GDP is a measure of output, not welfare.
Or, HOS meets the SNA

Nicholas Oulton

Centre for Macroeconomics, LSE, NIESR and ESCoE

March 2019

Email: n.oulton@lse.ac.uk

Contents

Abstract

What effect, if any, do changes in the terms of trade have on the level of output (GDP) or welfare? I examine this issue through two versions of a textbook, Heckscher-Ohlin-Samuelson (HOS), two-good model of a small, open economy. In the first version both goods are for final consumption. In the second, one good is an imported intermediate input into the other. In both versions, economic theory suggests that an improvement in the terms of trade raises welfare (consumption) but leaves aggregate output (GDP) unchanged. This follows from a continuous-time analysis using Divisia index numbers. I then show that a national income accountant applying the principles of the 2008 System of National Accounts (SNA) would reach the same conclusions.

Keywords GDP, welfare, SNA, Hecksher-Ohlin-Samuelson, terms of trade, Divisia

JEL codes E01, F11, C43, D60

1. Introduction[1]

In popular discussion GDP is often treated as a measure of welfare but national income accountants never tire of pointing out that it is designed to be a measure of output or income (e.g. European Commission et al. 2009). There are a number of well-known reasons why a measure of output may differ from one of welfare. For example GDP is gross of capital consumption, and the position of the production boundary is somewhat arbitrary: the imputed rent of owner-occupiers is included while unpaid house work and child care are excluded. Moreover the treatment of environmental assets is unsatisfactory. But in this paper I am concerned with a much simpler issue: how should changes in the terms of trade be treated in the national accounts?

This issue has been debated by both national income accountants and economists for decades. It is discussed in the volumes setting out both the 1993 and the 2008 System of National Accounts (SNA): see Commission of the European Communities et al. (1993) and European Commission et al. (2009). But there is no agreement even within these manuals about which price index should be used to compute the real gain or loss from changes in the terms of trade, the so-called trading gain. Nonetheless both these versions of the SNA agree on the distinction between real GDP and real Gross Domestic Income (GDI): real GDI equals real GDP plus the trading gain. The SNA manuals are also clear that real GDI is a measure of welfare, or at least a step on the road to a more comprehensive measure of welfare, while GDP is a measure of output. On the other hand, Diewert and Morrison (1986) have questioned this distinction between welfare and GDP, suggesting that an improvement in the terms of trade should be treated as a form of technical progress; Fox and Kohli (1998) have applied their approach to Australia, 1960-1992. The distinction between real GDI and real GDP is empirically important at least for some countries, e.g. Canada and Switzerland (Kohli 2006). The allegedly declining terms of trade of primary producers in the 1950s and 1960s (the Prebisch thesis), the recently ended commodity price boom, the oil price shocks of the 1970s and 1980s, and the gains to countries which can import ICT products at rapidly falling prices, all these make changes in the terms of trade a subject of perennial interest.

[1] I owe thanks to two anonymous referees, and to Marshall Reinsdorf and Kevin Fox for helpful comments. This paper benefited from being presented at an ESCoE seminar hosted by the ONS on 26 February 2019. I am also grateful to Jonathan Haskel who drew my attention to conflicting results in the literature on what is called here Model 2. This paper is also available at https://www.escoe.ac.uk/download/3608/ as an ESCoE discussion paper.

The approach of this paper is to consider some very simple models of trading economies and calculate from first principles the changes in output and welfare which follow from a change in the terms of trade. The first such model, the Hecksher-Ohlin-Samuelson (HOS) model of a small economy producing and trading consumption goods, predicts that (under certain assumptions) an improvement in the terms of trade, i.e. an increase in the price of exports relative to that of imports, raises economic welfare. I then ask whether a national income accountant, applying the principles of the SNA to this theoretical model, would agree.

The second simple model again has two goods but now one of them, the imported good, is an intermediate input into the other. This is the type of model used to analyse an oil price shock. Again I ask whether the theorist and the national income accountant would reach the same conclusions. These models are oversimplified and ignore many real world features. But considering them serves to illustrate the principals involved. And if we can't understand the relationship between GDP and welfare in these simple cases we will certainly fail to do so in more complicated ones.[2]

Figures 1(a) and 1(b) illustrate the first model and show the textbook analysis of the gain from an improvement in the terms of a trade in a small open economy. Figure 1(a) shows the original position and Figure 1(b) shows the position after the change in the terms of trade. In this simple form of the HOS model there are two goods and two factors of production which we can label land and labour, both inelastically supplied. Both goods are produced under constant returns to scale and are for final consumption. Hence there is a concave transformation curve or production possibility frontier showing possible combinations of output of each of the two goods given the factor endowments and the level of technology; this is the curve labelled PP′ in Figures 1(a) and (b). All markets are perfectly competitive so production takes place on the frontier. The country is a price taker in international trade, shown by the initial terms of trade line TT′. The point P_0 marks the initial production point (the tangency of the production possibility curve with the terms of trade line), and the point C_0 the initial consumption point (the tangency of the terms of trade line with the highest available indifference curve, labelled U_0U_0'). The country has comparative advantage in good 1, exporting CD of good 1 and importing AB of good 2 in exchange.

[2] Reinsdorf (2010) considers a similar range of issues from the perspective of discrete index numbers rather than continuous (Divisia) ones as here. He does not emphasise the output versus welfare question.

Now there is an exogenous rise in the relative price of the export good (good 1): in Figure 1(b) the terms of trade line rotates from TT′ to T″T‴. . So resources shift into good 1 and away from good 2; the production point moves from P_0 to P_1. Clearly this generates an improvement in *potential* welfare: The country can now consume at any point along the new terms of trade line. So potentially the country could choose a point like C_1 which lies to the north-east of the initial point C_0 and consume more of both goods. However to show that the change in the terms of trade generates an improvement in *actual* welfare requires more assumptions. The reason is that distributional issues cannot be ignored in the HOS model since goods prices determine factor prices. Suppose the export good is land-intensive. Then a rise in the relative price of good 1 raises the real rent on land and lowers the real wage (the Stolper-Samuelson theorem). To avoid these distributional issues assume that the population is composed of individuals who have equal shares in the endowments of land and labour. So they only care about their total income and not about factor prices per se. If all individuals have identical preferences and each maximises a conventional (strictly concave) utility function which depends on consumption of the two goods, we can draw representative indifference curves as in Figures 1(a) and (b) to indicate the actual level of welfare enjoyed before and after the change in the terms of trade. Clearly the country now enjoys a higher level of welfare since the representative consumer is now on a higher indifference curve, at point C_1 on the higher indifference curve $U_1 U_1'$ rather than at the initial point C_0.[3]

Suppose we apply the System of National Accounts (SNA) to the textbook economy of Figure 1. Will the conclusions reached on the basis of the figure be reflected in the national accounts? To answer this question, in the next section I write down the economic relationships lying behind Figure 1 in mathematical form. In Section 3 I set out the national accounts of the textbook economy. I consider whether the national income accountant, with access to all the necessary data, would reach qualitatively the same conclusions as the theorist. If so, the national income accountant can go one better than the theorist by actually quantifying the changes in output and welfare in Figure 1. Section 4 then extends the analysis to consider the case where one of the goods is an intermediate input into the other. Section 5 introduces some practical considerations and discusses the treatment of trading gains in the latest SNA manual. Section 6 concludes.

