Tax evasion as contingent debt*

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Abstract

This paper studies income-tax evasion in a quantitative incomplete-markets setting with heterogeneous agents. A central aspect is that, realistically, evaded taxes are a form of contingent debt. Since evasion becomes part of a portfolio decision, risk and credit considerations play a central part in shaping it. The model calibrated to match estimated average levels of evasion does a good job in producing observed cross-sectional average evasion rates that decline with age and with earnings. The model also delivers implications for how evasion varies in the cross sectional distribution of wealth and tax arrears. Evasion has substantial effects on macroeconomic variables and welfare, and agent heterogeneity and general equilibrium are very important elements in the explanation. The analysis also considers the response of evasion to a flat-tax policy reform. In spite of the direct incentives to evade less under a flat tax rate, the reform causes households to save more, rendering the change in overall evasion modest.

Keywords: tax evasion; contingent debt; incomplete markets with heterogeneous agents; portfolio choice; risk sharing; tax progressivity

JEL codes: E2, E62, H3

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1 Introduction

Motivation - Rapidly mounting levels of unpaid tax arrears underscore ongoing governmental efforts for improving tax collection in OECD countries (OECD (2013)). Gaining a quantitative understanding of the level and patterns of tax evasion acquires thus pressing policy relevance. Although tax evasion is notoriously hard to measure, there is evidence that non-compliance is substantial and varies with individual characteristics like age and income. The US Internal Revenue Service's comprehensive estimates point to a difference between the theoretical tax liability and the amount of taxes collected (i.e., the tax gap) in the order of 20% for 2008-2010. Empirical studies find that the ratio of under-reported tax to true-tax liability declines sharply with true income. On the other hand, younger tax payers seem to show higher average levels of non-compliance.[1] Understanding these facts and their implications is the subject of this paper.

Objective - We build a quantitative model to study tax evasion from a macroeconomic perspective that takes into account a rich cross section of individual characteristics, and where the evasion decision results in the household accumulating tax arrears as long-term contingent debt, a realistic feature of actual tax systems. We use this framework to address a number of fundamental questions. Under which circumstances can we explain measured levels of evasion given existing enforcement mechanisms? Can the explanation account for observed patterns of evasion over the life cycle and across income? Which inferences may one draw about evasion along other dimensions like individual wealth? We also seek to assess the significance of non-compliance for macroeconomic outcomes and welfare, and explore the effectiveness of a flat-tax reform for grappling with evasion, a possibility discussed in the context of actual experiences.[2]

Method - We introduce evasion in a general equilibrium life cycle model of heterogeneous households with idiosyncratic risk and incomplete markets.[3] There is a non-linear tax code on the income generated from households' labour supply and assets.[4] One novel aspect of our framework is that households can choose to underreport their tax liabilities and accumulate tax arrears as a consequence. These arrears are debt that becomes due in the event of the household being subject to an audit. Households are audited randomly and penalties are proportional to the value of unpaid tax arrears. Tax arrears decay only

[1] See, for instance, Johns and Slemrod (2010) on income, and Andreoni, Erard, and Feinstein (1998) or Cabral, Kotsogiannis, and Myles (2018) on age. The evidence will be discussed further in section 2 below.

[2] See, for example, Keen, Kim, and Varsano (2008).

[3] In the tradition of Aiyagari (1994) and Huggett (1996).

[4] Much like in Conesa, Kitao, and Krueger (2009) or Conesa and Krueger (2006).

gradually at some constant rate reflecting the length of the statutory prescription period.

Tax arrears resulting from concealing tax liabilities become therefore a form of long-term contingent debt. As such they are part of a joint portfolio choice that also includes the standard one period risk-free bond. Risk considerations will therefore become a central force for determining behaviour. Positive yet partial arrears may arise even for households that are not credit constrained. Borrowing via tax evasion carries an implicit interest smaller than the risk-free interest on savings but, since debt in arrears has an element of risk, it becomes optimal to evade only part of individual earnings. This mechanism for the determination of non-compliance rests only on plausible primitive assumptions about enforcement and penalties, without necessarily having to invoke other extraneous elements.[5]

Characterising the outcomes requires solving a non-standard portfolio choice problem subject to occasionally binding constraints with endogenous bounds on these constraints. The computational approach here will identify the kinks but pursue smooth function approximation elsewhere. In order to draw the essential intuition for the trade-offs at work, we also discuss an analytically tractable 2-period version of the model of the household.

Results - The full model is calibrated to match, in addition to standard macroeconomic aggregates, estimated levels of evasion, given a quantitatively realistic setting for tax enforcement in the US. This exercise indicates that the pecuniary penalty faced by the audited household must have a component above and beyond the statutory fine levied by the tax authority, a deadweight loss that, while large, may not be unreasonable. In this baseline setting, the distribution of evasion rates over taxes due is skewed—for which there is some supporting evidence—with more intensive evasion concentrated at the low end of the income and wealth distribution, involving the younger cohorts and also households near retirement. The implied degree of recovery is empirically plausible. By studying the policy functions along various household characteristics we uncover the varied mechanisms behind the evasion decision, thereby confirming that heterogeneity may matter.

Turning to specifically observable implications from the stationary distribution, this cali-

[5]Accounting for a degree of compliance is challenging. For instance, Bernasconi (1998) notes that it may require unrealistically high levels of risk aversion. In effect, non-pecuniary elements such as stigma, ethical norms, moral sentiments or some abstract contemporaneous cost of the act of evading, are found in much of the public finance literature, including Gordon (1989), Bordignon (1993), Erard and Feinstein (1994), and Chetty (2009). Our model does not include those elements. We think that, while a contemporary cost may be associated to compliance, or to avoidance like in Uribe-Teran (2015), it would be more arbitrary in the context of evasion. Here we choose to be parsimonious and focus only on a deadweight pecuniary cost conditional on detection, which we will measure in the calibration exercise. Our model contains a terminal end-of-life loss to arrears but its role, as we will explain, is merely quantitative, and only concerns the oldest cohort and will prove otherwise inconsequential.

brated model does a notable job at producing evasion rates that decline both with working-life age and with individual earnings, in a way that closely resemble the available evidence on the two dimensions. The model also delivers novel implications for how evasion varies in the cross sectional distribution of wealth and tax arrears, but empirical counterparts are scarcer in this area.

We investigate the significance of evasion by analysing the effect of eliminating it on macroeconomic variables, risk sharing, and ex-ante welfare. We find that zero-evasion implies considerable gains in welfare and in the level of consumption, with some reduction in capital accumulation, which all depend critically on the general-equilibrium downward adjustment of tax rates as the government budget position improves when evasion is eradicated. In contrast, in partial equilibrium, consumption and welfare decline, savings decrease steeply, yet consumption volatility declines as the riskiness associated with contingent arrears is removed. The sign of insurance changes varies across types of agents, with large losses from volatile consumption being experienced by agents relying more heavily on evasion, to be found mainly among the very young and the pre-retirement age groups.

Finally, we consider a flat-tax policy reform, and find that, in spite of the incentives to evade less under this less progressive system, the fact that the reform causes households to save more renders the change in overall evasion modest. Once again, the aggregate effects mask considerable variation across types of households.

Contribution to literature - This paper shares the approach of a strand of macroeconomics on taxation within general equilibrium models of heterogeneous households and incomplete markets, including Domeij and Heathcote (2004), Conesa and Krueger (2006), Conesa, Kitao, and Krueger (2009), Guner, Lopez-Daneri, and Ventura (2016), Karabarbounis (2016), Holter, Krueger, and Stepanchuk (2014) and Dyrda and Pedroni (2016). These papers study redistributive and macroeconomic aspects, and the optimal determination, of capital taxation and income tax progressivity, but do not incorporate evasion. Uribe-Teran (2015) introduces the related, but clearly distinct, issue of tax avoidance with a focus on the tax elasticity of labour supply. One contribution of the present paper is thus to consider evasion and accumulated arrears, and analyse their implications, within the framework characteristic of this literature.

Tax evasion has, on the other hand, long been the subject of work in public finance. Allingham and Sandmo (1972) already entertained the element of 'gambling' in non-compliance, which is akin to our idea of borrowing in contingent debt. In that vein, dynamic aspects appear in Andreoni (1992)'s analysis of tax compliance with binding borrowing constraints, Levaggi and Menoncin (2016) on the hedging role of the risk-free asset for evasion decisions as part of portfolio, Levaggi and Menoncin (2012) and Levaggi and Menoncin (2013) on the effect of tax rates on evasion, and Bernasconi (1998) on the role of risk aversion for the degree of compliance, and Chetty (2009) more recently. As it turns out, our analysis echoes all these themes.[6] However, these papers are in partial equilibrium, deal with representative or single agent, cannot study aggregate aspects, and so are not well suited for the type of analysis undertaken here. Our contribution is to build these ideas into a fully specified quantitative setting.[7]

Regarding the specific questions addressed, this paper, to the best of our knowledge, is the first study to use the available evidence on life cycle and cross sectional patterns of evasion to validate theory and provide an explanation with a quantitative model. As for the question on the macroeconomic and welfare consequences of evasion, the analytical framework developed here contains elements that are missing in previous studies on related questions. Finally, our question about the flat-tax reform is related to the link between tax rates and evasion considered in Levaggi and Menoncin (2013) and motivated by the empirical positive relationship reviewed in Cebula and Feige (2012); our distinct focus shows that general equilibrium, portfolio decisions, and heterogeneity are all important.

Plan - The plan of the paper is as follows. Section 2 documents available evidence on evasion. Section 3 sets out the model. Section 4 contains qualitative analysis of a simple two-period version of the model. Section 5 describes elements of the computation. Section 6 presents and assesses the calibrated benchmark. Section 7 studies the policy rules underlying the outcomes. Section 8 looks at the implications for evasion over the life-cycle and at the cross-sectional distribution of evasion over wealth and income. Section 9 studies macroeconomic implications of evasion. Section 10 analyzes a flat-tax reform. Section 11 summarizes and concludes.

[6] There is also work on dynamic penalties and monitoring (e.g., Niepelt (2005)), and on optimal enforcement policy with asymmetric information (e.g., Ravikumar and Zhang (2012), Armenter and Mertens (2013), Huang and Rios (2016), to cite but a few recent papers).

[7] There is, of course, a vast literature on non-compliance and policy-related issues. For surveys see, for instance, Cowell et al. (1990), Andreoni, Erard, and Feinstein (1998), Slemrod and Yitzhaki (2002), Slemrod (2007), and Hashimzade, Myles, and Tran-Nam (2013).

2 Evasion facts

This section reviews the evidence on the facts that will be used for calibration and validation of the model as well as elements that support some of the assumptions made.

2.1 Levels of evasion

Precise empirical measures of tax evasion must necessarily be elusive,[8] but available estimates indicate that non-compliance is substantial. The US Internal Revenue Service's estimates, for example in IRS (2016), point to a difference between the theoretical tax liability and the amount of taxes collected, the so called 'tax gap', near 16.9% for 2006 and 18.3% for the tax years 2008-10 (equivalent to $450 and $458 billion, respectively). Cebula and Feige (2012), using a general currency ratio model to measure unreported income, find that 18-23% of total reportable income may not properly be reported to the IRS. These measures suffer from well known problems but they all point towards an overall evasion rate of about 20%.

Of the $458 billion gap estimate, the IRS is expected to have uncovered $52 billion through its enforcement and collection activities, resulting in a net-tax-gap for tax years 2008-10 of $406 billion, which is 16.3% of the tax that should have been remitted (IRS, 2016).[9] The evidence thus indicates a recovery over tax due in the order of 2% to 2.5%, or 11% of tax evaded.

The above evidence speaks to the overall tax gap. In terms of its distribution the available evidence is that non-compliance is not uniformly distributed. For the UK, Advani, Elming, and Shaw (2017) report that 60 per cent of non-compliant individuals owe additional tax of £1,000 or less, whilst four per cent owe more than £10,000 but collectively account for more than 42 per cent of the revenue owed. This is a skewed distribution, with most evasion being of small size.

2.2 Compliance by age

Younger tax payers seem to show higher average levels of non-compliance. Andreoni, Erard, and Feinstein (1998) review evidence for the US in this direction. That income underre-

[8]For a discussion on the difficulties surrounding these estimate see Slemrod and Gillitzer (2013).

[9]For 2006, of the $450 billion estimate, the amount to uncover is $65, with a net-tax-gap of $385 billion, 14.5% of the tax due (IRS, 2012).

porting decreases with age is consistent with the findings in the literature on tax audits in the US using TCMP (Clotfelter (1983); Feinstein (1991)), and also in field experiments in Denmark (Kleven, Knudsen, Kreiner, Pedersen, and Saez (2011)).

For the UK, Cabral, Kotsogiannis, and Myles (2018) show that as taxpayers age they become more compliant. More specifically, if the household reference person is less than 35 years old, income is underreported by 27.3%, by 18.9% if between 35 and 45 of age, and by 14.2% if between ages 45 and 60. Advani, Elming, and Shaw (2017) also find that being older is negatively associated with evasion in the UK, with proportions of undeclared tax over tax due of 23%, 31% and 35% for the 65+, 35 to 65, and 35- age groups respectively. The meta analysis in Hofmann, Voracek, Bock, and Kirchler (2017) supports a significant positive relation between age and tax compliance.

2.3 Compliance by income/earnings

Johns and Slemrod (2010) find that the ratio of under-reported tax to true-tax liability declines sharply with true income.[10] Their misreporting calculations imply ratios of evasion to tax due of about 55%, 23% and 15% for tax payers in the three first <30%, 31-60% and 61%-90% income quantiles respectively. Christian (1994) reports, based on the 1988 TCMP study, that higher income individuals evade less than those with lower incomes, relative to the size of their true income.[11] However, interestingly, it appears that evasion levels do rise with income.[12]

For the UK, Advani, Elming, and Shaw (2017) similarly report that the estimated evaded tax as fraction of tax liabilities also declines markedly across income quintiles from about 51% at the bottom, through 35%, 22%, 18%, to 10% in the top quintile.

2.4 Audit process and penalties

The model features two key parameters, the probability of auditing taxpayers and the penalty imposed on them for detected non-compliance.[13] For the US, rounding up the

[10]See also Andreoni, Erard, and Feinstein (1998), Clotfelter (1983) and Feinstein (1991).

[11]Those with adjusted gross income above $500,000 on average reported 97.1 percent of their true incomes to the IRS, compared to just 78.7 percent for those with adjusted gross income between $5,000 and $10,000.

[12]In Johns and Slemrod (2010) the net misreporting percentage of income actually rises steadily with true income, peaking at 21% for incomes up to 1 million dollars and declines thereafter.

[13]This is a simplification since, in practice, neither of these elements can be aptly summarised in a single number.

examination coverage percentages, a probability of 4% corresponds to approximately the median of examination coverage percentage, and is a reasonable approximation for the typical audit probability.[14] Turning to the penalty, a fine of 75% of tax liabilities evaded proxies well the level of statutory fines.

It is also important to note that perceived probability may be different from the actual one. Households appear to substantially overestimate them. Beer, Kasper, Kirchler, and Erard (2018) and opinion survey data for the U.S. (Bernasconi (1998), Andreoni (1992)) point in that direction. Our model does not distinguish perceived and actual probabilities so a single parameter might create a tension in the quantitative fit. We refrain however from raising detection probabilities beyond reasonable objective ones.[15]

3 The model

The model extends the standard model of heterogeneous households with incomplete markets and idiosyncratic income shocks. This is an economy where households can evade income taxes, and the fiscal policy instruments include the tax schedule, the audit probabilities, and fines. As a result of evasion, a household holds a latent liability with the tax authority which becomes effectively due in the event of the household undergoing an audit. In this sense, tax arrears are a form of contingent debt which becomes part of the individual's financial portfolio choice. The model has three groups of agents, households, firms and a government, and competitive markets. We consider the households first.

3.1 Households

In terms of demographics, there is a continuum of households in each age group. Age is discrete and denoted by $j = 1, ..., J$. The retirement age is j_R. There is population growth across cohorts at the rate n. Survival probabilities are denoted ψ_j for each age j, with $\psi_J = 0$.

