0}^{\kappa}(1-\gamma)^j E_{t-j-1}(P_t^*) \quad (63)$$

Dividing equation (63) throughout by $(1-\gamma)$, I can obtain:

$$
\begin{aligned}
-\gamma\Bigg((P_t-(\frac{\gamma\alpha}{1-\gamma})Y_t\Bigg) = -\gamma^2\sum_{j=0}^{\kappa}(1-\gamma)^j E_{t-j-1}(P_t^*) \\
-\frac{\gamma^2}{1-\gamma}\sum_{j=\kappa+1}^{\infty}(1-\gamma)^j P_{t-j}(1+\pi^T)^j \quad (64)
\end{aligned}
$$

The last term in equation (63) can therefore be substituted out using equation (64) and the desired expression for the sticky-price Phillips Curve under bounded rationality can be obtained as follows:

$$
\begin{aligned}
\pi_t = (\frac{\alpha\gamma}{1-\gamma})Y_t + \gamma\sum_{j=0}^{\kappa}(1-\gamma)^j E_{t-j-1}(\pi_t+\alpha\Delta Y_t) \\
+\gamma\sum_{j=\kappa+1}^{\infty}(1-\gamma)^j(1+\pi^T)^j\Bigg(\pi_{t-j}+(\frac{\gamma}{1-\gamma})P_{t-j}\Bigg) \quad (65)
\end{aligned}
$$

6.3 Derivation of the expression for a dynamic path under the κ-augmented sticky-price new Keynesian Phillips Curve

Following Mankiw and Reis (2002) and Sargent (1987), an impulse response for prices can be obtained for the modified sticky-price new Keynesian Phillips Curve. Recall that the modified Phillips Curve is given by:

$$\pi_t = \frac{\alpha\lambda^2}{1-\lambda(1-\psi_\kappa)}Y_t + \frac{1-\lambda}{1-\lambda(1-\psi_\kappa)}E_t(\pi_{t+1}) + \frac{\psi_\kappa\lambda^2}{1-\lambda(1-\psi_\kappa)}P_t \quad (66)$$

I introduce new parameters μ and β such that:

$$\mu = \frac{\alpha\lambda^2}{1-\lambda(1-\psi_\kappa)} \quad (67)$$

$$\beta = \frac{\alpha\lambda^2}{1-\lambda} \quad (68)$$

Plugging in equation (67) and (68) into (66), inflation π_t can be expressed as:

$$\pi_t = \mu Y_t + \frac{\mu}{\beta}E_t\pi_{t+1} + \frac{\mu\psi_\kappa}{\alpha}P_t \quad (69)$$

Note in particular that as $\psi_\kappa = 0$, which occurs when $\kappa \to \infty$ as shown previously in section II, $\mu = \beta$ and equation (69) reduces to the benchmark model of the sticky-price new Keynesian Phillips Curve:

$$\pi_t = \frac{\alpha\lambda^2}{1-\lambda}Y_t + E_t(\pi_{t+1}) \quad (70)$$

I can rewrite the Phillips Curve expressed by equation (66) using the aggregate demand equation as follows:

$$\frac{(1-\lambda)\mu}{\alpha\lambda^2}E_t(P_{t+1}) + \left(\frac{\mu\psi_\kappa\lambda^2 - (1-\lambda)\mu - \mu\alpha\lambda^2 - \alpha\lambda^2}{\alpha\lambda^2}\right)P_t + P_{t-1} = -\mu M_t \tag{71}$$

Multiplying both sides of the equation by $\frac{\alpha\lambda^2}{(1-\lambda)\mu}$, equation (71) can be simplified into:

$$E_t(P_{t+1}) + \left(\frac{\psi_\kappa\lambda^2}{(1-\lambda)} - (1+\beta+\frac{\beta}{\mu})\right)P_t + \frac{\beta}{\mu}P_{t-1} = -\beta M_t \tag{72}$$

