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The endowment effect, discounting and the environment[☆]

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ABSTRACT

There is a considerable body of evidence showing that our preferences exhibit both reference dependence and loss aversion, a.k.a. the endowment effect. In this paper, we consider the implications of the endowment effect for discounting, with a special focus on discounting future improvements in the environment. We show that the endowment effect modifies the discount rate via (i) an instantaneous endowment effect and (ii) a reference-updating effect. Moreover we show that these two effects often combine to dampen the preference to smooth consumption over time. What this implies for discounting future environmental benefits may then depend critically on whether environmental quality is merely a factor of production of material consumption, or whether it is an amenity. On an increasing path of material consumption, dampened consumption smoothing implies a lower discount rate. But on a declining path of environmental quality and where we derive utility directly from environmental quality, it implies a higher discount rate. On non-monotonic paths, loss aversion specifically can give rise to substantial discontinuities in the discount rate.

"There is substantial evidence that initial entitlements do matter and that the rate of exchange between goods can be quite different depending on which is acquired and which is given up" (Tversky and Kahneman, 1991, p. 1039)

1. Introduction

The discounting debate is of enduring importance to environmental economics, because many investments in improving environmental quality provide pay-offs far into the future. Numerous aspects of environmental discounting have been discussed (e.g. Lind et al., 1982; Portney and Weyant, 1999; Gollier, 2012; Arrow et al., 2013). However, one that has been missing is the

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implications of the endowment effect.

In one of their classic experiments, Kahneman et al. (1990) endowed half of their subjects with a coffee mug and asked them for the lowest price at which they would sell it. The other half were asked how much they would pay for the same mug. Standard consumer theory would predict no difference between the selling and buying prices. However, subjects endowed with the mug were prepared to sell for more than twice as much as the remaining subjects were willing to pay (also see Knetsch, 1989, 1992).[1] Kahneman et al. therefore showed the initial endowment creates a reference point that matters, and in particular losses are ascribed more value than equivalent gains, which has been termed the 'endowment effect' (Thaler, 1980). As well as experiments, the endowment effect is consistent with a ubiquitous feature of contingent valuation studies into non-market goods, whereby stated willingness to accept compensation exceeds willingness to pay (Horowitz and McConnell, 2002). It is also consistent with studies of various sorts into status quo bias (e.g. Samuelson and Zeckhauser, 1988; Knetsch, 1989),[2] and has been demonstrated in field studies (e.g. Genesove and Mayer, 2001).

There are basic reasons why the endowment effect modifies the discount rate. The discount factor is the ratio of discounted marginal utility in the future to marginal utility today. The endowment effect calls on us to recast what determines utility by incorporating reference dependence and loss aversion (Tversky and Kahneman, 1991). Building on Bowman et al. (1999) and Kőszegi and Rabin (2006), our formal model of the endowment effect proposes that utility is an additive combination of standard '*consumption-level*' utility and '*gain/loss*' utility relative to a reference level, which depends on past consumption levels. This means that, at the margin, there are two sides to the endowment effect. The first is an '*instantaneous endowment effect*', which is just the marginal value of an instantaneous gain/loss in consumption from the reference level. The second is a '*reference-updating effect*', which is the marginal disvalue to forward-looking agents of increasing the reference level from which future gains/losses are valued. Overall marginal utility becomes the sum of marginal consumption-level utility and this marginal endowment effect, comprising the instantaneous endowment effect and the reference-updating effect. Loss aversion implies both the instantaneous endowment effect and the reference-updating effect are larger for losses than gains.

At the heart of our results lies the fact that, in many situations, the rate at which the marginal endowment effect is changing differs from the rate at which marginal consumption-level utility is changing. Take a rising consumption path. In the standard model – *sans* endowment effect – marginal consumption-level utility falls along the path, as preferences for consumption are gradually satiated. The consumption discount rate exceeds the pure time preference rate and we smooth consumption by bringing it forward to the relatively poor present. But, in some important cases, we find that the marginal endowment effect falls more slowly than marginal consumption-level utility, even not at all. Consequently the sum of marginal consumption-level utility and the marginal endowment effect does not fall as fast, and the consumption discount rate is lower. We become more patient. Conversely, on a falling consumption path, marginal consumption-level utility rises, the discount rate is lower than the pure time preference rate and we postpone consumption to the relatively poor future. But, again, sometimes the marginal endowment effect does not rise as fast, meaning the discount rate is higher. We become less patient.

Whether the marginal endowment effect falls more quickly or more slowly than marginal consumption-level utility depends on the interplay between the instantaneous endowment effect and the reference-updating effect. The former essentially captures our becoming habituated to higher consumption levels (e.g. Constantinides, 1990; Campbell and Cochrane, 1999). Once we become accustomed to a higher level of consumption, an extra unit is of similar value to an increment of consumption when we were poor. However, we also anticipate becoming habituated to higher levels of consumption, which is the reference-updating effect. This means habituation comes at a cost. In the cases just mentioned, it turns out that the instantaneous endowment effect is higher than the reference-updating effect, but we also identify some opposing cases, where the endowment effect can increase the discount rate on a positive growth path and decrease it on a negative growth path. We develop a sufficient statistic entitled the '*endowment factor*', which boils the endowment effect on the discount rate down to a single multiplying coefficient. In particular, taking the standard 'Ramsey rule' for discounting, we show that

$$r = \Delta + \theta\eta\frac{\dot{x}}{x},$$

where δ is the pure time preference rate, η is the elasticity of marginal utility of consumption (assumed constant), $\dot{x}/x$ is the growth rate of consumption and θ is the endowment factor. In the absence of the endowment effect, $\theta = 1$. We would like to know the sign and size of θ. If consumption is increasing, the endowment effect decreases the discount rate provided $\theta < 1$. Conversely if $\theta > 1$, the endowment effect increases the discount rate. If consumption is decreasing, θ has the opposite effect.

Loss aversion means that losses have greater (dis)value than gains at the margin. On some declining consumption paths, this can result in the endowment effect having a greater impact on the discount rate. But perhaps the most striking effect of loss aversion is on non-monotonic consumption paths. We examine a path in which consumption grows, then falls. Loss aversion implies gain/loss utility is discontinuous when gains/losses become vanishingly small. When growth moves from positive to negative territory, the instantaneous endowment effect thereby jumps. The reference-updating effect rises in anticipation of this. The discount rate first rises, then plummets, before eventually converging towards the rate in the standard model. Dis-

[1] To allay concerns that the disparity could have been due to differences in wealth between subjects, Kahneman et al. (1990) conducted a further experiment, in which, rather than being asked how much they would be willing to pay to buy the mug, subjects were given the option of being gifted the mug or a sum of money, and asked at what value they would choose money over the mug. Those endowed with the mug were still prepared to sell for over twice as much as the valuation put on the mug by those invited to choose.

[2] Status quo bias can, of course, be explained in other ways, such as the existence of search and transaction costs.

counted values can therefore be highly sensitive to the endowment effect in the region where growth switches sign, as this is where the discount rate can diverge widely from its equivalent in the standard model.

What are the implications of the endowment effect for discounting projects to improve the future environment? We argue the interpretation of our results depends importantly on whether environmental quality is a factor of production that enables greater material consumption, or an amenity that directly contributes to utility. In the former case, we might usually expect consumption to grow. Then if the endowment factor $\theta < 1$, the discount rate is lower, we are more patient and we invest more in the future environment. In the latter case, the literature on 'dual discounting' tells us that we should effectively discount the returns on a project to improve environmental quality at the rate pertaining to environmental quality, not material consumption (Weikard and Zhu, 2005; Hoel and Sterner, 2007; Sterner and Persson, 2008; Traeger, 2011). If environmental quality is falling and $\theta < 1$, the discount rate is higher, we become less patient and paradoxically we are less likely to invest in the future environment. The intuitive explanation is that we become habituated to lower environmental quality.

The rest of the paper is structured as follows. Section 2 sets out the basic elements of our model, shows how they affect the discount rate and defines the endowment factor. Section 3 then attempts to establish the sign and size of this endowment factor analytically on stylised growth paths. This theory is developed in the context of a generic consumption good. Section 4 offers an interpretation of the results by applying the theory to environmental discounting in both a single, material-good setting and where there are two consumption goods, one of which is environmental quality. Section 5 explores some realistic, but conceptually ambiguous, situations numerically, including the non-monotonic path described just above. Section 6 provides a discussion, including the broader question of whether the endowment effect *ought* to be considered in evaluating public environmental investments.

2. Basic theory

This paper is about discounting the future benefits and costs of projects to improve the environment. These projects are assumed to be marginal.[3]

2.1. Preferences

Welfare is discounted utilitarian,

$$J = \int_0^\infty U_t e^{-\Delta t} dt, \tag{1}$$

where instantaneous utility is discounted at the constant rate $\delta > 0$. Instantaneous utility depends on consumption of good $x \in [0, \infty)$. But it does not just depend on the level of consumption of x, it also depends on the difference between the level of consumption and a reference level. Our instantaneous utility function $U : \mathcal{R}_+ \times \mathcal{R} \to \mathcal{R}$ is

$$U(x_t, \underline{x}_t) = \overbrace{v\left(x_t\right)}^{\substack{\text{Consumption–}\\ \text{level}\\ \text{utility}}} + \overbrace{g\left(x_t - \underline{x}_t\right)}^{\substack{\text{Gain/loss}\\ \text{utility}}}, \tag{2}$$

where $\underline{x}$ is the reference level. Instantaneous utility therefore represents a mixed objective. The function v corresponds with the standard theory of preferences, in that individual utility remains directly responsive to the absolute level of consumption. This is what we refer to as consumption-level utility. We assume v is continuous, twice continuously differentiable, and that $v' > 0$ and $v'' < 0$.

By contrast, the gain/loss function g captures the endowment effect. It is assumed to be continuous and twice continuously differentiable except when $x_t - \underline{x}_t = 0$. We impose three further behavioural restrictions on g, as a formal representation of the famous value function in Kahneman and Tversky (1979).[4] Let $\Delta \equiv x_t - \underline{x}_t$:

Assumption 1. *[Bigger gains and smaller losses are weakly preferred]* $g'(\Delta) \geq 0$.

Assumption 1 just ensures the gain/loss function is weakly increasing over its entire domain.

Assumption 2. *[Loss aversion] If* $\Delta > 0$, *then* $g'(-\Delta) > g'(\Delta)$.

Assumption 2 represents loss aversion with respect to both large changes in consumption and small ones. The former property is of course shared with strictly concave consumption-level utility functions, but the latter property – specifically $\lim_{\Delta \to 0} g'(-\Delta)/g'(\Delta) > 1$ – is a distinctive feature of the gain/loss function, which was made famous by Kahneman and Tversky (1979).

[3] See Dietz and Hepburn (2013) on discounting non-marginal environmental improvements.

[4] Building on Bowman et al. (1999) and Kőszegi and Rabin (2006).

Assumption 3. *[Non-increasing sensitivity]* $g''(\Delta) \leq 0$ *for all* $\Delta > 0$*, and* $g''(\Delta) \geq 0$ *for all* $\Delta < 0$*.*

Assumption 3 ensures the gain/loss function is weakly convex over the domain of losses and weakly concave over the domain of gains. Diminishing sensitivity is required to represent a preference such as: "the difference between a yearly salary of $60,000 and a yearly salary of $70,000 has a bigger impact when current salary is $50,000 than when it is $40,000" (Tversky and Kahneman, 1991, p. 1048). Constant sensitivity implies the impact of the difference in salary does not depend on reference salary.

The reference level $\underline{x}$ depends on the history of consumption as in Ryder and Heal (1973):

$$\underline{x}_t = \alpha \int_{-\infty}^{t} e^{-\alpha(t-\tau)} x_\tau d\tau, \tag{3}$$

where the parameter $\alpha \in (0, \infty)$ represents the responsiveness of the reference level to changes in the level of consumption over time, i.e. it captures an individual's memory for past consumption. The smaller is α, the longer that memory is. In the limit as $\alpha \to 0$ the current reference level corresponds to mean consumption over the entire history of consumption. At the other extreme, as $\alpha \to \infty$ the current reference level is just consumption at the last instant. It is worth noting that there is empirical support for the idea that a long history of consumption levels determines the reference level (Strahilevitz and Loewenstein, 1998). Combining (2) and (3) means preferences are inter-temporally dependent.

