Market Power and Innovation in the Intangible Economy*

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Abstract

Productivity growth has stagnated over the past decade. This paper argues that the rise of intangible inputs (such as information technology) can cause a slowdown of growth through the effect it has on production and competition. I hypothesize that intangibles cause a shift from variable costs to endogenous fixed costs, and use a new measure to show that the share of fixed costs in total costs rises when firms increase ICT and software investments. I then develop a quantitative framework in which intangibles reduce marginal costs and endogenously raise fixed costs, which gives firms with low adoption costs a competitive advantage. This advantage can be used to deter other firms from entering new markets and from developing higher quality products. Paradoxically, the presence of firms with high levels of intangibles can therefore reduce the rate of creative destruction and innovation. I calibrate the model using administrative data on the universe of French firms and find that, after initially boosting productivity, the rise of intangibles causes a 0.6 percentage point decline in long-term productivity growth. The model further predicts a decline in business dynamism, a fall in the labor share and an increase in markups, though markups overstate the increase in firm profits.

Keywords: Business Dynamism, Growth, Intangibles, Productivity, Market Power

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1. Introduction

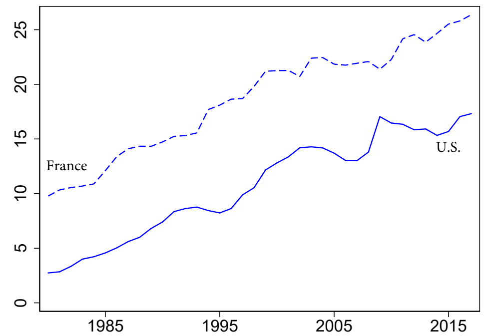
The decline of productivity growth has played a prominent role in the recent macroeconomic academic and policy debate. Fernald (2014) shows that average growth in the United States was 0.5% between 2005 and 2018, well below the long-term average of 1.2 percent.[1] The slowdown in productivity growth around 2005 followed after a decade of above-average growth, fueled by the rise of information technologies (IT). The initial surge and subsequent decline in productivity growth happened at a time that two other macroeconomic trends occurred: the slowdown of business dynamism and the rise of markups and firm concentration. Signs that dynamism is weakening include the decline in the rate at which workers reallocate to different employers (e.g. Davis et al. 2006, Decker et al. 2014) and the decline in the start-up rate (e.g. Pugsley and Șahin 2018). The rise of markups has recently attracted academic attention (e.g. De Loecker and Eeckhout 2017) and has been linked to the decline of the labor share in GDP (e.g. Karabarbounis and Neiman 2013, Kehrig and Vincent 2017). Though the timing of these trends differs across countries, they are visible across most advanced economies (Calvino et al. 2016, Diez et al. 2018). Despite the growing body of evidence detailing these trends, there is so far no consensus on what has caused them.

This paper claims that the trends in productivity growth, business dynamism, markups and the labor share can be explained jointly by a secular shift in the way that firms produce. Specifically, I claim that an increase in the use of intangible inputs in production (in particular the use of information technology/software) can drive these patterns. Figure 1 illustrates the dramatic rise of intangibles inputs: software alone is now responsible for 17% (26%) of all U.S. (French) corporate investments.[2] A key difference between intangible inputs and traditional (tangible) inputs is that intangibles *scale*; they can be duplicated at close to zero marginal cost (e.g. Haskel and Westlake 2017). This implies that when intangible inputs are used to produce a good, the cost structure of production changes. Firms need to invest in the development and maintenance of intangible inputs (which have depreciation rates upwards of 30%) but face minimal additional costs of using these intangibles when production is scaled up. An example of such an intangible input is Enterprise Resource Planning (ERP) software, which allows firms to automate business processes like supply chain and inventory management. ERP can automatically send invoices or order supplies, for example, therefore reducing the marginal cost of a sale. Alternatively, firms that sell products that include software (e.g. the operating system of a phone, the autopilot and drive-by-wire-system of a car), face no marginal costs of reproducing that software in additional units. The rise of intangibles has therefore shifted production away from variable and towards fixed costs.

[1] Adler et al. (2017) discuss evidence of similar trends across many advanced economies.

[2] This understates expenditure on software by firms, as an increasing part of software expenses is incurred 'as a service' (SaaS). This is expensed and not counted as investments.

Figure 1. Rise of Intangible Inputs: Software Investment in % of Total Investments

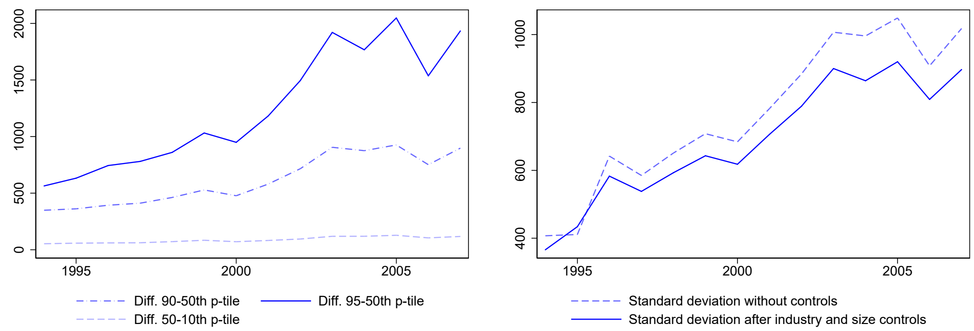


Software investments as a percentage of total private fixed investments, excluding residential investments, research and development, and entertainment. Solid line: U.S. data from BEA. Dashed line: French data (which includes database investments) from EU-KLEMS.

I show that intangible inputs modeled along these lines can explain the trends in productivity growth, business dynamism, markups and firm concentration. To do so, I develop and estimate an endogenous growth model that is both tractable and sufficiently rich to allow a quantification of the effect of intangibles. The model embeds intangibles as scalable production inputs in a framework with heterogeneous markups and endogenous entry/exit dynamics, in the spirit of Klette and Kortum (2004). Intangibles enable firms to reduce marginal costs, in exchange for a per-period cost to develop and maintain intangible inputs. These costs do not depend on the quantity that firms sell. Firms produce one or multiple products that are added or lost through creative destruction. They invest in research and development (R&D) to produce higher quality versions of goods that other firms produce. Successful innovation causes the innovator to become the new producer, while the incumbent loses the good. Firm-level innovation along this process drives aggregate growth through the step-wise improvement of random goods.

Intangible inputs introduce a new trade-off between *quality* and *price* to this class of models. In most Klette and Kortum (2004)-models, firms that innovate become the sole producer of the good when they develop a higher quality version. Other firms have the same marginal cost but are unable to produce the same quality, and hence cannot compete. Intangible inputs change this, if some firms are able to reduce their marginal costs by a greater fraction than others. Heterogeneity in the efficiency with which a firm adopts intangible inputs, for example, will cause some firms to produce their output at lower marginal costs and allow them to sell at lower prices. If a firm with less intangible-adoption develops a higher quality version of a good that one of these firms sells, the incumbent could undercut the innovator on price. Only if the quality difference is sufficiently large to offset the gap in marginal costs would the innovator become the new producer. The presence of firms with a high take-up of intangible inputs, therefore, deters other firms from entering new markets and from developing higher quality products. Paradoxically, the rise of firms with high intangible input productivity can therefore negatively affect economic growth.

Figure 2. Divergence in Intangible Inputs: Software Investments per Employee



(a) Inequality in Software Inv. per Employee (b) Standard Dev. of Software Inv. per Employee

The left figure depicts trends in the difference between various percentiles of the software-per-employee distribution. The right figure plots the standard deviation of software investments per employee after controlling for 5-digit industry fixed effects, size (3rd degree polynomial in log sales), and industry time-trends. Data: *Enquête Annuelle d'Entreprises* (EAE), discussed in Section 2.

Before quantifying the effect of scalable intangible inputs on productivity, business dynamism and markups, I provide micro evidence on the relationship between these variables. The analysis uses administrative data on the universe of French firms from 1994 to 2016, combined with survey data on innovation activities and intangible inputs. Using a new measure of fixed costs, I show that the share of fixed costs in total costs has gradually increased from 10 to 14.5%. There is a strong positive within and between-firm correlation between fixed costs and investments in software, as well as the adoption of intangible production technologies like ERP. Firms with a high fixed cost share also invest more in research and development and have higher average growth rates.

I then use the administrative data to quantify the model, and find that the rise of intangibles explains a considerable fraction of the productivity growth slowdown. To show this, I simulate the effect of an increase in the efficiency with which a fraction of firms deploy intangible inputs. In the baseline calibration, this causes steady-state growth to decline by 0.6 percentage points, while reallocation and entry decline by a third. Markups increase by 14 percentage points from 1.19 to 1.33, closely in line with the empirical increase from 1.18 to 1.29. Firms with high intangibles 'disrupt' sectors across the economy by investing more in research and development, which causes economic activity to concentrate disproportionately around these firms. As the economy transitions to the new balanced growth path, there is an initial increase in productivity as firms deploy more intangible inputs. This does not lead to an increase in wages, however, as the higher productivity is offset by higher markups. Firms without an increased intangible input efficiency have less incentive to innovate, as they are unable to offer a sufficiently low price if they enter new markets.

A central assumption of the model is that firms are heterogeneous in their use of intangible production inputs. A considerable literature explores why firms may differ in their use and efficiency of intangibles, in particular for IT. Before reviewing this, Figure 2 provides an illustration from the French data used to quantify the framework. It depicts spending on software (either developed in-house or purchased externally) per employee. Graph (a) shows that the 95th and 90th percentile to median ratios have increased strongly while spending at and below the median barely changed. Graph (b) shows that the polarization of intangible spending is also present within narrowly defined industries, even when adding flexible controls for size. Past evidence suggests that firm-heterogeneity in the productivity of IT might explain why seemingly similar firms differ in their adoption of IT. Bloom et al. (2012) show that European establishments become better at using IT if they are acquired by U.S. multinational firms, which supports the notion that IT productivity is a firm characteristic. Workplace organization and organization capital may also cause different levels of adoption, as they are important compliments to IT (e.g. Crespi et al. 2007, Bartel et al. 2007), but come at high adjustment costs (e.g. Bresnahan et al. 2002). Bloom et al. (2014) further show a tight relationship between IT adoption and structured management practices.

Related literature My theoretical framework builds on Schumpeterian growth models of creative destruction in the tradition of Aghion and Howitt (1992) and Grossman and Helpman (1993). In particular, I build on the strand of Schumpeterian models where firms produce multiple products (Klette and Kortum 2004). This framework is attractive as it is analytically tractable, yet able to replicate many empirical features of firm dynamics when quantified (Lentz and Mortensen 2008). The framework has recently been used to study the reallocation of innovative activity (Acemoglu et al. 2018), the effect of innovation policy (Atkeson and Burstein 2018) and to compare different innovation activities (Akcigit and Kerr 2018, Garcia-Macia et al. 2016). The model has also been used to analyze the effect of imperfect competition in a setting with heterogeneous markups (Peters 2018). I extend this framework with intangibles as scalable production inputs and show that, while preserving the framework's analytical flair and ability to replicate firm data, the model predicts the macroeconomic trends in productivity, business dynamism and market power.

This paper is most closely related to recent work that jointly explains low productivity growth, the fall in business dynamism and the rise of market power. Aghion et al. (2019) build a model where firms face convex overhead costs in size, which limits their expansion. Intangibles and ICT lower the overhead costs, which allows firms with ex-ante higher productivity to grow larger. My mechanism is distinct: intangibles are a production technology that allows firms to lower variable costs, at the expense of *higher* overhead (fixed) costs. This has a negative effect on innovation and business dynamism if firms differ in intangible productivity and adoption. High intangible adopters can undercut other firms on price, which has a negative effect on the innovative activity of other firms. Further, entry rates decline in my model because firms with high intangible productivity end up dominating a large fraction of all products, which makes the probability that an entrant can take over production from incumbents disproportionately low. Liu et al. (2019) relate low productivity and business dynamism to a decline in interest rates in a model where two firms compete for leadership through R&D. A lower interest rate increases investment incentives most strongly for the market leader, discouraging investments by the follower and lowering growth. Also in a two-firm setting, Akcigit and Ates (2019) find that intellectual property rights are used anti-competitively by leaders, which again discourages followers from investing. Both forms of discouragement differ from my framework, as the discouraging effect comes from the inability of low-intangible firms to compete on price. My framework is also different in that the rise of intangibles initially leads to a rise in productivity growth. It furthermore explains why markups could have increased rapidly at a time of low inflation: they were offset by a decline in marginal costs.

I also contribute to the literature on the static costs of markups. In a model where large firms charge higher markups, Edmond et al. (2018) find that markups reduce welfare by 7.5%. Baqaee and Farhi (2018) argue that eliminating markups would increase TFP by 20%, but also document that the upward trend of markups is driven by reallocation, which caused an increase in allocative efficiency. My model explains the rise of markups through a similar reallocation to high-intangible firms. The welfare effect of this is ambiguous: besides the static increase in productivity and markups, the reallocation creates a negative externality on entry and innovation by other firms.

This paper also relates to the recent literature that studies the trends in productivity and market power from a disaggregated perspective. As summarized by Van Reenen (2018), there is substantial heterogeneity in the extent to which firms are subject to these trends, causing productivity and profitability to diverge across firms. Andrews et al. (2016) for example show that productivity growth at the most productive firms within 2-digit industries has not declined across the OECD. Decker et al. (2018) similarly find an increase in productivity dispersion within the U.S. The rise in markups in De Loecker and Eeckhout (2017) is also strongest in the highest deciles, a result that has been confirmed for a number of countries by Diez et al. (2018) and Calligaris et al. (2018).[3] This paper contributes to this literature by showing that an increase in the ability to use intangibles by some firms can impose a negative externality on others, thereby driving the growing differences across firms as well as the aggregate trends.[4]

I also contribute to recent work that links the aggregate trends to intangibles. Crouzet and Eberly (2018) show that intangibles have caused an increase in market power and productivity for leading U.S. public firms. Similarly, McKinsey (2018) and Ayyagari et al. (2018) show that firms with high profitability and growth (which they refer to as superstar firms) invest more in software and R&D. Farhi and Gourio (2018) show that unmeasured intangibles can explain the rising wedge between the measured marginal product of capital and risk-free rates. At the sector level, Bessen finds a positive relationship between the rise of firm-concentration and the use of IT systems in the U.S. Differently from my approach, he stresses that the scalability of intangibles is advantageous to firms that are already large. Firm-level evidence on this is provided in Lashkari and Bauer (2018). Also at the sector level, Calligaris et al. (2018) find a positive correlation between the use of digital technologies and the rise of markups and concentration.

[3]There is also an increase in the disparity of pay across firms, as found by Berlingieri et al. (2017) and Song et al. (2018). Autor et al. (2017) and Kehrig and Vincent (2017) furthermore show that the decline in the aggregate labor share is primarily driven by a reallocation of economic activity towards firms with a low laborshare.

[4]An alternative mechanism is hypothesized by Brynjolfsson et al. (2018), who claim that new General Purpose Technologies initially require firms to invest in unmeasured intangible capital, causing measured productivity to decline. This does not, however, explain the coincidental reduction in business dynamism and rise in concentration.