Preferences of a household over their lifetime are represented by the sum of expected period utility

$$u(c, 1-l) = \frac{1}{1-\sigma} \left(c^{\gamma} (1-l)^{1-\gamma} \right)^{1-\sigma} \tag{1}$$

[14] See Table 9b of IRS (2015), page 27.

[15] Behavioural approaches consider this distinction. See, for instance, Hashimzade and Myles (2017).

defined over consumption c and leisure $1-l$, where l denotes hours worked, discounted at rate β. Labour supply is exogenous and constant over the working life.

Taxable household's income, denoted y, is defined as a function of their demographic and economic characteristics and given by

$$y(a,\eta,j,l) = \begin{cases} w\epsilon_j\eta l - 0.5\tau^{ss}\min\{w\epsilon_j\eta l, \overline{y}\} + r(a+Tr) & j < j_R \\ SS + r(a+Tr) & j \geq j_R, \end{cases}$$

where a is asset holdings, Tr is an accidental bequest transfer, η is individual idiosyncratic productivity, ϵ_j is the age-specific productivity component, τ^{ss} is a Social Security tax rate, and r and w are the aggregate rate of return and wage rate, respectively. The idiosyncratic component η follows a Markov chain with transition probabilities $\Gamma_{\eta,\eta'}$ of dimension N_η.

3.1.1 Taxes, tax arrears and the budget constraint

The household pays taxes as a function of income declared x by the amount $T(x)$. More specifically, we adopt the specification due to Gouveia and Strauss (1994)

$$T(x) = \kappa_0(x - (x^{-\kappa_1} + \kappa_2)^{-1/\kappa_1}). \tag{2}$$

The parameters κ_0 and κ_1 control for the level and progressivity of the tax code.[16]

Denote by $\mu \in [0,1]$ the proportion of taxable income y that is declared. A household may or may not be audited by the tax authorities. Consider first the case when the household is not audited. The budget constraint for a non-audited agent of working age $j < j_R$ is

$$(1+\tau^c)c + a' = y(a,\eta,j,l) - 0.5\tau^{ss}\min\{w\epsilon_j\eta l, \overline{y}\} + (a+Tr) - T(\mu y(a,\eta,j,l))$$

and, for a non-audited retired household of age $j \geq j_R$,

$$(1+\tau^c)c + a' = y(a,\eta,j,l) + (a+Tr) - T(\mu y(a,\eta,j,l))$$

where τ^c is a given consumption tax rate, and a' denotes the variable assets next period.

Denote by b the value of tax arrears currently outstanding. To capture expiration periods,

[16]See Conesa and Krueger (2006) for further discussion. Another possible choice would be the more parsimonious 2-parameter specification typical in public finance theory, and used recently in Heathcote, Storesletten, and Violante (2017).

it is assumed that, absent an audit, this debt is cleared in a gradual manner. It decays at rate $1-\nu$ so ν is the proportion of past tax arrears that remain into the next period, and $\nu = 0$ means past tax debts are erased after just one period. The choice of how much taxable income to report μ will determine how the value of household's unpaid tax debt evolves. Given the current b, the household will start next period with tax arrears resulting from adding the amount of current evasion to past arrears:

$$b' = [T(y(a,\eta,j,l)) - T(\mu y(a,\eta,j,l))] + \nu b.$$

The household can be the subject of a tax audit. In that event, they pay the taxes in arrears b and a cost in the proportion π^A of those arrears. This cost includes a penalty paid to the government as well as a deadweight loss borne by the household but not recovered by the government. In sum, the total cost of being audited to the household is $(1+\pi_A)b$, and must be added to their budget constraint. The audit also clears past arrears so, in the case of being audited, the law of motion of tax arrears becomes

$$b' = [T(y(a,\eta,j,l)) - T(\mu y(a,\eta,j,l))].$$

The above fully describes the budget set.[17] From now on, however, it will be convenient to use a more compact notation. Note first that we can drop the fraction of income declared μ since it is redundant with debt in arrears as $b' = T(y) - T(\mu y) + \nu b$. We also denote the audit status by $m \in \{0,1\}$, with $m = 1$ indicating that the household is audited, and define the debt service rate for tax liabilities as

$$\xi(m) \equiv (1-m)\nu + m(1+\pi^A). \tag{3}$$

This notation is useful in that the term $b' - \xi(m)b$ then denotes the value of resources to the household associated with tax arrears, $b' - \nu b$ if there is no audit, and $b' - (1+\pi^A)b$ if there is an audit

$$b' - \xi(m)b = (1-m)\underbrace{[b' - \nu b]}_{\text{no audit}} + m\underbrace{[b' - (1+\pi^A)b]}_{\text{audit}}.$$

[17]Note the analysis departs from voluntary repayments of tax arrears (i.e., repayments even when there is no audit). If such repayments were allowed, they would still be subject to the legal penalty, except if the taxpayer takes part in a voluntary disclosure agreement, a case not considered here.

This allows us to write the budget constraint as follows. For $j < j_R$,

$$(1+\tau^c)c = y(a,\eta,j,l) + (a+Tr) - T(y(a,\eta,j,l)) + b' - \xi(m)b - a' \\ - 0.5\tau^{ss}\min\{w\epsilon_j\eta l, \overline{y}\}, \quad (4)$$

and, for $j \geq j_R$,

$$(1+\tau^c)c = y(a,\eta,j,l) + (a+Tr) - T(y(a,\eta,j,l)) + b' - \xi(m)b - a' \quad (5)$$

where we have used that tax evasion in given by $b' - (1-m)\nu b$. The constraint $\mu \in [0,1]$ implies bounds on tax evasion (or arrears) given by

$$\underbrace{b' - (1-m)\nu b}_{\text{evasion}} \begin{cases} \leq & T(y(a,\eta,j,l)) - T(0) \\ \geq & 0. \end{cases} \quad (6)$$

In the absence of a bequest motive, the incentive of someone to meet tax liabilities when they are old is nil, and therefore the model would produce implausibly large evasion levels in old age. In the spirit of warm glow motives considered in much literature (e.g., De Nardi (2004)), we introduce a non-pecuniary cost of the arrears left after the last lifetime period as a function $\phi(b)$. It is specified as

$$\phi(b) = -\phi_0 b^{\phi_1}. \quad (7)$$

Its sole purpose is to contain the evasion levels at the very late ages, and will be of no significance otherwise.

3.1.2 Tax Audit

The tax authority today issues a strategy for auditing households tomorrow. We assume that the audit is purely random.[18] The probability of audit next period is $p^A(m)$, possibly a function of the audit status today m. So $p^A(0)$ is the probability if currently there is no audit; otherwise, if there is currently an audit then the probability of audit next period is $p^A(1)$. We assume $p^A(1) \geq p^A(0)$ if there is persistence. We also assume that auditing is

[18] The strategy could in general be conditioned on the individual characteristics (a, η, b, j) as well as on the outcome of past audits. While this would add more realism, how to specify it is not straightforward, and we leave it for future research.

costless, i.e, does not use material inputs.

3.2 Government

The government collects taxes, carries out public consumption, and pays social security transfers. The given social security payments to each retired SS are covered by payroll taxes levied at rate τ^{ss} to ensure balance in the social security system. Government consumption spending G and the consumption tax τ^c are given. Then the income-tax function $T(.)$ must balance the government budget, once recovery and penalties on total tax arrears, which we denote B, are also taken into account. The fine collected by the government through auditing is a fraction π^G of the total cost borne by the household π^A. (The remaining cost, $\pi^A - \pi^G$, is a deadweight cost born by the household but not recovered by the government.)

The aggregate level of tax in arrears held by households with audit type $m \in \{0, 1\}$, $B(m)$, is determined by the decisions of such individuals who survive. Tax arrears of those who exit go unpaid. The level of accidental transfers Tr to each household is determined by the level of savings of the individuals who exit via the mortality shock.

3.3 Technology, firms and markets

Aggregate output is produced by a representative firm operating a Cobb-Douglas production function of labour N and capital K inputs, with a share to capital α. Output can be consumed, spent on evasion costs or invested, and capital depreciates at the rate δ, so the aggregate resource constraint is

$$C + (\pi^A - \pi^G) \sum_{m=0,1} p^A(m)B(m) + K' + G = K^\alpha (ZN)^{1-\alpha} + (1-\delta)K,$$

where Z denotes aggregate productivity, and K' denotes next-period capital.

There is constant long-run productivity growth g. The market of inputs is competitive, and firms take as given the wage rate w and the rental rate of capital which, by arbitrage in financial markets, is $r + \delta$. Factor prices equal marginal products to the respective input hence $r + \delta = \alpha((ZN)/K)^{1-\alpha} - \delta$ and $w = (1-\alpha)Z(K/(ZN))^\alpha$. In equilibrium, aggregate labour N and capital K equate their supplies resulting from aggregating across all households.

3.4 Recursive equilibrium

The household's state is $s \equiv (a, b, \eta, i, j, m)$, where m is the indicator denoting whether they are audited, $m \in \{0, 1\}$. The recursive problem features time- and age-dependent value functions and decision rules, V_t, a'_t, b'_t, and c_t. The aggregate state Z_t evolves exogenously over time. There is a distribution of the population over individual states Φ_t that evolves in a away consistent with individual decisions and demographics. More formally, Φ_t belongs in the set of measures over a measurable space $(\mathcal{S}, \mathcal{Q})$, with $\mathcal{S} = R^+ \times R^+ \times \{\eta_1, ..., \eta_{N_\eta}\} \times \{1, ..., J\} \times \{0, 1\}$ the set of elements of the state, and $\mathcal{Q} = \mathcal{B}(R^+) \times \mathcal{B}(R^+) \times P(\{\eta_1, ..., \eta_{N_\eta}\}) \times P(\{1, ..., J\}) \times P(\{0, 1\})$ the product of the corresponding Borel algebras and power sets, and typical element $\mathcal{A} \times \mathcal{B} \times \mathcal{E} \times \mathcal{J} \times \mathcal{M}$.

We normalise per-capita variables by aggregate productivity Z_t, and aggregate variables by Z_t and by population size P_t. In terms of these detrended variables, the equilibrium definition includes the growth rates g and n, and removes Z_t accordingly. (Further details about de-trending are in the Appendix section A.)

In an equilibrium, the above policy and value functions, and distribution measure satisfy a number of conditions.

1. Household optimisation: Given $\{\tau_t^{ss}, SS_t, \tau_t^c, w_t, r_t, Tr_t, \overline{y}_t, T_t(.)\}_{t=0}^{\infty}$, for $j = 1, ..., J$,

$$V_t(a, b, \eta, j, m) = \max_{a', b'} \Big\{ u(c, 1 - l) + \beta \psi_j \sum_{\eta'} \Gamma(\eta, \eta') \\ [p^A(m) V_{t+1}(a', b', \eta', j + 1, 1) + (1 - p^A(m)) V_{t+1}(a', b', \eta', j + 1, 0)] \Big\},$$

subject to

- Taxable income

$$y_t(a, \eta, j, l) = \begin{cases} w_t \epsilon_j \eta l - 0.5 \tau_t^{ss} \min\{w_t \epsilon_j \eta l, \overline{y}_t\} + r_t(a + Tr_t) & j < j_R \\ SS_t + r_t(a + Tr_t) & j \geq j_R. \end{cases}$$

- The budget constraint

$$(1+\tau_t^c)c = \begin{cases} -0.5 \tau_t^{ss} \min\{w_t \epsilon_j \eta l, \overline{y}_t\} + y_t(a, \eta, j, l) & \\ +a + Tr_t - T_t(y_t(a, \eta, j, l)) + (1 + g)(b' - a') - \xi(m) b & j < j_R \\ y_t(a, \eta, j, l) + a + Tr_t - T_t(y_t(a, \eta, j, l)) + (1 + g)(b' - a') - \xi(m) b & j \geq j_R. \end{cases}$$

- Non-negative assets, $a' \geq 0$.
- Bounds on arrears such that $(1+g)b' - (1-m)\nu b \in [0, T_t(y_t(a,\eta,j,l)) - T_t(0)]$.
- Terminal condition: $V_t(a,b,\eta,J+1,m) = \phi(b)$ for all states.

2. Prices. Competitive firm's implies $r_t = \alpha((Z_t N_t)/K_t)^{1-\alpha} - \delta$ and $w_t = (1-\alpha)Z_t(K_t/(Z_t N_t))^{\alpha}$.

3. Social security balanced budget:

$$SS_t \int \Phi_t(da \times db \times d\eta \times \{j_R, ..., J\} \times dm) = \tau_t^{ss} \int \min\{w_t \epsilon_j \eta l, \overline{y}_t\} \Phi_t(da \times db \times d\eta \times \{1, ..., j_R - 1\} \times dm).$$

4. Government balanced budget: For $s \in \mathcal{S}$

$$G_t = \int \left(T_t(y_t(a,\eta,j,l)) - (1+g)b_t'(a,b,\eta,j,m) + (1-m)\nu b \right) \Phi_t(ds) + \tau_t^c \int c_t(a,b,\eta,j,m)\Phi_t(ds) + (1+\pi^G) \sum_{m=0,1} p^A(m) B_t(m),$$

where aggregate arrears evolve as

$$B_{t+1}(m) = \frac{1}{1+n} \int \psi_j b_t'(a,b,\eta,j,m) \Phi_t(da \times db \times d\eta \times dj \times m), \;\; m \in \{0,1\}.$$

5. Transfers. For $s \in \mathcal{S}$,

$$Tr_{t+1} \int \Phi_{t+1}(ds) = \frac{1}{1+n} \int (1-\psi_j) a_t'(s) \Phi_t(ds).$$

6. Market clearing. Capital, labour and final goods markets clear in the sense that $K_t = \int a\Phi_t(ds)$, $N_t = \int \epsilon_j \eta l \Phi_t(ds)$, and $\int c_t(s)\Phi_t(ds) + (\pi^A - \pi^G)\sum_{m=0,1} p^A(m) B_t(m) + (1+g)(1+n)K_{t+1} + G_t = K_t^{\alpha}(Z_t N_t)^{1-\alpha} + (1-\delta)K_t$, where $s \in \mathcal{S}$.

7. Distribution. The distribution measure evolves as

$$\Phi_{t+1} = H_t(\Phi_t),$$

where, for an element in the Borel algebra $(\mathcal{A}\times\mathcal{B}\times\mathcal{E}\times\mathcal{J}\times\mathcal{M})$, the transition function mapping H is defined as follows:

(a) For $1 \notin \mathcal{J}$,

$$\begin{aligned}&\Phi_{t+1}(\mathcal{A}\times\mathcal{B}\times\mathcal{E}\times\mathcal{J}\times\mathcal{M})\\ &=\frac{1}{1+n}\int Q_t(a,b,\eta,i,j,m;\mathcal{A}\times\mathcal{B}\times\mathcal{E}\times\mathcal{J}\times\mathcal{M})\psi_j\Phi_t(da\times db\times d\eta\times dj\times dm),\end{aligned}$$

where the probability transition Q_t is

$$Q_t(a,b,\eta,j,m;\mathcal{A}\times\mathcal{B}\times\mathcal{E}\times\mathcal{J}\times\mathcal{M}) = \Gamma(\eta,\eta')p^A(m')$$

if $j+1 \in \mathcal{J}$, $a'_t(s) \in \mathcal{A}$, $b'_t(s) \in \mathcal{B}$, and $\eta' \in \mathcal{E}$ and $m' \in \mathcal{M}$. Otherwise, it is 0.

(b) For $\mathcal{J} = \{1\}$,

$$\Phi_{t+1}(\mathcal{A}\times\mathcal{B}\times\mathcal{E}\times\mathcal{J}\times\mathcal{M}) = \frac{1}{1+n}\times\begin{cases}1 & 0\in\mathcal{A}, 0\in\mathcal{B}, 0\in\mathcal{M}, \overline{\eta}\in\mathcal{E}\\ 0 & \text{otherwise,}\end{cases}$$

where we define the mean endowment $\overline{\eta} = \sum \eta\Pi(\eta)$, with Π its invariant distribution.