This produces an expectational difference equation similar in structure to that found in Mankiw and Reis's (2002) original paper. Subsequently, I take expectations at time t and express these expectational variables with an asterisk. Using both the forward and lag operator F and L respectively, equation (72) can be re-expressed in the following manner:

$$\left(F^2 + \left(\frac{\psi_\kappa\lambda^2}{1-\lambda} - (1+\beta+\frac{\beta}{\mu})\right)F + \frac{\beta}{\mu}\right)LP_t^* = -\beta M_t^* \tag{73}$$

Observe that this reduces to the expectational difference equation of the benchmark model once again when $\psi_\kappa = 0$:

$$\left(F^2 - (2+\beta)F + 1\right)LP_t^* = -\beta M_t^* \tag{74}$$

Given the equation in (73), denote the roots of the quadratic $(x^2 + \left(\frac{\psi_\kappa\lambda^2}{1-\lambda} - (1+\beta+\frac{\beta}{\mu})\right)x + \frac{\beta}{\mu})$ by θ_1 and θ_2 such that:

$$F^2 + \left(\frac{\psi_\kappa\lambda^2}{1-\lambda} - (1+\beta+\frac{\beta}{\mu})\right)F + \frac{\beta}{\mu} = F^2 - (\theta_1+\theta_2)F + \theta_1\theta_2 \tag{75}$$

Comparing the coefficients of each term on both sides, it is clear that:

$$-(\theta_1+\theta_2) = \frac{\psi_\kappa\lambda^2}{1-\lambda} - (1+\beta+\frac{\beta}{\mu}) \tag{76}$$

$$\theta_1\theta_2 = \frac{\beta}{\mu} \tag{77}$$

Without a loss of generality, I pick $\theta_1 = \theta$ to be the smaller of the two positive roots and write equation (73) as:

$$(F-\theta)LP_t^* = \mu\theta(1 - \frac{\mu}{\beta}\theta F)^{-1}M_t^* \tag{78}$$

In order to expand the negative binomial term on the right, I impose the strict assumption that:

$$\frac{\theta\mu}{\beta} < 1 \tag{79}$$

An expression for the impulse response can thus be obtained as follows:

$$P_t = \theta P_{t-1} + \mu\theta\sum_{i=0}^{\infty}\left((\frac{\mu}{\beta})\theta\right)^i E_t(M_{t+i}) \tag{80}$$

It is easy to see that when $\psi_\kappa = 0$, the impulse response expressed in equation (80) reduces to the original impulse response of the benchmark model as seen in Mankiw and Reis (2002):

$$P_t = \theta P_{t-1} + (1-\theta)^2 \sum_{i=0}^{\infty} (\theta)^i E_t(M_{t+i}) \tag{81}$$

where the coefficient $(1-\theta)^2$ is given by $\beta\theta$ such that under the full rationality case when $\psi_\kappa = 0$,

$$\beta = \frac{(1-\theta)^2}{\theta} \tag{82}$$

Expressing μ and β in their original parameters, the dynamic path of prices for the κ-augmented sticky-price Phillips Curve is given by:

$$P_t = \theta P_{t-1} + (\frac{\alpha\lambda^2}{1-\lambda(1-\psi_\kappa)})\theta \sum_{i=0}^{\infty} \big((\frac{1-\lambda}{1-\lambda(1-\psi_\kappa)})\theta\big)^i E_t(M_{t+i}) \tag{83}$$

6.4 Derivation of inequality $f(\kappa,\lambda) \geq g(\kappa,\lambda)$ such that a real solution for θ exists

Begin with the equation below which rests within the square root of the quadratic formula applied to equation (73). For a real root to be defined, the following inequality must hold:

$$\left(\frac{\psi_\kappa \lambda^2}{1-\lambda} - (1+\beta+\frac{\beta}{\mu})\right)^2 - \frac{4\beta}{\mu} \geq 0 \tag{84}$$