Outline The remainder of this paper proceeds as follows. Section 2 introduces scalable intangible inputs empirically and summarizes the stylized facts. Section 3 presents the full growth model and a discussion of the main mechanism. The model is estimated in Section 4, and results are discussed in Section 5. Section 6 concludes.

2. Evidence

This section provides stylized facts on the rise of intangible inputs. To fix ideas, I outline a simple model of intangible inputs as a scalable technology. I then present evidence on three stylized facts in line with this model: 1) the share of fixed costs in total costs has increased, 2) there is a positive within and across-firm correlation between fixed costs and measures of software investments and technology adoption, and 3) there is a positive correlation between a firm's share of fixed costs and its innovative activity and rate of sales growth.

2.1. Intangibles as Scalable Inputs

Consider a first-degree homogeneous production function $f(z_{i,1}, z_{i,2}, .., z_{i,k}) \cdot \omega_i$ with k traditional (tangible) production factors and Hicks-neutral productivity ω_i. Firm i's marginal cost function is $\boldsymbol{c}(w_1, w_2, .., w_k)$, where w_k denotes the factor price of tangible production factor k. Intangible technologies allow firms to automate a part of their production process in order to reduce marginal costs by a fraction $s_i \in [0,1)$. Output therefore reads:

$$y_i = \frac{1}{1-s_i} \cdot f(z_{i,1}, z_{i,2}, .., z_{i,k}) \cdot \omega_i; \tag{1}$$

which is associated with marginal costs $mc_i = (1-s_i) \cdot \boldsymbol{c}(w_{i,1}, w_{i,2}, .., w_{i,k}) \cdot \omega_i^{-1}$.

In exchange for the reduction of marginal costs, s_i comes with greater expenditure on intangibles. The relationship between s_i and expenditure on intangibles is governed by a twice-differentiable function $S(\phi_i, s_i)$ which is strictly convex on the domain $s_i \in [0,1)$ and which satisfies $\partial S(\phi_i, s_i)/\partial \phi_i < 0$, $S(\phi_i, 0) = 0$ and $\lim_{s_i \to 1} S(\phi_i, s_i) = \infty$. The latter implies that the cost of automating production completely (and reducing marginal cost to 0) are infinite such that all firms have positive marginal costs. ϕ_i is a firm-specific parameter that captures the efficiency with which firms implement intangibles: firms with higher levels of ϕ_i are able to reduce their marginal costs by a greater fraction for a given expense on intangible inputs. Given that $S(\phi_i, s_i)$ does not directly depend on the amount that a firm sells, they represent a fixed cost.[5] The term fixed here is different from usual, in the sense that firms choose the level of $S(\phi_i, s_i)$ through a reduction in variable costs. Firms that do not increase their use of intangible inputs do not face an increase in fixed costs, and

[5]This does not mean that there is no correlation between $S(\phi_i, s_i)$ and output: firms with greater output have more incentives to reduce marginal costs and therefore choose a higher $S(\phi_i, s_i)$. The empirical analysis therefore includes controls for output.

intangibles do not cause entry costs to rise. Going forward, I also impose the following assumption on the demand function that firms face:

$$\frac{\partial \ln f(z_{i,1}, z_{i,2}, .., z_{i,k})}{\partial s_i} < 1 \tag{2}$$

This condition, which I view as mild, implies that a rise of intangibles does not cause a very large increase in demand. More formally, the condition implies that reducing marginal costs through s_i does not cause an increase of tangible input usage by a greater percentage than the increase in s_i.

This setup implies the following two results. First, optimal intangibles satisfy the following first order condition on the maximization of operating profits $\pi_i = (p_i - mc_i) \cdot y_i - S_i$:

$$\frac{\partial S(\phi_i, s_i)}{\partial s_i} = \frac{\partial p_i \cdot y_i}{\partial s_i} + \boldsymbol{c}(w_{i,1}, w_{i,2}, .., w_{i,k}) \cdot \omega_i^{-1} \cdot y_i - mc_i \cdot \frac{\partial y_i}{\partial s_i}$$

where the first term captures that intangibles may raise revenues, the second term captures the reduction in marginal costs on current output, and the third term captures the increase in operating costs due to additional output. It follows that expenditure on intangibles increases in the firm parameter ϕ, output, and the effect that cost-reductions have on revenue.

Second, it is straightforward to show that the ratio of fixed intangible costs as a percentage of total costs:

$$\frac{S(\phi_i, s_i)}{S(\phi_i, s_i) + (1 - s_i) \cdot \boldsymbol{c}(w_{i,1}, w_{i,2}, .., w_{i,k}) \cdot \omega_i^{-1} \cdot y_i} \tag{3}$$

is strictly increasing in s_i.[6]

2.2. Measurement

The framework in Section 2.1 has two testable implications. The first is the macro implication that, with the rise of intangibles, there is an endogenous shift in the cost structure of firms towards fixed costs. That causes the average of the ratio of fixed over total costs to increase over time. The second is the micro implication that firms which use more intangibles (or increase their use) should have a higher (or increasing) ratio of fixed to total costs. Testing these implications requires a measure of fixed costs, which is difficult to obtain as firms do not break down their expenses into fixed and variable costs on the income statement. Past work typically measures fixed costs through the sensitivity of operating costs or profits to sales, under the assumption that all variable costs are set freely. This is problematic when firms face adjustment costs for some variable inputs (for example when amending their labor force), and it allows for limited time-variation within firms.[7]

[6] This is formally shown in Appendix A.

[7] Adjustment costs incentivizes firms to retain constant employment when production needs vary, causing an artificially low correlation between shocks to sales and costs. This leads to an overestimation of fixed costs. Examples include Lev (1974) and García-Feijóo and Jorgensen (2010). Alternatively, De Loecker et al. (2018) assume that sales, general and administrative expenses on the income statement are fixed. Though appropriate for their purpose, it is likely that some of these costs are variable (like shipping costs and sales commissions).

Instead, I derive a time-varying measure of fixed costs from the difference between the marginal cost markup and a firm's profit rate, which equals operating profits divided by revenue. Under the first-degree homogeneity assumption of $f(z_{it,1}, z_{it,2}, .., z_{it,k})$, the profit rate is given by the following accounting definition:

$$\frac{\pi_{it}}{p_{it} \cdot y_{it}} = \frac{\left(p_{it} - mc_{it}\right) \cdot y_{it}}{p_{it} \cdot y_{it}} - \frac{F_{it}}{p_{it} \cdot y_{it}}$$

where fixed costs are expenditures on intangibles and other fixed costs η_i, such that $F_{it} = S(\phi_i, s_{it}) + \eta_{it}$. Isolating fixed costs and defining the marginal cost markup μ_i as the ratio of prices to marginal costs yields:

$$\frac{F_{it}}{p_{it} \cdot y_{it}} = \left(1 - \frac{1}{\mu_{it}}\right) - \frac{\pi_{it}}{p_{it} \cdot y_{it}} \tag{4}$$

I multiply the right hand side of (4) with revenues and divide by total operating costs to obtain fixed costs as a share of total costs. The intuition behind (4) is that markups capture the firm's marginal profitability, while profits capture the firm's average profitability. Because fixed costs are incurred regardless of sales, a firm with positive fixed costs should have a profit rate below the markup. This also implies that an increase in markups does not necessarily imply an increase in profitability.

Fixed costs in equation (4) are calculated from data on operating profits, revenues, and markups. Operating profits and revenues are obtained from the income statement. Markups are not directly observed, because income statement and balance sheet data lacks information on marginal costs and prices. Instead, I estimate markups using the method proposed by Hall (1988). He shows that markups are given by the product of the output elasticity β^m of a variable input m and the ratio of a firm's sales to its expenditure on that input. Formally:

$$\mu_{it} = \beta^m_{it} \cdot \left(\frac{p_{it} \cdot y_{it}}{w^m \cdot z^m_{it}}\right)$$

where the z^m_{it} denotes the quantity of m that firm i deploys in year t, and w^m denotes that input's unit cost. Revenue and expenditure on the input are observed on the income statement, while I obtain the output elasticity by estimating a translog production function using the iterative GMM procedure proposed by De Loecker and Warzynski (2012).[8]

2.3. Data

I analyze the behavior of fixed costs for the universe of French firms between 1994 and 2016. The data comes from two administrative datasets (FICUS from 1994 to 2007, FARE from 2008 to 2016) that rely on administrative data from DGFiP, the French tax authority, that are managed by the statistical office INSEE. French firms are required to submit their balance sheet and income statement

[8]Details are provided in Appendix C. The advantage of this approach to estimating markups is that it does not assume any form of market structure or competition, and is consistent with the framework in Section 2.1. Furthermore, markups are estimated based on a single variable input m. Other inputs may be fixed, variable, or a combination of both: as long as one freely-set variable input is observed the markup is estimated consistently.

Table 1: Descriptive Statistics

	Mean	Std. Dev.	Median	10th Pct.	90th Pct.	Obs.
Output						
Sales	4,684	103,285	617	149	4,996	9,913,058
Inputs						
Employment (headcount)	19	356	5	1	28	9,913,058
Wage bill	622	10,753	144	38	831	9,913,058
Capital	1,738	131,183	92	12	895	9,913,058
Intermediate inputs and raw materials	2,234	58,699	136	0	1,923	9,913,058
Other operating expenses	1,210	35,652	124	33	1168	9,913,058

Summary statistics for the merged FARE-FICUS database. Nominal figures in thousands of euros. Sales, profits, and materials are deflated with sector deflators from EU-KLEMS, wage bill is deflated with the GDP deflator.

annually. The data contains the full balance sheet and income statement, with detailed breakdowns of revenues and costs. INSEE extends these datasets with standard firm variables such as industry of operation, employment, and headquarter location. I append FICUS with FARE using a firm identifier (the *siren code*) that consistently tracks firms over time. The unit of observation is a legal entity (*unité légale*), although subsidiaries of the largest companies are grouped as a single firm. I restrict the sample to private firms and drop contractors, state-owned enterprises and non-profit organisations, as well as companies that receive operating subsidies in excess of 10% of sales. Firms in financial industries and firms with missing or negative sales, assets, or employment are also excluded. Details on variable definitions are provided in Appendix B. The remaining sample contains data on 1,087,726 firms across 651 NACE industries between 1994 and 2016. Summary statistics for the main firm characteristics are provided in Table 1. Appendix D replicates the macroeconomic trends that motivate this paper for the FARE-FICUS dataset, which confirms that France has incurred a decline in productivity growth and business dynamism, as well as an increase in markups and concentration.[9]

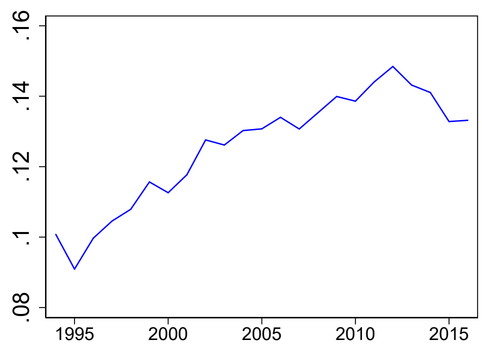
2.4. Analysis

Fact 1: Positive Trend in Fixed Costs Figure 3 depicts the average ratio of fixed to total costs measured from markups and profit rates along equation (4). In line with trends in intangible inputs like software, the measure shows a constant increase over the sample. Fixed costs made up 9% of costs at the lowest point over the sample, and close to 15% at its highest.[10]

[9]Access to the FICUS and FARE datasets was initially obtained for Burstein et al. (2019). The code to merge FICUS and FARE was developed for their project, and is partly provided by Isabelle Mejean. I thank them for their help in obtaining data access and for permission to use the code for this project.

[10]The level of the fixed cost measure mostly depends on the estimate of the supply elasticity that is used to calculate markups. Some estimations of these elasticities are consistently higher then the level used for fixed costs in Figure 3, and therefore imply a higher level of fixed costs. The trend was similar across estimations, however. Appendix C contains a full robustness check of all results in this section using different production function estimates.

Figure 3. Average Ratio of Fixed Costs to Total Costs over Time



Sales-weighted average of fixed costs over time, universe of French firms in FICUS-FARE, 1994-2016

The shift from variable costs to fixed costs occurs mainly within sectors. To show that, I perform a standard within-between decomposition:

$$\Delta \frac{F_t}{TC_t} = \sum_{j \in J} s_{jt-1} \cdot \Delta \frac{F_{jt}}{TC_{jt}} + \sum_{j \in J} \Delta s_{jt} \cdot \frac{F_{jt-1}}{TC_{jt-1}} + \sum_{j \in J} \Delta s_{jt-1} \cdot \Delta \frac{F_{jt}}{TC_{jt}}$$

where F_t/TC_t is the aggregate (sales-weighted) fixed cost share, F_{jt}/TC_{jt} the sector-level counterpart, and s_j the fraction of sales by sector j. The first term captures the change in aggregate fixed cost due to an increase in fixed costs within sectors. The second term captures the 'between' share: changes in aggregate fixed costs because of changes in the relative size of sectors. The last term is the interaction of both, also known as the reallocation share. I perform the decomposition annually and regress each term on the change in the aggregate fixed cost share. The resulting coefficients are presented in Table 2, which shows that 73% of changes in aggregate fixed costs are driven by changes within sectors. The left panel of Figure 4 illustrates the contribution of the within and between share over time, by plotting the development of aggregate fixed costs holding other contributors constant.

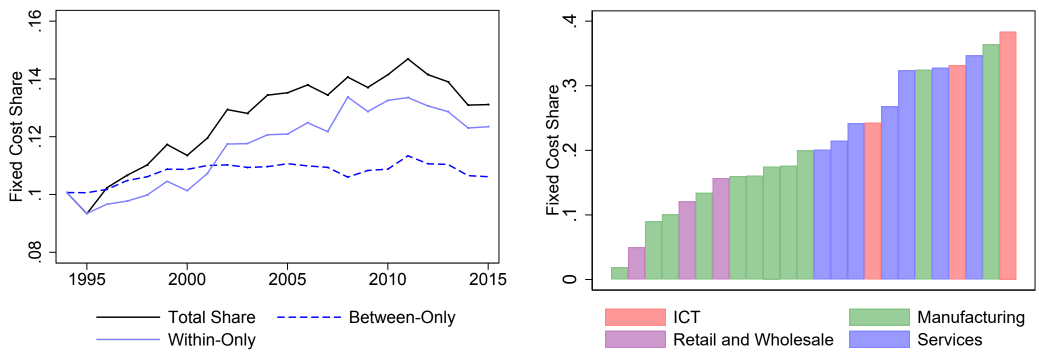
The right panel of Figure 4 provides a cross-sectional overview of fixed costs across major 1 and 2 digit industries. In line with the notion that fixed costs capture intangibles, service and ICT industries rely relatively strongly on fixed costs, while retail and wholesale industries rely mainly on variable costs. The manufacturing sector with the lowest fixed costs is food production, while industries such as transportation equipment manufacturing rely more strongly on fixed costs.

Table 2: Decomposition of Changes in Aggregate Fixed Cost Share

	Within	Between	Reallocation	Total
Contribution	0.73***	0.21***	0.06***	1
	(0.003)	(0.003)	(0.003)	

Standard errors in brackets. *** denotes significance at the 1% level.

