Frauds, Strategies and Complaints in Germany*

Walter R. Mebane, $\mathrm{Jr.}^\dagger$

Joseph Klaver[‡]

Blake Miller[§]

April 4, 2016

*Prepared for presentation at the 2016 Annual Meeting of the Midwest Political Science Association, Chicago, April 7–10. Work supported by NSF award SES 1523355.

[†]Professor, Department of Political Science and Department of Statistics, University of Michigan, Haven Hall, Ann Arbor, MI 48109-1045 (E-mail: wmebane@umich.edu).

[‡]Department of Political Science, University of Michigan, Haven Hall, Ann Arbor, MI 48109-1045 (E-mail: jrklav@umich.edu).

[§]Department of Political Science, University of Michigan, Haven Hall, Ann Arbor, MI 48109-1045 (E-mail: blakeapm@umich.edu).

Abstract

Many statistical methods that use low-level election vote count data to detect election frauds have the limitation that they have a hard time distinguishing distortions in vote counts that stem from voters' strategic behavior from distortions that originate with election frauds. Identifying latent components that underlie election forensics statistics and other contextual variables can help show the extent to which the statistics measure fraudulent as opposed to strategic behavior. We use an active-learning procedure with a support vector machine to classify complaints about German federal elections during 1949–2009 to show the diversity of the complaints, which we use as contextual data. We also use variables that measure strategic voting in those elections. For the elections of 2005 and 2009 we use latent variable methods to assess whether the parameters of a positive empirical model of frauds connect through latent variable structure to either the complaints or the strategic variables. Geographic ambiguity about the locations at which some complaints occur motivates embedding a geographic mixture structure in the latent variable model. The "fraud" parameters connect to both complaints and strategic behavior.

1 Introduction

Distinguishing empirical patterns that are caused by election frauds from those that result from strategic behavior is a core problem for election forensics. Election forensics uses statistical methods to try to determine whether the results of an election accurately reflect the intentions of the electors. The statistical methods that have been proposed to assess election accuracy, or merely to detect frauds (e.g. Myagkov, Ordeshook and Shaikin 2009; Levin, Cohn, Ordeshook and Alvarez 2009; Mebane 2010; Pericchi and Torres 2011; Cantu and Saiegh 2011; Deckert, Myagkov and Ordeshook 2011; Beber and Scacco 2012; Klimek, Yegorov, Hanel and Thurner 2012; Hicken and Mebane 2015; Mebane 2016), all have the potential to give ambiguous results—while they may be able to tell whether votes have been shifted, they are hard pressed to tell who moved them (e.g. Mebane 2013, 2014, 2016). Such methods attempt to form conclusions by analyzing exclusively election data such as vote counts and the number of eligible voters. Context is often invoked as helpful to evaluate the methods or to enhance conclusions (e.g. Shikano and Mack 2009; Breunig and Goerres 2011; Cantu and Saiegh 2011; Deckert, Myagkov and Ordeshook 2011), although only recently has there been an attempt formally to incorporate contextual information in the statistical analysis (Montgomery, Olivella, Potter and Crisp 2015).

Contextual information may also be ambiguous. In this paper we draw on election complaints submitted by Germans about German federal elections (Ziblatt 2009; Breunig and Goerres 2011; Mares and Zhu 2015) to try to assess whether the parameters of a positive empirical model of frauds (Klimek et al. 2012; Mebane 2016) actually measure frauds or respond to strategic behavior. Of course the parameters may do both. The complaints themselves may or may not be motivated by and refer to genuine frauds, by which we mean maleficent acts that distort the election. Postelection nullification petitions in Mexico have some similarities to the German complaints, although the Mexican petitions are submitted by political parties and not directly by citizens. The Mexican petitions relate in part to election-day problems, but they also clearly relate to parties'

tactical incentives (Mebane and Wall 2015). We will see that some of the German complaints address problems with administering the election but others take direct issue with features of the electoral system. While the former administrative problems may relate to frauds of kinds that election forensics is directly concerned with, complaints about the electoral system itself probably do not. All kinds of complaints may relate to citizens' strategic activites and understandings and not to frauds of any kind.

We use manually coded complaints data from 2005 and 2009 in a latent variable model to assess how both measures of strategic voting and the distribution of different types of complaints across districts relate to the distribution of the frauds model parameter estimates. We use a support vector machine (SVM) in an active learning framework to classify complaints from elections during 1949–2009 to get a broader perspective on the diversity of complaints. While we cannot say whether the fraud model's parameters would connect to complaints and measures of strategic voting in the same way in the earlier election periods as they do in 2005 and 2009—and the connections already vary between those two elections—we expect the relationships are similarly complex in the earlier years.

We use data from the 2005 and 2009 federal elections to help assess the efficacy of a likelihood implementation (Mebane 2016) of the Klimek et al. (2012) model. While postelection complaints in Germany do not necessarily concern what might be considered genuine frauds, they have face validity as imperfect measures of potentially serious irregularities. Ziblatt (2009) and Mares and Zhu (2015) use such complaints to measure the occurrence of election frauds in Germany during the years 1871–1912, and Breunig and Goerres (2011) make a similar usage with regard to more recent elections.

For data from 2005 and 2009 we use a latent variable model to assess how both measures of strategic voting and the distribution of different types of complaints across districts relate to the distribution of the fraud probability parameter estimates. The latent variable model features generalized linear associations between manifest variables and a set of latent variables that various manifest variables have in common. For some manifest

variables the relationship to common latent variables is simply linear, while for the binary variables that measure the incidence of complaints the relationship goes through a probit model and for the fraud probabilities the relationship goes through a Dirichlet link. Other fraud model parameters have log-Normal or inverse gamma links.

If fraud-detecting variables are genuinely ambiguous in the sense that they are triggered both by frauds and by strategic behavior, then common latent variables should connect the supposed fraud measures to the complaint and strategic variables.

2 Models

The current analysis features a latent variable model that we use with Monte Carlo Markov Chain (MCMC) Bayesian methods to relate a diverse set of manifest variables to a smaller set of common latent variables using an exploratory factor analysis model specification. First we describe the specifications of the latent variable model, which has a number of nonstandard features: various kinds of nonlinear "links" between manifest and latent variables; structure to accommodate uncertainty about the geographic locations of some observations.

Currently the complaints used to produce manifest variables for the latent variable model come from manually coded complaints from two elections (2005 and 2009), but we anticipate using codes produced with machine assistance for more years in future iterations of this analysis. After describing the specifications for the latent variable model, we describe the specifications and procedures for the active-learning classification framework we use to classify complaints during 1949–2009.

2.1 Latent Variable model

We use a latent variable model to study whether parameters in a model that estimates the frequency, type and magnitude of election "frauds" in each district relate more to the

citizens' complaints or to measures of strategic voting. These parameter estimates come from a finite mixture model (Mebane 2016) that implements the positive empirical frauds conception developed by Klimek et al. (2012). Estimates are based on the plurality rule votes (*Erststimmen*) at the polling stations in each single-member district. The parameter estimates we use for district i = 1, ..., 299 are: \hat{f}_{ii} , the probability of incremental fraud in district i; \hat{f}_{0i} , the probability of no fraud in district i; $\hat{\alpha}_i$, the parameter that for district idetermines whether fraud involves more vote manufacturing (from nonvoters) or vote stealing (from nonleading parties); and $\hat{\theta}_i$, the parameter that for district i determines the proportion of votes manufactured from nonvotes and helps determine the number of votes stolen from nonleading parties (see Mebane (2016) for details).¹ If $\hat{\alpha}_i < 1$ then vote stealing is estimated to be more important, and if $\hat{\alpha}_i > 1$ then manufacturing votes from nonvoters is more important (Mebane 2016, 8–9).

For the latent variable model there are K_C binary manifest variables that measure complaints, y_{ki} , $k = 1, ..., K_C$, i = 1, ..., 299. Each of the complaint variables relates to an unobserved continuous variable x_k that is itself related to J common latent variables ξ_{ℓ} through equations of the following form,

$$x_{ki} = c_k + \sum_{\ell=1}^{J} \lambda_{k\ell} \xi_{\ell i}, \quad k = 1, \dots, K_C,$$
 (1)

where the values of intercept c_k and of factor loading $\lambda_{k\ell}$ that are not constant² have Normal distributions and the ξ_ℓ are multivariate Normal with mean $\boldsymbol{\gamma} = (\gamma_i, \ldots, \gamma_J)'$ and precision matrix $\boldsymbol{\Upsilon}^3$. Usually the district associated with a complaint is known with

¹We do not use estimates for the probability of extreme fraud, \hat{f}_{ei} , because extreme fraud is sparse in the years for which we estimate the latent variable model: $\hat{f}_{ei} > 0$ only twice in 2009 and never in 2005. ²See the discussion on page 7.

³Adapting specifications given by Lee (2007), the prior for each mean γ_{ℓ} is Normal, and the prior for Υ is Wishart. Further details about the prior specifications are in the Model Appendix.

certainty, in which case the manifest variables y_k take the (probit) index variable form

$$\operatorname{Prob}(y_{ki}=0) = \int_{-\infty}^{0} \phi(x_{ki}, \psi_k) \, df \tag{2a}$$

$$\operatorname{Prob}(y_{ki}=1) = \int_0^\infty \phi(x_{ki}, \psi_k) \, df \tag{2b}$$

where $\phi(x, \psi)$ is the Normal density with mean x and precision ψ .⁴

But sometimes there is uncertainty about the assignment of a complaint to a district which leads to uncertainty about whether the observation y_{ki} should be $y_{ki} = 0$ or $y_{ki} = 1$. In this case we mix over the two possible values. Let v_1 and v_2 be the number of registered voters in the portion of each of the two districts for which the district assignment is uncertain.⁵ Generate $r_i \in \{0, 1\}$ using probabilities that have Dirichlet priors $\mathcal{D}([v_1, v_2])$ by

$$\pi_i \sim \mathcal{D}([v_1, v_2])$$
$$r_i \sim \text{multinom}(\{0, 1\}; \pi_i)$$

With this prior π_i has mean $\left(\frac{v_1}{v_1+v_2}, \frac{v_2}{v_1+v_2}\right)$. Assuming the probabilities in π_i refer to the events in the order $(y_{ki} = 1, y_{ki} = 0)$, let

$$\operatorname{Prob}(y_{ki}) = r_i \int_{-\infty}^0 \phi(x_{ki}, \psi_k) \, df + (1 - r_i) \int_0^\infty \phi(x_{ki}, \psi_k) \, df \,. \tag{3}$$

We generate π_i —separately for each pair of districts—because we are uncertain about the chances that any ambiguously locatable complaint should be associated with one of the two districts to which it could relate. The components of each π_i range over the unit interval and each r_i switches between the values zero and one as the MCMC algorithm proceeds.

The K_S manifest variables that relate to strategic behavior connect to the common latent variables straightforwardly. Denote these manifest variables by x_{ki} ,

⁴The precisions ψ_k have gamma priors.

⁵Details about how complaints are assigned to districts and about how v_1 and v_2 are measured are in the District Association paragraph of the Data Appendix in Mebane and Klaver (2015).

 $k = K_C + 1, \ldots, K_C + K_S$, and define

$$x_{ki} = c_k + \sum_{\ell=1}^{J} \lambda_{k\ell} \xi_{\ell i} + \epsilon_{ki} , \quad k = K_C + 1, \dots, K_C + K_S ,$$
(4)

where ϵ_{ki} is Normal with mean zero.⁶

For manifest variables \hat{f}_{ii} and $\hat{f}_{0i} = 1 - \hat{f}_{ii}$ we use M_i , the number of polling stations in district *i*, to specify a Dirichlet likelihood for the probability vector $(\hat{f}_{ii}, \hat{f}_{0i})$ that depends on the common latent variables.⁷ The loglikelihood is

$$\mathfrak{D}(\hat{f}_{ii}, \hat{f}_{0i}) = \log \Gamma\left(\sum_{j \in \{0, i\}} \zeta_{ji}\right) - \sum_{j \in \{0, i\}} \log \Gamma(\zeta_{ji}) + \sum_{j \in \{0, i\}} (\zeta_{ji} - 1) \log(\hat{f}_{ji})$$
(5a)

$$\zeta_{0i} = M_i \frac{1}{1 + \exp(x_{ki})}, \quad k = K_C + K_S + 1 \tag{5b}$$

$$\zeta_{ii} = M_i \frac{\exp(x_{ki})}{1 + \exp(x_{ki})}, \quad k = K_C + K_S + 1$$
(5c)

$$x_{ki} = c_k + \sum_{\ell=1}^{J} \lambda_{k\ell} \xi_{\ell i}, \quad k = K_C + K_S + 1.$$
 (5d)

Evidently x_{ki} in (5d) has the same form as x_{ki} in (1), also summarizing generalized linear connections to the common latent variables. If $\lambda_{k\ell}$, $k = K_C + K_S + 1$, for a latent variable is positive, then an increase in the latent variable tends to go with higher values of f_i and lower values of f_0 . Including M_i in the likelihood ensures that potentially important information about varying district sizes—measured by the numbers of polling stations—is not ignored.

The estimates $\hat{\alpha}_i$ and $\hat{\theta}_i$ are also manifest variables when they exist.⁸ For $\hat{\alpha}_i$ we specify

⁶The precisions of these variables' distributions have gamma priors.

⁷In the algorithm to estimate the finite mixture model that produces \hat{f}_{ii} and \hat{f}_{ei} , any value of \hat{f}_{ii} or \hat{f}_{ei} less than 10^{-9} is truncated to zero. Therefore for the latent variable model we set 10^{-9} as the smallest possible value of \hat{f}_{ii} or \hat{f}_{ei} . Therefore $2(10^{-9}) \leq \hat{f}_{0i} \leq 1 - 2(10^{-9})$. ⁸Recall that θ is undefined when $f_i = 0$ and α is undefined when $f_i = f_e = 0$.

a log-normal distribution and for $\hat{\theta}_i^{-1}$ we use a gamma distribution:

$$\log \hat{\alpha}_i \sim N(x_{ki}, \psi_k), \quad k = K_C + K_S + 2 \tag{6a}$$

$$\hat{\theta}_i^{-1} \sim \Gamma(x_{ki}, \psi_k), \quad k = K_C + K_S + 3 \tag{6b}$$

$$x_{ki} = c_k + \sum_{\ell=1}^{J} \lambda_{k\ell} \xi_{\ell i} , \quad k = K_C + K_S + 2, K_C + K_S + 3$$
(6c)

The scales and means of the latent variables are set by matching them to particular manifest variables, as follows. The factor loadings $\lambda_{k\ell}$ are fixed equal to zero when the manifest variable is not a measure of latent variable ξ_{ℓ} . For each latent variable there is one $\lambda_{k\ell}$ that is fixed equal to 1.0 (using a different k for each ℓ), thus establishing a unit of measurement (scale) for the latent variables. Each latent variable has $\lambda_{k\ell}$ fixed equal to 1.0 for one distinct value k; and for that value k, $\lambda_{k\ell'} = 0$ for all $\ell' > \ell$ while the $\lambda_{k\ell}$ are free to take on any value for all other combinations of manifest and latent variables. To set the mean of each latent variable we fix $c_k = 0$ for the values k that have $\lambda_{k\ell}$ fixed equal to 1.0. These lower triangular restrictions on $\lambda_{k\ell}$ and zero restrictions on c_k are sufficient to identify the parameters of an "exploratory" factor analysis model (Anderson and Amemiya 1988, 760).

2.2 Active-learning Classification Framework

We classify election complaints according to the type of issue or indicident they address. While for use in the current paper's latent variable model we use classifications performed manually for two recent election periods (Mebane and Klaver 2015), in order to process complaints from elections spanning 1949 through 2009 we use machine-assisted methods. In particular we use an active learning approach built on a SVM.

Prior to classification, we first preprocess the text of each document. This involves stemming, stop word removal, and the eventual transformation of text data into a numeric matrix. The first of these steps, stemming, reduces words to their base form, removing inflections and splitting up compound words. This is particularly important for German, which is more inflected and compounded than most other languages. Without first reducing the word space using stemming, the data would be unusually sparse and each inflection (e.g., "run," "ran," and "running") would count as a separate feature. After stemming, we remove stop words. Stop words are words we believe have low relative information value, or words we believe are uninformative to our classification task. Such words include "and," "the," "election," "complaint," etc. Finally, once we have preprocessed the words in each document, we use a "bag of words" model, converting each document into a document term matrix, where rows represent documents and columns represent each member word of the dictionary of all words in the corpus. Cell values represent the count of each word in the document.

Finally, we do one more transformation, called a TF-IDF transformation. TF-IDF transformations are used to weight word features from text data based on a measure of importance called "term frequency inverse document frequency." The idea is that words that appear very frequently across many documents will be less useful in separating documents into classes. For M terms and N documents, let $f(w_k, t_i)$ be the number of occurrences of term w_k in document $t_i, k = 1, \ldots, M$ and $i = 1, \ldots, N$. The "term document" portion of TF-IDF is

$$d_{i} = \frac{\sum_{i=1}^{N} f(w_{k}, t_{i})}{\sum_{j=1}^{M} \sum_{i=1}^{N} f(w_{k}, t_{i})},$$
(7)

or the proportion of terms in the corpus matching a given term w_i . Let $z_j(w_i) = 1$ if document j contains term w_i , otherwise $z_j(w_i) = 0$. The "inverse document frequency" measures how often a term occurs across all documents and is measured by

$$l_i = \log\left(\frac{N}{\sum_{j=1}^N z_j(w_i)}\right) \tag{8}$$

(Leopold and Kindermann 2002). The TF-IDF weight is $s_i = d_i l_i$. The end result is a

vector of TF-IDF weights S (Lan, Tan, Low and Sung 2005).

Our initial classificatory scheme was based on manually coded complaints from 2005 and 2009 (Mebane and Klaver 2015). Because we are expanding our coverage to include many more recorded election years, we first cluster complaint documents to facilitate reading a wide range of complaints from various years and of various complaint types. Along with careful reading of sampled documents from each cluster, we read from a distance using terms extracted from each cluster by TF-IDF weights. This process helps to validate and revise our classificatory scheme to better fit elections in all years. Because clusters match the complaint classes pretty well, we are able to label complaints from each year and complaint type more easily. We initially labeled 150 complaints for the algorithm, which was not a large enough training set to achieve our required accuracy.

Because we had very few labeled complaints to start with, we use active learning, an iterative supervised machine learning technique (Settles 2010). This framework uses uncertainty sampling to identify observations that we should label by hand to provide the most useful new input to the next iteration of the classifier. At each iteration, we train a SVM on labeled complaints.⁹ We use the distance from the SVM's separating hyperplane to measure model uncertainty. We then iteratively label the documents closest to the hyperplane and refit a model until an acceptable average precision, recall and F-measure are achieved.

3 Data

The manifest variables we use in our latent variable models are based on three kinds of measures. These are (1) codes representing postelection complaints filed with a committee of the *Bundestag*, (2) several measures of strategic behavior, and (3) fraud parameters estimated using a finite mixture likelihood variant of the Klimek et al. (2012) model.

⁹Eventually we also included all the complaints from 2005 and 2009, using each document's first EIRS+ code—adapting the codes to the labeling typology (see section 3.1.1 and Data Appendix sections 6.1.1 and 6.1.2)—for the label.

