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European Economic Review

journal homepage: www.elsevier.com/locate/euroecorev

Dynamic supply adjustment and banking under uncertainty in an emission trading scheme: The market stability reserve[☆]

Sascha Kollenberg^a, Luca Taschini^{b,c,*}^a University of Duisburg-Essen, Universitätsstraße 12, 45141 Essen, Germany^b The London School of Economics and Political Science, Grantham Research Institute on Climate Change and the Environment, Houghton St., WC2A 2AE, London, UK^c Università degli Studi di Verona - Dipartimento di Scienze Economiche, Via Cantarane, 24 - 37129 Verona VR, Italy

ARTICLE INFO

Article history:

Received 16 January 2018

Accepted 24 May 2019

Available online 4 June 2019

Keywords:

EU ETS Reform

Policy Responsiveness

Resilience

Supply Management Mechanism

Risk-Aversion

ABSTRACT

We study the impact of a supply management mechanism (SMM) similar to the Market Stability Reserve proposed in 2015 which preserve the overall emissions cap and we comment on the recent cap-changing amendments. We provide an analytical description of the conditions under which an SMM alters the emissions abatement paths, affecting the expected length of the banking period and its variability. While abatement strategies of risk neutral firms solely depend on the former, for risk-averse firms changes in the latter would lead to higher risk premia, accelerated depletion of the bank and, consequently, further reduction of abatement and allowance prices. Cancellation of part of the reserve could partially outweigh the effect on risk premia sustaining allowance prices.

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1. Introduction

Despite an emerging use of supply control mechanisms, in most existing cap-and-trade programmes the environmental reduction target (the cap) is fixed and the supply of allowances is inflexible and determined within a rigid allocation programme. In theory, as long as the regulator makes allowances available before they are needed, the programme will deliver a cost-effective solution [Hasegawa and Salant \(2015\)](#). However, observations from recent cap-and-trade schemes – in particular the European Union Emissions Trading System (EU ETS) – have raised concerns over excessive allowance price variability and price collapse. These maladies seem to stem from a problem of ‘over-supply’, wherein unexpectedly low levels of allowance demand have led to the accumulation of a significant *surplus* of allowances. An article in [The Economist \(2013\)](#) lamented a surplus of allowances equivalent to an average year’s emissions. This surplus is often attributed to two

[☆] Part of Kollenberg’s research was supported by the University of Duisburg-Essen. Part of Taschini’s research is supported by funding from the UK Economic and Social Research Council (ESRC) through the Centre for Climate Change Economics and Policy (CCCEP)[grant number ES/K006576/1] and the Grantham Foundation for the Protection of the Environment. This project has received funding from the Stiftung Mercator and from the European Union’s Seventh Programme for research, technological development and demonstration under grant agreement No 308481. Furthermore, the project was supported by the German Federal Ministry of Education and Research under promotional reference No. 01LA1115A. The authors would like to thank Federica Buricco, Dallas Burtraw, Corina Comendant, Harrison Fell, Carolyn Fischer, Rüdiger Kiesel, Andrei Marcu, Steve Salant, Peter Vis, Peter Zapfel, and other participants of the Workshop on EU ETS Market Stability Reserve policies held at DIW and CEPS. The usual disclaimers apply.

* Corresponding author at: The London School of Economics and Political Science, UK.

E-mail addresses: sascha.kollenberg@uni-due.de (S. Kollenberg), L.Taschini1@lse.ac.uk, luca.taschini@univr.it (L. Taschini).

effects. On the one hand, the economic recession and renewables-promoting policies have led to a significant drop in allowance demand; on the other, the system has been unable to respond to changes in economic circumstances and policies, see Grosjean et al. (2014) and Ellerman et al. (2015). The resultant drop in allowance prices has policy makers and other stakeholders concerned that the current imbalance in supply and demand, if left unchecked, could reduce incentives for low-carbon investment and ultimately impair the ability of the EU ETS to meet its targets.¹

There are already provisions within a cap-and-trade framework that, in theory, should compensate for unforeseen changes in allowance demand. For example, most ETSs have banking provisions that should provide firms with a tool to respond to demand shocks. Several studies have explored the effect of banking and borrowing provisions as cost ‘smoothing’ mechanisms which decrease allowance price variability; Hasegawa and Salant (2014) provide a comprehensive and critical review of the literature on bankable emissions allowances that has developed over the last two decades. Other studies demonstrate how hybrid systems, combinations of quantity- and price-based instruments, lower expected control costs ultimately mitigating allowance price variability (Fell and Morgenstern, 2010; Grull and Taschini, 2011; Fell et al., 2012b; 2012a).

However, these provisions alone may not be sufficient when the market is faced with severe demand shocks. This leads to the question of how to amend an existing ETS to deal with an unexpected under- or over-supply of allowances. Namely, how should the allowance allocation programme (the supply, which can be controlled by regulators) be changed to better cope with unexpected changes in allowance demand. In the case of the EU ETS, the European Commission (EC) has proposed a structural reform of the ETS, including the implementation of the Market Stability Reserve (MSR) that has started to operate in 2019 (EC, 2014a; 2014b; EP, 2015). The MSR amends the allowance allocation programme. In particular, it adjusts the number of allowances auctioned based on the size of the aggregate bank, i.e. the sum of firms’ individually held banks of allowances. Hereafter, we refer to quantities concerning the entirety of regulated firms as ‘aggregate’ whereas the respective quantities for each firm are referred to as ‘individual’. In a given year, if the aggregate bank of allowances exceeds 833 million, a pre-defined percentage of the size of the aggregate bank will be withheld from auctions and will be placed in a dedicated reserve. There are two intake rates: 24% from 2019 to 2023 and 12% from 2024. These allowances are returned to the market in batches of 100 million as soon as the aggregate bank drops below the threshold of 400 million. In its original 2015 design, the MSR changes the allowance allocation programme but leaves the total number of allocated allowances (the cap) unchanged within the regulatory period. As such, the reserve is temporary in nature and the initially proposed version of the MSR preserves the original cap. The alternatives of temporarily versus permanently placing allowances in the reserve have been heartily debated in the past years. In late 2017, after numerous stakeholder consultations and more than two years of negotiations, the European Commission decided that, starting in 2023 the volume of allowances that can be held in the reserve will be capped at the previous year’s auction volume. The resulting difference in the reserve will be cancelled, providing a mechanism for allowances to be retired and thus reduce the long-run supply of allowances.

In an earlier paper (Kollenberg and Taschini, 2016), we examined a similar dynamic allocation programme where the cap could be varied in response to exogenous shocks, as is the case for the 2017 version of the MSR. Accordingly, the applicability of this earlier framework to the original 2015 MSR is limited. With the additional objective to comment on the recent proposed MSR amendments (allowances cancellation), we focus our analysis on a generalised, cap-preserving supply management mechanism (SMM for short) similar to the original 2015 MSR legislation. The proposed SMM allows us to abstract from the operational details of the EC MSR² and to provide a conceptual framework that enable us to transparently illustrate (1) how firms’ abatement strategies vary in response to changes of the allowance allocation programme and (2) how an SMM affects the risk-premium associated to holding allowances or any equivalent investment in abatement. As such, we draw from and contribute to the literature on inter-temporal permit trading under uncertainty and to the emerging literature on the assessment of mechanisms that vary allowance allocation according to market conditions, such as indexed regulations (among others, Newell and Pizer, 2008; Kollenberg and Taschini, 2016; Lintunen and Kuusela, 2018), output-based allocation (Meunier et al., 2017), and price-based mechanisms (Aldy et al., 2017). In particular, the implications of the changes in risk-premia speak directly to the policy debate playing out among experts on no-cap adjustments versus cap adjustments. Our analysis ultimately suggests that a permanent cancellation of part of the reserve could keep in check the premium that risk-averse firms demand for abatement investments. As a by-product, the relevance of our findings extends to the policy debate in California, South Korea and the member States of the Regional Greenhouse Gas Initiative, where similar supply management mechanisms were adopted.

The results of previous theoretical and empirical analyses of intertemporal trading of emission allowances reveal that, under the usual assumption that marginal abatement costs are increasing in emissions reduction, firms start accumulating allowances and then draw them down, see Rubin (1996), Schennach (2000), Ellerman and Montero (2007), and Ellerman et al. (2015). Banking of allowances is thus a manifestation of the inter-temporal trading problem. The rationale for banking is quite intuitive: if tomorrow’s discounted expected cost is higher than today’s cost, it is worth banking allowances, whether obtained by abating more emissions today or by purchase, and either using them to cover some of tomorrow’s emissions or selling them later on. The expected duration of the banking period, i.e. the period of time during which firms prefer to hold allowances, depends on the amount of abatement implied by the cap and, as long

¹ In a 2015 official note, the European Parliament states that the surplus prevents “the EU ETS from delivering the necessary investment signal to reduce CO₂ emissions in a cost-efficient manner and from being a driver of low-carbon innovation...” [EC, 6th October 2015].