Figure 4. Analysis of Fixed Cost Share Across Sectors



(a) Within-Between Decomposition

(b) Average Ratio of Fixed Costs

Right figure: sales-weighted average of fixed cost share within EU-KLEMS 1 and 2 digit industries

Fact 2: Firm-Level Fixed Costs and Software Adoption While the trends and patterns in Figures (3) and (4) are in line with expectations, they do raise the question of whether they purely reflect intangibles. There may for instance be sector-specific heterogeneity in non-intangible fixed costs η_i. To affirm the relationship between intangibles and fixed costs, I merge the FICUS-FARE dataset with two surveys on software and IT adoption.[11] The first is the *Enquête Annuelle d'Entreprises* (EAE), which is an annual survey of around 12.000 firms between 1994 and 2007. The survey provides a comprehensive panel of firms with more than 20 employees, and samples smaller firms in most sectors. Following Lashkari and Bauer (2018), I use this survey to obtain the amount that firms spend on software, either developed in-house or purchased externally. Because the survey ends in 2007, I complement the EAE with data from the *Enquête sur les Technologies de l'Information de la Communication* (TIC). This survey contains questions on the use of IT systems from 2008 to 2016.

The relationship between investments in software and the ratio of fixed to total costs is presented in Table 3. The dependent variable is the ratio of fixed costs over total costs, the explanatory

Table 3: Relationship between Software Spending and Fixed Cost Share

Fixed Cost Share	I	II	III	IV	V	VI
Software Investments	5.60***	5.19***	3.03***	2.69***	1.45***	0.55***
	(0.235)	(0.235)	(0.242)	(0.242)	(0.138)	(0.127)
Year fixed effects	No	Yes	No	Yes	No	Yes
Firm fixed effects	No	No	No	No	Yes	Yes
Industry fixed effects	No	No	Yes	Yes	No	No
Size Poly.	Yes	Yes	Yes	Yes	Yes	Yes
Observations	136,208	136,208	136,208	136,208	136,208	136,208
R^2	0.125	0.132	0.289	0.295	0.073	0.196

Dependent variable is fixed costs as a percentage of total costs. Explanatory variable is software investments as a percentage of sales. Sales is deflated with the sector-specific gross output deflator, software with the investment input deflator from EU-KLEMS. Firm-clustered standard errors in parenthesis. *, **, *** denote significance at the 10, 5 and 1% level, respectively.

[11] The surveys do not cover the universe of firms, but contain sample weights to produce representative estimates.

variable is the ratio of software over sales. Both variables are winsorized at their 1% tails. The table shows a consistently positive relationship between software investments and fixed costs. The fact that this relationship is also present when controlling for firm-fixed effects suggests that fixed costs increase when firms increase their use of software. This confirms the idea that intangible inputs are scalable, and therefore are an endogenous fixed costs in production. The coefficients in Table 3 are meaningful: a firm that moves from the median to the 95th percentile of software investments increases its fixed cost share by 0.4 to 4 percentage points. Appendix Table A4 shows a similarly strong relationship between fixed costs and IT systems.[12]

Fact 3: Firm-Level Fixed Costs and Innovation, Growth To conclude this section, Table 4 presents the relationship between a firm's fixed cost share and its innovative activity and subsequent growth. Data on innovation is obtained from the *Enquête Communautaire sur L'Innovation* (CIS). The CIS was held in 1996 and 2000, and biannually since 2004. The main variable from this dataset is expenditures on research and development (R&D), including externally purchased R&D and expenditures on external knowledge or innovation-related capital expenditures. The upper panel shows that firms with higher fixed costs spend significantly more on R&D, while the lower panel shows that these firms subsequently grow faster. Both results are robust to detailed size controls and fixed effects.

Table 4: Fixed Costs, Research and Development, and Growth

	I	II	III	IV	V	VI
R&D Intensity						
Fixed Cost Share	0.024***	0.023***	0.015***	0.014***	0.027***	0.019**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.005)	(0.005)
Observations	92,536	92,536	92,536	92,536	92,536	92,536
R^2	0.007	0.012	0.104	0.113	0.003	0.016
Sales growth						
Fixed Cost Share	0.155***	0.155***	0.159***	0.159***	0.445***	0.514***
(1 lag)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
Observations	8,670,007	8,670,007	8,670,007	8,670,007	8,670,007	8,670,007
R^2	0.082	0.084	0.087	0.089	0.223	0.227
Year F.E.	No	Yes	No	Yes	No	Yes
Industry F.E.	No	No	Yes	Yes	No	No
Firm F.E.	No	No	No	No	Yes	Yes
Size Poly.	Yes	Yes	Yes	Yes	Yes	Yes

Notes: *, **, and *** denote significance at the 10, 5, and 1% level, respectively.

[12] Details on the cleaning of both surveys as well as variable definitions are provided in Appendix B.

3. Intangibles, Firm Dynamics, and Growth

I now embed scalable intangible inputs into a model of firm dynamics and endogeneous growth in the spirit of Klette and Kortum (2004). The model features endogenous entry and exit, multi-product firms, and heterogeneous markups. My goal is to quantitatively analyze the effect of a rise in the intangible input efficiency of a subset of firms. The framework is presented in this section, while the estimation is conducted in Section 4.

3.1. Preferences and Market Structure

Consider an economy where a unit mass of households maximizes the following utility function:

$$U = \int_0^\infty \exp(-\rho \cdot t) \cdot \ln C_t \, dt$$

where C_t is aggregate consumption and ρ is the discount factor. The household is endowed with a single unit of labor which is supplied inelastically. Time is continuous and indexed by t, which is suppressed when convenient.

Consumption is a composite of a continuum of intermediate goods, indexed by j. Each good can be produced by the set of firms I_j that own the production technology, a patent, to produce good j at a certain level of quality q_{ij}. The level of quality determines the value that each unit of a good produced by a firm $i \in I_j$ contributes to aggregate consumption. There is no capital, and intermediate goods are competitively aggregated with the following Cobb-Douglas technology:

$$\ln Y = \int_0^1 \ln \left(\sum_{i \in I_j} q_{ij} \cdot y_{ij} \right) dj$$

where $Y = C$ denotes GDP, and y_{ij} is the amount of good j that is produced by firm i.

The firms that own the patent to produce good j compete à la Bertrand. This implies that, while multiple firms own the patent to produce good j at some level of quality, only one firm will produce the good in equilibrium. In a model where firms have identical production technologies, this would always be the firm with the state-of-the-art patent that allows the firm to produce j and the greatest quality. In this paper's setup, intangibles create heterogeneity in production efficiency, which causes some firms to produce at lower marginal costs than others. It is optimal for the profit-maximizing aggregator to only demand good j from the firm that offers the highest combination of output and quality ($q_{ij} \cdot y_{ij}$) at a given expenditure. In other words, goods will be produced by the firm that is able to offer the greatest quality-to-price ratio q_{ij}/p_{ij}.

3.2. Firms and Intangibles

There is a continuum of firms, indexed by i. In the spirit of Klette and Kortum (2004), firms are able to produce all goods for which they have a patent in their portfolio $J_i = \{q_{ij} : j \in \text{patents owned by } i\}$.

This means that firms (potentially) produce more than one goods. Given the market structure, firms produce the set of goods $\tilde{J}_i \in J_i$ for which they are able to offer the highest quality-to-price ratio q_{ij}/p_{ij}. A firm that does not produce any good in its patent portfolio exits the economy.

In line with the general setup in Section 2, firms produce using tangible and intangible inputs. They choose the optimal fraction $s_{ij} \in [0,1)$ by which they reduce their marginal costs through the use of intangibles. Firms optimize this fraction separately for each good that they produce. To preserve tractability the only tangible input is labor, such that intangibles allow firms to cut the amount of labor required to produce an additional unit of output. The production function therefore reads:

$$y_{ij} = \frac{1}{1-s_{ij}} \cdot l_{ij} \tag{5}$$

where l_{ij} denotes labor dedicated by i to production of good j. The marginal cost of producing j therefore equals $mc_{ij} = (1-s_{ij}) \cdot w$, where w is the wage rate.

The reduction in marginal costs through the use of intangibles comes at a cost $S(\phi_i, s_{ij})$. This cost satisfies the properties of $S(\cdot)$ in Section 2, though I choose a specific functional form in order to quantify the model. The function reads:

$$S(\phi_i, s_{ij}) = \left(\left[\frac{1}{1-s_{ij}}\right]^{\psi} - 1\right) \cdot (1-\phi_i) \tag{6}$$

where ψ is a curvature parameter and ϕ_i captures the efficiency with which firms are able to implement intangible technologies. Firms draw their type ϕ_i from a known distribution $G(\phi)$ at birth and experience their level of intangible efficiency on each good that they produce.

Function (6) has a number of attractive properties. Firms that do not reduce their marginal costs through intangibles pay no fixed intangible costs, while the costs of reducing marginal costs fully ($s_{ij} \to 1$) are infinite. This implies that all firms will have positive marginal costs in equilibrium. Also note that firms pay the fixed costs of using intangible inputs during each instance of production. The motivation for that is twofold. First, Li and Hall (2016) estimate depreciation rates of software investments to range between 30 and 40% per year.[13] This implies that firms must spend considerable amounts each year to maintain a constant level of software. Second, an increasing share of enterprise software is sold *as a service* (SaaS), which means that firms pay periodic licence fees instead of an upfront cost for perpetual use.[14] Note that this does not mean that the model features no accumulation of intangible capital in the spirit of (e.g.) Corrado et al. (2009): firms also invest in research and development, and these investments have long-term effects on both firm size and GDP.

[13] Similar depreciation rates are found for computers and peripheral equipment, while computer system design is estimated to depreciate at close to 50%. A review of past evidence is found in Table 1 of Li and Hall (2016).

[14] For example, 35% of Microsoft's enterprise revenue in the second quarter of FY2019 came from cloud products, at a growth of 48% year-on-year.

3.3. Innovation

Firms expand their portfolio of patents by investing in research and development (R&D). When investing, firms choose the Poisson flow rate $x_i \geq 0$ with which a new patent is added to their portfolio. In exchange for the achieving x_i, firms employ researchers along:

$$rd^x(x_i) = \eta^x \cdot x_i^{\psi^x} \cdot n_i^{-\sigma} \tag{7}$$

where $\psi^x > 1$ and $\eta^x > 0$. The number of researchers that the firm employs is convex in the rate of innovation and declines in the number of goods that the firm produces, n_i. The latter is an assumption from Klette and Kortum (2004), and reflects that large firms have more in-house knowledge or organizational capital than small firms. Practically, the presence of n_i governs the relationship between firm size and firm growth. For $\sigma = \psi^x - 1$ the model satisfies Gibrat's law of constant firm growth, while for $\sigma = 0$ a firm's growth declines rapidly with size. Following Akcigit and Kerr (2018) I allow for an intermediate case between these two extremes, and estimate $\sigma \in [0, \psi - 1]$ by targeting the empirical relationship between size and growth in the data.

A firm that innovates successfully becomes the owner of a state-of-the-art patent for a random good j. Innovation is not directed, in the sense that firms are equally likely to innovate on all products. As in Aghion and Howitt (1992), the state-of-the-art patent allows firm i to produce its new good at a fraction λ above that of the current producer of the good:

$$q_{ij} = q_{-ij} \cdot (1 + \lambda_{ij})$$

where $-i$ denotes the incumbent of good j while λ_{ij} denotes the realized innovation step size, which is drawn from an exponential distribution with mean $\bar{\lambda}$. Initializing qualities to 1, the level of firm i's quality reflects the realization of all past innovations on that good:

$$q_{ij} = \prod_{c=1}^{|I_j|} (1 + \lambda_{cj})$$

In contrast to other endogenous growth models in Klette and Kortum (2004) spirit, a firm that innovates on a good will not necessarily become its new producer. The innovator will only become the producer if it can offer the best combination of quality and price. The lowest price that the incumbent and the innovator are willing to set are their respective choke prices. The choke price is the price at which, after payment of the fixed cost, firm profits are zero. If the incumbent has a lower choke price than the innovator, the incumbent can undercut the innovator on price if the quality of the innovator is sufficiently close to that of the incumbent. Formally, the innovator becomes the new producer if requires:

$$\frac{q_{ij}}{p^{choke}(\phi_i)} \geq \frac{q_{-ij}}{p^{choke}(\phi_i)}$$

Figure 5. Innovation with and without Intangible Inputs

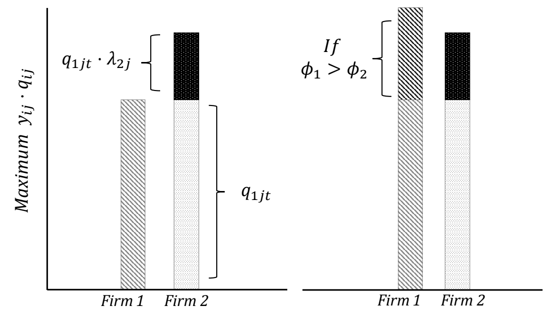


Illustration of case where Firm 2 develops higher quality version of j, currently produced by Firm 1. Left case is the model without intangibles, where Firm 2 always becomes the new producer. Right case is the model with intangibles, where Firm 1 remains the producer if $\phi_1 > \phi_2$ causes the choke price to be sufficiently lower for Firm 1.

where the choke price is a decreasing function of ϕ_i because firms with greater intangible ability are able to reduce their marginal costs by a greater fraction for a given expenditure on intangibles. Rewriting yields:

$$\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \tag{8}$$

The innovator is able to offer product *j* at a superior quality than the incumbent, but the incumbent can hold on to its product if it has a sufficiently low choke price. A greater difference between the choke prices is needed when the innovator has drawn a significant innovation (the realization of λ is high), and the innovator will always become the new producer if it has the same or a higher ϕ_i than the incumbent. The relationship is illustrated in Figure 5, which draws the hypothetical case where firm 1 is the incumbent on a product line on which firm 2 innovates. In a model without intangibles, firm 2 becomes the new owner of the product line because it is able to reduce at greater quality with the same marginal cost. That is not necessarily the case, however, in a model where firms are able to reduce their marginal costs through intangible inputs. If firm 1 is of a higher ϕ type than firm 2, the former is able to sell at a relatively lower price. That would allow consumers to compensate for the lower quality of the good by purchasing a greater quantity.

3.4. Quality and Intangibles

It is useful to briefly discuss the difference between quality and price in this framework. In most models of growth and innovation through creative destruction, the two are isomorphic. Prices reflect the ability of firms to produce at low marginal costs, that is: with high productivity. It may seem that this is equivalent to quality, in the sense that a firm can achieve a higher level of effective output $q_{ij} \cdot y_{ij}$ using the same tangible inputs either through selling at higher quality or through deploying a greater share of intangibles.

The difference between the two lies in the extent to which they contribute to long-term growth. Innovation leads to an increase in the state-of-the-art level of quality q_{ij} with which good j can be produced. If an innovating firm successfully takes over production, this has both a private and an economy-wide benefit. The private benefit is the stream of profit that the firm earns while it produces j. The economy-wide benefit is the fact that all future innovations on j are a percentage improvement over g_{ij}. The innovation by firm i allows good j to be produced at a permanently higher quality. This positive externality makes the step-wise improvement of quality across products the source of long-term economic growth. Intangibles do not come with a similar externality. They only improve production efficiency for the current producer. The fact that the incumbent uses clever software applications to reduce marginal costs does not benefit an innovating firm when it takes over production at some point in the future.