3.1 Postelection Complaints in Germany

As competitive elections are by definition contentious, that election disputes routinely arise should be unsurprising. States handle these issues in a variety of ways, utilizing an array of institutional forms. Disputed elections have a long history in Germany. A mandate for *Wahlprüfung* (Election Verification) was first instituted in Germany in 1871 as part of the constitution of the newly minted German *Reich* (Ziblatt 2009). This institution has evolved over time and its current constitutional form was developed as part of the 1949 *Grundgesetz* (Basic Law). The *Wahlprüfung* concept, in its post World War II configuration includes the *Bundestag* as initial arbiter for complaints and also allows for complaints to be further forwarded to the *Bundesverfassungsgericht* (Federal Constitutional Court). Since 1949, the *Ausschuss für Wahlprüfung, Immunität und Geschäftsordnung* (AWIG; Committee for Election Verification, Immunity and Rules of Procedure) of the *Bundestag* handles all complaints relating to federal elections in Germany¹⁰.

Any eligible voter may levy a complaint and the *Bundestag* investigates and issues a decision in each case, provided the complaints meet certain requirements. Throughout a given legislative session, the committee releases a series of recommendations that summarize the complaints received and suggest ways in which Germany's electoral laws could be improved. These case summaries are quite detailed (some stretch on for tens of thousands of words) and include a description of the events about which the citizen is complaining and the reasoned decision of the committee. The number of complaints in a given electoral period has varied tremendously since 1949, with the third *Bundestag* (elected in 1957) hearing a low of six complaints whereas the thirteenth *Bundestag* (elected in 1994) dealt with over 1400 complaints. There have typically been more complaints in each legislative session since reunification than previously.

In contrast to other election complaint systems—such as that used in Mexico—political

 $^{^{10}}$ For ease of reading, we use "Germany" throughout, even though between 1949 and *die Wende* (reunification) the *Bundestag* was only adjudicating disputes that took place in West Germany

parties in Germany do not play a central role in the complaint process. Complainants tend to be individuals who either directly experienced a failure of election administration or who are otherwise dissatisfied with the prevailing electoral system or political order more generally in Germany.

It is usually believed that election fraud is rare in Germany, but rare does not mean nonexistent. The AWIG and hence the *Bundestag* has never overturned any election results because of the complaints, although they did issue several recommendations on ways Germany's elections could be improved. One such situation involved the use of electronic voting machines in 2005. Several complaints in 2005 question the use of certain electronic voting machines due to their lack of a paper trail. The lack of a paper trail prevents voters from being able to positively ascertain that their vote has been recorded correctly. While the committee accepted the complaint as valid, it did not take any action to annul the results as the complainants had not actually demonstrated that the results had been manipulated using these machines. However, the committee did issue a recommendation that called on the government to investigate the allegations made by the complainants about the vulnerabilities of electronic voting (Bundestag 2000). The complainants would go on to lodge a successful complaint with the Federal Constitutional Court, which found the use of such electronic voting technology unconstitutional.

The AWIG has a standard that unless a complaint is shown to be "mandatsrelevant" the complaint will be rejected. To be mandatsrelevant, the circumstance a complaint addresses must demonstrably change the composition of the Bundestag or make this a possibility that cannot be ruled out (Ausschuss für Wahlprüfung, Immunität und Geschäftsordnung 2013). The Bundestag's failure to act on a complaint does not necessarily imply that the complaint is unfounded.

Facts motivating a complaint need not relate to election frauds in the sense of maleficent acts that distort votes. Because complaints generally come from citizens and not from political parties, they may be unlikely to result from partian motivations. But some

complaints are effectively partisan, such as those that relate to candidates' inability to get on the ballot, or would-be parties' frustration at not being treated as parties. Citizens may also act strategically with their complaints just as they do with their votes.

3.1.1 Classifying complaints

Previous efforts have examined only the number of complaints in a given district for a given election, without considering the nature of each dispute (Ziblatt 2009; Breunig and Goerres 2011; Mares and Zhu 2015).¹¹ As there are thousands of detailed complaints, manually coding them would have been impractical, so we developed the active-learning framework (see section 2.2) to machine-code these documents. The complaint labels are categories slightly changed from Mebane and Klaver (2015), which were themselves adapted from the coding scheme developed for the Election Incident Reporting System in the United States (Verified Voting Foundation 2005; Hall 2005; Johnson 2005).¹² The labels are as follows:

- Absentee-ballot related problem
- Registration related problem
- ID related problem
- Criminal status related problem
- Polling place problem
- Electoral System
- Party List Not on Ballot/Ballot Access Issues
- Problems with the creation of Party Lists
- Improper counting of the votes
- Improper Statistics
- Improper Campaign Activity/Allegations of Official Corruption
- Unspecified Other/No Subject

¹¹While the scope of this paper is the post-war period, similar complaint data are available for the *Reich-stagwahlen* between 1870 and 1911.

 $^{^{12}}$ Section 6.1.2 in the Data Appendix describes the earlier coding scheme.

Detailed descriptions of this labeling scheme can be found in section 6.1.1 of the Data Appendix. The performance of the SVM classifier by label class is reported in Table 1. Overall we achieve average precision, recall and F-measure of .77, .75 and .74 respectively. Categorizing the complaints by the type of violations they allege is productive because it can allow the researcher to distinguish between different types of electoral irregularities. Under this coding scheme, it is possible to ascertain whether the complainant was taking issue with a particular administrative failure that they actually experienced or if their complaint has more to do with the nature of the electoral system itself, i.e. they object to a particular aspect of the system without alleging that any laws were broken.

3.1.2 Description of Classified Complaints

Our model classified over 2500 complaints submitted to the *Bundestag* between 1949 (the beginning of the Federal Republic) and 2009. Several trends are immediately apparent. Most obviously, as observed by Breunig and Goerres (2011), the number of complaints received has drastically increased over time, especially since reunification in 1990 (*Wahlperiode* 12) (see Figure 1). For the first several decades of the Federal Republic election disputes were uncommon. While these complaints increased marginally during the 1970s and 1980s, it was not until 1994 that complaint activity reached its peak. What sort of electoral issues drove this increase?

Figure 2 displays the number of complaints received that fell into each complaint category. Even with the log scale used for the counts in Figure 2, it is immediately apparent that complaints relating to Germany's electoral system are the most prevalent. Complaints of this nature do not allege administrative wrongdoing, rather these complainants express objections to Germany's electoral laws themselves. These complaints span a range of issues, but predominantly feature complaints about Germany's 5% threshold, about overhang mandates, and about negative vote weight.¹³ In particular, it is

 $^{^{13}}$ Negative vote-weight as enabled by overhang mandates was found unconstitutional by the *Bundesver*fassungsgericht in 2008 (Behnke 2010).

clear that these concerns drove the massive influx of complaints seen in 1994.

While complaints about the electoral system have always been prominent, there were over 1000 complaints of this type received in 1994 alone. This could be due to any number of reasons, however it is notable that the 1994 election was the first time that overhang mandates appeared in consequential numbers.

Having noted that electoral system complaints make up a large portion of the dataset as a whole, it is worth exploring the nature of the complaints that allege administrative violations, such as issues with absentee ballots, registration issues, polling place problems, etc. Figure 3 omits electoral system complaints, allegations of official corruption, and complaints that contained no substantive information. Figure 4 shows the distribution of the types of administrative complaints as percentages of all administrative complaints from each election period since WP 12 (1990). Since reunification the most frequent administrative complaints tend to concern absentee ballot problems, registration problems and polling place problems. In contrast, the percentage distribution of all complaints in 1994 (WP 13) along with a similar proportional surge of complaints about improper campaign activity or allegations of official corruption in 2002 (WP 15).

3.1.3 Manually Coded Complaints: 2005 and 2009

Two unusual events relating to election administration dominated public perceptions of the federal election in 2005: mismatched *Briefwahl* (mail ballots) in Dortmund and a *Nachwahl* (late election) in Dresden. For more details about these incidents see Mebane and Klaver (2015). We capture the Dortmund and Dresden events as distinct types in the manually coded data for 2005 but not in the machine-classified data for those years for which the distributions are displayed in Figures 2–4. Note that the situation in Dresden was controversial due to the strategic advantage held by those voters who would cast their ballots already knowing the outcome in the rest of the country. That knowledge

encouraged conservative voters to behave strategically and cast a "coalition vote," i.e., to cast their *Erststimmen* for the CDU and their *Zweitstimmen* for the FDP (Behnke 2008).

Mebane and Klaver (2015) manually code the complaint documents from 2005 and 2009 using a scheme that as much as possible follows the Election Incident Reporting System (EIRS) coding scheme developed for elections in the United States (Verified Voting Foundation 2005; Hall 2005; Johnson 2005). We use these so-called EIRS+ coded data in the latent variable model. Eighteen EIRS+ types of complaints occur in 2005, plus types referring to either *Briefwahl in Dortmund* or *Nachwahl in Dresden*. Sixteen types of complaints occur in 2009.¹⁴

The manifest variable we use in the latent variable model for the complaints of type kin district i is y_{ki} , which is a binary indicator for whether at least one complaint of type koccurs for district i. As is more fully described in Mebane and Klaver (2015), sometimes ambiguity about the district to which a complaint refers produces an ambiguous count of the number of complaint instances.¹⁵ When the ambiguity is between a count of zero and a positive value, ambiguity is induced in y_{ki} .

The ambiguity also produces variety in the totals of the binary indicators across districts. Table 2 shows two total counts for each type of binary complaint indicator in each year, the least that can occur and the most. For most types the two counts are the same, and in a few instances the counts differ by one. Despite the variations, the types of EIRS+ complaints that are the most frequent in 2005 are the same: Electoral System; Absentee-ballot Related Problem; and Polling Place Problem. In 2009 Party List Not on Ballot is the second most frequent complaint type, behind Electoral System, and Absentee-ballot Related Problem is third.

 $^{^{14}}$ We describe the postelection complaint data collection and coding in more detail in section 6.1.2 of the Data Appendix.

 $^{^{15}\}mathrm{See}$ section 6.1.2 for description of the geographic location procedures.

3.2 Measures of Strategic Behavior

Ample evidence exists to demonstrate that strategic voting occurs in the mixed system used in German federal elections (Bawn 1999; Pappi and Thurner 2002; Gschwend 2007). The *Erststimmen*, being plurality votes for a single winner, are affected by "wasted vote" reasoning such as Cox (1994) analyzes. The proportional representation tier votes (*Zweitstimmen*) exhibit "threshold insurance" strategic behavior intended to insure that key smaller parties gain seats in the *Bundestag* (Herrmann and Pappi 2008; Shikano, Herrmann and Thurner 2009).

As measures of strategic behavior we use variables that have been argued to measure effects of strategic voting. Germany's mixed system gives opportunity to observe different distributions of votes being cast in the same district at the same time under both plurality (*Erststimme*) and proportional representation (*Zweitstimme*) rules. The difference between those votes is often used as a measure of strategic behavior (e.g. Cox 1997, 83; Bawn 1999). For each of the five most prominent parties, we use variables defined as the proportion of *Zweitstimmen* for a party in a district minus the proportion of *Erststimmen* in the same district. The variables are denoted by ze-SPD, ze-CDUCSU, ze-FDP, ze-Green and ze-Left.

With the plurality election results alone, measures such as the difference between each of the top two finisher's and the third-place candidate's votes connect to strategic behavior (Cox 1994). We use two margin difference variables: the difference between the first-place candidate's proportion of *Erststimmen* and the third-place proportion (\mathfrak{M}_{13}); and the difference between the second- and third-place proportions (\mathfrak{M}_{23}).

Pericchi and Torres (2011) argue that deviations in the distribution of the second significant digits of votes signal frauds, but Mebane (2013, 2014) finds that the conditional mean of the second significant digits of votes both for the winning and second-place candidates in a plurality election varies in relation both to strategic behavior and to district imbalances. We use the means of those two candidates' polling station vote counts as measures of strategic behavior. The second-digit mean for the winning candidate is

denoted \hat{j}_1 and the second-digit mean for the second-place candidate is denoted \hat{j}_2 . Mebane (2013, 2014) finds that patterns in which such means relate to strategic behavior and to district imbalances exhibit complicated nonlinearities, so the second-digit means may not fit well in our setting that imposes generalized linear functional forms. To the extent that the means measure frauds, essential nonlinearities in their relationships to frauds may reasonably be expected as well.

Some of the measures of strategic behavior seem to be related to one another, others not. Scatterplots of the Zweitstimmen minus Erststimmen variables in 2005, in Figures 6, show no apparent relationship across districts between ze-SPD and ze-CDUCSU, but as ze-SPD increases ze-Green decreases and as ze-CDUCSU increases ze-FDP decreases. Whether these patterns reflect the consequences of wasted-vote actions or of threshold-insurance actions is of course not clear from the scatterplots. ze-Left shows structure in its relationships to the Zweitstimmen minus Erststimmen differences for other parties that is not easy to summarize.

The margin and second-digit mean variables do not appear to relate to one another in any simple fashion. Scatterplots in Figure 7 show that in 2005 \mathfrak{M}_{13} relates somewhat positively to \mathfrak{M}_{23} . In addition the joint distribution of \mathfrak{M}_{13} and \mathfrak{M}_{23} appears to be roughly bimodal, as the equilibrium analysis of Cox (1994) suggests it should be. The second-digit mean variables \hat{j}_1 and \hat{j}_2 appear unrelated both to one another and to the margin variables.

Relating the Zweitstimmen minus Erststimmen, margin and second-digit mean variables to one another in 2005 shows signs of some linear relationship between ze-SPDand ze-CDUCSU, one the one hand, and \mathfrak{M}_{13} and \mathfrak{M}_{23} , on the other (Figure 8). Some relationship between these two kinds of variables is to be expected if wasted-vote actions are part of why the Zweitstimmen proportions differ from the Erststimmen proportions. But the relationships do not appear to be very strong, and the marginal distribution of ze-SPD and ze-CDUCSU lacks the bimodality that is apparent in the distribution of \mathfrak{M}_{13} and \mathfrak{M}_{23} . This may suggest that two very different kinds of strategies are at work in these

elections: one, within each district, that is tied to wasted vote logic; and one, spanning the whole election system, that connects to motives to ensure smaller parties' gain seats in the *Bundestag* and hence to threshold insurance. The second-digit mean measures appear unrelated to the other variables.

3.3 "Fraud" Parameters

We use polling station vote count data from the 2005 and 2009 elections (Bundeswahlleiter 2010*a*,*b*) to estimate parameters f_i , f_e , α and θ (Mebane 2016) for the *Erststimmen* (single-member district plurality rule votes) in each district.¹⁶ For each district *i*, $i = 1, \ldots, 299$, we obtain estimates \hat{f}_{ii} , \hat{f}_{ei} , $\hat{\alpha}_i$ and $\hat{\theta}_i$.

4 Latent Variable Model Estimation Results

The model specification we use features K = 6 common latent dimensions. Using more or fewer common latent dimensions produces posterior distributions for the mean $(c_k \text{ and } \boldsymbol{\gamma})$ and loading $(\lambda_{k\ell})$ parameters that are severely multimodal, featuring parameters with both positive and negative modes. Posteriors in the specifications we use are all unimodal and for the most part symmetric.

We use complaints variables to set the scales for three of the common latent variables (i.e., $\lambda_{k\ell} = 1$ and $c_k = 0$) and we use variables that measure strategic behavior to set the scales for the other three common latent variables. The variables we use to set the scales of each common latent variable are as follows: ξ_1 , AbsenteeB; ξ_2 , Electoral; ξ_3 , PollingPl; ξ_4 , ze-SPD; ξ_5 , ze-CDUCSU; ξ_6 , \mathfrak{M}_{13} . The exploratory factor analysis loading pattern means that while we use the named variables to set the scales of the common latent variables, we are not doing anything to make sure that "complaint" manifest variables and "strategic" manifest variables remain separated. A manifest variable we label as "strategic" may well

¹⁶Polling stations include both in-person (Urnenwahlbezirke) and mail (Briefwahlbezirke) vote districts.

have a significantly nonzero loading for a common factor that otherwise is associated primarily with "complaint" manifest variables, and vice versa. The interpretability of the common latent variables is not guaranteed by a prespecified factor loading pattern.

The common latent variables are for the most part not correlated with one another, but the nonzero covariances that do appear already raise questions about whether the complaints variables can be sharply distinguished from the strategic variables. The pattern of covariances between latent variables differs between 2005 and 2009. None of the posterior means of the covariances between latent variables are exactly zero (see $\Phi = \Upsilon^{-1}$ in Tables 3 and 4), but most of the covariances have 95% credible intervals that include zero. The posterior means for the covariances for which the credible interval does not include zero are shown in color: green for positive and red for negative. The four nonzero covariances in 2005 are Φ_{16} (positive) and Φ_{14} , Φ_{15} and Φ_{25} (negative). In 2009 the three nonzero covariances are Φ_{46} (positive) and Φ_{15} and Φ_{36} (negative).

Nominal "complaints" latent variables are correlated with nominal "strategic" latent variables. Expressed as correlations the posterior means of the covariances that are significantly different from zero in 2005 are $\mathbf{r}_{16} = .40$, $\mathbf{r}_{14} = -.40$, $\mathbf{r}_{15} = -.47$ and $\mathbf{r}_{25} = -.49$ and in 2009 they are $\mathbf{r}_{46} = .34$, $\mathbf{r}_{15} = -.56$ and $\mathbf{r}_{36} = -.49$. In 2005 one of the nominal "complaints" common latent variables (ξ_1 , whose scale is set by AbsenteeB) is correlated with all three of the nominally "strategic" common latent variables: positively with ξ_6 , whose scale is set by \mathfrak{M}_{13} , and negatively with ξ_4 , whose scale is set by ze-SPD, and with ξ_5 , whose scale is set by ze-CDUCSU. Also in 2005 nominal "complaints" variable ξ_2 , whose scale is set by Electoral, is negatively correlated with ξ_5 . In 2009 ξ_1 is negatively correlated with ξ_5 , while nominal "strategic" variable ξ_6 is negatively correlated with a with ξ_5 , while nominal "strategic" variable ξ_6 is negatively correlated with ξ_5 , whose scale is set by Electoral.

The patterns of factor loadings estimated for the two election periods differ, although in both cases it appears that the finite mixture model parameters \hat{f}_{ii} , $\hat{\alpha}_i$ and $\hat{\theta}_i$ are connected to strategic manifest variables via latent variables. In Tables 5–10, which show 95% credible intervals and posterior medians for the loading parameters $\lambda_{k\ell}$, loadings whose 95% credible intervals contain all positive values are highlighted in green, and loadings whose 95% credible intervals contain all negative values are highlighted in red. The loadings that are fixed to set the scales of the common latent variables are highlighted in gray.