² We discuss the implications of this modelling choice in Section 2.3.

as the original abatement path is feasible (see [Perino and Willner, 2016](#)), it is independent of the allowance allocation programme.

A banking model with no uncertainty and perfect competition would predict that during the banking period $[0, \tau)$ the price P_t of allowances will rise at the risk-free rate r , $dP_t/P_t = rdt$, where τ identifies the first instance when the aggregate bank is completely depleted and t represents time. In practice, however, firms cannot perfectly predict the number of allowances they will require in the future and, consequently, the market equilibrium price of allowances becomes subject to uncertainty. Holding allowances and investments in abatement are no longer risk-free. The evolution of the allowance price during the banking period is now governed by the no-arbitrage condition $\mathbb{E}[dP_t]/P_t = \mu_t dt$, where μ_t includes the possibly time-dependent risk premium ([Ellerman and Montero, 2007](#)). In effect, allowance prices and, accordingly, the required return on abatement investments will respond to changes in firms' expectation about future allowance demand and supply during the banking period, the length of which depends in turn on these expectations.

In the analysis that follows, we explore the impact of an SMM on firms' abatement strategies using a model of the inter-temporal pollution control and allowance trading. We consider the inter-temporal optimisation problem of each entity in a continuum of small regulated firms. At each point in time, each firm has to decide by how much she wants to offset her individual emissions, considering current and future costs of reducing emissions, as well as her existing individual bank of allowances and future allowance demand and allocations. The chief decision state variable is the firm's expected required individual abatement, the difference between counterfactual emissions (individual cumulative emissions in the absence of emissions restrictions) and the number of allowances individually allocated. Every firm adjusts her abatement and trading strategies at each time t based upon this state variable, taking into account her current bank of allowances and any change in the required abatement. Under uncertainty, changes in firm's expectation about the required individual abatement affect how much individual abatement and banking will occur in the future – and for how long. We thus frame our analysis of the impact of a cap-preserving mechanism that amends the allowance allocation programme, similar to the 2015 version of the MSR, in terms of two main state variables: firms' expectations about the required abatement and the length of the banking period.

Previous studies have demonstrated that firms' strategy adjustments and the overall efficacy of the 2015 MSR are highly dependent on the constraints on temporal provisions (i.e. limitations on borrowing) and on the design of the mechanism implemented to adjust the allowance allocation, see [Salant \(2016\)](#), [Perino and Willner \(2016\)](#), [Fell \(2016\)](#). In the absence of borrowing constraints, abatement decisions are independent of the temporal distribution of allowances. If firms can always borrow from future allocations, any change to the allocation programme that maintains the overall emissions cap is irrelevant. Firms will simply borrow the required allowances needed to remain on their original cost-minimised emissions path and the adjustments of the allowance allocation programme will have no influence. Under borrowing constraints, a change in the allocation programme can affect abatement and allowance price paths only when the amount of allowances presently available to firms to cover emissions is insufficient. This is the availability condition in [Salant \(2016\)](#) or the feasibility condition in [Perino and Willner \(2016\)](#).

Our analytical results are consistent with these results: an SMM can only change abatement and allowance price paths if and only if the onset of the SMM changes the expected required abatement, i.e. the expected future net demand of allowances. Specifically, abatement strategies are unaltered when neither the expectation about the length of the banking period (τ) nor the post-SMM expected required abatement change. Conversely, when the adjustments in the allowance allocation programme determined by an SMM affect the expected required abatement, the expected length of the banking period τ and its distribution vary.³ When considering the impact of previously unexpected changes (e.g. demand shocks), we note that changes to the timing of allowance allocation can affect the instantaneous likelihood of the event of an instantaneous depletion of the bank. We term this *instantaneous breakdown*. That is to say, changes to the distribution of τ by an SMM can change the probability that firms are not able to compensate for a demand shock with their current individual bank of allowances. This is related to the discussion of price variability in [Perino and Willner's](#) analysis. They show that the short-term scarcity produced by the (binding) MSR can drive prices up and increase price volatility when allowances are removed from the market. Our findings support the conclusion that a cap-preserving supply control mechanism increases price variability overall.

Crucially, changes in price volatility due to an SMM are immaterial for risk neutral firms. Their abatement strategies solely depend on the expected required abatement. However, for risk-averse firms, differences in price volatility matter and should be reflected in the risk premium demanded by those firms for holding allowances or for investing in abatement. Thus, we expand our analysis to risk-averse firms and show that changes in the probabilistic distribution of τ brought on by an SMM that lead to higher price variability (compared to no-SMM) generate higher risk premia. The higher the risk premium, the more quickly firms will deplete their bank, which leads to lower levels of abatement and lower prices. However, abatement and allowance prices are affected to a lower extent during different periods of the bank. Thus, compared to the no-SMM case, the consequence of higher price variability are more compelling when regulated firms are not perfectly risk-neutral. This could have significant implications for the overall impact of an SMM like the MSR proposed in 2015. While one of the goals some stakeholders attributed to the original MSR was to increase prices during periods of over-supply, the building up of the allowance reserve by a cap-preserving mechanism would have the opposite effect. When the behaviour

³ We use the term distribution in the sense of a distribution of variables that are subject to uncertainty.

of risk-averse firms is taken into account, the impact of an SMM is more striking: the rise in price volatility would lead to higher risk premia, accelerated depletion of the bank and, consequently, abatement and prices are reduced even further. Cancellation of part of the reserve could partially outweigh the effect on risk premia and sustain allowance prices.

The remainder of the paper is organised as follows. In Sections 2.0 and 2.1 we describe the model assumptions and define the key decision making variables for each of the agents on the allowance market. In Section 2.2 we present the market equilibrium in terms of aggregate quantities and provide an analytical description of the conditions under which an SMM alters the emissions abatement paths. In Section 2.3 we relax the assumption of risk neutrality and explore the effect of an SMM on a time-dependent risk premium. Section 3 concludes.

2. The model: firms' pollution control problem

Regulated firms are assumed to be atomistic in a perfectly competitive market for emission allowances. Firms face an inter-temporal optimisation problem where, at each point in time, they have to decide how much they want to offset their emissions (either by abating or by trading allowances), considering the current and future costs of reducing emissions. Each firm accounts for her current individual bank of allowances and the number of allowances she expects to be allotted in the future. In this context, the *required abatement*, the difference between the cumulated individual amount of emissions without abatement requirements (counterfactual individual emissions) and their future allocation, is the key quantity each firm has to assess at each point in time. Under uncertainty, changes in a firm's expectation about the required abatement affect how much abatement and banking will occur in the future – and for how long. Crucially, the impact of these changes is relevant only during the banking period. Once the bank is depleted, the inter-temporal problem *breaks down*: each firm uses every allowance available to cover contemporaneous individual emissions and instantaneously abates her residual individual emissions (Schennach, 2000).⁴ Thus, we focus our analysis on the banking period $[0, \tau]$ and investigate under which conditions an SMM can alter the length of the banking period τ and its probabilistic distribution.

Firm i 's dynamic cost minimisation problem is⁵

$$\begin{aligned} \min_{\alpha^i, \beta^i} \mathbb{E} \left[\int_0^\tau e^{-rt} v^i(\alpha_t^i, \beta_t^i) dt \right], \\ \text{s.t. } B_t^i = B_0^i + A^i(0, t) - E^i(0, t) + \int_0^t \alpha_s^i ds - \int_0^t \beta_s^i ds, \\ B_t^i > 0, \quad \text{and} \quad B_\tau^i = 0. \end{aligned} \quad (1)$$

where r is the risk-free rate; v^i denotes the cost function; B_0^i represents the firm's initial individual bank of allowances; $A^i(0, t)$ represents the sum of allowances allocated to firm i from time 0 to t ; and $E^i(0, t)$ represents firm i 's pre-abatement cumulated emissions during the same period. With an SMM the allowance allocation programme changes, thus both individual allocation and emissions may be subject to uncertainty. Finally, let α_t^i denote instantaneous abatement and β_t^i be the number of allowances sold ($\beta_t^i > 0$) or bought ($\beta_t^i < 0$). Later we will assume a specific functional form for the cost function $v(\cdot)$ and provide equilibrium results in closed form.