3.5 Stationary equilibrium

The exogenous policy variables are τ_t^c, τ_t^{ss}, G_t, the tax parameters κ_0 and κ_1. We will assume constant policy parameters. The endogenous policies are SS_t, Tr_t, and κ_2 and we will be focusing on situations where these endogenous variables are constant. This stationary equilibrium can be characterised by simply dropping all the time indexes t in the definition above. Outcomes will be described based on the resulting stationary distribution Φ over individual states.

4 Qualitative analysis with two periods

This section considers a simple version of the model in order to illustrate basic mechanisms and motivate further the quantitative analysis to follow. It shows that the contingent nature of tax arrears renders the portfolio choice well defined, and brings a clear association of evasion with income and savings, with an important role of liquidity constraints. Some of the observable implications that are the subject of this paper, however, are not obvious even in this simple setting. It justifies the quantitative analysis in the following sections.

The model is in partial equilibrium with two periods, and deterministic income. In the second period the optimal decision is full evasion since there is no future when an audit can happen. Consider the portfolio decision in the first period. To simplify, suppose current arrears are zero and so $b = 0$. Using some obvious short-hand notation, the problem can be written as

$$\max_{a',b'} \left\{ u(y(a) + b' - a') + \beta[(1 - p^A)u(y(a')) + p^A u(y(a') - (1 + \pi^A)b')] \right\},$$

with income consisting of an endowment e plus the interest on bonds, $y(a) = e + ra$, the constraint that arrears cannot exceed tax liabilities $b' \in [\underline{b}, \overline{b}] \equiv [0, T(y(a))]$, and the borrowing constraint $a' \geq 0$. If the agent is not liquidity constrained and the choice on assets a' is interior, the corresponding optimality condition is

$$u_c(y(a) + b' - a') = \beta(1 + r)[(1 - p^A)u_c(y(a')) + p^A u_c(y(a') - (1 + \pi^A)b')].$$

On the other hand the choice of b' satisfies

$$u_c(y(a) + b' - a') - \beta(1 + \pi^A)p^A u_c(y(a') - (1 + \pi^A)b') \begin{cases} < 0 & b' = \underline{b} \\ = 0 & b' \in (\underline{b}, \overline{b}) \\ > 0 & b' = \overline{b}. \end{cases}$$

Combining with the optimality condition for a', it can be rewritten as

$$(1 + r)(1 - p^A)u_c(y(a')) - (\pi^A - r)p^A u_c(y(a') - (1 + \pi^A)b') \begin{cases} < 0 & b' = \underline{b} \\ = 0 & b' \in (\underline{b}, \overline{b}) \\ > 0 & b' = \overline{b} \end{cases}$$

Clearly, if $(1 - p^A)(1 + r) < (\pi^A - r)p^A$, then there is no evasion $b' = 0$. In the contrary case, which is the more realistic situation where in expected terms the return on bonds dominates the return on arrears, evasion will be positive $b' > 0$. (Graphically, as functions of b', $(1+r)(1-p^A)u_c(y(a'))$ is constant, and $(\pi^A - r)p^A u_c(y(a') - (1 + \pi^A)b')$ is increasing, with the same u_c at $b' = 0$.)

If $b' > 0$ but does not hit the upper bound $T(y(a))$, the solution is interior with

$$\frac{u_c(y(a'))}{u_c(y(a') - (1+\pi^A)b')} = \frac{(\pi^A - r)p^A}{(1+r)(1-p^A)}.$$

This shows that higher a' implies a higher b', regardless of the current a. The idea is that of consumption smoothing across audit states: when a' goes up, higher b' is required to prevent a disproportionate increase of consumption in the low-consumption audit state.

Substituting into the optimality condition for a' yields

$$u_c(y(a) + b' - a') = \beta(1+r)\left[(1-p^A) + p^A \frac{(1+r)(1-p^A)}{(\pi^A - r)p^A}\right] u_c(y(a')).$$

For given a, this defines a mapping from a' to b'. Graphically, it has a positive slope, reflecting intertemporal smoothing. Although it shifts downwards with a (i.e., lower evasion the richer the agent), by our previous result, it is still the case that if higher a leads to higher a' then a higher b' will also follow. Therefore, when the agent is not liquidity constrained, evasion in the first period, if interior, is monotonically increasing and smooth as a function of a, at least as long as a' is so. The income endowment e would have the same type of effect on b' for exactly the same reasons.

Turning now to the case when the agent is liquidity constrained, $a' = 0$, the choice of evasion b' follows solely its own optimality condition. In this case, the sign of the relation between income $y(a)$ and evasion is negative, and it may well be that a poor enough agent becomes evasion-constrained in the sense that arrears are chosen at the upper bound $b' = T(y(a))$.

In sum, the sign of the relationship of the level of evasion with wealth and income will depend on the specific financial position of the household, and one would therefore need a framework with rich heterogeneity of household types. Furthermore, even in this simple case and with non-constrained households, the response of the rate of compliance, measured as the proportion of taxes evaded over taxes due, is not easy to establish, as it may depend of the fine details of the tax code and the specific shape of the individual decision rules over assets and evasion. For example, in the case of log utility and flat tax rate, the evasion rate is constant to changes in income and wealth. In order to address the observed data about variation in tax compliance rates a more general model is clearly needed. The assumption of deterministic income, while may be a reasonable approximation for retired households, may also be a limitation when studying data coming mainly from households of working age. It is thus that we turn now to the analysis of the full quantitative model from section 3.

5 Computation

The household's decision problem is solved iterating proceeding backwards from the last lifetime period. We approach the solution via direct maximisation of the household's objective.[19] Since this is a portfolio choice problem with two endogenous states, a and b, the specification of the grids and interpolation method will be important for accuracy and computational costs.

Policy functions should be smooth when outside of the kinks.[20] If the numerical approximation leads to some artificial kink in the policy rule for assets a', then the policy rule for evasion b' will be calculated with low accuracy. This is the case when using linear splines. So in order to preserve monotonicity in the solution for tax arrears b' we need to make a' a smooth function with continuous derivatives. The strategy will be to first, in an inner loop, interpolate the continuation values function in a' to find the maximizer of the objective for every given given b', and then perform the maximization over b' in the outer loop.

Regarding interpolation, predictably, linear and polynomial schemes do not provide enough smoothness to a'. A cubic spline works well, as long as the value function does not display sharp changes as cubic splines may not preserve concavity nor monotonicity. Cubic interpolation throughout does not work when utility can drop very fast, as when there is a sharp discontinuity. The reason for discontinuities of this sort is the existence of occasionally binding constraints. The occasionally binding constraints are of three types: $a > 0$, $a < a_{\max}$; $b' \in [\nu b, \nu b + T(a)]$ or $b' \in [0, T(a)]$; $c \geq 0$. One way of introducing these constraints is by imposing a sharp penalisation for non feasible options. The approach to deal with the resulting discontinuities here preserves the convenience of smooth interpolation over a' when away from the discontinuity states, and uses another scheme, say linear interpolation, when the constraints bind. Cubic splines appear to dominate over Schumaker's shape-preserving splines. Cubic splines are similarly used in the outer loop over the arrears choice b'.

Further details on the computation are in section B in the Appendix.

[19] An interesting alternative in this context will be to adapt the endogenous grid method. See, e.g., Hintermaier and Koeniger (2010).

[20] To see this, take the example of section 4. From the first order condition in an interior solution, intertemporal smoothing implies $u_c = \beta(+r)[(1-p^A)u'_{c_{NA}} + p^A u'_{c_A}]$, and cross-state smoothing $(1+r-\nu)u'_{c_{NA}} = (\pi^A - r)u'_{c_A}$, where $c = y(a') - a' + b' - T(a)$, $c'_A = y(a') - (1+\pi^A)b'$, and $c'_{NA} = y(a') - \nu b'$. The two sides of the cross-state smoothing condition, can be represented graphically as increasing functions of b', with the NA curve flatter than and starting above the A curve, which requires $(1+r-\nu)(1-p^A) > \pi^A - r$. It is easy to verify that as a' increases smoothly so does b'.

6 Calibration

A model's period corresponds to five years. We thus set lifetimes of length $J = 12$ and retirement age $j_R = 10$, implying that households start with 20 years of age, and retire at age 66, after a working life of 45 years.[21]

Several parameters, including some related to tax auditing and enforcement, are pinned down from direct observations. The remaining parameters, including unobservable ones related to the cost of evasion for the household, will be chosen so the model matches a number of empirical targets. Many of the choices made here are standard and follow, for example, Conesa, Kitao, and Krueger (2009) or Conesa and Krueger (2006). Other choices are less standard as they belong to the measurement and calibration of evasion and tax auditing and enforcement.

6.1 Parameters set directly

We begin with the parameters that are set directly and summarised in Table 1. The Markov chain for the stochastic productivity component η is chosen to approximate a standard AR(1) of the form $\log \eta' = \rho_\eta \log \eta + u'$, with $u' \sim N(0, \sigma_u^2)$. Estimates in the literature from PSID earnings data typically include also a transitory component, for example Storesletten, Telmer, and Yaron (2004). Since our model has only the persistent component of productivity, we will alternatively follow the approximation of Hintermaier and Koeniger (2011) on income data from various waves of the SCF, and aim at annual persistence 0.95, and variance of the innovation 0.048, implying an annual standard deviation $\log \eta$ of $\sqrt{\sigma_u^2/(1-\rho^2)} = 0.702$. For our purpose, these moments at annual frequency have to be converted to 5-year frequency moments so $\rho_\eta = 0.95^5$, and $\sigma_u^2 = 0.048 \times \sum_{j=0}^{2\times(5-1)} 0.95^j$.[22] We then approximate this process with a Markov chain with number of states $N_\eta = 7$ states using Rouwenhorst (1995) method (see Kopecky and Suen (2010) more recently). This results in values for the states, transition probabilities $\Gamma_{\eta,\eta'}$ and stationary distribution for η shown in section C in the Appendix.

The deterministic age-dependent component of productivity $\{\epsilon_j\}$ is calculated, in a way similar to Mateos-Planas (2010), as the 5-year means of the yearly profile in Conesa, Kitao, and Krueger (2009) based on Hansen (1993). This is displayed in section C in the Appendix.

[21] We have also considered versions where one period corresponds to one year and lifetimes are of length $J = 81$. Our present choice keeps computation times manageable even on a single processor.

[22] See, for example, Jorda and Marcellino (2004).

The aggregate parameters n, g are set to match U.S. long run 0.011 yearly population growth (0.056 5-yearly) and 0.0175 yearly output growth (0.0906 5-yearly), respectively. For the present purpose, we choose 100 per cent survival probabilities at no loss.

As discussed earlier in section 3, in this version the individual labour supply l is exogenous and constant, and we set it to a standard value of 1/3 of non-sleep time. The utility parameter γ is then just a normalization, but we choose a value that is consistent with the target labour supply when it is endogenous in Conesa, Kitao, and Krueger (2009). The chosen σ then implies a constant relative risk aversion of 2.

The payroll tax rate τ^{ss} is set to 12.4%.[23] The consumption tax τ^c is 5% following Mendoza, Razin, and Tesar (1994). The limit of income taxable by social security $\overline{y}$ is set to a value that is 2.305 times average income. This is the limit of earnings subject to payroll tax as a proportion of average income $87,000/37,748$ for the U.S. in 2003.

For the tax function, we adopt the specification given earlier in equation (2) whose parameters κ_0 and κ_1 have been estimated to fit the U.S. tax code. We use values that match the estimated average tax rates and progressivity of the US system in Gouveia and Strauss (1994). The remaining element of the tax function κ_2 is an equilibrium variable, not a parameter to pin down.

We now turn to directly determined parameters related to tax auditing and enforcement. The persistence or duration of arrears is chosen to match an average prescription period of 5 years, which means a rate 0.80 on an annual basis and, on five-year basis, $\nu = 0.80^5 = 0.32$.[24] The size of statutory fine for evasion as a proportion of outstanding tax arrears in on a range between 50% and 75%, and we pick the upper bound for π^G. The probability of audit p^A is harder to assess. We take it to be an annual 4% based on IRS, implying a probability of 20% on a 5-year basis. These three choices are based on the evidence from the IRS for the US discussed earlier in section 2.4.

Finally, the curvature of the terminal arrears penalty $\phi_1 = 1.5$ cannot be identified given the data. We proceed by choosing a value 1.5 that ensures convexity. Results of this paper will be robust to variation in this parameter.

[23] Incidentally, this will lead to a replacement rate close to 50% under equilibrium benefit payouts that balance the Social Security budget.

[24] For undeclared (and unaudited) tax liabilities there is a statue of limitation as tax authorities are limited in the tax years they can audit. For the US, the IRS can include returns filed within the last three years in an audit. The number of years may increase if substantial errors in tax returns are identified. A prescription period of five years should be typical.

Table 1: **Direct Parameters**

parameter	value	observation
lifetime length	$J = 12$	5-year periods
retirement age	$j_R = 10$	65 years
stochastic productivity persistence	$\rho_\eta - = 0.07740$	annual persistence 0.95
stochastic productivity variance	$\sigma_u^2 = 0.2433$	SCF annual var 0.048
life cycle component	$\{\epsilon_j\}$	Hansen (1993); See sec C
output growth	$g = 0.0906$	annual 1.75%
population growth	$n = 0.056$	annual 1.1%
labour supply	$l = 1/3$	1/3 working time
labour utility share	$\gamma = 0.377$	normalisation
utility curvature	$\sigma = 4$	2 CRRA
payroll tax	$\tau^{ss} = 0.124$	US 50% replacement
max payroll income	$r^{ss} = 2.305$	87,000/33,748 in 2003
consumption tax	$\tau^c = 0.05$	US 5% rate
income tax level	$\kappa_0 = 0.258$	US estimates
income tax progressivity	$\kappa_1 = 0.768$	US estimates
arrears persistence	$\nu = 0.32$	5-year prescription IRS
audit probability	$p^A = 0.20$	annual 4% IRS
statutory fine	$\pi^G = 0.75$	75% IRS
curvature terminal cost	$\phi_1 = 1.5$	choice

6.2 Parameters calibrated within the model

The five remaining parameters are the discount factor β, depreciation rate δ, government spending G, the audit penalty over arrears π^A, and the scale of the non-pecuniary terminal penalty ϕ_0. Their calibration is summarised in Table 2. These parameters are chosen so that in equilibrium the model delivers targeted values for five variables. Three of these variables are standard US macroeconomic aggregates which we take from Conesa, Kitao, and Krueger (2009). The corresponding target values are a capital to annual GDP ratio of 2.70, an investment to GDP ratio of 0.25, and a ratio of government spending to GDP of 0.17. They will mainly help identify β, δ and G.

The other two variables are measures of tax evasion. The first measure is the ratio of total tax evasion to total tax due which stands at 20% as discussed in section 2.1 and will help identify the pecuniary cost parameter π^A. The second measure is the mean ratio of evasion to tax due across households aged 65 and above. Based on Advani, Elming, and Shaw (2017) for the UK, we noted in section 2.2 that the figure is 24%. Since this variable across the overall UK population of 0.33 is very close to that implied by our model calibrated to the US (see below in Table 3), we infer that the figure of 24% for the older cohorts should be valid for the US too. This target will primarily serve to pin down the terminal cost parameter ϕ_0.

Table 2: **Calibrated Parameters**

parameter	value	targets
discount factor	$\beta = 0.9794$	2.7 annual K/GDP
depreciation rate	$\delta = 0.3113$	0.25 I/GDP
government spending	$G = 0.0680$	0.17 G/GDP
total audit penalty	$\pi^A = 3.449$	0.20 evasion/tax due US
level terminal cost	$\phi_0 = 57.53$	0.24 mean retired evasion/tax due (UK)

6.3 Properties and other moments

Here we offer some remarks that will help understand the determination of evasion in the model. We also discuss the implications of this benchmark model for some moments not targeted in the calibration.