[33] As drawn, consumers enjoy more of both goods at C_1. This is not a necessary outcome since the price of good 1 has risen. But even if consumption of good 1 fell there is still a rise in welfare on the assumptions made here.

2. Economic relationships in the HOS model

Figure 1 is an exercise in comparative statics. So the time period over which the terms of trade are supposed to change is left unclear. In the context of national income accounting, it is helpful to suppose that the change takes place continuously over a discrete time interval, 0 to *T*. As we shall see, this enables us to employ the powerful apparatus of Divisia indices to analyse the change.

Let us now write down the basic relationships of Figure 1 in mathematical terms.

(a) The production possibility frontier

The production possibility frontier (or transformation curve) can be defined implicitly as:

$$F(Y_1, Y_2; R, L, \tau) = 0 \tag{1}$$

Here Y_1 ,Y_2 are the outputs of the two goods; the endowments of land (*R*) and labour (*L*) are assumed constant as is the level of technology (τ). Differentiating with respect to time (*t*):

$$\frac{dY_2}{dt} = -\left(\frac{\partial F / \partial Y_1}{\partial F / \partial Y_2}\right)\frac{dY_1}{dt} \tag{2}$$

Here $(\partial F / \partial Y_1)/(\partial F / \partial Y_2)$ is the marginal rate of transformation between goods 1 and 2 and so in a perfectly competitive economy this equals the relative price of the two goods, P_1 / P_2. With a little bit of algebra, including dividing through by the total value of output $(P_1Y_1 + P_2Y_2)$, equation (2) becomes

$$s_{Y_1}^{GDP}\hat{Y}_1 + s_{Y_2}^{GDP}\hat{Y}_2 = 0 \tag{3}$$

where

$$s_{Y_1}^{GDP} := \frac{P_1Y_1}{P_1Y_1 + P_2Y_2},\ s_{Y_2}^{GDP} := \frac{P_2Y_2}{P_1Y_1 + P_2Y_2}$$

are the shares of each product in the total value of output (nominal GDP), a hat (^) denotes a growth rate, e.g. $\hat{Y}_1 = d\log Y_1 / dt$, and the symbol ":=" means "is defined to be".

(b) The trade balance

In the textbook model of Figure 1 trade always balances as there is no saving or investment:

$$B := P_1 X_1 - P_2 M_2 = 0 \tag{4}$$

Here B is the trade balance, X_1 is exports of good 1 and M_2 is imports of good 2. So differentiating with respect to time and dividing through by the value of output (*GDP*):

$$\frac{1}{GDP}\frac{dB}{dt} = \left[\frac{P_1 X_1}{GDP}\frac{1}{P_1}\frac{dP_1}{dt} - \frac{P_2 M_2}{GDP}\frac{1}{P_2}\frac{dP_2}{dt}\right] + \left[\frac{P_1 X_1}{GDP}\frac{1}{X_1}\frac{dX_1}{dt} - \frac{P_2 M_2}{GDP}\frac{1}{M_2}\frac{dM_2}{dt}\right] = 0$$

Defining $s_X^{GDP} := \frac{P_1 X_1}{GDP}$, $s_M^{GDP} := \frac{P_2 M_2}{GDP}$, whence $s_M^{GDP} = s_X^{GDP}$, as the shares of exports and imports in nominal GDP, the last equation can be rearranged as

$$s_M^{GDP}(\hat{P}_1 - \hat{P}_2) = -s_M^{GDP}(\hat{X}_1 - \hat{M}_2)$$

or

$$(\hat{P}_1 - \hat{P}_2) = -(\hat{X}_1 - \hat{M}_2) \tag{5}$$

In words: with balanced trade, if the terms of trade improve, then import volumes rise faster than export volumes. This relationship connects changes in volumes and prices and will be used below.

(c) Utility and welfare

Let the representative consumer's expenditure function be given by

$$x = c(P_1, P_2)u \tag{6}$$

where x is expenditure, $c(\cdot)$ is a strictly concave function of relative prices and u is utility. Here I am going a bit beyond what is strictly implied by Figure 1 since I am assuming that consumer demand is homothetic, in which case the expenditure function can be written in multiplicative form as in (6). Using (6) and selecting any arbitrary level of utility $\bar{u}$, a true cost-of-living (Konüs) index P_C at time t relative to time 0 is

$$\frac{P_C(t)}{P_C(0)} = \frac{c(P_1(t), P_2(t))\bar{u}}{c(P_1(0), P_2(0))\bar{u}} = \frac{c(P_1(t), P_2(t))}{c(P_1(0), P_2(0))} \tag{7}$$

In this case the Konüs price index is independent of the chosen level of utility.[4] Applying Shephard's Lemma (Varian 1992, page 74), the growth rate of this price index at any point t in the time interval (0,T) is

$$\hat{P}_C(t) = s_1^C(t)\hat{P}_1(t) + s_2^C(t)\hat{P}_2(t) \tag{8}$$

[4] The true cost-of-living index was introduced by Konüs (1939). On the relationship between homotheticity and true cost-of-living indices see Hulten (1973), Samuelson and Swamy (1974), and Deaton and Muellbauer (1980), chapter 7.

where s_1^C, s_2^C are the shares of goods 1 and 2 in the value of consumption:

$$s_1^C := \frac{P_1 C_1}{P_1 C_1 + P_2 C_2}, \quad s_2^C := \frac{P_2 C_2}{P_1 C_1 + P_2 C_2} \tag{9}$$