2.1. Required abatement under uncertainty

To capture the impact of uncertainty on banking in a cap-and-trade programme under the SMM, we identify two key state variables of the system: the time- t expectations of (i) the instant τ when the aggregate bank is completely depleted and (ii) the corresponding required aggregate abatement, that is counterfactual emissions over $[0, \tau]$ minus the total number of allowances allocated in the same period (including the initial aggregate bank of allowances). When new information becomes available, firms update their expectations and adjust their strategies. That is, abatement and trading strategies are adapted at each time t , taking into account the current aggregate bank of allowances and the change in the required aggregate abatement.

We express the time- t expectation of the instant when the aggregate bank is completely depleted as $\mathbb{E}_t[\tau]$.⁶ The aggregate abatement required over the period $[0, \tau]$ is represented by $Y = Y(0, \tau)$; we refer to its expected value as $\mathbb{E}_t[Y]$. Finally, $d\mathbb{E}_t[Y]$ represents changes in expectations about the required aggregate abatement. These three expressions are key to understanding how abatement and allowance prices change when firms' expectations change during the banking period.

⁴ Schennach (2000) investigates the effect of a permanent costless reduction in SO₂ emissions in a deterministic framework and provides approximate solution when the model is solved under uncertainty. In this context, our contribution extends the efforts of Schennach (2000) by deriving an exact analytical solution of the market equilibrium under uncertainty. This allows us to derive the equilibrium under risk-aversion, as described in Appendix B.

⁵ The instant τ of full depletion of the aggregate bank (in equilibrium) transfers directly to the individual firm's optimisation problem in form of the constraint $B_\tau^i = 0$, since after τ the price increases at a lower rate than the rate of interest. Thus, there is no individual incentive to bank allowances beyond τ .

⁶ By convention, $\mathbb{E}_t[\cdot] = \mathbb{E}[\cdot | \mathcal{F}_t]$ represents the conditional expectation where \mathcal{F}_t indicates the information available at time t . We refer to the Appendix for an analytical specification of \mathcal{F}_t .

2.2. Equilibrium solution for risk-neutral firms

We now consider the optimisation problem in (1) and characterise the market equilibrium under risk-neutrality. In order to have an analytically tractable model, we assume a linear functional form for the marginal abatement cost curve, $AC'(\alpha^i) = \Pi_t + 2\varrho\alpha^i$, where Π_t and ϱ represent the intercept⁷ and the slope of the marginal cost curve, respectively. Firms can sell and buy allowances $|\beta_t^i|$ at a price P_t ; they face costs $TC(\beta^i)$ for each trade.⁸

Firm i 's instantaneous costs of reducing emissions via abatement and trading are thus given by

$$v_t^i(\alpha_t^i, \beta_t^i) = \Pi_t\alpha_t^i + \varrho \cdot (\alpha_t^i)^2 + TC(\beta_t^i).$$

In [Appendix A](#) we solve the optimisation problem in (1) and obtain the market equilibrium as a triple $(\{\alpha^i, \beta^i\}_{i \in I}, P, \tau)$, where $P = (P_t)_{0 \leq t \leq \tau}$ is the equilibrium price process and τ denotes the length of the banking period in equilibrium. In what follows, we present the relevant analytical results in aggregate terms.

In equilibrium, the aggregate abatement at time t is given by

$$\alpha_t = re^{rt} \frac{\mathbb{E}_0[Y]}{e^{r\tau(0)} - 1} + re^{rt} \int_0^t \frac{d\mathbb{E}_s[Y]}{e^{r\tau(s)} - e^{rs}}, \quad (2)$$

where for legibility we replace $\mathbb{E}_t[\tau]$ with $\tau(t)$ and $\mathbb{E}_t[Y] = \mathbb{E}_t[Y(\tau)]$.

The first term on the right hand side of [Eq. \(2\)](#) is the expected abatement given the information available at time 0 (we compute this below). The expected required aggregate abatement $\mathbb{E}_0[Y(\tau)]$ is spread over the banking period and increases at the rate r . At each time t , new information about the future required aggregate abatement becomes available and adjustments in the equilibrium aggregate abatement may occur. This is represented by the second term on the right hand side of [Eq. \(2\)](#). When the expectation about the future required aggregate abatement changes, the corresponding adjustment $d\mathbb{E}_s[Y]$ is spread over the remainder of the banking period.⁹

In the following discussion we investigate the impact of changes in the expected required aggregate abatement and, ultimately, how an SMM affects the abatement and allowance price paths. We begin by considering the expected aggregate abatement path, computed at time $t = 0$ which provides a *static view* of the model results. The time-0 expectations of $d\mathbb{E}_t[Y]$ are all zero; hence the second summand of [Eq. \(2\)](#) vanishes when considering the time-0 expectation.¹⁰ Thus, we obtain

$$\mathbb{E}_0[\alpha_t] = re^{rt} \frac{\mathbb{E}_0[Y(\tau)]}{e^{r\tau} - 1}. \quad (3)$$

From this expression we can see that if (1) neither the time-0 expectation of τ (2) nor the time-0 expected required aggregate abatement $\mathbb{E}_0[Y(\tau)]$ change, the time-0 expected abatement is the same. Basically, the SMM has no effect on aggregate abatement and allowance prices solely when (post-SMM adjustment) firms' expectations about the future *net* demand of allowances do not change. Crucially, the SMM does not change the total number of allocated allowances, but changes the timing of the allocation of allowances. The conditions (1) and (2) mentioned above correspond to the no-violation of the availability condition in ([Salant, 2016](#)) or the feasibility condition in ([Perino and Willner, 2016](#)). The interpretation of these conditions is in line with previous studies on the impact of the 2015 version of the MSR.

We now consider the impact of *previously unexpected* changes to the required aggregate abatement and provide a *dynamic view* of our results. We investigate the impact of previously unexpected changes by looking at allowance prices (aggregate marginal abatement costs). [Eq. \(2\)](#) immediately yield the equilibrium price process

$$P_t = \Pi_t + 2\varrho\alpha_t = \Pi_t + 2\varrho re^{rt} \frac{\mathbb{E}_0[Y]}{e^{r\tau(0)} - 1} + 2\varrho re^{rt} \int_0^t \frac{d\mathbb{E}_s[Y]}{e^{r\tau(s)} - e^{rs}},$$

where price variability is generated by unanticipated changes to the required aggregate abatement $d\mathbb{E}_s[Y]$. Changes in the expected required aggregate abatement $d\mathbb{E}_s[Y]$ change the expected duration of the banking period too. Thus, the joint effect of possible changes $d\mathbb{E}_s[Y]$ and $\tau(s)$ determines the volatility of prices. We will see that this joint effect is in fact subject to changes in the programme of the allowance supply, such as the one introduced by the 2015 MSR. Such an effect has been explored in terms of a single random shock by ([Perino and Willner, 2016](#)). They conclude that such a mechanism increases price volatility when the shock occurs in the period when the reserve is building up.

The following analysis extends the efforts of these authors by studying how a cap-preserving supply control mechanism affects price volatility and – under risk aversion – the risk premium associated to the instant when firms prefer to deploy

⁷ The intercept Π_t is assumed to increase at the risk-free rate r .

⁸ In addition to the cost βP_t when buying (negative cost when selling) $|\beta|$ allowances, firms might face non-negligible transaction costs per trade. Among others, ([Frino et al., 2010](#)) and ([Medina et al., 2014](#)) document non-negligible transaction costs in the EU ETS. In our framework, we assume linear marginal trading costs, $TC'(\beta) = P_t - 2\nu\beta$. This ensures uniqueness of the equilibrium and allows us to derive the equilibrium in closed form. In aggregate terms, however, the equilibrium results are not affected by the level of ν and prevail for $\nu = 0$. We thereby consider negligible transaction costs, as is typically assumed in the environmental economics literature. Note that the impact of a specific distribution of firms' characteristics across their continuum can be studied by the individual strategies provided in our model results. However, this is not the focus of the present paper and is left for future research.

⁹ ([Ellerman and Montero, 2007](#)) investigate how the level of reversibility in abatement decisions affect these adjustments (abatement corrections) in current abatement.

¹⁰ This is a direct consequence of the tower property of the conditional expectation.