A positive last-period warm-glow cost of tax arrears $\phi(b')$ is not material to the characterisation of evasion or any of the results to follow. Without these costs, however, evasion of the retired ages will be too large, and at 100% in the final lifetime period. One reason is that with zero retirement income volatility, arrears become less of a risk for the household. Given the indicative evidence seen in section 2.2 that older groups seem to default with less intensity, these implications of the model would distort the calculation of the overall means. Our solution here via the cost ϕ is an expedient one given the lack of a sound model of evasion behaviour in older ages. As said, this fix is nonetheless largely inconsequential.[25]

We now turn to quantitative implications of this benchmark for non-targeted observable aggregate variables.

The total cost of being audited π^A includes the direct transfer fine π^G of three quarters of the tax evaded. The remaining component is not a transfer to the government. In order to match the 20% rate of evasion, the calibrated total audit penalty rate shown in Table 2 is well above the statutory fine component π^G. The deadweight loss rate is $\pi^A - \pi^G = 2.7$, or 1.5 times the payment to the tax administration.[26] This excess penalty is an efficiency loss for the economy. Given the outcomes, the size of this inefficiency is 1.83% of GDP,

[25] An alternative would be to simply calculate statistics from the population of working age only.

[26] We think of these costs as representing legal and court expenses and other disruptions experienced by the household in the course of enforcement proceedings, or inefficiency losses. Balafoutas, Beck, Kerschbamer, and Sutter (2015) show in experimental work that in transactions in which at least one partner was tax-dishonest, efficiency was up to 50% lower. Precise estimates for these costs are difficult to come by. In this paper we measure them via calibration.

Table 3: **Non targeted variables**

variable	model
Gini of wealth	0.65
sd log C	0.68
Aver tax rate	0.186
mean evas to tax due	0.27
evasion/GDP	0.0321
arrears/GDP	0.1696
recovered/GDP	0.0119
(evasion-recov)/GDP	0.02
deadweight loss/GDP	0.0183
return arrears vs savings	0.7473

roughly 2.70 deadweight rate times arrears over GDP 0.034 times the 0.20 probability of audit. This is one potential gain from fighting evasion.[27]

It is to be noted that in spite of the large detection penalty, the expected interest cost of arrears is still considerably smaller than the return on savings. Specifically, the effective ratio of rates of return of tax debt over bonds is $p_A(\pi^A - r)/((1-p_A)(1+r-\nu)) \approx 0.747$. Based on expected returns alone, everyone would be fully evading to fund savings. That evasion is nonetheless far less than 100% indicates that risk plays a central role in the portfolio choice. Note that this theory of partial evasion therefore does not rely on extraneous costs typical of some existing literature.[28]

The distributional and risk sharing implications of this benchmark look reasonable attending to the U.S. wealth Gini coefficient and volatility of consumption. The Gini coefficient of 0.65 is lower than the measure based on the SCF 2007 of 0.78 to 0.82 (see Kuhn and Rios-Rull (2016)). This is a known challenge of this simple setting in matching the tails of the distribution (see, for instance, Castaneda, Diaz-Gimenez, and Rios-Rull (2003)), and here we abstain from introducing elements geared to address this. Nonetheless the model does a decent job at approximating the central quantiles. As for consumption volatility, the model implies only a slight overestimation as observed measures range from 0.58 to 0.65 based on CEX 1994 and 2005 (see Storesletten, Telmer, and Yaron (2004) or Heathcote, Perri, and Violante (2010)).

Turning now to taxation and evasion, the average evasion to tax due across individuals of

[27]The size of the deadweight loss would be notably smaller if we were to increase the household's perceived probability of detection in light of the evidence discussed above.

[28]That is, except for the oldest households whose evasion would be 100% absent the terminal cost of arrears given by $\phi(b)$.

27% is larger than the measure of total evasion over total taxes due of 20%. This is an indication that heavy evaders have a relatively small weight in terms of total evasion and have therefore low tax liabilities and hence income. Those who evade a larger proportion of their taxes tend to be poorer households. This is consistent with the evidence in Advani, Elming, and Shaw (2017) of a skewed distribution of evasion in the UK discussed earlier in 2.1.

The average tax rate measures tax due over taxable income, and stands at about 19%. This is comparable with measures of how much the US government should be collecting. Of this target measure, a part equivalent to 3.2 per cent of GDP is revenue lost to evasion in any given period. At the same time, however, the government audit efforts result in recovered funds of about 7% of outstanding arrears per year, an amount equivalent to 1.2 per cent of GDP. These figures are not out of line with the IRS anticipated annual recovery of 11% discussed above in section 2. All in all, there is a net fiscal loss flow to the government associated with evasion in the order of 2% of GDP.

7 Policy functions

In this section we discuss the individual decision rules in the calibrated model primarily to appreciate how different factors affect evasion. This will be useful in interpreting equilibrium implications to be discussed later on. Evasion is part of a portfolio decision joint with savings. As we discussed in the context of the analytical model in section 4, higher savings requires a rise in arrears to keep the balance between marginal utility in the audit (i.e., high-marginal-utility) and no-audit states. In this way, one can think of evasion as a way a funding savings. It is interesting that a choice of higher evasion is associated with an optimal higher savings. More evasion requires a larger buffer to hedge the risk of audit in the future. Besides this risk considerations, agents near the borrowing limit may find it optimal to evade more heavily to prop up consumption.

We now turn to examine the resulting decisions in the calibrated model. As a baseline case, consider a prime-age worker, with about mean level of tax arrears and experiencing no audit. We represent the policy functions as functions of current assets in Figure 1.[29] Next-period arrears is increasing in current assets. In effect, for these average characteristics, evasion can be regarded as part of a joint portfolio choice which fits the intuition discussed earlier in section 4. More assets today implies higher assets tomorrow a'. Higher tax arrears next

[29]Assets and arrears in this and other figures are measured relative to mean annual income.

period b' optimally accommodate this increase in assets in order to strike a balance across the audit and non-audit states. Or, in a slightly different interpretation, wealthier agents can afford better the risk of evading taxes in terms of future penalties. Figure 1 also depicts the bounds for arrears in the next period, indicating that the choice of arrears is away from the limits. Correspondingly, the evasion rate as a proportion of taxes due is also interior, and increasing in wealth in the present case, although becoming flatter with the level of wealth. Evasion is positive even at very low levels of wealth, an indication that, at this income state and level of arrears, the fear factor of getting caught by the tax authorities is not very severe.

In contrast, consider now the behaviour of a household who may be liquidity constrained. Specifically, consider a low income realisation, in the audit event for a retired household. Figure 2 displays the corresponding policy functions alongside the ones seen earlier for the baseline state. At low levels of wealth, the household is liquidity constrained. In that type of situation, predictably, evasion is large and declining in wealth.

Figure 3 displays the consequences of changes in the stochastic productivity. Evasion generally rises when income declines. At low income, the profile of the default rate may become non-monotonic in wealth, being increasing at low values of debt and declining afterwards. So in principle the aggregate association between wealth and the evasion rate may be ambiguous. Note however, that total evasion and tomorrow'a arrears are always increasing.

Age also has an effect on the policy functions. Figure 4 displays the general fall in evasion with age. The specific cases displayed include working ages only, so the differences must arise from the different time horizon and the age-specific component of productivity, such that older save a lower amount, given current wealth.

As shown in Figure 5, starting from the baseline characteristics, a higher current level of arrears b reduces current evasion in a way that leaves next period arrears bounded from below, and reduces consumption and asset accumulation. The audit event $m = 1$ increases current evasion since the household tries to soften the blow of the material penalties experienced, yet reduces slightly next period arrears as old arrears are cleared, and leads to lower asset accumulation and consumption. See Fig. 6

It is also instructive to look at the different decision variables as joint functions of assets and arrears. Figures 7 and 8 represent the 3-dimensional graphs for the baseline household type and a low productivity shock respectively. This shows that the main associations discussed above hold more generally across the state space. For the baseline productivity in Figure 7, the evasion rate is generally increasing in assets and decreasing in current arrears but is flat

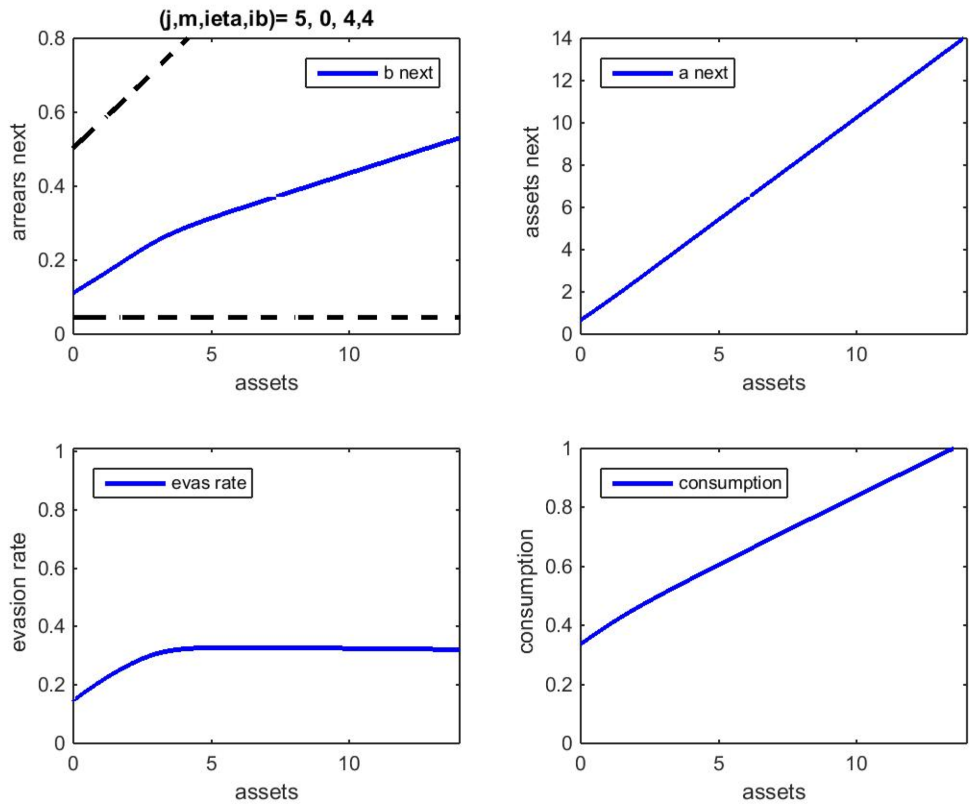

Figure 1: Baseline: age $j = 5$, mean η, mean b, $m = 0$.

at its lower bound in states of high arrears and low assets. For the low productivity states in Figure 8, the evasion rate may hit 100% in states of low current arrears and low wealth and then decreases as wealth increases, in contrast with the uniformly monotonic changes in the evasion rate under the baseline median productivity. This is an indication that in low-productivity states, evasion plays a role as insurance mechanism for current consumption. Therefore the effect of wealth on the individual rate of evasion may be positive or negative depending to an extent on the level of productivity.

The audit state and the obligation to pay backdated debts and the pecuniary cost may have an impact on the household. Figure 9 shows that in the audit state the agent will generally have to evade heavily, a manifestation of the role of evasion as insurance.

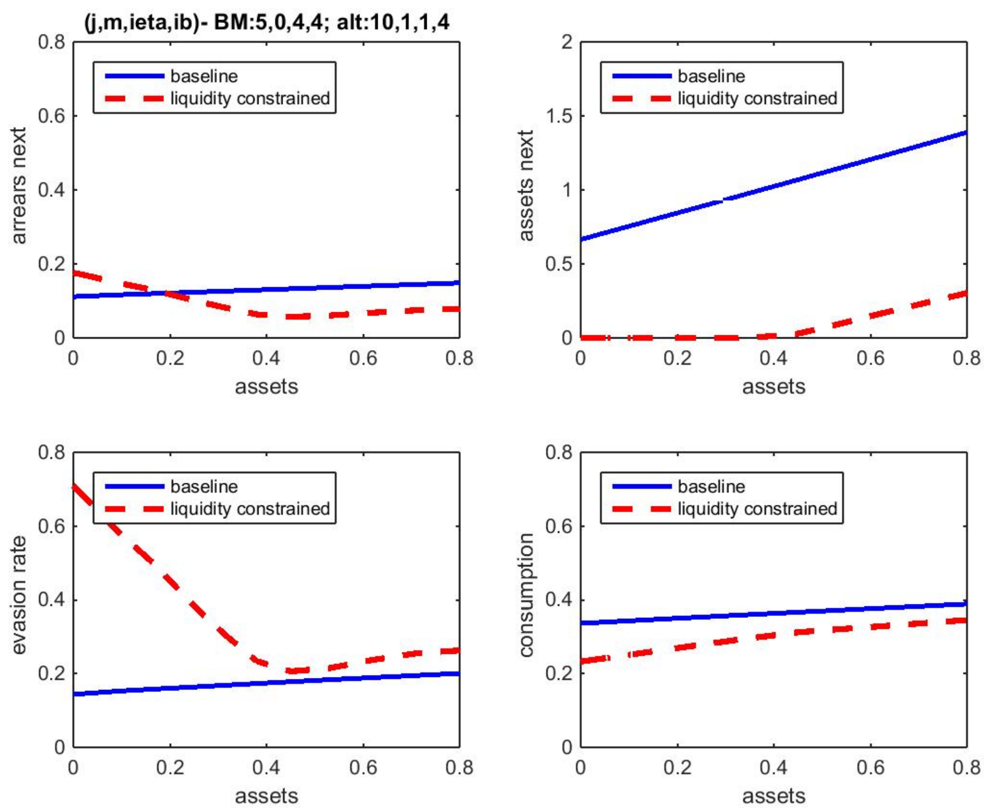

Figure 2: Liquidity constrained vs baseline. Liquidity constrained: age $j = 10$, bottom η, mean b, $m = 1$

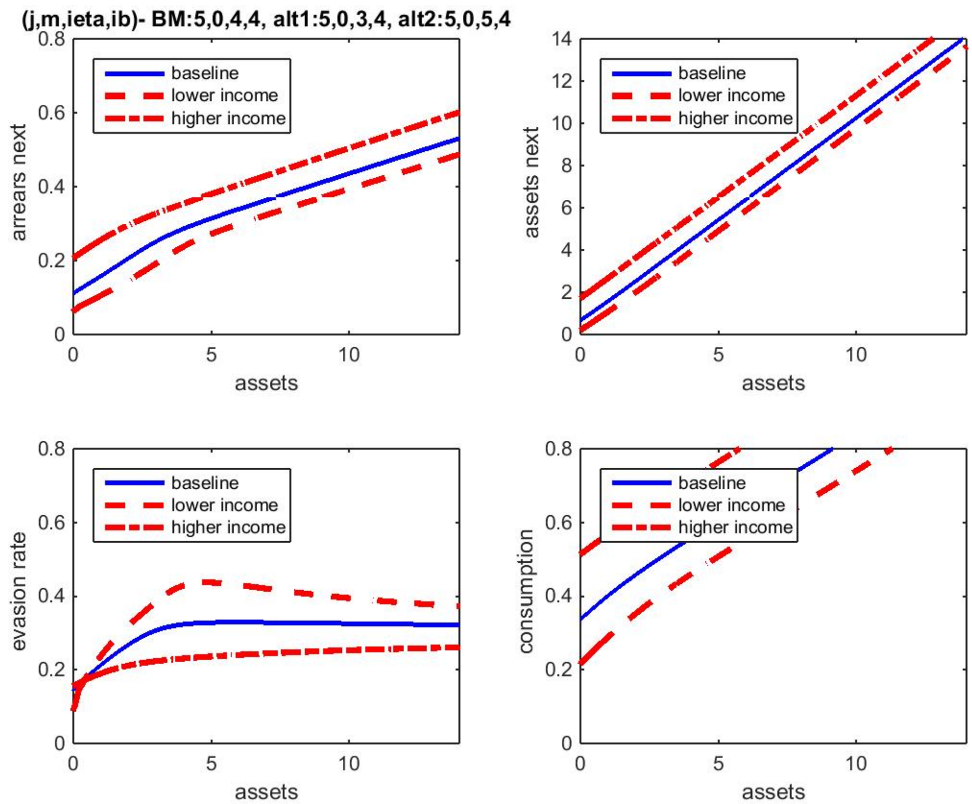

Figure 3: Productivity changes vs baseline. High and low income are productivity η 63% above and below the mean;