3.5. Entry

There is a mass of entrepeneurs that invest in R&D to obtain patents to produce goods that are currently owned by incumbents. The R&D cost function is analogous to the cost function for innovation by incumbents:

$$rd^e(e) = \eta^e \cdot e^{\psi^e} \tag{9}$$

where $rd^e(e)$ denotes the number of researchers employed by potential entrants to achieve start-up rate e, and where $\eta^e > 0$, $\psi^e > 1$.

Entrepeneurs that draw an innovation improve the quality of a random good that is currently produced by an incumbent. In similar spirit to models where firms draw idiosyncratic productivities at birth (e.g. Hopenhayn 1992, Melitz 2003), entrants then draw their intangible productivity ϕ_e, and learn about the type of the incumbent. They then enter the market if their innovation λ_{ej} is sufficiently large to overcome any difference between the choke price of both firms, along condition (8).

3.6. Creative Destruction

Successful innovation by entrants and existing firms cause the incumbents to lose production of the good that was innovated on. The rate at which this happens is the *creative destruction rate*, $\tau(\phi_i)$. This rate is endogenous and follows from the dynamic optimization of research and development decisions. Firms exit the economy when they lose production of their final product. The rate of creative destruction for a firm of type ϕ_i is given by:

$$\tau(\phi_i) = \sum_{\phi_h \in \Phi} \text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_h)}{p^{choke}(\phi_i)} - 1\right) \cdot \left[\sum_{n=1}^{\infty} M_{\phi_h,n} \cdot x(\phi_h, n) + e \cdot G(\phi_h)\right] \tag{10}$$

This expression is a sum over all possible intangible efficiencies, which is a discrete set Φ. The last term is this equation, $e \cdot g(\phi_i)$, captures the creative destruction caused by entrants. The entry rate e is multiplied by the probability $g(\phi)$ that the entrant is of type ϕ. The term $M_{\phi_h,n} \cdot x(\phi_h, n)$ measures the innovation activity by existing firms. As will be shown in Section 3.8, the optimal innovation rate x_i is a function of a firm's intangible efficiency ϕ_i and its number of products n_i. Innovation effort is equal to the innovation rate times the measure $M_{\phi_h,n}$ of firms with intangible efficiency ϕ_i that produce n products.

Innovation efforts by entrants and existing firms are multiplied by the probability that the innovating firm will become the new producer. This probability is a function of ϕ_i, because firms with higher intangible efficiency have a lower choke price. The probability that condition (8) is satisfied when i is the incumbent and h is the innovator is therefore:

$$\text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_h)}{p^{choke}(\phi_i)} - 1\right) = \bar{\lambda}^{-1}\exp\left(-\bar{\lambda}^{-1} \cdot \left[\frac{p^{choke}(\phi_h)}{p^{choke}(\phi_i)} - 1\right]\right)$$

where the right hand side is the CDF of the exponential distribution with mean $\bar{\lambda}$. The fact that high-ϕ firms are more likely to be able to compensate lower quality with lower prices means that this probability, and hence the creative destruction rate, is strictly lower for these firms.

3.7. Optimal Pricing and Intangibles

The firm maximizes operating profits for each good it produces by statically choosing the optimal price p_{ij} and the fraction of marginal costs it reduces through intangible inputs s_{ij}. Operating profits are given by:

$$\pi(\phi_j, q_{ij}) = (p_{ij} - mc_{ij}) \cdot y_{ij} - p^s \cdot S(\phi_i, s_{ij}) \tag{11}$$

where the fixed cost function (6) is multiplied by the intangibles price p^s.

The optimal price is determined by the wedge between the firm that produces good j and the efficiency of the second-best firm for that good. At the start of each instance t, firms observe the qualities q_{ij} and types ϕ_i of all firms with a patent to produce good j, and choose by which amount they reduce their marginal costs through intangibles. Firms that are unable to offer the highest quality-to-price ratio have no incentive to pay the fixed costs of intangibles, and therefore have a marginal cost of wage w. The demand for output from the firm with the highest quality-to-choke-price ratio has a unit demand elasticity: $\frac{y_{ij}}{Y} = p_{ij}^{-1}$. The profit maximizing price is therefore bound by the marginal cost of the second-best firm, adjusted for differences in quality between both firms:

$$p_{ij} = mc_{-ij} \cdot \frac{q_{ij}}{q_{-ij}}$$

where $-i$ identifies the second-best firm, $mc_{-ij} = w$, and $q_{ij}/q_{-ij} - 1$ is innovation realization λ_{ij}.

The markup is found by dividing the profit maximizing price by firm i's marginal cost $w \cdot (1 - s_{ij})$:

$$\mu_{ij} = \frac{1 + \lambda_{ij}}{1 - s_{ij}} \tag{12}$$

which implies that markups increase in the difference in quality between the producer and the second-best firm, as well as the firm's use of intangibles. Note that while intangibles increase the markup, profits do not increase proportionally because the firm incurs an expense on intangibles. Part of the increase in markups is therefore purely a compensation for fixed costs.

The optimal intangible fraction s_{ij} is found by inserting markups (12) into profit function (11):

$$s_{ij} = 1 - \left([1 + \lambda_{ij}] \cdot \frac{p^s}{Y} \cdot \psi \cdot (1 - \phi_i) \right)^{\frac{1}{\psi+1}} \tag{13}$$

or $s_{ij} = 0$ when the right hand side is negative. It follows that firms with lower intangible adoption costs reduce a greater fraction of their marginal costs and consequently have higher markups.

3.8. Equilibrium

I now characterize the economy's stationary equilibrium where productivity, output and wages grow at a constant rate g.

3.8.1. Optimal Innovation Decisions

Firms choose the optimal spending on research and development to maximize firm value. The associated value function, where notation is borrowed from Akcigit and Kerr (2018), reads:

$$r V_t(\phi_i, \tilde{J}_i) - \dot{V}_t(\phi_i, \tilde{J}_i) = \max_{x_i} \left\{ \begin{array}{c} \sum_{j \in \tilde{J}_i} \left[\begin{array}{c} \pi_t(\phi_i, \lambda_{ij}) + \\ \tau(\phi_i) \cdot \left[V_t(\phi_i, \tilde{J}_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, \tilde{J}_i) \right] \end{array} \right] \\ + x_i \cdot \text{prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \mathbb{E}_{\phi_i} \left[V_t(\phi_i, \tilde{J}_i \cup_+ \lambda_{ij}) - V_t(\phi_i, \tilde{J}_i) \right] \\ - w_t \eta_x (x_i)^{\psi_x} n_i^{\sigma} - F(\phi_i, n_i) \end{array} \right\}$$

The value function is split into two parts. The first part on right-hand side contains the sum of all good-specific items. The first line gives the contemporaneous profits for a firm that sets prices and the fraction of marginal costs reduced through intangibles optimally. The second line gives the change in firm value if the firm would cease production of good j because creative destruction by entrants or other incumbents have made them better at producing that good. The second part is not specific to product lines. The first line gives the expected increase in firm value from external innovation. $V(\phi_i, \tilde{J}_i \cup_+ \lambda_{ij})$ denotes the firm's value if it successfully takes product j from firm $-i$. The change in firm value is multiplied by both the innovation rate as well as the probability of success, because firms are only able to take over production on a fraction of the products they innovate in. The final line gives the costs of R&D spending on external innovation, as well as a fixed term $\mathrm{F}(\phi_i, n_i)$.

The value function admits a simple solution when imposing one technical restriction: to continue operating, firms must pay a cost $F(\phi_i, n_i)$ equal to the option value of external R&D. This restriction does not affect the main mechanisms introduced by this paper, but greatly improve the model's tractability. This assumption is taken form Akcigit and Kerr (2018) and assures that the value function scales linearly in the number of products that a firm produces.

Proposition 1. *The value function of a type ϕ in the stationary equilibrium firm can be written as:*

$$V(\phi_i, \tilde{J}_i) = \frac{\sum_{j \in \tilde{J}_i} \pi(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)}$$

which is increasing in ϕ. The optimal rate of of innovation in terms of the value function reads:

$$x(\phi_i, n_i) = \left(\text{prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \frac{\mathbb{E}_{\phi_i}\left[\frac{\pi(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)} \right]}{\eta^x \cdot \psi^x \cdot w_t} \right)^{\frac{1}{\psi^x - 1}} \cdot n_i^{\frac{\sigma}{\psi^x - 1}} \tag{14}$$

Proof: Appendix A.

The first order condition for innovation is intuitive. Firms engage in more innovation when the expected increase in the value function is larger, and invest less when the innovation cost-parameters are high. Innovation increases in the number of product lines n, though if $\sigma < \psi^x - 1$ the firm's expected growth rate will decline with size. Firms that are better at adopting intangible technologies (higher ϕ) choose a higher internal innovation rate because their ability to reduce marginal costs and raise markups increases contemporaneous profits. They furthermore experience a lower rate of creative destruction is lower, which decreases the effective discount factor. Firms with higher ϕ's also have a higher probability of becoming the new producer on products that they innovate on, which increases the expected profitability of external R&D. Jointly, these effects cause a positive relationship between ϕ and the rate of innovation.

Innovation by entrants is such that the marginal cost of increasing the entry rate e is equal to the expected value of producing a single good corrected, adjusted for the probability that the entrant is able to take over production from the incumbent by offering a low-enough choke price. Because entrants only learn about their type after they have drawn an innovation, the expected value of a product line is over the distribution of firm types at entry $G(\phi)$:

$$e = \left(\sum_{\phi_h \in \Phi} G(\phi_h) \cdot \text{prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_h)}{p^{choke}(\phi_{-i})} - 1 \right) \cdot \frac{\mathbb{E}_{\phi_h}\left[\frac{\pi(\phi_h, \lambda_{ij})}{r - g + \tau(\phi_i)} \right]}{\eta^e \psi^e w_t} \right)^{\frac{1}{\psi^e - 1}} \tag{15}$$

3.8.2. Dymamic Optimization by Households

Maximizing life-time utility with respect to consumption and savings subject to the standard budget constraint gives rise to the standard Euler equation combined with the standard transversality condition:

$$\frac{\dot{C}}{C} = r - \rho \tag{16}$$

Along the balanced growth path consumption grows at the same rate as output and productivity, such that:

$$r - g = \rho$$

3.8.3. Firm Measure and Size Distribution

The optimal innovation rate in (14) is a function of a firm's intangible input efficiency ϕ_i and the number of goods n it produces. The rate of creative destruction (and hence the growth rate of output and productivity), therefore depends on the equilibrium distribution of n and ϕ across firms.

Along the balanced growth path, this distribution is stationary. To find the stationary distribution, define the measure of firms of type ϕ_i that produces n goods $M_{\phi_i,n}$.[15] The law of motion for the measure of firms that produce more than 1 product is given by:

$$\begin{aligned} \dot{M}_{\phi,n} \quad &= \left(M_{\phi_i,n-1} \cdot x(\phi_i, n-1) - M_{\phi_i,n} \cdot x(\phi_i, n)\right) \cdot \\ &\text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1\right) + \left(M_{\phi_i,n+1} \cdot [n+1] - M_{\phi_i,n} \cdot n\right) \cdot \tau(\phi_i) \end{aligned} \tag{17}$$

where the first term captures entry into and exit out of mass $M_{\phi_i,n}$ through innovation by firms of type ϕ_h with $n-1$ products and n products, respectively. The second term captures entry and exit of firms with $n+1$ and n products that stopped producing on their products through creative destruction. For the measure of single product firms, the law of motion reads:

$$\dot{M}_{\phi_i,1} = \left(e \cdot G(\phi_i) - x(\phi_i, 1) \cdot M_{\phi_i,1}\right) \cdot \text{Prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1\right) + \left(2 \cdot M_{\phi_i,2} - M_{\phi_i,1}\right) \cdot \tau(\phi_i). \tag{18}$$

The stationary properties of the firm-size distribution follow from setting both equations to zero, which is done iteratively. The fraction of the unit measure of goods that is produced by firms with intangible efficiency ϕ_i is given by:

$$K(\phi_i) = \frac{\sum_{n=1}^{\infty} n \cdot M_{\phi_i,n}}{\sum_{\phi_i \in \Phi} \sum_{n=1}^{\infty} n \cdot M_{\phi_i,n}} \tag{19}$$

3.8.4. Labor Market Equilibrium

The solutions to the static and dynamic optimization problem of firms allow the labor market equilibrium conditions to be defined. Labor is supplied homogeneously by household at a measure standardized to 1. Equilibrium on the labor market requires that employment of workers on the various types of work in the economy satisfies:

$$1 = L^p + L^s + L^{rd} + L^e$$

[15] In a model with a discrete number of firms, the measure of firms $M_{\phi_i,n}$ would simply be the number of firms with efficiency ϕ that produce n products.

where L^p is the labor used to produce intermediate goods such that $L^p = \int_0^1 l_{ij} dj$. L^s is the labor involved with reducing marginal costs through the development and maintenance of intangible inputs S, L^{rd} is the labor involved with research and development by existing firms, while L^e is the labor involved with research and development by entrants. Labor demand for research and development by existing and entering firms is respectively given by:

$$L^{rd} = \sum_{\phi_i \in \Phi} \sum_{n=1}^{\infty} \left[M_{\phi_i,n} \cdot \eta^x \cdot x(\phi_i, n)^{\psi^x} \right], \; L^e = \eta^e \cdot e^{\psi^e}$$

where innovation rates $x(\phi_h, n)$ and e are the dynamically optimized along (14) and (15). Labor demand for the development and maintenance of intangible inputs is given by:

$$L^s = \int_0^1 S(\phi_i, s_{ij}) \, dj$$

where the producer of each good j incurs intangible expense $S(\phi_i, s_{ij})$ along the first order condition (13) for the optimal fraction s_{ij} with which it reduces marginal costs for that product.

3.8.5. Aggregate Variables

I can now characterize the economy's aggregate variables. The equilibrium wage is given by:

$$w = \exp\left(\int_0^1 \ln \left[\frac{q_{ij}}{1 - s_{ij}} \right] dj \right) \cdot \exp\left(\int_0^1 \ln \left[\frac{1 - s_{ij}}{1 + \lambda_{ij}} \right] dj \right) \tag{20}$$

Proof: Appendix A.

The first term of (20) is the standard CES productivity term. The second term is the inverse of the expected markup. Note that a rise in the use of intangibles has no effect on the level of the wage. While a firm that deploys more intangibles becomes productive, it is able to proportionally raise its markups. These have offsetting effects on the level of the wage. Aggregate output is given by:

$$Y = L^p \cdot \exp\left(\int_0^1 \ln \left[\frac{q_{ij}}{1 - s_{ij}} \right] dj \right) \cdot \frac{\exp \int_0^1 \ln \mu_{ij}^{-1} dj}{\int_0^1 \mu_{ij}^{-1} dj} \tag{21}$$

As in the model with heterogeneous markups and misallocation by Peters (2018), the last term captures the loss of efficiency due to the dispersion of markups. If all markups are equalized the term is equal to 1, while it declines as the variance of markups increases. Total factor productivity is the product of the second and last term in (21).