In 2005 the factor loadings show that most of the complaints manifest variables, some of the strategic manifest variables and one parameter of the finite mixture model positively depend on the first common latent variable. Most of the complaints manifest variables have significantly positive loadings on the first common latent variable, ξ_1 , including Dortmund and Dresden (Table 5). Also loading positively on ξ_1 are ze-SPD, ze-CDUCSU and $\hat{\alpha}_i$. \mathfrak{M}_{13} loads negatively on ξ_1 . ξ_1 relates both to many complaint variables and to two of the variables that measure vote switching between *Erststimmen* and *Zweitstimmen*—between votes cast under plurality rules and votes cast under proportional

representation rules. ξ_1 also relates to the finite mixture model parameter that calibrates how much vote stealing as opposed to vote manufacturing occurs. That parameter, α , is estimated to have a mean less than one—the posterior mean of $\hat{\alpha}_i = .7$ (see Table 11). But the positive value of λ_{131} means that as ξ_i increases $\hat{\alpha}_i$ increases, which implies a weaker tendency for votes to go to each district's leading party from the opposition parties in the district. As ξ_i increases ze-SPD and ze-CDUCSU tend to increase as well: even more proportional representation rule votes for the two largest parties than plurality rule votes.

The second common latent variable in 2005, ξ_2 , whose scale is set by Electoral, is measured by most of the other complaints manifest variables and one of the strategic variables (ze-CDUCSU). Loadings are in Table 5. All the loadings that have a definite sign are positive. No finite mixture model parameter is clearly related to ξ_2 .

The third common latent variable, ξ_3 , whose scale is set by PollingPl, is measured by several of the other complaints manifest variables, by \hat{j}_2 and by \hat{f}_{ii} and $\hat{\alpha}_i$ (see Table 6). Neither Dortmund nor Dresden load clearly on ξ_3 . The fact that the complaints about electronic voting technology are counted in the PollingPl variable may help focus

interpretation of ξ_3 as concerning problems with the accuracy of the results, but keep in mind that there was no demonstration that the electronic systems caused distortions. \hat{j}_2 ambiguously measures frauds and strategic behavior. The loadings for all the complaints that have a definite sign and for \hat{j}_2 are positive. Both \hat{f}_{ii} and $\hat{\alpha}_i$ have negative loadings. As ξ_3 increases incremental fraud becomes less likely, although the tendency for votes to go from opposition parties to the leading party in each district becomes stronger.

The fourth, fifth and sixth common latent variables in 2005 each have only few complaints manifest variables that load on them with a definite sign, but all are positively related to \hat{f}_{ii} , $\hat{\alpha}_i$ or both. All three latent variables have their scales set by a strategic manifest variable. ξ_4 has scale set by ze-SPD, relates positively to two complaints manifest variables (BallotRel and Disabilit) and relates positively to both \hat{f}_{ii} and $\hat{\alpha}_i$ (Table 6). ξ_5 has scale set by ze-CDUCSU, relates positively to two complaints manifest variables (PartyList and Registrat), negatively to Dresden and positively to both \hat{f}_{ii} and $\hat{\alpha}_i$ (Table 7).¹⁷ ξ_6 has scale set by \mathfrak{M}_{13} , relates positively to Dortmund and to \hat{f}_{ii} . The three nominally "strategic" latent variables all relate positively to parameters of the finite mixture model.

In 2009 the factor loading pattern is arguably simpler, if only because it varies less across common latent variables. $\hat{\theta}_i$ loads positively on all the common latent variables, and almost all the strategic manifest variables load positively on all the common latent variables as well (Tables 8–10). \hat{f}_{ii} loads with a definite sign (positive) only on the first common latent variable, and $\hat{\alpha}_i$ never loads with a definite sign on any common latent variable. As far as complaints manifest variables are concerned, all of them load positively on ξ_1 and most of them load positively on ξ_2 , ξ_3 and ξ_4 . Two of the complaints manifest variables load positively on ξ_5 and three of them load positively on ξ_6 . No complaints manifest variable has a loading that is definitely negative. Much as the strategic variables

¹⁷The negative loading for **Dresden** is understandable because the *Nachwahl in Dresden* presented a strategic opportunity for conservative voters who favored the CDU-CSU and the FDP but not for those with preferences more to the left.

are intricately related to the complaints variables via the common latent variable structure, so is the magnitude of "frauds" that would be estimated by the finite mixture model. The θ parameter is important for determining the magnitude estimated for frauds (Mebane 2016).

The substantial difference in the latent variable structure between 2005 and 2009 seems to go with a difference between the way strategic voting operated in the two elections. The 2005 election produced a Grand Coalition between the largest parties (CDU/CSU and SPD), while 2009 did not. In 2009 FDP gained in *Zweitstimmen* much more than they had in 2005. In terms of statistical indicators, the posterior mean of \mathfrak{M}_{23} is positive in 2005 but not in 2009 (Tables 11 and 12), which suggests there was more strategic vote switching in *Erststimmen* in 2005 than in 2009. \hat{j}_1 is significantly greater than 4.187 in 2005 but not in 2009, which has the same implication as do the means for \mathfrak{M}_{23} (Mebane 2013, 2014).

Overall, strategic dimensions of the data are related to dimensions that connect the various complaints. If the complaints manifest variables are plausibly interpreted as connected to frauds, then frauds are related to strategies in ways that the essentially linear relationships in the model can capture. Covariances occur between complaint-related and strategic common latent variables. Manifest variables of one broad type—complaint-related or strategic—load on common latent variables whose scale is set by a variable of the other broad type. The relations across broad types are not enough to make it impossible to characterize common latent variables as being essentially of one broad type or the other, but boundaries are not sharp.

The finite mixture model parameters relate to both complaint-related and strategic common latent variables. In the elections of 2005 and 2009, the finite mixture model measures both frauds—to the extent that the complaints can be considered as referring to frauds—and strategic behavior.

5 Discussion

If the complaints variables and the latent variables they have in common reflect real irregularities in the administration of the election, do the relationships between those latent variables and \hat{f}_{ii} , $\hat{\alpha}_i$ and $\hat{\theta}_i$ suggest that \hat{f}_{ii} , $\hat{\alpha}_i$ and $\hat{\theta}_i$ merit being described as "fraud" parameters? In both the 2005 and 2009 German federal elections, the loadings for these parameter estimates suggest that the estimates do relate in a meaningful way to irregularities. Whether these irregularities should be called *frauds* is an interpretive matter we will not try to resolve. But the parameters also relate to measures of strategic voting. The "fraud" parameters in the 2005 and 2009 elections are ambiguous. When \hat{f}_{ii} , $\hat{\alpha}_i$ or $\hat{\theta}_i$ are large, it is not clear whether the reason is that something went wrong with the voting or that voters themselves moved the votes around by acting strategically.

The parameters of the finite mixture model inspired by Klimek et al. (2012) that purport to measure the probability of election frauds sometimes also respond to strategic voting. The Klimek et al. (2012) parameters describe particular bimodal and trimodal distributions that are viewed as "unusual." But such distributions might arise as a matter of course, because of voters' strategic behavior (Mebane 2016). Strategic behavior being essential in politics, perhaps multimodal distributions should not be viewed as being generically odd.

Even though we performed analysis to relate complaints to the finite mixture model's parameters only for 2005 and 2009, we think our machine-assisted classification of complaints from 1949 through 2009 raises questions about how the complaints are interpreted by Breunig and Goerres (2011) and perhaps by Ziblatt (2009) and Mares and Zhu (2015), although the interpretation in the latter two pieces is supported by the historical research of Arsenschek (2003). Plainly a large proportion of the complaints during 1949–2009 express dissatisfaction with the electoral system and do not address specific failures in administering the election. Administrative failures can easily be or mask for election frauds, but it is more challenging to interpret the election system itself as

fraudulent. An election system may lack integrity (Norris 2014), but electoral integrity includes much more than administrative failures or frauds. Using the simple count of complaints to measure frauds fails to discriminate administrative problems from other concerns. It also introduces potential confusion due to many people complaining about the same incident—a problem we have tried to minimize in the latent variable analysis by reducing the complaints to binary indicators for each election district. We will use the same plan when we extend the latent variable analysis to other years' data.

Our latent variable analysis suggests that estimates of the "fraud" model parameters f_i , α and θ do relate meaningfully to the irregularities that provoke election complaints to the *Bundestag* in German elections. Whether the incidents that provoke the complaints should be described as "frauds" is a matter of interpretation, but the model that includes f_i , α and θ appear to be a valid but not perfect tool for measuring those incidents. That is, to be a bit more precise, the bimodal and trimodal distributions that the Klimek et al. (2012) model highlights appear to be valid but not perfect measures of the "frauds" that occur in German federal elections.

6 Appendices

The Data Appendix describes labels used in our classification algorithm and the original EIRS+ codes and data in greater detail. The Model Appendix reports BUGS code used for MCMC estimation of the latent variable models using OpenBUGS (Lunn, Spiegelhalter, Thomas and Best 2009; OpenBUGS 2013; Lunn, Jackson, Best, Thomas and Spiegelhalter 2013), along with tables that report credible intervals for means in the models.

6.1 Data Appendix

6.1.1 German complaints data (1949–2009)

Sources: One of the standing committees of the *Bundestag* is the *Ausschuss für Wahlprüfung, Immunität und Geschäftsordnung* (Committee for Election Verification, Immunity and Rules of Procedure). This committee deals with the rules of the *Bundestag*, possible criminal proceedings against *Bundestag* members and complaints about the administration of national elections (*Bundestagswahlen, Europawahlen*, etc.).

Complaint Typology Here is a list of all of the labels we applied to the complaint documents. They translate the nature of the complaint as presented in the long-form text into a more database-friendly form. Labels that are identical to a code in the EIRS+ scheme described in section 6.1.2 are marked.

- Absentee-ballot related problem: cases where complainants did not receive their absentee ballot, their absentee ballot came late, or where there were any other problems related to the preparation or administration of absentee voting. (EIRS+)
- Registration related problem: cases where complainants were not able to vote or request an absentee ballot due to problems with their registration (not registered at all or they were registered in a different location) or in cases where there were problems mailing the *Wahlbenachrichtungen* (letters that notify registered German voters when an upcoming election will take place and where they are supposed to vote). (EIRS+)
- ID related problem: as Germany does not have strict voter ID laws, many complainants demanded a more robust process for checking the identity of voters at the polling place. (EIRS+)
- Criminal status related problem: cases where problems with the administration of federal elections in prisons were alleged. (EIRS+)

- Polling place problem: includes problems related to the built environment of the polling place, the set-up of the voting booths and other temporary election structures, as well as problems with polling place workers. A few examples of issues with the built environment of a polling place would be the presence of surveillance cameras or an elevated balustrade that could hypothetically allow people to observe voters in the voting booths. Also includes complaints about political advertising displayed too close to a polling place and ballot related issues that made it difficult to protect electoral secrecy.
- Electoral System: includes complaints relating to aspects of the German electoral system (overhang mandates, the 5% threshold, the method used for turning votes into seats, etc.). Also includes complaints that do not criticize a specific aspect of the German electoral system, rather a broader issue that is related to the electoral system; these complaints generally do not allege wrongdoing, rather they signify dissatisfaction with the laws governing Germany's elections or political system more generally.
- Party List Not on Ballot/Ballot Access Issues: For the proportional representation portion of Germany's mixed electoral system, they use closed party lists at the state level. Election officials must approve parties in order for them to appear on the ballot, consequently many parties whose party lists were not recognized by the *Bundeswahlleiter* (and therefore did not appear as options under the second vote) complained about this impediment. This category also includes complaints from independent candidates about their location on the ballot or number of signatures required to get one's name on the ballot and complaints from candidates alleging that ballot design unfairly disadvantaged them.
- Problems with the creation of Party Lists: cases where complainants claim that party lists were improperly prepared. In Germany, the candidates and their order on the individual Landlists are determined by the parties themselves at a mass gathering of each party.
 (EIRS+)
- Counting of the votes: any complaint that alleges inconsistencies in vote counts or improper procedures in the preparation of those counts. (EIRS+)

- Improper Statistics: any complaint that alleges the violation of the secret ballot through the preparation of certain election statistics. In Germany, the most salient of these statistics is the *Repräsentative Wahlstatistik* (Representative Election Statistic). This determines the voting patterns of Germans differentiated by sex and age range, which is accomplished through sampling precincts throughout Germany by having them distribute marked ballots (these ballots indicate the voter's sex and age-range). This process is controversial, as numerous complainants objected to the perceived invasion of privacy. (EIRS+)
- Improper Campaign Activity/Allegations of Official Corruption: complaints that accuse government officials of involvement in various corruption schemes, previous activity in the *Ministerium für Staatssicherheit*, or otherwise challenges their eligibility to serve in the *Bundestag.* This category does not include allegations against poll workers, which would be labeled as a polling place problem. This category includes instances where the complainant felt that any of the parties' campaigns were conducted in an inappropriate manner. Finally, this category also includes complaints that allege improprieties or violations at the party conventions where they select their district candidates and finalize their party lists.
- Unspecified Other/No Subject: includes complaints where the nature of the complaint was not given (e.g. numerous complainants filed petitions consisting merely of "I dispute the 1990 *Bundestagswahl*" without any further elaboration) or documents in which the complainant never connects their complaint to any aspect of the electoral system or its administration).

6.1.2 Initial coding scheme (for 2005 and 2009 only)

Sources: All of the complaints data come from the archives of the *Bundestag*'s website. The "*Drucksache*" field represents the document number in the form "Election Period/Document Number." There is also a file number associated with every complaint in the form "WP XX/Election Year." The *Drucksache* field and the file number allow for the easy finding of the original complaint's text. The name, location, and reason fields are all taken directly from the original documents published by the relevant *Bundestag* committee. The "EIRS Coded Reason" borrows from the Election Incident Reporting System (Verified Voting Foundation 2005; Hall 2005; Johnson 2005), with a few additions necessitated by the vagaries of the German electoral system and the type of complaints that it precipitates. To determine the *Wahlkreis* (district) that corresponds with the zip code given for each case, we use a shapefile of German zip codes¹⁸ in conjunction with a shapefile that shows the district boundaries for the relevant election (occasionally, these borders were unclear, most likely due to projection differences between the two shapefiles). It should be noted that the locations given in the files are only the location of the complainant, i.e., it is entirely possible for someone to complain about an issue that they themselves did not experience—standing is not an issue.

EIRS+ **Codes:** Here is a list of the reason codes we applied to the hand-coded complaints data that differ from those used in the machine classification algorithm. All the categories previously marked "EIRS+" are also used. These codes translate the nature of the complaint as presented in the long-form text into a more database-friendly form.

- Improper Campaigning Influence: cases where the complainant encountered improper campaign advertising (for example, advertising too close to a polling place) or felt that any of the parties' campaigns were conducted in an otherwise inappropriate, if not necessarily illegal, manner.
- Disability access problem: cases where polling places or other voting-related buildings were not accessible to the disabled.
- Ballot related problem: all complaints related to the physical characteristics of the ballot and its design (the size of the ballot, the color of the ballot, the folding of the ballot, etc.)
- Polling place problem: includes problems related to the built environment of the polling place, the set-up of the voting booths and other temporary election structures, as well as problems with polling place workers. A few examples of problems with the built

¹⁸See http://arnulf.us/PLZ.

environment of a polling place would be the presence of surveillance cameras or an elevated balustrade that could hypothetically allow people to observe voters in the voting booths.

- Electoral System: includes complaints relating to specific aspects of the German electoral system (overhang mandates, the 5% threshold, the method used for turning votes into seats, etc.) Also includes complaints that do not criticize a specific aspect of the German electoral system, rather a broader issue that is related to the electoral system.
- Party List Not on Ballot/Other Ballot Access Issues: many parties whose party lists were not recognized by the *Bundeswahlleiter* (and therefore did not appear as options under the second vote) complained about this impediment. This category also includes complaints from independent candidates about their placement on the ballot.
- Improper District Boundaries: indicates a complaint that alleged improprieties in the drawing of district boundaries
- Allegations of Official Corruption: complaints that accuse various government officials of involvement in various corruption schemes (this does not include allegations of improprieties against poll workers)
- Police Harassment: complaints of this type allege that the police improperly interfered in some aspect of the electoral process. Specifically, the complaints in 2005 allege that the police impeded the legal activities of an aspiring political party
- Voter Intimidation: complaints of this type allege intimidation by polling place officials or other persons that occurred while the complainant was casting their ballot (whether in-person or via the *Briefwahl*)
- Unspecified Other: includes complaints where the nature of the complaint could not be ascertained or non-sequitur complaints.

Many of the codes that originate with EIRS are closely related (for example, many complaints coded under "Absentee-ballot related problem" involve problems with voter registration and as such could also be coded as "Registration related problems"). The same issue presents itself with many of the codes developed specifically for the German case, as they deal with numerous specific complaints about the electoral system.

To determine how to code the reason for the complaint, we consulted the "*Betreff*" (subject) field that is contained in the original *Bundestag* files, in the table of contents alongside the corresponding file number. This field gives an approximation of the nature of the complaint as parsed by the committee. The documents also contain the specifics of every complaint as well as the response of the committee. As such, both the complaints and the committee's responses can be quite lengthy. The four documents—17/2250, 17/3100, 17/4600 and 17/6300—that relate to the 2009 *Bundestagswahl* are 56, 212, 136, and 144 pages long respectively. The reason codes in the database take into account both the subject of the complaint as assigned in the table of contents and the broader enumeration of the complaint found in the body of the document.

Some complaints are assigned multiple "reason" codes (up to six). This is usually precipitated by a telltale "u.a." ("unter anderem," meaning among others) in the original *Betreff* of the complaint. The precise nature of each multifaceted complaint is completely enumerated in the main text of each complaint, as opposed to the *Betreff*. In order to receive multiple codes, a complaint had to enumerate multiple complaints that involved multiple EIRS+ codes, not simply multiple aspects of the same code. For example, in 2009 many people complained about overhang mandates, a complaint that was coded under "Electoral System." Many people also complained about the distribution of seats, which would also fall under an "Electoral System" complaint. In some cases, these complaints involved both the division of seats and overhang mandates, in which case the complaint was still simply coded as a single "Electoral System" complaint, regardless of the fact that there are two complaints about the electoral system. The codes are designed to show the subjects of the complaints, as opposed to their multiplicity.

Locating the complaints in electoral districts: To determine the district from which the complaint emanated, we use the postal codes given in the documents from the *Bundestag* in conjunction with several sets of shapefiles (one that shows the distribution of postal codes across Germany and the other that shows the division of the districts in a given *Bundestag* election). It should be noted that the locations given in the files are only the location of the complainant, i.e., it is entirely possible for someone to complain about an issue that they themselves did not experience—standing is not an issue.

Some complaints involved a more complex procedure for determining their geographic assignment. This is caused by the discrepancy in scale between the shapefile that displays all of Germany's zip codes and that which shows the boundaries of each district. There are significantly many more zip codes in Germany than districts, and occasionally it is difficult to discern whether the zip code in question is actually divided between districts or whether an apparent division is just an artifact of laying two differently scaled maps over one another. This problem is especially acute in the larger cities where there are often dozens upon dozens of zip codes. In some cases, due to the shape of both the district and zip code in question, it is clear that the discrepancy is purely a cartographic issue. In other cases, more investigation is required to determine whether a zip code straddles multiple districts.

To overcome this issue, we developed a procedure that takes this ambiguity into account. For zip codes that span multiple *Wahlkreise*, the relative portion of a given zip code's area that is in each *Wahlkreis* is used to develop weights that determine the likelihood of a complaint being located in a given district. These weights are determined by the size of the population in the portion of one zip code that is in a given *Wahlkreis*. The population numbers are the number of registered voters in each portion of the zip code that is coincident with a specific *Wahlkreis*. To determine the number of registered voters contained in a zip code and then subdivide this number between the *Wahlkreise* contained therein, we used a combination of zip code shapefiles and precinct shapefiles, augmented to include precinct-level voting data from the election in question. The procedures are

detailed in Mebane and Klaver (2015).