2.2. Discounting with the endowment effect

For an individual with preferences given by Equations (1)–(3), Appendix A shows that the marginal contribution to welfare of x at time t is given by

$$J_{x_t} = e^{-\Delta t}[v'(x_t) + \overbrace{\overbrace{g'\left(x_t - \underline{x}_t\right)}^{\substack{\text{Instantaneous}\\ \text{endowment}\\ \text{effect}}} - \alpha \overbrace{\int_{\tau=t}^{\infty} e^{-(\alpha+\Delta)(\tau-t)} g'\left(x_\tau - \underline{x}_\tau\right) d\tau}^{\substack{\text{Reference}-\text{updating}\\ \text{effect}}}}^{\substack{\text{Marginal endowment}\\ \text{effect}}}], \tag{4}$$

where the important feature is that J_{x_t} is a functional derivative, i.e. given that preferences are inter-temporally dependent, it describes the change in J with respect to a change in the consumption *path*.

As well as providing consumption-level utility v', a unit of consumption at time t provides a contemporaneous gain, the instantaneous endowment effect. In addition, a unit of consumption at time t affects the reference level from which gains are evaluated after time t. This is the reference-updating effect. In evaluating investment projects, forward-looking individuals will anticipate the effect that changes in consumption have on reference levels thereafter. The reference-updating effect is negative, because an increase in consumption today raises future reference levels, and thereby reduces future gains in consumption, or increases future losses. By how much an increase in consumption today raises future reference levels depends on the memory parameter α, and what effect this in turn has on welfare depends on the pure time discount rate δ. Loss aversion implies both the instantaneous endowment effect and the reference-updating effect will be higher on decreasing consumption paths, because g' will be higher. We refer to the sum of the instantaneous endowment effect and the reference-updating effect as the marginal endowment effect.

Using (4), the marginal rate of substitution between consumption at date 0 and date t, the discount factor, is

$$\begin{aligned} D(t,0) &\equiv J_{x_t}/J_{x_0}, \\ &= e^{-\Delta t}\frac{v'(x_t) + g'\left(x_t - \underline{x}_t\right) - \alpha \int_{\tau=t}^{\infty} e^{-(\alpha+\Delta)(\tau-t)} g'\left(x_\tau - \underline{x}_\tau\right) d\tau}{v'(x_0) + g'\left(x_0 - \underline{x}_0\right) - \alpha \int_{\tau=0}^{\infty} e^{-(\alpha+\Delta)\tau} g'\left(x_\tau - \underline{x}_\tau\right) d\tau}. \end{aligned} \tag{5}$$

From the discount factor we can derive an expression for the discount rate (see Appendix A):

$$\begin{aligned} r &\equiv -\frac{d}{dt}\ln D(t,0) = -\frac{\frac{d}{dt}J_x}{J_x}, \\ &= \Delta - \frac{\dot{v'} + \dot{g'} - \alpha\dot{\mu}}{v' + g' - \alpha\mu}, \end{aligned} \tag{6}$$

where μ is the shadow price of reference consumption:

$$\mu = \int_{\tau=t}^{\infty} e^{-(\alpha+\Delta)(\tau-t)} g' d\tau. \tag{7}$$

This is the marginal effect on welfare at time t of reducing the reference level, without changing the consumption path. Its time derivative is $\dot{\mu} = g' + (\alpha + \Delta) \int_t^{\infty} e^{-(\alpha+\Delta)(\tau-t)} g' d\tau$. Eq. (6) characterises the discount rate on an arbitrary consumption path. Appendix B shows that it also characterises the discount rate on an optimal path, where μ is the negative of the costate variable on reference consumption.

If we define the absolute value of the elasticity of consumption-level marginal utility as

$$\eta \equiv \frac{-v'' x}{v'},$$

then we obtain a more convenient and recognisable expression for r:

Definition 1. In the presence of the endowment effect as characterised by Eq. (2), the discount rate is

$$r = \Delta + \theta \eta \frac{\dot{x}}{x}, \tag{8}$$

where the 'endowment factor' is

$$\theta = \frac{1 + \frac{\dot{g}'}{v'} - \frac{\alpha \dot{\mu}}{v'}}{1 + \frac{g'}{v'} - \frac{\alpha \mu}{v'}}. \tag{9}$$

Equation (8) shows that the endowment effect modifies the discount rate through the factor θ.

3. The endowment factor on stylised growth paths

Given Definition 1, the crux of the paper is the question: what is the sign and size of the endowment factor θ? This section investigates the question in relation to some important, stylised trajectories for consumption of the good x. These trajectories have a common feature, which is that marginal gain/loss utility changes at a constant rate k:

$$g_t' = g_0' e^{kt}. \tag{10}$$

What kinds of path could be represented by (10)? Appendix C shows that, if the gain/loss function has a constant elasticity over the appropriate domain of gains/losses, then paths with constant k correspond with paths with a constant rate of consumption growth/decline. $k = 0$ can be a special case of this, where consumption is arithmetically increasing/decreasing, i.e. a linear consumption trajectory. But $k = 0$ is also consistent with any strictly increasing/decreasing consumption path, if and only if preferences obey constant sensitivity, i.e. $g''(\Delta) = 0$ for all $\Delta \neq 0$ (c.f. Assumption 3).

We can combine (10) with (9) to describe a functional relationship between θ and k,

$$\theta = \frac{1 + \frac{\dot{g}'}{v'} \left(\frac{\Delta - k}{\alpha + \Delta - k} \right)}{1 + \frac{g'}{v'} \left(\frac{\Delta - k}{\alpha + \Delta - k} \right)}. \tag{11}$$

Below we investigate this functional relationship in detail. We can also rewrite the discount rate as a function of k,

$$r = \Delta - \frac{\dot{v}' + \dot{g}' - \left(\frac{\alpha}{\alpha + \Delta - k} \right) \dot{g}'}{v' + g' - \left(\frac{\alpha}{\alpha + \Delta - k} \right) g'}, \tag{12}$$

which enables us to obtain a result that will be helpful in interpreting the analysis that follows.

Lemma 1. *The reference-updating effect is a fixed proportion $\alpha / (\alpha + \Delta - k)$ of the instantaneous endowment effect if k is constant.*

Lemma 1 implies that if $\delta > (<) k$, the instantaneous endowment effect is larger (smaller) than the reference-updating effect and therefore the marginal endowment effect is positive (negative).

3.1. *Constant marginal gain/loss utility ($k = 0$)*

First, consider the case where $k = 0$. Then

$$\theta = \frac{1}{\left[1 + \left(\frac{\Delta}{\alpha + \Delta} \right) \frac{g'}{v'} \right]}, \tag{13}$$

and we can say:

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Proposition 1. *[The endowment effect dampens consumption smoothing if marginal gain/loss utility is constant] On an arithmetically increasing or decreasing consumption path, or on any strictly increasing/decreasing consumption path with constant sensitivity,* $0 < \theta < 1$ *if and only if* $\delta > 0$.

Proof**.** *See Appendix D.*

Standard consumption-level utility makes us want to smooth consumption between dates at which the level of consumption is different. When consumption is increasing, marginal consumption-level utility falls. When consumption is decreasing, marginal consumption-level utility rises. Therefore in a growing economy we want to consume earlier by discounting the future at a higher rate. Conversely when consumption is falling we want to postpone it to the future by discounting at a lower rate. All of this is in the economist's DNA.

Yet the endowment effect interferes with these preferences. Proposition 1 considers two situations in which marginal gain/loss utility is constant. One is the behavioural assumption of constant sensitivity, allied with any strictly increasing/decreasing consumption path. The other is diminishing sensitivity along an arithmetically increasing/decreasing consumption path, because the linearity of the consumption trajectory ensures $k = 0$. If marginal gain/loss utility is constant, then both the instantaneous endowment effect and the reference-updating effect are constant, and so is the marginal endowment effect, i.e. the sum of the two. Moreover Lemma 1 implies that when $\delta > k = 0$, the instantaneous endowment effect is larger than the reference-updating effect and the marginal endowment effect is not only constant, it is positive. Since overall marginal utility is the sum of marginal consumption-level utility and the marginal endowment effect, the endowment effect causes overall marginal utility to decrease at a slower rate on an increasing path, and increase at a slower rate on a decreasing path. In other words, the endowment effect reduces the discount rate r on an increasing path, while it raises r on a decreasing path.

Positive pure time preference, however small, is an uncontroversial assumption. Even if the view is taken that the discount rate is derived from a social welfare functional and it should be impartial to the date at which utility is enjoyed, (very) small positive utility discounting still follows from taking into account the probability of extinction of society (e.g. Stern, 2007; Llavador et al., 2015). Therefore this result arguably has quite wide applicability.

3.2. Non-constant marginal gain/loss utility ($k \neq 0$)

Diminishing sensitivity implies marginal gain/loss utility falls, the larger is the gain/loss. On convex increasing and concave decreasing consumption paths, gains and losses respectively grow ever larger, so $k < 0$. By contrast, on concave increasing and convex decreasing consumption paths, gains and losses respectively grow ever smaller, so $k > 0$.

Strictly increasing consumption paths

Fig. 1 plots θ as a function of k on strictly increasing consumption paths. We always assume $\delta > 0$. Looking first at convex increasing paths, the following Proposition is established:

Proposition 2. *[The endowment factor on convex increasing paths] When consumption is strictly increasing and* $k < 0$,

$$\begin{cases} 0 < \theta \le 1 & \iff k \ge \dfrac{\dot{v}'}{v'} \\ \theta \ge 1 & \iff k \le \dfrac{\dot{v}'}{v'} \end{cases}.$$

On a convex increasing path, gains grow over time, hence g' falls over time and so do both the instantaneous endowment effect and the reference-updating effect. We know from Lemma 1 that the latter is a fixed fraction of the former. Moreover along a convex increasing consumption path $k < 0$, so the reference-updating effect is always smaller than the instantaneous endowment effect. This means the marginal endowment effect is positive and falls along the path.

What then becomes crucial is the relationship between the rate of decrease of marginal gain/loss utility, k, and the rate of decrease of marginal consumption-level utility, $\dot{v}'/v'$. When $k > \dot{v}'/v'$, marginal gain/loss utility falls more slowly than marginal consumption-level utility, hence overall marginal utility does not fall as quickly as it would otherwise do and $0 < \theta < 1$. The discount rate is consequently lower. By contrast, when $k < \dot{v}'/v'$, marginal gain/loss utility falls faster than marginal consumption-level utility, overall marginal utility falls more quickly than it would otherwise do and our preference to bring consumption forward is amplified, increasing the discount rate ($\theta > 1$).

On a concave increasing path, the sign of θ is ambiguous:

Proposition 3. *[The endowment factor on concave increasing paths] When consumption is strictly increasing and* $k > 0$,

$$\begin{cases} 0 < \theta \le 1 & \iff k \le \Delta \\ \theta \ge 1 & \iff \Delta \le k < \Delta + \alpha / \left(1 + g'/v'\right) \\ \theta < 0 & \iff k > \Delta + \alpha / \left(1 + g'/v'\right) \end{cases}.$$

The reference-updating effect is increasing in k. If $k < \delta$, the instantaneous endowment effect is larger than the reference-updating effect according to Lemma 1, which means that the marginal endowment effect is positive overall and this time it is

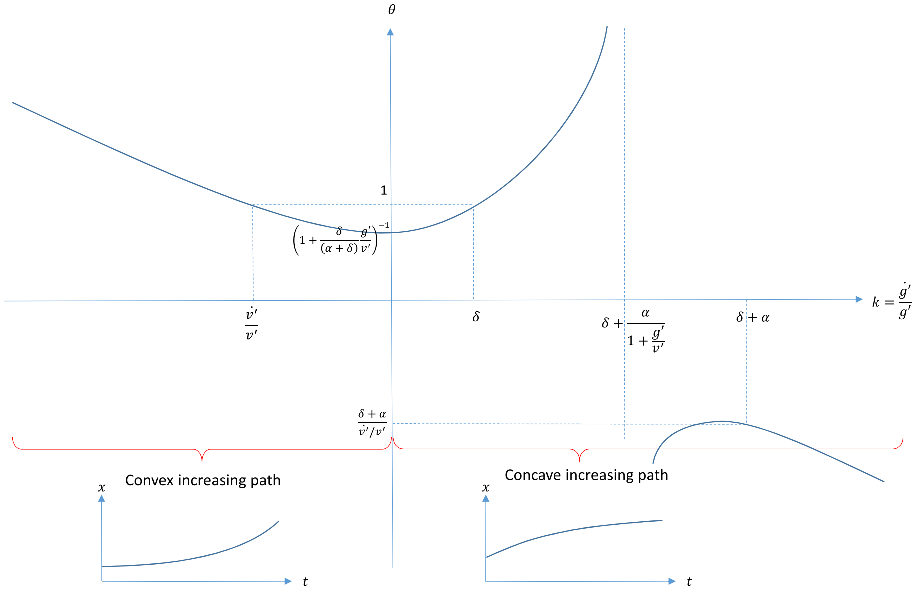

Fig. 1. The endowment factor as a function of k on strictly increasing consumption paths.

increasing. This gives us another case in which $0 < \theta < 1$. If $k > \delta$, the reference-updating effect is larger than the instantaneous endowment effect, which means that the marginal endowment effect is negative and increasing. This increases the discount rate, because the rate at which overall marginal utility falls is amplified ($\theta > 1$). In the limit as $k \to \Delta + \alpha/\left(1+g'/v'\right)$, θ becomes unbounded. This is because the reference-updating effect fully cancels out the sum of marginal consumption-level utility and the instantaneous endowment effect, overall marginal utility is zero and the discount rate explodes. To the right of the asymptote, the reference-updating effect is larger than the sum of marginal consumption-level utility and the instantaneous endowment effect, overall marginal utility is negative and $\theta < 0$.