Equation (21) reveals that a rise in the use of intangibles has two counteractive effects on the level of output. The spread of markups increases when the average s_{ij} increases along (12), because s_{ij} amplifies the heterogeneity in markups caused by the heterogeneous innovation steps. On the other hand, the increase in s_{ij} has a direct positive effect on total factor productivity because it increases the CES productivity index. As will be clear below, the second effect dominates the first effect in feasible calibrations. That means that a rise in the use of intangibles initially has a positive effect on the level of output and on total factor productivity. This may not be the case, however, for steady state growth.

3.8.6. Growth

The growth rate of total factor productivity and output is a function of creative destruction.

Proposition 2. *The constant growth rate of total factor productivity, consumption C, aggregate output Y, and wages w is given by:*

$$g = \sum_{\phi_i \in \Phi} K(\phi_i) \cdot \tau(\phi_i) \cdot \mathbb{E}_{-\phi_i}(\lambda_{hj}) \tag{22}$$

Proof: Appendix A.

where $\mathbb{E}_{-\phi_i}(\lambda_{hj})$ is the expected realization of innovation λ_{hj} when a firm with intangibles efficiency ϕ_i is the incumbent on a product line if a different firm h becomes the new producer after successful innovation. The equation states that growth equals product of the expected increase in quality if a good gets a new producer and the rate at which this happens, weighted by the fraction of product lines that firms of each intangible efficiency own.

Equation (22) shows the counteracting effects of an increase in ϕ at a subset of firms. On the one hand, firms with a higher ϕ have a greater incentive to invest in research and development, which causes the rate of creative destruction to increase. On the other hand, even at constant innovation rate, the presence of high-ϕ firms has a negative effect on the rate of creative destruction because firms with lower productivities ϕ have a lower probability of successfully becoming the new producer. This has a direct effect on growth at given innovation rates, and an indirect effect as these firms reduce their expenditure on research and development.

3.8.7. Equilibrium Definition

The economy's stationary equilibrium is defined as follows:

Definition 1. *The economy is in a balanced growth path equilibrium if for every t and for every intangible productivity $\phi \in \Phi$ the variables $\{r, e, L^p, g\}$ and functions $\{x(n_i, \phi_i), K_{\phi_i}, M_{\phi_i}, s(\phi_i, \lambda_{ij}), \tau(\phi_i)\}$ are constant, $\{Y, C, w, Q\}$, grow at the constant rate g that satisfies (22), aggregate output Y satisfies (21), innovation rates $x(n_i, \phi_i)$ satisfy (14), the entry rate e satisfies (15), firm distribution K_{ϕ_i} and measure M_{ϕ_i} are constant and satisfy (17) and (18), markups $\mu(\phi_i, \lambda_{ij})$ satisfy (12), the fraction of marginal costs reduced through intangibles $s(\phi_i, \lambda_{ij})$ satisfies (13) for all λ_{ij}, the rate of creative destruction $\tau(\phi_i)$ satisfies (10), and both the goods and labor market are in equilibrium such that $Y = C$ and $L^p = 1 - L^s + L^{rd} + L^e$.*

4. Quantification

In order to quantify the effect of a rise of intangibles, I now estimate the model using the micro data from Section 2. The estimation strategy and calibration are outlined in Section 4.1, while the fit fit are discussed in 4.2.

4.1. Calibration

Table 5 summarizes the baseline calibration. There are five parameters that are estimated using indirect inference, which are supplemented with parameters from the literature. The calibration targets moments in the first year of the data (1994), or the first available year for variables based on surveys. The estimation proceeds as follows. I solve the equilibrium of the model in line with definition 1 and obtain the equilibrium objects for innovation and entry rates, the firm-size distribution, rates of creative destruction and aggregate quantities such as the efficiency wedge, wages and output. I then simulate the economy for 8000 firms until the the distribution of s_{ij} has converged. I then simulate data for 5 years and collect moments from the simulated data.[16] Following Akcigit and Kerr (2018) I then compare the theoretical moments to data moments along the following objective function:

$$\min \sum_{i=1}^{5} \frac{|\,\text{model}_i - \text{data}_i\,|}{(|\,\text{model}_i\,| + |\,\text{data}_i\,|) \cdot 0.5} \cdot \Omega_i$$

where model_i and data_i respectively refer to the simulation and data for moment i with weight Ω.

I assume that initially all firms have the same intangible efficiency parameter ϕ. A higher ϕ incentivizes firms to increase their use of intangibles and hence causes a higher share of intangible (fixed) costs. To calibrate this parameter, I therefore target the ratio of fixed costs to variable costs calculated in Section 2 for the FARE-FICUS sample. The average share in the data was 10% in 1994, which is what I target in the baseline calibration.

The cost scalar of research and development by entrants (η^e) is estimated by targeting an entry rate of 10%. This is the fraction of firms that enter the FARE-FICUS dataset for the first time in 1995, the second year for which data is available. The cost scalar of innovation by existing firms (η^x) is estimated by targeting the fraction of researchers, employed by either incumbents or startups, in total employment. The empirical counterpart of this moment is the fraction of workers with a tertiary degree that is employed in science and technology as a percentage of total employment in 1994. Total employment is obtained from Fred, employment in science and technology is obtained from Eurostat.

[16] The firm simulation builds computationally on Akcigit and Kerr (2018) and Acemoglu et al. (2018).

Table 5: Overview of the Model's Parameters - 1994 Calibration

Parameter	Description	Method	Value
ρ	Discount rate	External	0.01
ψ	Intangibles cost elasticity	Match data	2
ψ^x	Cost elasticity of innovation	Match data	3
ψ^e	Cost elasticity of innovation	Match data	2
η^x	Cost scalar of innovation (incumbents)	Indirect inference	4.5
η^e	Cost scalar of innovation (entrants)	Indirect inference	8
$\bar{\lambda}$	Average innovation step size	Indirect inference	0.05
σ	Innovation-size scaling	Indirect inference	0.39
ϕ	Intangible efficiency	Indirect inference	0.72

Following Akcigit and Kerr (2018), I calibrate the parameter that governs the extent to which R&D scales with size (σ) by targeting a regression of size on growth. Specifically, I estimate the following regression with OLS:

$$\Delta_i \left(p \cdot y\right) = \alpha_s + \beta \cdot \ln\left(p_i \cdot y_i\right) + \varepsilon_i$$

where the left-hand side is the growth rate of sales using the measure of growth in Davis et al. (2006), α_s is a sector fixed effect, and data comes from 1994-1995. The estimated β is -0.035, which implies that a firm with 1% greater sales is expected to grow 0.035% less.[17]

The average innovation step-size $\bar{\lambda}$ is estimated by targeting a steady state growth rate 1.7%, which is the average growth rate of total factor productivity between 1964 and 1994 in the Penn World Tables. Note that the average innovation step-size of successful innovations (innovations that cause a new firm to produce) is higher when firms differ in intangible efficiency ϕ_i.

4.2. Fit of Moments

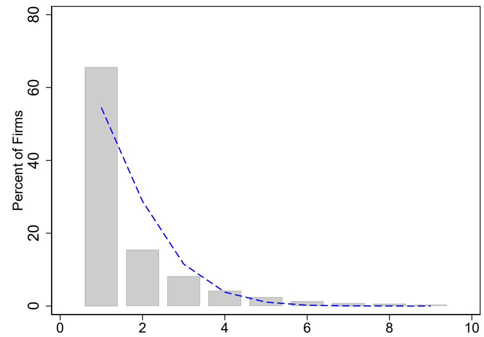
The remainder of this section assesses the extent to which the baseline calibration matches targeted and untargeted moments. A comparison of theoretical and empirical targeted moments is provided in Table 6. The first column lists the parameter that corresponds most closely to the moment. The table show that the model precisely matches the targeted long-term growth rate and the share of fixed costs, which are the most important for this paper's purpose. The model also

Table 6: Comparison of Empirical and Theoretical Moments - Baseline Calibration

Parameter	Moment	Value - Data	Value - Model
$\bar{\lambda}$	Long-term growth rate of productivity	1.7%	1.7%
ϕ	Fixed costs as a fraction of total costs	10%	10%
σ	Gibrat's Law: relation between firm growth and size	-0.035	-0.035
η^e	Entry rate (fraction of firms age 1 or less)	10%	10%
η^x	Employment in science and technology /w tertiary education	18%	21%

[17] Coincidentally, this is the same estimate is the one found for (log) employment on employment growth by Akcigit and Kerr (2018) for U.S. firms

Figure 6. Number of Products by Firm: Theory and Data



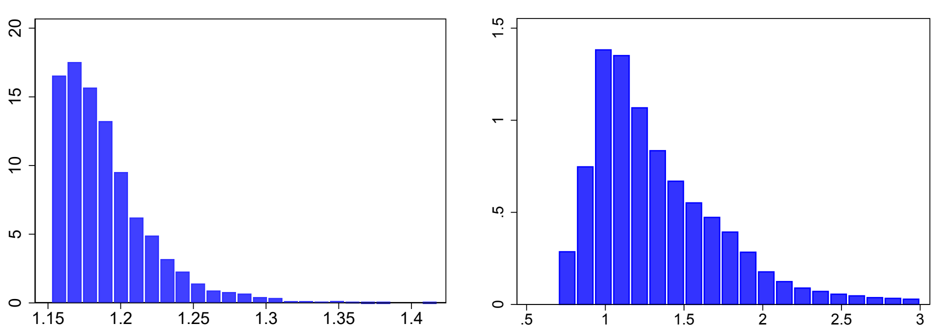
Bars represent the fraction of firms that produce that number of goods in 2009. Line plots the corresponding fraction in the baseline calibration. Data: *Enquête Annuelle de Production dans L'Industrie* (EAP).

matches the entry rate and the relationship between firm growth and firm size precisely, though it slightly overestimates the fraction of workers employed in research activities.

The firm size distribution is untargeted. As in most Klette and Kortum (2004) models, the Cobb-Douglas aggregator implies that a firm's revenue is determined by the number of goods that it produces. The distribution of this number is plotted in the grey bars of Figure 6. The blue line in the figure represents the data counterpart, which comes from the *Enquête Annuelle de Production dans L'Industrie* (EAP). This dataset is only available for firms in manufacturing, but contains product identifiers for each product that the firm sells. The first year of the survey is 2009, which is plotted here.[18] Results show that the distribution of products across firms is fitted well by the model, though the fraction of firms that produce a single product is slightly higher in the data.

The distribution of markups in the data and in the model is compared in Figure 7. While untargeted, the average markup in the model (1.19) is close to the average in the data (1.18) for 1994. The empirical markup has significantly more variance. The 90th and 10th percentile of the empirical

Figure 7. Distribution of Markups: Theory and Data



(a) Theory (baseline calibration)

(b) Data (1994)

[18] Further details are provided in Data Appendix B.

markup are 0.96 and 1.91, while the extremal values of the theoretical markup are 1.15 and 1.61, respectively. This difference is likely due to a combination of measurement error in the estimation of the production function, as well as due to unmodeled markup determinants.[19]

5. Analysis

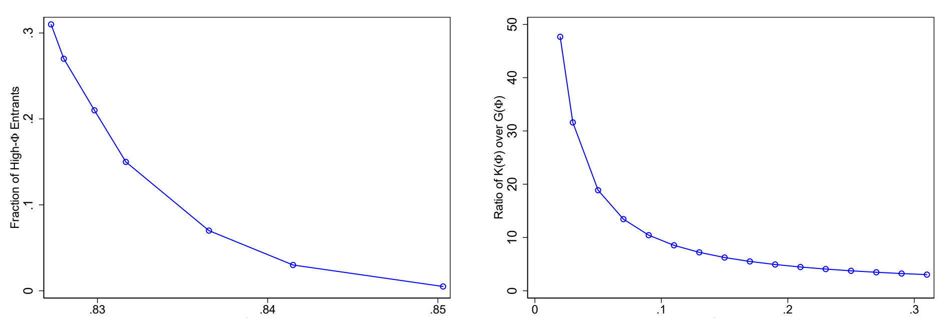
To model the rise of intangibles, I recalibrate the level and distribution of intangible efficiency ϕ. The recalibration embeds two features of the rise of intangibles. First, the recalibration causes an increase in the fixed cost share of total costs by 4.5 percentage points, which matches the rise in Figure 3. Second, the increase in intangibles after 1994 was not homogeneous (see Figure 2), even for firms of similar sizes in narrowly defined industries. The new calibration is motivated in Section 5.1. Steady states are compared in Section 5.2 while transitional dynamics are discussed in 5.3.

5.1. New Calibration

I increase the level of ϕ for a fraction of entrants. This means that the new calibration involves choosing two parameters: the level of ϕ of entrants that incur higher efficiency ($\bar{\phi}$), and the fraction $G(\bar{\phi})$ of entrants to which this applies. If the fraction of entrants that draws $\bar{\phi}$ is greater, a lower $\bar{\phi}$ is sufficient to achieve increase of the fixed cost share by 4.5 percentage point. The precise relationship between $\bar{\phi}$ and $G(\bar{\phi})$ under the baseline calibration is plotted in the left panel of Figure 8 for $G(\bar{\phi})$ between 0.5% and 30%.

While the relationship between $\bar{\phi}$ and $G(\bar{\phi})$ is clearly negative, the $\bar{\phi}$ that achieves an increase in the fixed cost share of 4.5 percentage points is very similar when only 0.5% of entrants receive a higher intangible efficiency to when 30% receives it (0.827 and 0.850, respectively). That is because the share of entrants that receive a higher ϕ is lower than the fraction of products that will be

Figure 8. Combinations of $\bar{\phi}$ and $G(\bar{\phi})$, $K(\bar{\phi})$ for 14.5% Intangible Share



(a) Combinations of $\bar{\phi}$ and $G(\bar{\phi})$ (b) Fraction of $\bar{\phi}$ across products and entrants

[19] The variance of markups would also increase if a process for innovation with a fatter tail was used, such as a truncated normal distribution with a sufficiently high variance.

produced by high-ϕ firms in the steady state. This is illustrated in the right panel of Figure 8. The horizontal axis plots $G(\bar{\phi})$, while the vertical axis plots the ratio of the fraction of products owned by high-ϕ firms, $K(\bar{\phi})$, and $G(\bar{\phi})$. The figure shows that the fraction of products owned by $\bar{\phi}$-type firms is almost 50 times larger than when only 2% of entrants receive $\bar{\phi}$. When 30% receive the higher share, this is close to just 3. It follows that even when there is substantial heterogeneity in ϕ, the vast majority of firms observed in the data will have high levels of intangible adoption. Formally, the following proposition applies:

Proposition 3. *In a setting with two firm types, ϕ^{top} and ϕ^{-top} with $\phi^{top} > \phi^{-top}$, the following holds:*

$$G(\phi^{top}) < K(\phi^{top})$$

and the relationship between $K(\phi^{top})$ and $G(\phi^{top})$ satisfies:

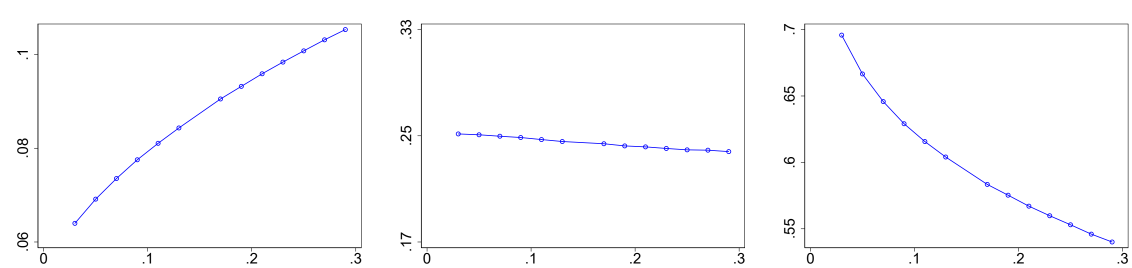
$$\frac{\partial K(\phi^{top})/G(\phi^{top})}{\partial \phi^{top}}\bigg|_{\phi^{top}>\phi^{-top}} > 0$$

Proof: Appendix A.