Expanding the square and substituting for $\mu = \frac{\alpha\lambda^2}{1-\lambda(1-\psi_\kappa)}$ and $\beta = \frac{\alpha\lambda^2}{1-\lambda}$, I can obtain the following inequality:

$$\frac{\psi_\kappa^2\lambda^4}{(1-\lambda)^2} - 2\psi_\kappa(\frac{\lambda^2}{1-\lambda})\left[2+\frac{\lambda(\psi_\kappa+\alpha\lambda)}{1-\lambda}\right] + 4 + \frac{4\lambda(\psi_\kappa+\alpha\lambda)}{1-\lambda} + \frac{\lambda^2(\psi_\kappa+\alpha\lambda)^2}{(1-\lambda)^2}$$
$$\geq (\frac{4}{1-\lambda})\left[1-\lambda(1-\psi_\kappa)\right] \tag{85}$$

Multiplying throughout by $(1-\lambda)^2$ and simplifying, equation (85) can be written as:

$$\lambda^2\psi_\kappa^2(1+\lambda^2) - 4\psi_\kappa\lambda^2(1-\lambda) + 2\alpha\psi_\kappa\lambda^3(1-\lambda) + 4\alpha\lambda^2(1-\lambda) + \lambda^3(\alpha^2\lambda - 2\psi_\kappa^2) \geq 0 \tag{86}$$

Recognising that $0 < \lambda < 1$ such that $\lambda^2 \geq 0$, divide the above by λ^2 and rearrange the equation to obtain:

$$\psi_\kappa^2(1-\lambda)^2 + 2(1-\lambda)\left[\psi_\kappa\alpha\lambda + 2\alpha - 2\psi_\kappa\right] + \alpha^2\lambda^2 \geq 0 \tag{87}$$

Dividing throughout by $(1-\lambda)^2$ subsequently yields:

$$\psi_\kappa^2 + 2\psi_\kappa(\frac{\alpha\lambda}{1-\lambda}) + (\frac{\alpha\lambda}{1-\lambda})^2 \geq \frac{4}{1-\lambda}(\psi_\kappa - \alpha) \tag{88}$$

This provides the expressions for $f(\kappa, \lambda)$ and $g(\kappa, \lambda)$ as seen in the text whereby

$$f(\kappa, \lambda) = (\psi_\kappa + \frac{\alpha\lambda}{1-\lambda})^2 = \left(\frac{\lambda(1-\lambda)^{k+1}(1+\pi^T)^{k+2}}{\lambda - \pi^T(1-\lambda)} + \frac{\alpha\lambda}{1-\lambda} \right)^2 \tag{89}$$

$$g(\kappa, \lambda) = \frac{4}{1-\lambda}(\psi_\kappa - \alpha) = \frac{4}{1-\lambda}\left(\frac{\lambda(1-\lambda)^{k+1}(1+\pi^T)^{k+2}}{\lambda - \pi^T(1-\lambda)} - \alpha \right) \tag{90}$$

6.5 Derivation of the expression for a dynamic path under the κ-augmented sticky-information Phillips Curve

I begin with the original model of price adjustment where κ is first assumed to be unbounded. This is given by:

$$P_t = \gamma \sum_{j=0}^{\infty} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] \tag{91}$$

This equation can be re-written as a summation of two terms as follows:

$$P_t = \gamma \sum_{j=0}^{t} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] + \gamma \sum_{j=t+1}^{\infty} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] \tag{92}$$

By expressing prices P_t in this manner, it is easy to see that P_t is now an aggregate of prices set by price setters who are aware of the shock (fall in aggregate demand at $t = 0$) and those who are not aware. The first term captures a geometrically weighted average of expectations formed beginning from $t = 0$ to $t = t$. These are the expectations formed ex-post to the change in aggregate demand. On the other hand, the second term captures a summation of expectations formed of prices from the beginning of time to exactly one period before the shock occurs at $t = 0$. These are the expectations formed ex-ante to the change in aggregate demand, and they represent the prices set by price setters who are not aware of the new path of aggregate demand.