Strictly decreasing consumption paths

Figs. 2 and 3 plot θ as a function of k on strictly decreasing consumption paths. The rate of decrease of consumption turns out to matter here, because the limit behaviour of θ in the region of the vertical asymptote depends on whether $\Delta + \alpha/\left(1+g'/v'\right) < (>) \dot{v}'/v'$. Fig. 2 depicts a setting of rapidly decreasing consumption, defined as $\dot{v}'/v' > \Delta + \alpha/\left(1+g'/v'\right)$. By contrast, Fig. 3 depicts the opposite setting of slowly decreasing consumption, where $\dot{v}'/v' < \Delta + \alpha/\left(1+g'/v'\right)$.[5]

Looking first at concave decreasing paths, the following Proposition is plain to see:

Proposition 4. *[The endowment factor is less than unity on concave decreasing consumption paths]* $\theta < 1$ *if consumption is strictly decreasing and* $k < 0$.

The preference to smooth consumption, in this case by postponing it, is also dampened on concave decreasing consumption paths. Indeed, if marginal gain/loss utility falls quickly enough, the endowment effect can actually reverse the preference to postpone consumption (although whether the discount rate changes sign depends on δ).[6]

An intuitive explanation for Proposition 4 draws once again on Lemma 1, which says that the reference-updating effect falls as a fixed proportion of the instantaneous endowment effect. The case of concave decreasing consumption is also one where the reference-updating effect is the smaller of the two effects ($k < 0$). On a concave decreasing consumption path, marginal consumption-level utility is increasing. On the other hand, the marginal endowment effect is positive and decreasing, so overall marginal utility increases more slowly. The effect could be sufficiently large that overall marginal utility itself is decreasing overall.

On a convex decreasing path, diminishing sensitivity instead results in an increase in marginal gain/loss utility over time. The instantaneous endowment and reference-updating effects grow in step with each other as will be familiar by now, but of course when $k > 0$ it is no longer assured that the reference-updating effect is smaller than the instantaneous endowment effect. It will be the case if $k < \delta$, a situation in which consumption decreases relatively slowly. If, in addition to this, $k < \dot{v}'/v'$,

[5] Note the exact placement of $\dot{v}'/v'$ on the Figures is arbitrary; we only know where it lies in relation to $\Delta + \alpha/\left(1+g'/v'\right)$. This affects the strength of the conclusions we can draw.

[6] Note that $\theta = 0$ when $k = \Delta + \frac{\alpha}{1+g'/v'}$.

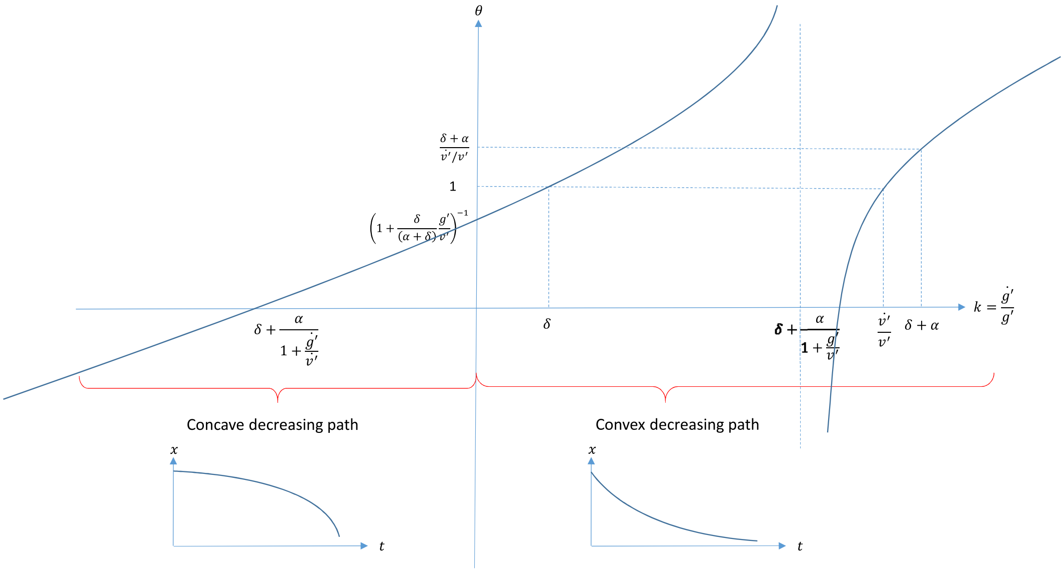

Fig. 2. The endowment factor as a function of k on strictly decreasing consumption paths, where $\dot{v}'/v' > \Delta + \alpha/(1+g'/v')$.

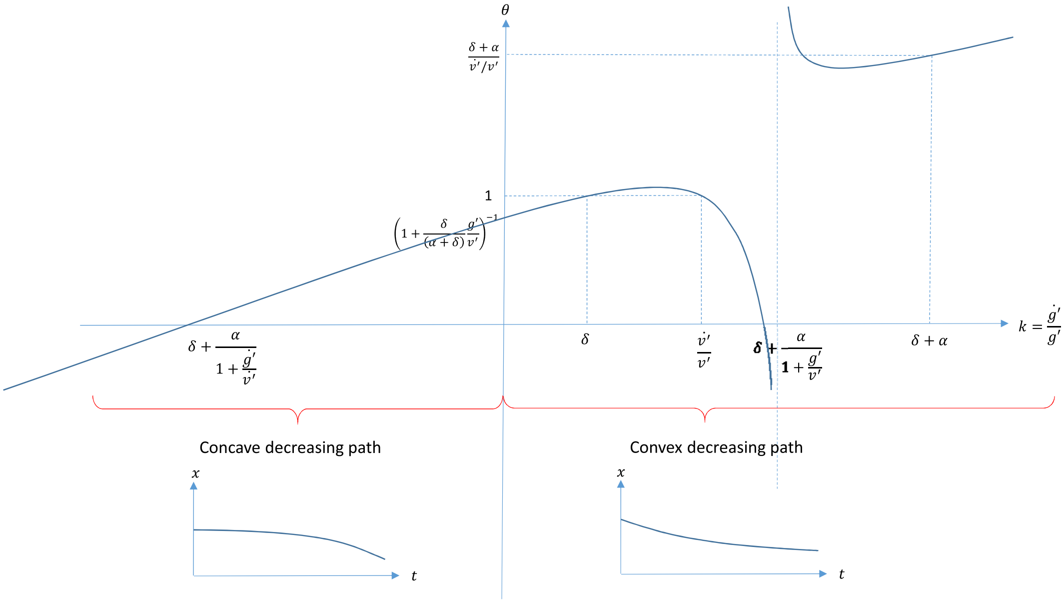

Fig. 3. The endowment factor as a function of k on strictly decreasing consumption paths, where $\dot{v}'/v' < \Delta + \alpha/(1+g'/v')$.

$0 < \theta < 1$ and the discount rate increases:

Proposition 5. *[On convex decreasing paths, the endowment effect dampens consumption smoothing if marginal gain/loss utility increases at a slower rate than pure time preference or marginal consumption-level utility, whichever is smaller] When consumption is strictly decreasing and* $k > 0$, $0 < \theta < 1$ *if* $k < \Delta < \dot{v}'/v'$.

If $k > \delta$ – consumption is decreasing more rapidly – the reference-updating effect is larger than the instantaneous endowment effect. As the Figures show, this can result in $\theta > 1$, but the picture is complicated by the asymptotic behaviour of θ, which depends on how rapidly consumption is falling.

3.3. θ in the long run

Although this analysis has been based on the assumption that k is constant over time, θ changes over time, because it depends on the evolution of v' and g'. As we show in Appendix E, in general either v' or g' comes to dominate in the long run. That is, either marginal consumption-level utility or the marginal endowment effect dominates in the long run, which enables us to establish some further useful results on the value of θ.

4. Discounting environmental projects

In order to interpret these analytical results in the context of discounting environmental projects, it is necessary to discuss what the appropriate discounting concept is.

4.1. Single-good setting

Suppose that the environment is a factor of production of material goods, but it does not directly affect utility. Then it is appropriate to discount environmental benefits in the future at the rate pertaining to consumption of material goods, C. This is a straightforward application of the theory above. Substituting C for x, the 'material discount rate' is

$$r^C = \Delta + \theta^C \eta^{CC} \frac{\dot{C}}{C},$$

where $\eta^{CC} \equiv -v'' C/v'$ and the 'material endowment factor' is

$$\theta^C = \frac{1 + \frac{\dot{g}'}{\dot{v}'} - \frac{\alpha\dot{\mu}}{\dot{v}'}}{1 + \frac{g'}{v'} - \frac{\alpha\mu}{v'}}.$$

Consumption of material goods is usually increasing. Propositions 1, 2, 3 showed that, when this is so, the endowment effect often reduces the material discount rate, because $\theta < 1$. Consequently our willingness to pay to provide future environmental improvements is increased. Important cases include, first, arithmetic growth (Proposition 1) and, second, exponential growth when either there is constant sensitivity with respect to gain utility (Proposition 1), or marginal gain/loss utility falls more slowly than marginal consumption-level utility (Proposition 2). The environmental production factor should of course be priced at its future marginal productivity, which may be higher, but this is unrelated to the endowment effect.

4.2. Two-good setting

Now suppose that, in addition to the composite produced good C, instantaneous utility also depends on the quality of the natural environment E. Preferences are a minimal extension of the single-good setting. The welfare functional remains discounted utilitarian as in (1). The instantaneous utility function $U : \mathcal{R}^2 \times \mathcal{R}^2_+ \rightarrow \mathcal{R}$ in period t is now

$$U_t(C_t, \underline{C}_t, E_t, \underline{E}_t) = v\left(C_t, E_t\right) + g\left(C_t - \underline{C}_t\right) + h\left(E_t - \underline{E}_t\right), \tag{14}$$

where $\underline{E}$ is the reference level of environmental quality.

We assume that the consumption-level utility function v is concave increasing in both material consumption and environmental quality. No restriction is placed on v_{CE}, i.e. the cross partial derivative. The gain/loss function h captures the endowment effect with respect to environmental quality. It is conditioned by the same assumptions as the gain/loss function with respect to the produced good g. By virtue of the additive way in which reference dependence enters the utility function, g' is assumed independent of the level or change in E, and h' is likewise assumed independent of the level or change in C.

We assume $\underline{E}$ evolves in just the same way as the reference level of material consumption, i.e.

$$\underline{E}_t = \alpha \int_{-\infty}^{t} e^{-\alpha(t-\tau)} E_\tau d\tau, \tag{15}$$

with the memory parameter α common to the evolution of both reference levels.

Following the same procedure as above, the *environmental* discount rate

$$r^E = \Delta - \frac{\dot{v}_E + \dot{h}' - \alpha\dot{\phi}}{v_E + h' - \alpha\phi}, \tag{16}$$

where the shadow price of reference environmental quality is

$$\phi = \int_{\tau=t}^{\infty} e^{-(\alpha+\Delta)(\tau-t)} h' d\tau. \tag{17}$$

Appendix B shows how the environmental discount rate r^E can be derived from an optimal control problem, in which environmental degradation has either a flow or stock character.