So in this case the Konüs price index is also a Divisia index.[5] The growth of the corresponding Divisia quantity index of consumption is

$$\hat{C}(t) = s_1^C(t)\hat{C}_1(t) + s_2^C(t)\hat{C}_2(t) \tag{10}$$

The total change in welfare over the period (0, *T*) can then be measured by the change in real consumption, i.e. nominal consumption deflated by the price index:

$$\log\left[\frac{C(T)}{C(0)}\right] = \log\left[\frac{P_1(T)C_1(T) + P_2(T)C_2(T)}{P_1(0)C_1(0) + P_2(0)C_2(0)}\right] - \log\left[\frac{P_C(T)}{P_C(0)}\right] \tag{11}$$

where from (8)

$$\log\left[\frac{P_C(T)}{P_C(0)}\right] = \int_0^T \left[s_1^C(t)\hat{P}_1(t) + s_2^C(t)\hat{P}_2(t)\right]dt \tag{12}$$

Alternatively the total change in real consumption can be expressed directly in terms of the quantity index:

$$\log\left[\frac{C(T)}{C(0)}\right] = \int_0^T \left[s_1^C(t)\hat{C}_1(t) + s_2^C(t)\hat{C}_2(t)\right]dt \tag{13}$$

Note that all the prices and quantities in equations (11), (12) and (13) are observable.[6]

So far we have viewed the Konüs price index as just an ideal cost-of-living index. We may note in passing that there is also an interpretation in terms of the compensating variation: the amount that a household must be paid (or taxed) after some change in prices to give it the same utility level as the one it started with. Following Hausman (2003) the representative consumer's compensating variation (CV) between two periods 0 and *T* for a given utility level $\bar{u}$ is:

[5] These indices were introduced by Divisia (1925-1926) and have been analysed by Hulten (1973) and Balk (2005). They were introduced into productivity analysis by Griliches and Jorgenson (1967). One of their great advantages is that the product of the Divisia price index and the Divisia quantity index is the value index. Another is that they are consistent in aggregation (though this latter property is not used in the present paper). The relationship between Konüs and Divisia price indices is analysed in Oulton (2008) and (2012).

[6] If demand is non-homothetic then Konüs and Divisia indices are not identical. Welfare measures now depend on the viewpoint: whose utility is to be the reference point when welfare changes are to be measured in monetary terms? See Oulton (2008) and (2012) for practical ways in which Konüs indices can be estimated from real world data. Non-homotheticity is also a problem for discrete index numbers such as chained Laspeyres or chained Fisher, making their interpretation problematic. The counterpart to non-homotheticity on the output side is non-constant returns to scale, assumed away in the models discussed here. Again, non-constant returns are a problem for both continuous (Divisia) and discrete index numbers.

$$CV = e(\mathbf{P}(T), \bar{u}) - e(\mathbf{P}(0), \bar{u})$$

where $e(\mathbf{P}(t), u)$ is the expenditure function, now allowed to be non-homothetic, and $\mathbf{P}(t)$ is the price vector at time *t*. The close connection with the Konüs price index is clear since the latter measures the price level in period *T* relative to period 0 by

$$P_C(T) = \frac{e(\mathbf{P}(T), \bar{u})}{e(\mathbf{P}(0), \bar{u})}$$

and the discrete growth rate of the Konüs price index between periods 0 and *T* is

$$P_C(T) - 1 = \frac{e(\mathbf{P}(T), \bar{u})}{e(\mathbf{P}(0), \bar{u})} - 1 = \frac{CV}{e(\mathbf{P}(0), \bar{u})}$$

That is, the growth rate of the Konüs index over a discrete period of time is the CV generated by the price change as a proportion of the original expenditure level. This shows that, contrary to a common view, the growth of a Konüs price index, and also that of a Divisia index when demand is homothetic, measures the change in consumer surplus resulting from price changes. So although the value of consumption does not include the level of consumer surplus, *changes* in real consumption (when measured by a Divisia index) do include *changes* in consumer surplus.

(d) Output

The Divisia index of aggregate output (*Y*) can be written as

$$\hat{Y} = s_{Y_1}^{GDP} \hat{Y}_1 + s_{Y_2}^{GDP} \hat{Y}_2 \tag{14}$$

and we have already seen from (3) that the right hand side of (14) is zero. Hence $\hat{Y} = 0$ in this model economy. So the total change in output over the interval [0, *T*] is also zero:

$$\log\left[\frac{Y(T)}{Y(0)}\right] = \int_o^T \hat{Y}(t)dt = 0 \tag{15}$$

Equations (3) and (14) say that a reallocation of factors, raising the output of one industry while reducing that of another, with endowments and technology held constant, leaves aggregate output unchanged. This makes perfectly good sense economically: only an increase in the endowment of one or both factors or an improvement in technology can increase aggregate output. In other words we are identifying an outward movement in the transformation curve (due say to technical progress, land reclamation or population growth) with an increase in aggregate output. But this does have an important implication: in the economy of Figure 1 welfare can increase while output (GDP) remains the same. Consequently, GDP must be interpreted as a measure of output but not of welfare.[7]

3. National income accounting in the HOS model

In this section I set out the national accounts of the HOS economy depicted in Figure 1. Here we must be careful to distinguish between relationships which derive entirely from the principles of the SNA and those which also rest on particular empirical features of the HOS model, such as that trade always balances.

A national income accountant measuring this economy would note the following supply-use relationships:[8]

$$P_1Y_1 = P_1C_1 + P_1X_1 \tag{16}$$

$$P_2Y_2 = P_2C_2 - P_2M_2 \tag{17}$$

The accountant would then go on to define nominal GDP from the expenditure and output sides as follows:

$$GDP(E) := P_E E \equiv P_1C_1 + P_2C_2 + P_1X_1 - P_2M_2 \tag{18}$$

$$GDP(O) := P_Y Y \equiv P_1Y_1 + P_2Y_2 \tag{19}$$

Here GDP(E) is conceived of as a price index (P_E) times a quantity index (*E*) and similarly GDP(O) is conceived of a price index (P_Y) times a quantity index (*Y*). Adding equations (16) and (17) shows that GDP(E) = GDP(O) or

$$P_E E = P_Y Y \tag{20}$$

National accountants are interested in growth rates as well as levels. So to obtain Divisia price and quantity indices, totally differentiate equations (18) and (19) with respect to time:

$$\hat{P}_E + \hat{E} = \left[s_1^{GDP}\hat{P}_1 + s_2^{GDP}\hat{P}_2 + s_X^{GDP}\hat{P}_1 - s_M^{GDP}\hat{P}_2) \right] + \left[s_1^{GDP}\hat{C}_1 + s_2^{GDP}\hat{C}_2 + s_X^{GDP}\hat{X}_1 - s_M^{GDP}\hat{M}_2) \right] \tag{21}$$

---

[7] Reinsdorf (2010), who employs a figure similar to Figure 1 by way of illustration, concludes too that aggregate output is constant in this case and for the same reason: there is a movement along the production possibility frontier but no shift in the frontier.