Now, suppose κ is no longer ∞ due to cognitive restrictions imposed by bounded rationality. Equation (92) is then expressed as a function of κ as shown below:

$$P_t = \gamma \sum_{j=0}^{t} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] \\ + \gamma \sum_{j=t+1}^{\kappa} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] + \gamma \sum_{j=\kappa+1}^{\infty} \varphi^j P_{t-j} \tag{93}$$

where $\varphi = (1-\gamma)(1+\pi^T)$. There are two important points to note when defining the limits of the summation operator in the above manner. First, observe that the expectational terms in the second term now sum from $E_{-1}(.)$ to $E_{t-\kappa}(.)$. To ensure that these expectations are formed ex-ante to the shock that occurs at $t = 0$, the restriction of $t < \kappa$ is imposed. This suggests that price setters who are not aware of the new path of aggregate demand are now restricted by the expectations that they formed $t-\kappa$ periods ago in the past of prices today. The third term captures the use of heuristics in place of expectations formed over a time horizon further away from $t-\kappa$. Second, the upper bound t in the first term is smaller than κ by definition or the expectational terms now become undefined as a result of bounded rationality. Following Mankiw and Reis (2002), the path of prices can be easily obtained such that

$$P_t = \begin{cases} \dfrac{-log(0.9)\left((1-\gamma)^{t+1}-(1-\gamma)^{\kappa+1}\right)+\gamma \sum\limits_{j=\kappa+1}^{\infty} \varphi^j P_{t-j}}{1-(1-\alpha)\left(1-(1-\gamma)^{t+1}\right)} & \text{for } t < \kappa \end{cases}$$

If κ is equal to 10, then prices from $t = 0$ to $t = 9$ thus follow the path outlined above.

What happens when $t > \kappa$? It is clear that prices can no longer follow the specified path outlined above, or they violate the expectational constraints imposed by equation (93). As a result, an alternative path for prices is required for all $t > \kappa$. I rewrite equation (93) in the following manner instead:

$$P_t = \gamma \sum_{j=0}^{\kappa} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + +\alpha M_t] + \gamma \sum_{j=\kappa+1}^{\infty} \varphi^j P_{t-j} \tag{94}$$

The first term represents price setters who are aware of the new path of aggregate demand and form ex-ante expectations of P_t and M_t contemporaneously up to κ periods ago. When $t > \kappa$, price setters with old information are now represented by the second term. In this world there are no longer price setters who set prices according to past information because the time horizon now prohibits the formation of expectations. Instead, these price setters who are not aware of the new path are left with the choice of simply referring to past prices P_{t-j} as an convenient anchor. Hence, the above equation can be written as:

$$P_t = P_t(1-\alpha)[1-(1-\gamma)^{\kappa+1}] + \gamma \sum_{j=\kappa+1}^{\infty} \varphi^j P_{t-j} \tag{95}$$

Denoting $\omega = 1-(1-\alpha)[1-(1-\gamma)^{\kappa+1}]$ to be a constant for a given κ, I can further rearrange the above equation as follows:

$$P_t = \frac{\gamma}{\omega} \sum_{j=\kappa+1}^{\infty} \varphi^j P_{t-j} \tag{96}$$

Taking out the first term and redefining the summation operator, equation (88) can be expressed as:

$$P_t = \frac{\gamma}{\omega}\varphi^{\kappa+1} P_{t-\kappa-1} + \varphi\left(\frac{\gamma}{\omega} \sum_{j=\kappa+1}^{\infty} \varphi^j P_{t-j-1}\right) \tag{97}$$

Recognising that the second term in parenthesis above is equivalent to P_{t-1}, the above equation can be expressed as a difference equation given by

$$P_t = \zeta P_{t-\kappa-1} + \varphi P_{t-1} \tag{98}$$

where $\zeta = \frac{\gamma}{\omega}\varphi^{\kappa+1}$ is a constant. Here, I see that the path of prices from $t > \kappa$ is now dependent fully on some weighted average of prices set one period ago and the furthest possible prices set up at κ periods ago. Noting a discontinuity of prices at $t = \kappa$, the desired paths of prices under policy experiment 1 is thus given by:

$$P_t = \begin{cases} \dfrac{-log(0.9)\left((1-\gamma)^{t+1}-(1-\gamma)^{\kappa+1}\right)+\gamma \sum_{j=\kappa+1}^{\infty} \varphi^j P_{t-j}}{1-(1-\alpha)\left(1-(1-\gamma)^{t+1}\right)} & \text{for } t \leq \kappa - 1 \\ \zeta P_{t-\kappa-1} + \varphi P_{t-1} & \text{if } t > \kappa \end{cases}$$