The wedge between $K(\bar{\phi})$ and $G(\bar{\phi})$ is driven by the fact that high-ϕ firms invest more in research and development. This has two causes. First, these firms are able to charge a higher markup as they produce at lower marginal costs, which raises the value of becoming the producer of a good. Second, these firms face a low rate of creative destruction, and hence discounts the value of producing a product at a lower rate. Both effects cause $\bar{\phi}$-firms to expand more strongly than low-ϕ firms. They therefore produce a disproportionate fraction of all firms.

Proposition 3 is key to understand the effect of a rise in ϕ on aggregate innovation. In particular, it explains the effect that a rise of intangibles has on the entry rate. Optimal investments by entrants is determined by the expected value of producing a good, multiplied by the probability that they successfully innovate. If a fraction $G(\bar{\phi})$ of entrants has a higher intangible efficiency than others,

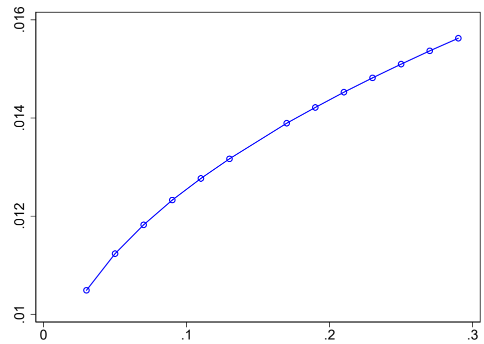
Figure 9. Innovation Rates by Source as a Function of $G(\bar{\phi})$



(a) Startup Rate (b) Innovation by Low-ϕ Firms (c) Innovation by High-ϕ Firms

Horizontal axis is the share of high-ϕ firms $G(\bar{\phi})$, where $\bar{\phi}$ is adjusted such that fixed costs are 14.5% of total costs. Innovation rates refer to $x(\phi_i, 1)$, that is, innovation rates for firms with a single product). All values are from separate balanced growth path equilibria.

Figure 10. Growth Rate as a Function of $G(\bar{\phi})$



Horizontal axis is the share of high-ϕ firms $G(\bar{\phi})$, where $\bar{\phi}$ is adjusted such that fixed costs are 14.5% of total costs.

the probability that entrants benefit positively from the higher productivity equals $G(\bar{\phi})$. When attempting to become a new producer, however, the entrants have a probability $K(\bar{\phi}) > G(\bar{\phi})$ of facing an incumbent with the higher productivity. Therefore if the wedge between $K(\bar{\phi})$ and $G(\bar{\phi})$ is sufficiently large, the rise of aggregate intangibles negatively affects the rate of entry.

To illustrate the distinct effect of intangibles on innovation by incumbents and by entrants, Figure 9 plots the start up rate and innovation rates of low and high ϕ firms on the vertical axis. Analogous to Figure 8, the horizontal axis contains the fraction $G(\bar{\phi})$, while $\bar{\phi}$ is calibrated such that the fixed cost share is 14.5%. The closer $G(\bar{\phi})$ is to 0, the greater the difference between $G(\bar{\phi})$ and the ϕ of other firms. Figure (a) plots the startup rate, which increases as the rise of intangibles is more 'inclusive'. In fact, if more than 30% of entrants draw $\bar{\phi}$, the rise of intangibles drives the startup rate above the previous steady state level of 10.5%. The startup rate falls to 6%, however, if only 2% of entrants draw $\bar{\phi}$. Note that in both cases, over 90% of products are produced by firms of $\bar{\phi}$-type in the steady state. The negative effect on the lack of entry is mitigated by the fact that $\bar{\phi}$-firms invest more in innovation as $\bar{\phi}$ increases. This is because their average markup is higher, because they are able to undercut innovators on price, and because the low startup rate declines the rate at which they are challenged in general. The innovation rate of low-ϕ firms does not depend strongly on $G(\bar{\phi})$ because the negative effect of high innovative investments by $\bar{\phi}$-firms is offset by the lower rate of creative destruction from entry.

The overall effect on growth is plotted in Figure 10. It affirms the result that, for a rise of intangibles to not harm growth, the rise must be 'inclusive'. Only if at least 30% of entrants benefit from a higher intangible productivity is growth not negatively affected, which coincides with the point where the startup rate falls below the previous steady state level.

5.2. Comparison of Steady States

Based on the results in Section 5.1, I recalibrate the model to match a rise of intangibles of 4.5 percentage points and a decline in the entry rate of 3.6 percentage points. This is achieved by setting $\bar{\phi}$ to 0.84, and $G(\bar{\phi})$ to 5%. A comparison between the old and the new steady state is provided in

Table 7: Comparison of Steady States Before and After Increase in Intangible Efficiency of Top Firms

Variable	Previous St. State	New St. State	Change (model)	Change (data)
Targeted				
Avg. fixed cost share	10%	14.5%	4.5 p.p.	4.5 p.p.
Entry rate	10.5%	6.9%	3.6 p.p.	3.6 p.p.
Untargeted				
Productivity growth	1.7 %	1.1 %	-0.6 p.p.	-1.5 p.p.
Reallocation rate	33%	20%	- 38%	-23%
Avg. markup	1.19	1.33	0.14 p.p.	0.11 p.p.

Table 7. The recalibration is able to match the targeted moments precisely, and is able to come both qualitatively and quantitatively close to untargeted moments. The steady state productivity growth rate falls from 1.7 to 1.1 percentage points. Because recent productivity growth in France has been close to 0, it is unsurprising that the model cannot explain the entire slowdown. It is able, however, to predict a meaningful fraction. The model predicts a large decline in the reallocation rate, measured through the rate of creative destruction. The decline exceeds the rate in the data, likely because all growth in the model is driven by creative destruction. The markup in the model increases from 1.19 to 1.33, which (both in levels and in change) closely resembles the increase of average French markups from 1.18 to 1.29.

5.3. Transition Dynamics

The analysis so far has studied the effect of a rise in intangibles in the long run. The short-term dynamics, however, are substantially different. The birth of firms with higher ϕ's leads to a higher fraction of marginal costs to be reduced by intangibles: while the average s_i in the previous steady state was 12%, the new steady state is characterized by an average s_i of 23%. The reallocation of production towards high-ϕ firms in the transition therefore causes an additional permanent increase of total factor productivity by 14%. This is consistent with the finding that above-average productivity growth from the mid 1990s to mid 2000s was primarily driven by IT (Fernald 2014). The increase in productivity is slightly mitigated by the increase in the variance of markups, but this effect is smaller than a percentage point.

The transitional dynamics predicted by the model can explain two recent macroeconomic puzzles. The first is why wages did not keep up with productivity growth in the past 20 years. While the entry of higher-ϕ firms leads to a reduction of marginal costs and an increase in productivity, there is no increase in wages because productivity is offset by higher markups. The second is why markups could have increased while inflation remained low. This is one common critique, for example, on the results of De Loecker et al. (2018). In my framework, markups increase proportionally in response to a reduction of marginal costs through intangibles. As prices are the product of the markup and marginal cost, they are therefore unaffected.

6. Conclusion

This paper proposes a unified explanation for the decline of productivity growth, the fall in business dynamism, and the growth of markups and firm concentration. I hypothesize that the rise of intangible inputs, in particular information technology and software, can explain these trends. Central to the theory is that intangible inputs change the way that firms compete and produce, as they cause a shift towards fixed costs. Using administrative income statement and balance sheet data on the universe of French firms, I calculate a new measure of fixed costs and show that the share of fixed costs in total costs has been steadily rising over time. Fixed costs have a positive within and across-firm correlation with software expenses and IT system adoption, suggesting that intangibles can be modeled as scalable inputs to production. I also find that firms with higher fixed costs invest more in research and development, and subsequently grow more.

I rationalize these findings in an endogenous growth model with heterogeneous multi-product firms, variable markups and realistic entry and exit dynamics. The model suggests that when only a subset of new firms become more efficient at using intangible inputs, the aggregate rise of intangibles comes with a decline in entry and a decline in long-term growth. I calibrate the model to match the empirical trends in fixed costs and entry rates, and find that intangibles cause a decline of long-term productivity growth of 0.6 percentage points, as well as a large decline in business dynamism, and an 11 percentage point increase in markups.

Future work on this paper involves a computational dynamic analysis of the transition between the old and new balanced growth path. That transition is, in contrast to the new steady state, characterized by *higher* productivity growth. This is because the entry of firms with high intangible efficiency causes a decline in marginal costs. This goes at the expense of an increase in markups, however, which means that wages do not increase in response to the boom. Markups also offset the effect of the decline in marginal costs on prices, which could explain why the last decade has been simultaneously characterized by rising markups and low inflation.

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Appendix A. Proofs and Derivations

Derivation of positive derivative of fraction (3)

The first order condition for intangibles implies that firms with lower adoption costs (higher ϕ) choose to reduce their marginal costs by a greater fraction s_i. To show that these firms also have a higher share of fixed (intangible) costs in total costs:

$$\frac{S(\phi_i, s_i)}{S(\phi_i, s_i) + (1 - s_i) \cdot \boldsymbol{c}(w_{i,1}, w_{i,2}, .., w_{i,k}) \cdot \omega_i^{-1} \cdot y_i}$$

I show that this fraction increases in the fraction of marginal costs automated (s_i). Define b_i as the log the share and take the derivative with respect to s_i:

$$\frac{\partial\, b_i}{\partial\, s_i} = \frac{\partial S(\phi_i, s_i)/\partial s_i}{S(\phi_i, s_i)} - \frac{\partial S(\phi_i, s_i)/\partial s_i + (1 - s_i) \cdot \boldsymbol{c}(..) \cdot \omega_i^{-1} \cdot (\partial y_i/\partial s_i) - y_i \cdot \boldsymbol{c}(..) \cdot \omega_i^{-1}}{S(\phi_i, s_i) + (1 - s_i) \cdot \boldsymbol{c}(..) \cdot \omega_i^{-1} \cdot y_i}$$

Grouping terms yields:

$$\frac{\partial\, b_i}{\partial\, s_i} = \frac{\partial S(\phi_i, s_i)}{\partial s_i} \cdot \left(S^{-1} - (S(\phi_i, s_i) + (1 - s_i) \cdot \boldsymbol{c}(..) \cdot \omega_i^{-1} \cdot y_i)^{-1}\right) + \frac{\boldsymbol{c}(..) \cdot \omega_i^{-1} \left[y_i - (1 - s_i) \cdot (\partial y_i / \partial s_i)\right]}{S(\phi_i, s_i) + (1 - s_i) \cdot \boldsymbol{c}(..) \cdot \omega_i^{-1} \cdot y_i}$$

All terms on the right hand side of this expression are positive, provided that $y_i \geq (1 - s_i) \cdot (\partial y_i / \partial s_i)$. Given that $y_i = (1 - s_i)^{-1} \cdot f(z_{i,1}, z_{i,2}, .., z_{i,k}) \cdot \omega_i^{-1}$, this condition can be written as:

$$f(z_{i,1}, z_{i,2}, .., z_{i,k}) \geq \frac{\partial f(z_{i,1}, z_{i,2}, .., z_{i,k})}{\partial s_i}$$

which is the condition set out in equation (2).

Proof of Proposition 1

The value function is given by the following Bellman equation:

$$r V_t(\phi_i, \tilde{J}_i) - \dot{V}_t(\phi_i, \tilde{J}_i) = \max_{x_i} \left\{ \begin{array}{c} \sum_{j \in \tilde{J}_i} \left[\begin{array}{c} \pi_t(\phi_i, \lambda_{ij}) + \\ \tau(\phi_i) \cdot \left[V_t(\phi_i, \tilde{J}_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, \tilde{J}_i)\right] \end{array} \right] \\ + x_i \cdot \text{prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1\right) \cdot \mathbb{E}_{\phi_i}\left[V_t(\phi_i, \tilde{J}_i \cup_+ \lambda_{ij}) - V_t(\phi_i, \tilde{J}_i)\right] \\ - w_t \cdot \eta_x \cdot (x_i)^{\psi_x} \cdot n_i^{\sigma} - F(\phi_i, n_i) \end{array} \right\}$$

Guess that the solution takes the following form:

$$V_t(\phi_i, \tilde{J}_i) = \sum_{j \in \tilde{J}_i} v_t(\phi_i, \lambda_{ij})$$

where $v_t(\cdot)$ (and hence V_t) grows at a constant rate g in the balanced growth equilibrium. Then $v_t(\phi_i, \lambda_{ij})$ can be written as:

$$\left[r - g + \tau(\phi_i)\right] \cdot v_t(\phi_i, \lambda_{ij}) = \pi_t(\phi_i, \lambda_{ij}) + \Gamma$$

where Γ is the option value of innovation adjusted for the fixed term $F(\phi_i, n_i)$:

$$\Gamma = \max_{x_i} \left[\frac{x_i}{n_i} \cdot \text{prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1\right) \cdot \mathbb{E}_{\phi_i}\left[v_t(\phi_i, \lambda_{ih})\right] - w_t \cdot \eta_x \cdot (x_i)^{\psi_x} \cdot n_i^{\sigma - 1} \right] - \frac{F(\phi_i, n_i)}{n_i} \quad (1)$$

Optimal investment in research and development is given by:

$$x(\phi_i, n_i) = \left(\text{prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1\right) \cdot \frac{\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i, \lambda_{ij}) + \Gamma}{r - g + \tau(\phi_i)}\right]}{\eta^x \cdot \psi^x \cdot w_t} \right)^{\frac{1}{\psi^x - 1}} \cdot n_i^{\frac{\sigma}{\psi^x - 1}}$$

which is a function Γ. In order for the value function to scale with size along the guess (which is needed to preserve a tractable model), Γ must not change with the number of goods that the firm produces. There are two ways to accomplish that. The first is to set $\sigma = \psi^x - 1$, as in the original model by Klette and Kortum (2004). In that case, the value function is additive in the number of goods that the firm produces when $F(\phi_i, n_i) = 0$ and:

$$\frac{x_i}{n_i} = \left(\text{prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1\right) \cdot \frac{\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i,\lambda_{ij})+\Gamma(\phi_i)}{r-g+\tau(\phi_i)}\right]}{\eta^x \cdot \psi^x \cdot w_t} \right)^{\frac{1}{\psi^x - 1}}$$

where the right hand side does not depend on n and where $\Gamma(\phi_i)$ follows from inserting x_i into (1). As described in the main text, I allow $\sigma \in [0, \psi^x - 1]$ in order to achieve an empirically accurate relationship between firm size and growth. Following Akcigit and Kerr (2018), I choose $F(\phi_i, n_i)$ such that $\Gamma = 0$. While this is an ad-hock assumption, it prevents firms with highest ϕ_i from producing all goods in equilibrium. To find the $F(\phi_i, n_i)$ that achieves this, use that the first order condition satisfies:

$$\text{prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1\right) \cdot \mathbb{E}_{\phi_i}\left[v_t(\phi_i, \lambda_{ih})\right] = \psi^x \cdot w_t \cdot \eta_x \cdot (x_i)^{\psi_x - 1} \cdot n_i^{\sigma}$$

such that if $\Gamma = 0$, the fixed term satisfies:

$$F(\phi_i, n_i) = (\psi^x - 1) \cdot w_t \cdot \eta_x \cdot \left[x(\phi_i, n_i)\right]^{\psi_x} \cdot n_i^{\sigma}$$

With this constraint, optimal research and development expenditures satisfy the equation in Proposition 1:

$$x(\phi_i, n_i) = \left(\text{prob}\left(\lambda_{ij} \geq \frac{p^{choke}(\phi_i)}{p^{choke}(\phi_{-i})} - 1\right) \cdot \frac{\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}\right]}{\eta^x \cdot \psi^x \cdot w_t} \right)^{\frac{1}{\psi^x - 1}} \cdot n_i^{\frac{\sigma}{\psi^x - 1}}$$

It follows that

$$V_t(\phi_i, \tilde{J}_i) = \frac{\sum_{j \in \tilde{J}_i} \pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)}$$

where operating profits satisfy:

$$\pi_t(\phi_i, \lambda_{ij}) = \left[1 - \frac{\left(\lambda_{ij} \cdot \frac{w}{Y} \cdot (1 - \phi_i)\right)^{\frac{1}{\psi+1}}}{\lambda_{ij}}\right] \cdot Y - w \cdot (1 - \phi_i) \cdot \left(\left[\lambda_{ij} \cdot \frac{w}{Y} \cdot (1 - \phi_i)\right]^{\frac{-\psi}{\psi+1}} - 1\right)$$

which increases at rate g along the balanced growth path, confirming the initial guess.