In the latent variable analysis we treat the complaints and their location in a binary way. The types of disputes are aggregated to the district level: if there is a positive number of complaints of a certain type then the whole district receives a "1" for that type, and if there are no complaints of a certain type in a district then it receives a "0". If the count of complaints is positive only because one of the possible locations of a geographically ambiguous complaint is a particular district, then we have an instance where we are uncertain about the assignment of a complaint to a district; in such instances we are uncertain about whether the observation y_{ki} should be $y_{ki} = 0$ or $y_{ki} = 1$ (recall page 5).

Despite the fact that there are districts with more than one complaint of the same type, we decided that considering the multiplicity of a particular type of complaint in a given district would be ill-advised for several reasons. The first is that the complaint data was coded not to represent the number of complaints of a certain type in a district, but to illuminate the type of complaints levied by citizens in a given district. In many cases, determining the precise number of disputes of the same type contained in a given complaint is a nebulous task, as often these disputes are inextricably related to one another. This is especially true of complaints that allege the unconstitutionality of various aspects of the electoral system. Second, considering the multiplicity of complaints could bias the analysis in favor of well-publicized problems with electoral administration. Just because more people filed complaints of a given nature does not mean that these complaints are more valid or serious than a complaint filed by just one individual. One example where the preceding logic would be less applicable would be in a situation where many individuals allege that their votes had not been counted or that they had otherwise been illegally barred from voting. This is not the case regarding the German disputes, however. Third, even in districts where relatively more complaints originated, the number of complainants still represents a tiny percentage of the population and reading too much into the sheer number of complaints risks distorting the resulting model.

6.2 Model Appendix

6.2.1 BUGS Code

The code we use with OpenBUGS (Lunn et al. 2009; OpenBUGS 2013; Lunn et al. 2013) to run the MCMC algorithms is as follows.

2005:

```
model{
  for(i in 1:N){
    for (j in 1:20) {
      # geo location indicator
     L[i,j] ~ dcat(pi[i,j,1:2])
      # prior for mixture probability vector
      alpha[i,j,1] <- w1[i,j]
      alpha[i,j,2] <- w2[i,j]
      pi[i,j,1:2] ~ ddirch(alpha[i,j,1:2])
    }
    for (j in 21:29) {
      z1[i,j+4] ~ dnorm(mu[i,j], psi[j])
    7
    alphapos[i] <- 1 + equals(z1[i,22],0)
    alphamu[i,2] <- 0
    alphapsi[i,2] <- 1
    alphamu[i,1] <- mu[i,22+9]
    alphapsi[i,1] <- psi[30]</pre>
    z1[i,22] ~ dnorm(alphamu[i, alphapos[i]], alphapos[i]])
    thetapos[i] <- 1 + equals(z1[i,23],0)</pre>
    thetamu[i,2] <- 1
    thetapsi[i,2] <- 1</pre>
    thetamu[i,1] <- exp(mu[i,23+9])
    thetapsi[i,1] <- psi[31]</pre>
    itheta[i] <- 1/z1[i,23]
    itheta[i] ~ dgamma(thetamu[i,thetapos[i]], thetapsi[i,thetapos[i]])
    #measurement equation model
    for(j in 1:20){
      r[i,j] <- L[i,j]-1
      y1[i,j]~dnorm(mu[i,j],psi[j])I(thd[1,z1[i,j]],thd[1,z1[i,j]+1])
      y2[i,j]~dnorm(mu[i,j],psi[j])I(thd[1,z2[i,j]],thd[1,z2[i,j]+1])
      y[i,j] <- r[i,j]*y1[i,j] + (1-r[i,j])*y2[i,j]
      ephat[i,j]<-y[i,j]-mu[i,j]</pre>
# "zero trick" Dirichlet likelihoods for fraud probabilities
    for (j in 1:1) {
```

```
p[i,j]<-exp(mu[i,j+29])/(1+exp(mu[i,30]))
  theta[i,j] <- p[i,j]*z1[i,24]
  lGtheta[i,j] <- loggam(theta[i,j])</pre>
  thp[i,j] <- (theta[i,j]-1)*log(z1[i,j+20])</pre>
  ephat[i,j+29]<-z1[i,j+20]-theta[i,j]/(theta[i,1]+theta[i,2])
}
theta[i,2] <- (1-p[i,1])*z1[i,24]
lGtheta[i,2] <- loggam(theta[i,2])</pre>
thp[i,2] <- (theta[i,2]-1)*log(1-z1[i,21])
logL[i] <- loggam(theta[i,1]+theta[i,2])-(lGtheta[i,1]+lGtheta[i,2])+(thp[i,1]+thp[i,2])</pre>
Zero[i] <- 0
Zero[i] ~ dpois(Dphi[i])
Dphi[i] <- -logL[i] + 100
# four factors
mu[i,1]<- xi[i,1]</pre>
mu[i,2]<- lam[1]*xi[i,1] + lam[29]*xi[i,2] + lam[56]*xi[i,3] + lam[82]*xi[i,4] +</pre>
  lam[107]*xi[i,5] + lam[131]*xi[i,6] +c[1] # Allegatio
mu[i,3]<- lam[2]*xi[i,1] + lam[30]*xi[i,2] + lam[57]*xi[i,3] + lam[83]*xi[i,4] +</pre>
  lam[108]*xi[i,5] + lam[132]*xi[i,6] +c[2] # Ballotrel
mu[i,4]<- lam[3]*xi[i,1] + lam[31]*xi[i,2] + lam[58]*xi[i,3] + lam[84]*xi[i,4] +</pre>
  lam[109]*xi[i,5] + lam[133]*xi[i,6] +c[3] # Countingo
mu[i,5]<- lam[4]*xi[i,1] + lam[32]*xi[i,2] + lam[59]*xi[i,3] + lam[85]*xi[i,4] +</pre>
  lam[110]*xi[i,5] + lam[134]*xi[i,6] +c[4] # Criminals
mu[i,6]<- lam[5]*xi[i,1] + lam[33]*xi[i,2] + lam[60]*xi[i,3] + lam[86]*xi[i,4] +</pre>
  lam[111]*xi[i,5] + lam[135]*xi[i,6] +c[5] # Disabilit
mu[i,7]<- lam[6]*xi[i,1] + xi[i,2]</pre>
mu[i,8]<- lam[7]*xi[i,1] + lam[34]*xi[i,2] + lam[61]*xi[i,3] + lam[87]*xi[i,4] +</pre>
  lam[112]*xi[i,5] + lam[136]*xi[i,6] +c[6] # IDrelated
mu[i,9]<- lam[8]*xi[i,1] + lam[35]*xi[i,2] + lam[62]*xi[i,3] + lam[88]*xi[i,4] +</pre>
  lam[113]*xi[i,5] + lam[137]*xi[i,6] +c[7] # ImproperC
mu[i,10]<-lam[9]*xi[i,1] + lam[36]*xi[i,2] + lam[63]*xi[i,3] + lam[89]*xi[i,4] +</pre>
  lam[114]*xi[i,5] + lam[138]*xi[i,6] +c[8] # ImproperD
mu[i,11]<-lam[10]*xi[i,1] + lam[37]*xi[i,2] + lam[64]*xi[i,3] + lam[90]*xi[i,4] +</pre>
  lam[115]*xi[i,5] + lam[139]*xi[i,6] +c[9] # ImproperS
mu[i,12]<-lam[11]*xi[i,1] + lam[38]*xi[i,2] + lam[65]*xi[i,3] + lam[91]*xi[i,4] +</pre>
  lam[116]*xi[i,5] + lam[140]*xi[i,6] +c[10] # PartyList
mu[i,13]<-lam[12]*xi[i,1] + lam[39]*xi[i,2] + lam[66]*xi[i,3] + lam[92]*xi[i,4] +</pre>
  lam[117]*xi[i,5] + lam[141]*xi[i,6] +c[11] # PoliceHar
mu[i,14] <-lam[13] *xi[i,1] + lam[40] *xi[i,2] + xi[i,3]</pre>
mu[i,15]<-lam[14]*xi[i,1] + lam[41]*xi[i,2] + lam[67]*xi[i,3] + lam[93]*xi[i,4] +</pre>
  lam[118]*xi[i,5] + lam[142]*xi[i,6] +c[12] # Problemwi
mu[i,16]<-lam[15]*xi[i,1] + lam[42]*xi[i,2] + lam[68]*xi[i,3] + lam[94]*xi[i,4] +</pre>
  lam[119]*xi[i,5] + lam[143]*xi[i,6] +c[13] # Registrat
mu[i,17]<-lam[16]*xi[i,1] + lam[43]*xi[i,2] + lam[69]*xi[i,3] + lam[95]*xi[i,4] +</pre>
  lam[120]*xi[i,5] + lam[144]*xi[i,6] +c[14] # Unspecifi
mu[i,18]<-lam[17]*xi[i,1] + lam[44]*xi[i,2] + lam[70]*xi[i,3] + lam[96]*xi[i,4] +</pre>
```

```
lam[121]*xi[i,5] + lam[145]*xi[i,6] +c[15] # Voterinti
  mu[i,19]<-lam[18]*xi[i,1] + lam[45]*xi[i,2] + lam[71]*xi[i,3] + lam[97]*xi[i,4] +</pre>
    lam[122]*xi[i,5] + lam[146]*xi[i,6] +c[16] # Dortmund
  mu[i,20]<-lam[19]*xi[i,1] + lam[46]*xi[i,2] + lam[72]*xi[i,3] + lam[98]*xi[i,4] +</pre>
    lam[123]*xi[i,5] + lam[147]*xi[i,6] +c[17] # Dresden
  mu[i,21]<-lam[20]*xi[i,1] + lam[47]*xi[i,2] + lam[73]*xi[i,3] + xi[i,4]</pre>
  mu[i,22]<-lam[21]*xi[i,1] + lam[48]*xi[i,2] + lam[74]*xi[i,3] + lam[99]*xi[i,4] +</pre>
    xi[i,5]
                                                 # CDUCSU
  mu[i,23]<-lam[22]*xi[i,1] + lam[49]*xi[i,2] + lam[75]*xi[i,3] + lam[100]*xi[i,4] +</pre>
    lam[124]*xi[i,5] + lam[148]*xi[i,6] +c[18] # FDP
  mu[i,24]<-lam[23]*xi[i,1] + lam[50]*xi[i,2] + lam[76]*xi[i,3] + lam[101]*xi[i,4] +</pre>
    lam[125]*xi[i,5] + lam[149]*xi[i,6] +c[19] # GR.NE
  mu[i,25]<-lam[24]*xi[i,1] + lam[51]*xi[i,2] + lam[77]*xi[i,3] + lam[102]*xi[i,4] +</pre>
    lam[126]*xi[i,5] + lam[150]*xi[i,6] +c[20] # DIE.LINKE
  mu[i,26]<-lam[25]*xi[i,1] + lam[52]*xi[i,2] + lam[78]*xi[i,3] + lam[103]*xi[i,4] +
    lam[127]*xi[i,5] + xi[i,6]
                                                 # fmt
  mu[i,27]<-lam[26]*xi[i,1] + lam[53]*xi[i,2] + lam[79]*xi[i,3] + lam[104]*xi[i,4] +</pre>
    lam[128]*xi[i,5] + lam[151]*xi[i,6] +c[21] # smt
  mu[i,28]<-lam[27]*xi[i,1] + lam[54]*xi[i,2] + lam[80]*xi[i,3] + lam[105]*xi[i,4] +</pre>
    lam[129]*xi[i,5] + lam[152]*xi[i,6] +c[22] # dwinner
  mu[i,29]<-lam[28]*xi[i,1] + lam[55]*xi[i,2] + lam[81]*xi[i,3] + lam[106]*xi[i,4] +</pre>
    lam[130]*xi[i,5] + lam[153]*xi[i,6] +c[23] # dsecond
  mu[i,30]<-lam[154]*xi[i,1] + lam[155]*xi[i,2] + lam[156]*xi[i,3] + lam[157]*xi[i,4] +</pre>
    lam[158]*xi[i,5] + lam[159]*xi[i,6] +c[24] # fi
  mu[i,31]<-lam[160]*xi[i,1] + lam[161]*xi[i,2] + lam[162]*xi[i,3] + lam[163]*xi[i,4] +</pre>
    lam[164]*xi[i,5] + lam[165]*xi[i,6] +c[25] # alpha
  mu[i,32]<-lam[166]*xi[i,1] + lam[167]*xi[i,2] + lam[168]*xi[i,3] + lam[169]*xi[i,4] +</pre>
    lam[170]*xi[i,5] + lam[171]*xi[i,6] +c[26] # theta
  #structural equation model
  xi[i,1:6]~dmnorm(u[1:6],phi[1:6,1:6])
}# end of i
#thresholds
for(j in 1:20){
  thd[j,1]<-alpbot
  thd[j,2]<-alpmid
  thd[j,3]<-alptop
}
for(i in 1:6){u[i]<-gam[i]}</pre>
#priors on loadings and coefficients
var.lam[1]<-4.0*psi[1]</pre>
                            var.lam[2]<-4.0*psi[2]</pre>
                                                         var.lam[3]<-4.0*psi[3]</pre>
```

```
      var.lam[1]<-4.0*ps1[1]</td>
      var.lam[2]<-4.0*ps1[2]</td>
      var.lam[3]<-4.0*ps1[3]</td>

      var.lam[4]<-4.0*ps1[4]</td>
      var.lam[5]<-4.0*ps1[5]</td>
      var.lam[6]<-4.0*ps1[6]</td>

      var.lam[7]<-4.0*ps1[7]</td>
      var.lam[8]<-4.0*ps1[8]</td>
      var.lam[9]<-4.0*ps1[9]</td>

      var.lam[10]<-4.0*ps1[10]</td>
      var.lam[11]<-4.0*ps1[11]</td>
      var.lam[12]<-4.0*ps1[12]</td>
```

```
var.lam[13]<-4.0*psi[13]</pre>
                                 var.lam[14]<-4.0*psi[14]</pre>
                                                                     var.lam[15]<-4.0*psi[15]</pre>
var.lam[16]<-4.0*psi[16]</pre>
                                 var.lam[17]<-4.0*psi[17]</pre>
                                                                     var.lam[18]<-4.0*psi[18]</pre>
var.lam[19]<-4.0*psi[19]</pre>
                                 var.lam[20]<-4.0*psi[20]</pre>
                                                                     var.lam[21]<-4.0*psi[21]</pre>
                                                                     var.lam[24]<-4.0*psi[12]</pre>
var.lam[22]<-4.0*psi[10]</pre>
                                 var.lam[23]<-4.0*psi[11]</pre>
var.lam[25]<-4.0*psi[13]
                                                                     var.lam[27]<-4.0*psi[15]
                                 var.lam[26]<-4.0*psi[14]</pre>
var.lam[28]<-4.0*psi[16]</pre>
                                 var.lam[29]<-4.0*psi[17]</pre>
                                                                     var.lam[30]<-4.0*psi[18]</pre>
var.lam[31]<-4.0*psi[19]</pre>
                                 var.lam[32]<-4.0*psi[20]</pre>
                                                                     var.lam[33]<-4.0*psi[21]</pre>
var.lam[34]<-4.0*psi[10]</pre>
                                 var.lam[35]<-4.0*psi[11]</pre>
                                                                     var.lam[36]<-4.0*psi[12]</pre>
var.lam[37]<-4.0*psi[13]</pre>
                                 var.lam[38]<-4.0*psi[14]</pre>
                                                                     var.lam[39]<-4.0*psi[15]</pre>
                                 var.lam[41]<-4.0*psi[17]</pre>
var.lam[40]<-4.0*psi[16]</pre>
                                                                     var.lam[42]<-4.0*psi[18]</pre>
var.lam[43]<-4.0*psi[19]</pre>
                                 var.lam[44]<-4.0*psi[20]</pre>
                                                                     var.lam[45]<-4.0*psi[21]</pre>
                                 var.lam[47]<-4.0*psi[11]
                                                                     var.lam[48]<-4.0*psi[12]</pre>
var.lam[46]<-4.0*psi[10]</pre>
var.lam[49]<-4.0*psi[13]</pre>
                                 var.lam[50]<-4.0*psi[14]</pre>
                                                                     var.lam[51]<-4.0*psi[15]</pre>
var.lam[52]<-4.0*psi[16]</pre>
                                 var.lam[53]<-4.0*psi[17]</pre>
                                                                     var.lam[54]<-4.0*psi[18]</pre>
var.lam[55]<-4.0*psi[19]</pre>
                                 var.lam[56]<-4.0*psi[20]</pre>
                                                                     var.lam[57]<-4.0*psi[21]</pre>
var.lam[58]<-4.0*psi[10]</pre>
                                 var.lam[59]<-4.0*psi[11]</pre>
                                                                     var.lam[60]<-4.0*psi[12]</pre>
var.lam[61]<-4.0*psi[13]</pre>
                                 var.lam[62]<-4.0*psi[14]</pre>
                                                                     var.lam[63]<-4.0*psi[15]</pre>
var.lam[64]<-4.0*psi[16]</pre>
                                 var.lam[65]<-4.0*psi[17]</pre>
                                                                     var.lam[66]<-4.0*psi[18]</pre>
var.lam[67]<-4.0*psi[19]</pre>
                                 var.lam[68]<-4.0*psi[20]
                                                                     var.lam[69]<-4.0*psi[21]</pre>
var.lam[70]<-4.0*psi[10]</pre>
                                 var.lam[71]<-4.0*psi[11]</pre>
                                                                     var.lam[72]<-4.0*psi[12]</pre>
var.lam[73]<-4.0*psi[13]</pre>
                                 var.lam[74]<-4.0*psi[14]</pre>
                                                                     var.lam[75]<-4.0*psi[15]</pre>
var.lam[76]<-4.0*psi[16]</pre>
                                 var.lam[77]<-4.0*psi[17]</pre>
                                                                    var.lam[78]<-4.0*psi[18]</pre>
var.lam[79]<-4.0*psi[19]</pre>
                                 var.lam[80]<-4.0*psi[20]
                                                                    var.lam[81]<-4.0*psi[21]</pre>
for (k in 82:106){var.lam[k]<-4.0*psi[13]}</pre>
for (k in 107:130){var.lam[k]<-4.0*psi[14]}</pre>
for (k in 131:153){var.lam[k]<-4.0*psi[15]}</pre>
for (k in 154:159){var.lam[k]<-4.0*psi[16]}</pre>
for (k in 160:165){var.lam[k]<-4.0*psi[17]}</pre>
for (k in 166:171){var.lam[k]<-4.0*psi[18]}</pre>
for(i in 1:171){lam[i]~dnorm(0.8,var.lam[i])}
var.b<-4.0*psi[1]</pre>
for(j in 1:6){gam[j]~dnorm(0.1,var.b)}
var.c<-4.0*psi[2]</pre>
  for(j in 1:26){c[j]~dnorm(0.1,var.c)}
for(j in 1:2){
  psg[j]~dgamma(10,8)
}
#priors on precisions
for(j in 1:P){
```

```
36
```

psi[j]^{dgamma(10,8)}
sgm[j]<-1/psi[j]</pre>

}

```
phi[1:6,1:6]~dwish(R[1:6,1:6], 30)
phx[1:6,1:6]<-inverse(phi[1:6,1:6])
} #end of model</pre>
```