For policy experiment 2, all ex-ante expectations $E_{t-j}P_t$ where $t-j < 0$ are equal to $0.025(t+1)$. Therefore, the desired path of prices are given by:

$$P_t = \begin{cases} \dfrac{0.025(t+1)\left((1-\gamma)^{t+1}-(1-\gamma)^{\kappa+1}\right)+\gamma \sum_{j=\kappa+1}^{\infty} \varphi^j P_{t-j}}{1-(1-\alpha)\left(1-(1-\gamma)^{t+1}\right)} & \text{for } t \leq \kappa - 1 \\ \zeta P_{t-\kappa-1} + \varphi P_{t-1} & \text{if } t > \kappa \end{cases}$$

To solve for an impulse response under policy experiment 3, equation (93) is written as follows:

$$\begin{aligned} P_t = \gamma \sum_{j=0}^{t+8} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] \\ + \gamma \sum_{j=t+9}^{\kappa+8} (1-\gamma)^j E_{t-j}[(1-\alpha)P_t + \alpha M_t] + \gamma \sum_{j=\kappa+9}^{\infty} \varphi^j P_{t-j} \end{aligned} \tag{99}$$

Observe that the limits of the summation operators are written in a way such that all ex-ante expectations are formed before period $t = -8$. For all $t - j < -8$, $E_{t-j}P_t = E_{t-j}M_t = 0.025(1 + t)$. These are inattentive price setters who are not aware of the announcement of forthcoming disinflation at period $t = 0$. For all $t - j \geq -8$, there are no uncertainty given the announcement. Price setters who are aware of the new path of prices thus set ex-post expectations such that $E_{t-j}P_t = P_t$. Therefore, for $t < \kappa$, prices follow the path given by:

$$P_t = \frac{0.025(t+1)(1-\gamma)^{t+9}\left(1-(1-\gamma)^{\kappa-t}\right) + \gamma \sum_{j=\kappa+9}^{\infty} \varphi^j P_{t-j}}{1-(1-\alpha)\left(1-(1-\gamma)^{t+9}\right)} \tag{100}$$

For $t > k$, equation (96) can be written instead as:

$$P_t = \frac{\gamma}{\eta} \sum_{j=\kappa+9}^{\infty} \varphi^j P_{t-j} \tag{101}$$

which can subsequently be written as a difference equation as follows:

$$P_t = \frac{\gamma}{\eta} \varphi^{\kappa+9} P_{t-\kappa-9} + \varphi P_{t-1} \tag{102}$$

where $\eta = 1 - (1-\alpha)(1-(1-\gamma)^{t+9})$.

6.6 Derivation of impulse responses for inflation in response to monetary policy shocks

6.6.1 Impulse responses for the κ-augmented sticky-price new Keynesian Phillips Curve

I begin by writing the growth of money supply M_t as an AR(1) process as follows

$$\Delta M_t = \rho \Delta M_{t-1} + \epsilon_t \tag{103}$$

where ϵ_t is a white-noise innovation. Under this model, the level of money supply is non-stationary but the growth rate of money supply is stationary for all$|\rho| < 1$. Given this conjecture, inflation follows a stationary process as Ill. The AR(1) process for inflation and prices can equivalently be written as a MA(∞) process as follows:

$$\pi_t = \sum_{j=0}^{\infty} \rho^j \epsilon_{t-j} \tag{104}$$

$$P_t = \sum_{\tau=0}^{\infty} \sum_{j=0}^{\infty} \phi_j \epsilon_{t-j-\tau} \tag{105}$$

Then, recall that the path of prices solved previously in subsection 4.3 is given by:

$$P_t = \theta P_{t-1} + \mu\theta \sum_{i=0}^{\infty} \left((\frac{\mu}{\beta})\theta \right)^i E_t(M_{t+i}) \tag{106}$$