[8] In principle the accountant would allow for the possibility that the country also exports good 2 and imports good 1 (two-way trade). For simplicity I ignore this since in the model this cannot happen as the goods are assumed to be homogeneous,

$$\hat{P}_Y + \hat{Y} = \left[s_{Y_1}^{GDP} \hat{P}_1 + s_{Y_2}^{GDP} \hat{P}_2 \right] + \left[s_{Y_1}^{GDP} \hat{Y}_1 + s_{Y_2}^{GDP} \hat{Y}_2 \right] \tag{22}$$

Here s_1^{GDP}, s_2^{GDP} are the shares of consumption of the two goods in nominal GDP and $s_{Y_1}^{GDP}, s_{Y_2}^{GDP}$ are the shares of output of the two goods in nominal GDP. Identifying terms in prices with the price indices and terms in quantities with the quantity indices we have:

$$\hat{P}_E = \left[s_1^{GDP} \hat{P}_1 + s_2^{GDP} \hat{P}_2 + s_X^{GDP} \hat{P}_1 - s_M^{GDP} \hat{P}_2 \right] \tag{23}$$

$$\hat{E} = \left[s_1^{GDP} \hat{C}_1 + s_2^{GDP} \hat{C}_2 + s_X^{GDP} \hat{X}_1 - s_M^{GDP} \hat{M}_2) \right] \tag{24}$$

$$\hat{P}_Y = \left[s_{Y_1}^{GDP} \hat{P}_1 + s_{Y_2}^{GDP} \hat{P}_2 \right] \tag{25}$$

$$\hat{Y} = \left[s_{Y_1}^{GDP} \hat{Y}_1 + s_{Y_2}^{GDP} \hat{Y}_2 \right] \tag{26}$$

Taking account again of equations (16) and (17) we conclude that

$$\hat{P}_E = \hat{P}_Y \tag{27}$$

and

$$\hat{E} = \hat{Y} \tag{28}$$

Since the growth rates are always equal the levels are always equal too provided that we choose the same reference period for the price indices (i.e. $P_E(r) = P_Y(r) = 1$ in some reference period *r*). In other words, real GDP(E) equals real GDP(O): $E(t) = Y(t)$, and $P_E(t) = P_Y(t)$, all *t*.[9]

The national income accountant would also wish to calculate the growth of real consumption which can be measured as nominal consumption deflated by the Consumer Price Index or directly by an index of real consumption. The CPI can be expressed as a Divisia price index and real consumption can be measured as a Divisia quantity index: see equations (8) and (10).

If the national income accountant carried out these calculations for the economy of Figure 1 over the interval (0, *T*), what conclusions would he or she reach? First of all, the accountant would note that, empirically, there is no change in real GDP over this period:

---

[9] The equality of real GDP(O) and real GDP(E) when Divisia indices are employed was proved in the more general case with many goods and with intermediate consumption in Oulton (2004). Both there and here the same price was assumed to apply for a given product whatever the use to which the product was put (e.g. exports, consumption or investment) and each industry was assumed to produce only one product. The more realistic case where industries and products are distinguished and where there is price discrimination or product heterogeneity is examined in Oulton et al. (2018) and the equality of real GDP(O) and real GDP(E) is shown to still hold.

$$\hat{Y}(t) = 0, \quad 0 \le t \le T \tag{29}$$

This follows from (3). Second, the accountant would note that real consumption is increasing. Empirically trade is balanced so from (18) and (19) $P_1C_1 + P_2C_2 = P_1Y_1 + P_2Y_2$. Hence

$$s_1^C = s_1^{GDP},\ s_2^C = s_2^{GDP},\ \text{and } s_M^{GDP} = s_X^{GDP}$$

Now from (5), (10), (24), and (28)

$$\begin{aligned} \hat{Y} &= \hat{C} + s_M^{GDP}(\hat{X}_1 - \hat{M}_2) \\ &= \hat{C} - s_M^{GDP}(\hat{P}_1 - \hat{P}_2) \end{aligned} \tag{30}$$

But as we have just seen, $\hat{Y} = 0$ so

$$\hat{C} = -s_M^{GDP}(\hat{X}_1 - \hat{M}_2) = s_M^{GDP}(\hat{P}_1 - \hat{P}_2) = s_M^{GDP}\hat{p} \tag{31}$$

putting $p := P_1 / P_2$, the terms of trade. So as long as the terms of trade are improving, real consumption is rising. More generally, we conclude that consumption (welfare) is rising faster than GDP if the terms of trade are improving:

$$\hat{C} > \hat{Y} \ \text{ if } \ \hat{p} > 0 \tag{32}$$

A subtle point here is that, empirically, the value of consumption is always equal to the value of GDP: $P_C C = P_Y Y$ because the balance of trade is zero. Nonetheless, the volume of consumption is growing faster than the volume of output. The explanation is that the weights in the two indices differ. The weight for the export good in the consumption index is lower than in the output index.

Summing up we have

Proposition 1 In the HOS model with two consumption goods, an improvement in the terms of trade increases consumption and welfare but leaves GDP unchanged.

In other words the theorist and the national income accountant would be in agreement about the effect of an improvement in the terms of trade in the HOS model.

4. Terms-of-trade effects when imports are not consumer goods

4.1 Imported intermediate inputs

What difference would it make if one of the goods served as an input into the production of the other? Let us consider the simplest possible case of an intermediate input. Suppose that the country is completely specialised in the production of good 1, part of whose output is exported to pay for imports of good 2 which is used as an intermediate input, say energy. This corresponds to the much analysed case of a country which imports but does not produce energy products like oil or gas. We continue to consider an improvement in the terms of trade (a fall in the relative price of energy).