Derivation of Aggregate Quantities and Proof of Proposition 2

The equilibrium wage is derived as follows. Start with the definition of aggregate output when each sector is in a betrand equilibrium:

$$\ln Y = \int_0^1 \ln\left(q_{ij} \cdot y_{ij}\right) dj$$

Inserting the firm's production function $y_{ij} = l_{ij}/(1-s_{ij})$ and demand function $y_{ij} = Y/p_{ij}$ yields:

$$\ln Y = \ln Y + \int_0^1 \ln \left(q_{ij} \cdot (w \cdot [1-s_{ij}])^{-1} \cdot \mu_{ij}^{-1}\right) dj$$

Isolating wage on the left hand side gives:

$$\ln w = \int_0^1 \ln \left[\frac{q_{ij}}{1-s_{ij}}\right] dj + \int_0^1 \ln \left[\frac{1-s_{ij}}{1+\lambda_{ij}}\right] dj$$

The derivation of GDP is as follows. Labor market equilibrium requires:

$$L^p = \int_0^1 l_{ij} dj$$

Inserting the firm's production function $y_{ij} = l_{ij}/(1-s_{ij})$ and demand function $y_{ij} = Y/p_{ij}$ yields:

$$L^p = \int_0^1 Y \cdot p_{ij}^{-1} \cdot \left(1-s_{ij}\right) dj$$

Isolate Y on the left hand side, insert the first order condition for pricing, and insert the equilibrium wage to obtain:

$$Y = L^p \cdot \exp\left(\int_0^1 \ln \left[\frac{q_{ij}}{1-s_{ij}}\right] dj\right) \cdot \frac{\exp \int_0^1 \ln \mu_{ij}^{-1} dj}{\int_0^1 \mu_{ij}^{-1} dj}$$

Define total factor productivity Q_t as the terms to the right of L^p in expression (A). A balanced growth path equilibrium is characterized by constant type-shares $K(\phi_i)$. Given that markups equation $\lambda_{ij}/(1-s_{ij})$ where s_{ij} is given by equation (13), the law of large numbers assures that the third term in (A) is constant. Hence $g \equiv \partial \ln Q/\partial t$ is given by:

$$g = \int_0^1 \frac{\partial \ln q_{ij}}{\partial t} dj = \sum_{\phi_i \in \Phi} K(\phi_i) \cdot \tau(\phi_i) \cdot \mathbb{E}_{-\phi_i}(\lambda_{hj})$$

which uses that $K(\phi_i) \cdot \tau(\phi_i)$ is the fraction of goods that changes producer each instance and where initially produced by ϕ_i-type firms.

Appendix B: Data

Appendix B1: Data Sources and Sample Selection

Balance Sheet and Income Statement The main firm-level datasets are FICUS from 1994 to 2007 and FARE from 2008 to 2016. I keep all firms in legal category 5, which means all non-profit firms and private contractors are excluded from the sample. I also drop firms with operating subsidies in excess of 10% of revenues. From 2004, INSEE starts to group firms that are owned by the same company in single *siren* codes. This treatment has been gradually extended over time, which means that data on groups in later years of the data contain more consolidated firms. From 2009 onwards, data is provided separately for the underlying firms (legal entities) and for the group. To have a consistent panel (and prevent an artificial increase in firm concentration), I group firms along the pre-2009 definitions and extend that treatment backwards and forwards.

Software and IT Data on software comes from the Annual Enterprise Survey (*Enquête Annuelle d'Entreprises*, EAE), which is an annual survey of around 12.000 firms between 1994 and 2007. There are separate surveys for major industries (agriculture, construction, manufacturing, services, transportation) which differ in variables and coverage. The survey is comprehensive for firms with at least 20 employees, and smaller firms are sampled for all sectors except manufacturing. The survey is merged to FARE-FICUS using the SIREN firm identifier. The level of observation is the legal unit, for firms that are aggregated prior to 2009 by INSEE as discussed in the main text. From 2008 onwards I use data from the E-Commerce Survey (*Enquête sur les Technologies de l'Information de la Communication* - TIC). This survey contains questions on the use of IT systems annually from 2008 to 2016. This dataset contains dummies on the adoption of specific IT systems such as Enterprise Resource Planning and Customer Resource Management.

Research and Development Data on R&D comes from the Community Innovation Survey (*Enquête Communautaire sur L'Innovation* - CIS). The CIS is carried out by national statistical offices throughout the European Union, and is coordinated by Eurostat. The survey is voluntary, but sample weights are adjusted for non-response to create nationally representative data. The French survey is carried out by INSEE, and contains consistent variables on research and development expenditures in 1996, 2000, 2004, 2006, 2008, 2010, 2012, 2014 and 2016.

Product Count The number of products by firm comes from the Annual Production Survey (*Enquête Annuelle de Production*, EAP). This survey is used for annual data on industrial production for the EU's PRODCOM statistics. The survey is available for manufacturing only, from 2009 to 2016. I count the number of unique products each year by firm, excluding products on which the firm acts as outsourcer, or was only involved in product design (M1 and M5).

Appendix B2: Variable Definitions

Revenue is total sales, including exports. In FICUS years this is CATOTAL, in FARE years this is REDI_R310. In regressions, firm-size is controlled for by a third degree polynomial of log revenue.

Employment Employment is the full-time equivalent of the number of directly employed workers by the firm averaged over each accounting quarter. In FICUS, the data is based on tax records for small firms, and on a combination of survey and tax data for large firms (variable name: EFFSALM). In FARE the variable is REDI_E200, which is based on the administrative DADS dataset.

Wage bill The wage bill is defined as the sum of wage payments (SALTRAI in FICUS, REDI_R216 in FARE) and social security contributions (CHARSOC in FICUS, REDI_R217 in FARE).

Direct production inputs are calculated as the sum of merchandise purchases (goods intended for resale) and the purchase of raw materials, corrected for fluctuations in inventory. In FICUS, the respective variables are ACHAMAR, ACHAMPR, VARSTMA, and VARSTMP. The corresponding variables in FARE are REDI_R210, REDI_R212,REDI_R211, and REDI_213.

Other purchases Other purchases are defined as purchases of services form other firms. This includes outsourcing costs, lease payments, rental charges for equipment and furniture, maintenance expenses, insurance premiums, and costs for external market research, advertising, transportation, and external consultants (AUTACHA in FICUS, REDI_R214 in FARE).

Operating profits is defined as revenue minus the wage bill, expenditure on direct production inputs, other purchases, import duties and similar taxes (IMPOTAX in FICUS, REDI_R215 in FARE) capital depreciation (DOTAMOR in FICUS), provisions (DOTPROV in FICUS), and other charges (AUTCHEX in FICUS). The sum of the wage bill, material input expenses, capital depreciation, provisions, and other charges is REDI_R201 in FARE.

Industry codes Industry codes are converted to NACE Rev. 2 codes using official nomenclatures. Firms that are observed before and after changes to industry classifications are assigned their NACE Rev. 2 code for all years, while other firms are assigned a code from official nomenclatures. Firms in industries without a 1-to-1 match in nomenclatures are assigned the NACE Rev. 2 that is observed most frequently for firms with their industry codes. Firms that switch industry codes are assigned their modal code for all years.

Research and Development R&D investments are measured as all innovative expenditures by firms as reported in the CIS. Subcategories of expenditures fluctuate with each version of the survey, but total expenditures seems consistently defined. In 2012 total expenditures are found in RALLX. In some year I add up underlying variables to create a similar variable. Details for each year are available upon request.

Software Investments The variable for software investments closely follows the definition in Lashkari and Bauer (2018). The underlying variables are observed from 1994 to 2007 in the EAE. The main variable for software is I460. This variable contains all software investments and is available for all sectors. Because missing observations are coded as 0, I drop these firm-years when analysing software. An additional sub-division into externally purchased and internally developed software is available for a subset of firms (I461, I462, I463, I464, I465). Where available, I use this to clean cases where I460 is smaller than I461-I465, and verify that summary statistics match Lashkari and Bauer (2018).

Appendix C: Production Function Estimation

This appendix summarizes the implementation of the the iterative GMM approach by De Loecker and Warzynski (2012) that is used to estimate the output elasticity of variable input m in Section 2.3. The production function estimation relies on codes developed for Burstein et al. (2019) who analyse the cyclical properties of French markups, and I thank the authors for permission to use the code for this project.

C1. Estimation Procedure

Because equation (1) contains both tangible (through $f(\cdot)$) and intangible inputs (through s_i), the framework in Section 2.1 implies a production function along $\tilde{f}(z_{it,1},..,z_{it,k};u_{it,1},..,u_{it,h})\cdot\omega_{it}$ with k tangible and h intangible inputs, Hicks neutral productivity ω_{it}, and potentially increasing returns to scale. I approximate this general production function by estimating a flexible translog function that contains the (squared) log of all observed inputs. I first estimate a production function with capital k, labor l and materials m for each 2-digit industry with at least 12 firms in the data, along:

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^m \cdot m_{it} + \beta^{mm} \cdot m_{it}^2 + \omega_{it} + \epsilon_t \tag{2}$$

where cross-terms are omitted to prevent measurement error in one of the inputs to directly affect the estimated elasticity of other inputs.[20] Capital is measured through fixed tangible assets, labor is the number of employees, and materials equal firm purchases. In contrast to (i.e.) U.S. Census data, data on materials is available annually for firms in all industries.

The three-factor production function is commonly used in the literature and is therefore the basis of estimates in the main text. To assess the robustness of these estimates, I also estimate a more extensive production function with four production factors. The FARE-FICUS dataset allows materials to be divided into direct production inputs v (intermediate goods for resale and expenses on primary commodities) and other purchases o, which include the purchase of external services like advertising. I estimate an additional production function that separates these factors along:

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^v \cdot v_{it} + \beta^{vv} \cdot v_{it}^2 + \beta^o \cdot o_{it} + \beta^{oo} \cdot o_{it}^2 + \omega_{it} + \epsilon_t \tag{3}$$

Because of the large number of firms in the data, I estimate this more extensive production function separately for each 4-digit industry.

All inputs but material are likely to be a combination of tangible and intangible inputs in the context of Section 2.1's model, with the exception of direct production inputs.[21] Direct production inputs are tangible, as they only include expenses on intermediate goods for resale or expenses on

[20] This follows De Loecker and Eeckhout (2017) in their treatment of capital.

[21] Labor may seem a tangible input, but if labor is used to develop or deploy software for production then the intangible input labor appears on the income statement through the wage bill.

primary commodities. An output elasticity can only be used to estimate markups when the factor is freely set each period, which seems most likely to hold for v. That is why I use the elasticity of output with respect to v to estimate markups from the four-factor production function.

Both production functions are estimated under the assumption that a firm's demand for material is an invertible function $m(\cdot)$ (or $v(\cdot)$) of the firm's productivity ω_{it} and capital and labor inputs. As a consequence, the production functions can be written as:

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^m \cdot m_{it} + \beta^{mm} \cdot m_{it}^2 + m^{-1}(\omega_{it}, l_{it}, k_{it}) + \epsilon_t \text{ and}$$

$$y_{it} = \beta^l \cdot l_{it} + \beta^{ll} \cdot l_{it}^2 + \beta^k \cdot k_{it} + \beta^{kk} \cdot k_{it}^2 + \beta^v \cdot v_{it} + \beta^{vv} \cdot v_{it}^2 + \beta^o \cdot o_{it} + \beta^{oo} \cdot o_{it}^2 + v^{-1}(\omega_{it}, l_{it}, k_{it}) + \epsilon_t$$

respectively. Under this assumption, I purge gross output y_{it} from measurement error by estimating:

$$y_{it} = h(l_{it}, k_{it}, m_{it}) + \varepsilon_{it} \text{ and } y_{it} = h(l_{it}, k_{it}, v_{it}, o_{it}) + \varepsilon_{it}$$

where h is a non-parametric function approximated by a third degree polynomial in the inputs.