Substituting the MA(∞) process for P_t into this solution yields:

$$\sum_{\tau=0}^{\infty} \sum_{j=0}^{\infty} \phi_j \epsilon_{t-j-\tau} = \theta \sum_{\tau=0}^{\infty} \sum_{j=0}^{\infty} \phi_j \epsilon_{t-1-j-\tau} + \mu\theta \sum_{i=0}^{\infty} \left((\frac{\mu}{\beta})\theta \right)^i \sum_{j=0}^{\infty} \sum_{\tau=max\{i-j,0\}}^{\infty} \rho^j \epsilon_{t+i-j-\tau} \tag{107}$$

as $E_t\{\epsilon_{t+i-j-\tau}\} = \epsilon_{t+i-j-\tau}$ for all $i - j \leq \tau$ and is zero otherwise. Subsequently, ϕ_j are coefficients to be determined. Using the same method of undetermined coefficients outlined by

Mankiw and Reis (2002), ϕ_υ can be solved by matching the coefficients on $\epsilon_{t-\upsilon}$ on both sides of the equation such that:

$$\sum_{j=0}^{\upsilon}\phi_j = \theta\sum_{j=0}^{\upsilon-1}\phi_j + \mu\theta\sum_{i=0}^{\infty}((\frac{\mu}{\beta})\theta)^i\sum_{j=0}^{\upsilon+i}\rho^j \tag{108}$$

The above equation can subsequently be simplified to yield $\{\phi_\upsilon\}$, the impulse response of inflation in response to monetary policy shocks.

$$\phi_\upsilon = (\theta-1)\sum_{j=0}^{\upsilon-1}\phi_j + (\frac{\mu\theta}{1-\rho})\left(\frac{1}{1-\frac{\mu}{\beta}\theta} - \frac{\rho^{\upsilon+1}}{1-\frac{\mu}{\beta}\theta\rho}\right) \tag{109}$$

Once again, observe that if $\kappa = \infty$, $\mu = \beta$ and the above impulse function reduces to that of the benchmark given by

$$\phi_\upsilon = (\theta-1)\sum_{j=0}^{\upsilon-1}\phi_j + \frac{(1-\theta)^2}{1-\rho}\left(\frac{1}{1-\theta} - \frac{\rho^{\upsilon+1}}{1-\theta\rho}\right) \tag{110}$$

6.6.2 Impulse responses for the κ-augmented sticky-information Phillips Curve

Similar to that for the sticky-price model, I begin by first defining a process for money and prices as follows:

$$M_t = \sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\rho^i\epsilon_{t-i-\varrho} \tag{111}$$

$$\Delta M_t = \sum_{i=0}^{\infty}\rho^i\epsilon_{t-i} \tag{112}$$

$$P_t = \sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i-\varrho} \tag{113}$$

$$\pi_t = \sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i} \tag{114}$$

Substituting the above processes into the κ-augmented sticky-information Phillips Curve outlined by equation (11) in the text yields:

$$\begin{aligned}\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i} = (\frac{\alpha\gamma}{1-\alpha})\left(\sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\rho^i\epsilon_{t-i-\varrho} - \sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i-\varrho}\right) + \\ \gamma\sum_{j=0}^{\kappa}(1-\gamma)^j E_{t-j-1}\left((1-\alpha)\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i} + \alpha\sum_{i=0}^{\infty}\rho^i\epsilon_{t-i}\right) + \\ \gamma\sum_{j=\kappa+1}^{\infty}\varphi^j\left(\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i-j} + (\frac{\gamma}{1-\gamma})\sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-j-i-\varrho}\right)\end{aligned} \tag{115}$$

To obtain an expression for the impulse response $\{\Psi_\upsilon\}$, all coefficients of the white noise innovation $\epsilon_{t-\upsilon}$ must be matched. However, note that the upper bound of the second term in the above expression is now changed to κ, when it was ∞ in the original sticky-information Phillips Curve. This results in a discontinuity of the moment of inflation between $\upsilon = \kappa$ and $\upsilon = \kappa + 1$.