Substitute $\dot{v}_E = v_{EE}\dot{E} + v_{EC}\dot{C}$ into (16) and define η^{EE} as the elasticity of consumption-level marginal utility of environmental quality with respect to environmental quality,

$$\eta^{EE} \equiv \frac{-v_{EE}E}{v_E},$$

and η^{EC} as the elasticity of consumption-level marginal utility of environmental quality with respect to consumption of material goods,

$$\eta^{EC} \equiv \frac{v_{EC}C}{v_E}.$$

In the presence of the endowment effect as characterised by Eq. (2), the environmental discount rate is

$$r^E = \Delta + \theta^E \left(\eta^{EE}\frac{\dot{E}}{E} - \eta^{EC}\frac{\dot{C}}{C} \right) \tag{18}$$

where the 'environmental endowment factor' is

$$\theta^E = \frac{1 + \frac{\dot{h}'}{\dot{v}_E} - \frac{\alpha\dot{\phi}}{\dot{v}_E}}{1 + \frac{h'}{v_E} - \frac{\alpha\phi}{v_E}}. \tag{19}$$

When it comes to trading off consumption of the produced good at date 0 with environmental quality at date t (or *vice versa*), this can be achieved in two ways, with identical results (Weikard and Zhu, 2005; Hoel and Sterner, 2007). Define the relative price of environmental quality at date t as $p_t \equiv J_{E_t}/J_{C_t}$. Then expand the term $\dot{p}/p$, the change in the relative price, to get

$$\begin{aligned}
\frac{\dot{p}}{p} &\equiv \frac{\frac{d}{dt}(J_E/J_C)}{J_E/J_C} \\
&= \frac{\dot{v}_E + \dot{h}' - \alpha\dot{\phi}}{v_E + h' - \alpha\phi} - \frac{\dot{v}_C + \dot{g}' - \alpha\dot{\mu}}{v_C + g' - \alpha\mu} \\
&= \theta^E(\eta^{EC}\frac{\dot{C}}{C} - \eta^{EE}\frac{\dot{E}}{E}) - \theta^C(\eta^{CE}\frac{\dot{E}}{E} - \eta^{CC}\frac{\dot{C}}{C}) \\
&= r^C - r^E,
\end{aligned} \tag{20}$$

where η^{CE} is the elasticity of consumption-level marginal utility of material goods with respect to environmental quality.

In the two-good setting, the material discount rate is not the appropriate discount rate for future environmental benefits, unless it is adjusted for changes in relative prices, $r^E = r^C - \dot{p}/p$ (Weikard and Zhu, 2005; Hoel and Sterner, 2007; Sterner and Persson, 2008; Traeger, 2011). Alternatively one can calculate the present value of environmental quality enjoyed at date t using r^E and then convert this into units of the produced good using the present relative price p_0. If we are in a setting where environmental quality is declining while consumption of material goods is increasing, $\eta^{EE} \cdot \dot{E}/E < 0$, which will reduce the discount rate. The sign of $\eta^{EC} \cdot \dot{C}/C$ is ambiguous *a priori*, but ordinarily we would expect $\eta^{EC} > 0$ so that, as environmental quality becomes relatively more scarce, the environmental discount rate is lower still (Hoel and Sterner, 2007; Traeger, 2011).

Parallel to the analysis of the single-good case, the endowment effect enters via the environmental endowment factor θ^E. When environmental quality is on a decreasing path, the endowment effect *increases* the environmental discount rate r^E if $\theta^E < 1$. Proposition 4 showed that $\theta^E < 1$ on concave decreasing consumption paths. Such paths might be of particular relevance to environmental discounting. For instance, in their exploration of the concept of the 'Anthropocene', Steffen et al. (2011) plotted the evolution of 12 global environmental indicators, ranging from the atmospheric stock of greenhouse gases and ozone, to the depletion of fisheries, forests and biological diversity. They showed that in all of the aforementioned cases environmental quality has been on a concave decreasing path since the beginning of the industrial revolution, branding the last 70 years in particular the 'Great Acceleration'.[7]

If $\theta^E < 1$, our willingness to pay to provide future environmental improvements may be reduced. However, it is not as clear as in the single-good setting. Not only does the endowment effect impact the discount rate in the two-good setting, it also impacts the initial accounting price $p_0 \equiv J_{E_0}/J_{C_0}$. Nonetheless we can say that for long-run investments in the environment, the effect of θ^E on r^E will dominate, and *vice versa* for short-run investments.

[7] Where environmental quality is the inverse of the stock of pollution (carbon dioxide and ozone), the stock of pollution has increased exponentially. The percentage of global fisheries fully exploited, and the percentage of global forest cover destroyed since 1700, have both increased exponentially. The rate of species extinctions has increased exponentially, with approximately no species additions. See Steffen et al. (2011), Fig. 1.

5. Numerical illustrations

Some numerical examples will be helpful at this point. They will enable us to quantify the endowment effect on the discount rate. We can also inspect cases similar to those above, where the sign and size of the endowment factor were found to be conceptually ambiguous. Finally, we can look at non-monotonic growth paths, where not only reference dependence but also loss aversion plays a prominent role.

5.1. Functional forms and parameter scheme

In the single-good setting, we specify a consumption-level utility function that is isoelastic, i.e.

$$v(C_t) = \frac{1}{1-\psi} C_t^{1-\psi}, \tag{21}$$

where $\psi > 0$ is the elasticity of marginal consumption-level utility. In the two-good setting, we combine this assumption of a constant elasticity of intertemporal substitution with a constant elasticity of substitution between the produced good and environmental quality (like Hoel and Sterner, 2007; Traeger, 2011):

$$v(C_t, E_t) = \frac{1}{1-\psi} \left[\gamma C_t^{1-1/\sigma} + (1-\gamma) E_t^{1-1/\sigma} \right]^{\frac{(1-\psi)\sigma}{\sigma-1}}, \tag{22}$$

where σ is the elasticity of substitution between the two goods.

For gain/loss utility we use a generalisation of the functional form proposed by Tversky and Kahneman (1992), which is consistent with Assumptions 1, 2, 3:

$$g(\Delta), h(\Delta) = \begin{cases} (\Delta + \omega)^\beta - \omega^\beta, & \Delta \geq 0 \\ -\lambda((-\Delta + \omega)^\beta - \omega^\beta), & \Delta < 0 \end{cases}, \tag{23}$$

where $\beta \in (0,1]$ and $\lambda \geq 1$. Compared with Tversky and Kahneman (1992), we introduce the parameter $\omega > 0$ to ensure marginal gain/loss utility is bounded from above as $\Delta \to 0$ in the limit.[8] The parameter ω enters twice in order to also satisfy the property that $g(0) = h(0) = 0$. Bounding marginal gain/loss utility becomes important when we consider non-monotonic paths later in this section. It does mean that the gain/loss functions exhibit a non-constant elasticity, which in turn means that k is not constant, but for $\Delta \gg 0$ it will be approximately constant.

In the single-good setting, a weighted sum of (21) and (23) makes up the instantaneous utility function. Assuming consumption is increasing, this would be written as

$$U_t(C_t, \underline{C}_t) = \frac{\zeta}{1-\psi} C_t^{1-\psi} + (1-\zeta) \left[\left(C_t - \underline{C}_t + \omega\right)^\beta - \omega^\beta \right]. \tag{24}$$

The parameter $\zeta \in [0,1]$ governs the value share of consumption-level utility. In order to calibrate ζ, we target the initial value share of consumption-level utility, Z:

$$Z \approx \frac{\zeta v_0' C_0}{\zeta v_0' C_0 + (1-\zeta) g' \left(C_0 - \underline{C}_0\right)}.$$

In the two-good setting, the value share of the produced good relative to environmental quality is determined by $\gamma \in [0,1)$. Assuming consumption of the produced good is increasing and environmental quality is falling, the instantaneous utility function would be written as

$$\begin{aligned} U_t(C_t, \underline{C}_t, E_t, \underline{E}_t) &= \frac{\zeta}{1-\psi} [\gamma C_t^{1-1/\sigma} + (1-\gamma) E_t^{1-1/\sigma}]^{\frac{(1-\psi)\sigma}{\sigma-1}} \\ &+ (1-\zeta)\gamma[(C_t - \underline{C}_t + \omega)^\beta - \omega^\beta] \\ &- (1-\zeta)(1-\gamma)\lambda[(-E_t + \underline{E}_t + \omega)^\beta - \omega^\beta]. \end{aligned}$$

In order to calibrate γ, we target the initial value share of the produced good, Γ:

$$\Gamma \approx \frac{\zeta v_{C_0} C_0 + (1-\zeta) g' \left(C_0 - \underline{C}_0\right)}{\zeta (v_{C_0} C_0 + v_{E_0} E_0) + (1-\zeta) \left[g' \left(C_0 - \underline{C}_0\right) + h' \left(E_0 - \underline{E}_0\right) \right]}.$$

[8] In a similar fashion to the bounding parameter in harmonic absolute risk aversion (HARA) functions (Gollier, 2001).

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Table 1
Default parameter values.

Parameter	Value
Z	0.75
Γ	0.9
ψ	1.5
σ	0.5
β	0.9
λ	2.25
δ	1.5%
α	0.5
ω	$1

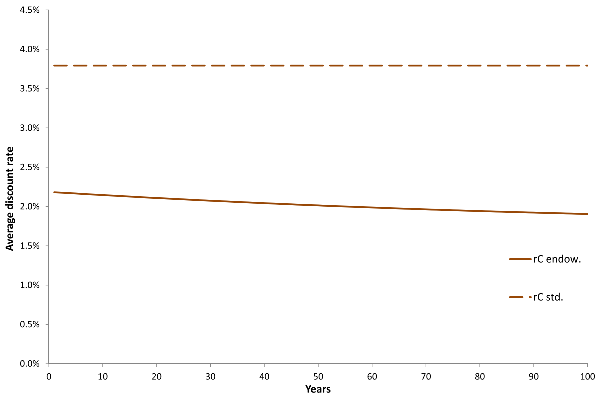

Fig. 4. Material discount rates with and without the endowment effect.

For numerical modelling it is natural to work in discrete time and in discrete time the reference levels form according to

$$\underline{C}_t = (1-\alpha)\underline{C}_{t-1} + \alpha C_{t-1},$$
$$\underline{E}_t = (1-\alpha)\underline{E}_{t-1} + \alpha E_{t-1},$$

where $\alpha \in [0,1]$ and t stands for one year. This is particularly convenient to interpret, because $\alpha = 1$ implies that the current reference level is exactly the last period's consumption level, $\alpha = 0$ implies that the current reference level is exactly the initial, exogenous reference level, and $\alpha = 0.5$ is exactly the intermediate case. We initialise the model 20 years in the past, so that by the time our discounting analysis begins (at $t = 0$), consistent reference consumption levels have formed.

Table 1 lists the default parameter values chosen for all of our numerical modelling. We choose typical values from empirical studies for the elasticity of marginal utility $\phi = 1.5$ and the parameters of the gain/loss function; $\beta = 0.9$ and $\lambda = 2.25$ (Barberis, 2013). Choosing the pure rate of time preference δ is particularly controversial: we opt for 1.5% here. We choose a middle-of-the-road value of $\alpha = 0.5$. Γ and Z are hard to pin down with empirical evidence, so we conservatively set $\Gamma = 0.9$ and $Z = 0.75$.

5.2. Single-good setting

Fig. 4 plots the time-averaged discount rate[9] in an illustration, in which annual consumption per capita grows at 1.5% per annum. This is the growth rate of global average household final consumption expenditure per capita over the last 30 years.[10] Discount rates with and without the endowment effect are shown.

Without the endowment effect, it is well known that the discount rate 'r^Cstd.' is given by the Ramsey rule; $r^C = \delta + \phi \cdot \dot{C}/C$, so $1.5 + 1.5*1.5 = 3.75\%$.[11] But, when the endowment effect is present, the discount rate 'r^Cendow.' is initially just 2.18%, and falls further to 1.94% in 100 years. Therefore the endowment effect makes a big difference in this empirically plausible example, resulting in a much more patient decision-maker who would be willing to pay more to improve the environment in the future,

[9] That is, the average rate of fall of the discount factor from time 0 to t.
[10] The initial value, which is also the reference value, is $3467. Both of these data points are taken from the World Bank's World Development Indicators.
[11] Technically the Ramsey rule will only be an approximation with a discrete time step; actually r^Cstd. = 3.79%.

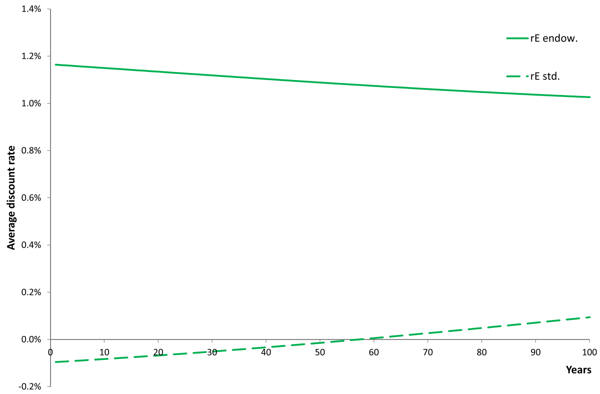

Fig. 5. Discount rates with and without the endowment effect.

in a single-good setting.

Since r^Cendow. is lower than r^Cstd. and falling relative to it, the material endowment factor $0 < \theta^C < 1$ and $\dot{\theta}^C < 0$. Indeed θ^C is initially 0.3 and it falls to 0.18 in 100 years. Consumption of the produced good is on a convex increasing path, so the implication is that we have an example here that is similar to $\dot{v}'/v' < k < 0$ above (Fig. 1 and Proposition 2). marginal gain/loss utility falls slowly enough that the endowment effect dampens our preference to smooth consumption of the produced good.