After purging gross output, the production function is estimated iteratively. The algorithm is as follows. First, I guess the coefficients of the production function using OLS estimates. Given (purged) output, inputs, and the production function, I calculate ω_{it}. The algorithm then estimates the autoregressive process of productivity along:

$$\omega_{i,t} = g' \left[1 \ \omega_{i,t-1} \ \omega_{i,t-1}^2\right]' + \xi_{i,t}$$

where residual ξ_{it} captures shocks to productivity not explained by (squared) lagged values of productivity, while g is a vector of coefficients obtained by minimizing the sum of squared residuals $\xi_{i,t}$:

$$g = \left(\begin{bmatrix} 1 & \omega_{t-1} & \omega_{t-1}^{\circ 2} \end{bmatrix} \begin{bmatrix} 1 \\ \omega_{t-1} \\ \omega_{t-1}^{\circ 2} \end{bmatrix} \right)' \left(\begin{bmatrix} 1 & \omega_{t-1} & \omega_{t-1}^{\circ 2} \end{bmatrix} \omega_t \right) \tag{4}$$

The algorithm iterates the production function coefficients until the errors of the AR(1) process for productivity satisfy:

$$\mathbb{E}\left(\xi_{it} Z_{i,t}\right) = 0 \tag{5}$$

where $Z_{i,t}$ is a vector of instruments:

$$Z_{i,t} = \left[l_{it-1} \ \ l_{it-1}^2 \ \ k_{it} \ \ k_{it}^2 \ \ m_{it-1} \ \ m_{it-1}^2\right]'$$

or for the four-factor production function:

$$Z_{i,t} = \left[l_{it-1} \ \ l_{it-1}^2 \ \ k_{it} \ \ k_{it}^2 \ \ v_{it-1} \ \ v_{it-1}^2 \ \ o_{it-1} \ \ o_{it-1}^2\right]'$$

Table A1: Summary Statistics on Estimated Markups

	Mean	Std. Dev.	Median	10th Pct.	90th Pct.	Observations
Basic production function	1.38	0.43	1.26	0.96	1.91	9,913,058
Extended production function	1.42	1.25	1.01	0.53	2.59	8,477,467

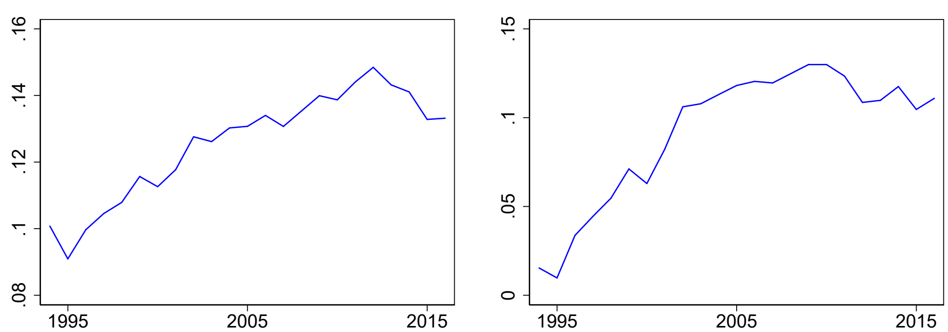
By instrumenting k with its current value, I assume that firms cannot increase capital in response to a contemporaneous productivity shock. By instrumenting l, m, v and o by their lagged value I assume that they are set freely each period, but require autocorrelation in factor prices.[22]

C2. Fixed Costs Results

The remainder of this Appendix shows that results in the main text are robust to using the more extensive four-factor production function. After estimating the industry-level production function coefficients, I calculate the firm-level markup along equation (2.2) and calculate the fixed cost share along (4). Markups at the firm-level are summarized in Table A1. The table shows that the extensive production function estimates a very similar average markup to the markup from the standard three-factor production function. The variance of markups, however, is significantly greater when using the four-factor production function. This is likely due to the additional parameters that need to be estimated at the 4-digit level, or because firms have some flexibility in what costs fall under direct production inputs v versus other purchases o.[23] The firm-level correlation coefficient between markups from both production functions is 0.35.

The trends of aggregate fixed costs are plotted in Figure Figure A1. The left figure is replicated from the main text and is for the three-factor standard production function, while the right figure

Figure A1. Trends in Aggregate Fixed Cost Share



(a) 3-Factor Production Function (b) 4-Factor Production Function

Sales-weighted average of fixed costs as a percentage of total costs, 1994-2016

[22] For France it is reasonable to assume that labor is, in fact, not set freely and could therefore be instrumented by contemporaneously. This turns out to have no significant effect on the estimated production function.

[23] To reduce the effects of measurement error, I winsorize markups from the four-factor production function at the 2% level in the remaining analysis, while markups from the standard production function are winsorized at the more conservative 1% level.

Table A2: Relationship between Software Spending and Fixed Cost Share

Fixed Cost Share	I	II	III	IV	V	VI
Software Investments	14.99***	13.89***	7.90***	7.04***	3.04***	1.37***
	(0.765)	(0.766)	(0.717)	(0.719)	(0.346)	(0.333)
Year fixed effects	No	Yes	No	Yes	No	Yes
Firm fixed effects	No	No	No	No	Yes	Yes
Industry fixed effects	No	No	Yes	Yes	No	No
Size Poly.	Yes	Yes	Yes	Yes	Yes	Yes
Observations	125,615	125,615	125,615	125,615	125,615	125,615
R^2	0.020	0.027	0.211	0.215	0.010	0.103

Dependent variable is fixed costs as a percentage of total costs. Explanatory variable is software investments as a percentage of sales. Sales is deflated with the sector-specific gross output deflator, software with the investment input deflator from EU-KLEMS. Firm-clustered standard errors in parenthesis. *, **, *** denote significance at the 10, 5 and 1% level, respectively.

uses the four-factor extensive production function. Both figures sho that the sales-weighted average fixed cost share has increased strongly over the 1994 to 2016 sample, with the largest increase occurring between 1994 and 2010, after which the increase moderates.

Table A2 replicates Table 3 from the main text for fixed costs derived from the estimation of the four-factor production function. Like before, results show a strong and robustly positive relationship between software investments and the share of fixed costs in total costs. Coefficients are larger than before, suggesting that an increase in software investment intensity from the median to the 95th percentile raises the fixed cost share between 1.2 and 12 percentage points. Results from Table 4 are replicated in Table A3, which again shows that the results in Section 2 are qualitatively robust to using the extensive production function.

Table A3: Fixed Costs, Research and Development, and Growth

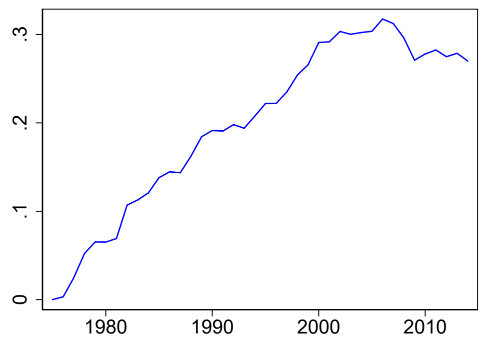
	I	II	III	IV	V	VI
R&D Intensity						
Fixed Cost Share	0.010***	0.023***	0.014***	0.013***	0.022***	0.012**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.005)	(0.005)
Observations	92,536	92,536	92,536	92,536	92,536	92,536
R^2	0.007	0.012	0.104	0.113	0.003	0.016
Sales growth						
Fixed Cost Share	0.010***	0.010***	0.002***	0.003***	0.043***	0.077***
(1 lag)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)
Observations	7,425,736	7,425,736	7,425,736	7,425,736	7,425,736	7,425,736
R^2	0.075	0.077	0.081	0.083	0.164	0.201
Year F.E.	No	Yes	No	Yes	No	Yes
Industry F.E.	No	No	Yes	Yes	No	No
Firm F.E.	No	No	No	No	Yes	Yes
Size Poly.	Yes	Yes	Yes	Yes	Yes	Yes

Notes: *, **, and *** denote significance at the 10, 5, and 1% level, respectively.

Appendix D: Macroeconomic Trends in France

The introduction summarizes three recent trends: the slowdown of productivity growth, the fall in business dynamism and the rise of corporate profits. This appendix gives an overview of the macroeconomics trends discussed in the introduction for France. The slowdown of productivity growth is depicted in Figure A2. It plots an index of the log of total factor productivity at constant prices, standardized to 0 in 1975. The figure shows that total factor productivity was growing at a rate close to 2% for most years between 1975 and 2000, and has not increased (and even modestly decreased) since.

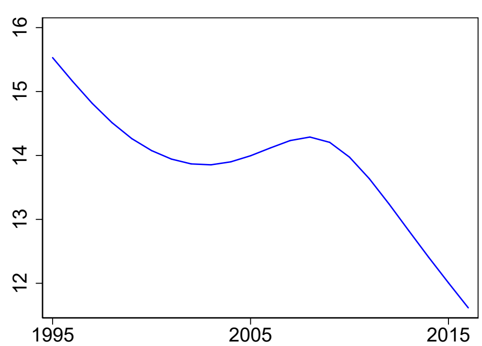
Figure A2. Total Factor Productivity in France



Log TFP at constant prices, 1975=0. Data: Penn World Tables.

The slowdown of business dynamism is summarized with three statistics, following the literature. The first is the reallocation rate in Figure A3, which is the sum of job destruction and creation rates. I calculate the reallocation rate across French firms using the FARE-FICUS dataset for 1994-2016. Because this sample coincides with the Great Recession, which brought a strong transitory increase in reallocation due to job destruction, I plot the HP trend.

Figure A3. Reallocation Rate in France

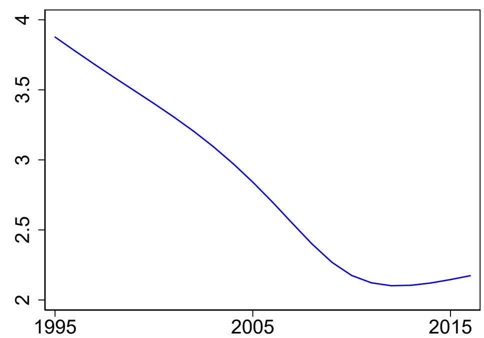


Sum of job creation and job destruction rates across companies. HP Trend.

Source: own calculations based for universe of French firms (FARE-FICUS)

The second fact is the decline of entry of new firms. Figure A4 captures this trend by plotting the fraction of employees that work for a firm that enters the FARE-FICUS dataset in a given year. Note that this may include firms that have undergone significant organizational changes that have caused their firm identifier to change. The figure shows that employment by entrants has declined by almost half within the 1994-2016 sample.

Figure A4. Fraction of Employment by Entrants in France

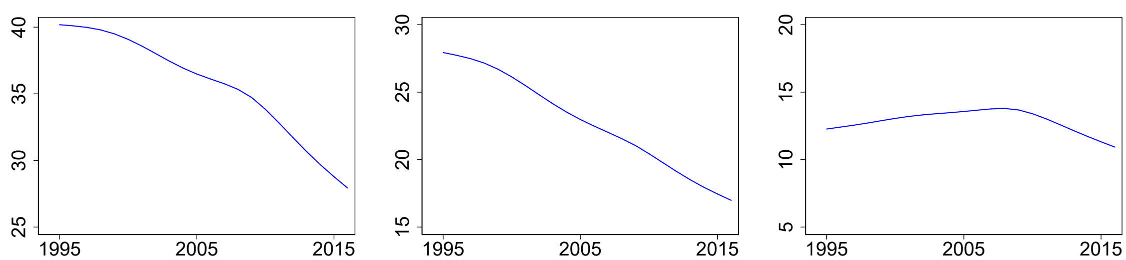


Percentage of employment by new firms (≤ 1yr) in private sector employment. HP trend

Source: own calculations based for universe of French firms (FARE-FICUS)

The third fact is the decline of skewness of the firm growth distribution. As discussed by Decker et al. (2017), small (young) high-growth firms have historically been an important contributor to productivity growth. They infer the decline in skewness of the growth distribution from the decline between the 90th and 10th, and between the 90th and 50th percentile of the growth distribution. Figure A5 shows that both have declined by around 40% between 1994-2016. The difference between the 50th and 10th percentile has remained flat, in line with evidence for the U.S.

Figure A5. Skewness of the Employment-Growth Distribution



(a) 90-10 Difference

(b) 90-50 Difference

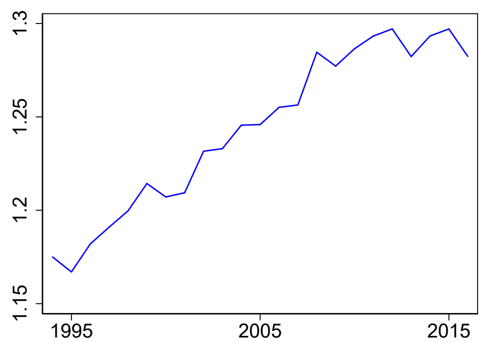
(c) 50-10 Difference

Difference (perc. point) in growth between percentiles of the employment-growth distribution. HP trend.

Data: universe of French firms (FARE-FICUS)

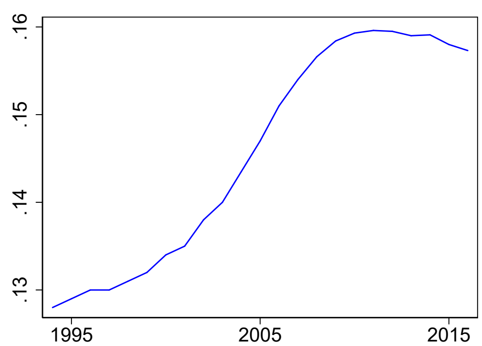
The rise of corporate profits is measured through the marginal cost markup. This is a measure of marginal rather than average profits, a distinction that is key in Section 2. Figure A6 plots the average sales-weighted markups for French firms between 1994 and 2016. The markups show a positive trend, which coincides with a positive trend in firm concentration. This is shown in Figure A7, which depicts the average Herfindahl Index across 5-digit industries. The rise of concentration has been linked to the decline in the labor share by Autor et al. (2017) through the reallocation of activity to low-laborshare firms. This result has been replicated for France for 1994-2007 by Lashkari and Bauer (2018).

Figure A6. Marginal Cost Markups for France



Sales-weighted marginal cost markups using the Hall (1988) equation with production function elasticities estimated with iterative GMM as in De Loecker and Warzynski (2012). Details in Appendix C. Data: universe of French firms (FARE-FICUS)

Figure A7. Firm Concentration



Average Herfindahl index across 5-digit NACE industries, weighted by value added.

Source: own calculations based for universe of French firms (FARE-FICUS)

Appendix E: Additional Tables

Table A4 presents the relationship between the use of IT systems the share of fixed costs. Data comes from the TIC, which has been held since 2007 and covers an annual sample of around 10.000 firms with at least 10 employees. All columns control for time and industry fixed effects and observations are weighted for representativeness.[24] ERP refers to enterprise resource planning, CRM to customer research management software, CAO to collaborative design tools, SCM to supply chain management software, RFID to radio frequency identification, and Skill refers to whether the firm employs IT specialists. Except for SCM they have a strong positive correlation with the share of fixed costs.

Table A4: Relationship between Technology Adoption and Fixed Cost Share

	Software Adoption					
Fixed Cost Share	ERP	CRM	CAO	SCM	RFID	Skill
Adoption Dummy	0.015***	0.006***	0.020***	0.004	0.023***	0.045***
	(0.002)	(0.002)	(0.006)	(0.003)	(0.006)	(0.004)
Year F.E.	Yes	Yes	Yes	Yes	Yes	Yes
Industry F.E.	Yes	Yes	Yes	Yes	Yes	Yes
Size Poly.	Yes	Yes	Yes	Yes	Yes	Yes
N	63,928	69,200	30,415	45,685	16,847	46,806
R^2	0.320	0.319	0.317	0.346	0.385	0.355

Dependent variable is fixed costs as a percentage of total costs. Explanatory variable is a dummy for the adoption of the technology specified in the column header (details provided in main text). Firm-clustered standard errors in parenthesis. *, **, *** denote significance at the 10, 5 and 1% level, respectively. The number of observations differ for each measure as not every measure was included in each survey year.

[24] The TIC samples different firms each year and is therefore not a panel, except when firms have been sampled multiple times. This is mainly the case for large firms, which makes the sample unrepresentative as a panel.