Now, consider what happens when $\upsilon \leq \kappa$. Firstly, the last term in the above expression no longer exists. Secondly, taking expectation $E_{t-j-1}(\epsilon_{t-i})$ yields ϵ_{t-j-i} if and only if $i \geq j+1$. Subsequently, the impulse response $\{\Psi_\upsilon\}$ from past expectations only exists for all $\upsilon \leq \kappa$ as it is only possible to replace the upper bound of the summation operator κ by υ if this constrain is satisfied. Redefining the summation operator such that $j = i - 1$, the coefficients of $\epsilon_{t-\upsilon}$ are thus given by:

$$\Psi_\upsilon = (\frac{\alpha\gamma}{1-\gamma})\Big(\sum_{i=0}^{\upsilon}\rho^i - \sum_{i=0}^{\upsilon-1}\Psi_i - \Psi_\upsilon\Big) + (\frac{\gamma}{1-\gamma})\Big(\sum_{i=0}^{\upsilon}(1-\gamma)^i - 1\Big)\Psi_\upsilon + \\ (\frac{\alpha\gamma}{1-\alpha})\sum_{i=1}^{\upsilon}(1-\gamma)^i\rho^\upsilon \quad (116)$$

What happens then, if $\upsilon \geq \kappa$? The impulse response arising from past expectations of inflation no longer exists now, as it is no longer possible to replace the upper bound κ with υ. Subsequently, all moments of inflation today will only arise from output, past inflation and some past prices. Now, consider the following innovation arising from past inflation given by:

$$\gamma \sum_{j=\kappa+1}^{\infty}\varphi^j\Big(\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i-j}\Big)$$

For there to be a $\epsilon_{t-\upsilon}$ term to feature in the above expression, there must be a restriction such that $i + j = \upsilon$. Satisfying this restriction implies that the coefficients of all $\epsilon_{t-\upsilon}$ terms are given by $\Psi_{\upsilon-j}$. By redefining the summation operator using this restriction, it is possible to rewrite all coefficients of $\epsilon_{t-\upsilon}$ embedded in the above expression as:

$$\frac{\gamma}{1-\gamma}\sum_{i=0}^{\upsilon-\kappa-1}\varphi^{\upsilon-i+1}\Psi_i \quad (117)$$

Similarly, consider the innovation to current inflation arising from past prices given by:

$$\gamma \sum_{j=\kappa+1}^{\infty}\varphi^j\Big((\frac{\gamma}{1-\gamma})\sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-j-i-\varrho}\Big)$$

All coefficients on $\epsilon_{t-\upsilon}$ terms embedded in the summation $\sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-j-i-\varrho}$ are given by $\sum_{i=0}^{\upsilon-j}\Psi_i$ with the restriction $j + i = \upsilon$. The impulse responses arising from past prices are thus given by:

$$\frac{\gamma^2}{1-\gamma}\sum_{j=\kappa+1}^{\upsilon}\varphi^j\Big(\sum_{i=0}^{\upsilon-j}\Psi_i\Big) \quad (118)$$

Using equations (117) and (118), the coefficients of $\epsilon_{t-\upsilon}$ are thus given by:

$$\Psi_\upsilon = (\frac{\alpha\gamma}{1-\gamma})\Bigg(\sum_{i=0}^{\upsilon}\rho^i - \sum_{i=0}^{\upsilon-1}\Psi_i - \Psi_\upsilon\Bigg) + \frac{\gamma}{1-\gamma}\sum_{i=0}^{\upsilon-\kappa-1}\varphi^{\upsilon-i+1}\Psi_i + \frac{\gamma^2}{1-\gamma}\sum_{j=\kappa+1}^{\upsilon}\varphi^j\Bigg(\sum_{i=0}^{\upsilon-j}\Psi_i\Bigg) \quad (119)$$