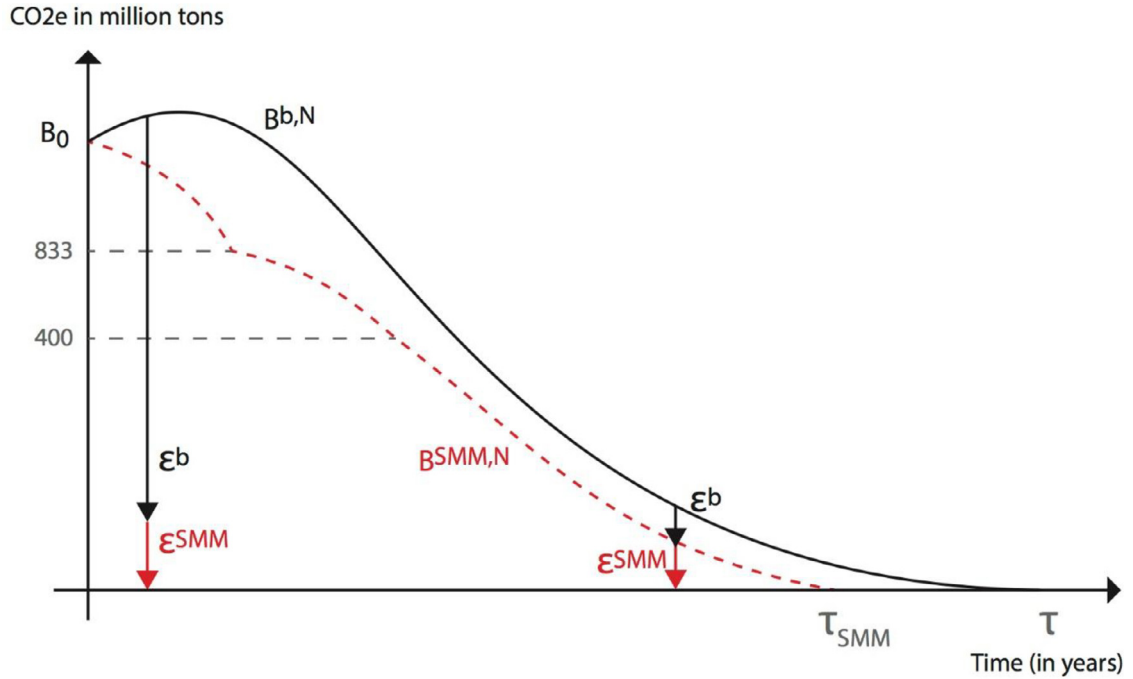


Fig. 1. The aggregate bank under risk-neutrality without the SMM (black line) and with the SMM (red dotted line) when the mechanism alters the expected banking period. With an SMM, shocks ε that could be absorbed by the bank, may lead to a higher likelihood of an unexpected breakdown. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

their bank. The rationale is the following: under risk aversion the impact of an SMM on price volatility is reflected in the risk premium and, consequently, in firms’ discount rate. The latter signals whether returns from allowance-related investments should promise higher or lower returns with consequent effects on allowance banking.

2.3. Changes in expectations: a dynamic view and risk-aversion

We now investigate how abatement and allowance prices respond to changes in time- t expectations. Recall that the required aggregate abatement Y represents counterfactual aggregate emissions over $[0, \tau]$ minus the total number of allowances allocated in the same period (including the initial aggregate bank of allowances). Recall also that at τ firms expect future allowance prices to increase at a rate lower than the prevailing market interest rate, so they prefer to use their bank to offset their emissions or, equivalently, to borrow allowances. Alternatively, and following the argument in Ellerman and Montero (2007), the full bank deployment occurs when the expected returns on allowances (and equivalently abatement investments) is deemed insufficient. The rest of the paper is devoted to the investigation of the impact of an SMM on firms’ expectations and, ultimately, on the evolution of the bank. With an SMM, changes in time- t expectations about the required aggregate abatement, $d\mathbb{E}_t[Y]$, will yield one of two scenarios. First, if the change in expected future net demand of allowances is such that allowance prices will continue to grow according to the no-arbitrage condition, then firms will prefer to continue to bank allowances and the time- t aggregate bank remains positive, $t < \mathbb{E}_t[\tau]$. If, however, the change in $d\mathbb{E}_t[Y]$ leads to the opposite situation, then firms will prefer to deploy their bank, $t = \tau$. We term this scenario *instantaneous breakdown*. Fig. 1 illustrates the case where the change in the expected required aggregate abatement brought on by the SMM alters the expected banking period. The aggregate bank with the SMM is weakly below the aggregate bank without the SMM (red and black line, respectively). Below we explore how an SMM influences the likelihood of this scenario and what conclusions we can draw in terms of policy implications about allowance cancellation.

We model time- t changes in expectations about the required abatement as $d\mathbb{E}_t[Y] = \sigma_t \sqrt{(\tau - t)} z_t$ where z_t are independent standard Gaussian shocks and $\sigma_t^2 \cdot (\tau - t)$ is the variance of $d\mathbb{E}_t[Y]$. The term $(\tau - t)$ captures a natural assumption: uncertainty about *cumulated* aggregate emissions, i.e. aggregate emissions over $[0, t]$, diminishes as time goes by and we approach τ . The term σ_t , on the other hand, represents the variance of unexpected changes to τ , which may be subject to the changes to the allocation programme as described further below.

At its inception, the SMM withholds allowances from auctions and places them in a dedicated allowance reserve – as long as the aggregate bank stays above a given upper threshold. Therefore, the level of the aggregate bank of allowances decreases when the reserve is building up. The smaller the aggregate bank, the larger the likelihood of an instantaneous breakdown. Later, allowances from the reserve are made available, adding to the aggregate bank. We capture these changes to the

likelihood of an instantaneous breakdown by modelling σ_t as a function of the current aggregate bank, $\sigma_t = \sigma(B_t) > 0$, where $\frac{\partial \sigma}{\partial B_t} < 0$. In effect, without an SMM, the likelihood of a breakdown would be smaller, in particular when the reserve is building up. As such, the SMM increases the risk associated to the expected abatement requirement, especially in the short run.

How do firms' abatement strategies change when adjustments in the allowance allocation programme determined by the SMM affect the time- t variance of τ ? In order to understand the implications of this effect, we consider an extension of the modelling framework where changes in the variance of τ are properly reflected in firms' abatement strategies. Thus, we relax the risk neutrality assumption and model risk averse firms who demand a risk premium against the risks associated to holding allowances and equivalent abatement investments. Modelling changes in the risk premium – in response to changes in the variance of τ – allow us to deepen our understanding of firms' reactions to a cap-preserving mechanism and, later, comment on the likely effect of allowances cancellation. With risk-aversion, firm i 's dynamic cost minimisation problem is

$$\min_{\alpha_t^i, \beta_t^i} \mathbb{E} \left[\int_0^\tau e^{-\mu_t \cdot t} v^j(\alpha_t^i, \beta_t^i) dt \right], \tag{4}$$

where the discount rate $\mu_t = r + q_t$ includes the risk-free rate r and a (time-dependent) risk-premium q_t .¹¹ Allowances (and related low-carbon investments) are perceived as risky investments and are discounted accordingly at the rate μ_t . If alternative investments promise higher returns (discounted according to their respective riskiness), firms would prefer to postpone abatement and use their bank of allowances to offset emissions. In turn, lower abatement levels will be reflected in lower prices. Intuitively, a high discount rate due to a positive risk premium q_t , or equivalently a risk-adjusted discount rate μ_t substantially higher than r , should yield the following market response: lower level of aggregate abatement and, consequently, lower aggregate bank and allowance prices. Similarly to the case of an instantaneous regulatory change in the gold market modelled by [Salant and Henderson \(1978\)](#), risk-neutral firms who face the possibility of an instantaneous breakdown should require the price of allowances to rise by more than the risk-free rate in order for them to hold allowances in the face of the possible losses. [Fell \(2016\)](#) and [Ellerman et al. \(2015\)](#) obtain similar market responses when studying the sensitivity analysis of the discount rate used in the cost minimisation problem.

As described in more detail in [Appendix B](#), we use our equilibrium results to analytically characterise the level of aggregate abatement under risk-aversion α_t^A and compare it to abatement under risk-neutrality α_t^N . We obtain the following identities:

$$\alpha_t^N - \alpha_t^A = \frac{q_t}{2Q} \frac{e^{r\tau^N} - e^{rt}}{r} e^{-rt} P_t^N > 0 \quad \text{for } t < \tau^A(t) \tag{5}$$

and

$$P_t^N - P_t^A = q_t \frac{e^{r\tau^N} - e^{rt}}{r} e^{-rt} P_t^N > 0 \quad \text{for } t < \tau^A(t),$$

where $\tau^A(t)$ denotes the expected instant when the aggregate bank is completely depleted under risk-aversion; and P_t^N and P_t^A denote the allowance price under risk-neutrality and risk-aversion, respectively. As expected, aggregate abatement under risk-aversion is strictly smaller than under risk-neutrality for $t < \tau^A$ and, consequently, the aggregate bank is depleted more quickly, $\tau^A < \tau^N$.