Consider first the national accounts. Supply and use of good 1 must be equal:

$$Y_1 = C_1 + X_1 \tag{33}$$

and nominal GDP is now

$$\begin{aligned} GDP(E) &:= P_E E \equiv P_1 C_1 + P_1 X_1 - P_2 M_2 \\ GDP(O) &:= P_Y Y \equiv P_1 Y_1 - P_2 M_2 \end{aligned} \tag{34}$$

Using (33) we see that $GDP(E) = GDP(O)$. Furthermore, if trade is balanced (equation (4)) then GDP(E) equals nominal consumption which also equals nominal value added or nominal GDP(O):

$$GDP(E) = P_E E = P_1 C_1 = P_1 Y_1 - P_2 M_2 = P_Y Y = GDP(O) \tag{35}$$

By totalling differentiating the relationships in equation (34) with respect to time, and separating terms in prices and quantities, we obtain

$$\hat{P}_E = \left[s_1^{GDP} \hat{P}_1 + s_X^{GDP} \hat{P}_1 - s_M^{GDP} \hat{P}_2 \right] = \hat{P}_1 + s_M^{GDP} (\hat{P}_1 - \hat{P}_2) \tag{36}$$

$$\hat{E} = \left[s_1^{GDP} \hat{C}_1 + s_X^{GDP} \hat{X}_1 - s_M^{GDP} \hat{M}_2) \right] = \hat{C}_1 + s_M^{GDP} (\hat{X}_1 - \hat{M}_2) \tag{37}$$

$$\hat{P}_Y = (1 + s_M^{GDP}) \hat{P}_1 - s_M^{GDP} \hat{P}_2 \tag{38}$$

$$\hat{Y} = (1 + s_M^{GDP}) \hat{Y}_1 - s_M^{GDP} \hat{M}_2 \tag{39}$$

using the fact that $s_M^{GDP} = s_X^{GDP}$ from (4). These last equations may be compared with (23)-(26) . As in the previous model

$$\hat{P}_Y = \hat{P}_E \text{ and } \hat{Y} = \hat{E} \tag{40}$$

For the price indices this follows directly from (36) and (38). The equality of the growth rates of the volume indices then follows since *GDP(E)* = *GDP(O)* from (35). (The equality of the volume indices can also be seen as a consequence of double deflation, implicit in equation (39)). These results rest solely on national income accounting principles together with the empirical facts that in this model economy good 1 is exported, good 2 imported, and trade is balanced. They make no use of economic theory.

However, if we want to answer substantive questions, such as, what is the effect of a fall in the price of imported energy on GDP?, then we need to invoke some theory. So assume a neo-classical production function for good 1:

$$Y_1 = Y_1(R, L, M_2, \tau) \tag{41}$$

Here as before τ indexes the level of technology. Dual to this is a price (or cost) function:

$$P_1 = P_1(P_R, P_L, P_2, \tau) \tag{42}$$

Suppose as before that *R* and *L* are fixed in supply and technology is constant. Then a lower price for energy encourages producers to move down the demand curve and increase energy input so that

$$\dot{Y}_1 = \frac{\partial Y_1}{\partial M_2}\dot{M}_2$$

or

$$\hat{Y}_1 = \left(\frac{s_M^{GDP}}{1+s_M^{GDP}}\right)\hat{M}_2 > 0 \tag{43}$$

assuming inputs are paid their marginal products, i.e. that in this case $\partial Y_1 / \partial M_2 = P_2 / P_1$, and noting that $P_2 M_2 / P_1 Y_1 = s_M^{GDP} / (1 + s_M^{GDP})$. Plugging (43) into (39) we find

$$\hat{Y} = 0 \tag{44}$$

i.e. real GDP is unchanged even though gross output of good 1 has risen. By differentiating the price function (42) and from (38) we see that the GDP deflator is constant too (relative to trend) while the price of good 1 falls: $\hat{P}_Y = 0$ and $\hat{P}_1 < 0$.

What about welfare? This is measured by the growth of consumption of good 1 which from (37) and (40) is

$$\hat{C}_1 = \hat{Y} - s_M^{GDP}(\hat{X}_1 - \hat{M}_2) > \hat{Y} \tag{45}$$

since from (5) $\hat{X} - \hat{M} < 0$ when the terms of trade are improving ($\hat{p} > 0$). Using (5) again and (44) this last equation can be written as

$$\hat{C}_1 = s_M^{GDP} \hat{p} > 0 \tag{46}$$

So consumption rises even though GDP is constant.

The marginal products of labour and land in terms of gross output of the consumption good have risen, assuming (as is usual) a positive relationship between the marginal products of the domestic inputs and the volume of the imported input, i.e. that $(\partial / \partial M_2)(\partial Y_1 / \partial R) > 0$ and $(\partial / \partial M_2)(\partial Y_1 / \partial L) > 0$. In other words, the real consumption wage (the money wage divided by the price of consumption) has risen and so has the real consumption rent on land. But the real product wage (the money wage divided by the price of value added) and the real product rent are both unchanged since (relative to trend) the GDP deflator is unchanged.

Summing up we have

Proposition 2 Suppose a 2-good HOS model where the country specialises in good 1 and imports good 2, and good 2 is an input into good 1. Then a fall in the relative price of good 2 raises consumption and welfare but leaves GDP and the GDP deflator (relative to trend) unchanged.

This conclusion is exactly the same as in the earlier model of two final consumption goods. So whether or not one of the goods is an intermediate input makes no difference. This may seem surprising given the considerable debate in the past about the effect of oil price rises on GDP and inflation, starting with Bruno and Sachs (1985) who argued that an oil price rise is a supply shock. In fact, Barsky and Kilian (2002) in re-visiting the Bruno-Sachs analysis reached the same conclusion as we have here, but they did so by making the restrictive assumption that the aggregate production function is separable into value added and energy. The argument of the present paper shows that this assumption is not necessary. Barsky and Kilian went too far however in claiming that an oil price rise is not a supply shock, i.e. it cannot change real GDP under any circumstances. Their model like the present one is static with fixed input supplies. Once we introduce the possibility of growth, i.e. if we drop the assumption of a fixed supply of land and allow capital to be accumulated, then effects on GDP are likely. The increase in energy input following an energy price fall raises the marginal product of both labour and capital. So an expansion of the capital stock is warranted together perhaps with an increase in labour supply. (A further qualification is that the model has nothing to say about any effects on GDP via aggregate demand but only considers aggregate supply.)