Therefore, the complete characterization of the stochastic process for inflation as seen in the text is given by:

$$\Psi_\upsilon = \frac{\alpha\gamma\Bigg[(1-\sum_{i=0}^{\upsilon-1}\Psi_i) + \sum_{i=1}^{\upsilon}\rho^i + \rho^\upsilon\sum_{i=1}^{\upsilon}(1-\gamma)^i\Bigg]}{1-\gamma(1-\alpha)\sum_{i=0}^{\upsilon}(1-\gamma)^i} \quad \text{for} \quad \upsilon \leq \kappa \quad (120)$$

$$\Psi_\upsilon = \frac{\alpha\gamma\Bigg[(1-\sum_{i=0}^{\upsilon-1}\Psi_i) + \sum_{i=1}^{\upsilon}\rho^i\Bigg] + \Pi}{1-\gamma(1-\alpha)} \quad \text{for} \quad \upsilon \geq \kappa + 1 \quad (121)$$

where $\Pi = \gamma\sum_{i=0}^{\upsilon-\kappa-1}\varphi^{\upsilon-i+1}\Psi_i + \gamma^2\sum_{j=\kappa+1}^{\upsilon}\varphi^j\Bigg(\sum_{i=0}^{\upsilon-j}\Psi_i\Bigg)$

What happens if $\kappa = 0$? Then, the stochastic process for inflation given by equation (115) is written instead as:

$$\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i} = (\frac{\alpha\gamma}{1-\alpha})\Bigg(\sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\rho^i\epsilon_{t-i-\varrho} - \sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i-\varrho}\Bigg) + \gamma E_{t-1}\Bigg((1-\alpha)\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i} + \alpha\sum_{i=0}^{\infty}\rho^i\epsilon_{t-i}\Bigg) + \gamma\sum_{j=1}^{\infty}\varphi^j\Bigg(\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i-j} + (\frac{\gamma}{1-\gamma})\sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-j-i-\varrho}\Bigg) \quad (122)$$

Realising that $E_{t-1}(\epsilon_{t-i}) = \epsilon_{t-i}$ for all $i \geq 1$ and 0 otherwise, the above equation can be expressed as:

$$\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i} = (\frac{\alpha\gamma}{1-\alpha})\Bigg(\sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\rho^i\epsilon_{t-i-\varrho} - \sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i-\varrho}\Bigg) + \gamma\Bigg((1-\alpha)\sum_{i=1}^{\infty}\Psi_i\epsilon_{t-i} + \alpha\sum_{i=1}^{\infty}\rho^i\epsilon_{t-i}\Bigg) + \gamma\sum_{j=1}^{\infty}\varphi^j\Bigg(\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-i-j} + (\frac{\gamma}{1-\gamma})\sum_{\varrho=0}^{\infty}\sum_{i=0}^{\infty}\Psi_i\epsilon_{t-j-i-\varrho}\Bigg) \quad (123)$$

Matching once again the coefficients on all ϵ_{t_v} terms, the impulse response $\{\Psi_v\}$ for inflation when $\kappa = 0$ is given as follows:

$$\Psi_v = \frac{\alpha\gamma\left[(1 - \sum_{i=0}^{v-1} \Psi_i) + \sum_{i=1}^{v} \rho^i + \rho^v \sum_{i=1}^{v} (1-\gamma)^i\right]}{1 - \gamma(1-\alpha) \sum_{i=0}^{v} (1-\gamma)^i} \quad \text{for} \quad v \leq \kappa \tag{124}$$

$$\Psi_v = \frac{\alpha\gamma\left[(1 - \sum_{i=0}^{v-1} \Psi_i) + \sum_{i=1}^{v} \rho^i + (1-\gamma)\rho^v\right] + \Gamma}{1 + (1-\alpha)(\gamma^2 - 2\gamma)} \quad \text{for} \quad v \geq \kappa + 1 \tag{125}$$

Such that $\Psi_0 = \frac{\alpha\gamma(2-\gamma)}{1+(1-\alpha)(\gamma^2-2\gamma)}$ and $\Gamma = (1-\gamma)\gamma \sum_{i=0}^{v-1} \varphi^{v-i}\Psi_i + \gamma^2 \sum_{j=1}^{v} \varphi^j \left(\sum_{i=0}^{v-j} \Psi_i\right)$.

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