We now explore what drives the difference $\alpha_t^N - \alpha_t^A$ and examine how the SMM affects abatement under risk-aversion. As time goes by and we approach the instant τ^A (when the non-borrowing constraint becomes binding, uncertainty about the required aggregate abatement is gradually reduced, making holding allowances and abatement investments less risky. Equally, as time goes by, the risk-premium q_t demanded by risk averse firms decreases. The risk premium enters linearly in the expression of the difference ($\alpha_t^N - \alpha_t^A$) and is multiplied by the term ($e^{r\tau^N} - e^{rt}$). This last term determines the influence of q_t on abatement and decreases in time as well. In words, the more time was left until the expected τ (the instant when allowance prices increase at a rate lower than μ_t and firms prefer to borrow allowances), the larger the potential losses associated to abatement investments. Conversely, the closer the expected τ , the smaller the potential losses. The value of abatement investments that is at risk hence decreases in time and consequently, the impact of risk-aversion on firms' strategies diminishes when approaching the expected τ . Since $e^{-rt} P_t$ is constant in expectation, the two expected abatement paths converge exponentially, when approaching the end of the banking period, as illustrated in [Fig. 2](#).

We now turn to the effect of changes to the allowance allocation by the SMM. [Fig. 3](#) illustrates the aggregate bank under risk-aversion with and without the SMM (red and blue line, respectively). The solid black line represents the aggregate bank without the SMM, when firms are risk-neutral, $q_t = 0$. As discussed earlier, the SMM adjusts the allowance allocation programme by initially removing allowances from the market and then returning them to the market. Accordingly, we model the volatility parameter $\sigma_t(g_t)$ as a function of the allowance allocation g_t at time t . The allocation of allowances decreases when an SMM removes allowances and increases when an SMM returns allowances to the market. In line with our previous discussion, an increase in g_t has a negative effect on volatility, $\frac{\partial \sigma_t(g_t)}{\partial g_t} < 0$. In order to examine how the changes in allowance

¹¹ There is growing empirical evidence of time-dependent risk premia, see [Gagliardini et al. \(2016\)](#) and references therein. In the financial econometric literature, moves in risk premia are often ascribed to changes in volatility or risk aversion.

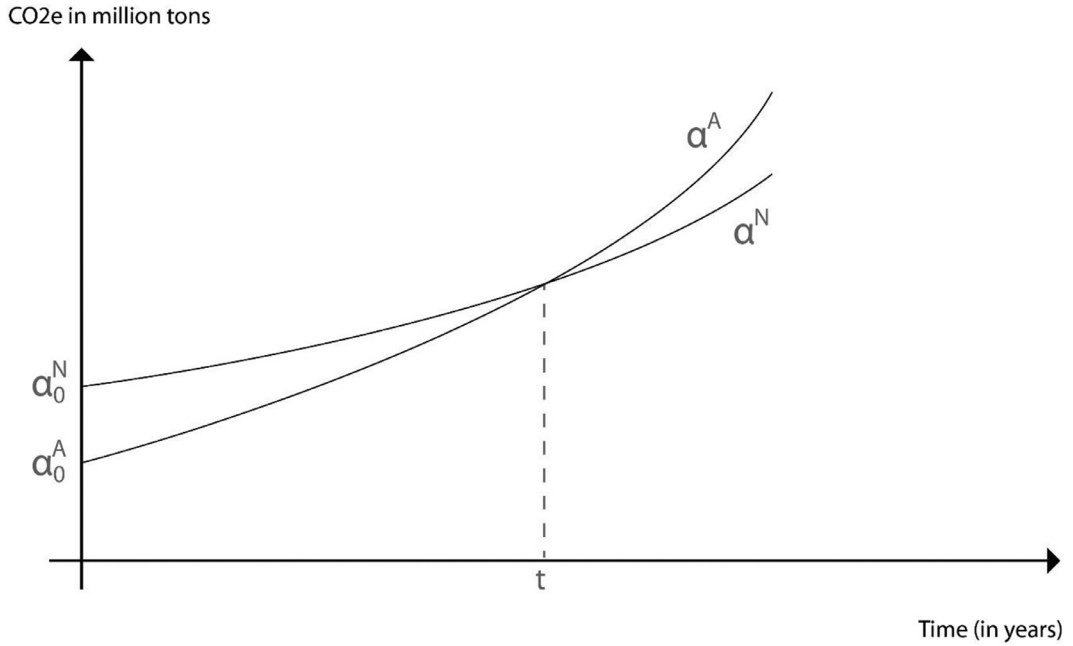


Fig. 2. Abatement curves for risk-neutral (α^N) and risk-averse firms (α^A). Total abatement under risk-aversion is smaller over the period $[0, \tau^A]$, where τ^A represents the end of the banking period under risk-aversion. The two abatement curves converge as time goes by and would intersect at time t . This event, however, will never be observed since under risk-aversion, the inter-temporal problem breaks down at time $t = \tau^A < \tau^N$.

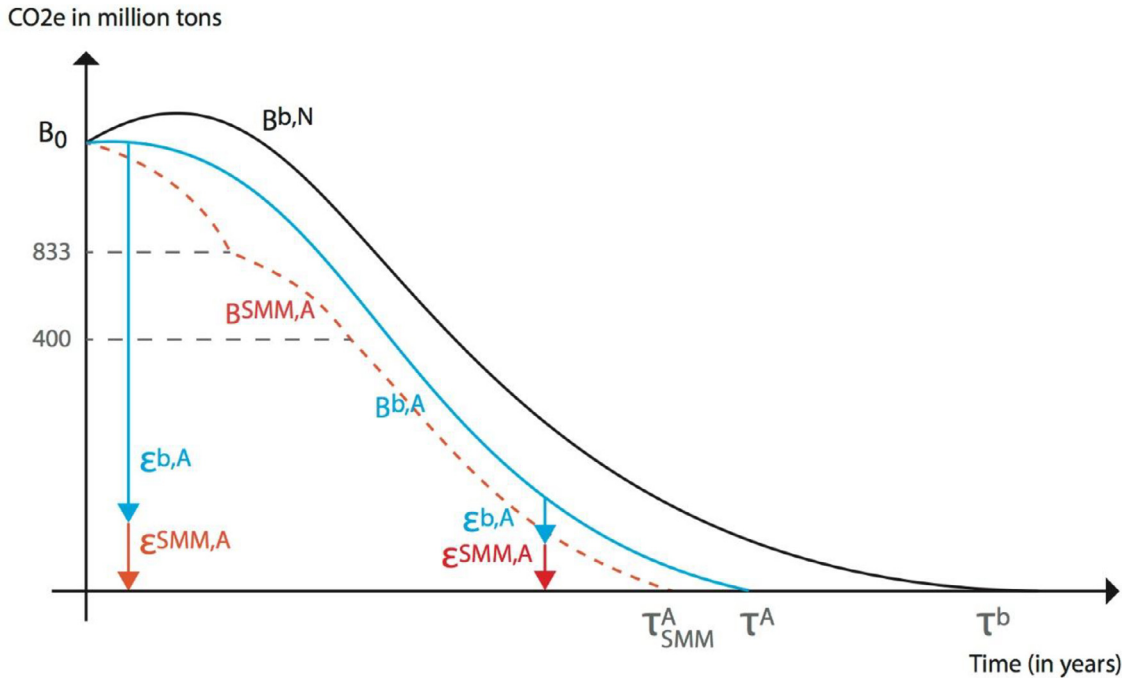


Fig. 3. The aggregate bank without an SMM under risk-neutrality (black line) and under risk-aversion (blue line); aggregate bank with the SMM under risk-aversion (red dotted line). The SMM decreases the aggregate bank in the short run and adds to it in the long run, when the reserve is re-injected. As in the risk-neutral case, the overall likelihood of an instantaneous breakdown is increased. However, the effect of risk-aversion on the slope of banking (dictated by α^A) decreases over time and hence short-term effects are amplified compared to long-term effects. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

allocation of the SMM affect abatement decisions under risk-aversion, we first consider the risk-premium q_t . As shown in Appendix B, the rate of change of q_t with respect to g_t is:

$$\frac{\frac{\partial q_t(g_t)}{\partial g_t}}{q_t} = \frac{\frac{\partial \sigma_t(g_t)}{\partial g_t}}{\sigma_t} \quad (6)$$

During the building up of the allowance reserve, the change in allowance allocation increases the likelihood of an instantaneous breakdown. This is reflected in an adjustment of the risk-premium q_t . More precisely, Eq. (6) reveals that a change in g_t generates a change in σ_t which equally transfers to a change in q_t . Consequently, risk-averse firms adjust their abatement behaviour, as quantified in Eq. (5). As previously discussed, the impact of q_t on abatement is larger in the short term, when the expected end to the banking period lies in the distant future and the SMM is removing allowances from the market. Thus, in the short run the potential losses associated to abatement investments due to an instantaneous breakdown are high. As time goes by, the expected time τ of complete depletion of the aggregate bank approaches and allowances from the reserve are released. As we can see from Eq. (5), the reduction in q_t is determined by the combined effect. Abatement and allowance prices increase, but to a lower extent than both were decreased earlier during the building up of the reserve.