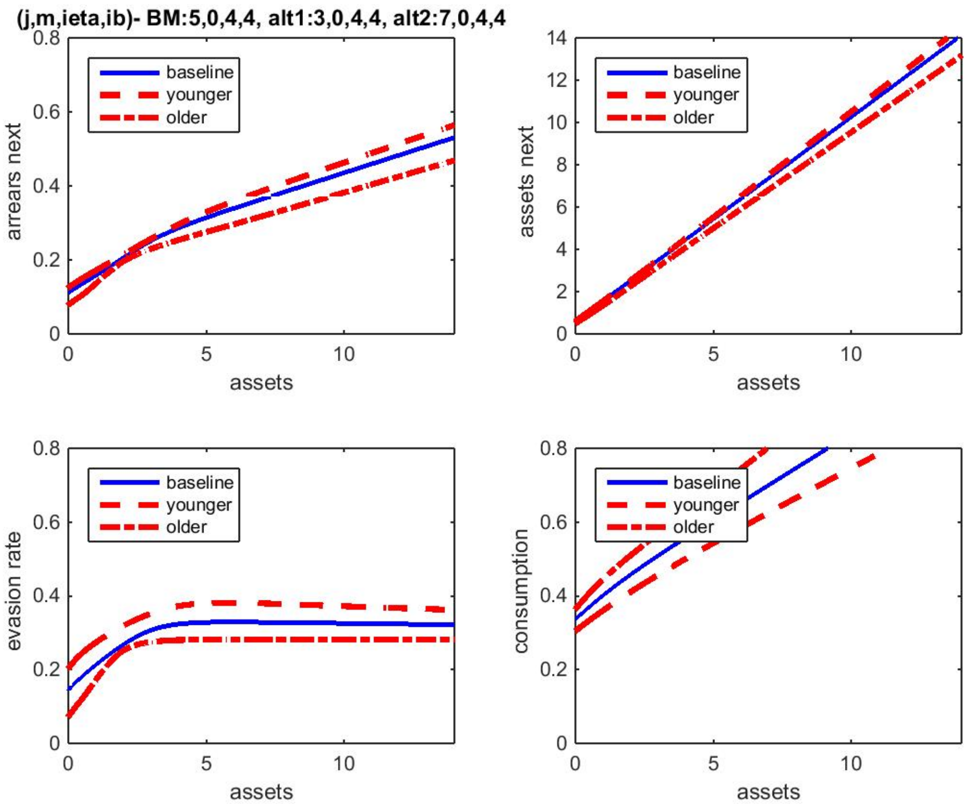

Figure 4: Age changes vs baseline. Younger is age 3; older is age 7

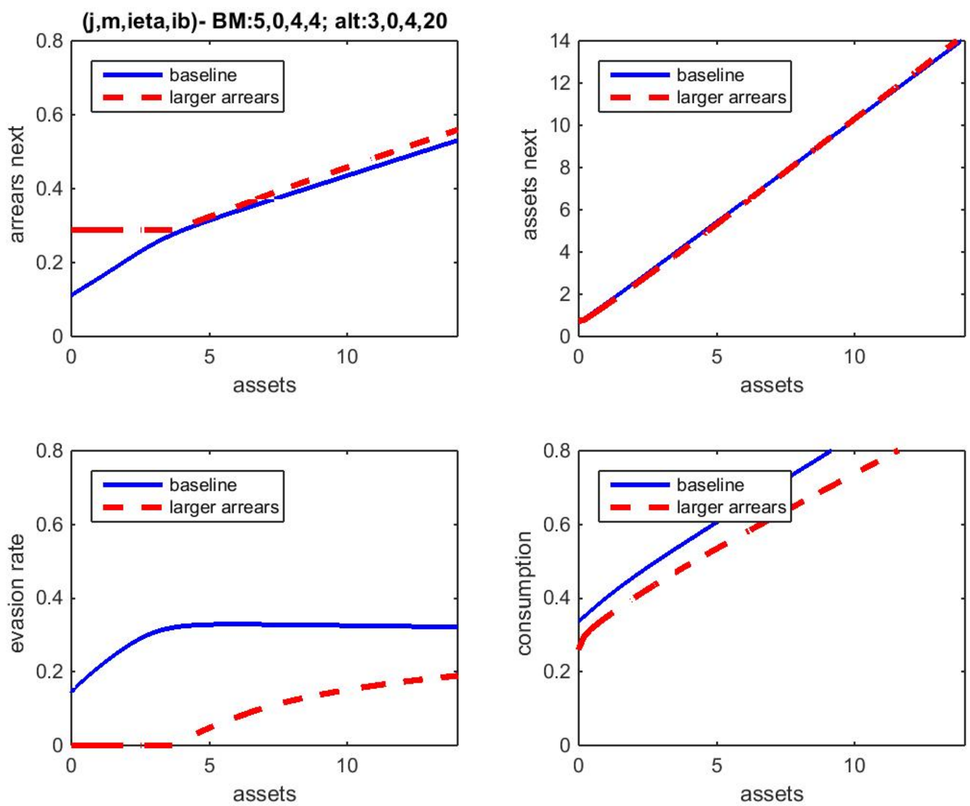

Figure 5: Tax arrears increased vs baseline.

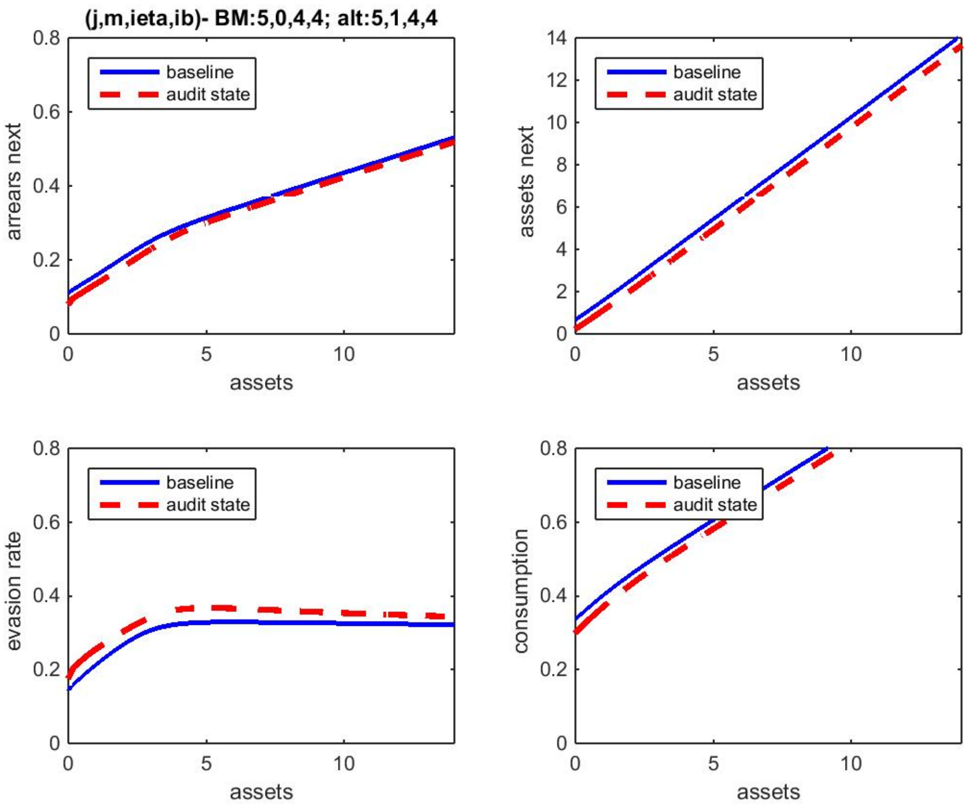

Figure 6: Audit state vs baseline.

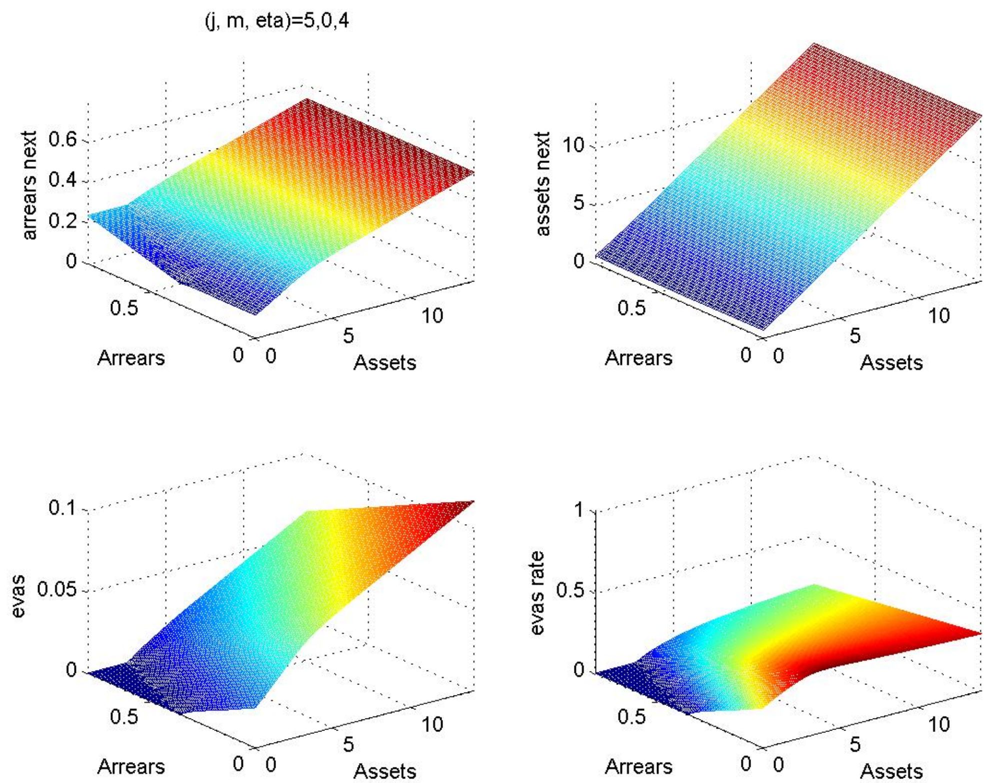

Figure 7: Baseline: age $j = 5$, mean η, $m = 0$.

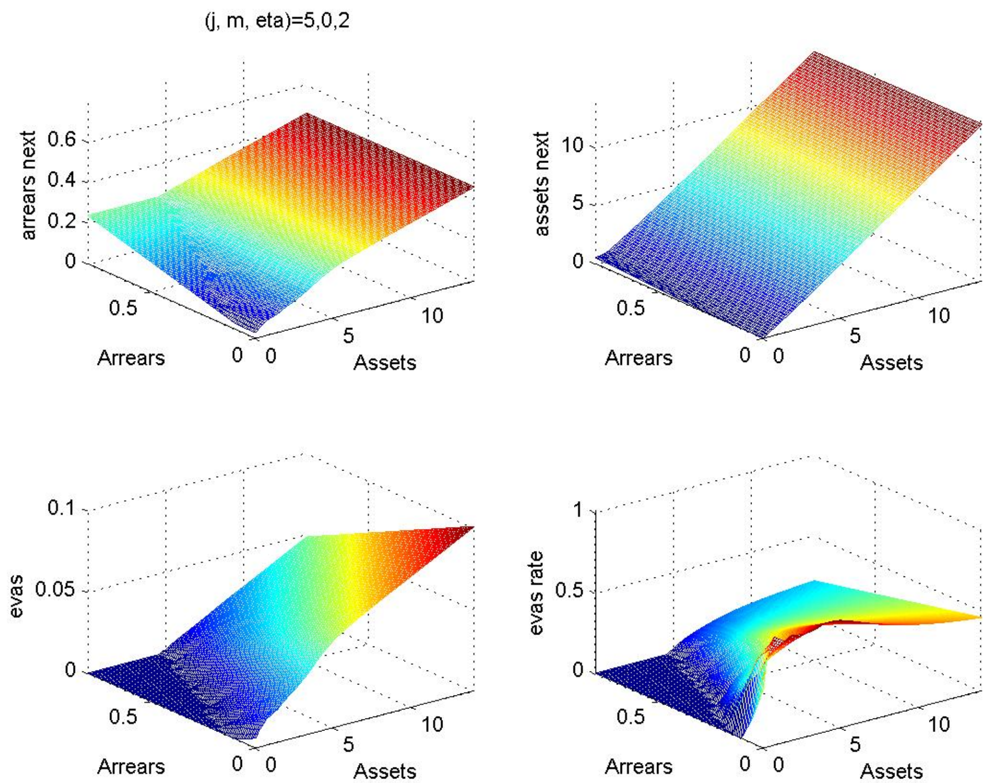

Figure 8: Lower productivity: η below mean by 127%.

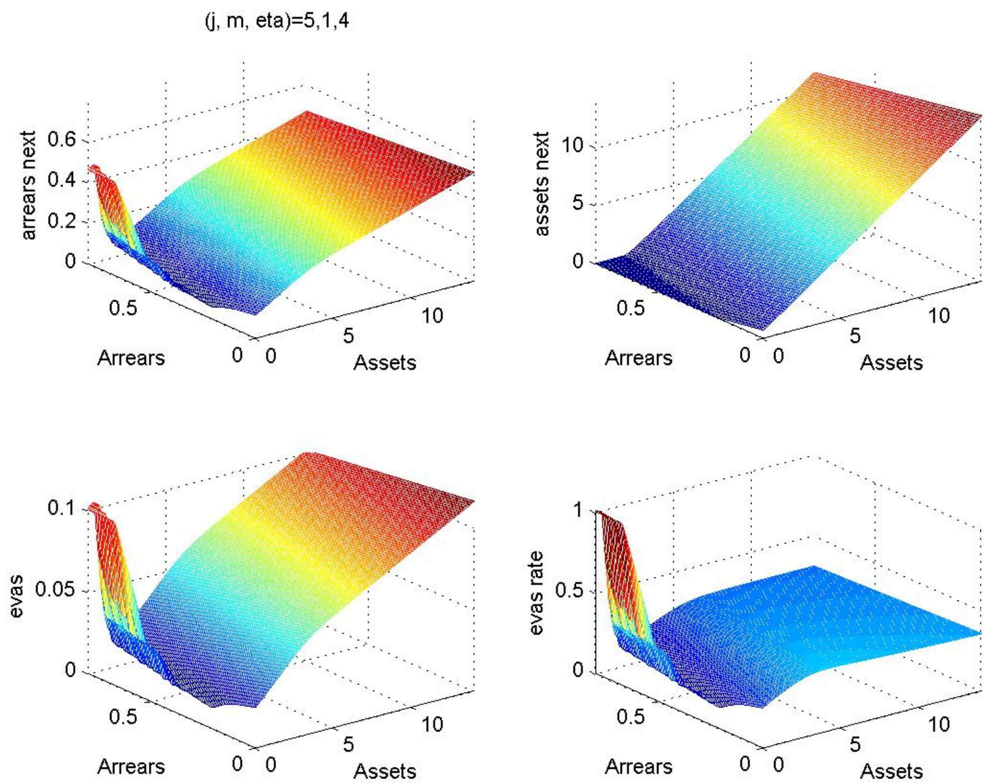

Figure 9: Audit state $m = 1$.

8 The distribution of evasion

We study the implications in the stationary equilibrium resulting from the above for variables over the life cycle and for the cross-sectional distribution of evasion across wealth and income, and discuss their empirical relevance in the light of existing evidence.

8.1 Evasion over the life cycle

The first panel of Figure 10 displays the average levels of assets and tax arrears at each age. Both display a typical hump shape with arrears increasing faster and peaking earlier than assets. The profile of arrears is driven by the shape of the level of evasion by age group, displayed in the next panel.