Appendix G presents a sensitivity analysis of θ^C to variation in the preference parameters.

5.3. Two-good setting

Fig. 5 plots the time-averaged environmental discount rate that results when annual material consumption per capita grows at 1.5% per year, as above, and when environmental quality falls at 0.5% per year. Environmental quality is hence on a convex decreasing path, which was a conceptually ambiguous case in Section 3.

Without the endowment effect, the average environmental discount rate 'r^Estd.' begins at −0.1% and nudges upwards to 0.09% in 100 years. If $U_t(E_t) = 1/1 - \phi E_t^{1-\phi}$, then according to the Ramsey rule the environmental discount rate would be $1.5 + 1.5^* - 0.5 = 0.75\%$, so the effect of including consumption-level utility from the produced good, $-\eta^{EC}\dot{C}/C$, is to pull the discount rate on environmental quality significantly downwards. Increases in environmental quality are more valuable when the produced good is relatively abundant; the relative prices story. For this to be the case, it must be that $\eta^{EC} > 0$, which can be verified for our parameter scheme.[12]

When the endowment effect is present, the average environmental discount rate 'r^Eendow.' starts at 1.16% and falls to 1.03% in 100 years, so the endowment effect does indeed increase the rate at which we would discount an environmental project, when environmental quality directly enters our utility function. The environmental endowment factor $0 < \theta^E < 1$, which implies the growth rate of marginal gain/loss utility is less than the pure rate of time preference, similar to the case highlighted in Proposition 5.[13] Initially $\theta^E = 0.21$, while after 100 years it is 0.34.

Appendix G also presents a sensitivity analysis of θ^E to variation in the preference parameters.

5.4. A non-monotonic path for environmental quality

Thus far the analysis has been entirely based on consumption trajectories that are strictly increasing/decreasing. Here we focus on an alternative path of environmental quality, whereby the initial growth rate is 0.5%, but the growth rate falls by 0.01 percentage points per year. Consequently environmental quality grows for the first thirty years, and then falls.

Since the average growth rate of environmental quality declines, so does r^Estd. (Fig. 6). By contrast, r^Eendow. follows a non-monotonic and discontinuous path. As $t \to 30$, it increases sharply to nearly 6%, before suddenly dropping far below 0%, and then increasing again to become close to r^Estd.. Loss aversion and diminishing sensitivity are the causes of this striking behaviour. As $t \to 30$, gains become ever smaller. Due to diminishing sensitivity, marginal gain/loss utility increases and it does so more than proportionally. This increases both the instantaneous endowment effect and the reference-updating effect. In this case, the

[12] In particular, given (22).

$$\eta^{EC} = \frac{(\gamma - 1)(\phi + \rho - 1)E^{\rho}}{[\gamma C^{\rho} + (1 - \gamma)E^{\rho}]},$$

where $\rho = 1 - 1/\sigma$. Hence $\eta^{EC} > 0 \Leftrightarrow (\gamma - 1)(\phi + \rho - 1) > 0$.

[13] Indeed marginal gain/loss utility g′ grows at only about 0.04% per year.

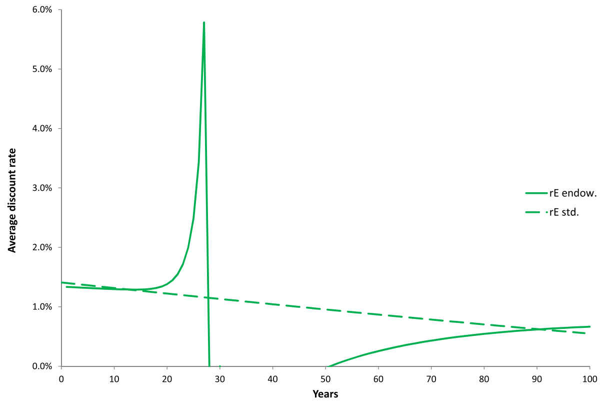

Fig. 6. The environmental discount rate when E follows an inverse-U shaped path.

latter effect is dominating the former and marginal utility falls faster than in the standard model. Indeed, the reference-updating effect is so large that the discount factor becomes negative as t gets close to 30. Hence the discount rate rises steeply to a peak (which is regulated by the bounding parameter ω).

At exactly $t = 30$, very small gains turn into very small losses. Due to the discontinuity in the gain/loss function, marginal gain/loss utility, which was already very high, jumps even higher. Accordingly, the discount factor jumps up to a large positive value and r^Eendow. jumps down to a very low trough. Thereafter losses become ever larger, marginal gain/loss utility falls and r^Eendow. becomes closer to r^Estd. again. Appendix H isolates the different effects of loss aversion and diminishing sensitivity by exploring various combinations of the loss aversion parameter λ and the elasticity of the gain/loss function β.

6. Discussion

Our analysis has shown that the endowment effect can make a substantial difference to the discount rate. In particular, we have shown that in many cases the endowment effect dampens our preference to smooth consumption over time, or indeed goes as far as reversing it. This is formalised in the idea that the endowment factor $\theta < 1$. In summary, we have found that $\theta < 1$ in at least the following cases:

1. If gain/loss utility conforms to constant sensitivity (Proposition 1);
2. If consumption is arithmetically increasing or decreasing (Proposition 1);
3. If consumption is convex increasing such that marginal gain/loss utility is decreasing at a constant rate k, and k is greater than the rate of change of marginal consumption-level utility (Proposition 2 and Section 5.2);
4. If consumption is concave increasing and $k < \delta$ or $k > \Delta + \alpha / \left(1 + g'/v'\right)$ (Proposition 3);
5. If consumption is concave decreasing (Proposition 4).
6. If consumption is convex decreasing and $k < \Delta < \dot{v}'/v'$ (Proposition 5 and Section 5.3).

We have also shown that the implications of this result may differ fundamentally, depending on whether the environment is simply a factor of production of material goods, or has amenity value and directly enters the utility function. In the former case, it is appropriate to discount improvements in the future environment at the material discount rate. Assuming material consumption is increasing, the preference to smooth consumption between dates contributes positively to the discount rate. That $\theta < 1$ makes us more patient if this is so. In the latter case, it is appropriate to discount improvements in the future environment at the environmental discount rate. Assuming environmental quality is falling, the preference to smooth consumption between dates contributes negatively to the discount rate. That $\theta < 1$ makes us *less* patient.

Especially the latter implication is perhaps surprising. One might have thought that the endowment effect would increase the value placed on an investment on a path where environmental quality is being lost. But remember that the exercise here is not to value the path itself, rather the discounting literature engages with the valuation of a marginal investment along a path. What matters is that, on a strictly decreasing path, environmental quality is being lost not only in the future, it is being lost today. If the marginal utility of losses today weighs more heavily on our welfare than the marginal utility of losses tomorrow, the endowment effect makes us less willing to postpone consumption to the future. It must also be borne in mind that, of the two elements of the endowment effect – reference dependence and loss aversion – only the former is at work on a strictly increasing/decreasing path. Hence we become accustomed – habituated – to lower environmental quality, such that future losses decrease our utility less.

Where loss aversion comes to the foreground is in valuing marginal investments along non-monotonic paths. Section 5.4 illustrated that the endowment effect on the discount rate can be very large along such paths, using the example of a non-monotonic path of environmental quality. This is because loss aversion introduces a discontinuity or kink in the gain/loss utility function when the change in consumption of a good $\Delta = 0$. On a non-monotonic path, which itself can be smooth as in our example, there will be a point in time when growth hits zero on its way from positive to negative territory, and *vice versa*. Around this point, the instantaneous endowment effect jumps, and the reference-updating effect changes rapidly in advance of the jump. The chief implication is that valuation of environmental investments is likely to be substantially modified by the endowment effect, when net benefits are incurred in the region of a turning point in the growth of environmental quality. It is clear, however, that the effect on valuations is context-specific.

Moving beyond a summary of our results to broader issues, there is naturally the question of whether the endowment effect ought to be considered in evaluating public environmental investments in the first place. There are at least two dimensions to this. First, there is the question of how strong the evidence behind the endowment effect is. Second, there is the question of whether preferences that represent the endowment effect should be afforded normative status, insofar as they are included in public/social decision-making.

On the first question, there is much empirical evidence that demonstrates the endowment effect in both laboratory and field settings (e.g. Camerer and Loewenstein, 2004; DellaVigna, 2009). This is not to deny the existence of dissenting evidence though. Most famously, List (2003) showed that experienced traders of a good do not exhibit the endowment effect with respect to that good, a result that is consistent either with those traders not being loss averse, or with those traders forming different reference points to inexperienced traders (DellaVigna, 2009). However, the preferences of people who trade baseball cards at least half a dozen times a month (i.e. an experienced trader) seem a poor analogy for those preferences of interest here, which are over future levels of overall material consumption and environmental quality. Other dissenting evidence has suggested that empirical regularities, which appear consistent with the endowment effect, are in fact due to other phenomena. For example, Chetty and Szeidl (2016) argue that a model of household-level adjustment costs explains the same empirical patterns of household consumption as do models of habit formation, while explaining other regularities that habit-formation models cannot be reconciled with.

On the second question, a simple application of the doctrine of consumer sovereignty would have it that, if the endowment effect characterises people's preferences, then the preferences of a social planner should include it too. However, objections can be raised to this position. It might be argued that the endowment effect is irrational, even from the point of view of individual consumer choice. Since the requirements of preferences are usually axioms or primitives, the yardstick of rationality is difficult to establish. Nonetheless one can find comparable objections to affording normative status to related phenomena, such as hyperbolic discounting (e.g. Hepburn et al., 2010) and ambiguity aversion (Al-Najjar and Weinstein, 2009; Gilboa et al., 2009). With hyperbolic discounting, the concern is that preferences are time-inconsistent and therefore explain patterns of behaviour, such as addiction and procrastination, which are not in the best interests of those who hold these preferences. However, it is important to highlight that models of habit formation such as ours do not lead to time-inconsistency, even though the utility function is not time-separable (Végh, 2013). A different objection might be based on the ethical implications of our results. In the case of the environmental discount rate and falling environmental quality, $\theta^E < 1$ implies that we should be less inclined to improve the environment for future generations, because they are accustomed to poor environmental quality. This might appear immoral. A related question, which is much more widely debated in the discounting literature, is to what extent features of individual preferences should inform discount rates applied to decisions with intergenerational consequences.[14] We feel that a proposed resolution to this debate is clearly beyond the scope of the present paper. At the very least, our results indicate how consumers who exhibit the endowment effect really do value future consumption. And if the endowment effect is judged not to be a legitimate feature of the social planner's preference, then there is a wedge between private and social discount rates that may require policy intervention.

Lastly, there are several extensions to the present work, which are worthwhile considering. First, different options could be pursued for modelling the endowment effect. For example, the formation of the reference level could be made a function of past consumption-level utility, rather than past consumption. Among other things, this would permit low consumption in the past to have a disproportionately high impact on the current reference level. By way of another example, the endowment effect could be applied to only one of the two goods. Second, Appendix B points the way towards an analysis of optimal control of pollution under the endowment effect. This will not be simple, however, given the large number of state variables implied by having reference levels and more than one good. Third, our results assume perfect foresight, a natural consequence of minimally extending standard preferences. In fact, this is likely to have important implications for our results, because the strength of the reference-updating effect rests on our anticipating the effect on future gain/loss utility of increments in consumption today. But what if we don't fully anticipate this effect, i.e. what if we succumb to projection bias? This would be worth looking into. Fourth, we have only examined the endowment effect in a riskless choice setting, in the tradition of Tversky and Kahneman (1991), even though reference dependence and loss aversion were first invoked to explain risky choices (Kahneman and Tversky, 1979). Therefore we could allow consumption of the two goods to follow a stochastic process. Again, this will not be wholly trivial, because the state space of future consumption levels could span the kink in marginal gain/loss utility that is implied by loss aversion. Under such circumstances, not only will there be familiar-looking results about the expectation of marginal gain/loss utility that derive from application of Jensen's inequality, there will also be a 'kink effect', so to speak.

[14] For example, non-constant (possibly hyperbolic) discounting is less obviously irrational in an intergenerational context, where different time discount rates might be applied within and across generations (Phelps and Pollak, 1968; Fujii and Karp, 2008; Ekeland and Lazrak, 2010).