Blinder and Rudd (2008) disputed the Barsky-Kilian conclusion. They based their analysis on an equation on page 13 of their paper (which they attribute to Bruno and Sachs (1985)). The right hand side of this equation can be written in my notation as $Y_1 - P_2 M_2 / P_1$. They claimed that the left hand side measures real GDP. They then show that a rise (fall) in the price of the imported input would lower (raise) what they call GDP. But reference to my equation (34) shows that the left hand side is $(P_Y Y / P_1)$ which is not equal to real GDP (*Y*); in fact, the left hand side is nominal GDP deflated by the price of consumption (more generally, the price of expenditure), not by the GDP deflator. Hence their conclusion is incorrect.[10]

4.2 Imported capital goods

For the sake of completeness it is worth mentioning too the case where the imported input is a capital good. This has been analysed by Oulton (2012b) who shows via a two-sector growth model that a continuing fall in the relative price of the imported capital good raises the growth rate of the stock of this good which in turn leads to faster growth of both GDP and consumption. So in contrast to the two models above an improvement in the terms of trade boosts GDP since it leads to faster capital accumulation. Welfare rises too but this is a result of the rise in GDP.

5. Coming down to earth

The discussion up to now may seem a bit rarefied. So here we come down to earth a bit and introduce some practical considerations.

[10] Their analysis actually misinterprets Bruno and Sachs (1985). The latter distinguish carefully between three concepts: (1) what they call "real income", which is the right hand side of the equation labelled real GDP by Blinder and Rudd; (2) "double deflated value added", which is a fixed base index; and (3) a Divisia index of value added which is the same as the one I use here (see their chapter 2, Appendix 2B). They show that the fixed base index is biased by comparison to the true index, the Divisia. This last finding is less relevant today when national statistical agencies have largely adopted chained indices.

5.1 Discrete approximations

In practice Divisia indices cannot be calculated since data are only available at discrete intervals rather than continuously. But they can be approximated by chained indices of which the most commonly used for volume changes are the annually chained Laspeyres, Fisher or Törnqvist. National income accountants generally use either the chained Laspeyres (mandated by Eurostat for EU countries) or the chained Fisher (as in Canada and the US). Economic modellers and productivity analysts (following Griliches and Jorgenson 1967 and Jorgenson et al. 1987) often use the Törnqvist.[11]

The 2008 SNA has a whole chapter (Chapter 15) devoted to price and volume measures. Unfortunately nowhere does it mention Divisia index numbers. Despite this I am arguing that real world price and volume indices are best thought of as (more or less good) approximations to the ideal, the Divisia index. This approach enables us to link economic theory to the practice of national income accounting without having to assume particular functional forms for the underlying relationships like utility functions or production functions. The alternative approach is to assume that economic behaviour can be explained exactly by utility or production functions which take the form of a "quadratic mean of order *s*", as in Diewert (1976). Furthermore economic agents make synchronised decisions at fixed, discrete intervals Then there is a superlative index number (dependent on *s*) which is exact for this particular functional form. The drawbacks to this approach are that the results are dependent on the choice of the parameter *s*, and that the attractive properties of the Divisia index – price index times volume index equals value index and consistency in aggregation – are either lost, or compel the choice of a particularly value for *s*. E.g. setting $s = 2$ results in the Fisher index which satisfies the first of these properties but not the second, consistency in aggregation. Setting $s = 0$ results in the Törnqvist index which satisfies neither property.

A final point on real world chain indices as approximations to ideal Divisia indices is that trends in official statistics seem to be moving in this direction. In the last 60 years quarterly GDP estimates have become widespread and there is now movement towards monthly estimates. So we can foresee a time when the frequency of available observations makes the approximation issue of minor importance.

[11] Both Fisher and Törnqvist are superlative indices but they are not the only ones. See Hill (2006) for some caveats here.

5.2 What does the SNA say about the terms of trade?

The 2008 SNA discusses the concept of the "trading gain" which measures the benefit from changes in the terms of trade. The trading gain is to be added to real GDP, a measure of output, to obtain real Gross Domestic Income, or GDI, a measure of welfare. The 2008 SNA (European Commission et al. 2009, chapter 15) states:[12]

"15.188 ***Real gross domestic income (real GDI) measures the purchasing power of the total incomes generated by domestic production***. It is a concept that exists in real terms only. When the terms of trade change there may be a significant divergence between the movements of GDP in volume terms and real GDI. The difference between the change in GDP in volume terms and real GDI is generally described as the "trading gain" (or loss) or, to turn this round, ***the trading gain or loss from changes in the terms of trade is the difference between real GDI and GDP in volume terms.*** … Trading gains or losses, TG, are usually measured by the following expression:

$$TG = \frac{P_X X - P_M M}{P} - (X - M) \qquad (19)"$$

(Bolded test as in the original. I have changed the notation to be more consistent with the present paper).

Here P_X is the price of exports, P_M the price of imports, and *P* is a general price index. The SNA volume is not very prescriptive on how *P* is to be defined. It suggests various possibilities: (i) the export price index; (ii) the import price index; (iii) an average of the export and import price index; or (iv) a general price index, e.g. the CPI or the price index for gross domestic final expenditure. Despite these uncertainties the SNA is in no doubt as to the importance of GDI, both conceptually and, for some countries at least, empirically:

"15.191 … a. Trading gains or losses, as defined above, should be treated as an integral part of the SNA[.]"

"15.192 These proposals are intended to ensure that the failure to agree on a common deflator does not prevent aggregate real income measures from being calculated. Some measure of the trading gain should always be calculated even if the same type of deflator is not employed by

[12] The 1993 SNA used very similar language (Commission of the European Communities at al. 1993).

all countries. When there is uncertainty about the choice of deflator, an average of the import and the export price indices is likely to be suitable."