In terms of the intensity of evasion as a proportion of taxes due, the third panel of Figure 10 shows that the proportion evaded tends to fall over the main age range before retirement. This is in accord with the effects of age studied in Section 7, Figure 4 for instance.

As mentioned in Section 2, there is firm evidence that evasion is decreasing in age, at least for for working age groups, and thus the model does well in this regard. Specifically, the profile of evasion rates from the model, although not directly comparable, aligns well with the quantitative estimates in Cabral, Kotsogiannis, and Myles (2018) on income undereporting by age groups in the UK reported above in Section 2.

It is worth pointing out that while the ratio of means of evaded taxes to due taxes is uniformly declining in the model, Figure 10 also shows that the mean across the individual ratios surges in the period before retirement. This reflects the sharp increase among individuals with low assets or income of that age group. The intuition is that uncertainty about future incomes dissipates before retirement and these particular agents may experience a general improvement of their economic situation after retirement, thus attenuating the riskiness of current evasion.

Figure 10 also reports the typical life-cycle profile for consumption volatility.

Consider now the subset of households who are constrained either because they choose to hold zero wealth $a' = 0$, about 2.5% of the population, or they are evading 100 per cent, about 2% of the total. Table 4 reports the mass of borrowing constrained and evasion constrained households, and how these constrained agents are distributed over the life cycle. Borrowing constrained agents are concentrated at young ages and also at the pre-retirement

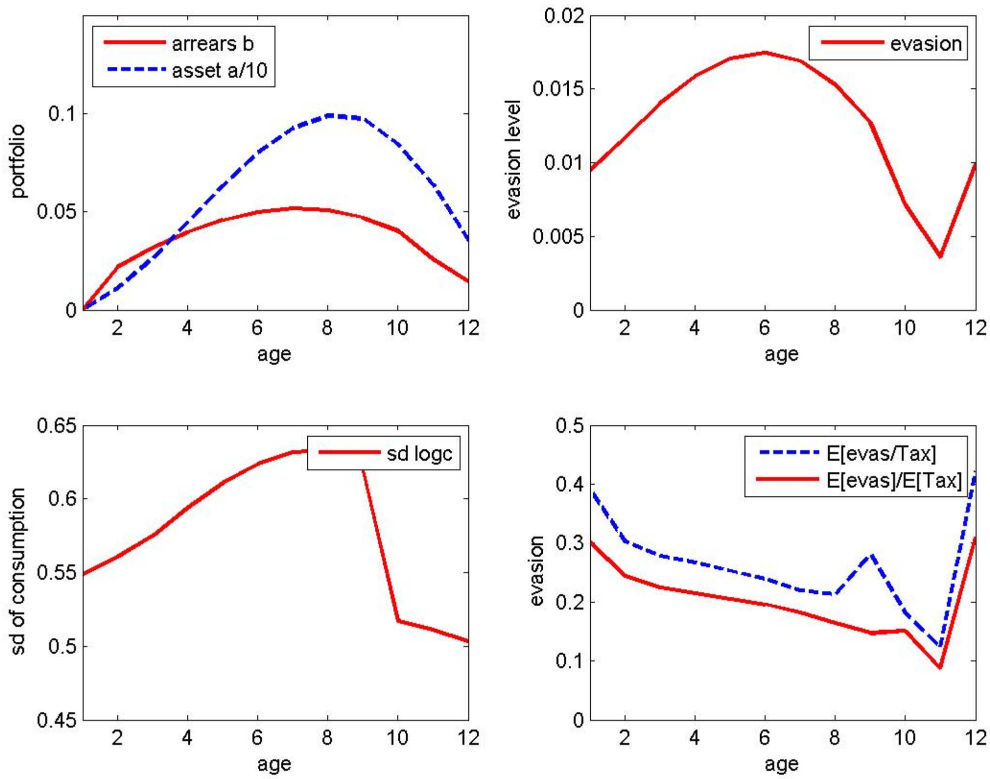

Figure 10: Life cycle.

period (age 9). A similar pattern is seen for the evasion constrained agents. Therefore in the pre-retirement period there is an abnormal presence of the poor households who rely on heavy disaving and/or on full evasion in order to smooth consumption on the anticipation of a relatively comfortable and predictable retirement income. These figures thus support the spike in the mean evasion rate at that age seen in Figure 10 as discussed in the previous paragraph.

Table 4: **Constrained households**

	credit constrained	evasion constrained
Total mass as pc of total	2.53%	2.02%
share of age 1	0.068	0.513
share of age 2	0.212	0.143
share of age 3	0.058	0.020
share of age 4	0.050	0.008
share of age 5	0.035	0.003
share of age 6	0.022	0.003
share of age 7	0.015	0.003
share of age 8	0.011	0.009
share of age 9	0.385	0.298
share of age 10	0.064	0.000
share of age 11	0.079	0.000

8.2 Evasion in the cross section

Table 5 displays arrears and measures of evasion by wealth decile. The level of evasion generally increases with the wealth decile. The only exception is at the lowest decile, an effect driven by the borrowing constrained young and pre-retirement individuals, whose evasion declines sharply when larger wealth relaxes that constraint and thus the desire to evade heavily. The positive association between wealth, savings and evasion seems to dominate otherwise. However, evasion as a proportion of taxes due decreases with wealth. This must be because wealth comes with higher earnings and age; as seen in section 7, the individual evasion rate decision– controlling for other characteristics– is not uniformly increasing nor decreasing with wealth.

Table 5: **Evasion and arrears by wealth deciles. Working age.**

wealth/Y	arrears/Y	evasion/Y	% evasion
0.002	0.007	0.020	0.303
0.104	0.046	0.016	0.294
0.324	0.075	0.020	0.273
0.755	0.122	0.027	0.237
1.220	0.143	0.029	0.226
2.008	0.200	0.038	0.215
3.206	0.253	0.044	0.206
5.064	0.325	0.054	0.185
11.230	0.501	0.077	0.156

Table 6 considers patterns in the distribution of outcomes over the idiosyncratic level of earnings. The evasion level increases with earnings yet the evasion rate decreases, with only a slight rise at the lowest income level.[30] This conforms the direction of the effect of productivity on evasion individual decision studied above in Section 7, Figure 3. The evidence in Johns and Slemrod (2010) discussed in section 2.3 confirms a similar pattern for the US: the ratio of underreported tax to true tax liability declines sharply with true income, yet the amount evaded rises. The approximated no-compliance rates across the bottom thirds of the income distribution 55%, 23% and 15%, compare well with the analogous 36%, 25% and 18% from the model data in Table 6.

Table 6: **Evasion and arrears by earnings. Working ages**

earnings	arrears/Y	% evasion	evasion/Y
0.149	0.039	0.395	0.018
0.281	0.066	0.412	0.046
0.530	0.104	0.309	0.081
1.000	0.161	0.253	0.151
1.888	0.239	0.200	0.255
3.565	0.344	0.155	0.403
6.731	0.497	0.126	0.662

We have also analysed the cross-sectional characteristics of the same households seen in Table 4 who are constrained either because the choose to hold zero wealth or are evading 100 per cent. The borrowing constrained are concentrated at low levels of arrears. This is consistent with precautionary behaviour. The evasion constrained are concentrated at low levels of assets and low levels of current arrears. This is consistent with heavy evaders being poorer.

9 Macroeconomic implications of evasion

In this section we study how the presence of evasion affects fiscal and economic outcomes, including personal savings and capital accumulation, consumption, risk sharing and overall welfare. We will compare the outcomes in the above calibrated economy with the outcomes arising in an economy identical to this benchmark except for the absence of evasion. This

[30]This is driven by younger age groups, up to age 5, whose evasion rate is contained by considerations of risk which are a more important concern as they are more likely to be credit constrained and face an uncertain future.

no-evasion economy obtains by, for instance, setting penalties of evasion that make evading prohibitively costly.

We will measure welfare as lifetime utility, in consumption equivalent units, in the initial lifetime period for a household with median starting productivity.

9.1 Aggregate effects

Table 7 displays the consequences of removing evasion for a number of key aggregate variables, both with and without general equilibrium effects. Consider first the changes in partial equilibrium, that is holding the interest rate r, the tax coefficient κ_2 and social security payments unchanged. There is a reduction in savings and capital of about -3.3%, and a slighter reduction in total level of consumption by about -0.4%. The volatility of the log consumption declines by nearly -3%. Overall utility declines by -0.49% in consumption equivalent units.

Table 7: **Aggregate effect of removing evasion**

	partial equilibrium	gral equilibrium
GDP	-1.19%	-0.42%
capital	-3.31%	-1.17%
consumption	-0.38%	+1.17%
sd log c	-0.026	+0.005
welfare	-0.49%	+2.33%
income tax rate $\times$100	-0.07	-2.16
(net gov rev/Y)$\times$100	+1.51	+0.17

Note the reduction in consumption in spite of the fact that the no-evasion scenario implies an efficiency gain which, as pointed out earlier, amounts to about 2% of GDP. Removing evasion leaves the household with a narrower set of options. Evasion permitted the household to afford higher levels of consumption and savings by avoiding tax payments.[31] Note the welfare losses from removing evasion do not come from worse insurance opportunities. In fact, evasion implied a riskier environment in the form of non-contingent debt so banning evasion reduces consumption volatility. The large decrease in household savings does partly follow from the reduced need of engaging in precautionary savings.

[31]This decline in consumption is not influenced by the reduction in aggregate output since in partial equilibrium prices and taxes remain unchanged for the household.

In partial equilibrium, the elimination of evasion brings a positive fiscal boost resulting in tax revenues rising by 6%, equivalent to an increase of 1.51 percentage points on GDP.[32]

We turn now to the general equilibrium where the interest rate, wage rate, and the level of taxes adjust to the removal of evasion. From Table 7, in general equilibrium there is a reduction in tax levels of 2.6 percentage points over income which directly benefits households, creating incentives for savings and capital accumulation that counter their fall in partial equilibrium. Compared to the response seen in partial equilibrium, there is a much smaller decrease in capital accumulation of -1.17% when removing evasion in general equilibrium, and a significant 1.17% increase, rather than a reduction, in overall consumption. Consumption volatility rises to a small degree after removing evasion in general equilibrium though, thereby overturning the visible reduction seen in partial equilibrium. This must be because the increased consumption pushes low-earning and low-assets individuals closer to the borrowing constraint since the tax windfall is less significant for those. On balance, preventing evasion in general equilibrium does raise welfare by over 2.3% equivalent consumption units, which is in sharp contrast with the negative response seen in partial equilibrium.

To sum up, although eliminating evasion may impact households negatively in a first instance, the ultimate net effects turn out to be positive and large once the fiscal boost is taken into account.

9.2 Effects by household types

To gain more insight into the mechanisms at work, consider the life-cycle pattern of changes following the elimination of evasion. The solid lines in Figure 11 are for the partial equilibrium response. The reduction in savings occurs at all age groups but the proportional size of the reduction clearly declines with age. Consumption also declines at all ages when removing evasion, and more for older agents, except for the very young. Volatility generally declines particularly for younger workers, except for the very young for whom the absence of the option to evade rises volatility substantially, and for poor households near retirement when, as discussed earlier in section 8.1, there is a surge in the recourse to evasion for consumption smoothing. This is again an indication that evasion helps as an insurance mainly for households who are less asset rich. For the remaining cohorts, removing evasion and arrears decreases volatility. The solid line in Figure 12 shows the partial equilibrium change

[32]The slight reduction in the average income tax rate follows from the fall in income and a progressive tax schedule.

in the savings and consumption decision rules as a response of the elimination of evasion for the typical baseline household (but, for comparability, with zero arrears), which illustrates the direction of the aggregate changes just shown.

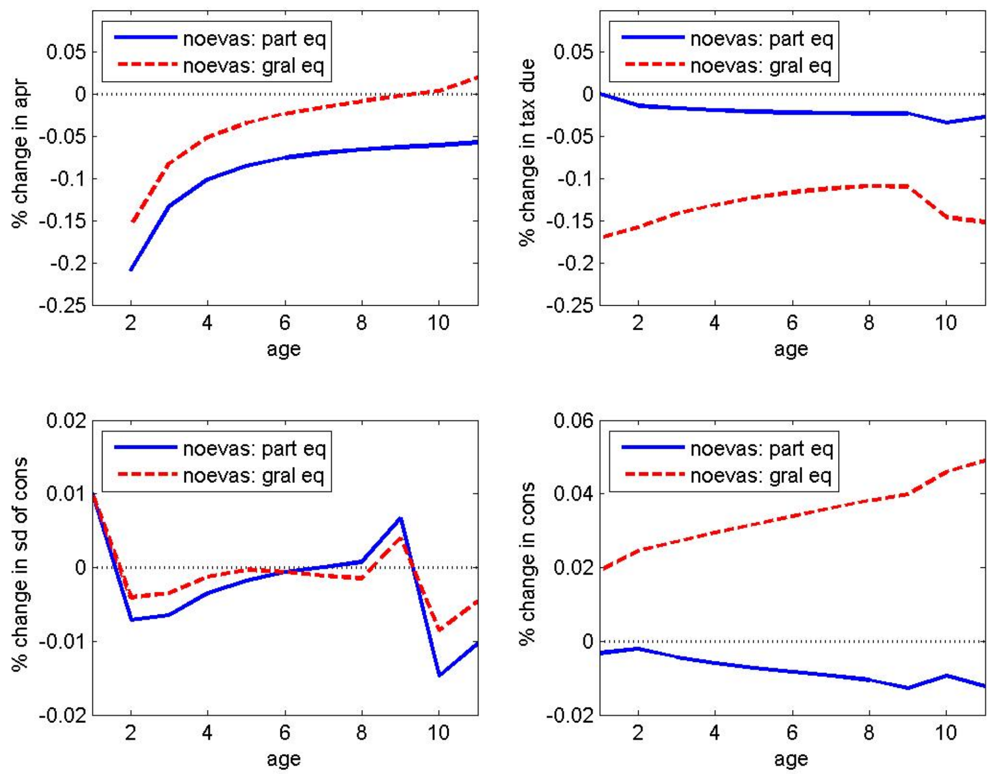

Figure 11: No evasion. Change in life cycle, Partial and general equilibrium.

Consider now the changes in general equilibrium. The dashed lines in Figure 11 show the responses by age group to removing evasion. The level of tax due decreases varies substantially, far more than it does under partial equilibrium. Savings decline for the young households but not nearly as much as in partial equilibrium and even increase for the older retired groups. This is why aggregate capital does not decline as much in general equilibrium as it does in partial equilibrium. Consumption in this case responds positively across all age groups. Volatility rises again for the very young, and shows much less of a reduction for most working-age groups than in partial equilibrium. This fits the interpretation given earlier for the overall rise in volatility in general equilibrium. The dashed lines in Figure 12

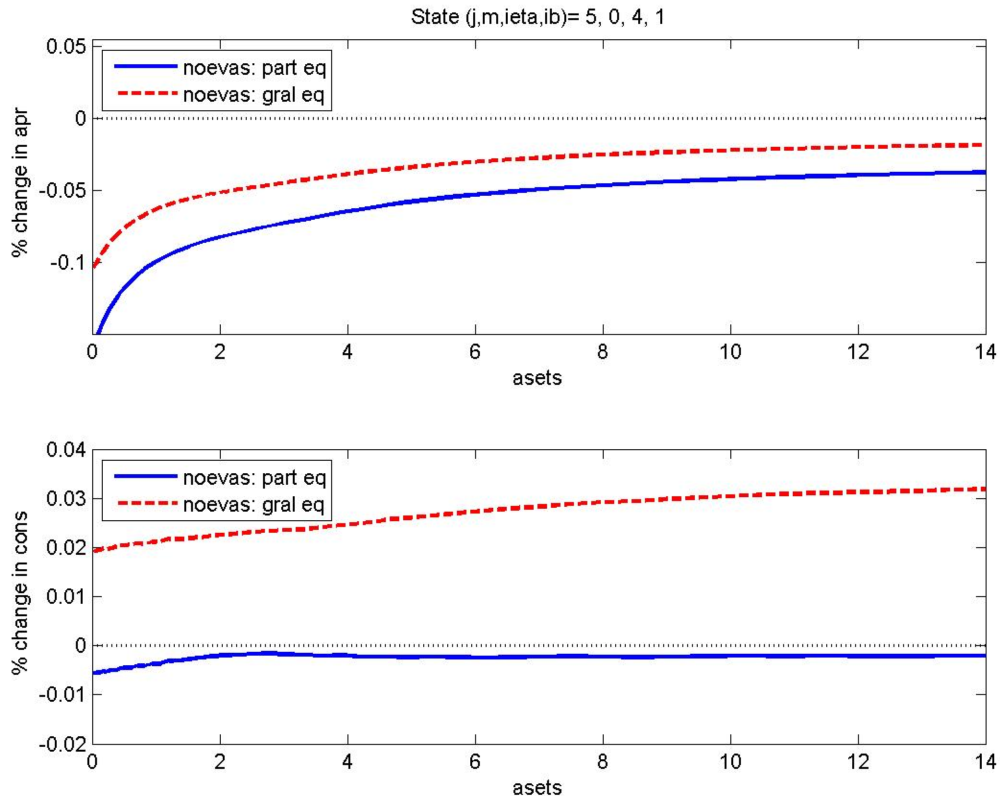

Figure 12: No evasion. Change in policy functions. Partial and general equilibrium.

display general-equilibrium changes in savings and consumption decision rules as a response to the elimination of evasion for the typical baseline household (with zero arrears), which illustrates the direction of the aggregate changes just discussed.