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A. Derivation of the discount factor and rate

Following Karp and Traeger (2009), the functional derivative $\widehat{J(x)}$ with respect to a perturbation in the consumption path $\widetilde{x}$ is

$$\widehat{J(x;\widetilde{x})} = \frac{d}{d\epsilon} J\left(x + \epsilon\widetilde{x}\right)\Big|_{\epsilon=0},$$
$$= \frac{d}{d\epsilon}\int_0^\infty e^{-\Delta t} U\left[x_t + \epsilon\widetilde{x}_t, \underline{x}\left(x+\epsilon\widetilde{x}\right)\right] dt\Big|_{\epsilon=0}.$$

Given utility function (2) and Eq. (3) describing the formation of the reference level,

$$\widehat{J(x;\widetilde{x})} = \frac{d}{d\epsilon}\int_0^\infty e^{-\Delta t}\left[v(x_t) + g\left(x_t - \underline{x}_t\right) + \left(v'(x_t) + g_x\left(x_t - \underline{x}_t\right)\right)\epsilon\widetilde{x}_t + g_{\underline{x}}\left(x_t - \underline{x}_t\right)\left(\underline{x}_t\left(x+\epsilon\widetilde{x}\right) - \underline{x}_t\left(x\right)\right)\right] dt\Big|_{\epsilon=0}.$$

In view of the fact that $\underline{x}_t\left(x+\epsilon\widetilde{x}\right) - \underline{x}_t\left(x\right) = \alpha\int_{-\infty}^t e^{-\alpha(t-\tau)}\left(x_\tau + \epsilon\widetilde{x}_\tau - x_\tau\right)d\tau$,

$$\widehat{J(x;\widetilde{x})} = \frac{d}{d\epsilon}\int_0^\infty e^{-\Delta t}\left[\left(v'(x_t) + g_x\left(x_t - \underline{x}_t\right)\right)\epsilon\widetilde{x}_t + g_{\underline{x}}\left(x_t - \underline{x}_t\right)\left(\alpha\int_{-\infty}^t e^{-\alpha(t-\tau)}\epsilon\widetilde{x}_\tau d\tau\right)\right] dt\Big|_{\epsilon=0},$$
$$= \int_0^\infty e^{-\Delta t}\left[\left(v'(x_t) + g_x\left(x_t - \underline{x}_t\right)\right)\widetilde{x}_t + g_{\underline{x}}\left(x_t - \underline{x}_t\right)\left(\alpha\int_{-\infty}^\infty 1_{\tau\le t} e^{-\alpha(t-\tau)}\widetilde{x}_\tau d\tau\right)\right] dt.$$

Since $g_{\underline{x}}\left(x_t - \underline{x}_t\right)$ is independent of τ,

$$\widehat{J(x;\widetilde{x})} = \int_0^\infty e^{-\Delta t}\left(v'(x_t) + g_x\left(x_t - \underline{x}_t\right)\right)\widetilde{x}_t dt + \alpha\int_0^\infty\int_{-\infty}^\infty e^{-\Delta t} g_{\underline{x}}\left(x_t - \underline{x}_t\right) 1_{\tau\le t} e^{-\alpha(t-\tau)}\widetilde{x}_\tau d\tau dt,$$

Switching the order of integration of the second term gives

$$\widehat{J(x;\widetilde{x})} = \int_{-\infty}^\infty 1_{t>0} e^{-\Delta t}\left(v'(x_t) + g_x\left(x_t - \underline{x}_t\right)\right)\widetilde{x}_t dt + \alpha\int_{-\infty}^\infty\int_0^\infty e^{-\Delta t} g_{\underline{x}}\left(x_t - \underline{x}_t\right) 1_{\tau\le t} e^{-\alpha(t-\tau)}\widetilde{x}_\tau dt d\tau.$$

Then switching t and τ in the second term allows us to write

$$\widehat{J(x;\widetilde{x})} = \int_{-\infty}^\infty\left[1_{t>0} e^{-\Delta t}\left(v'(x_t) + g_x\left(x_t - \underline{x}_t\right)\right) + \alpha\int_0^\infty e^{-\Delta\tau} g_{\underline{x}}\left(x_\tau - \underline{x}_\tau\right) 1_{\tau\ge t} e^{\alpha(t-\tau)} d\tau\right]\widetilde{x}_t dt. \tag{26}$$

Being a linear operator, the Fréchet derivative can also be written as the inner product of the consumption perturbation $\widetilde{x}$ and a density function $J_x(x,t)$, which is defined by the relationship

$$\widehat{J(x;\widetilde{x})} = \int_{-\infty}^\infty J_x(x,t)\widetilde{x} dt. \tag{27}$$

The density function $J_x(x,t)$ is also known as the Volterra derivative. While the Fréchet derivative is a functional that takes in two time paths as its arguments (i.e. x and $\widetilde{x}$), the Volterra derivative has the time path x and date t as its arguments. The value of the Volterra derivative at t can also be understood as the welfare effect of a marginal increase in the consumption path at t. Therefore it can be written as $J_x(x,t) = \widehat{J(x;\Delta_t)}$, where the delta distribution Δ_t is defined by $\int_{-\infty}^\infty \widetilde{x}_\tau\Delta_\tau d\tau = \widetilde{x}_t\ \forall\widetilde{x}\in x^\infty$, i.e. a functional that concentrates full weight on time t.

Combining (26) with (27) and considering only consumption perturbations after $t = 0$,

$$J_x(x,t) = e^{-\Delta t}\left[v'(x_t) + g_x\left(x_t - \underline{x}_t\right)\right] + \alpha\int_0^\infty e^{-\Delta\tau} g_{\underline{x}}\left(x_\tau - \underline{x}_\tau\right) 1_{\tau\ge t} e^{\alpha(t-\tau)} d\tau,$$
$$= e^{-\Delta t}\left[v'(x_t) + g_x\left(x_t - \underline{x}_t\right) + \alpha\int_t^\infty e^{-(\alpha+\Delta)(\tau-t)} g_{\underline{x}}\left(x_\tau - \underline{x}_\tau\right) d\tau\right].$$

Since $g_x = -g_{\underline{x}}$,

$$J_x(x,t) = e^{-\Delta t}\left[v'(x_t) + g'\left(x_t - \underline{x}_t\right) - \alpha\int_t^\infty e^{-(\alpha+\Delta)(\tau-t)} g'\left(x_\tau - \underline{x}_\tau\right) d\tau\right]. \tag{28}$$

The discount factor for transferring a unit consumption from time 0 to t is

$$D \equiv \frac{J_{x_t}}{J_{x_0}}. \tag{29}$$

Substituting (28) into (29) gives the expression for the discount factor in the main body of the paper, Eq. (5).

The corresponding discount rate at a given point in time is defined as

$$r \equiv -\frac{d}{dt}\ln D(t,0),$$

$$= \frac{d}{dt}\Delta t - \frac{d}{dt}\ln\left[v'(x_t) + g'\left(x_t - \underline{x}_t\right) - \alpha\int_{\tau=t}^{\infty} e^{-(\alpha+\Delta)(\tau-t)} g'\left(x_\tau - \underline{x}_\tau\right)d\tau\right] \tag{30}$$

$$+\frac{d}{dt}\left[v'(x_0) + g'\left(x_0 - \underline{x}_0\right) - \alpha\int_{\tau=0}^{\infty} e^{-(\alpha+\Delta)(\tau-0)} g'\left(x_\tau - \underline{x}_\tau\right)d\tau\right]. \tag{31}$$

The third term is independent of t. Using the chain rule to take the derivative of the second term we find that

$$r = \Delta - \frac{\dot{v'} + \dot{g'} - \alpha\dot{\mu}}{v' + g' - \alpha\mu}.$$

B. Derivation of the discount rate from an optimal control problem

The purpose of this Appendix is to link the analysis of discount rates in arbitrary economies, which is the focus of the main body of our paper, with discount rates in optimal economies. Suppose environmental quality is inversely related to the flow of pollution, $S = -E$. Following Brock (1973), we write production of the material good as a positive function of the flow of pollution. The production function is

$$Y = F(K,S),$$

where K is capital. We assume that $F_K > 0$ and $F_{KK} < 0$. For a given capital stock, production is also an increasing and strictly concave function of the pollution intensity of the capital stock, i.e. $F_S > 0$ and $F_{SS} < 0$. Production is either consumed or re-invested, so capital is accumulated according to

$$\dot{K} = F(K,S) - C,$$

where C is consumption and may be considered interchangeable with the generic consumption good x in Sections 2 and 3 of the main paper. Population and the production technology are assumed to be constant for simplicity, and for the same reason we omit capital depreciation.

The single-good setting

The single-good planning problem corresponding with this setting is

$$\max_{\{C,S\}} J = \int_0^\infty e^{-\Delta t}\left[v\left(C_t\right) + g\left(C_t - \underline{C}_t\right)\right]dt \tag{32}$$

$$\text{s.t.}\quad \dot{K} = F(K,S) - C, \tag{33}$$

$$\dot{\underline{C}} = \alpha\left(C - \underline{C}\right), \tag{34}$$

and initial K and $\underline{C}$. The current value Hamiltonian is defined as

$$\mathcal{H} = v(C) + g\left(C - \underline{C}\right) + \mu^K\left[F(K,S) - C\right] + \mu^{\underline{C}}\left[\alpha\left(C - \underline{C}\right)\right].$$

Notice that the costate variable on reference consumption of the material good $\mu^{\underline{C}} = -\mu$ in Eq. (7).

Necessary conditions for a maximum include

$$\mu^K = v' + g' + \mu^{\underline{C}}\alpha, \tag{35}$$

$$\frac{\dot{\mu}^K}{\mu^K} = \Delta - F_K, \tag{36}$$

$$\frac{\dot{\mu}^{\underline{C}}}{\mu^{\underline{C}}} = \Delta + \alpha + \frac{g'}{\mu^{\underline{C}}}. \tag{37}$$

The transversality conditions for K and $\mu^{\underline{C}}$ are

$$\lim_{t\to\infty} e^{-\Delta t}\mu^K = 0,$$

$$\lim_{t\to\infty} e^{-\Delta t}\mu^{\underline{C}} = 0. \tag{38}$$

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Substituting (35) into (36) leads to an extended version of the standard Euler equation, which shows that in an optimal economy the material discount rate in Eq. (6) must be equal to the marginal product of capital:

$$r^C = F_K = \Delta - \frac{\dot{v}' + \dot{g}' + \alpha\dot{\mu}^{\underline{C}}}{v' + g' + \alpha\mu^{\underline{C}}} = \Delta - \frac{\dot{v}' + \dot{g}' - \alpha\dot{\mu}}{v' + g' - \alpha\mu}.$$

Integrating (37) gives the general solution $\mu^{\underline{C}} = ke^{(\Delta+\alpha)t} - \int_t^\infty e^{-(\Delta+\alpha)(\tau-t)}g'd\tau$. The transversality condition (38) imposes a growth rate for the shadow price of reference consumption that is lower than δ. Therefore $k = 0$ and we obtain the shadow price of reference consumption as in (7).

The two-good setting

The two-good planning problem is

$$\max_{\{C,S\}} J = \int_0^\infty e^{-\Delta t}\left[v\left(C_t, E_t\right) + g\left(C_t - \underline{C}_t\right) + h\left(E_t - \underline{E}_t\right)\right] dt$$

subject to (33), (34), $\dot{\underline{E}} = \alpha\left(E - \underline{E}\right)$ and initial K, $\underline{C}$ and $\underline{E}$. The current value Hamiltonian in this case is

$$\mathcal{H} = v(C,E) + g\left(C - \underline{C}\right) + h\left(E - \underline{E}\right) + \mu^K\left[F(K,S) - C\right] + \mu^{\underline{C}}\left[\alpha\left(C - \underline{C}\right)\right] + \mu^{\underline{E}}\left[\alpha\left(E - \underline{E}\right)\right].$$

where $\mu^{\underline{E}} = -\phi$ in Eq. (17). Necessary conditions for a maximum include (35)–(37),

$$\mu^K F_S = v_E + h' + \alpha\mu^{\underline{E}} \text{ and} \tag{39}$$

$$\frac{\dot{\mu}^{\underline{E}}}{\mu^{\underline{E}}} = \Delta + \alpha + \frac{h'}{\mu^{\underline{E}}}. \tag{40}$$

Since we are dealing with a flow pollutant, the current-valued shadow price of environmental quality is just $-\mu^K F_S$. Therefore the environmental discount rate is

$$r^E = \Delta - \frac{\dot{\mu^K F_S}}{\mu^K F_S}.$$

Combined with Eq. (39), this gives

$$r^E = \Delta - \frac{\dot{v}_E + \dot{h}' + \alpha\dot{\mu}^{\underline{E}}}{v_E + h' + \alpha\mu^{\underline{E}}} = \Delta - \frac{\dot{v}_E + \dot{h}' - \alpha\dot{\phi}}{v_E + h' - \alpha\phi}, \tag{41}$$

which is equivalent to (18) with (19).