2009:

```
model{
  for(i in 1:N){
    for (j in 1:16) {
      # geo location indicator
      L[i,j] ~ dcat(pi[i,j,1:2])
      # prior for mixture probability vector
      alpha[i,j,1] <- w1[i,j]
      alpha[i,j,2] <- w2[i,j]
      pi[i,j,1:2] ~ ddirch(alpha[i,j,1:2])
    }
    for (j in 17:25) {
      z1[i,j+5] ~ dnorm(mu[i,j], psi[j])
    }
    alphapos[i] <- 1 + equals(z1[i,19],0)
    alphamu[i,2] <- 0
    alphapsi[i,2] <- 1</pre>
    alphamu[i,1] <- mu[i,27]
    alphapsi[i,1] <- psi[26]</pre>
    z1[i,19] ~ dnorm(alphamu[i, alphapos[i]], alphapos[i]])
    thetapos[i] <- 1 + equals(z1[i,20],0)</pre>
    thetamu[i,2] <- 1
    thetapsi[i,2] <- 1</pre>
    thetamu[i,1] <- exp(mu[i,28])</pre>
    thetapsi[i,1] <- psi[27]</pre>
    itheta[i] <- 1/z1[i,20]
    itheta[i] ~ dgamma(thetamu[i,thetapos[i]], thetapsi[i,thetapos[i]])
    #measurement equation model
    for(j in 1:16){
      r[i,j] <- L[i,j]-1
      y1[i,j]~dnorm(mu[i,j],psi[j])I(thd[1,z1[i,j]],thd[1,z1[i,j]+1])
      y2[i,j]~dnorm(mu[i,j],psi[j])I(thd[1,z2[i,j]],thd[1,z2[i,j]+1])
      y[i,j] <- r[i,j]*y1[i,j] + (1-r[i,j])*y2[i,j]
      ephat[i,j]<-y[i,j]-mu[i,j]</pre>
    ľ
# "zero trick" Dirichlet likelihoods for fraud probabilities
    for (j in 1:1) {
      p[i,j]<-exp(mu[i,j+25])/(1+exp(mu[i,26]))
      theta[i,j] <- p[i,j]*z1[i,21]
      lGtheta[i,j] <- loggam(theta[i,j])</pre>
```

```
thp[i,j] <- (theta[i,j]-1)*log(z1[i,j+16])</pre>
  ephat[i,j+16]<-z1[i,j+16]-theta[i,j]/(theta[i,1]+theta[i,2])</pre>
}
theta[i,2] <- (1-p[i,1])*z1[i,21]
lGtheta[i,2] <- loggam(theta[i,2])</pre>
thp[i,2] <- (theta[i,2]-1)*log(1-z1[i,17])</pre>
logL[i] <- loggam(theta[i,1]+theta[i,2])-(lGtheta[i,1]+lGtheta[i,2])+(thp[i,1]+thp[i,2])</pre>
Zero[i] <- 0
Zero[i] ~ dpois(Dphi[i])
Dphi[i] <- -logL[i] + 100
# three factors
mu[i,1]<- xi[i,1]</pre>
mu[i,2]<- lam[1]*xi[i,1] + lam[25]*xi[i,2] + lam[48]*xi[i,3] + lam[70]*xi[i,4] +
  lam[91]*xi[i,5] + lam[111]*xi[i,6] +c[1] # Allegatio
mu[i,3]<- lam[2]*xi[i,1] + lam[26]*xi[i,2] + lam[49]*xi[i,3] + lam[71]*xi[i,4] +</pre>
  lam[92]*xi[i,5] + lam[112]*xi[i,6] +c[2] # Ballotrel
mu[i,4]<- lam[3]*xi[i,1] + lam[27]*xi[i,2] + lam[50]*xi[i,3] + lam[72]*xi[i,4] +</pre>
  lam[93]*xi[i,5] + lam[113]*xi[i,6] +c[3] # Countingo
mu[i,5]<- lam[4]*xi[i,1] + lam[28]*xi[i,2] + lam[51]*xi[i,3] + lam[73]*xi[i,4] +</pre>
  lam[94]*xi[i,5] + lam[114]*xi[i,6] +c[4] # Criminals
mu[i,6]<- lam[5]*xi[i,1] + lam[29]*xi[i,2] + lam[52]*xi[i,3] + lam[74]*xi[i,4] +</pre>
  lam[95]*xi[i,5] + lam[115]*xi[i,6] +c[5] # Disabilit
mu[i,7]<- lam[6]*xi[i,1] + xi[i,2]</pre>
mu[i,8]<- lam[7]*xi[i,1] + lam[30]*xi[i,2] + lam[53]*xi[i,3] + lam[75]*xi[i,4] +</pre>
  lam[96]*xi[i,5] + lam[116]*xi[i,6] +c[6] # IDrelated
mu[i,9]<- lam[8]*xi[i,1] + lam[31]*xi[i,2] + lam[54]*xi[i,3] + lam[76]*xi[i,4] +</pre>
  lam[97]*xi[i,5] + lam[117]*xi[i,6] +c[7] # ImproperC
mu[i,10]<-lam[9]*xi[i,1] + lam[32]*xi[i,2] + lam[55]*xi[i,3] + lam[77]*xi[i,4] +</pre>
  lam[98]*xi[i,5] + lam[118]*xi[i,6] +c[8] # ImproperD
mu[i,11]<-lam[10]*xi[i,1] + lam[33]*xi[i,2] + lam[56]*xi[i,3] + lam[78]*xi[i,4] +</pre>
  lam[99]*xi[i,5] + lam[119]*xi[i,6] +c[9] # ImproperS
mu[i,12]<-lam[11]*xi[i,1] + lam[34]*xi[i,2] + lam[57]*xi[i,3] + lam[79]*xi[i,4] +
  lam[100]*xi[i,5] + lam[120]*xi[i,6] +c[10] # PartyList
mu[i,13]<-lam[12]*xi[i,1] + lam[35]*xi[i,2] + xi[i,3]</pre>
mu[i,14]<-lam[13]*xi[i,1] + lam[36]*xi[i,2] + lam[58]*xi[i,3] + lam[80]*xi[i,4] +</pre>
  lam[101]*xi[i,5] + lam[121]*xi[i,6] +c[11] # Problemwi
mu[i,15]<-lam[14]*xi[i,1] + lam[37]*xi[i,2] + lam[59]*xi[i,3] + lam[81]*xi[i,4] +</pre>
  lam[102]*xi[i,5] + lam[122]*xi[i,6] +c[12] # Registrat
mu[i,16]<-lam[15]*xi[i,1] + lam[38]*xi[i,2] + lam[60]*xi[i,3] + lam[82]*xi[i,4] +
  lam[103]*xi[i,5] + lam[123]*xi[i,6] +c[13] # Unspecifi
mu[i,17]<-lam[16]*xi[i,1] + lam[39]*xi[i,2] + lam[61]*xi[i,3] + xi[i,4]</pre>
mu[i,18]<-lam[17]*xi[i,1] + lam[40]*xi[i,2] + lam[62]*xi[i,3] + lam[83]*xi[i,4] +</pre>
  xi[i,5]
                                               # CDUCSU
mu[i,19]<-lam[18]*xi[i,1] + lam[41]*xi[i,2] + lam[63]*xi[i,3] + lam[84]*xi[i,4] +</pre>
  lam[104]*xi[i,5] + lam[124]*xi[i,6] +c[14] # FDP
mu[i,20]<-lam[19]*xi[i,1] + lam[42]*xi[i,2] + lam[64]*xi[i,3] + lam[85]*xi[i,4] +</pre>
```

```
lam[105]*xi[i,5] + lam[125]*xi[i,6] +c[15] # GR.NE
  mu[i,21]<-lam[20]*xi[i,1] + lam[43]*xi[i,2] + lam[65]*xi[i,3] + lam[86]*xi[i,4] +
    lam[106]*xi[i,5] + lam[126]*xi[i,6] +c[16] # DIE.LINKE
  mu[i,22]<-lam[21]*xi[i,1] + lam[44]*xi[i,2] + lam[66]*xi[i,3] + lam[87]*xi[i,4] +</pre>
    lam[107]*xi[i,5] + xi[i,6]
                                                      # fmt
  mu[i,23]<-lam[22]*xi[i,1] + lam[45]*xi[i,2] + lam[67]*xi[i,3] + lam[88]*xi[i,4] +</pre>
    lam[108]*xi[i,5] + lam[127]*xi[i,6] +c[17] # smt
  mu[i,24]<-lam[23]*xi[i,1] + lam[46]*xi[i,2] + lam[68]*xi[i,3] + lam[89]*xi[i,4] +</pre>
    lam[109]*xi[i,5] + lam[128]*xi[i,6] +c[18] # dwinner
  mu[i,25]<-lam[24]*xi[i,1] + lam[47]*xi[i,2] + lam[69]*xi[i,3] + lam[90]*xi[i,4] +</pre>
    lam[110]*xi[i,5] + lam[129]*xi[i,6] +c[19] # dsecond
  mu[i,26]<-lam[130]*xi[i,1] + lam[131]*xi[i,2] + lam[132]*xi[i,3] + lam[133]*xi[i,4] +</pre>
    lam[134]*xi[i,5] + lam[135]*xi[i,6] +c[20] # fi
  mu[i,27]<-lam[136]*xi[i,1] + lam[137]*xi[i,2] + lam[138]*xi[i,3] + lam[139]*xi[i,4] +</pre>
    lam[140]*xi[i,5] + lam[141]*xi[i,6] +c[21] # alpha
  mu[i,28]<-lam[142]*xi[i,1] + lam[143]*xi[i,2] + lam[144]*xi[i,3] + lam[145]*xi[i,4] +</pre>
    lam[146]*xi[i,5] + lam[147]*xi[i,6] +c[22] # theta
  #structural equation model
  xi[i,1:6]~dmnorm(u[1:6],phi[1:6,1:6])
}# end of i
#thresholds
for(j in 1:16){
  thd[j,1]<-alpbot
  thd[j,2]<-alpmid
  thd[j,3]<-alptop
}
for(i in 1:6){u[i]<-gam[i]}</pre>
#priors on loadings and coefficients
var.lam[1]<-4.0*psi[1]</pre>
                               var.lam[2]<-4.0*psi[2]</pre>
                                                              var.lam[3]<-4.0*psi[3]</pre>
var.lam[4]<-4.0*psi[4]</pre>
                              var.lam[5]<-4.0*psi[5]</pre>
                                                              var.lam[6]<-4.0*psi[6]</pre>
var.lam[7]<-4.0*psi[7]</pre>
                              var.lam[8]<-4.0*psi[8]</pre>
                                                              var.lam[9]<-4.0*psi[9]</pre>
                                                                var.lam[12]<-4.0*psi[12]</pre>
var.lam[10]<-4.0*psi[10]</pre>
                              var.lam[11]<-4.0*psi[11]</pre>
                                                                var.lam[15]<-4.0*psi[15]</pre>
var.lam[13]<-4.0*psi[13]</pre>
                              var.lam[14]<-4.0*psi[14]</pre>
var.lam[16]<-4.0*psi[16]</pre>
                              var.lam[17]<-4.0*psi[17]</pre>
                                                                var.lam[18]<-4.0*psi[18]</pre>
                                                                var.lam[21]<-4.0*psi[21]
var.lam[19]<-4.0*psi[19]</pre>
                               var.lam[20]<-4.0*psi[20]</pre>
var.lam[22]<-4.0*psi[10]</pre>
                               var.lam[23]<-4.0*psi[11]</pre>
                                                                var.lam[24]<-4.0*psi[12]</pre>
var.lam[25]<-4.0*psi[13]</pre>
                              var.lam[26]<-4.0*psi[14]</pre>
                                                                var.lam[27]<-4.0*psi[15]</pre>
var.lam[28]<-4.0*psi[16]</pre>
                               var.lam[29]<-4.0*psi[17]</pre>
                                                                var.lam[30]<-4.0*psi[18]
var.lam[31]<-4.0*psi[19]</pre>
                               var.lam[32]<-4.0*psi[20]</pre>
                                                                var.lam[33]<-4.0*psi[21]</pre>
                              var.lam[35]<-4.0*psi[11]</pre>
var.lam[34]<-4.0*psi[10]</pre>
                                                                var.lam[36]<-4.0*psi[12]</pre>
var.lam[37]<-4.0*psi[13]</pre>
                              var.lam[38]<-4.0*psi[14]</pre>
                                                                var.lam[39]<-4.0*psi[15]</pre>
var.lam[40]<-4.0*psi[16]</pre>
                               var.lam[41]<-4.0*psi[17]</pre>
                                                                var.lam[42]<-4.0*psi[18]</pre>
var.lam[43]<-4.0*psi[19]</pre>
                               var.lam[44]<-4.0*psi[20]</pre>
                                                                var.lam[45]<-4.0*psi[21]
```

```
var.lam[46]<-4.0*psi[10]</pre>
                                  var.lam[47]<-4.0*psi[11]</pre>
                                                                     var.lam[48]<-4.0*psi[12]</pre>
  var.lam[49]<-4.0*psi[13]</pre>
                                  var.lam[50]<-4.0*psi[14]</pre>
                                                                     var.lam[51]<-4.0*psi[15]</pre>
                                                                     var.lam[54]<-4.0*psi[18]</pre>
  var.lam[52]<-4.0*psi[16]</pre>
                                  var.lam[53]<-4.0*psi[17]</pre>
  var.lam[55]<-4.0*psi[19]</pre>
                                  var.lam[56]<-4.0*psi[20]</pre>
                                                                     var.lam[57]<-4.0*psi[21]</pre>
  var.lam[58]<-4.0*psi[10]</pre>
                                  var.lam[59]<-4.0*psi[11]</pre>
                                                                     var.lam[60]<-4.0*psi[12]
  var.lam[61]<-4.0*psi[13]</pre>
                                  var.lam[62]<-4.0*psi[14]</pre>
                                                                     var.lam[63]<-4.0*psi[15]
  var.lam[64]<-4.0*psi[16]</pre>
                                  var.lam[65]<-4.0*psi[17]</pre>
                                                                     var.lam[66]<-4.0*psi[18]</pre>
  var.lam[67]<-4.0*psi[19]</pre>
                                  var.lam[68]<-4.0*psi[20]</pre>
                                                                     var.lam[69]<-4.0*psi[21]</pre>
  var.lam[70]<-4.0*psi[10]</pre>
                                  var.lam[71]<-4.0*psi[11]</pre>
                                                                     var.lam[72]<-4.0*psi[12]</pre>
  var.lam[73]<-4.0*psi[13]</pre>
                                  var.lam[74]<-4.0*psi[14]</pre>
                                                                     var.lam[75]<-4.0*psi[15]</pre>
  var.lam[76]<-4.0*psi[16]</pre>
                                  var.lam[77]<-4.0*psi[17]</pre>
                                                                     var.lam[78]<-4.0*psi[18]
  var.lam[79]<-4.0*psi[19]</pre>
                                  var.lam[80]<-4.0*psi[20]</pre>
                                                                     var.lam[81]<-4.0*psi[21]</pre>
  for (k in 82:106){var.lam[k]<-4.0*psi[13]}</pre>
  for (k in 107:130){var.lam[k]<-4.0*psi[14]}</pre>
  for (k in 131:135){var.lam[k]<-4.0*psi[15]}</pre>
  for (k in 136:141){var.lam[k]<-4.0*psi[16]}</pre>
  for (k in 142:147){var.lam[k]<-4.0*psi[17]}
  for(i in 1:147){lam[i]~dnorm(0.8,var.lam[i])}
  var.b<-4.0*psi[1]</pre>
  for(j in 1:6){gam[j]~dnorm(0.1,var.b)}
  var.c<-4.0*psi[2]</pre>
    for(j in 1:22){c[j]~dnorm(0.1,var.c)}
  #priors on precisions
  for(j in 1:P){
    psi[j]~dgamma(10,8)
    sgm[j]<-1/psi[j]</pre>
  }
  phi[1:6,1:6]~dwish(R[1:6,1:6], 30)
  phx[1:6,1:6]<-inverse(phi[1:6,1:6])
} #end of model
```

6.2.2 Credible Intervals for Additional Model Parameters

*** Tables 11, 13 about here ***

References

- Anderson, T.W. and Yasuo Amemiya. 1988. "The Asymptotic Distributions of Estimators in Factor Analysis under General Conditions." Annals of Statistics 16:759–771.
- Arsenschek, Robert. 2003. Der Kampf um die Wahlfreiheit im Kaiserreich: zur parlamentarischen Wahlprüfung und politischen Realität der Reichstagswahlen 1871–1914. Düsseldorf: Droste. Beiträge zur Geschichte des Parlamentarismus und der politischen Parteien, Bd. 136.
- Ausschuss für Wahlprüfung, Immunität und Geschäftsordnung, Deutscher Bundestag.
 2013. "Bilanz der Tätigkeit des 1. Ausschusses: 17. Wahlperiode (2009–2013)." Berlin: Deutscher Bundestag, 2013. Web. 15 July 2014.
- Bawn, Kathleen. 1999. "Voter Responses to Electoral Complexity: Ticket Splitting, Rational Voters and Representation in the Federal Republic of Germany." British Journal of Political Science 29(3):487–505.
- Beber, Bernd and Alexandra Scacco. 2012. "What the Numbers Say: A Digit-Based Test for Election Fraud." *Political Analysis* 20(2):211–234.
- Behnke, Joachim. 2008. "Strategisches Wählen bei der Nachwahl in Dresden zur Bundestagswahl 2005." *Politische Vierteljahresschrift* 49(4):695–720.
- Behnke, Joachim. 2010. "Negatives Stimmgewicht, Erfolgswert und Uberhangmandateeinige Anmerkungen." Kritische Vierteljahresschrift für Gesetzgebung und Rechtswissenschaft (1):3–28.
- Breunig, Christian and Achim Goerres. 2011. "Searching for Electoral Irregularities in an Established Democracy: Applying Benford's Law Tests to Bundestag Elections in Unified Germany." *Electoral Studies* 30(3):534–545.
- Bundestag, Deutscher. 2000. "Fünfte Beschlussempfehlung und Bericht." Drucksache 16:19.
- Bundeswahlleiter, Der. 2010a. Wahl zum 16. Deutschen Bundestag am 18. September 2005.
 Wiesbaden: im Auftrag der Herausgebergemeinschaft. Ergebnisse der Wahlbezirksstatistik: Die Wahlleiter des Bundes und der Länder, Auszugsweise Verviefältigung und Ver-

breitung mit Quellenangaben gestattet.