Regarding the borrowing constrained households, when evasion is eliminated, the mass of such households increases by several fold compared to the baseline in Table 4, to 8.2% and 7.1% in partial and general equilibrium respectively. Evasion did indeed serve as insurance for those households in tight financial position. We also see that the borrowing constrained become more concentrated at the first age group (45% and 53% respectively). Incidentally, the high presence of borrowing constrained households on the pre-retirement period observed in the benchmark economy disappears with the impossibility of evasion, suggesting that the option to use evasion was one reason why those less well off would decide not to save for retirement.

10 A flat tax reform

The flat-tax reform is a shift from the benchmark progressive tax schedule described in equation (2) to a proportional tax function $T(x) = \kappa_2 x$, where κ_2 is the flat tax rate. The focus will be on the consequences for evasion behaviour.

We look first at the partial equilibrium effects, with the flat rate at the level of the equilibrium average tax rate in the benchmark equilibrium 18.62%, so $\kappa_2 = 0.1862$, and holding unchanged the wage rate and the interest rate. In order to isolate key incentive effects, we will in a first instance also suppose that the savings policy functions of households are the same as in the benchmark economy. Then we will consider the changes when savings behaviour responds optimally to the tax reform. Having considered these partial equilibrium responses, we can study the outcomes in general equilibrium where the level of the tax rate, as well as wages and interest rates and transfers, respond to clear markets and meet the government budget constraint. Table 8 shows key variables under those three different scenarios as well as in the pre-reform benchmark as the central reference.

Table 8: **Flat tax reform**

	Benchmark	Flat tax partial equil fixed bond decis	Flat tax partial equil	Flat tax gral equil
Aver tax rate	0.1862	0.1862	0.1862	0.1835
Capital/Y	2.70	2.72	2.84	2.80
evasion/Y	0.0321	0.0305	0.0355	0.0314
(mean evas)/(mean tax due)	0.200	0.188	0.215	0.198
mean(evas/tax due)	0.269	0.254	0.227	0.211

The ultimate general equilibrium outcomes are in the last column of Table 8. The tax reform implies a predictable increased incentive to save, and a small reduction in the level of overall evasion over GDP. The increased capital accumulation and lower evasion afford accordingly a reduction in the tax rate, albeit a tiny one. Looking more closely at evasion, there is an only slightly lower overall evasion over tax due (19.8% down from 20%), but a more substantial reduction in the average individual evasion intensity (21% down from 27%). Although the impact on overall evasion is small, the reform brings about a change in the distribution of evasion intensity across types of households in a way that tempers the asymmetry present in the pre-reform benchmark where low-income households were evading a much large fraction. The reform decreases the fraction evaded among low income agents as they face an increased level of tax liabilities after the reform.

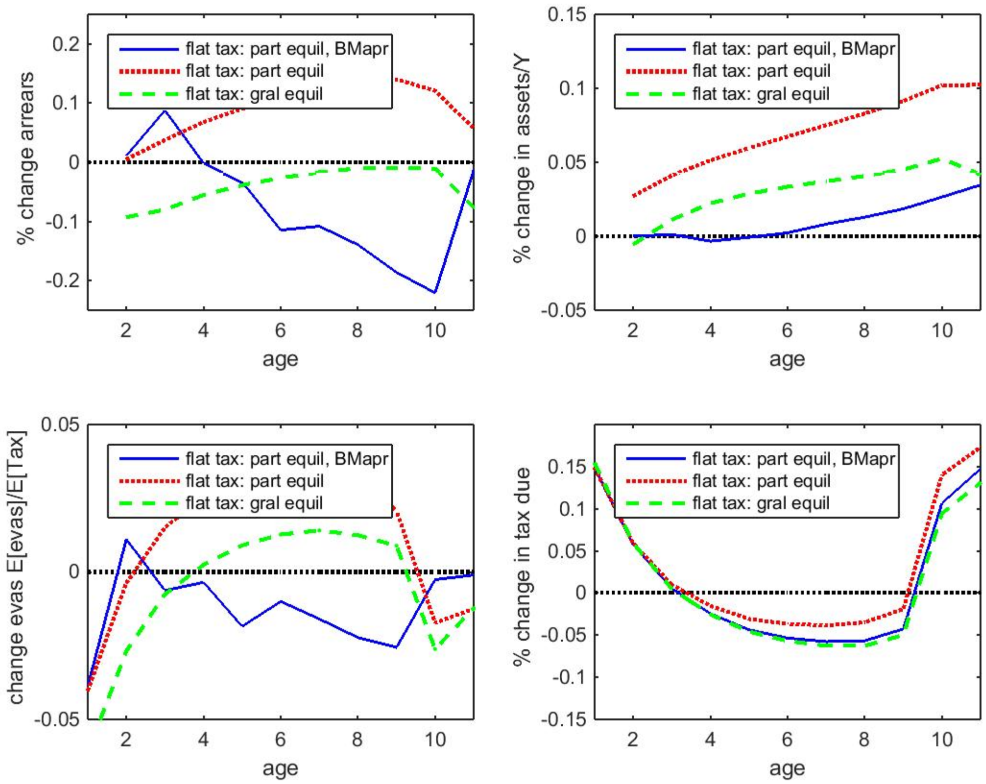

Figure 13: Changes from flat tax reform relative to benchmark

In order to uncover the various factors at work, consider the outcomes under partial equilibrium and constant savings policy functions in the second column of Table 8. By design, the experiment will reflect the direct pure effect of the tax change on tax compliance incentives. Overall, it turns out that the channel that dominates is that, with a lower marginal rate for some households, there is less reason to evade. The two measures of evasion to tax due do in effect decrease to a similar degree. These net aggregate figures must however mask variation across different households since, while reducing the marginal rate for some, the tax reform must also have increased rates for other taxpayers. To help understand these various responses, Figure 13 draws changes in variables by age group compared to the pre-reform benchmark. The bottom right graph, solid line, shows that tax liabilities fall for the main range of working age population; correspondingly, mean evasion intensity in the bottom left graph declines at those ages. For the youngest group, however, the evasion rate decreases in spite of the heavier taxation they experience; this is in order to hedge the risk of heavy audit fines which is more of a concern at this low-asset early stage of the life cycle.

The third column in Table 8 shows the outcome when the positive response of households' savings is taken into account. This has the effect of raising the overall proportion of evasion to taxes due, an indication that tax arrears via evasion is part of the joint portfolio choice. The graphs in Figure 13 (dotted lines) support this by showing an overall upward shift in the life cycle profile of evasion associated with faster capital accumulation. All in all, in partial equilibrium higher savings are conducive to higher evasion.

Interestingly, in contrast with the increased overall evasion rate, the mean of the rate across households declines markedly in partial equilibrium as a result of households adjusting their savings decisions, implying a reduction in the skewness of the individual distribution of evasion rates.

As discussed in regards with the fourth column of Table 8, in general equilibrium overall evasion decreases thus overturning the partial-equilibrium response. Figure 13 (dashed lines) shows that this follows from the fact that the increase in savings is dampened in general equilibrium, featuring a wider range of ages, the very young and the retired, who on average evade less after the tax reform.

In sum, the response of evasion to a flat-tax reform reflects various intervening mechanisms and varies across types of households in a way that render the net response small. Nonetheless an understanding of the underlying heterogeneity is necessary for drawing any conclusions.

11 Conclusions and final remarks

This paper studies tax evasion in a quantitative incomplete-markets heterogeneous-agents setting. A central aspect is that evasion is a form of contingent debt, a view motivated by realistic description of auditing and enforcements provisions. In this way, the analysis echoes ideas entertained in public finance theory while introducing the analysis of evasion, and the related portfolio choice problem, into an area of vibrant research in macroeconomics.

The calibrated model is confronted with available evidence about patterns of evasion across household types, a novel aspect of this paper, and it appears to do a notable job on this front. We also learn that evasion may have quantitatively important macroeconomic implications, and, as a policy illustration exercise, the consequences of a flat tax reform for non compliance might well be modest. In these exercises, heterogeneity and general equilibrium responses prove very significant.

One hindrance to this type of study is that inevitably empirical evidence of evasion behaviour by household individual characteristics is not readily available. This paper highlights areas where progress in data collection would be very valuable for improving models and their practical relevance.

Its reasonable performance notwithstanding, the model used here is a first step and as such presents limitations. The presence of a realistic bankruptcy code or social insurance will be important for understanding evasion, particularly for the less well off. The distinction between employed and self-employed individuals facing different opportunities for non-compliance, and the related occupational choice, will introduce additional realism. Incorporating the extensive margin in non-compliance, as well as an endogenous labour supply will be a necessary extension to support the evaluation tax reforms, and for further work going forward. Promising such work will include the study of optimal taxation in the presence of evasion, a stepping stone towards the study of the joint determination of tax enforcement and tax rates.

A Equilibrium de-trending

In terms of the original model's variables, in an equilibrium the policy and value functions, and distribution measure satisfy the following conditions.

1. Household optimisation: Given $\{\tau_t^{ss}, SS_t, \tau_t^c, w_t, r_t, Tr_t, \overline{y}_t, T_t(.)\}_{t=0}^{\infty}$, for $j = 1, ..., J$,

$$V_t(a, b, \eta, j, m) = \max_{a', b'} \Big\{ u(c, 1-l) + \beta\psi_j \sum_{\eta'} \Gamma(\eta, \eta')$$
$$[p^A(m)V_{t+1}(a', b', \eta', j+1, 1) + (1 - p^A(m))V_{t+1}(a', b', \eta', j+1, 0)] \Big\},$$

subject to

- Taxable income
$$y_t(a, \eta, j, l) = \begin{cases} w_t\epsilon_j\eta l - 0.5\tau_t^{ss} \min\{w_t\epsilon_j\eta l, \overline{y}_t\} + r_t(a + Tr_t) & j < j_R \\ SS_t + r_t(a + Tr_t) & j \geq j_R \end{cases}$$
- The budget constraint
$$(1+\tau_t^c)c = \begin{cases} -0.5\tau_t^{ss} \min\{w_t\epsilon_j\eta l, \overline{y}_t\} + y_t(a, \eta, j, l) & \\ +a + Tr_t - T_t(y_t(a, \eta, j, l)) + b' - a' - \xi(m)b & j < j_R \\ y_t(a, \eta, j, l) + a + Tr_t - T_t(y_t(a, \eta, j, l)) + b' - \xi(m)b - a' & j \geq j_R \end{cases}$$
- Non-negative assets, $a' \geq 0$.
- Bounds on arrears such that $b' - (1-m)\nu b \in [0, T_t(y_t(a, \eta, j, l)) - T_t(0)]$.
- Terminal condition: $V_t(a, b, \eta, J+1, m) = \phi(b)$ for all states.

2. Prices. Competitive firm's implies $r_t = \alpha((Z_tN_t)/K_t)^{1-\alpha} - \delta$ and $w_t = (1-\alpha)Z_t(K_t/(Z_tN_t))^{\alpha}$.

3. Social security balanced budget:

$$SS_t \int \Phi_t(da \times db \times d\eta \times \{j_R, ..., J\} \times dm) =$$
$$\tau_t^{ss} \int \min\{w_t\epsilon_j\eta l, \overline{y}_t\}\Phi_t(da \times db \times d\eta \times \{1, ..., j_R - 1\} \times dm)$$

4. Government balanced budget: $s \in \mathcal{S}$

$$G_t = \int \big(T_t(y_t(a, \eta, j, l)) - b_t'(a, b, \eta, j, m) + (1-m)\nu b\big)\, \Phi_t(ds)$$
$$+ \tau_t^c \int c_t(a, b, \eta, j, m)\Phi_t(ds)$$
$$+ (1 + \pi^G) \sum_{m=0,1} p^A(m)B_t(m),$$

where aggregate arrears evolve as

$$B_{t+1}(m) = \int \psi_j b_t'(a,b,\eta,j,m)\Phi_t(da \times db \times d\eta \times dj \times m), \ \ m \in \{0,1\}.$$

5. Transfers. For $s \in \mathcal{S}$,

$$Tr_{t+1} \int \Phi_{t+1}(ds) = \int (1-\psi_j)a_t'(s)\Phi_t(ds).$$

6. Market clearing. Capital, labour and final goods markets clear in the sense that $K_t = \int a\Phi_t(ds)$, $N_t = \int \epsilon_j \eta l \Phi_t(ds)$, and $\int c_t(s)\Phi_t(ds) + (\pi^A - \pi^G)\sum_{m=0,1} p^A(m)B_t(m) + K_{t+1} + G_t = K_t^{\alpha}(Z_t N_t)^{1-\alpha} + (1-\delta)K_t$, where $s \in \mathcal{S}$.

7. Distribution. The distribution measure evolves as

$$\Phi_{t+1} = H_t(\Phi_t),$$

where, for an element in the Borel algebra $(\mathcal{A}\times\mathcal{B}\times\mathcal{E}\times\mathcal{J}\times\mathcal{M})$, the transition function mapping H is defined as follows:

(a) For $1 \notin \mathcal{J}$,

$$\begin{aligned}&\Phi_{t+1}(\mathcal{A}\times\mathcal{B}\times\mathcal{E}\times\mathcal{I}\times\mathcal{J}\times\mathcal{M})\\ &\quad = \int Q_t(a,b,\eta,i,j,m;\mathcal{A}\times\mathcal{B}\times\mathcal{E}\times\mathcal{J}\times\mathcal{M})\psi_j\Phi_t(da\times db\times d\eta\times dj\times dm),\end{aligned}$$

where the probability transition Q_t is

$$Q_t(a,b,\eta,j,m;\mathcal{A}\times\mathcal{B}\times\mathcal{E}\times\mathcal{J}\times\mathcal{M}) = \Gamma(\eta,\eta')p^A(m')$$

if $j+1 \in \mathcal{J}$, $a_t'(s) \in \mathcal{A}$, $b_t'(s) \in \mathcal{B}$, and $\eta' \in \mathcal{E}$ and $m' \in \mathcal{M}$. Otherwise, it is 0.

(b) For $\mathcal{J} = \{1\}$,

$$\Phi_{t+1}(\mathcal{A}\times\mathcal{B}\times\mathcal{E}\times\mathcal{J}\times\mathcal{M}) = (1+n)^t \times \begin{cases} 1 & 0\in\mathcal{A}, 0\in\mathcal{B}, 0\in\mathcal{M}, \overline{\eta}\in\mathcal{E} \\ 0 & \text{otherwise}\end{cases}$$

where we define the mean endowment $\overline{\eta} = \sum \eta\Pi(\eta)$, with Π its invariant distribution.