In the case of a stock pollutant, where $\dot{E} = -S - \omega E$ and ω is the decay rate of the pollutant in the environment, stock pollution requires an additional costate equation,

$$\frac{\dot{\mu}^{\underline{E}}}{\mu^{\underline{E}}} = \Delta + \omega - \frac{v_E + h' + \mu^{\underline{E}}\alpha}{\mu^{\underline{E}}}, \tag{42}$$

and Eq. (39) becomes just

$$\mu^{\underline{E}} = \mu^K F_S. \tag{43}$$

The appropriate discount rate to trade off a marginal unit of stock pollution over time is therefore defined as

$$r^E = \Delta - \frac{\dot{\mu}^{\underline{E}}}{\mu^{\underline{E}}}.$$

Combined with Eq. (43), this gives the following environmental discount rate:

$$r^E = -\omega + \frac{v_E + h' + \mu^{\underline{E}}\alpha}{\mu^{\underline{E}}},$$

which differs from Eq. (41), because it includes the fact that adding a unit of pollution at a given date will affect the quality of the environment at future dates.

C. Interpreting *k*

Convex exponential paths

Suppose that consumption grows exponentially at rate j:

$$x_t = x_{t_0}e^{j(t-t_0)}. \tag{44}$$

When $j > 0$ consumption is convex increasing; when $j < 0$ it is convex decreasing. Setting the current time to t_0 and substituting Eq. (44) into the definition of the reference level yields

$$\underline{x}_t = \alpha \int_{-\infty}^{t} e^{-\alpha(t-\tau)} x_t e^{-j(t-\tau)} d\tau = \alpha x_t \int_{-\infty}^{t} e^{-(\alpha+j)(t-\tau)} d\tau. \tag{45}$$

Let us first consider $j > -\alpha$. Given that $\alpha \in [0,1]$, this covers all convex increasing paths, and convex decreasing paths as long as the rate of decrease is not too large. If $j > -\alpha$, (45) simplifies to

$$\underline{x}_t = \frac{\alpha}{j+\alpha} x_t, \tag{46}$$

such that the current reference level is a fixed proportion of current consumption and it exhibits the same exponential growth as consumption:

$$x_t - \underline{x}_t = \frac{j}{j+\alpha} x_t = \frac{j}{j+\alpha} x_{t_0} e^{j(t-t_0)}$$

$$\Rightarrow j = \frac{\dot{x}_t - \dot{\underline{x}}_t}{x_t - \underline{x}_t}.$$

The elasticity of marginal gain/loss utility is

$$\eta_t^g \equiv -\frac{\partial g'/g'}{\partial \Delta/\Delta} = -\frac{g''}{g'}(\Delta),$$

where as before $\Delta \equiv x_t - \underline{x}_t$. In Section 3 we used k to denote the rate of change of marginal gain/loss utility, therefore in general $k_t = \dot{g}'/g'$. This immediately implies that

$$k_t = -\eta_t^g j,$$

in other words the rate of change of marginal gain/loss utility at time t is the product of the negative of the elasticity of marginal gain/loss utility and the consumption growth rate.

Section 3 specifically deals with paths that conform to constant k over the time period $[t, \infty)$. It turns out that if $j > 0$ and the gain/loss function g exhibits constant elasticity on the domain $[x_t - \underline{x}_t; \infty]$, then within the time interval $[t, \infty)$ we have the special case of

$$k = -\eta^g j.$$

This is also true if $-\alpha < j < 0$, but only if $g(.)$ exhibits constant elasticity on the domain $\left[x_t - \underline{x}_t; 0\right]$, which would imply $\lim_{\Delta \to 0} g'(\Delta) = \infty$. In case this is felt to be undesirable, we might consider convex decreasing paths that are approximated by (44), but where $\dot{j} > 0$ for $t > T$, with T being the time at which the loss enters the non-constant elasticity domain. Another option is to assume that the reference-updating effect is not infinitely forward-looking, rather it only extends to a finite date T far in the future.

What about if $j \leq -\alpha$? From Eq. (45) we can see that $\underline{x}_t = \infty$. Since we 'forget' our past consumption levels at a slower rate than consumption is falling, the reference level is determined by the infinite consumption we once enjoyed on this strictly decreasing path.

Concave exponential paths

Suppose instead that consumption evolves according to

$$x_t = \Upsilon - \Upsilon e^{j(t-T)}. \tag{47}$$

When $j > 0$, consumption is concave decreasing from a horizontal asymptote at $x_{-\infty} = \Upsilon$ to $x_T = 0$. On the whole path, $t - T < 0$. When $j < 0$, consumption is concave increasing from $x_T = 0$ and converges asymptotically to $x_\infty = \Upsilon$. On the whole path, $t - T > 0$.

Consider $j > 0$. Substituting (47) into the definition of the reference level, this time we obtain

$$\underline{x}_t = \alpha \int_{-\infty}^{t} e^{-\alpha(t-\tau)} (\Upsilon - \Upsilon e^{j(\tau-T)}) d\tau = \Upsilon - \frac{\alpha}{\alpha+j} \Upsilon e^{j(t-T)}, \tag{48}$$

which again means that $j = \left(\dot{x}_t - \dot{\underline{x}}_t\right) / \left(x_t - \underline{x}_t\right)$ and so, if the gain/loss function is isoelastic on the domain $]-\infty; C_t - \underline{C}_t]$, $k = -\eta^g j$.

Now consider $-\alpha < j < 0$. The path begins at $x_T = 0$, in which case

$$\underline{x}_t = \alpha \int_{T}^{t} e^{-\alpha(t-\tau)} (\Upsilon - \Upsilon e^{j(\tau-T)}) d\tau = \Upsilon(1 - e^{-\alpha(t-T)}) - \frac{\alpha}{\alpha+j} \Upsilon (e^{j(t-T)} - e^{-\alpha(t-T)}).$$

For $t >> T$ this solution converges to Eq. (48), so again $k = -\eta^g j$. However, this case requires that the gain/loss function is isoelastic on the domain $[0; x_t - \underline{x}_t]$, which encounters the same possible objection and proposed solutions as the case of convex decreasing paths above.

There is no tractable relation between k and g when $j \leq -\alpha$ and consumption evolves according to (47).

D. Proof of Proposition 1

Proof. We begin by proving the endowment factor is given by (13). In the case of diminishing sensitivity, but where the consumption path is decreasing arithmetically, $x_t = x_{t_0} + \kappa(t - t_0), \kappa < 0$ for any arbitrary date in the past $t_0 \in (-\infty, t]$ and we can write Eq. (15) as

$$\underline{x}_t = \alpha \int_{-\infty}^{t_0} e^{-\alpha(t-\tau)} x_\tau d\tau + \alpha \int_{t_0}^{t} e^{-\alpha(t-\tau)} [x_{t_0} + \kappa(\tau - t_0)] d\tau,$$

$$= \alpha \int_{-\infty}^{t_0} e^{-\alpha(t-\tau)} x_\tau d\tau + \alpha e^{-\alpha t} \int_{t_0}^{t} e^{\alpha\tau} x_{t_0} d\tau + \alpha\kappa e^{-\alpha t} \int_{t_0}^{t} e^{\alpha\tau} (\tau - t_0) d\tau,$$

$$= \alpha \int_{-\infty}^{t_0} e^{-\alpha(t-\tau)} x_\tau d\tau + x_{t_0}(1 - e^{-\alpha(t-t_0)}) + \kappa(t - t_0) - \frac{\kappa}{\alpha} + \frac{\kappa}{\alpha} e^{-\alpha(t-t_0)}. \tag{49}$$

Taking the limit as t_0 goes to minus infinity we obtain

$$\lim_{t_0 \to -\infty} \underline{x}_t = x_{t_0} + \kappa(t - t_0) - \frac{\kappa}{\alpha} = x_t - \frac{\kappa}{\alpha}.$$

Therefore $g\left(x_t - \underline{x}_t\right) = g\left(\frac{\kappa}{\alpha}\right)$, which is constant over time. If consumption follows an arithmetically increasing path instead, t_0 is taken to be the time when consumption was zero, $x_{t_0} = 0$. This eliminates the first two terms in Eq. (49). Since we cannot take the limit as t_0 goes to minus infinity, we approximate the same result if t_0 is sufficiently far in the past: $\underline{x}_t \approx x_{t_0} + \kappa(t - t_0) - \frac{\kappa}{\alpha}$. In this case $g\left(x - \underline{x}_t\right) = g\left(\frac{\kappa}{\alpha}\right)$ too. Therefore on a linear path the instantaneous endowment effect g' is constant, which also means that $\dot{g}' = 0$. This is self-evidently true if preferences obey constant sensitivity, as long as consumption is strictly increasing or decreasing, in other words the increase/decrease need not be linear. Either way, since g' is constant over time, the reference-updating effect

$$\alpha \int_t^{\infty} e^{-(\alpha+\Delta)(\tau-t)} g' d\tau = \alpha g' \left[\frac{e^{-(\alpha+\Delta)(\tau-t)}}{-\alpha - \Delta}\right]_t^{\infty} = \frac{\alpha}{\alpha + \Delta} g'.$$

Substituting this result into (19) results in Eq. (13). From (13), it is clear that $\delta > 0$ is a necessary and sufficient condition for $0 < \theta < 1$.

E. θ in the long run

This Appendix develops insights into the long-run behaviour of the discount rate under the endowment effect.

Constant marginal gain/loss utility

On an arithmetically increasing consumption path, or on any strictly increasing consumption path with constant sensitivity, it is clear from Eq. (13) that $\lim_{t \to \infty} v' = 0$, $\theta = 0$ and $r = \delta$. On such paths, marginal consumption-level utility falls over time, while the marginal endowment effect is constant. As we get more and more affluent, absolute levels of consumption matter less and less, but gains from the reference level remain a source of utility. Eventually the (undiscounted) marginal contribution of an extra unit of consumption is constant over time, so we discount at the pure time preference rate. Although this is also the case on an arithmetically increasing consumption path without the endowment effect, the endowment effect speeds up the convergence process (per Eq. (12), $\dot{v}' / \left[v' + \Delta g' / (\alpha + \Delta)\right]$ converges to zero faster than $\dot{v}'/v'$).

Conversely on an arithmetically decreasing consumption path, or a strictly decreasing consumption path with constant sensitivity, $\lim_{t \to \infty} v' = \infty$, $\theta = 1$ and $r = \Delta - \dot{v}'/v'$. On such paths, marginal consumption-level utility increases over time and eventually dominates the constant marginal endowment effect.[15]

Convex paths

Suppose like Appendix C that consumption grows at a constant rate j, such that $x_t = x_0 e^{jt}$, and that the gain/loss utility function has a constant elasticity η^g. Then $k = -\eta^g j$, with $g'_t = g_0 e^{(-\eta^g jt)}$. Suppose that the consumption-level utility function

[15] On an arithmetically decreasing consumption path, there will of course be a point at which consumption is zero and we would normally restrict consumption to be non-negative. Still, for the class of strictly concave consumption-level utility functions, it is true that, as consumption approaches zero in finite time, marginal consumption-level utility is arbitrarily large.

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also has a constant elasticity, denoted η as in Eq. (8). Then $\dot{v}'/v' = -\eta j$, with $v'_t = v_0 e^{(-\eta j t)}$. Substituting these into (11) allows us to describe θ as an explicit function of time,

$$\theta = \frac{1 + \left(\frac{\Delta - k}{\alpha + \Delta - k}\right) \frac{g_0}{v_0} e^{(-\eta^g + \eta) j t} \frac{\eta^g}{\eta}}{1 + \left(\frac{\Delta - k}{\alpha + \Delta - k}\right) \frac{g_0}{v_0} e^{(-\eta^g + \eta) j t}}.$$

Consider convex increasing paths, where $j > 0$. When the elasticity of the gain/loss function is smaller than the elasticity of the consumption-level utility function, $\eta^g < \eta$, the second term grows over time, both in the numerator and the denominator, such that $\lim_{t\to\infty} \theta = \eta^g/\eta$, the marginal endowment effect dominates in the long run and we discount at $r = \delta + \eta^g j$. Empirical evidence might point to the relevance of this case, as discussed in Section 5. It corresponds to the region between $k = \dot{v}'/v'$ and $k = 0$ in Fig. 1.

Conversely when $\eta^g > \eta$, $\lim_{t\to\infty} \theta = 1$, marginal consumption-level utility dominates in the long run and we discount at the standard rate. Both marginal consumption-level utility and the marginal endowment effect decrease over time, but the marginal endowment effect decreases faster (as can be seen in Eq. (12)). This corresponds to the region left of the point $k = \dot{v}'/v'$ on the horizontal axis in Fig. 1.

On convex decreasing paths, constant η^g would give rise to infinite marginal gain/loss utility at the reference point, which may be argued to be an undesirable property. If, however, we constrain marginal gain/loss utility to be constant for very small gains/losses, then on a convex decreasing consumption path we have another case where $\lim_{t\to\infty} \theta = 1$ and $r = \Delta - \dot{v}'/v'$. In the numerical analysis of Section 5, we bound marginal gain/loss utility at zero as the gain/loss approaches zero in the limit.