- Bundeswahlleiter, Der. 2010b. Wahl zum 17. Deutschen Bundestag am 27. September 2009.
 Wiesbaden: im Auftrag der Herausgebergemeinschaft. Ergebnisse der Wahlbezirksstatistik: Die Wahlleiter des Bundes und der Länder, Auszugsweise Verviefältigung und Verbreitung mit Quellenangaben gestattet.
- Cantu, Francisco and Sebastián M. Saiegh. 2011. "Fraudulent Democracy? An Analysis of Argentina's Infamous Decade Using Supervised Machine Learning." *Political Analysis* 19(4):409–433.
- Cox, Gary W. 1994. "Strategic Voting Equilibria Under the Single Nontransferable Vote." American Political Science Review 88:608–621.
- Cox, Gary W. 1997. Making Votes Count: Strategic Coordination in the World's Electoral Systems. New York: Cambridge University Press.
- Deckert, Joseph, Mikhail Myagkov and Peter C. Ordeshook. 2011. "Benford's Law and the Detection of Election Fraud." *Political Analysis* 19(3):245–268.
- Gschwend, Thomas. 2007. "Ticket-splitting and Strategic Voting under Mixed Electoral Rules: Evidence from Germany." European Journal of Political Research 46(1):1–23.
- Hall, Joseph Lorenzo. 2005. "Towards Work-Practice Modeling of Elections and the Election Incident Reporting System (EIRS)." URL https://josephhall.org/papers/eirs_ brahms_mims.pdf (accessed July 5, 2006).
- Herrmann, Michael and Franz Urban Pappi. 2008. "Stategic Voting in German Constituencies." *Electoral Studies* 27(2):228–244.
- Hicken, Allen and Walter R. Mebane, Jr. 2015. "A Guide to Election Forensics." Working paper for IIE/USAID subaward #DFG-10-APS-UM, "Development of an Election Forensics Toolkit: Using Subnational Data to Detect Anomalies".
- Johnson, Jeff. 2005. "The EIRS 2004 and 2005 Voting Problem Incident Data: A Valuable Public Resource." URL http://triton.towson.edu/~jlazar/EIRS_factsheet.doc (accessed July 10, 2014).

- Klimek, Peter, Yuri Yegorov, Rudolf Hanel and Stefan Thurner. 2012. "Statistical Detection of Systematic Election Irregularities." *Proceedings of the National Academy of Sciences* 109(41):16469–16473.
- Lan, Man, Chew-Lim Tan, Hwee-Boon Low and Sam-Yuan Sung. 2005. A comprehensive comparative study on term weighting schemes for text categorization with support vector machines. In Special interest tracks and posters of the 14th international conference on World Wide Web. ACM pp. 1032–1033.
- Lee, Sik-Yum. 2007. Structural Equation Modelling: A Bayesian Approach. New York: Wiley.
- Leopold, Edda and Jörg Kindermann. 2002. "Text Categorization with Support Vector Machines. How to Represent Texts in Input Space?" Machine Learning 46:423–444.
- Levin, Inès, Gabe A. Cohn, Peter C. Ordeshook and R. Michael Alvarez. 2009. "Detecting Voter Fraud in an Electronic Voting Context: An Analysis of the Unlimited Reelection Vote in Venezuela." 2009 Electronic Voting Technology Workshop/Workshop on Trustworthy Elections, Montreal, August 10–11.
- Lunn, David, Christopher Jackson, Nicky Best, Andrew Thomas and David J. Spiegelhalter. 2013. The BUGS Book: A Practical Introduction to Bayesian Analysis. Boca Raton: CRC Press, Taylor & Francis Group.
- Lunn, David, David J. Spiegelhalter, Andrew Thomas and Nicky Best. 2009. "The BUGS project: Evolution, Critique and Future Directions (with Discussion)." Statistics in Medicine 28:3049–3082.
- Mares, Isabela and Boliang Zhu. 2015. "The Production of Electoral Intimidation: Economic and Political Incentives." Comparative Politics 48(1):23–43.
- Mebane, Jr., Walter R. 2010. "Fraud in the 2009 Presidential Election in Iran?" *Chance* 23:6–15.
- Mebane, Jr., Walter R. 2013. "Election Forensics: The Meanings of Precinct Vote Counts' Second Digits." Paper presented at the 2013 Summer Meeting of the Political Methodology

Society, University of Virginia, July 18–20, 2013.

Mebane, Jr., Walter R. 2014. "Election Forensics." book MS.

- Mebane, Jr., Walter R. 2016. "Election Forensics: Frauds Tests and Observation-level Frauds Probabilities." Paper presented at the 2016 Annual Meeting of the Midwest Political Science Association, Chicago, IL, April 7–10.
- Mebane, Jr., Walter R. and Jonathan Wall. 2015. "Election Frauds, Postelection Legal Challenges and Geography in Mexico." Paper presented at the 2015 Annual Meeting of the American Political Science Association, San Francisco, CA, September 3–6.
- Mebane, Jr., Walter R. and Joseph Klaver. 2015. "Election Forensics: Strategies versus Election Frauds in Germany." Paper presented at the 2015 Annual Conference of the European Political Science Association, Vienna, Austria, June 25–27, 2015.
- Montgomery, Jacob M., Santiago Olivella, Joshua D. Potter and Brian F. Crisp. 2015. "An Informed Forensics Approach to Detecting Vote Irregularities." *Political Analysis* forthcoming.
- Myagkov, Mikhail, Peter C. Ordeshook and Dimitry Shaikin. 2009. *The Forensics of Election Fraud: With Applications to Russia and Ukraine*. New York: Cambridge University Press.
- Norris, Pippa. 2014. *Why Electoral Integrity Matters*. New York: Cambridge University Press.
- OpenBUGS. 2013. "OpenBUGS." URL http://www.openbugs.net/w/Downloads (version 3.2.2 downloaded November 24, 2013).
- Pappi, Franz Urban and Paul W. Thurner. 2002. "Electoral Behaviour in a Two-vote System: Incentives for Ticket Splitting in German Bundestag Elections." *European Journal* of Political Research 41(2):207–232.
- Pericchi, Luis Raúl and David Torres. 2011. "Quick Anomaly Detection by the Newcomb-Benford Law, with Applications to Electoral Processes Data from the USA, Puerto Rico and Venezuela." *Statistical Science* 26(4):502–516.
- Settles, Burr. 2010. "Active learning literature survey." from Computer Sciences Technical

Report 1648 http://burrsettles.com/pub/settles.activelearning.pdf.

- Shikano, Susumu, Michael Herrmann and Paul W. Thurner. 2009. "Strategic Voting under Proportional Representation: Threshold Insurance in German Elections." West European Politics 32(3):634–656.
- Shikano, Susumu and Verena Mack. 2009. "When Does the Second-Digit Benford's Law-Test Signal an Election Fraud? Facts or Misleading Test Results." Jahrbücher fur Nationalökonomie und Statistik 231(5-6):719–732.
- Verified Voting Foundation. 2005. "Election Incident Reporting System." URL https: //voteprotect.org/index.php?display=EIRMapNation&tab=ED04 (accessed July 5, 2006).
- Ziblatt, Daniel. 2009. "Shaping Democratic Practice and the Causes of Electoral Fraud: The Case of Nineteenth-Century Germany." American Political Science Review 103(1):1–21.

Class	Precision	Recall	F-Measure	Support
Absentee ballot related problem	0.85	0.92	0.88	12
Counting of the votes	1.00	0.70	0.82	10
Criminal status related problem	1.00	0.67	0.80	3
Electoral System	0.68	0.90	0.78	52
ID problem	0.80	0.89	0.84	9
Improper Campaign Influ-	0.56	0.45	0.50	11
ence/Allegations of Official Corruption				
Improper Statistics	1.00	0.83	0.91	6
Other/No subject	0.56	0.71	0.63	7
Party List Ballot Access	0.67	0.36	0.47	11
Polling Place Problem	0.92	0.67	0.77	18
Problem with the creation of party	1.00	0.40	0.57	5
lists/district nominations				
Registration related problem	0.83	0.77	0.80	13
Ave. Total	0.77	0.75	0.74	15

 Table 1: SVM Classifier Performance

		20	005	20	09
Type	Description	\mathbf{I}^{a}	Π^{b}	I^a	Π^{b}
AbsenteeB	Absentee-ballot Related Problem	29	29	16	17
Electoral	Electoral System	67	68	54	54
PollingPl	Polling Place Problem	24	24	15	15
Allegatio	Allegations of Official Corruption	8	8	3	3
BallotRel	Ballot Related Problem	6	6	3	3
Countingo	Counting of the Votes	6	6	6	6
CriminalS	Criminal Status Related Problem	5	5	3	3
Disabilit	Disability Access Problem	2	2	2	2
IDrelated	Identification Related Problem	6	6	13	13
ImproperC	Improper Campaigning Influence	11	11	15	15
ImproperD	Improper District Boundaries	1	1	2	2
ImproperS	Improper Statistics	4	4	8	8
PartyList	Party List Not on Ballot	19	20	22	23
Problemwi	Problems with the Creation of Party Lists	2	2	9	9
Registrat	Registration Related Problem	20	20	5	5
Unspecifi	Unspecified Other	10	10	5	5
PoliceHar	Police Harassment	1	1	0	0
VoterInti	Voter Intimidation	1	1	0	0
Dortmund	Briefwahl in Dortmund	12	12	0	0
Dresden	Nachwahl in Dresden	35	36	0	0

Table 2: Frequency of Postelection Complaint Types, Germany 2005 & 2009

Note: Number of districts that have each type of complaint in the hand-coded data. a minimum number of districts with at least one complaint. b maximum number of districts with at least one complaint.

Source: Compiled from archives of the Bundestag's website for the Ausschuss für Wahlprüfung, Immunität und Geschäftsordnung (see the Appendix). Types are described in the Appendix.

	0.3656	-0.2159	-0.3264	-0.2854	-0.342	0.01923
	-0.2159	0.4709	-0.6819	-0.3549	-0.4654	-0.3618
$lower^a$	-0.3264	-0.6819	0.2111	-0.05212	-0.04705	-0.1264
lower	-0.2854	-0.3549	-0.05212	0.06322	-0.002089	-0.05512
	-0.342	-0.4654	-0.04705	-0.002089	$-0.002089 \\ 0.06657$	-0.05844
	0.01923	-0.3618	-0.1264	-0.05512	-0.05844	0.06489

Table 3: Common Latent Variable Covariance Matrix (Φ) , Germany 2005

	1.034	0.1246	-0.0438	7 -0.1283	3 -0.17	01 <mark>0</mark>	.1339
	0.1246	1.401	-0.206	5 -0.1	469 -0.20	26	-0.08673
mean^b	-0.04387	-0.2065	0.4714	0.022	215 0.0	4816	-0.02801
mean	-0.1283	-0.1469	0.0221	5 0.1009	0.	0382 -	-0.006862
	-0.1701	-0.2026	0.0481	6 0.03	382 0.124 2	2 –	-0.008273
	0.1339	-0.08673	-0.0280	1 - 0.0068	862 - 0.00	8273 <mark>0</mark>	.1077
	[1.595	0.5239	0.1949	-0.01666	-0.03854	0.299	9]
	0.5239	2.456	0.07471	0.02649	-0.0207	0.152	3
uppor ^c	0.1949	0.07471	0.8163	0.1167	0.1998	0.0685	4
upper ^c	-0.01666	0.02649	0.1167	0.165	0.0914	0.0308	9
	-0.03854	-0.0207	0.1998	0.0914	0.2154	0.038	3
	0.2999	0.1523	0.06854	0.03089	0.0383	0.181	7

Note: $\Phi = \Upsilon^{-1}$. n = 299. ^{*a*} 95% credible interval elementwise lower bounds, ^{*b*} elementwise posterior means, ^{*c*} 95% credible interval elementwise upper bounds.

	0.4378	-0.356 -	-0.3709	-0.1241	-0.3966	-0.2368]
	-0.356	0.2194 -	-0.4862	-0.1762	-0.3587	-0.2347
1. <i>a</i>	-0.3709	-0.4862	0.2981	-0.1853	-0.2617	-0.3611
$lower^a$	-0.1241	-0.1762 -	-0.1853	0.06024	-0.007534	0.001031
	-0.3966	-0.3587 -	-0.2617 -	-0.007534	0.1042	-5.856e-4
	-0.2368	-0.2347 -	-0.3611	0.001031	-5.856e-4	0.09324
	L					-
	0.715	-0.1012	-0.08506	-0.0265	67 -0.2116	-0.06679]
	-0.1012	0.4609	-0.192	-0.041	1 -0.0585	-0.04846
b	-0.08506	-0.192	0.5575	-0.063	6 - 0.07529	-0.1581
mean^b	-0.02657	-0.0411	-0.0636	0.08986	0.03489	0.04346
	-0.2116	-0.0585	-0.07529	0.0348	9 0.2003	0.07811
	-0.06679	-0.04846	-0.1581	0.04346	0.07811	0.1838
	L					
	[1.045	0.1588	0.1509	0.06354	-0.08428	0.06973]
	0.1588	1.06	0.03202	0.0317	0.06286	0.07649
C	0.1509	0.03202	1.01	0.04118	0.06995 –	0.03158
$upper^{c}$	0.06354	0.0317	0.04118	0.1467	0.1028	0.1096
	-0.08428	0.06286	0.06995	0.1028	0.4082	0.2326
	0.06973	0.07649 -	-0.03158	0.1096	0.2326	0.3345
	-					-

Table 4: Common Latent Variable Covariance Matrix (Φ), Germany 2009

 $\begin{bmatrix} 0.06973 & 0.07649 & -0.03158 & 0.1096 & 0.2326 & 0.3345 \end{bmatrix}$ Note: $\Phi = \Upsilon^{-1}$. n = 299. ^a 95% credible interval elementwise lower bounds, ^b elementwise posterior means, c 95% credible interval elementwise upper bounds.

Manifest		Latent '	Variable 1		Latent Variable 2			
Variable	load.	$lower^a$	mean^{b}	$upper^{c}$	load.	$lower^a$	mean^{b}	$upper^{c}$
AbsenteeB	λ_{101}		1.0^{d}					
Electoral	λ_{102}	0.4044	0.6986	0.9981	λ_{202}		1.0^d	
PollingPl	λ_{103}	1.542	2.877	5.145	λ_{203}	-0.6614	0.1395	0.8013
Allegatio	λ_{104}	0.8335	1.476	2.152	λ_{204}	0.3919	1.145	1.953
BallotRel	λ_{105}	0.01674	0.8887	2.009	λ_{205}	0.03748	0.6303	1.266
Countingo	λ_{106}	0.001618	0.6058	1.26	λ_{206}	0.3973	0.9294	1.515
CriminalS	λ_{107}	0.7967	1.46	2.265	λ_{207}	-0.2748	0.3151	0.8733
Disabilit	λ_{108}	-0.1848	0.5202	1.18	λ_{208}	0.4476	0.744	1.037
IDrelated	λ_{109}	0.4909	0.9613	1.441	λ_{209}	-0.3552	0.33	0.9996
ImproperC	λ_{110}	-0.2004	0.5099	1.168	λ_{210}	0.7748	1.472	2.3
ImproperD	λ_{111}	0.6577	1.277	1.976	λ_{211}	0.03197	0.6333	1.265
ImproperS	λ_{112}	-0.2531	0.4875	1.311	λ_{212}	-0.07786	0.9081	2.044
PartyList	λ_{113}	0.3802	1.092	1.846	λ_{213}	0.6646	1.654	2.902
PoliceHar	λ_{114}	0.1467	0.784	1.431	λ_{214}	0.3483	1.187	2.096
Problemwi	λ_{115}	-1.144	0.1448	1.36	λ_{215}	-0.9509	0.02823	0.9549
Registrat	λ_{116}	1.61	2.559	3.941	λ_{216}	-0.6108	0.1292	0.8474
Unspecifi	λ_{117}	-0.659	0.1652	1.103	λ_{217}	0.3817	0.9536	1.513
VoterInti	λ_{118}	-0.336	0.4671	1.239	λ_{218}	0.05514	0.6832	1.307
Dortmund	λ_{119}	1.217	2.139	3.334	λ_{219}	0.4132	0.7019	0.9886
Dresden	λ_{120}	0.436	1.126	1.879	λ_{220}	1.041	1.835	2.735
ze-SPD	λ_{121}	0.008381	0.1012	0.2273	λ_{221}	-0.03039	0.09112	0.1942
ze-CDUCSU	λ_{122}	0.05329	0.1488	0.2495	λ_{222}	$7.185e{-4}$	0.1263	0.2406
ze-FDP	λ_{123}	-0.04376	$8.449e{-4}$	0.04522	λ_{223}	-0.03266	0.003869	0.04075
ze-Green	λ_{124}	-0.04361	0.001176	0.04598	λ_{224}	-0.03353	0.00293	0.04002
ze-Left	λ_{125}	-0.04238	0.001429	0.0454	λ_{225}	-0.03426	0.002278	0.03966
\mathfrak{M}_{13}	λ_{126}	-0.3168 -	-0.1878	-0.0616	λ_{226}	-0.1072	0.06351	0.2123
\mathfrak{M}_{23}	λ_{127}	-0.08977 -	-0.02991	0.02317	λ_{227}	-0.03861	0.01093	0.06037
$\hat{j}_1 \\ \hat{j}_2$	λ_{128}	-0.08178 -	-0.00776	0.06031	λ_{228}	-0.06288 -	-0.004008	0.05523
\hat{j}_2	λ_{129}	-0.08512	0.009695	0.1015	λ_{229}	-0.07033	0.001829	0.07703
\hat{f}_{ii}	λ_{130}	-0.2921	0.3338	0.9947	λ_{230}	-0.1477	0.4642	1.092
$\hat{\alpha}_i$	λ_{131}	0.0226	0.229	0.4553	λ_{231}	-0.05474	0.1664	0.3876
$\hat{ heta}_i$	λ_{132}		-0.01288	0.1563	λ_{232}	-0.1344	0.01592	0.1661
- ı	· ·152	0.1011		0.2000	• \$252	0.1011	3.0100 -	0.1001

Table 5: Six-LV Model Factor Loadings, Latent Variables 1 and 2, Germany 2005

Manifest		Latent	Variable 3		Latent Variable 4			
Variable	load.	$lower^a$	mean^b	$upper^{c}$	load.	$lower^a$	mean^b	$upper^{c}$
PollingPl	λ_{303}		1.0^{d}					
Allegatio	λ_{304}	0.5171	1.247	2.024	λ_{404}	-2.42	-0.1295	1.568
BallotRel	λ_{305}	0.4723	0.7775	1.088	λ_{405}	0.1178	2.005	4.346
Countingo	λ_{306}	0.2572	1.027	1.854	λ_{406}	-0.4266	1.323	3.321
CriminalS	λ_{307}	0.6238	1.493	2.463	λ_{407}	-0.7963	0.9529	2.8
Disabilit	λ_{308}	-0.00121	0.6688	1.354	λ_{408}	0.336	2.008	4.002
IDrelated	λ_{309}	0.6703	1.835	3.319	λ_{409}	-0.726	1.087	3.096
ImproperC	λ_{310}	1.276	2.825	4.774	λ_{410}	-1.395	0.7695	2.574
ImproperD	λ_{311}	-0.3167	0.7851	1.92	λ_{411}	-1.463	0.4489	2.038
ImproperS	λ_{312}	-1.292	0.62	2.194	λ_{412}	-1.58	0.4718	2.615
PartyList	λ_{313}	-0.7126	0.2651	1.195	λ_{413}	-3.649	-0.7689	1.258
PoliceHar	λ_{314}	0.2603	1.27	2.351	λ_{414}	-0.9938	0.7623	2.413
Problemwi	λ_{315}	0.2615	1.019	1.826	λ_{415}	-0.7423	1.005	2.917
Registrat	λ_{316}	0.333	1.007	1.706	λ_{416}	-2.24	0.2331	2.006
Unspecifi	λ_{317}	0.6336	0.9281	1.227	λ_{417}	-0.5778	1.588	4.118
VoterInti	λ_{318}	0.2888	1.126	2.028	λ_{418}	-0.2724	1.344	3.085
Dortmund	λ_{319}	-1.26	-0.3197	0.5522	λ_{419}	-1.577	0.5669	2.331
Dresden	λ_{320}	-0.2468	0.4668	1.149	λ_{420}	-2.073	0.005899	1.736
ze-SPD	λ_{321}	-0.1584	-0.002915	0.16	λ_{421}		1.0^d	
ze-CDUCSU	λ_{322}	-0.2026	-0.04014	0.1189	λ_{422}	-0.2871	-0.01134	0.2798
ze-FDP	λ_{323}	-0.04358	0.004635	0.05416	λ_{423}	-0.1309	-5.263e-4	0.1307
ze-Green	λ_{324}	-0.04409	0.004454	0.05422	λ_{424}	-0.1433	-0.01223	0.1191
ze-Left	λ_{325}	-0.04449	0.003212	0.05253	λ_{425}	-0.1266	0.002623	0.134
\mathfrak{M}_{13}	λ_{326}	-0.1466	0.07904	0.2805	λ_{426}	-0.3942	-0.08156	0.2171
\mathfrak{M}_{23}	λ_{327}	-0.03267	0.03192	0.1022	λ_{427}	-0.2096	-0.04519	0.1176
\hat{j}_1	λ_{328}	-0.05774	0.017	0.1006	λ_{428}	-0.2555	-0.05274	0.1525
$\hat{j}_1 \\ \hat{j}_2$	λ_{329}	0.01042	0.1017	0.2225	λ_{429}	-0.3472	-0.08234	0.1872
$\hat{f}_{\mathrm{i}i}$	λ_{330}	-1.846	-1.09	-0.2519	λ_{430}	1.736	3.857	5.846
$\hat{\alpha}_i$	λ_{331}	-0.7191	-0.4249	-0.1305	λ_{431}	0.2724	1.053	1.739
$\hat{ heta}_i$	λ_{332}	-0.006307	0.1889	0.4166	λ_{432}	-0.6457	-0.1235	0.4039
•	002				102			

Table 6: Six-LV Model Factor Loadings, Latent Variables 3 and 4, Germany 2005

Note: n = 299. ^a 95% credible interval lower bound, ^b posterior mean, ^c 95% credible interval upper bound. d fixed parameter. Loading parameters not shown with a value are fixed at zero.