In conclusion, we find that under risk aversion, the changes brought on by the SMM to the distribution of the instant when firms prefer to use their entire aggregate bank, lead to higher price variability – compared to the risk-neutral case – and, consequently, higher risk premia. However, we note that the impact of a change in the likelihood of an instantaneous breakdown affects prices and abatement more in the short run than it does in the long run. These findings corroborate concerns about rising price volatility, as raised by Perino and Willner (2016) and Fell (2016).

The conceptual framework proposed here makes it possible to readily describe also the impact of various policy changes that are particularly relevant to the EU ETS market. Most crucially, the amendment of the MSR that allows the mechanism to permanently remove allowances, thus changing the cap. Recall that the modelled cap-preserving supply management mechanism first places allowances in the reserve and later returns them all to the market.¹² Intuitively, a permanent reduction in the cap corresponds to a permanent positive shock to the expected required aggregate abatement. Keeping counterfactual emissions fixed, this would result in a downward shift in $\mathbb{E}_t[Y]$, higher abatement and higher (future) prices, ultimately reducing the risk premium required for holding allowances or investing in abatement.

3. Conclusions

The supply of allowances in the European Union Emissions Trading System (EU ETS) has been inflexible and determined within a rigid allocation programme. As such, the system lacked provisions to address severe imbalances in demand and supply of allowances resulting from economic shocks. In 2015 the European Commission proposed a structural reform of the EU ETS, including the implementation of a Market Stability Reserve (MSR), operative since 2019. The MSR will adjust the allowance allocation programme based on the aggregate bank of allowances: In times of a large bank, allowances are transferred to a dedicated reserve to be released in times of scarcity. In its original 2015 design, the MSR preserves the total number of allowances issued over the regulatory phase. After two years of negotiations and an extensive impact assessment, the European Commission decided that allowances held in the reserve above the previous year's auction volume will no longer be valid. The findings of our work support the decision for regular cancellation of excess allowances.

We develop a stochastic equilibrium model of inter-temporal trading of emission allowances to investigate under which conditions a supply management mechanism (SMM) similar to that proposed for the EU ETS can alter allowance price and emissions abatement paths. Similar mechanisms were adopted in California and South Korea. We show that the timing of allocation is largely irrelevant as long as changes in expected net demand of allowances are such that the resulting bank remains essentially unaltered. Conversely, when the transitory scarcity brought on by the SMM changes the net allowance demand, the mechanism affects the expected abatement and the price paths. In this context, we consider unexpected changes in firms' expectations that triggers an instantaneous depletion of the bank of allowances (what we termed unexpected breakdown).

Risk neutral firms are indifferent to changes in the variability of this event. However, when firms account for the risk in the change of the variability of their future required abatement – i.e. counterfactual emissions minus total number of allowances allocated over the same period – and equivalently, risk in the variability of the value of their abatement investments, adjustments in the allowance allocation programme matter. We then expand our analysis to study how risk-averse firms' strategies are affected by an SMM at different points in time of the banking period. We show that changes in the distribution of the time of the unexpected breakdown brought on by the SMM lead to higher price variability and, consequently, higher risk premia. The higher the risk premium associated with holding allowances, the more quickly firms will deplete their bank, which is associated with lower levels of abatement and, importantly, lower allowance prices.

This has clear policy implications for the current debate on cap adjustments vs. no-cap adjustments: the influence of a generalized, cap-preserving supply management mechanism like the 2015 version of the MSR could be counter-productive,

¹² In practice, under the original 2015 MSR it was possible that the rate at which allowances from the reserve were released was insufficient to cover present emissions. Under this scenario, some allowances might be in the reserve at τ . This possibility means that the modelled SMM is not allowance-preserving as the mechanism modelled in Perino and Willner (2016).

especially when the behaviour of risk-averse firms is considered. Importantly, while increased price variability in the short run may prevail even under the amended MSR, (the anticipation of) a permanent cancellation of part of the reserve will, at the very least, lead to lower risk of low-carbon investments (such as purchase of allowances) and, accordingly, higher prices in the short run with lower but less risky long-run returns. The late 2018 increase in allowances prices to almost three times its value since the amendment of the MSR may well be attributed to a market perception of such decreased risk.

Appendix

In the following sections, we provide the derivations of the key results.

Appendix A. The model under risk-neutrality

We consider an inter-temporal optimisation problem where, at each point in time, every firm has to decide how much she wants to offset her emissions (either by abating or by trading allowances), considering the current and future costs of reducing emissions. We model the banking period $[0, \tau]$ of a secondary emissions allowances market in a partial equilibrium framework under perfect competition. Firms are assumed to be atomistic, that is, firm $i \in I$ individual quantities x^i are continuously distributed under a measure m^x such that aggregate quantities can be obtained by integration, $x = x^I = \int_I x^i dm^x(i)$. Each firm continuously minimises expected individual abatement and trading costs at each point in time $t \in [0, \tau^i]$, where τ^i denotes the first instance when the individual bank B^i is completely depleted,

$$\tau^i = \min\{t \geq 0, B_t^i = 0\}$$

and her instantaneous cost function is given by

$$v^i(\alpha_t^i, \beta_t^i) = \Pi_t \alpha_t^i + \varrho \cdot (\alpha_t^i)^2 - P_t \beta_t^i + \nu \cdot (\beta_t^i)^2.$$

That is, each firm has the same marginal abatement cost curve $\Pi_t + 2\varrho\alpha_t^i$, where we assume that the intercept Π_t increases by the risk-free rate r and $\varrho > 0$ is constant. Firms face non-negligible transaction costs per trade. More specifically, we assume marginal trading costs to be linear in the number of allowances sold ($\beta_t^i > 0$) or bought ($\beta_t^i < 0$). The parameter ν represents the magnitude of transaction costs and P_t denotes the time- t allowance price. We define $\tau = \tau^I$ as the first time when the aggregate bank of allowances is completely depleted,

$$\tau = \min\{t \geq 0, B = B^I = 0\}.$$

By definition of τ , there is no incentive for the aggregate market in equilibrium to hold allowances beyond the end of the banking period. Therefore, at time $t > \tau$ the incentive to hold allowances vanishes and allowance prices increase at a rate less than the risk-free rate. Since firms are atomistic and face the same marginal abatement cost function, it is always suboptimal for each individual firm to hold allowances beyond the time τ . We represent this by the requirement

$$\tau^i = \tau, \quad \text{i.e.} \quad B_\tau^i = 0.$$

For convenience, we write $\varpi_t^i = \mathbb{E}_t[Y^i(t, \tau)]$, where $Y^i(t, \tau)$ is the time- t residual offsetting requirement:

$$Y^i(t, \tau) = Y^i(0, \tau) - \int_0^t \alpha_s^i ds + \int_0^t \beta_s^i ds.$$

We note that

$$dE_t[Y^i(t, \tau)] = (\beta_t^i - \alpha_t^i)dt + dE_t[Y^i(0, \tau)].$$

Furthermore, for $t = \tau$ we have

$$B_\tau^i = -Y^i(\tau, \tau).$$

Hence, we can replace the requirement $B_\tau = B_{\tau,i} = 0$ with the constraint $Y_\tau^i = 0$.

The equilibrium consists of abatement- and trading strategies α_t^i and β_t^i for each firm i , the market clearing price process P_t and the equilibrium length (duration) τ of the banking period. In equilibrium, individual deviations from the equilibrium do not yield expected additional cost savings for any firm. The market is assumed to be free of arbitrage and complete. We can therefore postulate the existence of a martingale measure \mathbb{Q} that is equivalent to the real-world measure \mathbb{P} . We first assume that firms are risk-neutral. Accordingly, all expectations in this section are taken under the measure \mathbb{Q} . In [Appendix B](#), we transfer our results to risk-averse firms by deriving the change of measure from \mathbb{Q} to \mathbb{P} .