A.1 Detrending

We normalise per-capita variables by aggregate productivity Z_t, and aggregate variables by Z_t and by population size P_t. That is, redefine per-capita variables $w_t = w_t/Z_t$, $\bar{y}_t = \bar{y}_t/Z_t$, $Tr_t = Tr_t/Z_t$, $T_t(.) = T_t(.)/Z_t$, $SS_t = SS_t/Z_t$, $y_t(.) = y_t(.)/Z_t$, $a = a/Z_t$, $b = b/Z_t$, $a' =$

a'/Z_{t+1}, and $b' = b'/Z_{t+1}$. Regarding aggregates, redefine $K_t = K_t/(Z_t P_t)$, $G_t = G_t/(Z_t P_t)$, $B_t(.) = B_t(.)/(Z_t P_t)$, $N_t = N_t/P_t$, and $\Phi_t = \Phi_t/P_t$.

The tax function now has $\kappa_2 = \kappa_{2,t} Z_t^{\kappa_1}$, where the original $\kappa_{2,t}$ will adjust over time to keep κ_2 constant. The utility function is now $z_t^{\gamma(1-\sigma)} u(c_t, l_t)$, so the discount rate must be redefined as

$$\beta^t = \beta z_t^{\gamma(1-\sigma)} = [\beta(1+g)^{\gamma(1-\sigma)}]^t.$$

In terms of these detrended variables, the equilibrium definition includes the growth rates g and n and removes Z_t accordingly as in the main text.

B Further details on computation

A solution to the household's decision problem is characterized by functions of the states $s = \{a, b, \eta, m)$ and age j, denoted $b'(s,j)$, $a'(s,j)$, and $c(s,j)$ and value functions $v(s,j)$. For short-hand notation, as functions defined on continuum support we can denote them b', a', and v.

In the final period $j = J$, the assets choice is $a' = 0$, and the decision on b' maximises current utility minus the warm-glow terminal cost $\phi(b')$, a function defined directly over real numbers. The derivatives can be evaluated at corner choices to determine whether the solution is interior. If it is interior, it can be found via a standard optimisation procedure like the golden search rule.

For the other periods $j = J-1, J-2, ..., 1$, denote by bi', ai', and vi the corresponding arrays on the discrete support indexed by i (grid). We proceed backwards by solving the mapping $vi \to vi, bi', ai'$. Maximisation requires specifying choices over interpolation schemes and optimisation procedures.

In this paper we compute the joint decision by splitting the problem into two nested problems, an inner loop for the choice on a', and an outer loop for the choice on b'. The goal is to achieve the smoothness in the policy functions for a' that is required for accuracy in the decision over b'. Procedures based on direct joint local optimisation methods do not appear very stable and often deliver unseemly outcomes.

For the inner loop, conditional on each point ib' on the grid for b', we build the objective $RHS_a(a', ib')$ (i.e., the right-hand side of the Bellman equation) over the continuum support for a', and then find the maximum $ai'_b(ib')$ and associated value $RHSi_b(ib') = RHS_a(ai'_b(ib'), ib')$. The objective $RHS_a(a', ib')$ as a function of a' is constructed via cubic spline interpolation of the continuation values above a certain threshold, but linear interpolation for a' below that cutoff point. The split in interpolation schemes is in order to avoid the typical internodal oscillations associated with cubic splines when the objective presents kinks or sharp changes. These type of situations arise near the liquidity constraint or in situations close to zero consumption when, as it the case here, violation of these occasionally binding constraints is penalised with an arbitrarily large utility loss. Using Schumaker's shape preserving splines instead of cubic splines does not improve performance.

For the outer loop, based on the discrete $RHSi_b(ib')$ obtained in the inner loop we build the objective over the continuum of values b', $RHSb(b')$, again via cubic splines below a cutoff point, and– linear interpolation for b' above that point. In this problem, the bounds on b' imply occasionally bindings constraints. Since these bounds do not generally lie on the grid, we evaluate the objective at those points with extrapolation at points outside of these bounds. The optimality of a corner solution in b' is established by checking the local slope of the objective at that point.

C Calibration details

Income process. Discretised stochastic productivity vector:

$$\eta = \begin{pmatrix} 0.14856 & 0.28050 & 0.52963 & 1.00 & 1.88812 & 3.56501 & 6.73117 \end{pmatrix}'$$

Transition probabilities:

$$\Gamma_{\eta,\eta'} = \begin{pmatrix} 0.48665358 & 0.37239204 & 0.11873250 & 0.02019006 & 0.00193120 & 0.00009852 & 0.00000209 \\ 0.06206534 & 0.52623108 & 0.32042173 & 0.08044247 & 0.01017713 & 0.00064583 & 0.00001642 \\ 0.00791550 & 0.12816869 & 0.55075007 & 0.26044107 & 0.04852507 & 0.00407085 & 0.00012875 \\ 0.00100950 & 0.02413274 & 0.19533081 & 0.55905390 & 0.19533081 & 0.02413274 & 0.00100950 \\ 0.00012875 & 0.00407085 & 0.04852507 & 0.26044107 & 0.55075007 & 0.12816869 & 0.00791550 \\ 0.00001642 & 0.00064583 & 0.01017713 & 0.08044247 & 0.32042173 & 0.52623108 & 0.06206534 \\ 0.00000209 & 0.00009852 & 0.00193120 & 0.02019006 & 0.11873250 & 0.37239204 & 0.48665358 \end{pmatrix}$$

Stationary distribution:

$$\Pi(\eta) = \begin{pmatrix} 0.015625 & 0.09375 & 0.234375 & 0.3125 & 0.234375 & 0.09375 & 0.015625 \end{pmatrix}'$$

Life-cycle productivity component:

$$\epsilon = \begin{pmatrix} 1.14314 & 1.4897 & 1.74738 & 1.881 & 1.96404 & 1.9726 & 1.9623 & 1.9238 & 1.77012 \end{pmatrix}'$$

Internally calibrated parameters. The parameter values in Table 3 minimise the sum of squared proportional deviations of the model moments from their corresponding data moments. The algorithm is based on the Software BOBYQA, authored by M. J. D. Powell, to minimize sum of squares with bound constraints by combing trust region method and Levenberg-Marquardt method.

Equilibrium variables. Given the calibrated parameters, the equilibrium pins down κ_2, SS, r and N, which are sufficient to derive all equilibrium prices and aggregate and individual allocations. Their values are displayed in Table 9. Individual households receive a pension which amounts to a 40% replacement over average per capita income; the interest rate is near 6% in annual terms.

Table 9: **Equilibrium variables**

N	0.566
r	0.355 (annual 0.063)
SS/Y	0.398
κ_2	3.201

References

Advani, A., W. Elming, and J. Shaw (2017): "The dynamic effects of tax audits," Discussion paper, WP17/24 Institute for Fiscal Studies.

Aiyagari, S. R. (1994): "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, 109(3), 659–684.

Allingham, M. G., and A. Sandmo (1972): "Income tax evasion: a theoretical analysis," *Journal of Public Economics*, 1(3-4), 323–338.

Andreoni, J. (1992): "IRS as loan shark tax compliance with borrowing constraints," *Journal of Public Economics*, 49(1), 35–46.

Andreoni, J., B. Erard, and J. Feinstein (1998): "Tax compliance," *Journal of economic literature*, 36(2), 818–860.

Armenter, R., and T. M. Mertens (2013): "Fraud deterrence in dynamic Mirrleesian economies," *Journal of Monetary Economics*, 60(2), 139–151.

Balafoutas, L., A. Beck, R. Kerschbamer, and M. Sutter (2015): "The hidden costs of tax evasion.: Collaborative tax evasion in markets for expert services," *Journal of Public Economics*, 129, 14–25.

Beer, S., M. Kasper, E. Kirchler, and B. Erard (2018): "Do audits deter future noncompliance? Evidence on self-employed taxpayers," Paper presented at the Conference on New Perspectives on Tax Administration, CESifo.

Bernasconi, M. (1998): "Tax evasion and orders of risk aversion," *Journal of Public Economics*, 67(1), 123–134.

Bordignon, M. (1993): "A fairness approach to income tax evasion," *Journal of Public Economics*, 52(3), 345–362.

Cabral, A. C. G., C. Kotsogiannis, and G. Myles (2018): "Self-Employment Income Gap in Great Britain: How Much and Who?," *CESifo Economic Studies*.

Castaneda, A., J. Diaz-Gimenez, and J.-V. Rios-Rull (2003): "Accounting for the US earnings and wealth inequality," *Journal of political economy*, 111(4), 818–857.

Cebula, R. J., and E. L. Feige (2012): "Americas unreported economy: measuring the size, growth and determinants of income tax evasion in the US," *Crime, Law and Social Change*, 57(3), 265–285.

Chetty, R. (2009): "Is the taxable income elasticity sufficient to calculate deadweight loss? The implications of evasion and avoidance," *American Economic Journal: Economic Policy*, 1(2), 31–52.

Christian, C. W. (1994): "Voluntary compliance with the individual income tax: results from the 1988 TCMP study," *The IRS Research Bulletin*, 1500(9-94), 35–42.

Clotfelter, C. T. (1983): "Tax evasion and tax rates: An analysis of individual returns," *The review of economics and statistics*, pp. 363–373.

Conesa, J. C., S. Kitao, and D. Krueger (2009): "Taxing capital? Not a bad idea after all!," *The American economic review*, 99(1), 25–48.

Conesa, J. C., and D. Krueger (2006): "On the optimal progressivity of the income tax code," *Journal of Monetary Economics*, 53(7), 1425–1450.

Cowell, F. A., et al. (1990): "Cheating the government: The economics of evasion," *MIT Press Books*, 1.

De Nardi, M. (2004): "Wealth inequality and intergenerational links," *The Review of Economic Studies*, 71(3), 743–768.

Domeij, D., and J. Heathcote (2004): "On the distributional effects of reducing capital taxes," *International economic review*, 45(2), 523–554.

Dyrda, S., and M. Pedroni (2016): "Optimal fiscal policy in a model with uninsurable idiosyncratic shocks," in *2016 Meeting Papers*, vol. 1245.

Erard, B., and J. S. Feinstein (1994): "Honesty and evasion in the tax compliance game," *The RAND Journal of Economics*, pp. 1–19.

Feinstein, J. S. (1991): "An econometric analysis of income tax evasion and its detection," *The RAND Journal of Economics*, pp. 14–35.

Gordon, J. P. (1989): "Individual morality and reputation costs as deterrents to tax evasion," *European economic review*, 33(4), 797–805.

Gouveia, M., and R. P. Strauss (1994): "Effective federal individual income tax functions: An exploratory empirical analysis," *National Tax Journal*, pp. 317–339.

Guner, N., M. Lopez-Daneri, and G. Ventura (2016): "Heterogeneity and Government Revenues: Higher Taxes at the Top?," *Journal of Monetary Economics*.

Hansen, G. D. (1993): "The cyclical and secular behaviour of the labour input: Comparing efficiency units and hours worked," *Journal of Applied Econometrics*, 8(1), 71–80.

Hashimzade, N., and G. Myles (2017): "Risk-based Audits in a Behavioral Model," *Public Finance Review*, 45(1), 140–165.

Hashimzade, N., G. D. Myles, and B. Tran-Nam (2013): "Applications of behavioural economics to tax evasion," *Journal of Economic Surveys*, 27(5), 941–977.

Heathcote, J., F. Perri, and G. L. Violante (2010): "Unequal we stand: An empirical analysis of economic inequality in the United States, 1967–2006," *Review of Economic dynamics*, 13(1), 15–51.

Heathcote, J., K. Storesletten, and G. L. Violante (2017): "Optimal tax progressivity: An analytical framework," *The Quarterly Journal of Economics*, 132(4), 1693–1754.

Hintermaier, T., and W. Koeniger (2010): "The method of endogenous gridpoints with occasionally binding constraints among endogenous variables," *Journal of Economic Dynamics and Control*, 34(10), 2074–2088.

——— (2011): "On the evolution of the US consumer wealth distribution," *Review of Economic Dynamics*, 14(2), 317–338.

Hofmann, E., M. Voracek, C. Bock, and E. Kirchler (2017): "Tax compliance across sociodemographic categories: Meta-analyses of survey studies in 111 countries," *Journal of Economic Psychology*, 62, 63–71.

Holter, H. A., D. Krueger, and S. Stepanchuk (2014): "How does tax progressivity and household heterogeneity affect Laffer curves?," Discussion paper, National Bureau of Economic Research.

Huang, J., and J. Rios (2016): "Optimal tax mix with income tax non-compliance," *Journal of Public Economics*, 144, 52–63.

Huggett, M. (1996): "Wealth distribution in life-cycle economies," *Journal of Monetary Economics*, 38(3), 469–494.

IRS (2016): "Tax Gap Estimates for Tax Years 2008-2010," Discussion paper, Internal Revenue Service.

Johns, A., and J. Slemrod (2010): "The distribution of income tax noncompliance," *National Tax Journal*, 63(3), 397.

Jorda, O., and M. Marcellino (2004): "Time-scale transformations of discrete time processes," *Journal of Time Series Analysis*, 25(6), 873–894.

Karabarbounis, M. (2016): "A road map for efficiently taxing heterogeneous agents," *American Economic Journal: Macroeconomics*, 8(2), 182–214.

Keen, M., Y. Kim, and R. Varsano (2008): "The flat tax (es): principles and experience," *International Tax and Public Finance*, 15(6), 712–751.

Kleven, H. J., M. B. Knudsen, C. T. Kreiner, S. Pedersen, and E. Saez (2011): "Unwilling or unable to cheat? Evidence from a tax audit experiment in Denmark," *Econometrica*, 79(3), 651–692.

Kopecky, K. A., and R. M. Suen (2010): "Finite state Markov-chain approximations to highly persistent processes," *Review of Economic Dynamics*, 13(3), 701–714.

Kuhn, M., and J.-V. Rios-Rull (2016): "2013 Update on the US earnings, income, and wealth distributional facts: A View from Macroeconomics," *Quarterly Review, Federal Reserve Bank of Minneapolis, April*, pp. 1–75.

Levaggi, R., and F. Menoncin (2012): "Tax audits, fines and optimal tax evasion in a dynamic context," *Economics Letters*, 117(1), 318–321.

——— (2013): "Optimal dynamic tax evasion," *Journal of Economic Dynamics and Control*, 37(11), 2157–2167.

——— (2016): "Optimal dynamic tax evasion: A portfolio approach," *Journal of Economic Behavior & Organization*, 124, 115–129.

Mateos-Planas, X. (2010): "Demographics and the politics of capital taxation in a life-cycle economy," *American Economic Review*, 100(1), 337–63.

Mendoza, E. G., A. Razin, and L. L. Tesar (1994): "Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption," *Journal of Monetary Economics*, 34(3), 297–323.

Niepelt, D. (2005): "Timing tax evasion," *Journal of Public Economics*, 89(9-10), 1611–1637.

OECD (2013): *Tax Administration 2013-Comparative Information on OECD and Other Advanced and Emerging Economies.*

Ravikumar, B., and Y. Zhang (2012): "Optimal auditing and insurance in a dynamic model of tax compliance," *Theoretical Economics*, 7(2), 241–282.

Rouwenhorst, K. G. (1995): "Asset Pricing Implications of Equilibrium Business Cycle Models," *Frontiers of business cycle research*, pp. 294–330.

Slemrod, J. (2007): "Cheating ourselves: The economics of tax evasion," *Journal of Economic Perspectives*, 21(1), 25–48.

Slemrod, J., and C. Gillitzer (2013): *Tax systems.* MIT Press.

Slemrod, J., and S. Yitzhaki (2002): "Tax avoidance, evasion, and administration," in *Handbook of public economics*, vol. 3, pp. 1423–1470. Elsevier.

Storesletten, K., C. Telmer, and A. Yaron (2004): "Consumption and risk sharing over the life cycle," *Journal of Monetary Economics*, 51(3), 609–633.

Uribe-Teran, C. (2015): "Down the Rabbit–Hole: Tax Avoidance and the Elasticity of Taxable Income," Discussion paper, Mimeo.