Manifest		Laten	t Variable 5		Latent Variable 6			
Variable	load.	$lower^a$	mean^b	$upper^{c}$	load.	$lower^a$	mean^{b}	$upper^{c}$
Allegatio	λ_{504}	-1.414	0.7969	3.09	λ_{604}	-0.8809	0.7731	2.307
BallotRel	λ_{505}	-1.555	0.8447	3.382	λ_{605}	-0.5827	1.344	3.421
Countingo	λ_{506}	-0.9664	1.351	3.557	λ_{606}	-2.06	0.1582	1.941
CriminalS	λ_{507}	-1.912	0.2792	2.432	λ_{607}	-1.303	0.6334	2.275
Disabilit	λ_{508}	-0.8613	1.327	3.657	λ_{608}	-0.1439	1.398	3.079
IDrelated	λ_{509}	-3.053	-0.5094	1.76	λ_{609}	-1.67	0.3841	2.096
ImproperC	λ_{510}	-0.7132	1.893	4.459	λ_{610}	-1.052	0.9306	2.715
ImproperD	λ_{511}	-0.5963	1.352	3.403	λ_{611}	-0.6771	0.8688	2.347
ImproperS	λ_{512}	-3.957	-1.238	1.166	λ_{612}	-1.073	0.6603	2.465
PartyList	λ_{513}	2.337	4.721	7.202	λ_{613}	-1.258	0.881	3.164
PoliceHar	λ_{514}	-0.5226	1.567	3.811	λ_{614}	-1.125	0.5364	1.938
Problemwi	λ_{515}	-0.782	1.522	3.969	λ_{615}	-1.088	0.6379	2.111
Registrat	λ_{516}	0.1978	2.356	4.668	λ_{616}	-0.5016	1.223	2.881
Unspecifi	λ_{517}	-2.271	0.6953	3.544	λ_{617}	-2.928 -	-0.2634	2.023
VoterInti	λ_{518}	-0.3939	1.725	3.99	λ_{618}	-0.9415	0.721	2.199
Dortmund	λ_{519}	-3.006	-0.6016	1.479	λ_{619}	0.06662	1.635	3.474
Dresden	λ_{520}	-5.624	-3.151	-0.8895	λ_{620}	-0.9668	0.7021	2.653
ze-CDUCSU	λ_{522}		1.0^d					
ze-FDP	λ_{523}	-0.1298	0.004239	0.1385	λ_{623}	-0.1097	0.01827	0.1476
ze-Green	λ_{524}	-0.1298	0.004022	0.1405	λ_{624}	-0.1174	0.01224	0.142
ze-Left	λ_{525}	-0.13	0.003642	0.1383	λ_{625}	-0.121	0.006687	0.1328
\mathfrak{M}_{13}	λ_{526}	-0.4024	-0.0907	0.2297	λ_{626}		1.0^{d}	
\mathfrak{M}_{23}	λ_{527}	-0.1885	-0.02121	0.1478	λ_{627}	-0.006731	0.1486	0.3092
\hat{j}_1	λ_{528}	-0.2776	-0.05999	0.1571	λ_{628}	-0.1066	0.09821	0.3071
\hat{j}_2	λ_{529}	-0.2572	0.03001	0.3223	λ_{629}	-0.2064	0.08452	0.3642
$\hat{j}_1 \ \hat{j}_2 \ \hat{f}_{ii}$	λ_{530}	0.567	1.975	3.564	λ_{630}	0.5779	2.287	3.925
$\hat{\alpha}_i$	λ_{531}	0.2655	0.9125	1.547	λ_{631}	-0.6616	0.01666	0.704
$\hat{ heta}_i$	λ_{532}	-0.5451	-0.01161	0.5227	λ_{632}	-0.3386	0.1662	0.6728
- <i>i</i>	1.002	0.0101	0.01101	0.01	1.032	0.0000		5.0.20

Table 7: Six-LV Model Factor Loadings, Latent Variables 5 and 6, Germany 2005

Manifest		Latent V	ariable 1			Latent	Variable 2	
Variable	load.	$lower^a$	mean^b	$upper^{c}$	load.	$lower^a$	mean^b	$upper^{c}$
AbsenteeB	λ_{101}		1.0^{d}					
Electoral	λ_{102}	0.8418	1.366	1.871	λ_{202}		1.0^{d}	
PollingPl	λ_{103}	0.9178	1.488	2.258	λ_{203}	-0.7344	-0.007504	1.05
Allegatio	λ_{104}	0.8051	1.608	2.485	λ_{204}	-0.4512	0.4076	1.208
BallotRel	λ_{105}	1.433	3.322	5.903	λ_{205}	-0.4061	0.74	2.122
Countingo	λ_{106}	0.4446	1.277	2.233	λ_{206}	-4.047	-1.41	2.161
CriminalS	λ_{107}	0.1736	0.8585	1.593	λ_{207}	-0.5664	0.5042	1.568
Disabilit	λ_{108}	0.4504	1.1	1.793	λ_{208}	0.4593	0.7559	1.05
IDrelated	λ_{109}	0.434	0.9559	1.484	λ_{209}	0.4958	0.8	1.097
ImproperC	λ_{110}	0.7163	1.27	1.794	λ_{210}	0.4992	0.7554	1.018
ImproperD	λ_{111}	0.5321	0.9761	1.464	λ_{211}	0.576	0.8053	1.038
ImproperS	λ_{112}	0.6438	1.292	2.0	λ_{212}	0.5445	0.7829	1.027
PartyList	λ_{113}	1.086	1.766	2.576	λ_{213}	-0.5006	0.489	1.679
Problemwi	λ_{114}	0.3654	1.078	1.733	λ_{214}	0.09511	0.9762	2.062
Registrat	λ_{115}	1.323	2.319	3.56	λ_{215}	-0.4602	0.4889	1.392
Unspecifi	λ_{116}	1.42	2.71	4.261	λ_{216}	-0.3083	0.7208	1.721
ze-SPD	λ_{117}	-0.05333	0.09781	0.2459	λ_{217}	0.03443	0.2139	0.3722
ze-CDUCSU	λ_{118}	0.3192	0.4563	0.6027	λ_{218}	0.277	0.4651	0.6511
ze-FDP	λ_{119}	0.1202	0.2406	0.3743	λ_{219}	0.158	0.3043	0.4602
ze-Green	λ_{120}	0.1482	0.2653	0.3987	λ_{220}	0.1858	0.3302	0.4897
ze-Left	λ_{121}	0.2062	0.3259	0.4565	λ_{221}	0.2745	0.415	0.57
\mathfrak{M}_{13}	λ_{122}	0.4265	0.5782	0.7355	λ_{222}	0.6006	0.7699	0.9481
\mathfrak{M}_{23}	λ_{123}	0.1766	0.2973	0.4325	λ_{223}	0.2678	0.4104	0.5662
\hat{j}_1	λ_{124}	0.1695	0.3276	0.4969	λ_{224}	0.2883	0.4961	0.7047
\hat{j}_2	λ_{125}	0.2592	0.4314	0.6104	λ_{225}	0.3832	0.6094	0.8463
$\hat{f}_{\mathrm{i}i}$	λ_{126}	0.2653	0.9946	1.876	λ_{226}	-1.929	-0.6045	1.01
$\hat{\alpha}_i$	λ_{127}	-0.09342	0.3593	0.7906	λ_{227}	-0.5755	0.03064	0.6397
$\hat{ heta}_i$	λ_{128}	0.4629	0.6332	0.8105	λ_{228}	0.6094	0.8081	1.014
ı	120				-220			

Table 8: Six-LV Model Factor Loadings, Latent Variables 1 and 2, Germany 2009

Note: $n=299.~^a$ 95% credible interval lower bound, b posterior mean, c 95% credible interval upper bound.

Manifest		Latent V	variable 3			Latent Va	ariable 4	
Variable	load.	$lower^a$	mean^b	$upper^{c}$	load.	$lower^a$	mean^b	$upper^{c}$
PollingPl	λ_{303}		1.0^{d}					
Allegatio	λ_{304}	-0.09753	0.6294	1.324	λ_{404}	-0.09609	0.8746	1.864
BallotRel	λ_{305}	-0.2694	0.534	1.361	λ_{405}	-0.24	0.6977	1.613
Countingo	λ_{306}	-1.545	0.401	1.72	λ_{406}	-0.00955	0.7574	1.5
CriminalS	λ_{307}	-3.214	-0.7638	1.497	λ_{407}	0.1405	1.019	1.905
Disabilit	λ_{308}	0.4826	1.554	2.568	λ_{408}	-0.4531	0.8681	2.375
IDrelated	λ_{309}	0.4553	0.7541	1.06	λ_{409}	2.635	5.287	8.156
ImproperC	λ_{310}	0.5629	0.8592	1.144	λ_{410}	0.5087	2.411	4.222
ImproperD	λ_{311}	0.5372	0.7746	1.013	λ_{411}	0.4965	0.792	1.086
ImproperS	λ_{312}	0.6474	0.8801	1.116	λ_{412}	0.4601	0.7534	1.04
PartyList	λ_{313}	0.4331	0.6802	0.9237	λ_{413}	0.5652	0.8109	1.058
Problemwi	λ_{314}	-0.00584	1.228	2.237	λ_{414}	0.5811	0.8185	1.059
Registrat	λ_{315}	-0.3566	0.5342	1.388	λ_{415}	0.5659	0.805	1.048
Unspecifi	λ_{316}	0.2548	0.9457	1.656	λ_{416}	-0.1697	0.7027	1.563
ze-SPD	λ_{317}	0.05089	0.2233	0.4223	λ_{417}		1.0^d	
ze-CDUCSU	λ_{318}	0.2085	0.44	0.6909	λ_{418}	0.06478	0.3621	0.6707
ze-FDP	λ_{319}	0.149	0.3039	0.4784	λ_{419}	0.02741	0.2314	0.455
ze-Green	λ_{320}	0.1813	0.3322	0.5036	λ_{420}	0.03449	0.2434	0.4706
ze-Left	λ_{321}	0.2746	0.423	0.5901	λ_{421}	0.07558	0.3041	0.5457
\mathfrak{M}_{13}	λ_{322}	0.6675	0.8309	1.01	λ_{422}	0.09544	0.4381	0.787
\mathfrak{M}_{23}	λ_{323}	0.2746	0.4193	0.5788	λ_{423}	0.05485	0.283	0.5175
\hat{j}_1	λ_{324}	0.3408	0.5194	0.7028	λ_{424}	0.00281	0.3166	0.6236
\hat{j}_2	λ_{325}	0.4463	0.6334	0.8254	λ_{425}	0.0802	0.4489	0.8054
$\hat{f}_{\mathrm ii}$	λ_{326}	-2.123	-0.4915	2.333	λ_{426}	-1.943	0.5988	3.889
$\hat{\alpha}_i$	λ_{327}	-0.7082	0.04048	1.135	λ_{427}	-0.5502	0.3471	1.315
$\hat{ heta}_i$	λ_{328}	0.6208	0.8106	1.008	λ_{428}	0.4711	0.7368	1.003
-	020		-		120		-	

Table 9: Six-LV Model Factor Loadings, Latent Variables 3 and 4, Germany 2009

Manifest		Latent Va	ariable 5			Latent V	Variable 6	
Variable	load.	$lower^a$	mean^b	$upper^{c}$	load.	$lower^a$	mean^b	$upper^{c}$
Allegatio	λ_{504}	-0.04466	0.8096	1.645	λ_{604}	-0.224	1.053	2.465
BallotRel	λ_{505}	0.01977	0.8608	1.732	λ_{605}	0.07904	1.218	2.497
Countingo	λ_{506}	-0.1218	0.7773	1.629	λ_{606}	-0.5269	0.8456	2.242
CriminalS	λ_{507}	-0.04332	0.7807	1.653	λ_{607}	0.08225	1.322	2.928
Disabilit	λ_{508}	-0.2392	0.6324	1.453	λ_{608}	-1.202	0.3236	1.61
IDrelated	λ_{509}	-0.3149	0.5676	1.534	λ_{609}	-0.06706	1.364	3.228
ImproperC	λ_{510}	-0.8142	0.302	1.209	λ_{610}	-1.215	0.2993	1.968
ImproperD	λ_{511}	0.1084	0.8555	1.606	λ_{611}	-0.2573	0.792	1.855
ImproperS	λ_{512}	-0.6059	0.4769	1.361	λ_{612}	-2.108	-0.6542	0.5467
PartyList	λ_{513}	-0.1466	0.8672	2.127	λ_{613}	0.8841	2.801	4.668
Problemwi	λ_{514}	-0.6109	0.2824	1.223	λ_{614}	-0.6878	0.8329	2.301
Registrat	λ_{515}	-0.21	0.6576	1.485	λ_{615}	-0.1692	1.056	2.451
Unspecifi	λ_{516}	-0.1756	0.7788	1.631	λ_{616}	-1.323	0.03473	1.184
ze-CDUCSU	λ_{518}		1.0^{d}					
ze-FDP	λ_{519}	0.1601	0.3601	0.571	λ_{619}	0.08785	0.2464	0.41
ze-Green	λ_{520}	0.2027	0.3981	0.6116	λ_{620}	0.1126	0.2667	0.4297
ze-Left	λ_{521}	0.2896	0.4907	0.6987	λ_{621}	0.1852	0.3417	0.5061
\mathfrak{M}_{13}	λ_{522}	0.4952	0.8099	1.122	λ_{622}		1.0^d	
\mathfrak{M}_{23}	λ_{523}	0.2468	0.4514	0.667	λ_{623}	0.1932	0.3555	0.5235
\hat{j}_1	λ_{524}	0.08235	0.3869	0.7359	λ_{624}	0.2073	0.4693	0.7151
$\hat{j}_1 \ \hat{j}_2$	λ_{525}	0.2506	0.57	0.9149	λ_{625}	0.1733	0.4808	0.769
$\hat{f}_{\mathrm ii}$	λ_{526}	-1.974	3.6	5.736	λ_{626}	-4.027	-1.77	1.812
\hat{lpha}_{i}	λ_{527}	-0.2673	1.215	2.037	λ_{627}	-1.438	-0.5921	0.7518
$\hat{ heta}_i$	λ_{528}	0.4872	0.7484	1.004	λ_{628}	0.5304	0.7791	1.023

Table 10: Six-LV Model Factor Loadings, Latent Variables 5 and 6, Germany 2009

Note: n = 299. ^a 95% credible interval lower bound, ^b posterior mean, ^c 95% credible interval upper bound. d fixed parameter. Loading parameters not shown with a value are fixed at zero.

variable	mean	$lower^a$	mean^{b}	$upper^{c}$
latent variable 1	γ_1	-1.949	-1.597	-0.9905
latent variable 2	γ_2	-0.5198	-0.07572	0.3442
latent variable 3	γ_3	-1.413	-0.7029	-0.1967
latent variable 4	γ_4	-0.03215	0.1225	0.342
latent variable 5	γ_5	-0.091	0.1495	0.3442
latent variable 6	γ_6	-0.0117	0.1396	0.3241
Allegatio	c_4	-7.73	-5.039	-2.561
BallotRel	c_5	-2.712	-1.503	-0.2856
Countingo	c_6	-2.948	-1.649	-0.4649
CriminalS	c_7	-2.272	-1.075	0.2022
Disabilit	c_8	-3.936	-2.458	-1.212
IDrelated	c_9	-2.438	-0.9749	0.4922
ImproperC	c_{10}	-3.55	-1.652	0.2259
ImproperD	c_{11}	-4.117	-2.666	-1.295
ImproperS	c_{12}	-5.29	-3.026	-1.39
PartyList	c_{13}	-3.195	-1.673	-0.2086
PoliceHar	c_{14}	-7.074	-4.256	-2.141
Problemwi	c_{15}	-7.055	-4.358	-2.31
Registrat	c_{16}	-1.477	-0.3469	1.042
Unspecifi	c_{17}	-3.715	-2.16	-0.7097
VoterInti	c_{18}	-5.238	-3.378	-1.824
Dortmund	c_{19}	-3.44	-2.092	-0.9539
Dresden	c_{20}	-2.857	-1.626	-0.5399
ze-FDP	c_{23}	-0.02222	0.05226	0.1268
ze-Green	c_{24}	-0.04377	0.03102	0.1066
ze-Left	c_{25}	-0.06396	0.00975	0.08374
\mathfrak{M}_{23}	c_{26}	0.1087	0.1966	0.287
\hat{j}_1 \hat{j}_2	c_{28}	4.264	4.377	4.491
\hat{j}_2	c_{29}	4.155	4.306	4.473
\hat{f}_{ii}	c_{30}	-7.704	-6.76	-5.869
$\hat{\alpha}_i$	c_{31}	0.3245	0.7096	1.066
$\hat{\theta}_i$	C_{32}	-0.234	0.07241	0.386
$\sim \iota$	032	0.201	0.01211	0.000