Clearly the SNA intends that GDI should be at least a step on the road to an aggregate measure of welfare though it clearly thinks further adjustments to GDP are required. The endpoint is "real net national disposable income" which is real GDP plus the trading gain, plus real net primary incomes from abroad, minus real net transfers to and from abroad, and minus "consumption of fixed real capital in volume terms" (paragraph 15.193). On how to deflate nominal primary incomes and transfers the SNA states (paragraph 15.194): "There may be no automatic choice of price deflator, but it is recommended that the purchasing power of these flows should be expressed in terms of a broadly based numeraire, specifically the set of goods and services that make up gross domestic final expenditure. This price index should, of course, be defined consistently with the volume and price indices for GDP." [13]

Let us focus on the trading gain. Within the context of this paper the first term in the formula above vanishes since trade always balances in the two models considered earlier. The SNA volume recommends a chained Laspeyres approach to measuring volumes. So in terms of contributions to the growth of GDI the second term in the trading gain formula above translates as

$$\begin{aligned} &-\left[\frac{P_X(t-1)X(t-1)}{GDP(t-1)}\right]\left(\frac{\Delta X(t)}{X(t-1)}\right)+\left[\frac{P_M(t-1)M(t-1)}{GDP(t-1)}\right]\left(\frac{\Delta M(t)}{M(t-1)}\right) \\ &=-\left[\frac{P_M(t-1)M(t-1)}{GDP(t-1)}\right]\left[\frac{\Delta X(t)}{X(t-1)}-\frac{\Delta M(t)}{M(t-1)}\right] \end{aligned} \tag{47}$$

on the assumption of balanced trade. This can be seen to a be a discrete, Laspeyres-type analogue of the continuous time formulas on the right hand sides of equations (30) and (45). In other words what the SNA calls real GDI is the same as real consumption in the two models studied in the previous sections. So the analysis of these models is quite consistent with the latest SNA.

Let us now revert to the first term in the SNA's equation (19) defining the trading gain:

$$\frac{P_X X - P_M M}{P}$$

[13] Reinsdorf (2010) reviews the various alternative suggestions for P.

This term measures the accumulation (or decumulation) of foreign assets and can only arise in an economy which saves and invests. If we are seeking a welfare measure this suggests that the trade balance should be deflated by the consumer price index or better still, a price index which in addition to private consumption covers at least some public consumption (such as on health). This approach would be in the spirit of Weitzman (1976) who suggested net national income deflated by a consumer price index as a welfare measure. Sefton and Weale (2006) also conclude that deflating by a consumer price index is appropriate for what they call real income, which is a monetary measure of welfare. So SNA 2008 is on the right lines but errs in recommending the deflator for gross domestic final expenditure (which includes investment) rather than the deflator for consumption. The treatment of the trading gain in ESA 2010 (Eurostat 2013, chapter 10) is very similar to that in SNA 2008. But though again not very prescriptive about the price index, it does recommend that the trade balance should be deflated by an average of export and import price indices. Compared to SNA 2008, this is a retrograde step.

5.3 A different approach

Diewert and Morrison (1986) argue that an improvement in the terms of trade has the same effect as an advance in technology. They devise indices of output and productivity for any number of products and inputs, including imported ones. They describe their indices as welfare measures. The present paper does not disagree in fundamentals with their approach or their conclusions, only about the usefulness of distinguishing between output and welfare as concepts, and stemming from this, between utility functions and production functions. It is thus more consistent with the approach to welfare measurement in Jorgenson and Schreyer (2017). Changes in the terms of trade may have the same effect as changes in technology, but the same is true of other things like climate change. It does not follow that climate change and technical progress are the same thing or that it is useful to produce measures which do not distinguish between them.

6. Conclusions

This paper has argued first, that the conclusions of economic theory about the effect of some exogenous change, like a change in the terms of trade, on aggregates like output, consumption or welfare can be translated into statements about the effects on Divisia index numbers. Second, Divisia index numbers provide a clear conceptual foundation for national income accounting. Third, the System of National Accounts provides the means to measure the effects studied in economic models, at least approximately. Fourth, I have shown that there is no conflict between the conclusions from textbook models about the effects of changes in the terms of trade and what a national income accountant would conclude by applying the principles of the SNA.

The SNA provides a practical approach to measuring output and welfare. It can be viewed as providing approximations to theoretical concepts like Divisia index numbers which cannot be measured exactly. And the distinction which the SNA makes between the concepts of output and welfare is supported by economic theory. Nevertheless the treatment of changes in the terms of trade in the latest version of the SNA is not perfect. SNA 2008 makes no firm recommendation as to how a non-zero trade balance should be deflated, though favouring the deflator for total final expenditure rather than the deflator for consumption as argued for here. SNA 2008 is however better than the European version, ESA 2010, which recommends an average of export and import price indices.

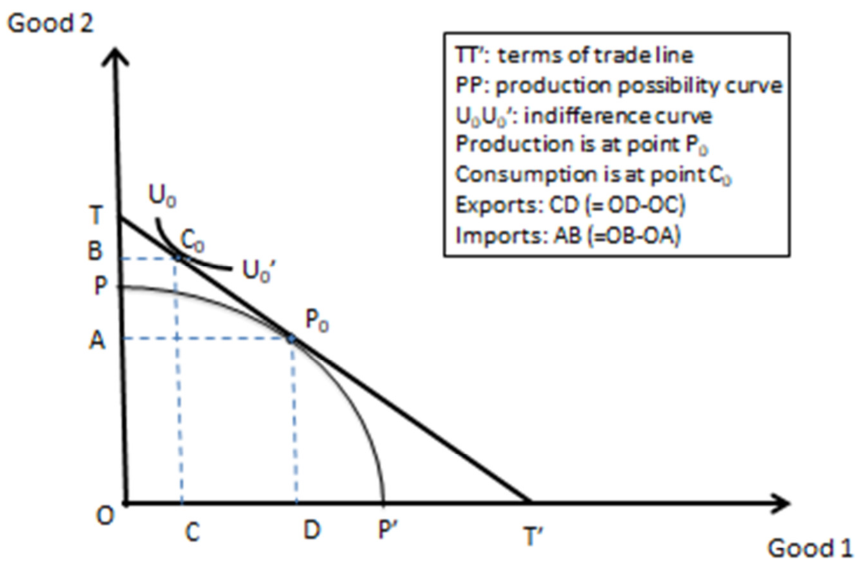

Figure 1(a). Equilibrium in the HOS model of a small open economy.