We begin by assuming Markovian strategies $\alpha^j = \alpha(Z_t^j)$, $\beta^j = \beta(Z_t^j)$ for every firm $j \in I \setminus \{i\}$ except for i . These strategies are given as functions of each firm's individual state processes Z_t^j , which will be specified later. We show that it is optimal for firm i to replicate the other firms' strategies given below, as a function of her own state process Z_t^i . For convenience, we

define

$$h_t = \frac{re^{rt}}{e^{r\tau(t)} - e^{rt}}, \quad \text{where } \tau(t) = \mathbb{E}_t[\tau].$$

For each firm $j \in I \setminus \{i\}$, let her abatement and trading strategies be given by

$$\alpha_t^j = \frac{P_t - \Pi_t}{2(\nu + \varrho)} + \frac{\nu}{\nu + \varrho} h_t \varpi_t^j \quad \text{and} \quad \beta_t^j = \frac{P_t - \Pi_t}{2(\nu + \varrho)} - \frac{\varrho}{\nu + \varrho} h_t \varpi_t^j.$$

The market clearing condition $\beta^j = 0$ yields

$$P_t = \Pi_t + 2\varrho h_t \varpi_t^j. \tag{7}$$

Substituting for the strategies α_t^j, β_t^j above, we obtain the dynamics for the process ϖ_t^j :

$$d\varpi_t^j = (\beta_t^j - \alpha_t^j) dt + d\mathbb{E}_t[Y^j(0, \tau)] = -\frac{re^{rt}}{e^{r\tau(t)} - e^{rt}} \varpi_t^j dt + d\mathbb{E}_t[Y^j(0, \tau)].$$

Solving the above, we obtain:

$$\varpi_t^j = \varpi_0^j \frac{e^{r\tau(0)} - e^{rt}}{e^{r\tau(0)} - 1} + (e^{r\tau(t)} - e^{rt}) \int_0^t \frac{d\mathbb{E}_t[Y^j(0, \tau)]}{e^{r\tau(s)} - e^{rs}}.$$

Integrating over I yields, together with Eq. (7) that

$$P_t = \Pi_t + 2\varrho h_t \varpi_t^I = \Pi_t + 2\varrho \frac{re^{rt}}{e^{r\tau(0)} - 1} \varpi_0^I + 2\varrho re^{rt} \int_0^t \frac{d\mathbb{E}_t[Y^I(0, \tau)]}{e^{r\tau(s)} - e^{rs}}.$$

In particular, we observe that P has the following dynamics

$$dP_t = rP_t dt + 2\varrho h_t d\mathbb{E}_t[Y^I(0, \tau)].$$

Let the random shocks to $\mathbb{E}_t[Y^I(0, \tau)]$ be governed by a driftless diffusion

$$d\mathbb{E}_t[Y^I(0, \tau)] = \sigma_t^I dW_t^{\mathbb{Q}},$$

where σ_t^I is deterministic and $W_t^{\mathbb{Q}}$ is a Brownian motion under the measure \mathbb{Q} .

The process σ_t^I describes how changes in the expected future net-supply of allowances are distributed across the set of firms I . We abstract from specific assumptions about the form of σ_t^I . However, we note that it is reasonable to assume different σ_t^i for different firms, since pre-abatement emissions levels and allowances allocations can vary depending on the individual firm and the type of industry under consideration.

We consider changes in pre-abatement individual allowances demand and in the (possibly contingent) supply of allowances. Their degree of impact on firms can vary. However, all firms are subject to systemic shocks. Hence, we consider the same Brownian motion $W_t^{\mathbb{Q}}$ for each $i \in I$, whereas differences in size, technology etc. are captured by the distribution of σ_t^i across I . Accordingly, shocks to $\mathbb{E}_t[Y^i(0, \tau)]$ are represented by

$$d\mathbb{E}_t[Y^i(0, \tau)] = \sigma_t^i dW_t^{\mathbb{Q}}.$$

We now consider the problem of optimal pollution control and allowance trading for firm i . Let p denote an observed allowance price and let $P^{t,p}$ denote the price process with time- t value $P_t^{t,p} = p$. Analogously, let $\Pi_t^{t,\pi} = \pi$. At time t , the firm i has to bear costs v_t^i given by

$$v_t^i(\alpha_t^i, \beta_t^i) = \Pi_t^{t,\pi} \alpha_t^i + \varrho \cdot (\alpha_t^i)^2 - P_t^{t,p} \beta_t^i + \nu \cdot (\beta_t^i)^2.$$

Firm i 's problem is to find (Markovian) abatement- and trading strategies α^i and β^i respectively, such that, for all $t \in [0, \tau)$ the cost function J , given by

$$J(t, \varpi^i, p, \pi, \alpha^i, \beta^i) = \mathbb{E} \left[\int_t^\tau e^{-rs} v_s^i(\alpha_s^i, \beta_s^i) ds \right],$$

is minimised by α^i, β^i for all $\pi > 0, p \geq 0$, and such that the constraint $\mathbb{E}_t[\varpi_t^i] = 0$ is satisfied for all $t \in [0, \tau)$. Let $w(t, \varpi^i, p, \pi) = \inf_{(\alpha^i, \beta^i)} J(t, \varpi^i, p, \pi, \alpha^i, \beta^i)$ denote the value function for firm i .

Firm i 's observes the state process $Z_t^i = (\varpi_t^i, P_t, \Pi_t)$, where

$$\begin{aligned} d\varpi_t^i &= (\beta_t^i - \alpha_t^i) dt + d\mathbb{E}_t[Y^i(0, \tau)] = (\beta_t^i - \alpha_t^i) dt + \sigma_t^i dW_t^{\mathbb{Q}}, \\ dP_t &= rP_t dt + 2\varrho h_t d\mathbb{E}_t[Y^I(0, \tau)] = rP_t dt + 2\varrho h_t \sigma_t^I dW_t^{\mathbb{Q}}, \\ d\Pi_t &= r\Pi_t dt. \end{aligned}$$

Let the firm's filtration $(\mathcal{F}_t^i)_{t \geq 0}$ be generated by the process Z^i and accordingly, let (\mathcal{F}_t^I) , generated by Z^I , denote the aggregate filtration. The Hamilton–Jacobi–Bellman (HJB) equation associated to the minimisation problem above is given by

$$\begin{aligned} 0 &= D_t w + \inf_{a,b} \left[\mathcal{L}^{(a,b)} w + e^{-rt} v_t^i(a, b) \right] \\ &= D_t w + \inf_{a,b} \left[(b-a) D_{\varpi^i} w + r p D_p w + r \pi D_\pi w + \frac{1}{2} \text{tr}(\Sigma \Sigma' D_z^2 w) + e^{-rt} (\pi a + \varrho a^2 - p b + v b^2) \right] \\ &= D_t w + r p D_p w + r \pi D_\pi w + \frac{1}{2} \text{tr}(\Sigma \Sigma' D_z^2 w) + \inf_{a,b} \left[(b-a) D_{\varpi^i} w + e^{-rt} (\pi a + \varrho a^2 - p b + v b^2) \right], \end{aligned}$$

where Σ is the vector

$$\Sigma = \begin{pmatrix} \sigma_t^i \\ 2\varrho h_t \sigma_t^i \end{pmatrix}$$

which implies that

$$\text{tr}(\Sigma \Sigma' D_z^2 w) = (\sigma_t^i)^2 D_{\varpi^i}^2 w + 2\varrho h_t \sigma_t^i \sigma_t^i D_p D_{\varpi^i} w + 2\varrho h_t \sigma_t^i \sigma_t^i D_{\varpi^i} D_p w + 4\varrho^2 h_t^2 (\sigma_t^i)^2 D_p^2 w.$$

We notice that the minimisers a, b in the above equation have to satisfy

$$a = \frac{1}{2\varrho} (e^{rt} D_{\varpi^i} w - \pi) \quad \text{and} \quad b = \frac{1}{2v} (p - e^{rt} D_{\varpi^i} w). \quad (8)$$

Furthermore, we notice that the second-order condition is satisfied for all a, b . This yields the following

Lemma 1. *The HJB equation can be rewritten as*

$$0 = e^{rt} (D_t w + r p D_p w + r \pi D_\pi w) + \frac{e^{rt}}{2} \text{tr}(\Sigma \Sigma' D_z^2 w) - \frac{1}{4\varrho} (e^{rt} D_{\varpi^i} w - \pi)^2 - \frac{1}{4v} (p - e^{rt} D_{\varpi^i} w)^2. \quad (9)$$

In order to enforce the constraint $\mathbb{E}_t[\varpi_\tau^i] = 0$ for all t , we impose the singular terminal condition

$$\lim_{t \nearrow \tau} w(t, \varpi^i, p, \pi) = \begin{cases} 0 & : \varpi^i = 0, \\ \infty & : \varpi^i \neq 0. \end{cases} \quad (10)$$