Table 11: Six-LV Model Means, Germany 2005

Mean parameters not shown with a value are fixed at zero.

variable	mean	$lower^a$	mean^b	$upper^{c}$
latent variable 1	γ_1	-2.481	-2.08	-1.616
latent variable 2	γ_2	0.4517	1.519	2.239
latent variable 3	γ_3	-0.1992	0.3975	1.166
latent variable 4	γ_4	-0.6606	-0.2676	0.002976
latent variable 5	γ_5	-0.24	0.09635	0.7126
latent variable 6	γ_6	-0.5059	0.01921	0.5589
Allegatio	c_4	-7.269	-4.349	-1.805
BallotRel	c_5	-3.024	-0.5553	1.227
Countingo	c_6	-3.673	0.9101	3.074
CriminalS	c_7	-4.956	-2.639	-0.3185
Disabilit	c_8	-4.144	-2.497	-1.064
IDrelated	c_9	-2.771	-0.9271	0.8228
ImproperC	c_{10}	-1.23	-0.1532	0.8794
ImproperD	c_{11}	-3.216	-2.049	-1.049
ImproperS	c_{12}	-2.675	-1.446	-0.3596
PartyList	c_{13}	-1.512	0.5528	2.234
Problemwi	c_{14}	-4.151	-2.505	-1.025
Registrat	c_{15}	-4.541	-2.489	-0.5136
Unspecifi	c_{16}	-2.462	-0.7468	0.8823
ze-FDP	c_{19}	-0.1504	-0.01647	0.107
ze-Green	C_{20}	-0.184	-0.05289	0.06991
ze-Left	c_{21}	-0.2366	-0.09503	0.03465
\mathfrak{M}_{23}	C_{22}	-0.1561	-0.00657	0.1237
\hat{j}_1	c_{24}	3.838	4.069	4.257
$\hat{j}_1 \\ \hat{j}_2$	c_{25}	3.589	3.854	4.086
$\hat{f}_{\mathrm{i}i}$	c_{26}	-3.505	-2.042	0.6826
$\hat{\alpha}_i$	c_{27}	1.102	1.648	2.422
$\hat{ heta}_i$	c_{28}	-0.7849	-0.3802	-0.009104

Table 12: Six-LV Model Means, Germany 2009

Mean parameters not shown with a value are fixed at zero.

variable	var.	$lower^a$	mean^b	$upper^{c}$
AbsenteeB	ψ_1^{-1}	0.2968	0.5544	0.819
Electoral	a/,-1	0.2092	0.3094	0.4493
PollingPl	ψ_{3}^{-1}	2.416	5.862	11.17
Allegatio	ψ_{A}	8.573	14.41	22.99
BallotRel	ψ_{5}^{-1}	0.4114	0.7681	1.407
Countingo	ψ_{6}^{-1}	0.4745	0.9108	1.698
CriminalS	ψ_{7}^{-1}	0.4021	0.7089	1.223
Disabilit	ψ_{8}^{-1}	0.4121	0.7406	1.306
IDrelated	ψ_{9}^{-1}	0.4141	0.7506	1.353
ImproperC	$\begin{array}{c} \psi_{10}^{-1} \\ \psi_{11}^{-1} \\ \psi_{12}^{-1} \\ \psi_{13}^{-1} \\ \psi_{14}^{-1} \\ \psi_{15}^{-1} \end{array}$	0.3642	0.627	1.07
ImproperD	ψ_{11}^{-1}	0.551	1.018	1.814
ImproperS	ψ_{12}^{-1}	0.6347	1.249	2.313
PartyList	ψ_{13}^{-1}	0.3658	0.5799	0.9079
PoliceHar	ψ_{14}^{-1}	1.294	3.157	7.527
Problemwi	$\begin{array}{c} \psi_{15}^{-1} \\ \psi_{16}^{-1} \\ \psi_{17}^{-1} \\ \psi_{18}^{-1} \\ \end{array}$	0.9961	2.36	6.993
Registrat	ψ_{16}^{-1}	1.359	2.651	4.737
Unspecifi	ψ_{17}^{-1}	0.6195	1.094	1.906
VoterInti	ψ_{18}^{-1}	0.7988	1.446	2.542
Dortmund	$\psi_{19}^{-1} \\ \psi_{20}^{-1}$	0.4351	0.7121	1.151
Dresden	ψ_{20}^{-1}	0.3829	0.6049	0.9472
ze-SPD	ψ_{21}^{-1}	0.08372	0.1008	0.1206
ze-CDUCSU	ψ_{22}^{-1}	0.07329	0.08881	0.1067
ze-FDP	ψ_{23}^{-1}	0.04442	0.05192	0.06074
ze-Green	ψ_{24}^{-1}	0.04463	0.05218	0.06105
ze-Left	$\begin{array}{c} \psi_{25}^{-1} \\ \psi_{25}^{-1} \\ \psi_{26}^{-1} \\ \psi_{27}^{-1} \\ \end{array}$	0.04428	0.05182	0.06068
\mathfrak{M}_{13}	ψ_{26}^{-1}	0.07582	0.09163	0.1104
\mathfrak{M}_{23}	ψ_{27}^{-1}	0.0538	0.06305	0.07387
$\hat{j}_1 \\ \hat{j}_2$	ψ_{28}^{-1}	0.0754	0.08831	0.1034
\hat{j}_2	ψ_{29}^{-1}	0.09657	0.1135	0.1332
\hat{lpha}_i	ψ_{31}^{-1}	0.2355	0.2921	0.3559
$\hat{ heta}_i$	ψ_{32}^{-1}	4.981	5.977	7.153

Table 13: Six-LV Model Uniqueness Variances, Germany 2005

variable	var.	$lower^a$	mean^b	$upper^{c}$
AbsenteeB	ψ_1^{-1}	0.6516	1.196	1.792
Electoral	ψ_{2}^{-1}	0.2241	0.3714	0.8538
PollingPl	$\psi_2^{-1} \\ \psi_3^{-1}$	0.444	0.772	1.344
Allegatio	ψ_{4}^{-1}	5.218	9.212	15.52
BallotRel	ψ_5^{-1}	0.4372	0.8138	1.507
Countingo	ψ_{6}^{-1}	0.3904	0.6812	1.178
CriminalS	ψ_{7}^{-1}	0.4065	0.7136	1.237
Disabilit	ψ_{8}^{-1}	0.4275	0.7508	1.307
IDrelated	ψ_{9}^{-1}	0.317	0.5095	0.814
ImproperC	ψ_{10}^{-1}	0.3104	0.5301	0.9192
ImproperD	ψ_{11}^{-1}	0.5198	0.9659	1.756
ImproperS	$\psi_{12}^{-1} \\ \psi_{13}^{-1}$	0.4875	0.8944	1.62
PartyList	ψ_{13}^{-1}	0.3256	0.5481	0.9473
Problemwi	ψ_{14}^{-1}	0.652	1.791	4.227
Registrat	$\psi_{15}^{-1} \\ \psi_{16}^{-1}$	2.709	7.155	13.52
Unspecifi	ψ_{16}^{-1}	0.7338	1.811	3.705
ze-SPD	ψ_{17}^{-1}	0.07453		0.1084
ze-CDUCSU	ψ_{18}^{-1}	0.07123	0.08564	0.1026
ze-FDP	ψ_{19}^{-1}	0.05239	0.06196	0.07322
ze-Green	$\psi_{20_1}^{-1}$	0.05004	0.05911	0.06965
ze-Left	ψ_{21}^{-1}	0.05085	0.06031	0.07134
\mathfrak{M}_{13}	ψ_{22}^{-1}	0.07149	0.0862	0.1036
\mathfrak{M}_{23}	ψ_{23}^{-1}	0.05152	0.06064	0.07137
\hat{j}_1	ψ_{24}^{-1}	0.07986	0.09399	0.1103
$\hat{j}_1 \\ \hat{j}_2$	ψ_{25}^{-1}	0.1079	0.1276	0.1508
\hat{lpha}_i	ψ_{27}^{-1}	0.2197	0.2679	0.3238
$\hat{ heta}_i$	ψ_{28}^{-1}	7.661	9.215	11.12

Table 14: Six-LV Model Uniqueness Variances, Germany 2009

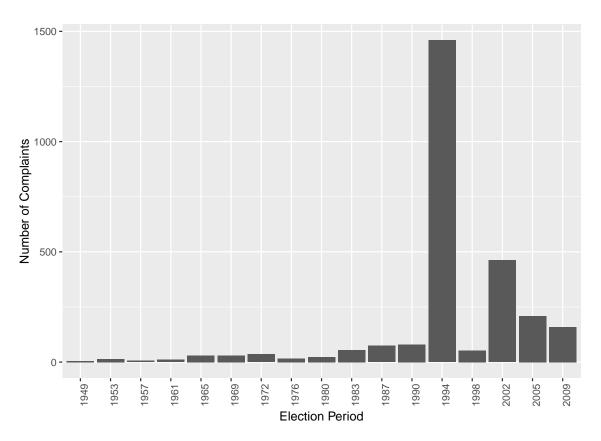


Figure 1: Election Complaints by Election Year

Source: archives of the *Bundestag*'s website for the *Ausschuss für Wahlprüfung, Immunität* und Geschäftsordnung (see the Appendix).

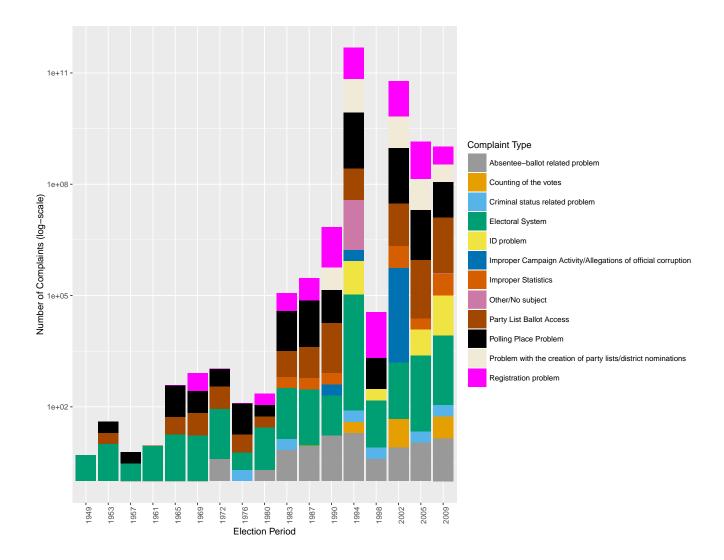


Figure 2: Election Complaints by Type and Election Year

Note: The type for each complaint is the first class assigned to the complaint by our classification algorithm.

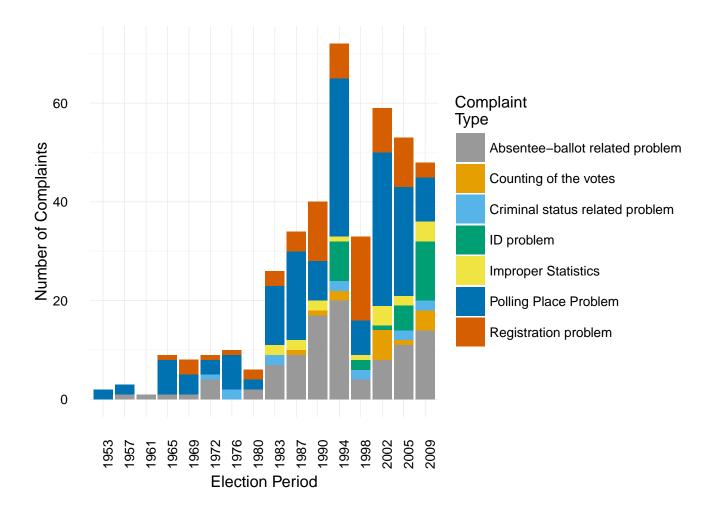


Figure 3: Administrative Complaints by Type and Election Year

Note: The type for each complaint is the first class assigned to the complaint by our classification algorithm.

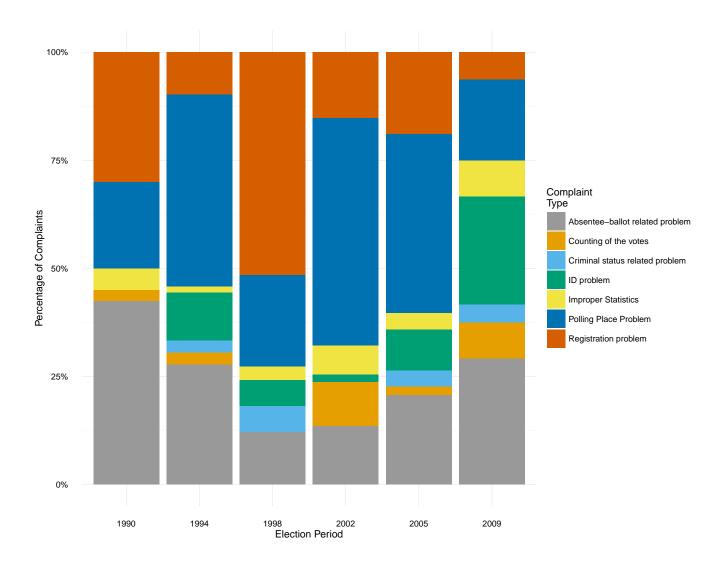


Figure 4: Administrative Complaints by Type and Year, Percentages Since Reunification

Note: The type for each complaint is the first class assigned to the complaint by our classification algorithm. Percentages are computed separately for each election period.

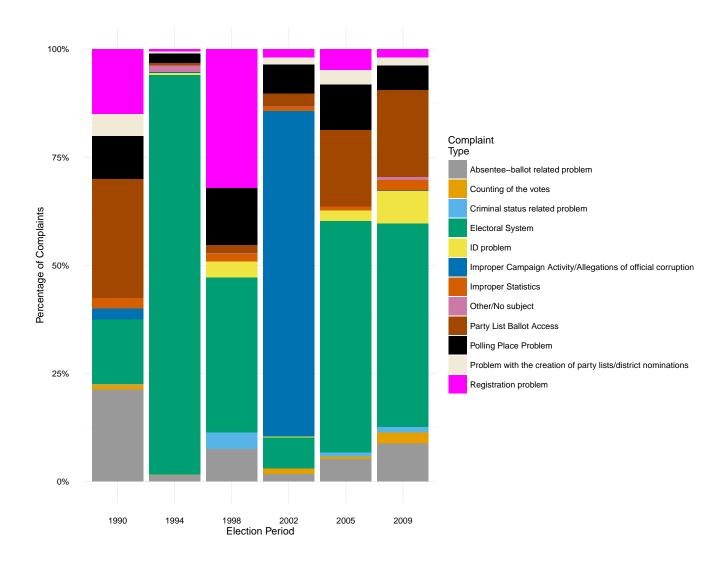


Figure 5: Election Complaints by Type and Year, Percentages Since Reunification

Note: The type for each complaint is the first class assigned to the complaint by our classification algorithm. Percentages are computed separately for each election period.

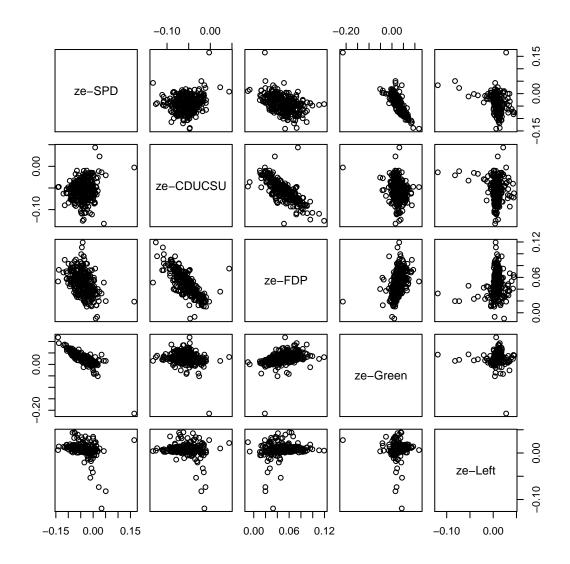


Figure 6: Strategic Voting Measures, Germany 2005: Zweitstimmen Minus Erststimmen

Note: ze-SPD, ze-CDUCSU, ze-FDP, ze-Green and ze-Left refer to the differences between the proportion of *Zweitstimmen* and of *Erststimmen* received by the referent party in each district.

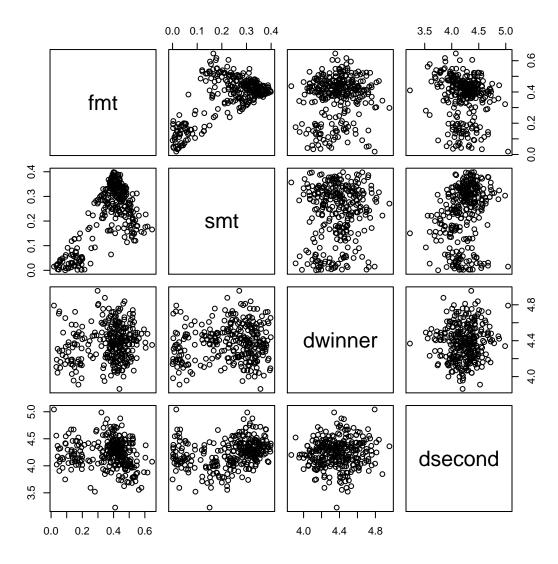


Figure 7: Strategic Voting Measures, Germany 2005: Margins and Second Digit Means

Note: fmt refers to the difference between the proportions of *Erststimmen* received by the first- and third-place parties in each district (denoted \mathfrak{M}_{13} in Tables 5–11). smt refers to the difference between the proportions of *Erststimmen* received by the second- and third-place parties (denoted \mathfrak{M}_{23} in Tables 5–11). dwinner is the mean of the second significant digits of the first-place party's polling place vote counts in each district (denoted \hat{j}_1 in Tables 5–11). dwinner is the mean of the first-place party's polling place vote counts in each district (denoted \hat{j}_2 in Tables 5–11).

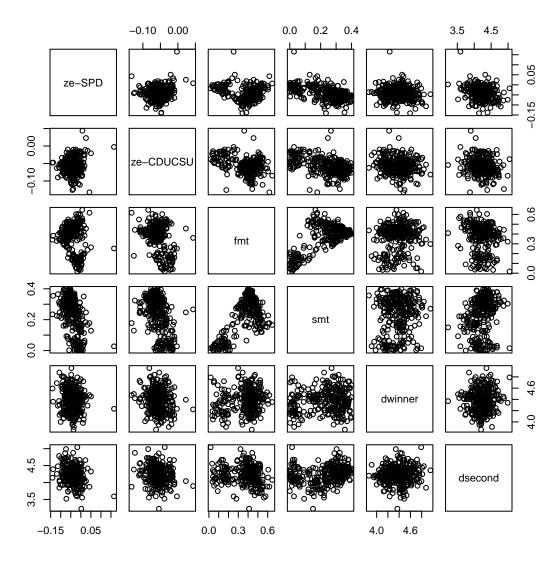


Figure 8: Strategic Voting Measures, Germany 2005

Note: see the notes in Figures 6 and 7 for descriptions of the variables.