Concave paths

Assume like Appendix C that x evolves according to $x_t = \Upsilon - \Upsilon e^{j(t-T)}$. Then the growth rate is $\dot{x}/x = j/\left[1 - e^{-j(t-T)}\right]$. Assuming an iso-elastic consumption-level utility function, we have

$$\dot{v}'/v' = -\eta \frac{\dot{x}}{x} = \frac{\eta j}{e^{-j(t-T)} - 1}.$$

Integrating this gives

$$v'_t = v_0 \left(\frac{e^{j(t-T)} - 1}{e^{jT} - 1}\right)^{-\eta}.$$

Substituting these two equations into (11) gives

$$\theta = \frac{1 + \left(\frac{\Delta - k}{\alpha + \Delta - k}\right) \frac{g'}{v'} \frac{-\eta^g \left(e^{-j(t-T)} - 1\right)}{\eta}}{1 + \left(\frac{\Delta - k}{\alpha + \Delta - k}\right) \frac{g'}{v'}},$$

with

$$\frac{g'}{v'} = \frac{g'_0 e^{-\eta^g j t}}{v'_0} \left[\frac{e^{j(t-T)} - 1}{e^{jT} - 1}\right]^{\eta}.$$

For $j < 0$, the path is concave increasing, x is zero at time T and evolves to a steady state $x = \Upsilon$. We have

$$\lim_{t\to\infty} \theta = \infty \text{ if} \frac{\Delta - k}{\alpha + \Delta - k} < 0,$$

$$\lim_{t\to\infty} \theta = -\infty \text{ if} \frac{\Delta - k}{\alpha + \Delta - k} > 0.$$

Note that if marginal gain/loss utility is constrained to be constant for very small gains/losses, then on a concave increasing path we have another case where $\lim_{t\to\infty} \theta = 0$ and $r = \delta$.

For $j > 0$, the path is concave decreasing, reaching zero at time T. For $t = T$, we have $\theta = 1$. g' decreases over time, while v' increases. The shadow price of the reference point is defined with respect to the indefinite future. Therefore the result is based on the assumption that the path further declines (in negative territory) after $t = T$. If the path is instead characterised by $x = 0$ after T, the formula for θ will be a bad approximation when t approaches T. In the limit, we are back in the constant marginal gain/loss utility case.

F. Asymptotic behaviour of θ

Equation (11) can also be written in the following way:

$$\theta = \frac{\alpha + \Delta - k + \frac{g'}{v'} k(\Delta - k)}{\alpha + \Delta - k + \frac{g'}{v'}(\Delta - k)}.$$

To understand the sign of the denominator, consider θ in the neighbourhood of the vertical asymptote at $k = \Delta + \frac{\alpha}{1+g'/v'} + \epsilon$ with arbitrarily small ϵ. Substituting this value of k into the denominator gives

$$\alpha - \frac{\alpha}{1+\frac{g'}{v'}} - \epsilon - \frac{g'}{\dot{v}'}\left(\frac{\alpha}{1+\frac{g'}{v'}} + \epsilon\right) = -\epsilon - \frac{g'}{v'}\epsilon.$$

The denominator will therefore be positive to the left of the asymptote and negative to the right of it.

The numerator is positive in the neighbourhood of the asymptote at $k = \Delta + \frac{\alpha}{1+g'/v'}$ if

$$\alpha - \frac{\alpha}{1+\frac{g'}{v'}} + \frac{g'}{v'}\left(\Delta + \frac{\alpha}{1+\frac{g'}{v'}}\right)\left(\frac{-\alpha}{1+\frac{g'}{v'}}\right) > 0$$

$$\Leftrightarrow \frac{v'}{\dot{v}'}\left(\Delta + \frac{\alpha}{1+\frac{g'}{v'}}\right) < 1.$$

Therefore the numerator is positive if $\frac{\dot{v}'}{v'} < 0$ or $\frac{\dot{v}'}{v'} > \Delta + \frac{\alpha}{1+\frac{g'}{v'}}$. As a result, θ jumps from infinity to minus infinity as k increases beyond the asymptote. On the contrary, the numerator is negative if $0 < \frac{\dot{v}'}{v'} < \Delta + \frac{\alpha}{1+\frac{g'}{v'}}$, with the result that θ jumps from minus infinity to infinity as k passes the asymptote.

G. Sensitivity analysis

Single-good setting

Fig. 7 analyses the sensitivity of θ^C in Section 5.2 to variation in the preference parameters at a maturity of 50 years, a typical horizon for a long-run environmental project. The gain/loss parameters $\alpha, \beta$and$Z$ are varied over their entire possible ranges, while $\Delta \in [0.001, 0.02]$ and $\psi \in [0.5, 4]$. The value of θ^C that corresponds with the default parameter settings is 0.22 at $t = 50$. The material endowment factor is most sensitive to Z, the initial value share of consumption-level utility, followed by β, the parameter determining the curvature of the gain/loss function, the memory parameter α and the elasticity of marginal consumption-level utility ψ. The material endowment factor is insensitive to the rate of pure time preference δ. If $Z = 0$ so that preferences only depend on gains and losses, $\theta^C = 0.07$ at $t = 50$, and the discount rate is close to the pure rate of time preference.[16] Observe that $0 < \theta^C < 1$ for all values of ψ that we investigated, despite the fact that ψ bears upon $\dot{v}_C/v_C$, which we know to be important in the case of convex increasing consumption.

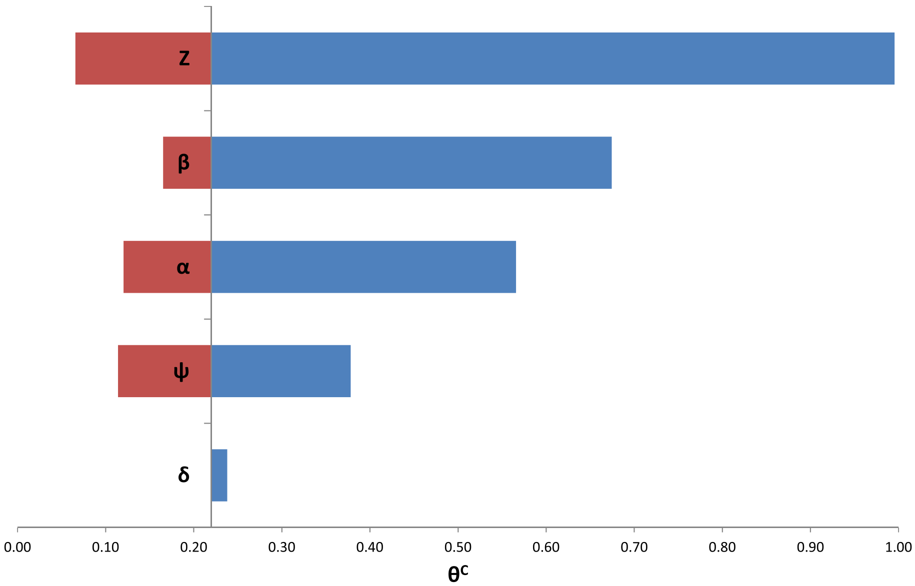

Fig. 7. Sensitivity of θ^C to parameters at $t = 50$.

[16] Recall Z is the initial value share of consumption-level utility, so by t = 50 the contemporaneous value share of consumption-level utility has increased slightly.

Two-good setting

Fig. 8 analyses the sensitivity of θ^E in Section 5.3 to preference parameters at $t = 50$. The range of elasticity of substitution between the two goods is $\sigma \in [0.04, 100]$, running from approximately perfect substitutes to perfect complements, while the loss aversion parameter $\lambda \in [1, 5]$. In the two-good setting, not all values of Z are feasible when $\Gamma = 0.9$ and conversely not all values of Γ are feasible when $Z = 0.75$. For this analysis $Z \in [0.6, 1]$ and $\Gamma \in [0.8, 1)$. The endowment factor is most sensitive to the elasticity of substitution between consumption and environmental quality, σ, followed by Z, even over its limited range in this case.

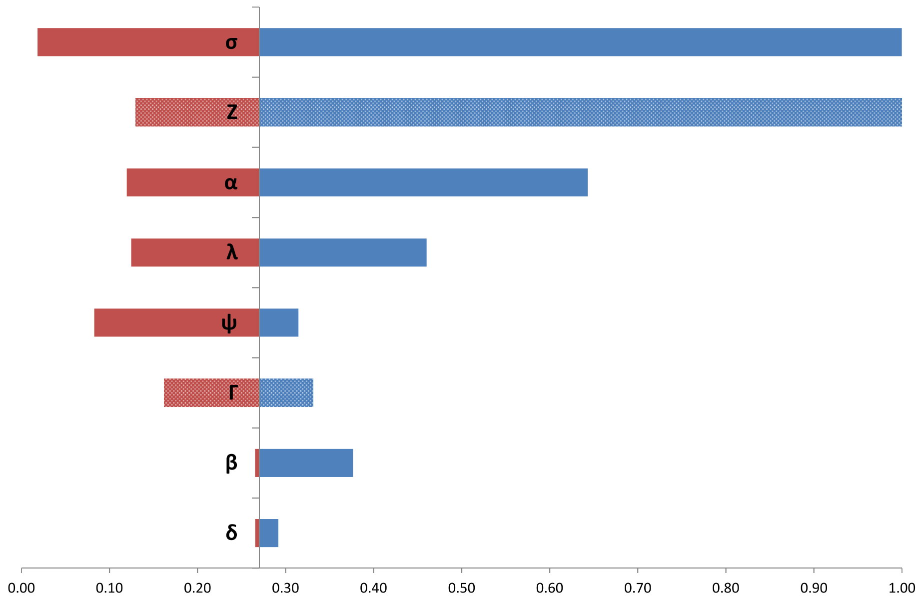

Fig. 8. Sensitivity of θ^E to parameters at $t = 50$.

H. Further analysis of a non-monotonic path for environmental quality

As well as r^Estd. and the default parameterisation of r^E endow. ($\lambda = 2.25; \beta = 0.9$), we aid interpretation of the results in Section 5.4 by providing additional plots of r^E endow., which are generated by omitting loss aversion ($\lambda = 1$) and/or assuming constant sensitivity ($\beta = 1$).

In the absence of loss aversion ($\lambda = 1$), there is no large jump in the discount rate. Under diminishing sensitivity but without loss aversion – in other words when the marginal gain/loss function is a smooth sigmoid – there is a trough in the discount rate around the turning point in environmental quality, just because marginal gain/loss utility becomes large when the change in environmental quality is small. Moreover the discount rate only appears discontinuous due to the effect of the bounding parameter ω. As one would expect, when neither loss aversion nor diminishing sensitivity is present ($\lambda = 1; \beta = 1$), so that the marginal gain/loss function is linear, the discount rate does not deviate from a declining path.

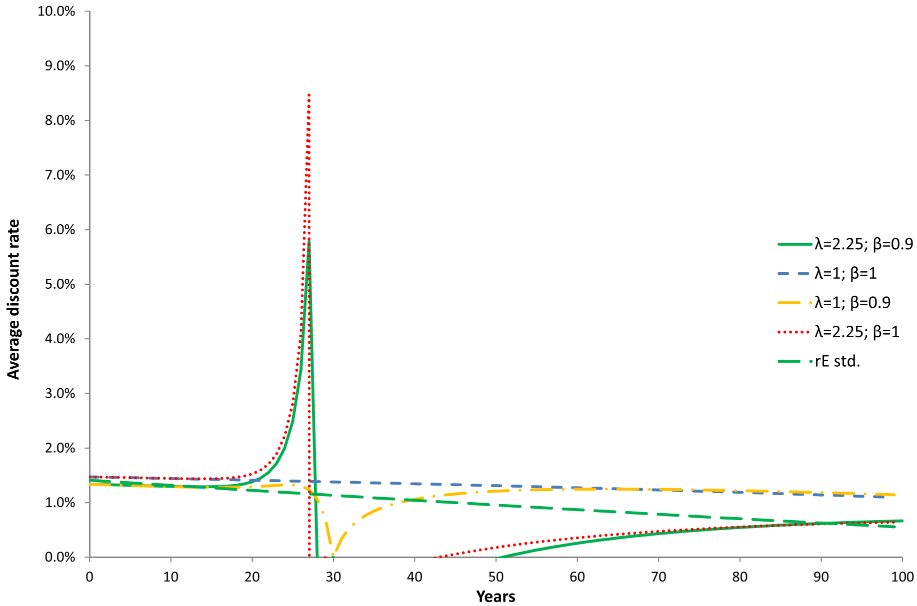

Fig. 9. Additional plots of r^Eendow. on a non-monotonic path of E, varying λ and β.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jeem.2019.01.010.

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