Theorem 1. *The HJB Eq. (9), together with the terminal condition (10) is solved by*

$$w(t, \varpi^i, p, \pi) = \frac{rv\varrho(\varpi^i)^2}{(e^{r\tau} - e^{rt})(v + \varrho)} + e^{-rt} \left(\pi + \frac{\varrho(p - \pi)}{v + \varrho} \right) \varpi^i + \frac{(1 - e^{r(\tau-t)})p^2}{4re^{rt}(v + \varrho)} + \int_t^\tau C_s ds$$

where

$$C_s = \frac{rv\varrho(\sigma_s^i)^2}{(e^{r\tau} - e^{rs})(v + \varrho)} + \frac{2\varrho^2 h_s \sigma_s^i \sigma_s^i e^{-rs}}{v + \varrho} + \frac{\varrho^2 h_s^2 (\sigma_s^i)^2 (1 - e^{r(\tau-s)})}{re^{rs}(v + \varrho)} \quad \text{for } t \leq s < \tau.$$

The above theorem can be proved by simple differentiation. The verification argument for w is straightforward but lengthy. Thus, we omit the full proof. We note that standard arguments of verification confirm α^i, β^i as the firm's optimal strategies. Substituting $D_{\varpi^i} w$ in Eq. (8) yields

$$\alpha_t^i = \frac{P_t - \Pi_t}{2(v + \varrho)} + \frac{v}{v + \varrho} h_t \varpi_t^i \quad \text{and} \quad \beta_t^i = \frac{P_t - \Pi_t}{2(v + \varrho)} - \frac{\varrho}{v + \varrho} h_t \varpi_t^i.$$

This proves the equilibrium strategies α^i, β^i to be given as above for all $i \in I$. Furthermore, the aggregate abatement is given by

$$\alpha_t = re^{rt} \frac{\varpi_0^I}{e^{r\tau(0)} - 1} + re^{rt} \int_0^t \frac{d\mathbb{E}_s[Y^I]}{e^{r\tau(s)} - e^{rs}}$$

and, accordingly, the market-clearing price process is given by

$$P_t = \Pi_t + 2\varrho \frac{re^{rt}}{e^{r\tau(0)} - 1} \varpi_0^I + 2\varrho re^{rt} \int_0^t \frac{d\mathbb{E}_s[Y^I]}{e^{r\tau(s)} - e^{rs}}.$$

Let ϵ_t^I and g_t^I denote time- t aggregate emissions before abatement and aggregate allocations, respectively. At τ , the intertemporal optimisation problem breaks down and thus $\alpha_\tau^I = \epsilon_\tau^I - g_\tau^I$. Also, by the definition of τ and the zero-borrowing constraint, we have that $\min_t B_t^I = B_\tau^I = 0$, which yields the first order condition $dB_t^I = (g_t^I - \epsilon_t^I + \alpha_t^I) dt = 0$ for $t = \tau$. Both conditions coincide and yield

$$\mathbb{E}_t[\alpha_\tau^I] = re^{r\tau(t)} \left(\frac{\varpi_0^I}{e^{r\tau(0)} - 1} + \int_0^t \frac{d\mathbb{E}_s[Y^I]}{e^{r\tau(s)} - e^{rs}} \right) = \mathbb{E}_t[\epsilon_\tau^I - g_\tau^I].$$

This implies that $\tau(t)$ is given by

$$\tau(t) = \frac{1}{r} \left(\ln(\mathbb{E}_t[\epsilon_\tau^I - g_\tau^I]) - \ln \left(r \frac{\varpi_0^I}{e^{r\tau(0)} - 1} + r \int_0^t \frac{d\mathbb{E}_s[Y^I]}{e^{r\tau(s)} - e^{rs}} \right) \right).$$

In particular, $\tau(0)$ is given in terms of the implicit function

$$\tau(0) = \frac{1}{r} \ln \left(\frac{\mathbb{E}_0[\epsilon_\tau^I - g_\tau^I](e^{r\tau(0)} - 1)}{r\varpi_0^I} \right),$$

where ϖ_0^I depends on $\tau(0)$.

Appendix B. The model under risk-aversion

Recall that

$$h_t = \frac{re^{rt}}{e^{r\tau(t)} - e^{rt}}, \quad \text{where } \tau(t) = \mathbb{E}_t^\mathbb{Q}[\tau].$$

Under risk-aversion, q_t represents the (possibly time-dependent) risk-premium and the allowance price is

$$dP_t = (r + q_t)P_t dt + 2\varrho h_t \sigma_t dW_t^\mathbb{P}.$$

Recalling that under the risk-neutral measure \mathbb{Q} we have

$$dP_t = rP_t dt + 2\varrho h_t \sigma_t dW_t^\mathbb{Q},$$

we obtain the change of measure by requiring

$$dW_t^\mathbb{Q} = dW_t^\mathbb{P} + \underbrace{\frac{q_t}{2\varrho h_t \sigma_t} P_t}_{=: \zeta_t} dt. \tag{11}$$

We then obtain the Radon–Nikodým density of \mathbb{Q} with respect to \mathbb{P} , restricted on \mathcal{F}_t as

$$\frac{d\mathbb{Q}}{d\mathbb{P}} \Big|_{\mathcal{F}_t} = L_t = \exp \left(- \int_0^t \zeta_s dW_s^\mathbb{P} - \frac{1}{2} \int_0^t \zeta_s^2 ds \right),$$

since $L = (L_t)_{t \geq 0}$ is in fact a martingale. Recall that the expected residual abatement requirement for the aggregate market follows the dynamics

$$d\varpi_t = -\alpha_t dt + \sigma_t dW_t^\mathbb{Q}.$$

Let α_t^A denote the aggregate abatement under risk-aversion. By definition of the aggregate abatement requirement, we obtain that

$$d\varpi_t = -\alpha_t^A dt + \sigma_t dW_t^\mathbb{P}.$$

The two equations above imply

$$dW_t^\mathbb{Q} = dW_t^\mathbb{P} + \frac{\alpha_t - \alpha_t^A}{\sigma_t} dt.$$

Comparing this to Eq. (11) reveals that the following must hold:

$$\alpha_t - \alpha_t^A = \frac{q_t}{2\varrho h_t} P_t. \tag{12}$$

This shows that aggregate abatement under risk-aversion is strictly smaller than under risk-neutrality, whenever $P_t > 0$ and $t < E_t^\mathbb{Q}[\tau]$. However, notice that

$$\alpha_t^A \rightarrow \alpha_t \quad \text{for } t \rightarrow \mathbb{E}_t^\mathbb{Q}[\tau].$$

Furthermore, notice that since $P_t = \Pi_t + 2\varrho\alpha_t$ we can directly relate α_t to α_t^A :

$$\alpha_t^A = \alpha_t - \frac{q_t}{2\varrho h_t} (\Pi_t + 2\varrho\alpha_t) = \frac{h_t - q_t}{h_t} \alpha_t - \frac{q_t \Pi_t}{2\varrho h_t}.$$

We now want to examine how the market responds to cap-preserving changes to the allowance allocation programme and how this is captured by varying risk premia. First notice that, given the real-world measure \mathbb{P} , the risk-neutral measure is parameterised by the risk-premium q_t . And, conversely, q_t becomes an implicit function of \mathbb{Q} . Therefore, we can fix \mathbb{Q} in order to see how q_t is affected by the time- t allocation of allowances, denoted by g_t . Since the timing of allocation does

not affect the equilibrium in \mathbb{Q} -expectation, σ_t and q_t are the only parameter that are then affected by g_t . Fixing \mathbb{Q} in Eq. (11) then yields

$$\frac{\partial}{\partial g_t} \mathbb{E}^{\mathbb{Q}} \left[\frac{q_t}{2\rho h_t \sigma_t} P_t \right] = \frac{\mathbb{E}^{\mathbb{Q}} P_t}{2\rho h_t \sigma_t} \frac{\partial q_t(g_t)}{\partial g_t} - \frac{q_t \mathbb{E}^{\mathbb{Q}} P_t}{2\rho h_t \sigma_t^2} \frac{\partial \sigma(g_t)}{\partial g_t} = 0,$$

from which we obtain

$$\frac{\frac{\partial q_t(g_t)}{\partial g_t}}{q_t} = \frac{\frac{\partial \sigma(g_t)}{\partial g_t}}{\sigma_t},$$

that is, changes to σ through adjustments to the allowance allocation programme are equally reflected in changes to the risk-premium q_t .

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