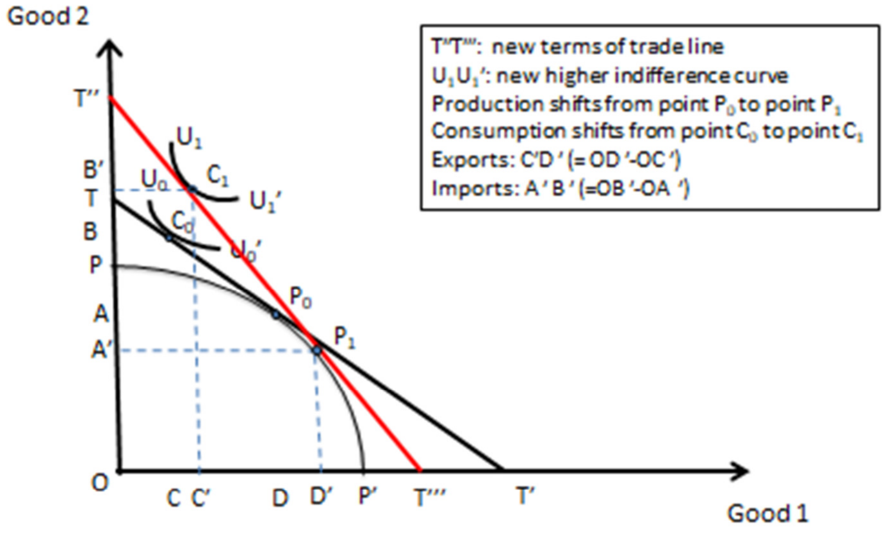

Figure 1(b). The new equilibrium in the HOS model after an improvement in the terms of trade. The new terms of trade are given by the slope of the red line T″T‴.

References

Balk, Bert M. (2005). "Divisia Price and Quantity Indices: 80 Years After." *Statistica Neerlandica*, 59: 119-158.

Barsky, Robert B. and Lutz Kilian (2002). "Do we really know that oil caused the Great Stagflation?". *NBER Macroeconomics Annual 2001*, volume 16.

Blinder, Alan S. and Jeremy B. Rudd (2008). "The supply-shock explanation of the Great Stagflation revisited". CEPS Working Paper No. 176.

Bruno, Michael, and Jeffrey D. Sachs. (1985). *Economics of Worldwide Stagflation*. Cambridge, MA: Harvard University Press.

Commission of the European Communities, International Monetary Fund, Organisation for Economic Cooperation and Development, United Nations, and World Bank (1993). *System of National Accounts 1993.* Brussels/Luxembourg, etc.

Deaton, Angus, and John Muellbauer (1980). *Economics and Consumer Behaviour*. Cambridge: Cambridge University Press.

Diewert, W. Erwin (1976). "Exact and superlative index numbers". *Journal of Econometrics*, 4, 115-46.

Diewert, W. Erwin and Catherine J. Morrison (1986) "Adjusting Output and Productivity Indexes for Changes in the Terms of Trade," *Economic Journal,* vol. 96, pp. 659-679.

Divisia, François (1925-1926). "L'indice Monétaire et la Théorie de la Monnaie." *Revue d'Economie Politique*, 39(4): 842-864; 39(5): 980-1008; 39(6): 1121-1151 [1925]; 40(1): 49-81 [1926].

European Commission, International Monetary Fund, Organisation for Economic Cooperation and Development, United Nations, World Bank (2009). *System of National Accounts 2008.* New York.

Eurostat (2013). *European System of Accounts: ESA 2010.*

Fox, Kevin J. and Ulrich Kohli (1998). "GDP growth, terms-of-trade effects, and total factor productivity". *Journal of International Trade and Economic Development*, 7:1, pp. 87-110.

Griliches, Zvi and Dale W. Jorgenson (1967). 'The explanation of productivity change', *Review of Economic Studies*, 34, 249-283. Reprinted in D.W. Jorgenson, *Productivity: Volume 1: Postwar U.S. Economic Growth.* Cambridge, MA: The MIT Press.

Hausman, Jerry (2003). “Sources of bias and solutions to bias in the Consumer Price Index”. *Journal of Economic Perspectives*, 17(1), Winter, pp. 23-44.

Hill, Robert J. (2006). “Superlative Index Numbers: Not All of Them Are Super”. *Journal of Econometrics*, 130: 25-43.

Hulten, Charles R. (1973). “Divisia Index Numbers.” *Econometrica*, 41: 1017-1025.

Kohli, Ulrich (2006). “Real GDP, Real GDI, and Trading Gains: Canada, 1981-2005”. *International Productivity Monitor*, Number 13, Fall, pp. 46-56.

Jorgenson, Dale W., Frank M. Gollop, and Barbara M. Fraumeni. (1987). *Productivity and U.S. Economic Growth*. Cambridge, MA: Harvard University Press.

Jorgenson, Dale W. and Paul Schreyer (2017). “Measuring Individual Economic Well-Being and Social Welfare within the Framework of the System of National Accounts”. *Review of Income and Wealth,* Series 63, Supplement 2 (December), pp. S460-S477.

Konüs, Alexander A. (1939). “The Problem of the True Index of the Cost-of-Living.” *Econometrica*, 7(1): 10-29. (English translation; first published in Russian in 1924).

Oulton, Nicholas (2004). “A statistical framework for the analysis of productivity and sustainable development”. Appendix to the *Allsopp Review of Statistics for Economic Policymaking*.
http://webarchive.nationalarchives.gov.uk/20100407010852/http://www.hm-treasury.gov.uk/d/allsopp_oulton_140.pdf. Also available at:
http://cep.lse.ac.uk/pubs/download/dp0629.pdf.

Oulton, Nicholas (2008). “Chain Indices of the Cost of Living and the Path-Dependence Problem: An Empirical Solution.” *Journal of Econometrics*, 144: 1 (May): 306-324.

Oulton, Nicholas (2012a). “How to measure living standards and productivity”. *Review of Income and Wealth,* vol. 58, issue 3 (September), 424-456.

Oulton, Nicholas (2012b). “Long term implications of the ICT revolution: applying the lessons of growth accounting and growth theory”. *Economic Modelling*, vol. 29, 1722-1736, 2012.

Oulton, Nicholas, Ana Rincon-Aznar, Lea Samek and Sylaja Srinivasan (2018). “Double deflation: theory and practice” Economic Statistics Centre of Excellence, Discussion Paper, ESCoE DP 2018-17.

Reinsdorf, Marshall B. (2010). “Terms of trade effects: theory and measurement”. *Review of Income and Wealth*, Series 56, Special Issue 1 (June), S177-S205.

Samuelson, Paul A. and Subramanian Swamy (1974). “Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis.” *American Economic Review*, 64: 566-593.

Sefton, J.A. and M.R. Weale (2006). "The Concept of Income in a General Equilibrium". *Review of Economic Studies*, 73, 219-249.

Varian, Hal R. (1992). *Microeconomic Analysis.* Third edition. New York: W.W. Norton and Company, Inc.

Weitzman, Martin L. (1976). "On the welfare significance of national product in a dynamic economy". *Quarterly Journal of Economics*, vol. XC, 156-62.